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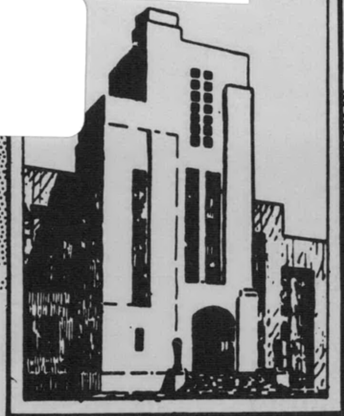

DEPARTMENT OF THE NAVY
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HYDROMECHANICS

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# THE EXCITING FORCES AND WETTING OF SHIPS IN WAVES

**(Vozmushchayushchie Sili i Zalivaemost' Sudov na Volnenii)**



by

M. D. Haskind

Izvestia Akademii Nauk SSSR
Otdelenie Tekhnicheskikh Nauk,
No. 7, 1957, pp. 65–79

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## ABSTRACT

General formulas are established for the exciting forces and moments acting on a ship in arbitrary waves. It is shown that for the diffraction of waves due to isolated singularities, such as sources, dipoles, pressure points, etc., the exciting forces and moments depend only on the ship's radiation functions, which characterize the wave radiation in a heavy fluid due to forced oscillations of the ship with unit velocity amplitude in calm water.

In the case of diffraction of regular progressive wave systems, it is shown that the exciting forces and moments, the damping coefficients, and the estimated wetting of ships in waves can all be expressed in terms of the asymptotic characteristics of the radiation functions.

The general results obtained are employed to calculate the exciting forces and moments in specific examples.

## 1. GENERAL FORMULAS FOR THE EXCITING FORCES AND MOMENTS

In the linear hydrodynamical theory of ship oscillations in the presence of regular waves of frequency $\sigma_0$, the velocity potential $\Phi\,(x,\,y,\,z,\,t)$ for the motion of a heavy fluid can be represented in the form[1,2]

$$\Phi\,(x, y, z, t) = \phi\,(x, y, z)\,e^{i\sigma_0 t}, \quad \phi = \boldsymbol{\Phi}_1 \cdot \boldsymbol{v} + \boldsymbol{\Phi}_2 \cdot \boldsymbol{\omega} + \phi_0 + \phi_7$$
$$(\boldsymbol{V} = \boldsymbol{v}e^{i\sigma_0 t}, \quad \boldsymbol{\Omega} = \boldsymbol{\omega}e^{i\sigma_0 t}) \qquad [1.1]$$

where $\boldsymbol{V}$ and $\boldsymbol{\Omega}$ are the translational and rotational velocity vectors,

$\Phi_0 = \phi_0 \exp i\sigma_0 t$ is the velocity potential of the given incident wave system, and

the vectors $\boldsymbol{\Phi}_1 = (\phi_1, \phi_2, \phi_3)$ and $\boldsymbol{\Phi}_2 = (\phi_4, \phi_5, \phi_6)$ and the function $\phi_7$ satisfy the conditions

$$\frac{\partial \boldsymbol{\Phi}_1}{\partial n} = \boldsymbol{n}, \quad \frac{\partial \boldsymbol{\Phi}_2}{\partial n} = \boldsymbol{r} \times \boldsymbol{n}, \quad \frac{\partial \phi_7}{\partial n} = -\frac{\partial \phi_0}{\partial n} \text{ on } S \qquad [1.2]$$

$$\frac{\partial \phi_m}{\partial z} - k\phi_m = 0 \text{ for } z = 0 \left(k = \frac{\sigma_0^2}{g}, \; m = 0, 1, \ldots, 7\right) \qquad [1.3]$$

$$\lim_{R \to \infty} \sqrt{R}\left(\frac{\partial \phi_m}{\partial R} + ik\phi_m\right) = 0 \quad (R^2 = x^2 + y^2, \; m = 1, \ldots, 7) \qquad [1.4]$$

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[1]References are listed on page 23.

where $\boldsymbol{n}$ is the exterior unit vector normal to the ship's surface $S$,

$\boldsymbol{r}$ is the radius vector to any point on this surface, and

$g$ is the gravitational acceleration constant.

The functions $\phi_m(x, y, z) \exp i\sigma_0 t$ ($m = 1, \ldots . 6$) represent the velocity potentials for the motion of a heavy fluid due to the oscillations of the ship with unit velocity amplitude.* These functions determine the form of the radiated waves in a heavy fluid, and we shall call them *radiation functions.* The function $\phi_7(x, y, z) \exp i\sigma_0 t$ (*the scattering function*) determines the solution of the diffraction problem.

For any harmonic function $\phi_m(x, y, z)$ ($m = 1, \ldots 7$) satisfying the radiation condition Equation [1.4], we have the relation

$$\phi_m = -\frac{1}{4\pi}\iint_S \left(\frac{\partial \phi_m}{\partial n} G - \phi_m \frac{\partial G}{\partial n}\right) dS \qquad [1.5]$$

where for an infinitely deep fluid, the source function $G$ can be represented by[3]

$$G = \frac{1}{r_1} + \frac{1}{r_2} + 2ke^{kz}\int_\infty^z \frac{e^{-kz}}{r_2} dz - 2\pi ik \exp k(z + \zeta) H_0^{(2)}(kr_0) \qquad [1.6]$$

$$(r_1^2 = r_0^2 + (z - \zeta)^2,\ r_2^2 = r_0^2 + (z + \zeta)^2,\ r_0^2 = (x - \xi)^2 + (y - \eta)^2)$$

with $H_0^{(2)}(x)$ the Hankel function. For a fluid of finite depth, the source function $G$ is defined in a somewhat different form.[4]

From the expression for the source function and the general formula Equation [1.5], we have the following asymptotic representations, valid when $R \to \infty$, for infinite and finite depth, respectively:

$$\phi_m = -\left(\frac{k}{2\pi R}\right)^{1/2} H_m(k, \theta) \exp\left[kz - i\left(kR + \frac{\pi}{4}\right)\right] + O\left(\frac{1}{R}\right) \quad (m = 1, \ldots, 7) \qquad [1.7]$$

$$\phi_m = -\frac{g}{2u_0\sigma_0}\ \frac{chk_0(z+h)}{chk_0 h}\left(\frac{k_0}{2\pi R}\right)^{1/2} H_m(k_0, \theta) \exp - i\left(k_0 R + \frac{\pi}{4}\right) + O\left(\frac{1}{R}\right) (m=1, \ldots, 7) \qquad [1.8]$$

$$c = \frac{\sigma_0}{k_0},\quad u_0 = \frac{1}{2} c\left(1 + \frac{2k_0 h}{sh2k_0 h}\right) \qquad [1.9]$$

*Translator's note — The radiation functions correspond to the problem of forced oscillations in calm water. The scattering function corresponds to the problem of wave diffraction by a restrained ship.

where $h$ is the fluid depth,

$c$ and $u_0$ are the phase and group velocities, respectively, of a wave with wave number $k_0$, determined from the equation

$k_0 \; th \; k_0 h = k = \sigma_0^2/g$, and the asymptotic characteristics

$H_m$ can be expressed in the form*

$$H_m = \iint_S \left( f \frac{\partial \phi_m}{\partial n} - \phi_m \frac{\partial f}{\partial n} \right) dS \quad (m = 1, \ldots, 7) \qquad [1.10]$$

$$f(k, \theta) = \exp\left[kz + ik(x \cos\theta + y \sin\theta)\right]$$

$$f(k_0, \theta) = \frac{chk_0(z+h)}{chk_0 h} \exp ik_0 (x \cos\theta + y \sin\theta) \qquad [1.11]$$

For the determination of the exciting forces and moments in the general case, we have the following expression for the fluid pressure, giving rise to these forces and moments

$$p = -\rho i \sigma_0 (\phi_0 + \phi_7) e^{i\sigma_0 t} \qquad [1.12]$$

where $\rho$ is the fluid density.

On this basis, the components of the exciting force vector $X_1$, $X_2$, $X_3$ and the exciting moment vector $X_4$, $X_5$, $X_6$ are determined in the form

$$X_m = \rho i \sigma_0 e^{i\sigma_0 t} \iint_S (\phi_0 + \phi_7) \frac{\partial \phi_m}{\partial n} \, dS \quad (m = 1, \ldots, 6) \qquad [1.13]$$

We shall utilize the transposition principle, which was proved in References 1 and 2,

$$\iint_S U \frac{\partial W}{\partial n} \, dS = \iint_S W \frac{\partial U}{\partial n} \, dS \qquad [1.14]$$

---

*The functions $H_m(k_0, \theta)$ are connected with the functions $M_m(k_0, \theta)$ of Reference 4 by the equation

$$H_m(k_0, \theta) \; ch \; k_0 h = M_m(k_0, \theta)$$

The functions $H(k, \theta)$ were introduced in the theory of wave motion in a heavy fluid by N.E. Kochin.[5, 6]

and which holds for any two harmonic functions $U$ and $W$, satisfying the boundary condition, Equation [1.3], and the radiation condition, Equation [1.4]. Applying this principle to the functions $\phi_7$ and $\phi_m$ and imposing the condition of Equation [1.2], we finally obtain

$$X_m = \rho i\sigma_0 e^{i\sigma_0 t} \iint_S \left( \phi_0 \frac{\partial \phi_m}{\partial n} - \phi_m \frac{\partial \phi_0}{\partial n} \right) dS \quad (m = 1, \ldots, 6) \qquad [1.15]$$

Thus the exciting forces and moments are completely determined by the radiation functions $\phi_m$ for an arbitrary given incident wave system ($\phi_0$ exp $i\sigma_0 t$).

In particular, we shall consider the diffraction of waves around a ship due to the radiation from a source. Thus the function $\phi_0$ takes on the following form

$$\phi_0 = - \frac{Q_0}{4\pi} \cdot G\,(x, y, z, x_1, y_1, z_1) \qquad [1.16]$$

where $x_1$, $y_1$, $z_1$ and $Q = Q_0$ exp $i\sigma_0 t$ are the coordinates and strength, respectively, of the source.

From the general equations [1.5] and [1.15] with the substitution of Equation [1.16], we obtain the simple expression

$$X_m = \rho \phi_m\,(x_1, y_1, z_1) \frac{dQ}{dt} \quad (m = 1, 2, \ldots, 6) \qquad [1.17]$$

Now let us consider the case of a concentrated pressure $P = P_0$ exp $i\sigma_0 t$ at the point $(x_1, y_1, 0)$. Then, using similar arguments,[3] we obtain

$$X_m = - k\phi_m\,(x_1, y_1, 0)\,P \quad (m = 1, \ldots, 6) \qquad [1.18]$$

It is evident that for incident waves radiated from a dipole or higher order singularity, the exciting forces and moments can be obtained from Equation [1.17] by differentiation. For the effect of a distributed system of singularities, these forces are obtained by means of summation from the expressions in Equations [1.17] and [1.18].

In the case of diffraction of a regular progressive wave system, the function $\phi_0$ has the form

$$\phi_0 = i \frac{g}{\sigma_0} r_0 \exp\,[kz - ik\,(x \cos \epsilon + y \sin \epsilon)]$$

$$\phi_0 = i \frac{g}{\sigma_0} r_0 \frac{chk_0\,(z + h)}{chk_0 h} \exp - ik_0\,(x \cos \epsilon + y \sin \epsilon) \qquad [1.19]$$

for infinite or finite depths, respectively, where $2r_0$ is the incident wave height and $\epsilon$ is the angle between the direction of propagation of the waves and the $x$-axis. Substituting Equation [1.19] in Equation [1.15] and utilizing Equations [1.10] and [1.11], we obtain

$$X_m = -\rho g r_0 H_m (k, \epsilon \pm \pi) e^{i\sigma_0 t}$$

$$X_m = -\rho g r_0 H_m (k_0, \epsilon \pm \pi) e^{i\sigma_0 t} \quad (m = 1, \dots, 6) \tag{1.20}$$

for infinite and finite depths, respectively.

Hence, in the case of diffraction of a progressive regular wave system, the exciting force and moment can be expressed in terms of the asymptotic characteristics $H_m$ of the radiation functions $\phi_m$. These same characteristics also determine the damping coefficients.[1, 2, 4]

The same results hold also in the two-dimensional case. Indeed, the radiation and scattering functions $\phi_m (y, z)$ $(m = 2, 3, 4, 7)$ can be expressed, corresponding to Equation [1.5], by the formula[7]

$$\phi_m (y, z) = -\frac{1}{2\pi}\int_{L_0} \left(\frac{\partial\phi_m}{\partial n} G - \phi_m \frac{\partial G}{\partial n}\right) dl \quad (m = 2, 3, 4, 7) \tag{1.21}$$

where $L_0$ is the contour of a transverse section of the floating cylindrical ship and $G(y, z, \eta, \zeta)$ is the source function corresponding to the two-dimensional problem.[3, 7]

It follows from Equation [1.21] that the asymptotic representations for $y \to \pm\infty$ in a fluid of infinite or finite depth are of the form

$$\phi_m = iB_m^{\pm}(k) \exp k (z \mp iy)$$

$$\phi_m = i \frac{g}{2u_0\sigma_0} B_m^{\pm}(k_0) \frac{ch k_0 (z + h)}{ch k_0 h} \exp \mp ik_0 y \tag{1.22}$$

where the asymptotic characteristics $B_m^{\pm}(k)$ and $B_m^{\pm}(k_0)$ are determined by the formulas*

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*The values of $B_m^{\pm}(k_0)$ are related to the quantities $M_m^{\pm}$ of Reference 7 by the expression $B_m^{\pm} ch\, k_0 h = M_m^{\pm}$.

$$B_m^{\pm} = \int_{L_0} \left( f_0 \frac{\partial \phi_m}{\partial n} - \phi_m \frac{\partial f_0}{\partial n} \right) dl, f_0\,(k) = \exp k\,(z \pm iy)$$

$$f_0\,(k_0) = \frac{chk_0\,(z+h)}{chk_0 h} \exp \pm ik_0 y \qquad [1.23]$$

For plane waves progressing in the positive or negative direction along the $y$-axis, $\phi_0 = ir_0 f_0 g/\sigma_0$. Thus from the general formula of Equation [1.15] and Equation [1.23], it follows that the exciting force and moment are determined by the simple equation

$$X_m = -\rho g r_0 B_m^{\pm} e^{i\sigma_0 t} \quad (m = 2, 3, 4) \qquad [1.24]$$

where the upper sign corresponds to waves progressing in the negative direction along the $y$-axis and the lower sign corresponds to waves progressing in the positive direction.

For the diffraction of disturbances radiated from two-dimensional singularities, and for plane waves, the expressions for the exciting forces and moments retain the same form as in the three-dimensional case. For example, the source and concentrated pressure point results are given by Equations [1.17] and [1.18].

Analogous results can be found for the case of diffraction of oblique waves around a moving cylindrical ship. In this case for the pressure, giving rise to the exciting forces and moments, we have the expression

$$p = -\rho i\sigma_0 \exp i\,(\sigma t - kx \cos \epsilon)\,(\psi_0 + \psi_7)\,(\sigma = \sigma_0 - ku \cos \epsilon)$$

where $\sigma$ is the frequency of encounter,

$u$ is the forward velocity of the ship,

$\psi_0$ is the function representing the incident wave, and

$\psi_7$ is the function corresponding to the scattered wave.

The exciting forces and moments acting on a section of the cylindrical ship are given, analogously to Equation [1.15], by the formula

$$X_m = \rho i\sigma_0 \exp i\,(\sigma t - kx \cos \epsilon) \int_{L_0} \left( \psi_0 \frac{\partial \psi_m}{\partial n} - \psi_m \frac{\partial \psi_0}{\partial n} \right) dl \quad (m = 2, 3, 4) \qquad [1.25]$$

where $\psi_m$ $(m = 2, 3, 4)$ are the radiation functions characterizing the radiated waves in a heavy fluid due to bending oscillations of a cylindrical ship with unit velocity amplitude.

For plane progressive waves, the function $\psi_0$ is given for infinite and finite depth of fluid by the expressions

$$\psi_0 = i \frac{g}{\sigma_0} r_0 \exp k (z - iy \sin \epsilon), \quad \psi_0 = i \frac{g}{\sigma_0} r_0 \frac{chk_0 (z+h)}{chk_0 h} \exp - ik_0 y \sin \epsilon \qquad [1.26]$$

and corresponding to these we obtain the equations

$$\begin{aligned} X_m &= - \rho g r_0 \exp i (\sigma t - kx \cos \epsilon) D_m {}^{\pm} (k) \\ X_m &= - \rho g r_0 \exp i (\sigma t - k_0 x \cos \epsilon) D_m {}^{\pm} (k_0) \end{aligned} \qquad [1.27]$$

where $D_m{}^{\pm}$ denote the asymptotic characteristics of the radiation functions $\psi_m$:

$$D_m {}^{\pm} = \int_{L_0} \left( f_1 \frac{\partial \psi_m}{\partial n} - \psi_m \frac{\partial f_1}{\partial n} \right) dl \quad (m = 2, 3, 4) \qquad [1.28]$$

$$\begin{aligned} f_1 (k) &= \exp kz \pm ik_2 y \quad (k_2 = k \, |\sin \epsilon|) \\ f_1 (k_0) &= \frac{chk_0 (z+h)}{chk_0 h} \exp \pm ik_2' y \quad (k_2' = k_0 \, |\sin \epsilon|) \end{aligned} \qquad [1.29]$$

The plus sign is to be taken for $\sin \epsilon < 0$ and the minus sign for $\sin \epsilon > 0$.

Thus, if one finds simply the radiation functions or their asymptotic characteristics,[3, 8–11] then the general damping coefficients and the exciting forces and moments are determined simultaneously. Furthermore, we note that the above-mentioned results can be extended to the case of arbitrary irregular waves by means of operational transforms.[2]

## 2. APPROXIMATE DETERMINATION OF THE ASYMPTOTIC CHARACTERISTICS

In practical calculations of the mechanical characteristics of ship motions, it is of interest to determine the exciting forces and moments due to the diffraction of plane progressive waves.

In order to determine the approximate asymptotic characteristics of the functions $\phi_m$ and thus the exciting forces and moments, we shall employ the method of averaging which is applied to the two-dimensional case in Reference 7. We note that the value of the functions $\phi_m$ on the surface of an ellipsoid with the three axes $L$, $B$, and $T$ situated in an infinite fluid can be represented in the form[12]

$$\phi_1 = -c_1 x, \quad \phi_2 = -c_2 y, \quad \phi_3 = -c_3 z$$
$$\phi_4 = -c_4 yz, \quad \phi_5 = -c_5 zx, \quad \phi_6 = -c_6 xy \qquad [2.1]$$

where

$$c_j = \frac{\mu_{jj}}{\rho D} \ (j = 1, 2, 3), \quad c_4 = \frac{\mu_{44}}{\rho I_{xx}} \frac{1 + s_1^2}{1 - s_1^2}, \quad c_5 = \frac{\mu_{55}}{\rho I_{yy}} \frac{s_2^2 + 1}{s_2^2 - 1}$$
$$c_6 = \frac{\mu_{66}}{\rho I_{zz}} \frac{s_3^2 + 1}{s_3^2 - 1}, \quad s_1 = \frac{2T}{B}, \quad s_2 = \frac{2T}{L}, \quad s_3 = \frac{L}{B} \qquad [2.2]$$

Here $\mu_{jj} (j = 1, \ldots 6)$ are the coefficients of added mass,

$D$ is the volume of the ellipsoid, and

$I_{xx}$, $I_{yy}$, and $I_{zz}$ are the moments of inertia of this volume, with respect to the $x$, $y$, and $z$ axes.

By the argument stated in Reference 7, in order to approximate the functions $H_m$, we shall make use of the relations in Equation [2.1], but substitute for $c_j$ $(j = 1, \ldots 6)$ the values appropriate to the given ship surface $S$. Specifically, for the value of $C_5$ we have

$$c_5 = \frac{\mu_{55}}{\rho I}, \quad I = \iint_S xz\,(z \cos(n, x) - x \cos(n, z))\, dS = I_s - I_\omega$$
$$J_s = \frac{DT^2}{3\,(3 - 2\chi_0)}, \quad J_\omega = \frac{DL^2}{12\,(3 - 2\phi_0)} \qquad [2.3]$$

where $J_s$ is the moment of inertia of the waterline,

$J_\omega$ is the moment of inertia of the midship section,

$\chi_0$ and $\phi_0$ are the corresponding coefficients of vertical and longitudinal fineness, and

$L$, $B$, and $T$ are the length, beam, and draft of the ship.

Substituting Equation [2.1] in Equation [1.10] and using the Gauss theorem, we can then approximate all the functions $H_m$ $(m = 1, \ldots 6)$ for any $k$.

Thus, for example, for a ship with longitudinal symmetry, the functions $H_3(k, \theta) \approx H_{30}(k, \theta)$ and $H_5(k, \theta) \approx H_{50}(k, \theta)$ are determined in the form

$$H_{30}(k, \theta) = \frac{S_0}{\alpha}\left\{-\int_{-T}^{0}\int_{0}^{1} e^{kz}\left(Z'(z)\, X\left(\frac{xL}{2}\right)\cos p + kc_3 \frac{\sin p}{\gamma \sin \theta}\right)\cos qx\, dx\, dz\right\} \qquad [2.4]$$

$$H_{50}(k, \theta) = \frac{iBL^2}{2}\int_{-T}^{0}\int_{0}^{1} e^{kz}\left(Z'(z)\, X\left(\frac{xL}{2}\right)x \cos p + kc_5 x \frac{\sin p}{\gamma \sin \theta}\right)\sin qx\, dx\, dz$$

$$+ ik(1 + c_5)\, BL \cos \theta \int_{-T}^{0}\int_{0}^{1} ze^{kz} \cos qx \frac{\sin p}{\gamma \sin \theta}\, dx\, dz \qquad [2.5]$$

$$\left(q = \frac{kL}{2}\cos \theta,\ \gamma = \frac{kB}{2},\ p = \gamma Z(z)\, X\left(\frac{xL}{2}\right)\sin \theta\right)$$

where $S_0$ is the waterline area,

$\alpha$ is the waterline area coefficient, and

$y = \pm \frac{1}{2} BZ(z)\, X(x)$ is the equation of the ship's surface.

If $X(x)$ and $Z(z)$ are given in analytic form, then the functions $H_{30}$, $H_{50}$, and the remaining functions $H_{m0}$ can be calculated for any $\theta$ and $k$. In particular, to within a term containing $Bp^2$, we obtain from Equations [2.4] and [2.5]

$$H_{30}(k, \theta) = -S_0\left[\kappa_2(kT) - kT\beta c_3 \kappa_1(kT)\right] K_1(q) \qquad [2.6]$$

$$H_{50}(k, \theta) = ik \cos \theta \{I_y \left[\kappa_2(kT) + kT\beta c_5 \kappa_1(kT)\right] K_3(q) - - (1 + c_5)\, Db_0 \kappa_4(kT)\, K_1(q)\} \qquad [2.7]$$

where $\beta$ is the coefficient of area of the midship section and is equal to $\chi_0$ in the case considered,

$I_y$ is the moment of inertia of the waterline area about the $y$-axis,

$b_0$ is the depth of submergence of the centroid, and

the coefficients $K_1$, $K_3$, $\kappa_1$, $\kappa_2$, and $\kappa_4$ are given in Reference 7.

Corresponding to Equation [1.20] for the exciting force $X_3$ and moment $X_5$, we have the expressions

$$X_{30} = \rho g r_0 S_0 \, [\kappa_2 (kT) - kT\beta c_3 \kappa_1 (kT)] \, K_1 (q_\epsilon) \, e^{i\sigma_0 t} \qquad [2.8]$$

$$X_{50} = \rho g i k \cos \epsilon \, \{ I_y \, [\kappa_2 (kT) + kT\beta c_5 \kappa_1 (kT)] \, K_3 (q_\epsilon) -$$
$$- (1 + c_5) \, D b_0 \kappa_4 (kT) \, K_1 (q_\epsilon) \} \, e^{i\sigma_0 t} \left( q_\epsilon = \frac{kL}{2} \cos \epsilon \right) \qquad [2\ 9]$$

which are valid with the above indicated accuracy and, in the case $\epsilon = 0$ or $\epsilon = \pi$, are in agreement with the results implied from Equations [2.4] and [2.5].

If we set $c_3 = 0$ and $c_5 = 0$ in Equations [2.8] and [2.9], then we obtain the values of the exciting force $X_{30}^0$ and moment $X_{50}^0$ corresponding to the hypothesis of A.N. Krilov (the hypothesis that the wave is not influenced by the presence of the ship). For the ratio $\xi = X_{30}^0 / X_{30}$ we find

$$\xi = \frac{\kappa_2 (kT)}{(1 + c_3) \, \kappa_2 (kT) - c_3} \quad \left( \kappa_1 (kT) = \frac{1}{kT\beta} \, [1 - \kappa_2 (kT)] \right)$$

Figure 1 shows the dependence of $\xi$ on $T/\lambda$ ($k = 2\pi/\lambda$) for the ship model $S_1^{13}$ in two cases: the dotted line corresponds to the calculation for $c_3 = c_3(\infty)$, i.e., where $\mu_{33}(k)$ is replaced by the limiting value $\mu_{33}(\infty)$, and the solid line is obtained from the computed value of $\mu_{33}(k)$ for this model.[13] The same graph shows the individual points plotted from the experimental values of $\xi$ taken from Reference 13. Note also that for the model considered, $L/T = 16$. Thus the relatively long waves ($\lambda > L$) are more favorable for the Krilov hypothesis.

The graph in Figure 1 shows sufficiently good agreement between the theoretical and experimental values of the variable $\xi$. Furthermore it follows that even for favorable values $\lambda > L$, the Krilov hypothesis is subject to a significant error since $\xi$ differs from unity by 40 percent.

The fact that in utilizing the relations of Equation [2.1], we take the real part of the values of the functions $\phi_m$ on the ship surface $S$ means that for calculating $H_m$, the approximation takes into account only the inertia effects and completely neglects the damping effects in a heavy fluid. It is evident that such an assumption is valid for very small and large frequencies, for which the effect of radiated waves is insignificant. This is obvious from the above comparison of theoretical and experimental values of the exciting force $X_3$. In the case of intermediate frequencies of oscillation, significant radiation of waves takes place in a heavy fluid and thus for the approximate determination of the functions $H_m(k, \theta)$, it is necessary to consider also the imaginary part of the functions $\phi_m$ on the surface $S$.

In order to consider more completely the inertial and damping effects, we substitute the value of $\phi_m$ from Equation [2.1] in Equation [1.5] and thus we find a first approximation to the function $\phi_m$. Repeating this process, we can obtain the second and higher approximation. Using this method, we shall compute the first approximation to the imaginary part of the function $\phi_m$ for an infinitely deep fluid. From Equation [1.6] we have

$$\operatorname{Im} G = -2\pi k \exp k(z+\zeta) J_0(kr_0) =$$

$$= -ke^{k(z+\zeta)} \int_{-\pi}^{+\pi} \exp -ik[(x-\xi)\cos u + (y-\eta)\sin u]\, du \qquad [2.10]$$

Using this relation together with Equations [2.1] and [1.5], we obtain

$$\operatorname{Im} \phi_m = \frac{k}{4\pi} \int_{-\pi}^{+\pi} \exp k(z - ix\cos u - iy\sin u) H_{m0}(k, u)\, du \qquad [2.11]$$

Substituting this expression and Equation [2.1] in Equation [1.10], we find the second approximation for the functions $H_m(k, \theta)$

$$H_m(k, \theta) = H_{m0}(k, \theta) - \frac{ik^2}{4\pi} \int_{-\pi}^{+\pi} \dot{H}_{m0}(k, u) N(k, \theta, u)\, du \qquad [2.12]$$

where

$$N = \frac{S_0}{\alpha} \left\{ - \int_{-T}^{0} \int_{0}^{1} e^{2kz} Z'(z) X\left(\frac{xL}{2}\right) \cos p_0 \cos q_0 x\, dx\, dz + \right.$$

$$\left. + k[\cos(\theta - u) - 1] \int_{-T}^{0} \int_{0}^{1} e^{2kz} \frac{\sin p_0 \cos q_0 x}{\gamma(\sin\theta - \sin u)}\, dx\, dz \right\} \qquad [2.13]$$

$$(p_0 = \gamma Z(z) X(\tfrac{1}{2}xL)(\sin\theta - \sin u),\ q_0 = \tfrac{1}{2}kL(\cos\theta - \cos u),\ \gamma = \tfrac{1}{2}kB)$$

Within the accuracy of terms containing $Bp_0^2$, Equation [2.13] takes the much simpler form

$$N = -S_0 \{\kappa_2(2kT) + kT\beta\kappa_1(2kT)[1 - \cos(\theta - u)]\} K_1(q_0) \qquad [2.14]$$

which is correct for elongated ships.

Similar expressions can be obtained for the two-dimensional asymptotic characteristics $B_m \pm$. For example, using Reference 7 and taking into account the approximate inertia effects in the form shown in Equation [2.1], we find for $B_{30} \pm (k)$

$$B_{30} \pm (k) = -b\left\{\frac{\sin \gamma_0}{\gamma_0} - k\,(1 + c_3') \int_{-T}^{0} e^{kz}\,\frac{\sin [\gamma_0 Z\,(z)]}{\gamma}\,dz\right\} \qquad [2.15]$$

$$\left(\gamma_0 = \frac{kb}{2}, \quad c_3' = \frac{\mu_{33}'}{\rho\omega_0}\right)$$

where $y = \pm \frac{1}{2}\, bZ(z)$ is the equation of the section $L_0$, and $\omega$ and $\mu_{33}'$ are, respectively, the area of this section and its added mass coefficient.

In the special cases $\beta = 1$ and $\beta = 0.5$, the integral in Equation [2.15] can be evaluated* and thus we obtain

$$B_{30} \pm (k) = -b\,\frac{\sin \gamma_0}{\gamma}\,e^{-kT}\,(1 + c_3' - c_3' e^{kT}) \qquad (\beta = 1)$$

$$B_{30} \pm (k) = -\frac{b}{\gamma_0^2 + (kT)^2}\,\left[(\gamma_0^2 - (kT)^2\, c_3')\,\frac{\sin \gamma_0}{\gamma_0} - kT\,(1 + c_3')\,(e^{-kT} - \cos \gamma_0)\right]$$

$$(\beta = 0.5) \qquad [2.16]$$

For normal values of $\alpha_0$, the expression in Equation [2.15] can be evaluated from a power series in $\alpha_0$. With an accuracy of degree $\gamma_0^4$ we have

$$B_{30} \pm (k) = -b\left\{\kappa_2\,(kT) - kT\beta c_3'\kappa_1\,(kT) - \frac{\gamma_0^2}{6}\,[\kappa_3\,(kT) - kb_1 c_3'\kappa_5\,(kT)]\right\}$$

$$\left(\kappa_5\,(kT) = \frac{1}{kb_1}\left(e^{-kT} - \kappa_3\,(kT)\right)\right. \quad \left(b_1 = \int_{-T}^{0} Z^3\,(z)\,dz\right) \qquad [2.17]$$

where the coefficient $\kappa_3\,(kT)$ is given in Reference 7.

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*Translator's note – By $\beta = 1$ and $\beta = 0.5$, the author implies rectangular and triangular sections, respectively.

In a similar way we can calculate the values of $B_{20}^{\pm}$, which are pure imaginary for sections symmetrical about the $z$-axis. Accounting for the damping effects leads to comparatively simple formulas. We have

$$B_3^{\pm} = B_{30}^{\pm}\left[1 + \frac{i}{2}\,(kb\kappa_2\,(2kT) + \sin kb)\right]$$
$$B_m^{\pm} = B_{m0}^{\pm}\left[1 + \frac{i}{2}\,(kb\kappa_2\,(2kT) - \sin kb)\right] \qquad (m = 2, 4) \qquad [2.18]$$

From Equations [2.17] and [2.18] we find that with an accuracy of $\gamma_0^3$ ($kc_3'$ is of the order of $\gamma_0$), the asymptotic characteristics $B_3^{\pm}$ can be represented in the form

$$B_3^{\pm} = -b\left\{\kappa_2\,(kT) - kT\beta c_3'\kappa_1\,(kT) - \frac{\gamma_0^2}{6}\,\kappa_3\,(kT) + \right.$$
$$\left. + i\gamma_0\,[1 + \kappa_2\,(2kT)]\,[\kappa_2\,(2kT) - kT\beta c_3'\kappa_1\,(kT)]\right\} \qquad [2.19]$$

For two-dimensional problems, with an accuracy of $\gamma_0^3$, we have

$$B_3^{\pm} = -b\left(1 + ikb - k\,\frac{\mu_{33}'}{\rho b}\right) \qquad [2.20]$$

Taking into account the values of $\mu_{33}'$ for small $kb$ and a flat plate,[7, 14] it follows that Equation [2.20] is consistent with the value of $B_3^{\pm}$ which follows from the exact solution.[14]

The approximate asymptotic characteristics $D_m^{\pm}$ can be calculated by the same method to determine the exciting forces and moments for diffraction of oblique waves past a moving cylindrical ship. Using Reference 7 we find, analogously to Equation [2.18], the formulas

$$D_3^{\pm}(k) = D_{30}^{\pm}(k)\left\{1 + \frac{i}{2\,|\sin\epsilon|}\left[kb\kappa_2(2kT) + \frac{\sin k_2 b}{|\sin\epsilon|} - \right.\right.$$
$$\left.\left. - \frac{2k_1^2}{k_2}\int_{-T}^{0} e^{2kz}\sin[k_2 bZ(z)]\,dz\right]\right\}$$

$$D_m^{\pm}(k) = D_{m0}^{\pm}(k)\left\{1 + \frac{i}{2\,|\sin\epsilon|}\left[kb\kappa_2(2kT) - \frac{\sin k_2 b}{|\sin\epsilon|} + \right.\right.$$

$$+ \frac{2k_1^2}{k_2} \int_{-T}^{0} e^{2kz} \sin [k_2 bZ (z)]\, dz \Big]\Big\} \qquad [2.21]$$

$$(m = 2, 4,\ k_1 = k \mid \cos \epsilon \mid,\ k_2 = k \mid \sin \epsilon \mid)$$

which reduce to Equation [2.18] for $\epsilon = \pm \frac{1}{2}\pi$.

However, in the case considered here, the radiation functions $\psi_m$ are not harmonic, and for an infinite fluid, they satisfy the conditions

$$\frac{\partial \psi_m}{\partial z} - k\psi_m = 0 \text{ for } z = 0, \quad \frac{\partial^2 \psi_m}{\partial y^2} + \frac{\partial^2 \psi_m}{\partial z^2} - k_1^2 \psi_m = 0 \qquad [2.22]$$

$$\psi_m = i \frac{k}{k_2} D_m^{\pm} (k) \exp kz \mp ik_2 y \text{ for } y \to \pm \infty \qquad (m = 2, 3, 4) \qquad [2.23]$$

$$\frac{\partial \psi_2}{\partial n} = \cos (n, y), \quad \frac{\partial \psi_3}{\partial n} = \cos (n, z), \quad \frac{\partial \psi_4}{\partial n} = y \cos (n, z) - z \cos (n, y) \text{ on } L_0 \qquad [2.24]$$

Thus the coefficients $Cm(k)$, characterizing the inertial effects, are not known precisely. These coefficients can be found from numerical analysis of the exact solutions[8] or from special experiments in which cylindrical bodies are given oscillatory deformations.

As an approximate estimate of the influence of inertial effects, we examine a simple case: let us take a circular cylinder of radius $a$, the axis of which is submerged at a sufficiently large depth $h_0$, such that the influence of the free surface can be neglected. Then, taking the origin of coordinates on the cylinder axis, we can approximate the value of the vector $\psi$ $(\psi_1, \psi_2, \psi_3)$ at points on the surface with the following expressions[7]

$$\psi = - c (\eta)\, a \boldsymbol{e}_r, \quad \boldsymbol{e}_r = \boldsymbol{j} \cos \theta + \boldsymbol{k} \sin \theta, \quad c(\eta) = \frac{\mu}{\rho \pi a^2} = \frac{K_1 (\eta)}{\eta K_0 (\eta) + K_1 (\eta)} \qquad [2.25]$$

where $\theta$ is the polar angle and $K_0$ and $K_1$ are the Bessel functions of imaginary argument ($\eta = k_1 a$).

On the basis of Equation [1.27] – Equation [1.29], we obtain the vector of the exciting force:

$$\boldsymbol{F} = -\rho g r_0 a \exp - kh_0 +$$

$$+ i\,(\sigma t - kx \cos \epsilon\,) \left\{ \int_{-\pi}^{+\pi} \exp \delta\,(\sin \theta - i \sin \epsilon \;\cos \theta)\, \boldsymbol{e}_r d\theta + \right. \tag{2.26}$$

$$\left. + \delta c\,(\eta)\, \frac{\partial}{\partial \delta} \left[ \int_{-\pi}^{+\pi} \exp \delta\,(\sin \theta - i \sin \epsilon \;\cos \theta)\, \boldsymbol{e}_r d\theta \right] \right\} \qquad (\delta = ka)$$

In order to proceed further, we expand

$$\exp - i\omega \cos \theta = J_0\,(\omega) + 2 \sum_{m=1}^{\infty} (-i)^m\, J_m\,(\omega) \cos m\theta \qquad (\omega = \delta \sin \epsilon\,) \tag{2.27}$$

and use the following formulas

$$\begin{aligned} (-1)^n\, I_{2n}\,(\delta) &= \frac{1}{\pi} \int_0^{\pi} ch\,(\delta \sin x) \cos 2nx\,dx, \\ (-1)^n\, I_{2n}\,(\delta) &= \frac{1}{\pi} \int_0^{\pi} sh\,(\delta \sin x) \sin\,(2n+1)\,x\,dx \end{aligned} \tag{2.28}$$

where $J_m(\omega)$ and $I_n(\delta)$ are the Bessel functions of real and imaginary argument, respectively.

Carrying out the necessary reductions, we find

$$\begin{aligned} X_2 &= \frac{4\pi i}{\delta}\, \rho g r_0 a \exp\,[-kh_0 + i\,(\sigma t - kx \cos \epsilon\,)] \sum_{n=0}^{\infty} \Big\{ J_{2n+1}\,(\delta \sin \epsilon\,)\, I_{2n+1}\,(\delta) \\ &\qquad + \delta c\,(\eta)\, \frac{d}{d\delta}\,[J_{2n+1}\,(\delta \sin \epsilon\,)\, I_{2n+1}\,(\delta)] \Big\} \\ X_3 &= -2\pi \rho g r_0 a \exp\,[-kh_0 + i\,(\sigma t - kx \cos \epsilon\,)] \sum_{n=0}^{\infty} \lambda_n \Big\{ J_{2n}\,(\delta \sin \epsilon\,)\, I_{2n}\,(\delta) \\ &\qquad + \delta c\,(\eta)\, \frac{d}{d\delta}\,[J_{2n}\,(\delta \sin \epsilon\,)\, I_{2n}\,(\delta)] \Big\} \\ &\qquad (\lambda_0 = 1,\ \lambda_n = 2) \end{aligned} \tag{2.29}$$

For $\epsilon = 0$ or $\epsilon = \pi$, Equation [2.29] takes on a simpler form. We then have

$$X_2 = 0,\ X_3 = 2\mu\,(0)\ \zeta\,(\delta)\ \frac{dV_{0z}}{dt}\ ,\ \frac{dV_{0z}}{dt} = -\sigma_0^2\, r_0\, \exp\,[-kh_0 + i\,(\sigma t \mp kx)]$$
$$\zeta(\delta) = \frac{1}{\delta\,[\delta K_0\,(\delta) + K_1\,(\delta)]},\quad \mu\,(0) = \pi\rho a^2 \qquad [2.30]$$

where $dV_{0z}/dt$ is the vertical component of acceleration of the incident wave at the cylinder axis.

Figure 2 shows the dependence of $\xi(ka)$ and for comparison also shows (by the dotted line) the dependence of $\xi_1 = I_0\,(ka)$, characterizing the influence of inertial effects from the theory of an unbounded fluid ($c = 1$). As is shown, accounting for these theoretical inertial effects overestimates the values of the exciting force $X_3$.

In the case of a thin cylindrical ship of small beam $b$, one can approximate the asymptotic characteristics $D_3\,\pm$ with full regard to the inertial and damping effects. Indeed, from Equations [1.28] and [1.29] we find the approximate expressions

$$D_3\,\pm\,(k) = -\,b\left\{\kappa_2\,(kT) - k\int_{-T}^{0}\psi_3\,(0,z)\,e^{kz}\,[Z\,(z) + k\sin^2\epsilon\ Z\,(z)]\,dz\right\} \qquad [2.31]$$

which are correct to an accuracy of order $b^3$. The function $\psi_3\,(0, z)$ is easily found from the source function for this problem.[7] We have

$$\psi_3\,(0,z) = \frac{b}{2\pi}\int_{-T}^{0} Z'(\zeta)\left\{K_0|k_1\,(z-\zeta)| + K_0|k_1\,(z+\zeta)| + \right.$$
$$\left. + \,2ke^{kz}\int_{\infty}^{z} e^{-kv}\,K_0\,|k_1\,(v+\zeta)|dv - 2\pi i\,\frac{k}{k_2}\,\exp k\,(z+\zeta)\right\}d\zeta \qquad [2.32]$$

Equations [2.31] and [2.32] determine the values of $D_3\,\pm(k)$ for a thin cylindrical ship, and in particular

$$\mathrm{Im}\,[D_3\,\pm\,(k)] = -\,\frac{kb^2}{|\sin\epsilon\,|}\ \kappa_2\,(kT)\,[\kappa_2\,(2kT) + kT\beta\,\sin^2\epsilon\ \kappa_1\,(2kT)] \qquad [2.33]$$

as can also be obtained from the general formula Equation [2.21] for small $kb$.

The results obtained above are easily generalized to the case of finite depth of fluid. From Equations [1.28] and [1.29] we have

$$D_3^{\pm}(k_0) = -b\left\{\kappa_2' - k_0 \int_{-T}^{0} \psi_3(0, z)\left[Z'(z)\frac{sh\, k_0(z+h)}{ch\, k_0 h} + \right.\right.$$
$$\left.\left. + k_0 \sin^2\epsilon\; Z(z)\frac{ch\, k_0(z+h)}{ch\, k_0 h}\right] dz\right\} \qquad [2.34]$$

$$\kappa_2' = \int_{-T}^{0} Z'(z)\frac{ch\, k_0(z+h)}{ch\, k_0 h}\, dz \qquad [2.35]$$

The function $\psi_3(y, z)$ can, as above, be obtained from the corresponding source function, but it is considerably simpler to use the method of othogonal functions. Since $\psi_3(-y, z) = \psi_3(y, z)$, it is sufficient to determine the function $\psi_3$ for the domain $y \geq 0$ and $-h < z < 0$, while satisfying for $y = 0$ the condition

$$\frac{\partial\psi_3}{\partial y} = \begin{cases} -\frac{1}{2}\, bZ'(z) & (-T < z < 0) \\ 0 & (-h < z < -T) \end{cases} \qquad [2.36]$$

For the solution of this problem, we consider the system of solutions

$$\phi_n = \frac{\cos q_n(z+h)}{\cos q_n h}\exp - y\sqrt{q_n^2 + k_1^2} \qquad (n = 0, 1, 2, \ldots;\; k_1 = k_0\,|\cos\epsilon|) \qquad [2.37]$$

which satisfy Equations [2.22] and the condition $\partial\phi_n/\partial z = 0$ on $z = -h$, where $q_n$ is the root of the equation $q_n\, tg\, q_n h = -k$, with $q_1, q_2, \ldots$ real and $q_0 = ik_0$ ($k_0$ is the wave number of a progressive wave.)

It is easily seen that the functions $\phi_n(0, z)$ ($n = 0, 1, 2, \ldots$) form a complete set of orthogonal functions in the interval $-h < z < 0$. Thus the function $\psi_3(y, z)$ can be expressed in the following expansion as a series of the functions $\phi_n$:

$$\psi_3(y, z) = \sum_{n=0}^{\infty} a_n\phi_n(y, z) \qquad [2.38]$$

Imposing the condition of Equation [2.36], we obtain

$$a_n = \frac{b}{2d_n\sqrt{q_n^2 + k_1^2}} \int_{-T}^{0} Z'(z)\, \frac{\cos q_n\,(z + h)}{\cos q_n h}\, dz$$

$$d_n = \int_{-h}^{0} \phi_n^2\,(0,\, z)\; dz = \frac{1}{2q_n^2}\,[h\,(k^2 + q_n^2) - k] \qquad [2.39]$$

Equations [2.38], [2.39], and [2.34] determine the values of the asymptotic characteristics $D_3^{\pm}(k_0)$.

## 3. THE APPROXIMATE EVALUATION OF THE WETTING OF SHIPS IN WAVES

The characteristics of wetting of ships in waves depends on the level of the fluid along the length of the ship, according to the following formulas

$$\mathfrak{z}_r = \mathfrak{z}_0 - \zeta_e, \quad \mathfrak{z}_0 = -\frac{1}{g}\left(\frac{\partial \Phi}{\partial t} - u\,\frac{\partial \Phi}{\partial x}\right)_{z=0}, \quad \zeta_e = \zeta_0 - \psi_0 x + \theta_0 y \qquad [3.1]$$

where $\zeta_e$ is the total vertical displacement of a point on the waterline,
$\zeta_0$ is the vertical displacement of the centroid of the waterline,
$\psi_0$ and $\theta_0$ are the angles of pitch and roll, respectively, and
$\mathfrak{z}_0$ is the free-surface elevation above the plane $z = 0$.

The value of $\mathfrak{z}_r$ for elongated ships[7] is obtained from the formula

$$\mathfrak{z}_r\,(x) = -\frac{i\sigma}{g}\,[v_2\phi_2\,(y,\,0,\,\nu) + v_3\phi_3\,(y,\,0,\,\nu) + v_4\phi_4\,(y,\,0,\,\nu)]\,e^{i\sigma t} -$$

$$-\frac{i\sigma_0}{g}\,[\psi_0\,(y,\,0,\,k) + \psi_7\,(y,\,0,\,k)]\,\exp i\,(\sigma t - kx\cos\epsilon) - \zeta_e \qquad \left(\nu = \frac{\sigma^2}{g}\right) \qquad [3.2]$$

which, obviously, is valid for average frequencies of roll of thin ships, and where $v_2$, $v_3$, and $v_4$ are the complex amplitudes of the translational and rotational velocities.

It is evident from Equations [3.1] and [3.2] that for a complete evaluation of the wetting of the ship in waves, it is necessary to know the values of all of the displacements as well as the radiation and scattering functions. Consequently the complete determination of the value of $\mathfrak{z}_r$ from the Equation [3.2] can be found only with the help of the results of the explicit solution.[8, 9] Nevertheless, we can try to estimate the value of $\mathfrak{z}_r$ based on the approximate expressions for the velocity potential of the fluid motion.

Numerical analysis of the solution of the corresponding plane diffraction problem shows[15, 16] that for comparatively small distances from the obstacle, upon which the regular wave system is incident, the asymptotic formula for determining the free-surface elevation becomes valid very quickly. Similar properties hold also in the three-dimensional case.[17] Therefore one can suppose that the application of the asymptotic formula will lead to an approximate evaluation of $\varsigma_r$ in which, to some degree, the hydrodynamic interaction is accounted for. On the other hand, application of the Krilov hypothesis of penetration of waves and calculations of the initial elevation for a ship moving in calm water do not lead to affirmative results[18] when compared with experimental values.

Using these arguments in Equation [3.2], we have the following approximate expression for a fluid of infinite depth:

$$\begin{aligned} \varsigma_r(x) = r_0 \exp i\,[\sigma t - k\,(x\cos\epsilon + y\sin\epsilon)] + \frac{\sigma}{g}\,[v_2 B_2{}^{\pm}(\nu) + v_3 B_3{}^{\pm}(\nu) \\ + v_4 B_4{}^{\pm}(\nu)]\exp i\,(\sigma t \mp \nu y) - \frac{\sigma_0}{g}\, iC^{\pm} \exp i\,(\sigma t - kx\cos\epsilon \mp k_2 y) - \zeta_e \end{aligned} \qquad [3.3]$$

where $C^{\pm}$ are the asymptotic characteristics of the scattering function $\psi_7(y, z, k)$.

In particular, we shall consider longitudinal oscillations ($\zeta_e = \zeta_0 - x\psi_0$) with $\epsilon = 0$ or $\epsilon = \pi$. In this case the effective radiation of the scattered wave is insignificant, and we can assume that $C^{\pm} \approx 0$. Furthermore, using the above statements, we find

$$X_3(\nu, x) = -\rho g_0 r_0 B_3{}^{\pm}(\nu)\, e^{i\sigma t}, \quad v_3 e^{i\sigma t} = i\sigma\zeta_e, \quad \zeta_e = r_0\,(F_\zeta - xF_\psi)\, e^{i\sigma t} \qquad [3.4]$$

Here $F_\zeta$ and $F_\psi$ are the transfer functions of the mechanical characteristics of $\zeta_0$ and $\psi_0$, determined from the complete dynamical equations for these motions, and $X_3(\nu, x)$ is the exciting force acting on a section with abscissa $x$ for incident waves of wave number $\nu = \sigma^2/g$.

Substituting Equation [3.4] in Equation [3.3] we finally obtain

$$\begin{aligned} \varsigma_r(x) = r_0 \exp i\,(\sigma t \mp kx) - \zeta_e - \frac{i\nu}{\rho g}\,(F_\zeta - xF_\psi)\, X_3(\nu, x) \exp - i\gamma\xi(x) \\ \left(\gamma = \frac{\nu B}{2}\right) \end{aligned} \qquad [3.5]$$

where $y = \pm \frac{1}{2} B\,\xi(x)$ is the waterline equation.

Equation [3.5] provides an approximate expression for the relative wave elevation, valid for the longitudinal oscillations of a thin ship at normal frequencies, at least for $\gamma > 1$. For these values, the asymptotic formulas are known to be valid. For the bow and stern parts of the waterline, Equation [3.5] is unsuitable, since the influence of the bow and stern is a three-dimensional effect.

In the absence of ship motion, it is easy to estimate the three-dimensional effects, using the asymptotic formula

$$\zeta_0 = r_0 \exp i\,[\sigma_0 t - kR \cos(\theta - \epsilon)] + + \frac{\sigma_0}{g}\left(\frac{k}{2\pi R}\right)^{1/2} H(k, \theta) \exp i\left(\sigma_0 t - kR + \frac{\pi}{4}\right) + O\left(\frac{1}{R}\right)$$
$$\left(H(k, \theta) = \sum_{m=1}^{6} v_m H_m(k, \theta) + H_7(k, \theta)\right) \qquad [3.6]$$

Substituting $R = L/2$ and $\theta = 0$, we obtain the approximate value of $\zeta_0$ in the vicinity of the bow. For longitudinal oscillations of the ship and $\epsilon = \pi$, we have

$$\zeta_0 = r_0 \exp i\left(\sigma_0 t + \frac{kL}{2}\right) - \frac{i\sigma_0}{g}\left(\frac{k}{\pi L}\right)^{1/2}\left[\frac{\sigma_0}{\rho g} F_\zeta X_3(k) + \frac{\sigma_0}{\rho g} F_\psi X_5(k) - - iH_z(k, 0)\, e^{i\sigma_0 t}\right] \qquad [3.7]$$

where $X_3(k)$ and $X_5(k)$ are the exciting force and moment, and using the source approximations, the function $H_z(k, 0)$ is given by the formula

$$H_z(k, 0) = i\sigma_0 r_0 S_0 K_1(kL) \qquad [3.8]$$

For wave lengths of order $L$, the value of $K_1(kL)$ is very small and the hydrodynamic interactions in Equation [3.7], in the bow, are determined from the values of $X_3$ and $X_5$.

The relations in Equations [3.1] and [3.8] determine the approximate value of the transfer function for the wetting of ships in waves. For longitudinal oscillations of thin ships (and for $\epsilon = 0$ or $\epsilon = \pi$), the source method or the method of orthogonal functions permits the determination of more accurate formulas for these functions in the general case involving a moving ship and finite depth of fluid. With the help of these transfer functions, the statistical characteristics of the exciting forces can be obtained for ships in irregular waves, examined as a stationary random process[19] in a similar way as for calculating the characteristics of oscillating mechanical systems.

Along with the linear characteristics, we can also find the approximate value of the nonlinear characteristics of the wetting of ships in waves. Accounting for second-order quantities, the free-surface elevation is given by the formula[2]

$$\zeta = -\frac{1}{g}\left\{\frac{\partial_a \Phi (x, y, 0, t)}{\partial t} + \zeta_0 \frac{\partial_a^2 \zeta_0}{\partial t^2} + \frac{1}{2}\left[\left(\frac{\partial \Phi}{\partial x}\right)^2_{z=0} + \left(\frac{\partial \Phi}{\partial y}\right)^2_{z=0} + \left(\frac{\partial_a \zeta_0}{\partial t}\right)^2\right]\right\}$$

$$\left(\frac{\partial_a}{\partial t} = \frac{\partial}{\partial t} - u \frac{\partial}{\partial x}\right) \qquad [3.9]$$

where $\zeta_0$ is the elevation of the free surface as obtained from the linear theory.

Equation [3.9] permits us to express the average value of the expression for $\zeta$ in a period of oscillation by means of the value determined from the linear theory. From the asymptotic expression[7] valid for normal frequencies of motion of a thin ship, we obtain

$$\zeta^* = \zeta_1 + \zeta_2,$$

$$\zeta_1 = -\frac{\sigma_0 kr}{2g} \sin \epsilon (\sin \epsilon \mp |\sin \epsilon|)\ Re\ [iC^{\pm} \exp i\,(ky \sin \epsilon \mp k_2 y)]$$

$$\zeta_2 = \frac{\sigma}{2g}\ Re \left\{\left[r_0 \left(\nu + k - \frac{\sigma\sigma_0}{g}(1 \pm \sin \epsilon)\right) - \right.\right.$$

$$\left.\left. - i \frac{\sigma_0}{g} C^{\pm} \left(\nu + k - \frac{\sigma\sigma_0}{g}(1 + |\sin \epsilon|)\right)\right] B^{\pm} \exp \pm i\,(\nu y - k_2 y) - ikx \cos \epsilon\right\} \qquad [3.10]$$

$$\left(\zeta^* = \frac{1}{T}\int_t^{t+T} \zeta(t)\, dt,\ T = \frac{2\pi}{\sigma}\right)$$

For longitudinal oscillations of a thin ship (and for $\epsilon = 0$ or $\epsilon = \pi$) ($C^+ \approx 0$), we find

$$\zeta^*(x) = -\frac{1}{2} \nu r_0 \left(\nu + k - \frac{\sigma\sigma_0}{g}\right) Re\left[\frac{i}{\rho g}(F_\zeta - xF_\psi)\, X_3(\nu, x) \exp i\,(\sigma t - \gamma \xi (x) \pm kx)\right] \qquad [3.11]$$

In the three-dimensional case, on the basis of Equations [3.6] and [3.9] we have

$$\zeta^* = \frac{\sigma_0}{2g} kr_0 \left(\frac{k}{2\pi R}\right)^{1/2} [1 - \cos(\theta - \epsilon)]\ Re\ [H(k, \theta)\, e^{i\chi}]$$

$$\left(\chi = kr\,[\cos(\theta - \epsilon) - 1] + \frac{\pi}{4}\right) \qquad [3.12]$$

In a similar manner we can also find $\zeta^*$ in the other cases.

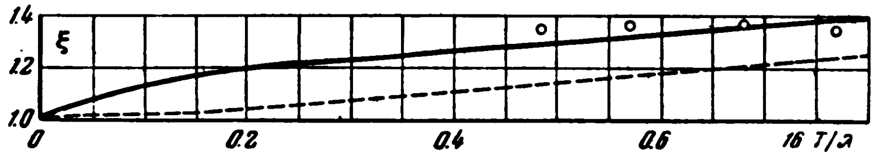


Figure 1

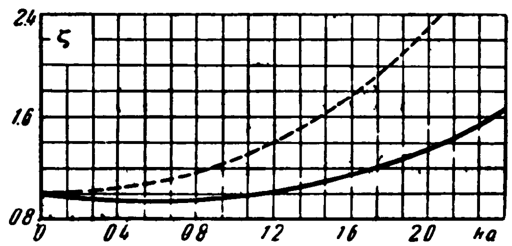


Figure 2

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General formulas are established for the exciting forces and moments acting on a ship in arbitrary waves. It is shown that for the diffraction of waves due to isolated singularities, such as sources, dipoles, pressure points, etc., the exciting forces and moments depend only on the ship's radiation functions, which characterize the wave radiation in a heavy fluid due to forced oscillations of the ship with unit velocity amplitude in calm water.

1. Ships--Motion--Exciting forces
2. Water waves--Diffraction--Mathematical analysis

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III. T: Wetting of ships in waves

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