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SPHERICAL SHELL WEAKENED BY TWO UNEQUAL CIRCULAR HOLES

by

G. N. Savin, G. A. Van Fo Fy,
and V. N. Buivol

Translated from Russian by. O. Lomacky

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ABSTRACT

The method of successive approximations is used to obtain a solution of the problem of the distribution of stresses around two unequal circular holes in a spherical shell loaded by a constant internal pressure and constant shear forces along the contours of the holes.

Finding the successive approximations is reduced to a solution of Equation [1] under appropriate boundary conditions. Two approximations are found, a numerical example is discussed, and the boundaries of the applicability of the solution are indicated

The problem of stress concentration around openings of arbitrary shape was formulated and, in principle, solved in References 1 and 2.* These references contain basic differential equations of the problem as well as the general integrals of the basic equations. In this investigation, by using the results of References 1 and 2, we offer a solution of the problem of the stress distribution around two unequal circular openings in spherical shell.** We assume that the shell of radius R and thickness h is subjected to constant internal pressure, q = const, and that the openings are closed by hatches of such construction that only the shear forces are transmitted to the shell.

If the "reduced" polar coordinates x, θ connected with the center of one of the openings are utilized, then, as was shown in Reference 2, the state of stress and deformation in the vicinity of one opening is obtained from the solution of the basic differential equation of the form:

$$\nabla^2\nabla^2\Phi + i\nabla^2\Phi = 0 \qquad [1]$$

*References are listed on page 8.

**Translator's Note: This solution is limited to small holes. See Reference 2, Equation [3.5] for size limitations.

Here

$$\Phi = \omega + ig\varphi$$

where ω and φ are, respectively, the deflection and the stress functions,

$i = \sqrt{-1}$,

$$g = \frac{\sqrt{12\ (1 - \nu^2)}}{Eh^2}$$

E and ν are, respectively, Young's modulus and Poisson's ratio, and ∇^2 is the Laplacian operator in polar coordinates x, θ where the "reduced" polar radius x is connected with the polar radius ρ* by the relations $x = \varkappa \rho$ and $\varkappa^2 = \frac{\sqrt{12\ (1 - \nu^2)}}{Rh}$.

If the shell is weakened by two holes sufficiently far apart that the zones of perturbations introduced into the basic state of stress by such openings do not overlap at the edge of each opening, then the state of stress around the openings can be represented by the function

$$\Phi_0 = ig\varphi^0 + \Phi^{(1)} + \Phi^{(2)} \qquad [2]$$

Here φ^0 is the stress function for the shell without openings, and the functions $\Phi^{(1)}$ and $\Phi^{(2)}$ have the form

$$\Phi^{(k)} = iC^{(k)} \ln x_k + (A^{(k)} + iB^{(k)})\ H_o^{(1)}\ (x_k\sqrt{i}) \quad (k = 1,2) \qquad [3]$$

where $A^{(k)}$, $B^{(k)}$, $C^{(k)}$ are arbitrary constants that can be determined from the boundary conditions on the contours of the openings, and $H_o^{(1)}\ (x_k \sqrt{i})$ are the Hankel's functions of first kind and zero order.

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*Translator's Note: The radius ρ is referred to the center of the opening.

If the shear forces on the contours of the openings are constant, then Equation [3] takes the following form: (We omit index for variable x.)

$$\Phi = \frac{q(\beta + i\alpha)}{2\varkappa^4 D\,[\beta\ \mathrm{hei}'\ (x_0) - \alpha\ \mathrm{her}'\ (x_0)]}\, x_0 H_0^{(1)}\ (x\sqrt{i})$$

Here D is the cylindrical stiffness,

her$'$ and hei$'$ are the derivatives of the real and the imaginary parts of the Hankel functions of zero order, and

α and β are the boundary values of the functions

$$\alpha = \mathrm{hei}(x) + \frac{1-\nu}{x}\,\mathrm{her}'\ (x) \qquad \text{and } \beta = \mathrm{her}(x) - \frac{1-\nu}{x}\,\mathrm{hei}'\ (x)$$

evaluated at $x = x_0$ where x_0 is the "reduced" radius of the opening.

In the case of the closely spaced openings, Equation [2] can be regarded only as the zero-order approximation of the solution since it does not permit the full satisfaction of the boundary conditions. To decrease the "mismatch" on the contours of the openings in the first-order approximation, it is necessary in Equation [2] to add the "correcting" functions $\Phi_{12}^{(1)}$ and $\Phi_{21}^{(1)}$ for the first and the second openings, respectively. These functions should be the solutions of the basic Equation [1]. If one takes into account the conditions at infinity and the symmetry of the solution with respect to the line connecting the centers of the openings, then such solutions will have the form:

$$\Phi = igC\ \ln x + \sum_{n=1}^{\infty} (A_n + iB_n)x^{-n} \cos n\theta + \sum_{n=0}^{\infty} (C_n + iD_n)H_n^{(1)}\ (x\sqrt{i}) \cos n\theta \quad [4]$$

Arbitrary constants that appear in Equation [4] are determined as the result of the solution of the system of algebraic equations that is obtained from the boundary conditions. We note that in the case of $n = 1$

it is necessary to consider displacements since the number of equations turns out to be less than the number of unknowns.*

In Equation 4 by separating the real and the imaginary components and utilizing the relations between the stress resultants and the stress and deflection functions, we obtain the "corrections" of the first-order approximations:**

$$T_x^{(1)} = \frac{\varkappa^2}{gx} \sum_{n=0}^{\infty} \left(\varphi_n' - \frac{n^2}{x} \varphi_n \right) \cos n\theta$$

$$T_\theta^{(1)} = \frac{\varkappa^2}{g} \sum_{n=0}^{\infty} \varphi_n'' \cos n\theta$$

$$S^{(1)} = - \frac{\varkappa^2}{gx} \sum_{n=0}^{\infty} n \left(\frac{1}{x} \varphi_n - \varphi_n' \right) \sin n\theta \qquad [5]$$

$$G_x^{(1)} = \varkappa^2 D \sum_{n=0}^{\infty} \left[\omega_n'' + \frac{\upsilon}{x} \left(\omega_n' - \frac{n^2}{x} \omega_n \right) \right] \cos n\theta$$

$$G_\theta^{(1)} = \varkappa^2 D \sum_{n=0}^{\infty} \left[\frac{1}{x} \left(\omega_n' - \frac{n^2}{x} \omega_n \right) + \upsilon \omega_n'' \right] \cos n\theta$$

$$\widetilde{Q}_x = - \varkappa^3 D \sum_{n=0}^{\infty} \left[\frac{\partial \nabla^2 \omega_n}{\partial x} + \frac{n^2}{x^2} \left(\frac{3 - \upsilon}{x} \omega_n - (2 - \upsilon) \omega_n' \right) \right] \cos n\theta$$

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*Translator's Note: From this consideration it can be shown that $A_1 = 0$. See "Stress Concentration around Two Openings in Spherical Shell," G. N. Savin, G. A. Van Fo Fy, and V. N. Buivol. Dopovidi A. N. URSR No. 11, 1961. Figure 1 is taken from the same source.

**Translator's Note: $T_x^{(1)}$, $T_\theta^{(1)}$, $S^{(1)}$, $G_x^{(1)}$, $G^{(1)}$, and $\widetilde{Q}_x$ correspond to unit normal forces, unit shear in the middle surface of the shell, unit bending moments, and unit shear normal to the middle surface of the shell, respectively.

Here the following designations were used:

$$\begin{aligned}
\omega_0 &= C_0\ \mathrm{her}(x) + D_0\ \mathrm{hei}(x);\\
\varphi_0 &= gC\ \ln x + D_0\ \mathrm{her}(x) - C_0\ \mathrm{hei}(x);\\
\omega_n &= A_n x^{-n} + C_n\ \mathrm{her}_n(x) + D_n\ \mathrm{hei}_n(x);\\
\varphi_n &= B_n x^{-n} + D_n\ \mathrm{her}_n(x) - C_n\ \mathrm{hei}_n(x).
\end{aligned} \qquad [6]$$

If in Equations [6] we utilize the constants A_n, B_n, C_n, D_n, and C which were found from the boundary conditions at the first opening, then Equations [5] determine the correction of the first-order approximation associated with the correcting function $\Phi_{12}^{(1)}$, which accounts for the influence of the second opening on the state of stress around the first opening. If, on the other hand, these constants are found from the boundary conditions at the second opening, then Equations [5] yield the corrections of the first-order approximation associated with the correcting function $\Phi_{21}^{(1)}$ on the state of stress around the second opening.

In this manner the solution of the given problem including the first-order approximation, will be given by the following function:

$$\Phi_1 = ig\varphi^{\circ} + \Phi^{(1)} + \Phi^{(2)} + \Phi_{12}^{(1)} + \Phi_{21}^{(1)}$$

We recall that $\Phi^{(1)}$ and $\Phi^{(2)}$ are given by Equation [3] and $\Phi_{12}^{(1)}$ and $\Phi_{21}^{(1)}$ by Equation [4].

Similarly, one should construct the successive approximations $\Phi_{12}^{(k)}$ and $\Phi_{21}^{(k)}$ ($k = 2, 3 \ldots$). It is not possible, however, to accomplish this by means of the indicated method since in the construction of such functions it is necessary to use formulas which relate the two coordinate systems referred to the centers of the openings. This complicates the arguments of the Bessel's functions. The theory[3] of addition for such functions permit the separation of variables only for the functions of the zero order. Construction of the subsequent approximations involves functions of higher orders.

Since we have limited ourselves to the construction of the zero- and the first-order approximations, it is necessary to establish the bounds of applicability of the obtained solution. The results of the numerical calculations have shown that the obtained solution gives good accuracy provided the distance between the contours of the openings is not less than the radius of the smaller opening. Analysis of these results shows that the perturbation zones caused by the openings do not extend beyond the distance from the contour equal to their diameters.

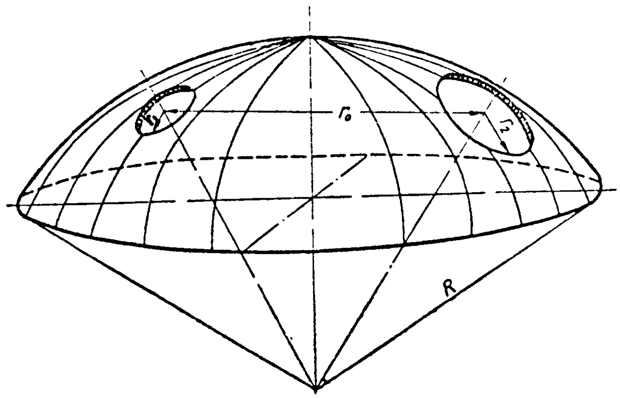

Figure 1

Furthermore, by considering all stress components it is seen that the stress resultant T_θ plays the dominant role. It is interesting to note that there exists some proportionality between the opening size and the maximum values of T_θ, G_x, and $\tilde{Q}_x$, but that the components T_x and G_θ are not appreciably influenced by the size of the opening. Also, in spite of the large difference (several hundredfold) between the maximum values of the stress resultants and the moments, the corresponding stresses differ by much smaller factors (tenfold).

In conclusion, we note that these conclusions are valid if the openings are closed by hatches of the indicated construction and if the distances between the openings are not too small.

Figures 2 through 6 show curves of the variation of stress resultants and moments along the section between the openings for the case of $R = 200$ cm, $h = 0.2$ cm, opening radii 10 and 20 cm, distance between the centers of the openings* 50 cm, $E = 7.2 \times 10^5$ kg/cm,^{2} and $\nu = 0.3$. The solid lines are drawn for the case of a single opening. The case of two openings is represented by the dash-point lines for the zero-order approximation and by the dash lines for the first-order approximation.

*Translator's Note: This is the distance r_0 shown in Figure 1.

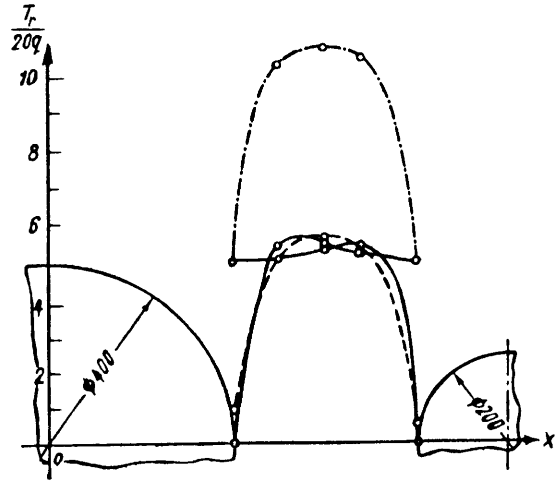

Figure 2

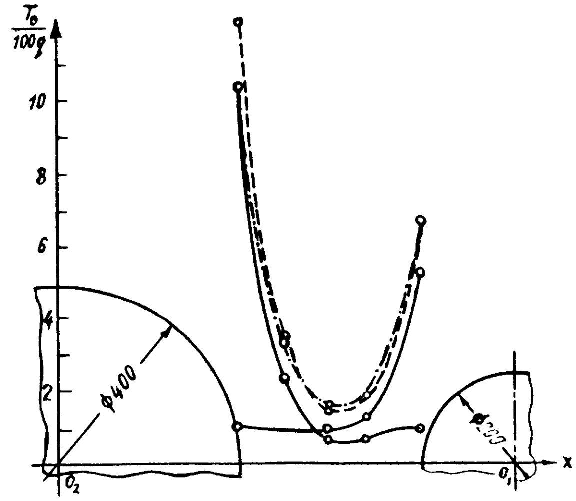

Figure 3

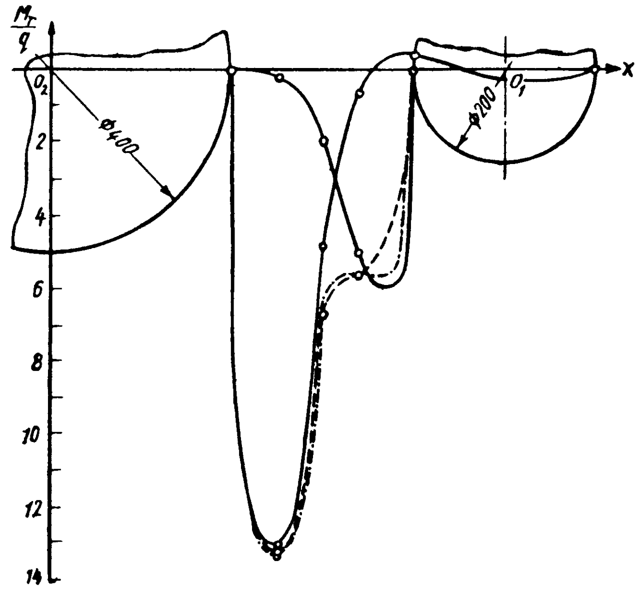

Figure 4

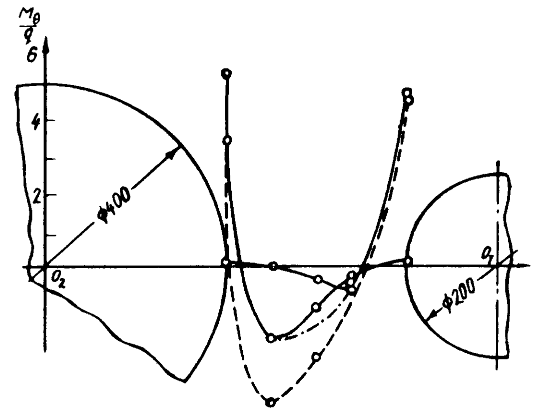

Figure 5

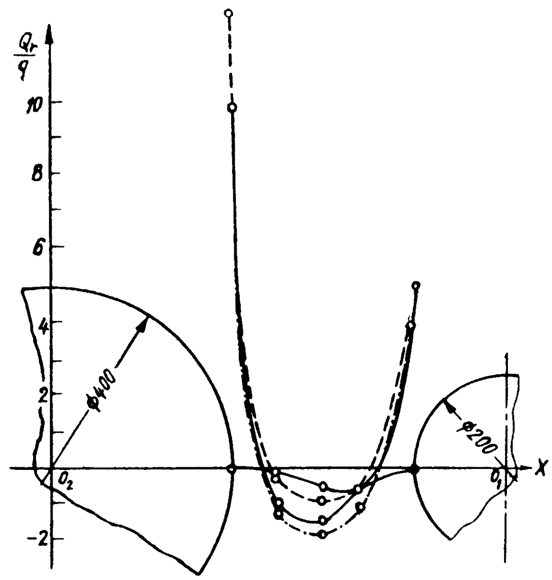

Figure 6

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Finding the successive approximations is reduced to a solution of Equation [1] under appropriate boundary

1. Spherical shells--Openings--Stresses
2. Openings--Stresses--Mathematical analysis
3. Approximate computation
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