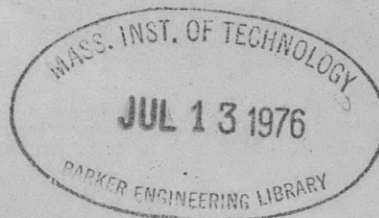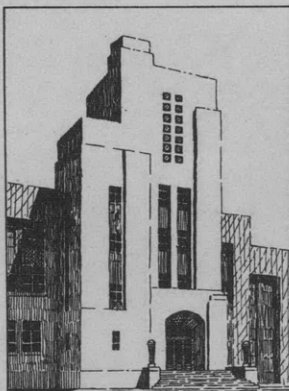# THE DAVID W. TAYLOR MODEL BASIN

UNITED STATES NAVY

THE DETERMINATION OF PRESSURE IN ATMOSPHERIC SHOCK WAVES DUE TO BLASTING AND THE FIRING OF GUNS
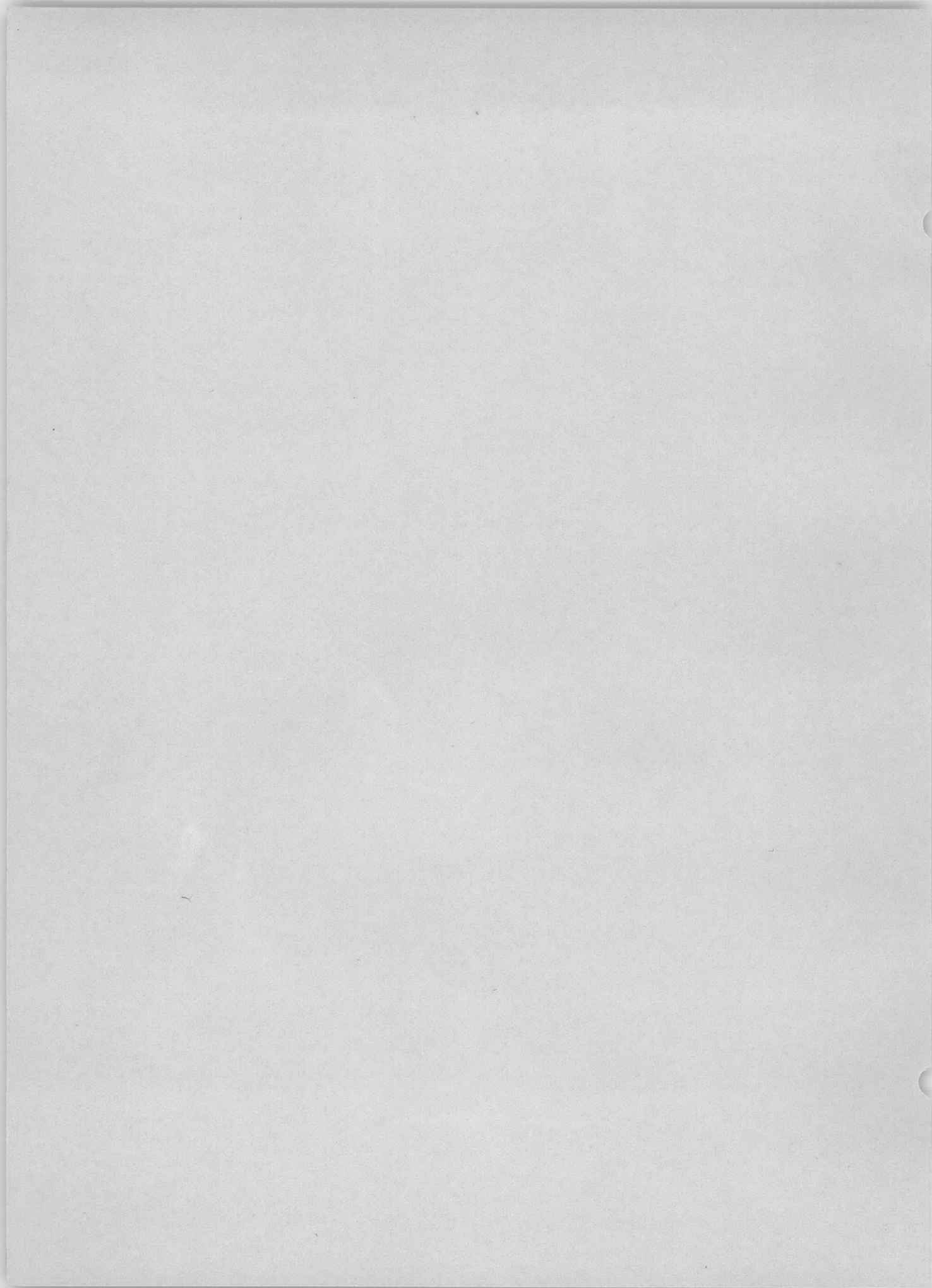
BY WILHELM SCHNEIDER

APRIL 1943

RESTRICTED

TRANSLATION 102

THE DETERMINATION OF PRESSURE IN ATMOSPHERIC SHOCK WAVES
DUE TO BLASTING AND THE FIRING OF GUNS

(UNTERSUCHUNGEN ZUR BESTIMMUNG DES DRUCKES IN DEN BEIM SPRENGEN
UND SCHIESZEN ENTSTEHENDEN LUFTSTOSZWELLEN)

by

Wilhelm Schneider

Translated by F.A. Raven

The David W. Taylor Model Basin
Bureau of Ships
Navy Department, Washington, D.C.

April 1943

Translation 102

The atmospheric shock waves caused by blasting and the firing of guns are strongly damped in comparison to the sound waves generally studied in acoustics. For this reason the mass of information which has been experimentally obtained on the subject of undamped sound waves cannot be used to solve the problems encountered in the study of atmospheric shock waves. Particular attention must therefore be paid to the transient stage of the behavior of vibratory systems under the influence of atmospheric shock waves, and the studies must be extended to analytical expressions which correspond approximately to the course of the atmospheric shock waves.

Rapidly converging analytical formulas most suitably answer this purpose. Studies of this particular type are not to be found in the literature. They are needed, however, to clarify the behavior of atmospheric shock waves.

A study was made of the conditions necessary to obtain accurate pressure-time records of air blast pressures by the use of diaphragm-type pressure gages. The paper is divided into two parts, theoretical and practical.

In the first part the diaphragm considered is idealized to the extent that it is assumed to be a rigid piston supported by a spring so that it can be treated as a simple system having one degree of freedom. The basis of the theoretical part of the paper is the well-known equation

$$m \frac{d^2x}{dt^2} + 2\delta \frac{dx}{dt} + cx = p(t)$$

In this equation x is the motion or deflection of the diaphragm, which is the indication of output of the instrument, and $p(t)$ is the blast force acting on the diaphragm, which is the input to the instrument. In an ideal instrument the following relationship would hold

$$cx = p(t)$$

and static calibration would suffice for correct interpretation of transient records. The existence of the inertia force $m\frac{d^2x}{dt^2}$ and the damping force $2\delta \frac{dx}{dt}$ spoil this ideally desired condition. This equation is put into a simpler form which uses a dimensionless time τ, by defining the following quantities

$$\omega_0 = \sqrt{\frac{c}{m}}; \quad \gamma = \frac{\omega}{\omega_0}; \quad \tau = \omega_0 t; \quad \alpha = \frac{\delta}{m\omega_0}$$

The general equation then becomes

$$\frac{d^2x}{d\tau^2} + 2\alpha \frac{dx}{d\tau} + x = p(\tau)$$

In the absence of a definite, complete physical theory of blast pressure waves $p(\tau)$ is unknown. The general features of a blast wave are known to be: 1. the pressure rises to a maximum value very rapidly, 2. the pressure dies down from the

maximum more slowly, 3. there is at least one negative swing of pressure after the positive phase is over.

From among the large number of functions which include the salient features enumerated in the foregoing

$$p(\tau) = e^{-\mu_1\gamma_1\tau}\cos\gamma_1\tau - e^{-\mu_2\gamma_2\tau}\cos\gamma_2\tau$$

is chosen to represent the blast pressure.

Taking $10\gamma_1 = \gamma_2'$

$\mu_1 = 0.5$

$\mu_2 = 0.8$
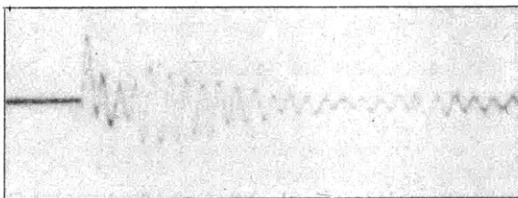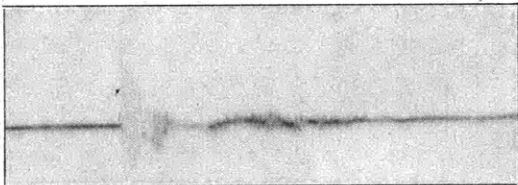
$\alpha = 0.2$

and $\gamma_2 = 0.1, 0.2, 0.3, \ldots 1.5$

The solution of the equation
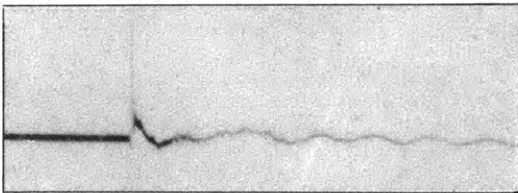
$$\frac{d^2x}{d\tau^2} + 2\alpha\frac{dx}{d\tau} + x = e^{-\mu_1\gamma_1\tau}\cos\gamma_1\tau - e^{-\mu_2\gamma_2\tau}\cos\gamma_2\tau$$

is discussed. In this equation x is in reality the ratio between the deflection of the diaphragm and the static deflection of the diaphragm under the maximum blast pressure.

The conclusion is reached that a diaphragm which will give an undistorted report of a transient blast pressure wave must have its natural period T_0 so small that

$$T_0 \leqq 0.6\, t_m$$

where t_m is the time taken for the pressure to rise from zero, or atmospheric, to its maximum value.

Based on this conclusion a short table of diaphragm natural frequencies necessary to achieve undistorted records for various values of t_m is given.

a) Natural Frequency of the Diaphragm
1300 cycles per second

b) Natural Frequency of the Diaphragm
2400 cycles per second

c) Natural Frequency of the Diaphragm
8000 cycles per second

Figure 17 – Experimental Recordings of Atmospheric Shock Waves

Diaphragms were at 10 m (32.80 feet) distance from the point of detonation. Charge: 1 kg (2.20 pounds) of trinitrotoluol.

t_m (10^{-4} second)	$1/T_0$ (cycles per second)
10	1660
5	3340
2	8400
1	16 500
0.5	33 000
0.2	83 500

Previous work in this field is briefly discussed and a description is given of the author's instrumental developments for measuring blast waves. The diaphragm type of instrument used is a condenser microphone which modulates a high-frequency current in a slightly under-tuned resonant circuit. The modulated current, amplified, is fed into an oscillograph. The natural frequency of the element is given as 12,000 cycles per second. Diaphragms with natural frequencies varying from 1300 cycles per second up to 10,000 cycles per second were used. According to the criterion developed in the theoretical discussion the 10,000-CPS diaphragm is capable of recording faithfully any blast pressure which has a time of rise to maximum pressure equal to or greater than 170×10^{-6} seconds.

Records of explosion pressures from 1 kg (2.2 pounds) of TNT in air are shown in Figures 17 and 20. Figure 17 shows records taken by various recording diaphragms placed 10 meters (32.8 feet) away from the charge. The natural frequencies of the diaphragms used were 1300, 2400, and 8000 cycles per second respectively. The two lower-frequency diaphragms give mainly records of their own natural frequency oscillations. The 8000-CPS gage gave a record which evidently is closer to the true picture of pressure variation. It is concluded from these records that the time of rise to maximum pressure was certainly less than 200×10^{-6} second. Figure 20 shows two records of explosion pressures from 1 kg (2.2 pounds) of TNT in air. These records were taken with a 10,000-CPS condenser microphone placed at 160 meters (525 feet) and 240 meters (787 feet) from the exploding charge. These records are entirely free of 10,000-cycle distortions.
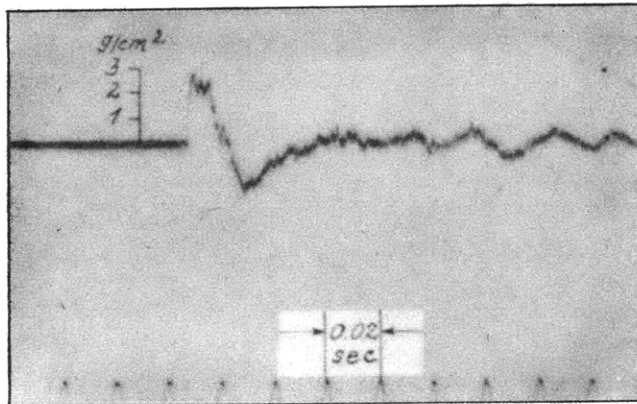
a) Distance from the Point of Detonation 160 m (524.93 feet)
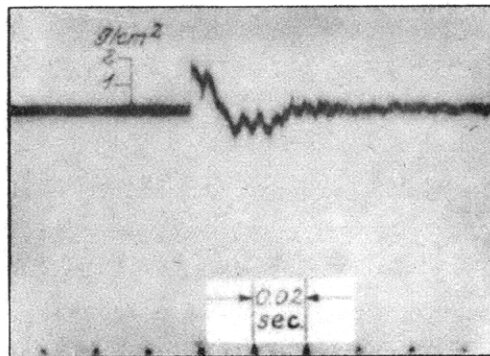
b) Distance from the Point of Detonation 240 m (787.39 feet)

Figure 20 - Atmospheric Shock Waves Recorded with a Condenser Microphone

Detonating Charge: 1 kg (2.20 pounds) of trinitrotoluol

Lt. Comdr. J. Ormondroyd, USNR

THE DETERMINATION OF PRESSURE IN ATMOSPHERIC SHOCK WAVES
DUE TO BLASTING AND THE FIRING OF GUNS

ABSTRACT

The time function of atmospheric shock waves has been approximated in the simplest possible terms to ascertain the behavior of vibratory systems under the influence of these waves. Particular attention was given to the transient stages, and, since it is difficult to get a perspective of this stage from the analytical formulas, much study has been devoted to the calculation and presentation of specific cases. The report presents a correlation between the natural frequency required to obtain a diaphragm record without distortion, and the characteristic duration of the initial pressure rise of atmospheric shock waves. It will henceforth permit an analysis of the effect of waves of this kind on vibratory systems. It will also obviate the uncertainties and errors of former methods based on the results of studies of undamped sound waves.

A recording apparatus devised for use at greater distances from the point of detonation is described; this also produces undistorted records of atmospheric shock waves near the point of detonation. The apparatus represents a great improvement over instruments in use up to this time. Important conclusions can, however, be derived from distorted recordings of atmospheric shock waves with the aid of the procedure described.

These studies have not only improved the determination of pressure in atmospheric shock waves, but they can be advantageously used wherever these waves affect buildings or structures of any type, and where the effects must be analyzed before or after occurrence.

I. THE SCIENTIFIC AND PRACTICAL IMPORTANCE OF ATMOSPHERIC SHOCK WAVES

The atmospheric shock waves caused by blasting and the firing of guns can be considered as sound waves of a particular type. This concept is supported by the fact that both ordinary sound waves and atmospheric shock waves are contained in the basic equations of aerodynamics. Assuming that the density change and velocity of flow are very small compared to the normal state, the integration of these basic equations gives the equations of ordinary sound waves. Without these assumptions (1)* the integration shows violent and sudden density changes similar to those encountered in atmospheric shock waves produced by blasting and gun fire.

The propagation of atmospheric shock waves, the decrease of their intensity with relation to the distance from point of origin, and the change of their time function during propagation obey definite laws, which differ from those pertaining to ordinary sound waves. These special laws can, in part, be derived for plane waves

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* Numbers in parentheses indicate references on page 32 of this translation.

from the hypothesis that the mass per unit time, the accompanying impulse, and the accompanying energy, which traverse two arbitrarily chosen planes which are perpendicular to the direction of propagation, must always be of equal size.

The five characteristic quantities of an atmospheric shock wave are the velocity of propagation of the wave, the velocity of flow, and the pressure, density, and temperature within the wave. The velocity of propagation is relatively easy to measure, while the measurement of the others presents difficulties. To calculate the other quantities of an atmospheric shock wave from only one of these quantities, for instance the easily determined velocity of propagation, it is necessary to make assumptions concerning the specific heat and the behavior of gases at the pressures and temperature changes occurring in the atmospheric shock wave, in addition to the special laws for their behavior previously mentioned. While the validity of these hypotheses is not subject to any restriction because of their fundamental meaning in physics, the correctness of the assumptions which have been made by various people engaged in this research has not yet been proved for the total range of pressure and temperature. A satisfactory proof would be possible if three of the quantities which characterize the atmospheric shock wave could be determined. A check within a limited range is possible, however, if two quantities, for example the velocity of propagation and the pressure, are measured. The determination of the pressure is therefore very important for a deeper insight into the physical behavior of this type of wave.

Determination of pressure in atmospheric shock waves is important from a practical as well as a scientific viewpoint. For instance, factors of safety must be established for the construction of munitions buildings and explosives factories, the effective range of aerial bombs must be determined for civilian protection, and claims which arise from damage to buildings by firing heavy artillery must be judged, appraised, and settled.

On account of the manifold importance of atmospheric shock waves it has long been attempted to determine the magnitude and the time function of the pressure in them. Up to this time no results which are universally satisfactory have been obtained. The reason for this lies in the peculiar nature of atmospheric shock waves, and in the particular circumstances under which they must be measured. It is proposed here to examine more closely the questions and difficulties which arise in measuring the pressure in atmospheric shock waves and to review the techniques hitherto used to measure them.

II. SOME DIFFERENCES BETWEEN ORDINARY SOUND WAVES
AND ATMOSPHERIC SHOCK WAVES

The detonations necessary to produce atmospheric shock waves can be undertaken only in large open areas. For this reason certain special features are required in the measuring devices. They must be easily portable and relatively insensitive to the influences of moisture and temperature. The instruments must also be insensitive

to concussion for use close to the point of detonation. Moreover, very costly instruments should not be used near the point of detonation, because they may be destroyed by concussion or by flying stones and clods of earth.

Considering these circumstances all methods hitherto used for recording atmospheric shock waves are basically similar; a vibratory system, i.e., a diaphragm, is excited by the impact of the atmospheric shock waves. Therefore it will not detract from the exhaustiveness of this investigation if the effect of atmospheric shock waves on diaphragms is permitted to form the central theme of discussion.

The movement of a diaphragm resulting from the impact of an atmospheric shock wave can be recorded by radically different methods. Depending upon the inertia and the friction of the diaphragm, this record gives a more or less exact image of the magnitude and time function of the pressure. The question soon arises as to what conditions a diaphragm must fulfill so that its movement will most precisely correspond to the pressure function which it is recording.

To answer this question it is assumed, as in the case of a "piston diaphragm," that all points on a diaphragm perform the same movements, or that the movements at all points on a diaphragm can be determined with sufficient accuracy from the movement of its midpoint. Hence these studies can be based upon an equation of movement

$$m \frac{d^2 x}{d t^2} + 2\delta \frac{dx}{dt} + cx = p(t) \qquad [1]$$

of a material point instead of the usual partial differential equation valid for the movement of a diaphragm.

In similar investigations in acoustics, or in analogous ones concerned with the precision of oscillographic recordings, the exciting function $p(t)$ has often been developed in a Fourier series. This procedure is suitable and justified in acoustics, for in that field the waves, consisting of fundamentals and overtones, are damped either but slightly or not at all. A Fourier series is the best means to express such phenomena analytically, for it likewise divides the phenomena to be expressed into fundamentals and overtones. If the period of the fundamental is correctly determined, then the individual terms of the Fourier series have a clear physical significance. They represent just those fundamentals and overtones of which the train of waves is composed. The Fourier coefficients correspond to the amplitudes, and their size is a measure of the intensity of the individual vibrations contained in the train of waves. It is not difficult to determine the total behavior of such a train of waves, as, for example, its propagation or its impact upon a sound recorder, from the behavior of the component vibrations.

Atmospheric shock waves are strongly damped in contrast to the ordinary sound waves regularly studied in acoustics. The questions arise whether physical processes of short duration can also be developed in a Fourier series; whether the individual terms still retain a physical significance; and particularly whether the process as a whole can also be judged in this case from its individual components.

A decaying phenomenon can ordinarily not be expressed in a Fourier series without special assumptions. It must rather be considered as repeated after each successive cessation, and the total duration of the diminishing process must be regarded as the period T_0 of the fundamental. With these assumptions, its development in a Fourier series is possible. By means of a Fourier series an analytical expression can be obtained for a damped wave which is only graphically given. This offers advantages in many respects, as, for example, in mathematical calculations. This representation, however, has meaning only in the interval from 0 and T_0, and the individual terms have, as a rule, no physical significance. The assumption that the process is to be repeated after each successive cessation introduces periods which in reality do not exist. Since the duration of a decaying phenomenon is not sharply defined, there is further a certain arbitrariness in the choice of the period of the fundamental which determines that of all the others; it appears either shorter or longer, depending upon the sensitivity of the measuring instruments. A simple example will illustrate this.

Let a damped sinusoidal vibration

$$f(t) = e^{-\epsilon t}\, \sin\!\left(\frac{2\pi t}{T}\right)$$

be developed in a Fourier series. The coefficients a_λ for the cosine terms and b_λ for the sine terms are stated in the following equations*

$$a_\lambda = \frac{1}{2\pi n}\left[\frac{1 + \frac{\lambda}{n}}{\frac{1}{k^2} + \left(1 + \frac{\lambda}{n}\right)^2} - \frac{1 - \frac{\lambda}{n}}{\frac{1}{k^2} + \left(1 - \frac{\lambda}{n}\right)^2}\right]\left[1 - e^{\frac{-2\pi n}{k}}\right]$$

$$b_\lambda = \frac{1}{2\pi n k}\left[\frac{1}{\frac{1}{k^2} + \left(1 - \frac{\lambda}{n}\right)^2} - \frac{1}{\frac{1}{k^2} + \left(1 + \frac{\lambda}{n}\right)^2}\right]\left[1 - e^{\frac{-2\pi n}{k}}\right]$$

λ signifies the series of all positive whole numbers. n is the ratio of the period T_0 of the fundamental assumed for the Fourier development to the period of the damped vibrations T, hence $n = (T_0/T)$. k is the ratio of the circular frequency of the damped sine vibration $2\pi/T = \omega$ to the damping factor ϵ, therefore $k = \omega/\epsilon$. If the sine and cosine terms of the same period are combined they can be reduced to $C_\lambda = \sqrt{a_\lambda^2 + b_\lambda^2}$. Figure 1 shows the curve for C_λ. The values of n and k were chosen as follows: $n = 5$; $k = 3$. According to Figure 1, C_λ reaches a gradual peak at $\lambda = 5$. According to Fourier's series
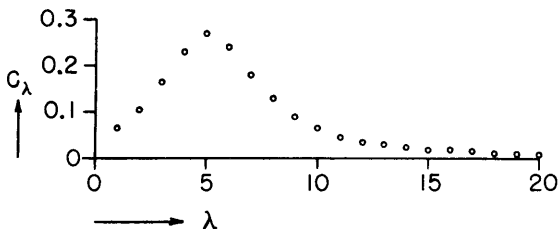

Figure 1 - The Fourier Coefficients of a Damped Sinusoidal Vibration

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* Translator's Note: The first equation, in the original German text, appears to contain two errors in sign which have been corrected.

the amplitude which corresponds to the period of the damped sinusoidal vibration has the greatest value; the amplitudes corresponding to the periods which are contiguous on either side have considerable values also. A damped sinusoidal vibration which is inherently simple is divided into a large number of individual vibrations by the Fourier development. Even if the individual vibrations, whose amplitudes are less than 10 per cent of the largest, are considered irrelevant for the total process and hence disregarded, the Fourier development still gives 13 individual vibrations for a simple damped wave. Such a representation serves no purpose. Beyond this, however, the individual vibrations have, with one exception, no physical significance, since they are not contained in the original process at all. An attempt to judge the behavior of the complete wave from the behavior of the individual component vibrations derived by means of the Fourier series could lead to erroneous conclusions. The possibility of the resonance resulting from the impact of an atmospheric shock wave upon a building would, for example, be greatly overestimated.

Basic doubts exist whether a Fourier series offers a correct basis for investigation of the conditions which a diaphragm must fulfill to furnish undistorted records of an atmospheric shock wave. For this reason the numerous investigations which have been undertaken to establish the forcing function based upon a Fourier series, cannot be used to solve the present problem. Other means must be sought.

From numerous direct and indirect observations it is known that atmospheric shock waves consist essentially of a positive pressure impact which increases very rapidly and is followed by a negative pressure of comparatively long duration but of smaller magnitude. On account of this peculiar behavior of atmospheric shock waves, particular attention must be paid to the initial stage of vibration of a diaphragm. The investigations must be extended also to analytical expressions which correspond approximately to the pattern of atmospheric shock waves. Rapidly diminishing analytical expressions are most suited to this purpose.

In the literature on this subject investigations of this kind are rare. H. Martin (2) has treated the subject of the initial stage of vibration of a vibratory system under the influence of sinusoidal vibrations. He confined his study to undamped forces and to the boundary line cases of an undamped or aperiodically damped vibratory system. Only a short reference by Kalähne (3) concerning the behavior of vibratory systems under the influence of decaying forces is to be found in the literature. It is therefore necessary to begin the investigation from the ground up. For this purpose the simplest possible mathematical basis was devised for the study of atmospheric shock waves, and the initial stage of these waves was particularly treated.

III. THE BEHAVIOR OF VIBRATORY SYSTEMS UNDER THE INFLUENCE
     OF UNDAMPED SINUSOIDAL FORCES

In order to relate this study to familiar material, knowledge of the effect of undamped sinusoidal forces on vibratory systems is used as a point of departure.

Then the effect of damped sinusoidal and cosinusoidal forces will be treated.* Finally by the combination of two damped cosinusoidal functions an analytical expression is derived, which closely approximates the character of the pressure curve in the atmospheric shock wave. The investigations respecting the behavior of a vibratory system under the influence of such a pressure change then yield particularly valuable information.

For these investigations the following non-dimensional mathematical quantities are introduced into the general equation of motion.**

$$\tau = \omega_0 t; \quad \omega_0 = \sqrt{\frac{c}{m}}; \quad \alpha = \frac{\delta}{m \omega_0}; \quad \beta = \sqrt{1 - \alpha^2}; \quad \gamma = \frac{\omega}{\omega_0}$$

For an undamped sinusoidal vibration as the exciting function the general equation of motion then becomes

$$\frac{d^2 x}{d\tau^2} + 2\alpha \frac{dx}{d\tau} + x = a \sin \gamma \tau \qquad [2]$$

where a is the static displacement.

The solution of this equation is

$$x = \frac{a}{\rho} \sin(\gamma\tau - \psi) + A e^{-\alpha\tau} \sin(\beta\tau + \varphi) \qquad [3]$$

ρ and ψ are defined by the following equations

$$\rho = \sqrt{(1 - \gamma^2)^2 + 4\alpha^2 \gamma^2} \qquad [3a]$$

$$\psi = \frac{2\alpha\gamma}{1 - \gamma^2} \qquad [3b]$$

From the initial conditions $x = \frac{dx}{d\tau} = 0$, for $\tau = 0$, it follows

$$\tan \varphi = \frac{2\alpha\beta}{\gamma^2 - 1 + 2\alpha^2} \qquad [3c]$$

$$A = \frac{a}{\rho} \frac{\sin\psi}{\sin\varphi} = \frac{a}{\rho} \cdot \frac{\gamma}{\beta} \qquad [3d]$$

If the value for A is substituted in Equation [3], we get

$$x = \frac{a}{\rho} \left[\sin(\gamma\tau - \psi) + \frac{\gamma}{\beta} e^{-\alpha\tau} \sin(\beta\tau + \varphi) \right] \qquad [4]$$

Because of the inertia and frictional forces the motion of a diaphragm can never exactly correspond to the pressure change that excites it. Consideration of the effect of inertia and friction leads to a combination of two analytical expressions for the forced vibrations of a diaphragm, Equation [4]. With the exception of a

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* The cosinusoidal or cosine curve is the same curve as the sinusoidal, but with the y-axis through the point $\pi/2$.

** Translator's Note: Several obvious typographical errors in the original German have been corrected in this equation.

phase-constant, the same analytical expression serves for the first member of this summation as that which is valid for the exciting pressure change. The second member corresponds in its form to the free vibration, i.e., to the natural vibration of the diaphragm. This formal agreement is used to differentiate between the two components of the forced vibrations of a diaphragm. Therefore in the following discussion the individual terms representing the forced vibrations of a diaphragm will be referred to simply as the forced vibration and the free vibration.

Equation [4] shows that after a certain time, which is termed the initial stage of vibration, the free vibration becomes vanishingly small as compared to the forced vibration. From this point on, the knowledge of the magnification factor $1/p$ which stands in front of the bracket and the angle of phase ψ is sufficient to evaluate the motion of a diaphragm resulting from a sinusoidal pressure change. For the steady condition the magnification factor indicates how many times greater the amplitude of a forced sinusoidal vibration is than that of a vibration caused by a uniform pressure equal to the maximum for the sinusoidal vibration. In Figure 2 the magnification factor is plotted as a function of the frequency ratio γ for various values of the damping factor α. Figure 3 shows the curve of the angle of phase ψ. From Figures 2 and 3 in conjunction with Equation [4] it is clearly evident that after the initial
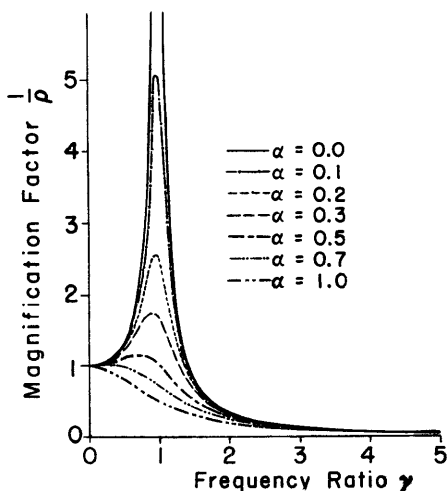

Figure 2 - The Magnification Factor
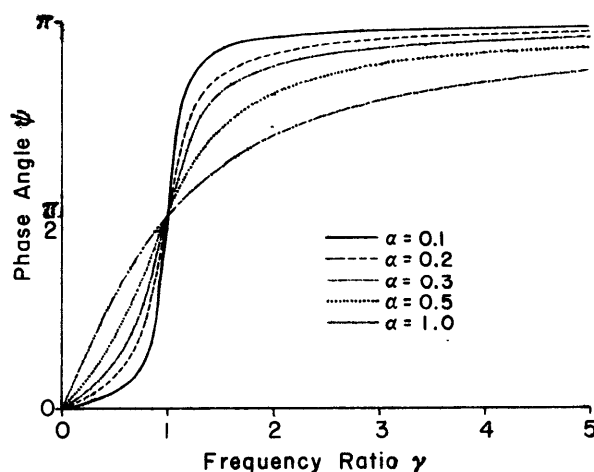for Undamped Sinusoidal or
Cosinusoidal Excitation

Figure 3 - The Phase Angle of the Forced
Vibration for Undamped Sinusoidal or
Cosinusoidal Excitation

vibration period the distortion of the recordings made by a diaphragm is less the smaller the ratio of the frequency of the pressure change to be recorded to the natural frequency of the diaphragm. With respect to the foregoing, the general equation of motion has already been treated so often that a mere reference to literature on the subject will suffice (4). Hence the phenomenon termed the transient stage of vibration will be immediately considered. For this purpose the free vibration must also

be studied. Its amplitude and the phase φ depend upon the frequency ratio γ and the damping factor α. The amplitude of the free vibration with respect to that of the forced vibration, Equation [4] is

$$\frac{\gamma}{\sqrt{1-\alpha^2}} = \frac{\gamma}{\beta}$$

Hence, if γ increases, it can assume very large values. However, since it is a function of the magnification factor, its absolute value decreases as γ increases. Because the amplitude depends upon $\beta = \sqrt{1-\alpha^2}$, the free vibration requires a further special treatment for the case where $\alpha = 1$. Hence we write

$$\frac{\gamma}{\beta} e^{-\alpha\tau}\sin(\beta\tau + \varphi) = \frac{\gamma}{\beta} e^{\alpha\tau}\left[\sin\beta\tau\cos\varphi + \cos\beta\tau\sin\varphi\right]$$
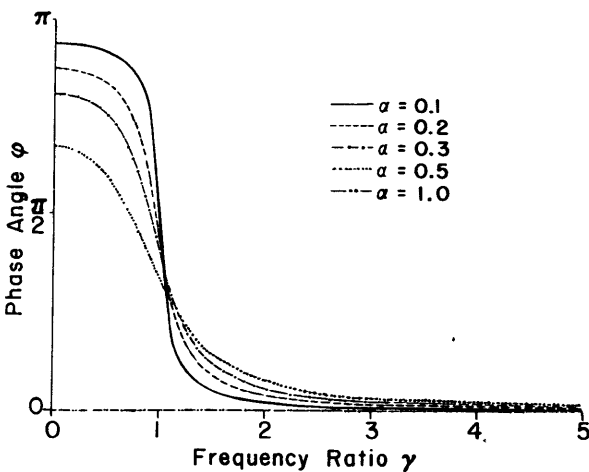
For very small values of β, i.e., for values of α which lie near 1, the following expression can be written

$$\frac{\gamma}{\beta} e^{-\alpha\tau}\left[\beta\tau + \frac{2\alpha\beta}{\gamma^2 - 1 + 2\alpha^2}\right] = \gamma e^{-\alpha\tau}\left[\tau + \frac{2\alpha}{\gamma^2 - 1 + 2\alpha^2}\right]$$

Hence it is evident that the indeterminate nature of the expression for the free vibration when $\alpha = 1$, which is apparent at first glance in Equation [4], in reality does not exist.

Figure 4 shows the curve of the angle of phase of the free vibration φ, Equation [3c], as a function of γ for varying values of α. The tangent of the angle of phase φ is only slightly changed for small values of γ, and tends toward the value $\tan \varphi = 2\alpha\beta/(2\alpha^2 - 1)$ when $\gamma = 0$. This limiting value is zero for $\alpha = 0$ and $\alpha = 1$, and infinity for $\alpha = \sqrt{0.5}$. The angles, which correspond to these limiting values, are $\varphi = \pi$ when $\alpha = 0$, $\varphi = 0$ when $\alpha = 1$ and $\varphi = \pi/2$ when $\alpha = \sqrt{0.5}$.

The curve of the angle φ as a function of γ is

$$\left.\begin{array}{l} \varphi = \pi \\ \varphi = 0 \end{array}\right\} \quad \text{for } \alpha = 0; \quad \begin{array}{l} \gamma < 1 \\ \gamma > 1 \end{array}$$

$$\varphi = 0 \qquad \text{for } \alpha = 1; \quad 0 \lesseqgtr \gamma \lesseqgtr \infty$$

For values of γ which lie near 1, $\tan \varphi$ changes radically and tends toward zero for larger values of γ and for all values of α which come into practical consideration.

Little more information can be derived from the analytical expressions as to the initial stage of the movement of a diaphragm composed of

——— $a = 0.1$
------ $a = 0.2$
——·· $a = 0.3$
········ $a = 0.5$
—··· $a = 1.0$

Figure 4 - The Phase Angle of the Free
Vibration for Undamped Sinusoidal Excitation

two vibrations of different amplitude and phase. For this reason Equation [4] was calculated for the initial stage and for various values of γ and represented graphically in Figure 5. These calculations are based on the value $\alpha = 0.5$ for the damping of the diaphragm. This damping coefficient corresponds approximately to the ratio $\eta = 5$ between two consecutive amplitudes.

Curve a of Figure 5 represents the first two vibrations of an undamped sinusoidal pressure change, curves b to l show the motion of a diaphragm caused by this pressure during the initial stage. The smaller γ, the ratio of the frequency of the exciting force to the natural frequency of the diaphragm, the closer will be the agreement between the movement of the diaphragm during the initial stage and the pressure change. For the case where

Figure 5 - Forced Vibrations of a Diaphragm due to Undamped Sinusoidal Excitation

$\gamma = 0.17$, curve b, the movement of the diaphragm records the pressure change without distortion. In this case, then, the amount of pressure can be determined from the amplitude of the vibration with the help of static calibration. The movement of the diaphragm however has an angle of lag φ with respect to the pressure change. This phase lag becomes smaller with further decrease in γ (see Figure 3). When $\gamma = 0.35$ to 0.87 (curves c to f) the diaphragm records the magnitude of pressure with sufficient exactness for many practical cases. However, the phase lag is already greater. Thus the diaphragm shows a deceptive pressure rise, which takes place much more slowly than in reality. With a further increase of γ increasing differences appear in the motion of the diaphragm with respect to the pressure change, as much in the amplitude as in the phase. This is clearly evident from curves g to l of Figure 5.

The solution of the displacement equation in the case of undamped sinusoidal vibrations which was reported by H. Martin, is more general than that stated herein in
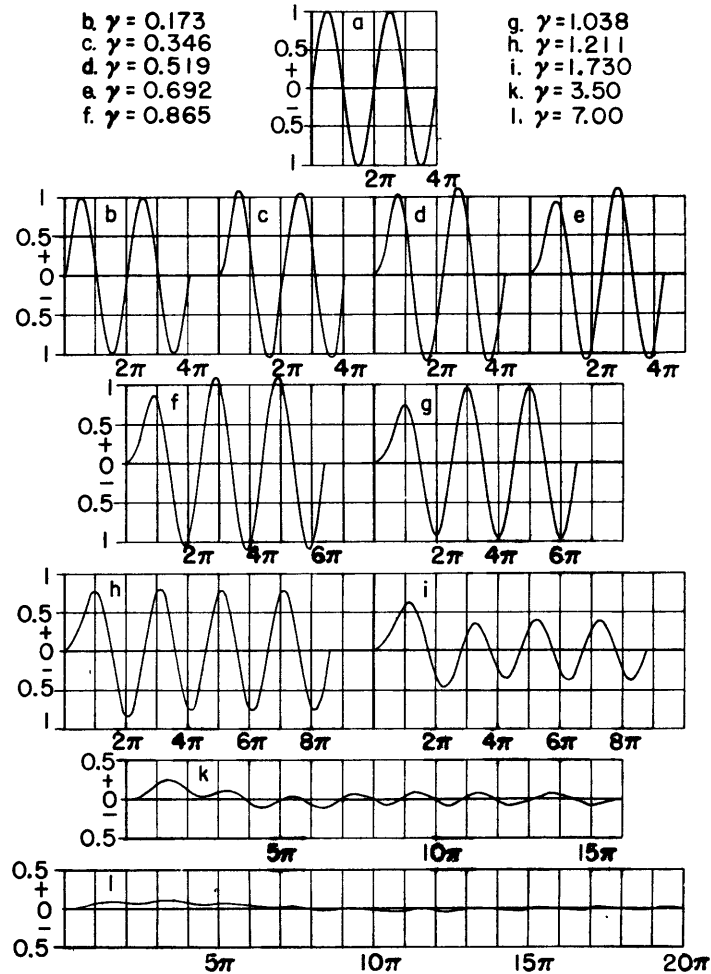
Equation [4]. It is valid, in fact, for exciting sinusoidal vibrations which begin with an arbitrary phase. The damping factors $\alpha = 0$, and $\alpha = 1$ form the basis for the initial-stage curves published by Martin. Hence they form a valuable complement to Figure 5. Since Martin wished to give particular emphasis to the origin of the forced vibrations in his diagrams, he plotted the exciting vibrations in a direction opposite to the forced vibrations. Martin's curves have correct physical significance only if one of the two vibrations is reversed around the point π. The lack of agreement which results from a comparison of Martin's curves with Figure 5 thus has no deeper reason but is simply a question of the most suitable representation.

If it is desired to derive information from these results concerning the natural frequency which a diaphragm must have to record without distortion an atmospheric shock wave, the initial pressure rise in the atmospheric shock wave can be regarded as one-fourth of the wave length of an undamped sinusoidal vibration. According to this a diaphragm whose natural period is smaller than about three times the time of the initial pressure rise in the atmospheric shock wave would be able to record an atmospheric shock wave without distortion. Too much weight cannot, however, be attached to this conclusion, because the pressure curve used as a basis only approximately coincides for a small period of time with the pressure curve in the atmospheric shock wave. Therefore the movement of a diaphragm under the excitation of a damped sinusoidal force will now be treated.

IV. THE BEHAVIOR OF VIBRATORY SYSTEMS UNDER THE INFLUENCE
OF DAMPED SINUSOIDAL FORCES

We write

$$\frac{d^2x}{d\tau^2} + 2\alpha\frac{dx}{d\tau} + x = be^{-\mu\gamma\tau}\sin\gamma\tau \tag{5}$$

where b is the static displacement.

The solution of this equation is

$$x = \frac{b}{\rho}e^{-\mu\gamma\tau}\sin(\gamma\tau - \psi) + Ae^{-\alpha\tau}\sin(\beta\tau + \varphi) \tag{6}$$

ρ and ψ are determined by the following equations

$$\rho = \sqrt{(1 - \gamma^2 + \mu^2\gamma^2 - 2\alpha\mu\gamma)^2 + 4\gamma^2(\alpha - \mu\gamma)^2} \tag{6a}$$

$$\tan\psi = \frac{2\gamma(\alpha - \mu\gamma)}{1 - \gamma^2(1 - \mu^2) - 2\alpha\mu\gamma} \tag{6b}$$

Since the diaphragm should be at rest before being acted upon by the exciting force, the initial conditions $x = 0, \frac{dx}{d\tau} = 0$ are valid when $\tau = 0$. From this the the following equations result for the fixed magnitudes A and φ

$$A = \frac{b}{\rho}\frac{\sin\psi}{\sin\varphi} = \frac{b}{\rho}\cdot\frac{\gamma}{\beta} \tag{6c}$$

$$\tan\varphi = \frac{2\beta(\alpha - \mu\gamma)}{2\alpha(\alpha - \mu\gamma) + \gamma^2(1 + \mu^2) - 1} \tag{6d}$$

By considering Equation [6c], Equation [6] may be written as follows

$$x = \frac{b}{\rho}\left[e^{-\mu\gamma\tau}\sin(\gamma\tau - \psi) + \frac{\gamma}{\beta}e^{-\alpha\tau}\sin(\beta\tau + \varphi)\right]$$ [7]

Just as was the case in an undamped sinusoidal exciting force, the forced motion of a diaphragm is here, too, the sum of the forced and free vibrations. Both vibrations are damped. Hence, a condition in which the one vibration remains unchanged after the other has faded out cannot develop. Therefore, the magnification factor $1/\rho$, considered by itself, does not have the same significance as in the case of an undamped sinusoidal excitation. The magnitudes ρ, ψ, and φ depend not only upon γ and α, but also upon μ, which is the damping factor of the exciting force.

As in the case of an undamped sinusoidal excitation, the amplitude of the free vibration is γ/β. Limiting conditions when $\alpha = 1$ have also a finite value.

For different values of α, i.e., the damping factor of the diaphragm, Figure 6 shows the curve of the magnification factor $1/\rho$ as a function of the frequency ratio γ. The damping factor of the exciting force μ was based upon the value 0.5. The curve of the magnification factor is similar to that in an undamped sinusoidal excitation. It approaches unity when γ is very small, increases as γ increases and at a certain ratio between α and μ when γ has a definite value it can even become infinite. Where γ is very large, the magnification factor is inversely proportional to γ^2. The particular case where $\alpha = 0$ is expressed

$$\frac{1}{\rho} = \frac{1}{\sqrt{(1 - \gamma^2 + \mu^2\gamma^2)^2 + 4\gamma^2\mu^2}}, \quad \alpha = 0$$

Figure 6 - The Magnification Factor for Damped Sinusoidal or Cosinusoidal Excitation

In this case then, as long as μ is other than zero, the magnification factor has a finite value for all values of γ. It is striking that in a damped sinusoidal excitation the magnification factor remains finite when $\alpha = 0$, but that it can become infinite when $\alpha \neq 0$ and when there is a definite ratio between α and μ. This seems to mean, then, that the amplitude of a damped vibratory system excited by a damped sinusoidal exciting force can become infinite, while that of an undamped one cannot. If such a result should in fact follow from our mathematical investigations, it would contradict physical observations. In reality, however, this paradox does not exist, as later investigations will prove.

A further particular value for the magnification factor results from $\mu^2\gamma^2 - 2\alpha\mu\gamma = 0$. With this assumption the magnification factor becomes

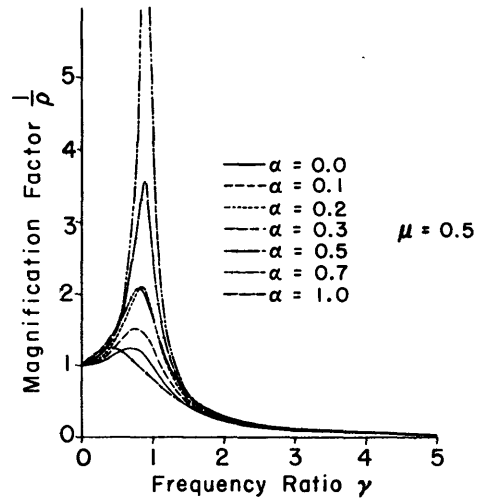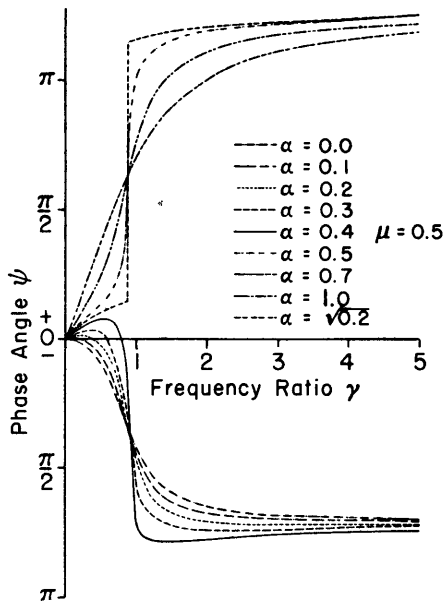$$\frac{1}{\rho} = \frac{1}{\sqrt{(1 - \gamma^2)^2 + 4\gamma^2\alpha^2}}$$

Figure 7 - The Phase Angle of the
Exciting Vibration for a Damped
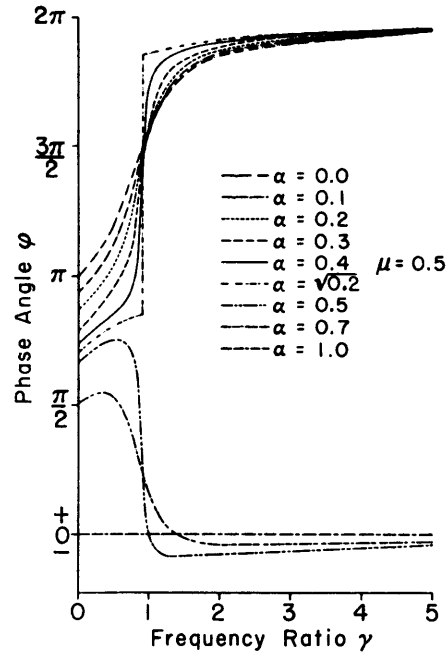Sinusoidal or Cosinusoidal Excitation

Figure 8 - The Phase Angle of the
Free Vibration for a
Damped Sinusoidal Excitation

It then has the same value for this special case as it has for an undamped sinusoidal excitation.

Figure 7 represents the curve of the phase angle ψ as a function of γ for different values of α. The value 0.5 was chosen for μ. For very small values of γ, ψ approaches 0; for larger values of γ the value of ψ changes greatly, and at still greater values of γ it approaches a fixed value which is found by the formula $\tan \psi = \frac{-2\mu}{\mu^2 - 1}$. In a damped sinusoidal excitation the curve of ψ as a function of γ is fundamentally different from that in an undamped sinusoidal excitation. Thus the angle ψ is not restricted to the first and second quadrants, but extends also into the third and fourth. For values of $\alpha < \frac{\mu}{\sqrt{1 + \mu^2}}$ the curve of ψ is chiefly in the third and fourth quadrants. When $\alpha > \frac{\mu}{\sqrt{1 + \mu^2}}$, the curve of ψ is continuous in the first, second, and third quadrants. The curve of ψ as a function of γ is constant with one exception. This exception exists for the case where $\alpha = \frac{\mu}{\sqrt{1 + \mu^2}}$. It becomes evident that for this value of α and for $\gamma = \frac{1}{\sqrt{1 + \mu^2}}$ Equation [6] becomes indefinite. Limiting conditions for this case given $\tan \psi = \frac{-\mu}{-1}$ if the critical point is approached from the direction of γ and $\tan \psi = \frac{+\mu}{+1}$ if the boundary transition is completed from the direction of small values of γ. The phase angle ψ thus jumps from a value in the first quadrant to one in the third. The numerical value of the tangent is the same for both angles.

Figure 8 represents the curve of the phase angle φ according to Equation [6c] for different values of α. μ is likewise based upon the value 0.5. In many respects the curve of φ shows similarity to that of ψ in Figure 7. Thus φ appears in all four quadrants. With one exception the curve of φ is continuous just as is that of ψ. The upper and lower limits for φ at this point of discontinuity are stated in the formulas.

$$\tan \varphi = \frac{-\mu}{+1} \quad \text{and} \quad \tan \varphi = \frac{+\mu}{-1}$$

In contrast to ψ, φ does not tend toward 0 when $\gamma = 0$, but toward an angle which is determined by the equation $\tan \varphi = \frac{2\beta\alpha}{2\alpha^2 - 1}$.

In order to get a clear picture of the motion of a diaphragm due to excitation by a damped sinusoidal force, several displacement-time curves are calculated and plotted. These are represented in Figure 9. These calculated curves are based upon the values $\alpha = 0.2$, and $\mu = 0.5$. Curve **a** of Figure 9 shows the assumed form of the pressure acting upon a diaphragm; it is a damped sinusoidal vibration. Curves **b** to **i** represent the movement of the diaphragm under the influence of this pressure, and are based on various values for the natural frequency of the diaphragm. If the ratio of the frequency of the exciting pressure to the natural frequency of the diaphragm is small, the diaphragm gives an undistorted image of the pressure curve, as shown by curve **b** of Figure 9, which corresponds to a frequency ratio of $\gamma = 0.1$. A distortion shows up in curve **c** where $\gamma = 0.2$. This distortion becomes more marked in curves **d** and **e**, whose values of γ are 0.3 and 0.4 respectively. Curves c, d, and e still depict pressure forms which approximate reality. Curves f, g, h, and i, in contrast, scarcely show anything of the true nature of the pressure change affecting the diaphragm. They are essentially the natural vibrations of the diaphragm, which are the more distinct and characteristic, the greater γ becomes. Beginning at a certain frequency ratio, which is about 0.6 when $\alpha = 0.2$ and $\mu = 0.5$, a diaphragm is chiefly excited to natural vibrations by damped sinusoidal forces. This hypothetically derived result fully agrees with experimental findings
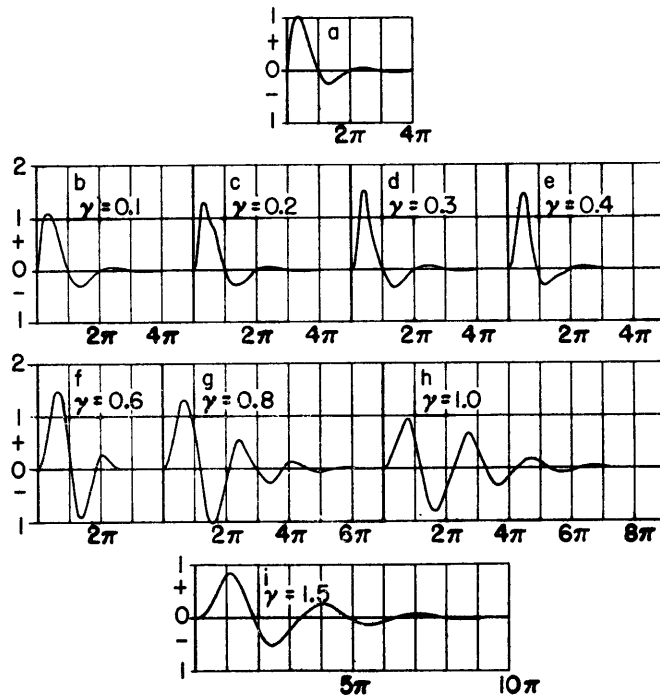

Figure 9 - Forced Vibrations of a Diaphragm Caused by a Damped Sinusoidal Excitation

regarding the measurement of pressure in atmospheric shock waves. This will be reserved for later discussion.

A comparison of Figure 9 with Figure 5 shows that the natural frequency of a diaphragm must be substantially higher to record a damped sinusoidal pressure change without distortion than is the case for an undamped sinusoidal pressure change. For sufficiently practical exactness it was determined that a frequency ratio of $\gamma < 0.9$ is requisite to record undamped sinusoidal forces without distortion. For a sufficiently precise determination of damped sinusoidal forces, however, a frequency ratio of $\gamma < 0.1$ is necessary.

At frequency ratios which are greater than 0.1 but less than unity, curves c to g, the amplitude of the diaphragm is still noticeable. If the maximum pressure were to be determined from these amplitudes alone with the help of the static calibration, it would appear greater than it really is.

It has already been pointed out that the solution of the general equation of motion seems to have no finite value in the case of a damped sinusoidal excitation, Equation [7], for a certain definite ratio between α and μ, although there is no clear physical basis for this. Hence Equation [7] must be subjected to a special investigation for this case.

If $\gamma = \frac{\alpha}{\mu}$ in Equation [6a] it becomes

$$\rho = 1 - \frac{\alpha^2}{\mu^2}(1 - \mu^2) - 2\alpha^2 = 1 - \alpha^2 \left(\frac{1 + \mu^2}{\mu^2}\right)$$

From this it follows that when $\alpha = \frac{\mu}{\sqrt{1 + \mu^2}}$ the magnification factor $\frac{1}{\rho}$ becomes infinite. Since $\beta = \sqrt{1 - \alpha^2}$, $\gamma = \beta$ and $\alpha = \mu\beta$ for this special case. The solution in the form of Equation [7] fails then, if the period and damping of the exciting force coincide with the period and damping of the natural frequency of the diaphragm. In order to determine whether the solution of the general equation of motion really has a finite value, as is to be expected from physical observations, Equation [7] will be examined for a value of γ which is only slightly different from β; i.e., $\gamma = \beta + \delta$. Assuming that $b = 1$, Equation [7] can then be stated

$$x = \frac{\beta^2}{\delta \sqrt{4\beta^4 + \delta^2 + 4\delta\beta^3}} \left[e^{-(\alpha + \mu\delta)\tau} (\sin[(\beta + \delta)\tau - \psi] + \frac{\beta + \delta}{\beta} e^{-\alpha\tau} \sin(\beta\tau + \varphi)\right] \quad [8]$$

The following proportions can easily be derived from Equations [6b] and [6d]

$$\frac{\sin\psi}{\sin\varphi} = \frac{\beta + \delta}{\beta}; \quad \frac{\cos\psi}{\cos\varphi} = \frac{-2\beta - \delta + \mu^2\delta}{2\beta - \delta + \mu^2\delta}$$

By developing Equation [8], substituting these proportions, and considering that for sufficiently small δ we can write $\cos \delta\tau = 1$, $\sin \delta\tau = \delta\tau$, and

$$e^{-(\alpha + \mu\delta)\tau} = e^{-\alpha\tau} (1 - \mu\delta\tau)$$

it follows that

$$x = \frac{\beta^2 e^{-\alpha\tau}}{\delta\sqrt{4\beta^4 + \delta^2 + 4\delta\beta^3}} \left[1 - \mu\delta\tau\right]\left[\delta\tau\cos(\beta\tau - \psi) \ \delta\sin\beta\tau\cos\varphi\left(\frac{1}{\beta} + \frac{2\mu^2}{2\beta + \delta + \mu^2\delta}\right)\right] \quad [8a]$$

In Equation [8a] the δ in the denominator can be dropped. If δ then approaches zero, the following equation results

$$x_{\delta \to 0} = \frac{e^{-\alpha\tau}}{2}\left[\tau\cos(\beta\tau - \psi) + \frac{1}{\beta^2}\sin\beta\tau\right] \qquad [8b]$$

Equation [8b] shows that the solution of the general equation of motion leads to a finite value for the particular case where

$$\gamma = \beta = \frac{1}{\sqrt{1 + \mu^2}} \text{ and } \alpha = \mu\beta$$

Therefore Equation [7] is not really indeterminate as it appears to be at first glance. For $\alpha = 0$, Equation [8b] becomes the familiar formula for resonance for undamped sinusoidal excitation, i.e.,

$$x = \frac{1}{2}\left[\tau\cos\tau - \sin\tau\right]$$

According to Equation [8b] the amplitude can never become infinite, since $e^{\alpha\tau}$ increases more rapidly than τ. The ratio of the amplitudes for values of τ, for which the second term of Equation [8b] can be neglected, is relatively obvious. In fact, it becomes the ratio of two consecutive amplitudes

$$\epsilon = \frac{e^{\alpha\pi}}{1 + \frac{\pi}{\tau}}$$

Therefore, ϵ is not constant, but depends upon τ. Excluding those values of α which correspond to an aperiodic or nearly aperiodic damping, and which are of slight practical interest, ϵ becomes less than unity for small values of τ, gradually increases, reaches unity, and finally approaches the value of $e^{\alpha\pi}$ which is always greater than unity. According to Equation [8b], the amplitude first increases, then,· after a certain time, it in turn decreases. The smaller the damping factor α, the longer the increase of the amplitudes lasts, and the later the fading out begins. The smaller the damping, the greater become the maximum amplitudes.

In Figure 10 the dotted curve represents the exciting force, and the solid curve represents the forced motion of the diaphragm according to Equation [8b]. The value 0.2 is assumed for α. From this it follows that $\mu = 0.204$. This illustration shows the rising and falling of the amplitudes for a special case, as previously described.

The gradual rising of the amplitudes shown in Equation [8b] appears only when the damping as
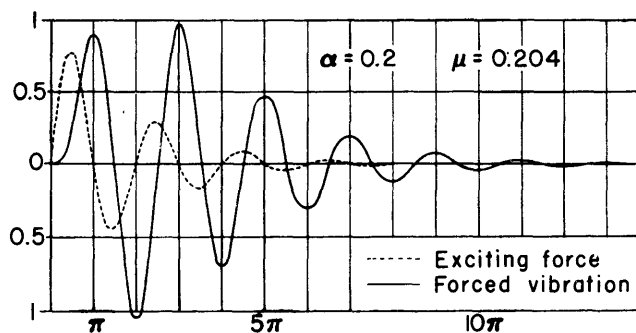

Figure 10 – Resonance Phenomenon in a Damped Sinusoidal Excitation

well as the period of the exciting force coincides with the corresponding values of the vibratory system. The probability that this rising will appear in damped excitations is therefore much less than in the case of undamped ones.

V. THE BEHAVIOR OF VIBRATORY SYSTEMS UNDER THE INFLUENCE OF COSINUSOIDAL FORCES

Various criteria for the evaluation of pressure from the movement of a diaphragm have been obtained by the foregoing observations. The time of the pressure rise is the same as that of the pressure drop in both of the cases observed. The pressure rise upon impact occurs in a much shorter time than does the pressure drop in atmospheric shock waves. Hence the general equation of motion still remains to be examined to determine the existence of analytical expressions which satisfy the time relation of atmospheric shock waves.

Pressure impacts whose initial pressure rise occurs in an infinitely short time, i.e., which begin at maximum pressure, will first be studied. The cosine function is a suitable method of expressing these. A damped cosine vibration, besides, shows many of the characteristics of atmospheric shock waves. For purposes of completeness the solution of the general equation of motion in an excitation by an undamped cosine vibration will be briefly stated, in order to permit a more searching study of the solution to be made for a damped cosine vibration.

Under the assumption of an undamped cosinusoidal pressure change the general equation of motion is

$$\frac{d^2x}{d\tau^2} + 2\alpha\frac{dx}{d\tau} + x = b\cos\gamma\tau \qquad [9]$$

By considering the initial conditions $x = \frac{dx}{dt} = 0$, the solution when $\tau = 0$ is

$$x = \frac{b}{\gamma}\left[\cos(\gamma\tau - \psi) - \frac{1}{\beta}e^{-\alpha\tau}\cos(\beta\tau + \varphi)\right] \qquad [10]$$

The expressions for ρ and ψ are the same as those used under the assumption of an undamped sine vibration as the disturbing function. Their curve can therefore be derived from Equations [3a] and [3b] and Figures 2 and 3.

The phase angle φ of the free vibration of the diaphragm is determined by the equation

$$\tan\varphi = -\frac{\alpha}{\beta}\left[\frac{1 + \gamma^2}{1 - \gamma^2}\right] \qquad [10a]$$

Figure 11 shows the curve of φ as a function of γ for different values of α. It is to be noted that the amplitude of the free vibration does not depend upon γ and β, as it does under the influence of a sinusoidal vibration, but upon β alone. The apparent indeterminate nature of the factor $1/\beta$ for $\alpha = 1$ does not really exist; this can easily be proved by the limiting conditions.
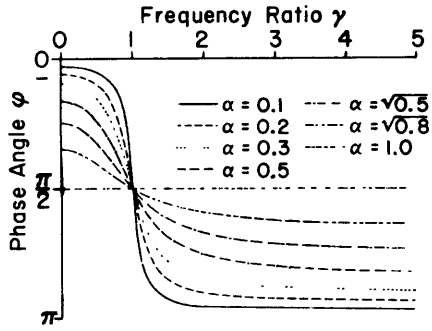
Figure 11 - The Angle of Phase of
the Free Vibration in an
Undamped Cosine Excitation

Assuming a damped cosine pressure wave, the general equation of motion takes the form

$$\frac{d^2x}{d\tau^2} + 2\alpha\frac{dx}{d\tau} + x = b\,e^{-\mu\gamma\tau}\cos\gamma\tau \qquad [11]$$

By considering the initial conditions $x = \frac{dx}{dt} = 0$, when $\tau = 0$, the solution is

Figure 12 - The Angle of Phase of
the Free Vibration in a Damped
Cosine Excitation

$$x = \frac{b}{\rho}\left[e^{-\mu\gamma\tau}\cos(\gamma\tau - \psi) - \frac{\sqrt{1 + \mu^2\gamma^2 - 2\alpha\mu\gamma}}{\beta}\,e^{-\alpha\tau}\cos(\beta\tau + \varphi)\right] \qquad [12]$$

ρ and ψ depend upon γ, α and μ in the same way as they did in the studies of the effect of damped sinusoidal forces. Hence it will suffice to refer to [6a], [6b], and to Figures 6 and 7.

The angle of phase φ of Equation [12] is expressed

$$\tan\varphi = \frac{\mu\gamma - \alpha}{\beta}\left[\frac{1 + \gamma^2 + \mu^2\gamma^2 - 2\alpha\mu\gamma}{1 - \gamma^2 + \mu^2\gamma^2 - 2\alpha\mu\gamma}\right] \qquad [12a]$$

Figure 12 shows the curve of φ as a function of γ for different values of α, and a fixed value of $\mu = 0.5$. For very small values of γ, φ approaches an angle whose tangent has the value $\tan\varphi = -\frac{\alpha}{\beta}$. The quantity φ is only slightly variable in the range of small or very large values of γ. Within a range which includes $\gamma = \beta$ = $\frac{1}{\sqrt{1 + \mu^2}}$, φ varies sharply. Here also, as in Figure 8, there appears a cyclic change in the angle of phase φ when $\gamma = \beta$.

If the critical point is approached from large values of γ, $\tan\varphi = \frac{+\mu}{-1}$; if this approach is from small values of γ, $\tan\varphi = \frac{-\mu}{+1}$.

Figure 13 gives a perspective of the movement of a diaphragm excited by a damped cosine pressure change. Curve a represents the time curve of the exciting pressure. The curves b to k show the time-displacement curves of the diaphragm
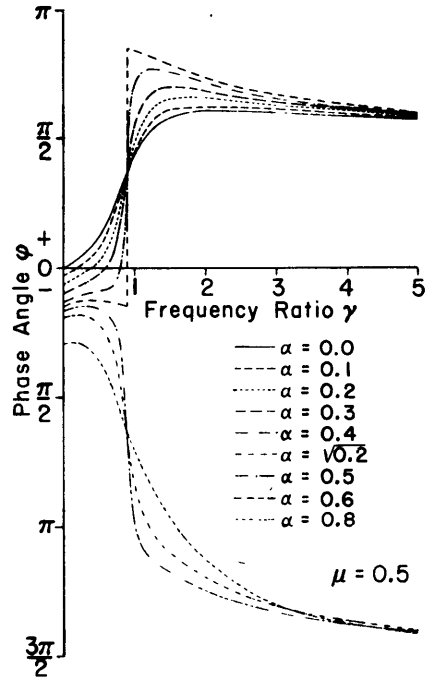
Figure 13 - The Forced Vibrations
of a Diaphragm Produced by
a Damped Cosine-Shaped Excitation

corresponding to various frequency ratios. The free vibrations of the diaphragm stand out in all the time-distance curves. In the case of very small values of γ they distort only the first part of the pressure curve, while the remainder is faithfully reproduced; see curves b and c. When γ increases, curves d and e, their influence is gradually extended to the entire pressure curve. In the case of still greater γ, finally, the movements of a diaphragm have almost nothing more in common with the exciting pressure wave. The time-displacement curves f to k essentially represent free vibrations of the diaphragm, to which they were excited by the pressure acting on the diaphragm. Therefore, pressures of the impact type, which begin directly with their maximum value, corresponding to a damped cosine vibration, cannot be recorded without distortion even by diaphragms whose natural frequencies are very high. This can be readily recognized from Equation [12] in conjunction with Figures 6, 7, and 12, for values of γ still smaller than those which the time-distance curves in Figure 13 represent. However, in special cases the pressure curve can be determined from partially distorted recordings. It is to be understood from Equations [12] and [12a], that the amplitude of the free vibration assumes the value of unity, for small values of γ and α, while the angle of phase φ decreases. If γ, then, is so small that the initial vibrations of the diaphragm occur in a period of time in which the cosine pressure wave, which begins at its highest value, has not yet appreciably changed, the pressure curve can readily be determined from the vibrations of the diaphragm. It is only necessary, then, to divide the line drawn through two consecutive peaks into components a and b having the ratio $\frac{a}{b} = e^{\alpha\pi}$. If a number of vibrations are thus divided, a series of points is obtained which when combined form the pressure curve.

It is not to be assumed that the pressure rise in atmospheric shock waves occurs with infinite speed, i.e., instantaneously, but rather within a finite period of time. It might be inferred from this that the results derived herein have no

practical value. However, the time consumed by the pressure rise near the point of detonation is certainly so small that it may practically be regarded as infinitely small considering the vibratory range of the diaphragms currently at our command. Finally, Riemann proved that discontinuous pressure changes are contained in the basic equations of aerodynamics. Hence the study of damped cosine excitations is rightly a problem of practical interest.

VI. THE BEHAVIOR OF VIBRATORY SYSTEMS UNDER THE INFLUENCE OF A PRESSURE CHANGE COMPOSED OF TWO COMBINED COSINE VIBRATIONS

The pressure curves studied up to this point were based upon analytical expressions, in which the time of the pressure rise either equalled that of the pressure drop or the waves began at maximum pressure. Hence with respect to atmospheric shock waves these expressions represent borderline cases which must be supplemented.

It was found imperative to seek a simple analytical expression for the pressure change in atmospheric shock waves, which would be valid for the most widely varying investigations. This was first attempted by combining several sine and cosine vibrations. These attempts failed because the expressions which they yielded were too complicated and moreover, failed to furnish sufficiently close approximations to the pressure change in atmospheric shock waves. A very close approximation can be attained by functions of the following type:

$$p = \tau a e^{-\epsilon \tau} \cos \gamma \tau$$

By assuming such a function as the exciting function the solution of the general equation of motion is to be found according to the principle of the variation of the constants. Aside from an extensive calculation, this method presents no difficulties. However, the solution is a combination of several expressions, which makes it difficult to survey conditions as a whole, and the calculation is too involved for purposes of practical analysis. The sum of two damped cosine vibrations has proved to be the most favorable method; the sum, moreover, of two such vibrations of equal amplitude but of opposite phase, so that when $\tau = 0$, the pressure is zero. The one vibration should have a small damping factor and a low frequency in ratio to the other, so that it predominates in the theoretical pressure curve. For practical purposes the other influences only the initial pressure rise.

$$\frac{d^2 x}{d\tau^2} + 2\alpha \frac{dx}{d\tau} + x = e^{-\mu_1 \gamma_1 \tau} \cos \gamma_1 \tau - e^{-\mu_2 \gamma_2 \tau} \cos \gamma_2 \tau^* \qquad [13]$$

The solution of this differential equation is**

$$x = \frac{e^{-\mu_1 \gamma_1 \tau}}{\rho_1} \cos(\gamma_1 \tau - \psi_1) - \frac{e^{-\mu_2 \gamma_2 \tau}}{\rho_2} \cos(\gamma_2 \tau - \psi_2) + A e^{-\alpha \tau} \cos(\beta \tau + \varphi) \qquad [14]$$

---

* Translator's Note: In Equation [13] x no longer has the significance of diaphragm displacement, but is now the dimensionless ratio $\frac{\text{diaphragm displacement}}{\text{static deflection}}$. This change of meaning is not mentioned in the text.

** Translator's Note: Various typographical errors in Equations [14], [14a], and [14b] in the original have been corrected in the translation.
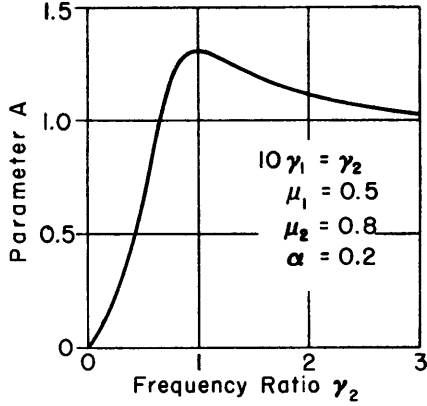
Figure 14 - The Amplitude of the Free Vibration in an Excitation Produced by the Sum of Two Damped Cosine Vibrations

Figure 15 - The Angle of Phase of the Free Vibration in an Excitation Produced by the Sum of Two Damped Cosine Vibrations

Equations [6a] and [6b] are valid for ρ_1, ρ_2, ψ_1, and ψ_2. When $\tau = 0$, there results from the initial conditions $x = \dfrac{dx}{d\tau} = 0$

$$A = \frac{1}{\cos\varphi}\left[\frac{\cos\psi_2}{\rho_2} - \frac{\cos\psi_1}{\rho_1}\right] \qquad [14a]$$

$$\tan\varphi = \frac{\dfrac{\gamma_1\sin\psi_1}{\rho_1} - \dfrac{\mu_1\gamma_1\cos\psi_1}{\rho_1} + \dfrac{\mu_2\gamma_2\cos\psi_2}{\rho_2} - \dfrac{\gamma_2\sin\psi_2}{\rho_2}}{\beta\left(\dfrac{\cos\psi_2}{\rho_2} - \dfrac{\cos\psi_1}{\rho_1}\right)} \frac{\alpha}{\beta} \qquad [14b]$$

So that the sum of the vibrations expressed in the right-hand term of [13] will sufficiently approximate the pressure change in atmospheric shock waves, γ_1 must be small with respect to γ_2. Then $n\gamma_1 = \gamma_2$ for $n > 0$. In addition, the conditions $n \geq 10$ and $\gamma_2 \leq 3$ are introduced. All the previous assumptions can be expressed by

$$n\gamma_1 \leq \gamma_2 \leq 3 \qquad [15]$$

These assumptions permit considerable simplification of Equations [14], [14a], and [14b], without limiting this investigation, since when $n = 10$, the ratio of the duration of the pressure rise to that of the pressure drop is about 1:8. This ratio is attained in atmospheric shock waves only at great distances from the point of detonation. At the distances used in practice this ratio is always smaller. Moreover, let $\alpha = 0.2$, $\mu_1 = 0.5$, $\mu_2 = 0.8$. Considering the assumptions which have been made, $\rho_1 \approx 1$, $\varphi \approx 0$. Hence, Equations [14a] and [14b] can be restated as

$$A = \frac{\cos\psi_2 - \rho_2}{\gamma_2\cos\varphi} \qquad [16]$$

$$\tan\varphi = \frac{n_2\gamma_2\cos\varphi_2 - \gamma_2\sin\varphi_2 n_2\gamma_1\rho_2}{\beta(\cos\varphi_2 - \rho_2)} \frac{\alpha}{\beta} \qquad [16a]$$

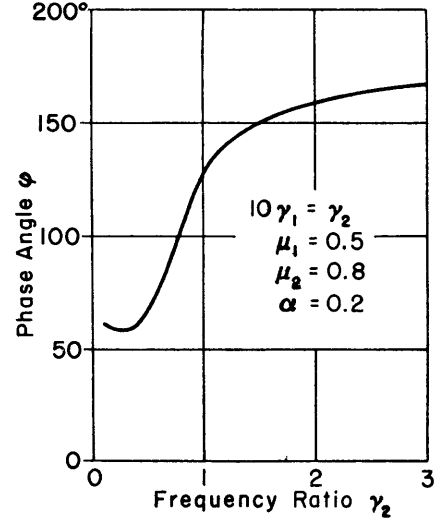Figure 14 shows the curve of the amplitude of the free vibration as a function of γ_2,

corresponding to Equation [16] and Fig-
ure 15 shows the angle of phase φ which
is derived from Equation [16a]. Figure
16 shows the behavior of a diaphragm ex-
cited by a pressure, which corresponds
to Equation [13].

The calculations were based
upon the values $10\gamma_1 = \gamma_2$, $\alpha = 0.2$,
$\mu_1 = 0.5$, $\mu_2 = 0.8$.

It should be noted that the
amplitude of the free vibration tends
toward zero for small values of γ_2, in
contrast to the corresponding amplitude
expressed in [12], which approaches the
value $\frac{1}{\beta}$ for small values of γ.

Curve a of Figure 16 repre-
sents the time curve of the exciting
pressure. Curves b to i are the time-
deflection curves for various natural
frequencies of the diaphragm. For the
time-deflection curves b and c, $\gamma_2 =$
0.1 and 0.2 respectively. They repro-
duce the pressure curve without dis-
tortion. In curve d ($\gamma_2 = 0.3$) the
influence of the free vibration begins

Figure 16 - The Forced Vibrations of a
Diaphragm in an Excitation Produced by
the Sum of Two Damped Cosine Vibrations

to appear. It is so small, however, that at this frequency the diaphragm movement
practically corresponds to the pressure curve. In curve e ($\gamma_2 = 0.4$) the influence
of the free vibration is somewhat more evident in the region of the first maximum
value. That portion of the curve which follows the peak is undistorted. In curves
f to i, Figure 16, the influence of the free vibration becomes more apparent. Con-
sequently these curves deviate more and more from the curve of the exciting pressure.

According to the chosen analytical expression for the pressure curve and
to the limitations of [15], the time necessary to reach the first maximum value t_m
is somewhat less than half the period of the vibration corresponding to γ_2. If the
latter is designated as $\frac{T_2}{2}$, then

$$t_m \approx \frac{T_2}{2} \qquad [17]$$

According to Figure 16, the frequency ratio γ_2 must be less than 0.3 to
get an undistorted recording. If the period of the diaphragm is designated by T_0,
then

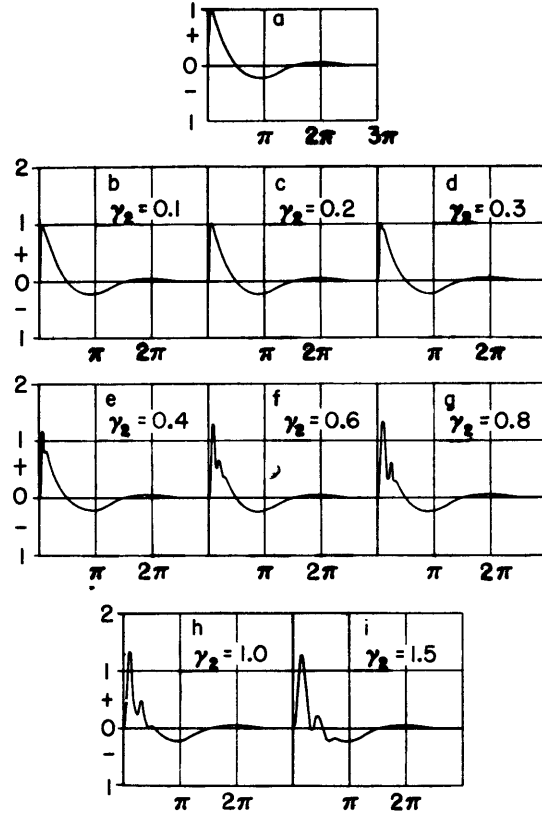$$\frac{T_0}{T_2} \leqq 0.3 \qquad [18]$$

By substituting the value of T_2 from [17] it is obvious that

$$T_0 \leqq 0.6 \, t_m \qquad\qquad [19]$$

which is necessary for an undistorted recording. Strictly speaking, this result follows from the assumption that $10\gamma_1 = \gamma_2$, and therefore is valid only for a definite ratio between the time consumed in the pressure rise and the pressure drop. This ratio is about 1:8. However, Equation [19] remains valid for practical purposes when $n\gamma_1 = \gamma_2$, if $n > 10$. The following considerations should prove this.

The chosen pressure curve satisfies the function p.

$$p = e^{-\mu_1 \gamma_1 \tau} \cos\gamma_1\tau - e^{-\mu_2 \gamma_2 \tau} \cos\gamma_2\tau$$

Assuming $10\gamma_1 = \gamma_2$, $\mu_1 = 0.5$ and $\mu_2 = 0.8$, the second term, for practical purposes, covers only the range in which the first term has not yet appreciably changed. The influence of the second term here extends only from $\tau = 0$ to the time point of the first maximum value τ_m. If the values of μ remain constant and if it is assumed $n\gamma_1 = \gamma_2$, this is all the more valid, as long as $n > 10$. In Figure 16 the curve of p, assuming that $n\gamma_1 = \gamma_2$ ($n > 10$), changes practically only in the time interval from 0 to τ_m, i.e., only the pressure rise changes. In this process the ordinate values τ are shifted toward $\frac{10}{n}\tau$. Likewise, the time point τ_m corresponding to the highest value moves toward $\frac{10}{n}\tau_m$. The change of p with respect to n is evident practically only in the initial rise of p. The fall of the pressure curve remains the same, so that with respect to n, the ratio of the time of pressure rise to the time of pressure fall changes. As n increases, this ratio becomes smaller.

The variations in the time-displacement curves produced by such a variable pressure are clearly evident also. The influence which γ_1 exerts on the angle of phase φ, Equation [16a], is already so small when $10\gamma_1 = \gamma_2$ that no important change occurs from the assumption that $n\gamma_1 = \gamma_2$, (when $n > 10$). The same holds true for the amplitude A according to Equation [16]. Therefore the change in the displacement-time curves which are determined by the new assumption, when $\gamma_2 \leqq 0.3$, is only evident when the ordinate values are shifted from τ to $\frac{10}{n}\tau$. Here also, this shifting extends practically to the time point τ_m. Hence, the displacement-time curves for $\gamma_2 \leqq 0.3$ record the pressure change without distortion when $n\gamma_1 = \gamma_2$ ($n > 10$). Accordingly Equation [19] is valid for every ratio of the duration of pressure rise to that of fall which is smaller than 1:8. The following is a summary of the natural frequencies of diaphragms at varying periods of pressure rise, which are, according to [19], necessary for undistorted recordings of atmospheric shock waves.

t_m (10^{-4} sec)	10	5	2	1	0.5	0.2
$1/T_0$ (cycles/sec)	1660	2300*	8400	16500	33000	34000*

* Translator's Note: These figures are obviously erroneous. Recalculation at the Taylor Model Basin shows that 2300 should be 3340, and 34000 should be 835000.

VII. RESULTS OF INVESTIGATION ON ATMOSPHERIC SHOCK WAVES

Investigation of the behavior of vibratory systems under the influence of damped sinusoidal vibrations produced Equation [19]. This relationship between the characteristic duration of the initial pressure rise in atmospheric shock waves and the natural period of a diaphragm required for undistorted recordings is extremely important. Formerly, lacking other means, the natural frequency of a diaphragm necessary to make undistorted recordings of atmospheric shock waves was determined in the same way as is customary in the case of undamped waves. It was shown at the outset of this investigation that this method can lead to erroneous conclusions. In addition the method of determining the frequencies ascribed to an atmospheric shock wave is somewhat arbitrary. These uncertainties are now obviated. By the derivation of Equation [19] the attempt was made for the first time to use a method which conforms to the characteristic curve of atmospheric shock waves.

The essential results of these studies will be briefly and definitively summarized. Other similar studies will be included.

The behavior of vibratory systems during the transient stage of vibration produced by undamped sinusoidal forces form the point of departure for these investigations. H. Martin (2) made similar studies in the field of seismometry. The difference between Martin's curves and those of the present study is explained on pages 9 and 10. Although these studies are necessarily based chiefly on damped sinusoidal forces which correspond to the pressure change in atmospheric shock waves, the behavior of vibratory systems influenced by undamped sinusoidal excitations is treated for two reasons. First, to contrast the different behavior of a diaphragm during the transient stage of vibration with that of the subsequent period of steady vibration; second, to facilitate a comparison with the behavior of vibratory systems excited by damped sinusoidal forces. No studies of excitation by damped sinusoidal vibrations have been undertaken up to the present, to judge by the literature. Kalähne (3) gives the solution for the general equation of motion when the exciting force is a damped sinusoidal vibration. He begins with the statement

$$\frac{d^2 x}{d t^2} + \frac{2\delta\, dx}{d t} + n^2 x = A e^{-\epsilon t} \sin k t$$

Since the damping of the exciting force is independent of its frequency in his equation, Kalähne's solution is valid for those exciting forces whose amplitudes are limited by the function $\pm A e^{-\epsilon t}$. Hence, the damping of the exciting force varies inversely as the frequency k. With respect to the phase displacement, Kalähne says that the damping of the exciting force is therefore equal to that of the natural vibrations of the excited system when $\epsilon = \delta$. According to Kalähne's statement, this conclusion is not generally valid, but only for the special case when the frequencies of both vibrations are equal.

No further studies are known. Aside from a brief reference in "Handbuch der Experimentalphysik," vol. 17, p. 59, there is nothing to be found in the latest

handbooks of physics which report on the most recent state of research. The fore-going reference states: "Conditions remain essentially the same for an excitation by a damped sinusoidal vibration as for an undamped one, considering the first moments of the initial and final vibration periods are kept in view. They become complicated by the appearance of fluctuations since the transition between the exciting and the excited vibrations becomes active." This opinion, which is very general, appears to need supplementation to prevent the development of erroneous ideas. The present investigation gives a suitable basis for this purpose.

The behavior of vibratory systems under the influence of decaying sinusoidal forces differs from that of undamped excitations in many respects. In an undamped excitation a condition develops in which the vibrations remain steady, if a damping of the vibratory system is present. When this condition is reached, the further vibrations of the system coincide in period with those which are exciting it. The phase and amplitude can still be different. Hence, by properly shifting the time axis and by using a suitable scale on the ordinate axis of the recording of the forced vibrations, the exciting vibrations are obtained at the same time. The determination of the phase and of the conversion factor offer no difficulties in these cases, since all the values necessary for their calculation can be ascertained.

In contrast to this, no condition in which the vibrations remain steady develops in the case of excitation by damped vibrations. Hence it is impossible to measure the exciting forces by waiting for extinction of the free vibration. An undistorted recording of damped vibrations is only possible, if the free vibration is negligible from the very outset. Hence, the natural frequency of a vibratory system, which is to serve as a measure of damped sinusoidal vibrations, must be much higher than is necessary for one measuring similar undamped vibrations.

The curve of the angles of phase ψ and φ as a function of the frequency ratio γ likewise differs in undamped excitations from that of damped ones. These angles are only continuous in two quadrants in undamped excitations, but are continuous in all four in damped excitations. A gradual increase of the amplitudes, similar to the resonance phenomenon of an undamped excitation, appears also in the operation of damped sinusoidal forces or vibrations. However, in such case, the amplitudes cannot increase indefinitely, but after a given time which depends on the damping, they again decrease. The maximum amplitude is governed by the damping and varies inversely as the damping. This phenomenon occurs in damped sinusoidal excitations if the damping factor and the frequency of the exciting force are equal to the damping factor and the damped natural frequency of the vibratory system.

The results of this investigation will be welcomed, perhaps, by some branch of physics or other, and some things, which were not particularly stressed in the general summary, will seem important, depending on practical needs. Conforming to the stated object of this study, an evaluation with respect to atmospheric shock waves, however, must follow this summary.

The demands made upon a diaphragm for the undistorted recording of atmospheric shock waves are greater than could be deduced from the theory of the action of undamped forces. For an undistorted recording of atmospheric shock waves, the period of vibration of a diaphragm, T_0, must be smaller than the duration of the initial pressure rise. This is expressed by

$$T_0 \leqq 0.6 \, t_m$$

If this condition is not fulfilled, the recordings of a diaphragm give no true picture of the atmospheric shock wave acting on it. Vibrations appear which are not contained in the atmospheric shock wave but which represent natural vibrations of the diaphragm. The smaller t_m becomes with respect to T_0, the more marked the natural vibrations become. Finally, only natural vibrations will be set up in a diaphragm acted upon by an atmospheric shock wave, over which the shock wave itself will be superposed with hardly noticeable effect.

For the special case in which the pressure in the atmospheric shock wave rises very fast but subsequently changes very little within a given time, i.e., when the time curve roughly corresponds to a damped cosine vibration, the pressure curve can be ascertained rather simply, if the damping factor of the diaphragm is known.

The maximum pressure in an atmospheric shock wave can be satisfactorily determined solely by means of static calibration only if the condition $T_0 \leqq 0.6 \, t_m$ is satisfied. If it is not satisfied two cases arise, which are distinct from each other. First, if T_0 is only slightly larger than $0.6 \, t_m$, the maximum pressure derived from the amplitude is greater than in reality. Second, if T_0 is much greater than $0.6 \, t_m$, then the maximum pressure derived by the same method is smaller than in reality.

Still another result of this investigation deserves special emphasis. A glance at Figure 13 shows that above a given frequency ratio, as shown by curves g to k, the movement of a diaphragm excited by shock loads produces comparatively steady vibrations. By means of only a small number of equidistant ordinates, such vibrations can be expressed with good approximation by a Fourier series. The pressure curve can then be derived by differentiation. This possibility has been widely used in practice, and will be taken up later.

It scarcely needs to be mentioned that these studies can be effectively used wherever atmospheric shock waves affect buildings or other structures, and where the effects must be judged either before or afterward, for example, to predict the effect of aerial bombs on buildings, or to judge and settle purported claims of damage to buildings arising from artillery practice, when these buildings are far from the artillery range. As experience has shown in many damage suits arising from the firing of large guns, the problem of the resonance phenomena in the action of atmospheric shock waves was of great importance. This problem was closely studied on page 10, section IV. Formerly, through lack of thorough studies of this problem, the wildest

opinions were advanced. Pursuit of this question further would lead to too great digression and it will therefore be reserved for special treatment.

VIII. QUALITATIVE PROOFS OF GOOD AGREEMENT BETWEEN THE CURVES
USED FOR CALCULATION AND THOSE IN ATMOSPHERIC SHOCK WAVES

These studies would find no extensive application without experimental proofs of good approximation between the assumed pressure curve and the pressure variation in atmospheric shock waves. No quantitative proof for this agreement in close proximity to the detonating point can yet be given. Hence a qualitative proof will have to suffice at this point.

Figure 17 shows the records of experiments in the measurement of pressure at a distance of 10 m (32.80 feet) from the point of detonation. The charge detonated was 1 kg (2.20 pounds) of trinitrotoluol (TNT). The movement of the diaphragm was optically recorded by a mirror. Figure 17a shows the results obtained by using a diaphragm with a natural frequency of 1300 cycles per second. Figures 17b and 17c refer to diaphragms with natural frequencies of 2400 and 8000 cycles per second respectively. The close similarity between the calculated curves and the recorded ones indicates a qualitative agreement of the assumed pressure curve with that appearing in atmospheric shock waves.

Moreover, it is evident from Figures 17a and 17b that the atmospheric shock wave has excited the diaphragm to practically only natural vibrations, superposed over which the atmospheric shock wave is hardly noticeable. This result is to be expected from the foregoing theoretical observations on diaphragms of comparatively low natural frequency. In Figure 17c the pressure curve of the atmospheric shock wave is correctly reproduced up to the initial pressure impact, which agrees with theoretical studies of diaphragms with
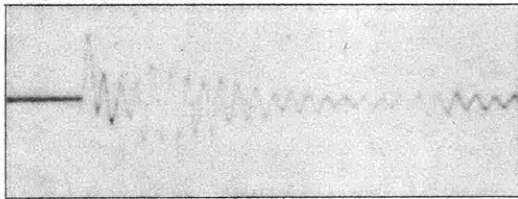
a) Natural Frequency of the Diaphragm
1300 cycles per second
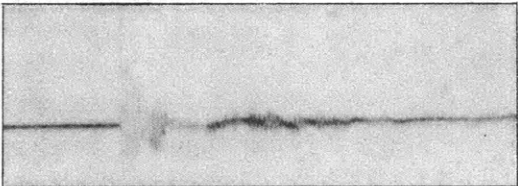
b) Natural Frequency of the Diaphragm
2400 cycles per second
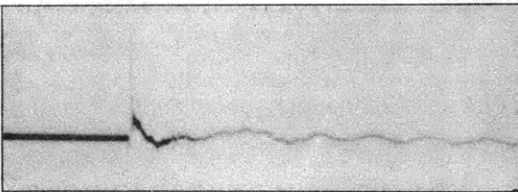
c) Natural Frequency of the Diaphragm
8000 cycles per second

Figure 17 - Experimental Recordings of
Atmospheric Shock Waves

Diaphragms were at 10 m (32.80 feet) distance
from the point of detonation. Charge: 1 kg
(2.20 pounds) of trinitrotoluol.

higher natural frequencies. The first pressure impact, however, is very sharply distorted by natural frequencies of the diaphragm.

The knowledge that a diaphragm with a natural frequency of 8000 cycles still does not record the pressure curve of an atmospheric shock wave without distortion can be used, with the help of Equation [19], to determine an upper limit for the initial pressure rise. Hence, at a distance of 10 m (32.80 feet) from the point of detonation, with an explosive charge of 1 kg (2.20 pounds) of trinitrotoluol (TNT), the time needed for the initial pressure rise in the atmospheric shock wave is certainly less than 2×10^{-4} second. It is evident, therefore, that with the help of the present theoretical investigation very important information can be derived from the recorded curves of atmospheric shock waves, which are still quite defective near the point of detonation.

The question now arises why diaphragms with even higher natural frequencies have not been used to record atmospheric shock waves. The calculated curves, compared to the actual records, Figure 17, obviously require the use of such diaphragms.

The use of diaphragms of higher natural frequency leads to other difficulties. As the frequency increases the sensitivity decreases sharply. Basically, this decrease of sensitivity can be removed by inserting a system of levers between the diaphragm and the axis of the mirror. This method, however, is not feasible in the immediate vicinity of the detonating point for recording atmospheric shock waves. The magnification of the masses participating in the movement of the diaphragm, which occurs when a system of levers is inserted, renders the apparatus sensitive to explosive concussions transmitted through the ground or caused by the impact of the wave.

The low sensitivity of high frequency diaphragms can also be compensated by using electronic tubes. However, previous experiments have shown that the tubes themselves are so violently shaken by the impact of atmospheric shock waves that obvious field distortions appear. This is true even when the tubes are mounted in the best shock-proof manner. In any event, it frequently happens that at the moment of impact of the wave large displacements in one direction occur in the record, and then gradually diminish. This phenomenon cannot be traced to overloading the tubes, for it is not observable at a correspondingly high steady-state load.

The author has only recently overcome these difficulties. Experiments using a condenser microphone with a solid dielectric have produced great progress in the measurement of the pressure in atmospheric shock waves. This will now be treated in detail.

IX. THE MEASUREMENT OF PRESSURE IN ATMOSPHERIC SHOCK WAVES
     AT GREATER DISTANCES FROM THE POINT OF DETONATION

At greater distances from the point of detonation, where the atmospheric shock wave has already lost much of its initial power and where conditions to record

it are more favorable, the measurement of its pressure is more readily possible. The first measurements of this type were performed by W. Wolff (5). The motion of the midpoint of the rubber diaphragm which he used was transmitted to a scriber by a system of levers and recorded on a drum upon which was stretched paper coated with lamp black. The corrections for inertia and frictional forces of the diaphragm devised by Wolff to correct his recordings later proved unreliable.

F. Ritter (6) (7) subsequently recorded atmospheric shock waves at distances of from 500 to 2000 m (1640 to 6560 feet) from the point of detonation with improved apparatus, and determined their pressure readily and correctly. In constructing his apparatus he utilized the condition previously mentioned, that the time-displacement curves are comparatively smooth at definite natural frequencies of diaphragms. The apparatus which Ritter constructed is a diaphragm device equipped with lamp-black recording. The instrument consists of an airtight case, made in the shape of a parallelepiped. A round, wood diaphragm is fitted into one side of the case. The actual recording mechanism is housed inside the box. It consists of a drum, on which paper coated with lamp black is stretched; the drum is turned by clockwork. By a system of levers, the movement of the midpoint of the diaphragm is magnified about ten times and transmitted to a scriber which rests on the drum. The system of levers is equipped with a damper vane which is immersed in a vessel containing oil.

Even at great distances from the detonating point, which vary from a few hundred to a few thousand meters depending on the size of the explosive charge, the pressure rise of the first pressure impact of the atmospheric shock wave still occurs so swiftly that records made with Ritter's apparatus give no true picture of the atmospheric shock wave. Therefore Ritter (7) constructed a special mechanism for dynamic calibration, in order to evaluate correctly these distorted recordings. This device permits the maximum pressure in the atmospheric shock wave to be determined. However, during the first pressure impact, at least, the time-pressure curve remains out of phase. At the suggestion of F. Ritter, I then attempted to ascertain whether the pressure curve of atmospheric shock waves could not be calculated from distorted recordings by a method similar to that used for recoil measurements (8). The following reflections form the basis for these studies.

In cyclic sound pressures the acceleration and velocity of the diaphragm assume considerable proportions. Therefore, in addition to the third, the first two terms of the general equation for motion, Equation [1], must be considered. To do this, the recorded curve is developed in a Fourier series and the acceleration and velocity are determined by differentiating the time-displacement curve. By this method the time curve of the atmospheric shock wave can be determined up to the very swiftly rising initial pressure impact. The pressure in the initial impact is far greater than in the subsequent portions of the atmospheric shock wave. Hence it represents the maximum pressure, which is determined by a special method. First, the maximum pressure is assumed to be that one which is obtained by extrapolation of the

pressure curve derived from the time-displacement curve by differentiation. The total pressure curve thus obtained is then developed in a Fourier series; the time-displacement curve belonging to it is calculated and compared to the recorded one. If the calculated and recorded time-displacement curves still do not agree, the maximum pressure obtained by extrapolation is successively changed until complete agreement is achieved. This method of calculation is described in detail in the Zeitschrift für Physik (9); hence, it will not be treated further here. By this method, the pressure curve has been determined in numerous cases, resulting in a deeper knowledge of the phenomena in the propagation of atmospheric shock waves.

Endeavoring to cut down the labor of calculation required by the method mentioned, new experiments were undertaken whose object was to develop a new instrument to record atmospheric shock waves at greater distances from the point of detonation. By considering the aggregate advantages and disadvantages of the various methods under consideration, the use of a condenser microphone seemed most advantageous. Its wide use in the field of electro-acoustics, which proved favorable to research on condenser microphones, is the reason for this. Moreover, the techniques for its dynamic calibration have been worked out to high frequencies.

Tests with a standard measuring instrument, used in the field of electro-acoustics, proved unsatisfactory. First the sensitivity was too great for our purposes, and second, the recorded curves showed too great a variation with frequency. Then, at our suggestion, the central laboratory of the firm of Siemens built condenser microphones to our specifications. These, too, did not at first record the cyclic progress of atmospheric shock waves. Finally, after many tests and gradual improvement, a measuring apparatus was developed, whose wiring diagram is shown in Figure 18. The pressure wave recording apparatus contains the condenser microphone connected in a high frequency circuit. This high frequency hook-up was chosen, in order to effect static calibration to check on the instrument. The apparatus consists essentially of an oscillating circuit, a tank circuit, and an amplifying circuit. A high frequency alternating current which is transmitted to the tank circuit by a loose coupling is induced in the oscillator. The microphone is connected in parallel to a suitable variable condenser in the tank circuit.
The variable condenser is installed in such a way that its capacity corresponds to a point on the resonance curve, which is as steep as possible and linear for a short distance. Generally, this is the case at about two-thirds of the height of the maximum of resonance. When an atmospheric shock wave impinges on the microphone, the change in capacity
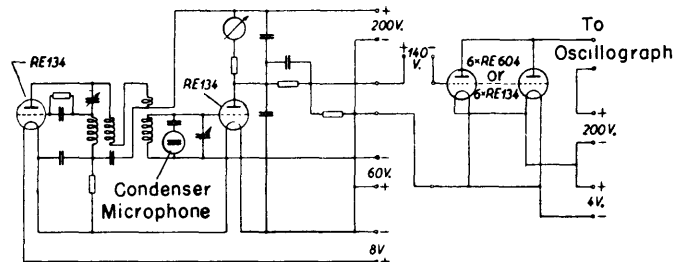

Figure 18 - Measuring Apparatus for Pressure Waves with Condenser Microphone in a High Frequency Circuit
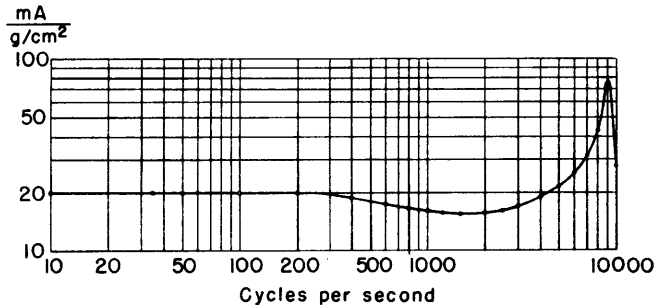
Figure 19 - Calibration Curve for the Pressure
Wave Measuring Apparatus with a Condenser
Microphone in a High Frequency Circuit

results in a change in the current in the resonance circuit. The voltage change, which is proportional to the current change, is transmitted to the grid of a vacuum tube. The low frequency anode discharge of this tube is amplified to such an extent that it can be . recorded by an oscillograph. The oscillograph element used had a frequency of 12000 cycles per second.

The complete measuring instrument including the amplifying unit and the oscillograph element was dynamically calibrated by the firm of Siemens. Figure 19 shows the calibration curve finally attained by gradual improvement of the original apparatus. It can be concluded from this curve that pressure fluctuations of frequencies from zero to 4500 cycles per second are recorded with a margin of error of ± 10 per cent. What an improvement this apparatus signifies can be best shown if it is compared to that described by Zettel in the 29th annual volume of the Zeitschrift für das gesamte Schiesz- und Sprengstoffwesen. The calibration curve reproduced in that article under the title of Figure 6 shows that Zettel's measuring device is one-third as sensitive for frequencies of 100 to 1000 cycles and only one-fifth as sensitive for a frequency of 2000 cycles, as it is for frequencies of about 5 to 40 cycles. It is impossible to record without distortion the impact-like atmospheric shock waves with a measuring apparatus whose response to frequency is so variable. For that reason Zettel's recordings do not show the characteristic curve of atmospheric shock waves. The very rapid rise of the initial pressure impact is suppressed, and the amplitudes of the subsequent slow pressure changes occurring in the atmospheric shock wave are recorded greater than they really are.

Our first experiments with condenser microphones yielded very similar results. Recordings of atmospheric shock waves were successfully made with an apparatus whose calibration curve is shown in Figure 19. Figure 20 shows two of these curves. The distances from the point of detonation were 160 m (524.93 feet) and 240 m (787.39 feet) respectively; the detonating charge was 1 kg (2.20 pounds) of trinitrotoluol. In contrast to Zettel's recordings, the characteristic pressure curve is clearly evident. A very rapid pressure rise is followed by a slower pressure drop. The latter is followed by a negative pressure. The positive pressure is greater than the negative. However, the duration of the positive pressure is less than that of the negative.

One kilogram (2.20 pounds) of trinitrotoluol was exploded and the maximum pressure recorded. At 160 m (524.93 feet) from the point of detonation the pressure

was 2.7 g/cm² (0.039 pound per square inch); see Figure 20a; at 240 m (787.39 feet) it was 1.7 g/cm² (0.0258 pound per square inch); see Figure 20b. The decrease of pressure between these two distances is quite closely proportional to the distance.* Pressure changes of 1000 cycles per second are still superposed on the atmospheric shock wave at both distances. A satisfactory explanation of the origin of these vibrations is not yet possible. However, it can be reliably assumed that they are inherent in atmospheric shock waves and were not the deceptive result of peculiar properties of the measuring apparatus, for there are no parts of the measuring apparatus subject to vibration whose natural frequency is approximately 1000 cycles per second. The closest natural frequency to 1000 cycles present in the measuring apparatus is that of the microphone; its frequency is about 10,000 cycles.

a) Distance from the Point of Detonation
160 m (524.93 feet)

b) Distance from the Point of Detonation
240 m (787.39 feet)

Figure 20 - Atmospheric Shock Waves
Recorded with a Condenser Microphone

Detonating Charge: 1 kg (2.20 pounds)
of trinitrotoluol.

On account of the special conditions under which the recordings have to be made it is difficult to use a higher film speed than that here used. The rapidity of the initial pressure rise can no longer be read with certainty at the film speed used. The initial pressure rise takes place from 3×10^{-4} second to 4×10^{-4} second. In this respect it must be considered that in spite of the sensitive diaphragm and limited movement of about 1μ of the condenser microphone used, a certain phase lag will still persist. In the case of undamped sound vibrations, the phase lag could easily be calculated from the frequency of the sound vibrations and the natural frequency of the diaphragm. There is still no satisfactory method known of calculating the phase lag for atmospheric shock waves.
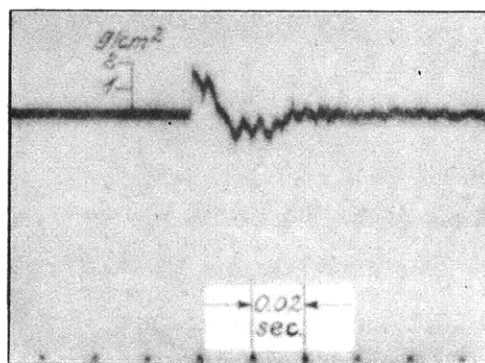
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* Translator's Note: The maximum pressures in these two records satisfy the condition

$$P_{max} = \frac{20.4}{L}$$

where P_{max} is the maximum pressure in pounds per square inch and L is the distance from the charge in feet.
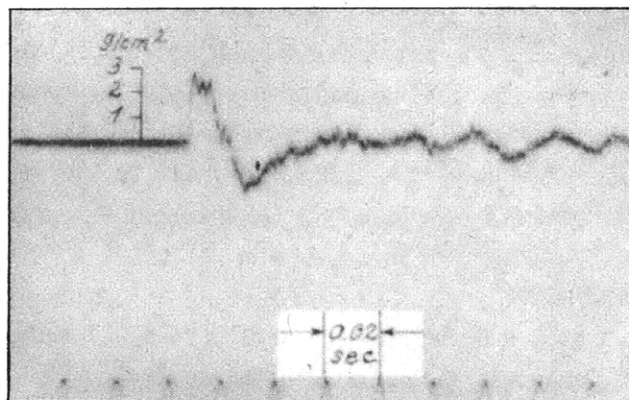
The present investigation permits a deeper insight into the behavior of atmospheric shock waves, which was previously difficult to survey. A number of problems have been cleared up by theoretical study, and progress has been made in the technique of measurement. That several questions remain unanswered is due to the peculiar character of atmospheric shock waves and to the special difficulties which attend their measurement.

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