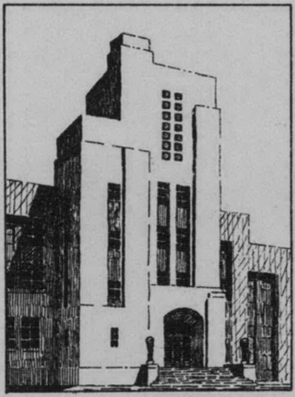
THE DAVID W. TAYLOR MODEL BASIN

UNITED STATES NAVY

NATURAL VIBRATION AND DAMPING OF GAS BUBBLES IN LIQUIDS

BY ERWIN MEYER AND KONRAD TAMM



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NATURAL VIBRATION AND DAMPING OF GAS BUBBLES IN LIQUIDS

(EIGENSCHWINGUNG UND DÄMPFUNG VON GASBLASEN IN FLÜSSIGKEITEN)

by

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ABSTRACT

The natural frequencies and damping decrements of gas bubbles in liquids, especially in water and glycerin, have been measured in the frequency ranges of 1.5 to 7 kilocycles and from 15 to 35 kilocycles. The results agree with calculations by Minnaert and Smith with respect to the resonance frequency f_0. This is found by the expression $f_0 = \frac{0.66}{d}$ kilocycles, where d is the diameter of the bubble in centimeters.

The decrement is composed of two parts, the constant component of radiation damping and a component proportionate to the frequency. At 25 kilocycles the decrement amounts to about 0.9.

The measuring methods were as follows: In the range of 15 to 35 kilocycles the release of bubbles electrolytically produced on an electrode under conditions of sound transmission was observed, and the speed along the surface of the bubble as a function of the diameter was measured with a type of ribbon microphone. In the frequency range of 1.5 to 7 kilocycles the amplitude of the air bubbles as a function of the frequency was photoelectrically determined. Finally, the sonic spectra during the rise of air bubbles from a nozzle were recorded with a supersonic analyzer.

INTRODUCTION

It is known that liquids containing a gas possess higher sound damping characteristics than those not containing gas. Sörensen (1)* made measurements on stationary waves in the supersonic range from 200 to 1000 kilocycles and showed that at 200 kilocycles tap water immediately after being drawn possessed a damping factor 1.8 times as high as water that had been allowed to stand, and 2.4 times as high as boiled water, i.e., water not containing air. This fact can easily be proved by attempting to produce standing sound waves in pipes filled with water. Fresh tap water or still water containing air bubbles along the walls of the pipe, especially water having a high content of carbon dioxide such as soda water, produces no waves or only weak standing waves. Sound absorption occurs when the gas dissolved in the liquid is released in the form of bubbles. As the size of the bubbles increases the audio-frequency range is reached.

An obvious explanation of this phenomenon is that gas bubbles in a liquid are mechanical systems capable of vibration with attendant damping.

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* Numbers in parentheses indicate references on page 12 of this translation.

Minnaert (2) indicates that gas bubbles in liquid have properties of resonance which are determined by the elasticity of the gas bubble and by the mass of the entrained liquid. He calculated the position of the point of resonance, and by subjectively listening to the pitch of gas bubbles which were released into a liquid through a jet he determined agreement with the formula derived. In this way he verified that air bubbles actually represent a system capable of vibration, and by his experiments he further proved that they do not behave as cavity resonators. F.D. Smith (3) likewise calculated the resonance frequency and introduced a damping factor. For this he assumes only radiation damping but he indicates other causes of damping. In the course of his calculation Smith points out the possibility of explaining the destruction of living organisms by supersonic vibrations. He shows, in fact, that gas bubbles which are released by the effect of supersonic vibrations and whose size remains below that at resonance can exert considerable tangential stresses. According to Smith these stresses are about 15,000 times greater than the corresponding stresses in the undisturbed sonic field.

The purpose of this study was to trace quantitatively the natural vibration of gas bubbles in liquids over a wide range of frequency, i.e., from 1.5 to 35 kilocycles. The natural frequency and decrement were measured.

THE CALCULATION OF VIBRATIONS OF GAS BUBBLES

Before the experiments are described, the theory will be briefly examined. The value of the natural frequency of a gas bubble is $\omega_0 = \sqrt{\frac{1}{MF}}$. Since the bubble represents a radiating agent of zero order, the total entrained mass M of the medium is $\rho r S$, when the circumference $2\pi r$ of the spherical radiating agent with a surface S is small compared to the wave length λ, and ρ denotes the density of the liquid. The elasticity of the gas bubble is given by the expression $F = \frac{1}{S} \frac{r}{3P\kappa}$, where P is the hydrostatic pressure and κ the ratio of the specific heats of the gases. The effect of the surface tension in the interface of air-water can be neglected in the range of frequencies investigated. If the atmospheric pressure (1.02×10^6 dyne/cm^2) is substituted for P and d is the diameter of the bubble in centimeters, the natural frequency is

$$\omega_0 = \frac{1}{r}\sqrt{\frac{3\kappa P}{\rho}}$$

The equation for the resonance frequency f_0 in kilocycles when $\kappa = 1.41$ and $\rho = 1$, as in water, is

$$f_0 d = 0.657 \text{ kc} \times \text{cm}$$

Hence the product of the diameter and the resonance frequency is constant.

If the resistance to radiation $R = S\rho c \frac{4\pi^2}{\lambda^2} r^2$ only is used as a basis for damping, then the decrement is expressed by

$$\theta_0 = \frac{\pi \rho_0 r}{c}$$

where c denotes the velocity of sound in the fluid. Therefore, for bubbles in resonance the decrement becomes independent of the frequency. For water, $\theta_0 = 0.045$. A super-resonance of 80 corresponds to this; in other words it is a very undamped vibration. Other damping factors are disregarded, such as losses caused by thermo-exchange by the air and water, as well as energy losses resulting from the viscosity of the medium.

The experiments covered two frequency ranges, one from 15 to 35 kilocycles and the other from 1.5 to 7 kilocycles. A different method was used in each range to determine clearly the coefficient of actual vibration. In addition, a second method was developed for each range to measure both the natural frequency and the decrement.

MEASUREMENTS IN THE SUPERSONIC RANGE, 15 TO 35 KILOCYCLES

The first measurement of the resonance of gas bubbles in water was based on the following observation, which was made without a knowledge of Minnaert's theory. Gas bubbles were produced by electrolysis, in a small glass vessel in which weak supersonic waves of 15 to 35 kilocycles were generated by a magneto-striction oscillator. The gas bubbles (oxygen or hydrogen, depending upon the polarity) were produced along a short horizontal platinum wire fused into a thin glass tube. Directly after the point where the wire was fused into the tube the latter bent directly upward. The gas bubbles formed collected in the bend to form a continuously growing bubble which became detached after a short time. Under microscopic observation it became apparent that at the time the bubbles detached, their size was always the same at identical sonic frequency. As the frequency increased, the critical diameter of the bubble decreased. The results of this first observation on oxygen and hydrogen are tabulated in Table 1. The controlled frequency f, the measured diameter of the bubble d_0, and their product fd_0 are included.

The product fd_0 is not a function of the frequency and very closely approaches the calculated value of 0.66. The mean value of all observations for hydrogen is practically equal to that for oxygen, as should be expected from theoretical consideration. Explanation of the observed phenomena is simple. The gas bubble produced in the bend of the glass tube and which continuously increases in size begins to vibrate strongly as soon as it reaches the resonance diameter, and thus becomes detached from its base. For an exact measurement of the resonance diameter the sonic intensity must be fixed as

TABLE 1

Resonance Diameter of Hydrogen and Oxygen Bubbles in Water at 15 to 35 kilocycles

	Hydrogen (Mean of 4 values)		Oxygen (Mean of 2 values)	
f kc	d_0 cm	$f \times d_0$ kc × cm	d_0 cm	$f \times d_0$ kc × cm
16	0.0403	0.645	0.042	0.67
19	0.0325	0.62	0.0345	0.655
23	0.0265	0.61	0.0275	0.63
27	0.0235	0.635	0.0245	0.665
32	0.020	0.64		
35	0.0185	0.65	0.0190	0.665

small as possible. Otherwise the vibrating forces destroy the adhesive forces between the gas bubble and the glass surface at rather small diameters.

Using an improved test setup, the second experiment in the supersonic range was designed to permit a quantitative determination of the resonance curve. The concept involved was as follows: The bubble which is to be measured is brought in contact with a microphone, and its reaction, which is a function of the frequency, is determined. It is immediately obvious that the microphone area must be as small as possible. A small disk-shaped Rochelle salt microphone with an area of 1 cm^2, however, recorded no measurable difference with or without air bubbles. If the bubble was clinging to a thin rod which was connected to a microphone located outside of the liquid no effect occurred, even though the rod was protected against sonic effects by a layer of air and a shielding tube up to its point. Finally, a microphone was constructed which showed not the pressure but the speed on the surface of the gas bubble. Assuming that the air bubble is not located in the compression node of a standing wave, the speed on the surface of the air bubble is great whereas the speed in the liquid is small. Hence an effect was to be expected with this equipment.

A type of ribbon microphone as shown in Figure 1 was used as a speed receiver. This instrument consisted of a platinum wire 1 cm long and 15 μ* in diameter in a magnetic field. A glass tube was used as a vessel to

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* Translator's Note: Micron, a unit of length equal to 0.001 mm (0.0000394 inch). In physical chemistry it is used to denote the size of particles whose diameter lies between 0.01 and 0.0001 mm (0.000394 to 0.00000394 inch).

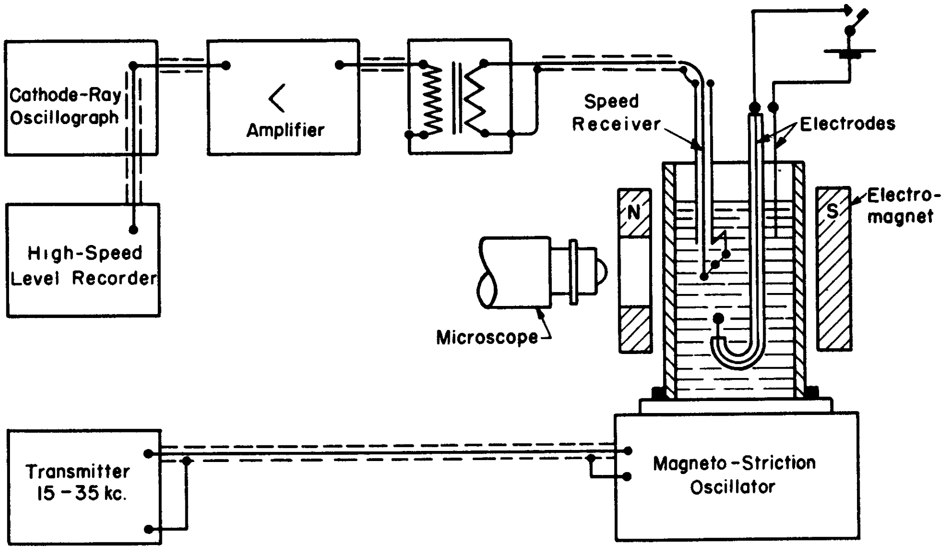

Figure 1 - Arrangement to Record Velocity on the Surface of a Gas Bubble as the Diameter Increases

contain the liquid. As a transmitting instrument a magneto-striction oscillator was again used. The EMF produced by the microphone was amplified and recorded with Neumann's high-speed level recorder.

The bubbles were generated by electrolysis on an electrode at a short distance beneath the receiver. As they rose they were trapped by the wire. This was fitted with a small plate of mica during the first experiments and later with a small sphere of sealing wax. The diameter of the bubbles was determined with a microscope.

Recording the resonance curves of the bubbles as a function of the frequency with this apparatus was unsuccessful, because the sonic pressure showed exceptionally strong fluctuations with the frequency as a result of the natural frequencies of the transmitter, of the tube, and of the column of water. The method was only sufficiently inclusive to permit finding the points of resonance of the bubbles themselves. This phenomenon could, moreover, be directly recognized by the fact that the surface of the bubble, which was otherwise shiny and able to reflect, appeared dull and blurred as a result of the strong vibration at the resonating point. Hence to determine the decrement it remained necessary only to keep the frequency constant and to permit the gas bubbles to grow slowly. Figure 2 shows a photographic record of this feature. The scale is interpolated according to the diameters marked. It is clearly evident that the voltage produced by the small ribbon microphone traverses a maximum as the diameter of the bubble increases.

A series of photographs of this type could be made only as far as the point of resonance itself, since many bubbles did not increase beyond the

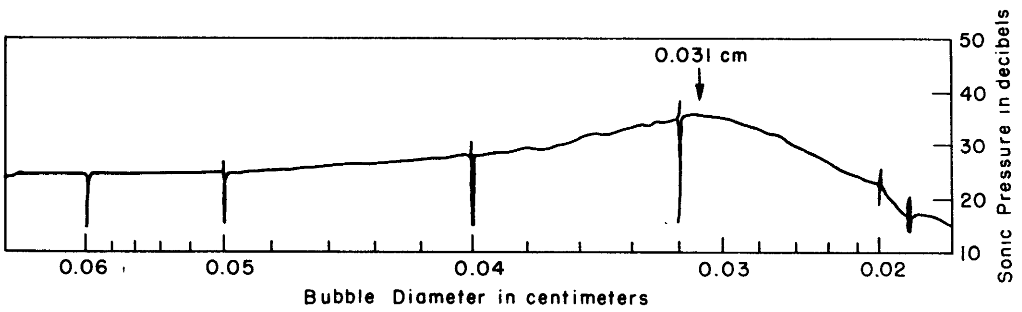

Figure 2 - Curve of Velocity at Increasing Bubble Diameters
f = 21 kc $d_{0\,\text{meas.}}$ = 0.031 cm fd_0 = 0.69 kc × cm

resonance diameter. The sonic intensity in these experiments had to be kept small also, to prevent breaking away of the bubble.

Table 2 contains a few of the values measured. The controlled frequency f, the resonance diameter d_0, their product fd_0, and the decrement θ are included. It is evident that, disregarding two cases where observation was uncertain, the mean value of the product closely approaches the theoretical value. The decrement is considerably larger than the damping radiation alone would demand. It increases with the frequency and amounts to about 0.9 at 25 kilocycles. Owing to the blurred aspect of the bubbles in the neighborhood of resonance, as previously mentioned, and to the smallness of the movement of the bubble as a whole, it was possible to measure the exact diameter only with difficulty. Therefore, a certain element of uncertainty or error is present in the measurements of the decrement.

TABLE 2

Resonance Diameter and Damping Decrement of Gas Bubbles in Water at 15 to 35 kilocycles

f kc	d_0 cm	$f \times d_0$ kc × cm	θ
21	0.031	0.65	0.69*
	0.032	0.68	0.72
22	0.029	0.64	0.82
	0.0263	0.58	0.88
	0.0307	0.675	0.88
25	0.027	0.665	0.82*
27	0.0225	0.61	1.04
30	0.0176	0.53	0.85

* Both sides of the resonance curve are present.

The wire upon which the air bubble clings has a negligible effect on the natural frequency. The entrained mass of the medium is about three times as great as the total mass of the platinum wire for the smallest bubble. Hence, it is certainly great compared to the vibrating mass of wire.

MEASUREMENT OF GAS BUBBLES IN THE FREQUENCY RANGE OF 1.5 TO 7 KILOCYCLES

The measuring method just described is not applicable at lower frequencies because, as a result of their buoyancy, the larger bubbles do not cling to the thin wire of the microphone. Therefore, it is easier to measure the amplitude of the bubble surface directly.* For this purpose a photoelectric method was selected. The illuminated gas bubble is projected on the sensitive layer of a gas-filled photoelectric cell by a lens. In the electric circuit of the photoelectric cell an alternating current is generated which is proportional to the change of the cross section of the bubble. This alternating current is recorded with Neumann's high-speed level recorder. The air bubbles are produced by a nozzle and collected on the point of a thin needle, the point of which is coated with a little sealing wax to hold them. Smaller bubbles are almost spherical, larger ones are somewhat distorted by their buovancy. Very large air bubbles of diameters of about 5 mm must be held in a thin wire annulus. A cylindrical glass tube of 5-cm diameter and 10.5-cm length was used as a container. It was filled to a depth of 7.5 cm. The sonic excitation was produced by an electromagnetic telephone which sealed the glass tube at its base. The natural damping factor of the telephone diaphragm is large. This measuring device was chosen after numerous preliminary tests, because the sonic pressure showed the least fluctuation with frequency.

The sonic pressure at the point where the air bubble was located was measured with a small quartz microphone tuned to about 300 kilocycles. Figure 3 shows such a pressure measurement. The point of high resonance at 700 cycles is caused by the diaphragm or the tube. Figure 4 represents a frequency curve for air bubbles of diameter d = 0.395 cm, on a logarithmic scale. In analysis of the curve, the location of the maximum gives the natural frequency and the amplitude at half-width gives the damping. The product of bubble diameter times natural frequency is 0.65; the decrement θ is 0.10. Numerous records were obtained in the same way as Figure 4. Moreover, a few

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* According to A. Schoch and E. Ganitta, the volumetric change of the air bubbles at strong sonic excitation, in low frequencies below resonance, can be shown in an admirable demonstrative experiment. Stroboscopic illumination is used for this purpose. If the experiment is made in standing waves, the air bubbles at the pressure node move to and fro and at the pressure loop there is a pulsating movement.

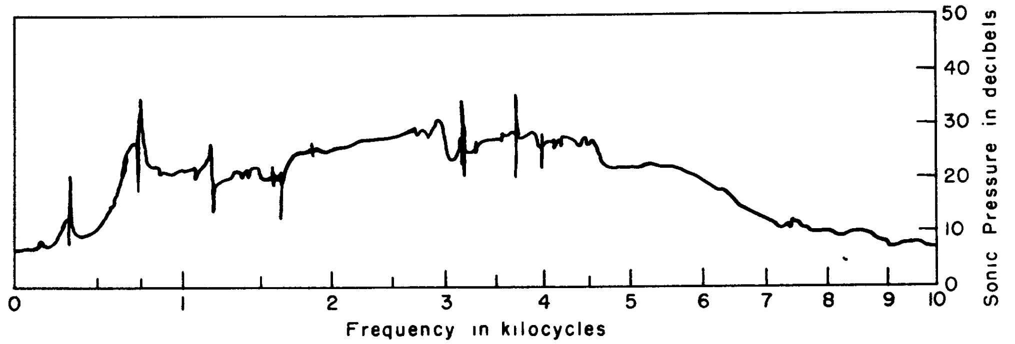

Figure 3 - Pressure Curve in the Liquid as Frequency Decreases

The microphone was located 2.3 cm above the telephone diaphragm in the vicinity of the needle point holding the air bubble.

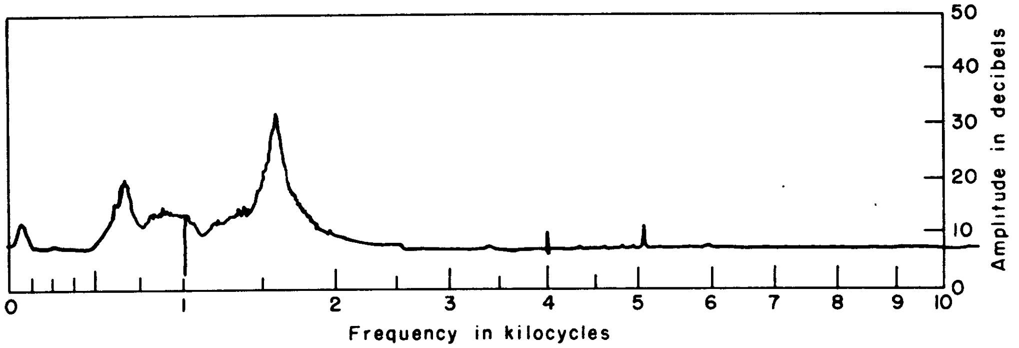

Figure 4 - Amplitude on the Surface of an Air Bubble at Decreasing Frequency

$f_{0\ \text{meas.}} = 1.65$ kc $\quad d = 0.395$ cm $\quad f_0 d = 0.65$ kc × cm

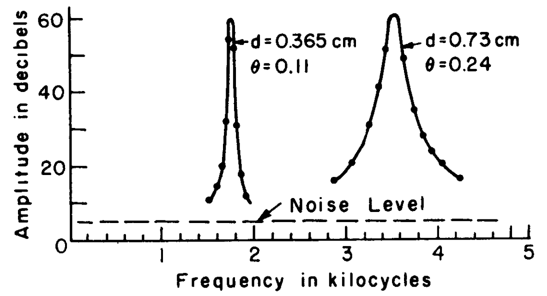

Figure 5 - Resonance Curves for Two Air Bubbles in Water on a Linear Scale

$d = 0.365$ cm $\quad \theta = 0.11$

$d = 0.173$ cm $\quad \theta = 0.24$

measurements were made with a line-recording instrument, of which Figure 5 contains two examples. The diameters of the air bubbles corresponding to the curves are 0.365 cm and 0.173 cm. The values of the decrements are 0.11 and 0.24. These two records clearly show the rise of damping as the frequency increases, corresponding to the data in Table 3, where all the measurements in the low frequency range are tabulated. The natural frequency agrees well with the theoretical value; the mean of the product of the frequency times the diameter, $f_0 d$, is 0.63. The decrement rises from 0.11 at 1.6 kilocycles to 0.27 at 6.5 kilocycles.

TABLE 3

Resonance Frequency and Damping Decrement of Air Bubbles in Water at 1.5 to 7 kilocycles

d cm	f_0 kc	$f_0 \times d$ kc × cm	θ
0.097	6.5	0.63	0.265
0.097	6.5	0.63	0.214
0.118	5.3	0.63	0.25
0.128	5.0	0.64	0.20
0.133	4.8	0.64	0.213
0.145	4.15	0.60	0.25
0.15	4.0	0.60	0.186
0.155	4.0	0.62	0.164
0.170	3.75	0.64	0.198
0.173	3.8	0.66	0.24
0.175	3.6	0.63	0.22
0.185	3.65	0.675	0.17
0.19	3.35	0.64	0.173
0.24	2.5	0.60	0.183
0.24	2.5	0.60	0.175
0.33	2.0	0.66	0.18
0.365	1.75	0.64	0.113
0.365	1.75	0.64	0.114
0.395	1.65	0.65	0.10
Calculated		0.66	$\theta_0 = 0.045$

To study the effect of the viscosity of the liquid, some measurements were made in a 43 per cent glycerin-water mixture and in pure glycerin. Although the viscosity of these liquids is about 400 or 500 times that of water the measured decrement was only doubled, as shown in Table 4.

The small increase of the decrement becomes comprehensible if it is considered that neighboring particles perform identical movements in a pulsating vibration. Therefore, no shear friction forces arise as, for example, in the experiment with a falling sphere for the determination of viscosity. The viscosity can exert an effect here only in connection with the volumetric change. This type of viscosity obviously does not vary greatly in water and in glycerin. A corroborating fact is that the standing sonic waves can be observed in glycerin or oil not containing gas, as well as in water. The

TABLE 4

Resonance Frequency and Damping Decrement of Air Bubbles in Glycerin at 1.5 to 7 kilocycles

d cm	f_0 kc	$f_0 \times d$ kc × cm	θ	Glycerin
0.315	2.0	0.63	0.225	43 per cent
Calculated		0.63		ρ = 1.11
0.107	4.7	0.50	0.375	100 per cent
0.122	4.6	0.56	0.36	ρ = 1.25
0.155	3.8	0.56	0.332	
0.293	2.03	0.595	0.263	
0.295	2.07	0.61	0.25	
Calculated		0.59	θ_0 = 0.03	

product $f_0 d$ for glycerin also coincides to a good degree with the theoretical value 0.59.

The decrement as a function of the frequency for all the measurements previously discussed is summarized in Figure 6. The curve can be extrapolated to very low frequencies and the calculated value of the damping radiation, calculated on page 2, can be obtained to a close degree of approximation. This value, θ_0, is only 0.03 for glycerin because of the greater sonic velocity in that medium. It may be observed that the damping of the gas bubbles apparently consists of two components, a somewhat constant component which can be traced to the radiation damping (in water $\theta_0 = 0.045$) and a component proportionate to the frequency (in water $\theta_1 = 0.035 \times f$). By experiments performed up to the present it cannot be determined whether the second component is to be traced to the viscosity of the medium or whether thermal loss inside the air bubble is a decisive factor. Experiments with gases having different capacities of thermal conductivity might permit this problem to be solved.

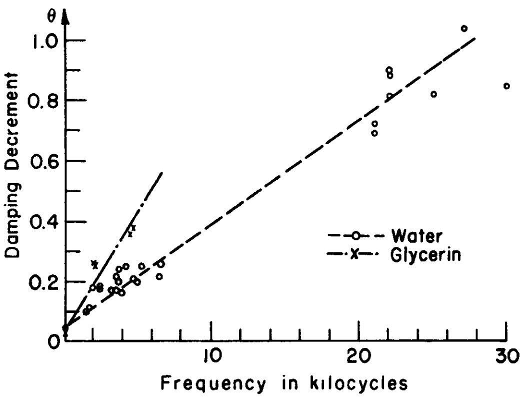

Figure 6 - Frequency as a Function of Damping Decrement of Gas Bubbles in Water and in Glycerin

In conclusion, experiments similar to those of Minnaert were made to determine directly the pitch of air bubbles as they arose from a nozzle or jet. This problem is

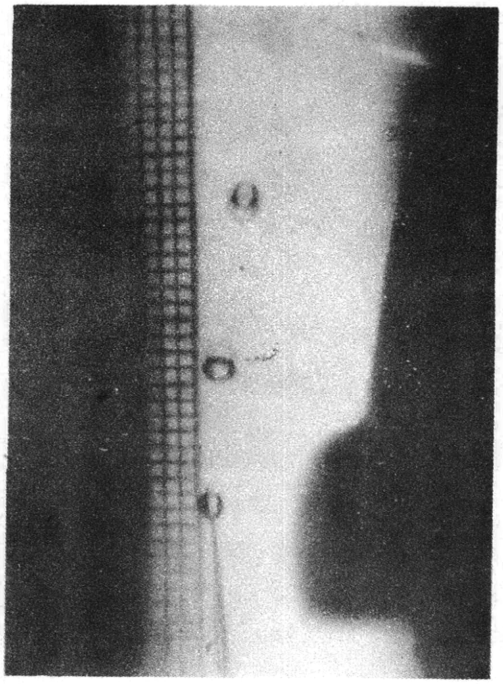

Figure 7 - Air Bubbles Rising from a Nozzle in Stroboscopic Illumination
d = 0.195 cm $f_{0\,\text{cal.}}$ = 3.4 kc
(see Figure 8c)

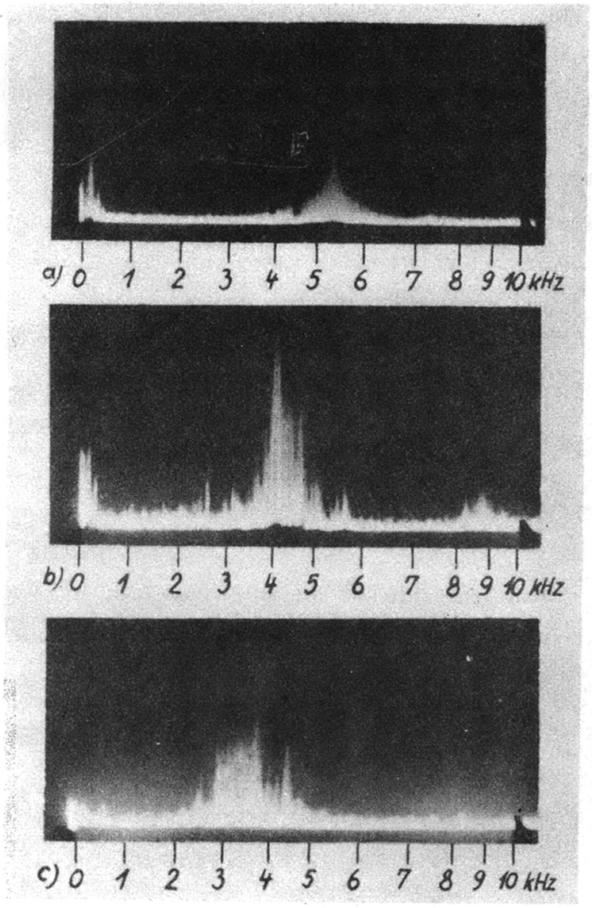

Figure 8 - Sound Spectra of Air Bubbles Rising from a Nozzle
(a) d = 0.12 cm $f_{0\,\text{cal.}}$ = 5.5 kc
(b) 0.165 cm 4.05 kc
(c) 0.195 cm 3.4 kc

significant from the standpoint of measuring technique, for frequently in laboratories where sound in water is studied, a jet of air blown into the water is often used as a source of sound for calibrating microphones, and other purposes. By the use of a suitably designed nozzle and a constant air pressure the bubbles are discharged from the nozzle at a quite constant rate. Hence with stroboscopic illumination their shape and diameter can be observed or photographed.

For this experiment air bubbles were produced in a glass vessel; wall vibrations were damped. A small quartz crystal microphone received the sound. Its initial voltage was amplified and transmitted to a high-pitch analyzer.* Since the size of the bubble changes continuously within certain limits, it must be expected that the sound spectrum does not contain a single resonance frequency but a rather broad band of them.

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* The high-pitch analyzer developed by G. Buchmann and G. Kossatz contains a mechanical band filter whose width is about 40 cycles at a mean frequency of 20 kilocycles. The spectrum is made visible on the screen of a cathode-ray tube having an extended afterglow (Nachleuchtzeit).

Figure 7 is a photograph of a few air bubbles, ellipsoidal in form as a result of their ascension. Figures 8a, 8b, and 8c are spectrographs recorded for bubbles of differing size whose diameters were 0.12, 0.165, and 0.195 cm. The resonance frequencies calculated at 5.5, 4.05, and 3.4 kilocycles lie in the strongly emphasized band of the spectrum. The various bubble sizes were produced by changing the air pressure and the shape of the nozzle. If various nozzles are united, a practically complete and continuous sonic spectrum can be produced.

The natural vibrations of air bubbles apparently play a part in the noises which are produced by drops of water with entrained air falling into a liquid and in the noises of effervescing liquids such as carbonated water.

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