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SPEED OF RISE OF AIR BUBBLES IN LIQUIDS

BY T. BRYN, OSLO

JUNE 1949

TRANSLATION 132

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SPEED OF RISE OF AIR BUBBLES IN LIQUIDS

[STEIGGESCHWINDIGKEIT VON LUFTBLASEN IN FLÜSSIGKEITEN]

by

T. Bryn, Oslo

(Forschung, Vol. 4, No. 1, Jan-Feb 1933)

Translated by F.A. Raven

Navy Department
David Taylor Model Basin
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NOTATION

Test Liquid

s	Specific weight, in grams per cubic centimeter
ν	Kinematic viscosity, in square centimeters per second
K	Capillarity constant, in milligrams per millimeter
$\sqrt{2K/s} = a$	Laplace's constant, in millimeters

Air Bubble

v	Volume, in cubic millimeters
$\sqrt[3]{6v/\pi} = d$	Ideal spherical diameter, in millimeters
D	Horizontal diameter, in millimeters
w	Speed of rise, in centimeters per second

SPEED OF RISE OF AIR BUBBLES IN LIQUIDS*

by

T. Bryn, Oslo

OBJECT OF THE INVESTIGATION

Allen (1),** Hoefer (2), and others have already measured the phenomenon of air bubble rising in liquids. Allen measured only very small bubbles whose diameters were less than 0.4 mm (0.01576 in.); Hoefer's measurements were restricted entirely to water, and consequently his experiments could supply no information respecting the effect of the properties of the liquid on the speed of rise of bubbles.

The object of this investigation was to determine the speed of an air bubble rising in a very extensive, calm liquid, as a function of the bubble size and properties of the liquid. Omitting detailed exposition of theoretical principles, it will merely be stated at this point that the kinematic viscosity ν and Laplace's constant can be assumed as the primarily decisive properties of the liquid.

EXPERIMENTS

The test liquids used were glycerin-water and alcohol-water mixtures.

The glycerin-water mixtures showed an almost constant value of a, whereas each pair of alcohol-water mixtures revealed the same value for ν; see Table 1.

TABLE 1

Number	Glycerin					Alcohol				Water
	1	2	3	4	5	1a	1b	2a	2b	
Approximate weight, percent	81.0	68.0	56.0	42.0	19.0	92.5	13.0	70.0	28.0	100.0
Mean temperature t_m, in ° C	17.0	18.0	18.5	18.5	18.5	19.5	17.5	20.0	20.5	18.0
Kinetic viscosity ν, in cm^2/s	0.64	0.19	0.08	0.039	0.018	0.019	0.019	0.027	0.027	0.01
Specific cohesion a^2, in mm^2	11.1	11.4	11.8	12.4	13.5	6.0	9.8	6.3	7.6	14.8

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*Published in Forschung, Vol. 4, No. 1, Jan-Feb 1933. It is a summary of the results of an experimental investigation performed in the machinery laboratory of the Technische Hochschule in Dresden.

**Numbers in parentheses indicate references at the end of this translation.

Three methods were used to produce bubbles of varying size. Very small bubbles whose diameters *d* were within the range of 0.5 to 3.5 mm (0.0197 to 0.1378 in.) were produced with glass capillaries which had been drawn to a fine tip. As stated by Hoefer (2), uniform bubble size was maintained by packing the capillaries with cotton almost to the orifices.

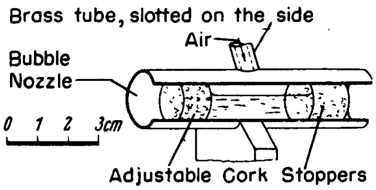

Figure 1 - Bubble Generator for Producing Bubbles of Medium Size

Larger bubbles whose diameters ranged from 2 to 20 mm (0.0787 to 0.7874 in.) were generated with a nozzle especially designed for the purpose; see Figure 1. Air was fed from above to a horizontal brass tube, provided with a wide slot and corked at the ends. The air escaped through the slot, in the form of bubbles whose size could be regulated by shifting the cork stoppers.

Very large bubbles, with diameters up to 120 mm (4.7244 in.), were generated with the aid of a "tilt-chamber" (Kippgefäsz); see Hoefer (2) and Figure 4.

To measure size, the bubbles were caught near the surface in a glass bell filled with the liquid. Buoyancy was determined with a balance. A rather large number of the very small bubbles were intercepted and counted. The size of the individual bubbles was found by taking the mean of the total buoyancy.

The speed of rise was measured either by timing with a stop watch for a distance of 1 m (39.37 in.) of rise, or photographically. The photographic method consisted in recording the bubble on a moving strip of bromide

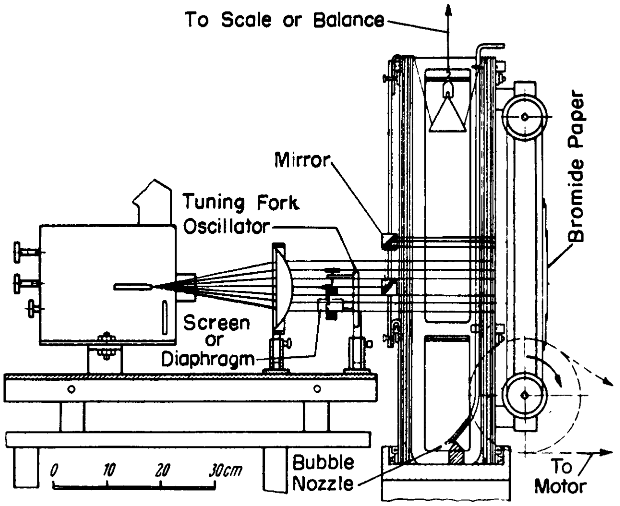

Figure 2 - Apparatus for Photographic Method of Measuring Size, Shape, and Speed of Rise of Large Air Bubbles

paper at two different heights and simultaneously recording time calibrations on the paper with a tuning fork.

Some of the measurements were made in a glass tube 140 cm (55.11 in.) long with an inside diameter of 70 mm (2.75 in.), and others in a tank fabricated of plane-parallel glass such as used for mirrors. This tank, which was square, measured 15 by 15 cm (5.90 by 5.90 in.). Figure 2 shows the test setup for measurements in this tank. The right side of the tank was coated with black paper wherein two slits were cut.

After the arc lamp was switched on and the buzzer was set to oscillate at the required frequency, the motor which drove the belt carrying the strip of bromide paper was started; but the belt was not allowed to move immediately. Next, a bubble was produced by the nozzle merely by opening the air-feed cock. While the bubble was rising, the belt was released. This permitted the bromide paper to pass the slits, where it was exposed twice. The bubble cast a shadow on the paper through each slit, producing an image on the paper. From the distance between these images and with the aid of the time calibration recorded simultaneously, the speed of rise of the bubble can be calculated. The time calibrations are produced by the oscillator as follows: The tuning-fork oscillator, which is provided with a slit diaphragm or screen, is synchronized so that only half the lower slit in the black paper is exposed or illuminated when the tines of the fork separate.

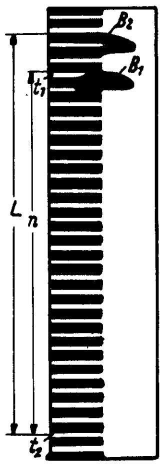

Figure 3 - Photograph of Rising Bubbles

B_1 and B_2 are images of a bubble behind slits 1 and 2 at the time calibrations t_1 and t_2. The transverse black stripes are time calibrations. The distance between two stripes corresponds to a time $T = 0.00986$ sec. The speed of rise $w = L/nT$ is found from the number of time calibrations n between B_1 and B_2 and the distance between images of the bubble L.

Figure 3 shows an exposed strip of bromide paper with two bubble images, B_1 and B_2. The images of bubbles thus obtained could also be used to determine bubble deformation numerically. Only very small bubbles are spherical; larger ones are flattened. Hence, their horizontal diameter D is greater than the diameter d of a sphere. The ratio D/d can be assumed as a scale of deformation.

The behavior of extremely large bubbles, i.e., where $v = 400$ cm^3 (24.41 cu in.), was studied in Lake Mylla, near Oslo, where these bubbles were measured also. The tilt-chamber **a**, Figure 4, used to produce large bubbles, was submerged to a depth of 10 m (32.80 ft); then air was pumped into it. The time of rise of the bubbles was determined by a telephone which responded to the contact on the tilt-chamber and a stop watch.

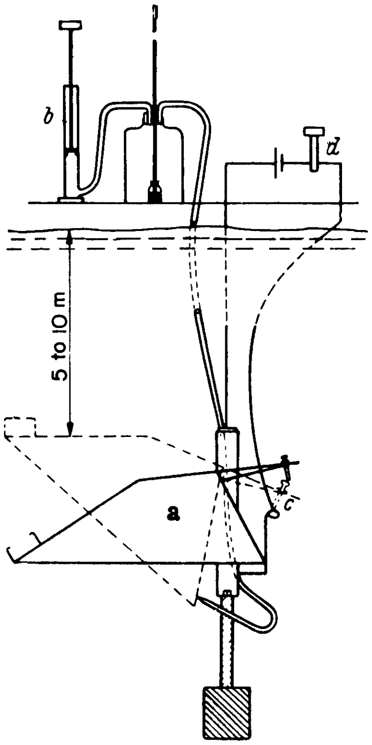

Figure 4 - Apparatus for Measuring the Speed of Rise of Very Large Bubbles, up to 400 cm^3 (24.41 in.), Used in Lake Mylla, near Oslo

a Tilt chamber, b Air pump, c Contact switch, d Telephone.

EXPERIMENTAL RESULTS AND EVALUATION

Classification of Bubbles

Tests showed that the movement of bubbles can be so various from a purely qualitative approach that it is advisable to classify the bubbles into several groups according to the type of movement. Therefore three classifications were established. Moreover, these categories considered both the shape of the bubble and the type of motion. The shape of the bubble is determined primarily by the ratio d/a, whereas the type of movement is determined by the ratio d/ν.

1. The "small bubbles" are approximately spherical and rise on a straight line. For water at room temperature, this group will include all bubbles whose diameter d is less than 1.5 mm (0.0591 in.).

2. The "medium-sized bubbles" are more or less flattened and rise with rocking, pendulum-like, oscillating, rotary, or spiral movements. (Water: 1.5 mm $< d <$ 8 mm [0.0591 in. $< d <$ 0.3150 in.].)

3. The "large bubbles" are very greatly deformed, assuming a mushroom-like shape. They rise relatively straight but are quite unstable and easily break down into numerous similar bubbles. (Water: 8 mm $< d <$ 120 mm [0.3150 in. $< d <$ 4.7244 in.].)

Graphic Analysis

The measurements were analyzed by plotting the speed of rise w, in centimeters per second, as the ordinate against diameter d, in millimeters, as the abscissa for each test liquid, and drawing a smooth curve through the plotted points. Figure 5 shows two curves for 42 per cent glycerin in water by weight and for water at 18° C (64.4° F). These curves show how greatly the behavior of the large bubbles differs from that of the small ones.

Large bubbles. For $(d/a) < 2$ the velocity curves can be represented very well by straight lines. Moreover, it was shown that the speed of rise is determined conclusively by the size of the bubbles and by Laplace's

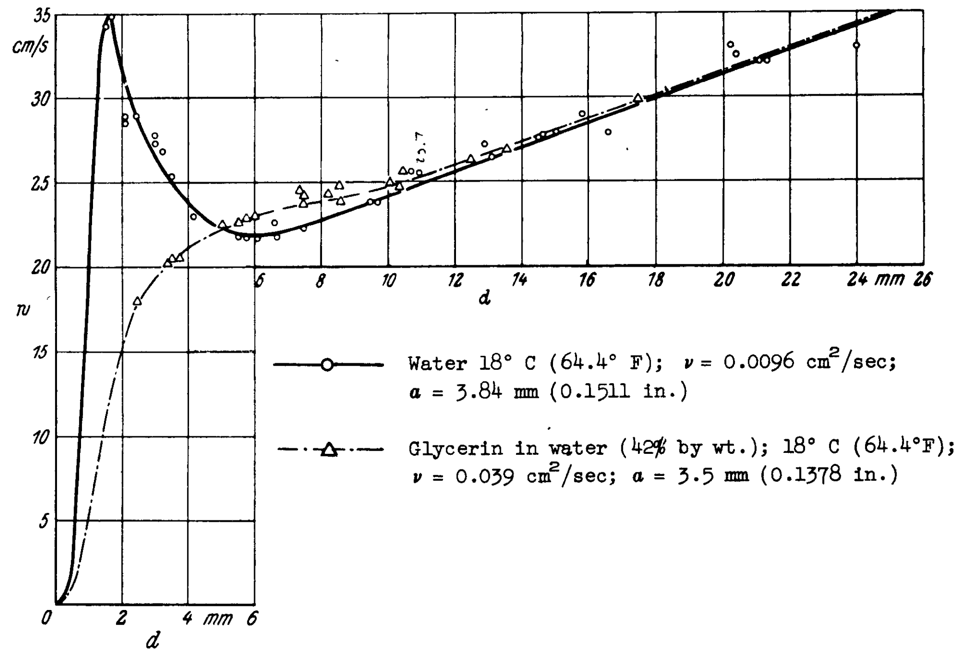

Figure 5 - Speed of Rise of Air Bubbles *w* as a Function of Their Ideal Spherical Diameter *d* and of the Viscosity of the Liquid

constant and that it is entirely independent of the viscosity of the liquid. The empirical formula

$$w = A\sqrt{a} + B\frac{d}{\sqrt{a}}$$

could be established where a and d are measured in millimeters and w is given in centimeters per second. A and B are constants whose values were found to be 9.26 and 1.33 respectively.

The range of validity of this formula extends through $2 < (d/a) < 20$ and Reynolds numbers $\mathrm{Re} > 60$.

Small bubbles. According to Allen, the same law of resistance holds for small bubbles as for solid spheres. By introducing a nondimensional coefficient of resistance c = resistance to flow/(hydrostatic pressure times cross-sectional area) and by introducing Reynolds number $\mathrm{Re} = wd/\nu$, the law of resistance for spheres can be represented by a single curve in the c, Re diagram.

To carry out the comparison with spheres for small bubbles, the velocity curves for small bubbles were converted and plotted in a log c, log Re diagram.

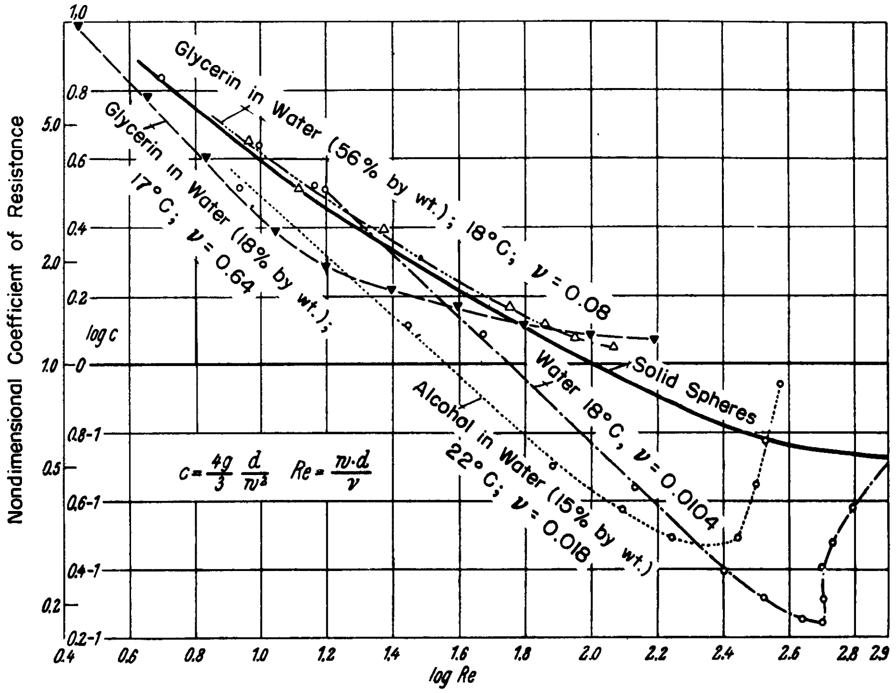

Figure 6 - Nondimensional Representation of the Resistance to Rise of Air Bubbles in Liquids of Varying Viscosity

The resistance of solid spheres is also plotted for comparison.

Figure 6 shows such a diagram with four curves according to measurements in four different test liquids. For comparison, the resistance curve for solid spheres according to measurements in Göttingen (3) is included.

It is clearly evident from Figure 6 that air bubbles cannot be considered as solid spheres even if they are so small and rigid that they are spherical. Only very small bubbles or small bubbles in very viscous liquids behave hydrodynamically as solid spheres.

It is surprising that the resistance of a bubble in water, for example, can be very much less than the resistance of a solid sphere of equal volume. It is just as astonishing, however, that this is true for only a few liquids such as alcohol-water mixtures, whereas it is not true for other mixtures such as 56 per cent glycerin by weight in water.

The reason for this difference must be sought in the behavior of the boundary layer. It is probable that the boundary layer around the bubble is partially replaced while the bubble is rising (4). If this replacement occurs very rapidly, the bubble "glides," so to speak, through the liquid, whereas if this replacement is slower the bubble resembles a solid sphere because it entrains liquid in the vicinity of its surface.

Space does not permit more detailed treatment of this interesting problem. It will suffice merely to state here that in 1926 Kleinmann (5) and later others succeeded in determining by measurement that a completely replaced surface possesses a much greater surface tension than an old one. From the foregoing it follows that the replacement of surface requires a greater expenditure of energy at the top of the bubble than is regained at the bottom. Accordingly, the relaxation time of the surface tension is an important property of the liquid affecting the movement of the bubble, but a property of which little is known at present.

The group comprising medium-sized bubbles is very difficult to treat theoretically because the shape of the bubbles varies greatly and because the movement of the bubbles is affected both by gliding or shifting of the boundary layer and by turbulence.

SUMMARY

The speed of rise of air bubbles of different volumes was measured in liquids of varying viscosity. Bubbles of various shapes were photographed. Differentiation into "small," "medium-sized," and "large" bubbles can be made on the basis of the behavior of the bubbles. The "small" bubbles are spherical, the "large" ones are mushroom-shaped. The shape of the "medium-sized" bubbles varies greatly. The speed of rise of "large" bubbles is determined conclusively by their size and Laplaces' constant, whereas the viscosity of the liquid has no effect. In spite of their spherical shape, "small" bubbles do not behave as solid spheres. Their resistance to flow and therefore their speed of rise depends largely on the manner in which replacement of their boundary layers occurs.

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(2) VDI-Forschungsheft 138, Berlin 1913; article by K. Hoefer. [Compare "Das Aufsteigen von Gasblasen in Flüssigkeiten" (Rise of Gas Bubbles in Liquids), by F. Pickert, in Forschung auf dem Gebiete des Ingenieurwesens (B), Vol. 3, Nov-Dec 1932, pp. 308-309, Translation 146. Reference copy available only in TMB Library.

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