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INVESTIGATIONS OF SHIPS IN STEERING MANEUVERS

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A. Introduction

The incentive for the investigation carried out in the present paper was supplied by the fact that up to this day it is not yet possible to compute in advance and with requisite accuracy the forces acting upon a rudder. The reason for this is that although the nature of the factors governing the forces is known, the order of magnitude of their effect is not. Thus there exists a need requiring to be filled.

Since measurements of rudder torques carried out on several vessels of the German Navy were available, and since in addition it was possible by means of further tests to determine the values still needed to formulate an opinion it seemed worth while to utilize these tests for determination of the numerical influence of the individual factors, and beyond this, for setting up a universally valid method of calculation.

The determination is carried out by comparing the rudder moment values known from measurements with those found by calculation. Since for one thing it was necessary to carry out the first calculation with certain assumptions as to the basic behavior of the various factors, comparison between measured and calculated values permitted a conclusion as to the correctness of the assumptions made. Using the assumptions found to be correct for one steering mechanism, comparative calculations were then carried out for the other mechanisms. Some of the various ships and their steering mechanisms differed from one another in essential points, so that it was possible by this procedure to eliminate the influences peculiar to the various mechanisms from the factors governing the rudder forces and moments, and to find their basic curves.

These comparative calculations showed that it is possible to formulate a generally valid method of calculating loads in steering mechanisms in advance if the time curve of the so-called "operating conditions" of a ship in a steering test is known. By "operating conditions" are understood the following values: Velocity of the ship, v, in m/sec; the turning speed of the ship, ω, in $\sphericalangle^{\circ}$/sec; the position of the center of curvature of the course at any given moment, M; the position of the apparent center of rotation of the ship, D and the rudder angles: α in $\sphericalangle^{\circ}$. Since heeling has no influence as far as these considerations are concerned, it will not be considered.

Therefore, before it was possible to begin calculating the rudder torques with some prospect of success, it was first necessary to determine the time curve of the running characteristics during a steering test. To do this it was required to develop test methods, some of them new. Furthermore, the test accuracy to be attained with these and the method already familiar had to be investigated.

At the same time it was necessary to investigate the test accuracy of the various test methods for determining the rudder torque, in order to be able to obtain an idea of the differences resulting in comparison of calculated and measured values.

In order to avoid having the paper become too voluminous, no description nor study of the test methods used for determining the running characteristics is given in the present publication. Only results are given. On the other hand, the test methods for determining the rudder torque are discussed in detail.

Results obtained by the research are:

1. A method of calculating the loads in steering mechanisms while running forward or backing; also:

2. Study of the test methods now in use for determining the rudder torque,

3. Determination of the angle of repose in the main pieces of the rudder, and

4. Studies of torsional vibrations in the rudder post.

B. Effect of Running Characteristics on Rudder Force

As is shown in the paper, the pressure of the rudder can be computed as a force resulting from the lift and drag force of the rudder considered as a section in flow. To compute these forces it is necessary to know in addition to the actual rudder characteristics, – area, aspect ratio, section, – the velocity of the inflowing water both as to direction and magnitude. Since the rudder values are fixed by the design of the rudder, the ultimate problem of the test is to determine the velocity of flow at the rudder both as to magnitude and direction.

Measurement of these values directly on the rudder is difficult, and probably in no wise possible with sufficient accuracy.

The velocity is therefore measured at another point on the ship more favorable for measurement. This value and the speed of advance of the propellers determined from the shaft R.P.M. permit a posteriori conclusions as to the water velocity at the rudder.

Much more difficult is the determination of the direction of flow into the rudder. The angle between the longitudinal axis of the ship and the flow changes rapidly during the course of a steering test and is susceptible of measurement only with difficulty. Therefore it is determined indirectly by measuring several auxiliary values. This calculation is based on the following reasoning:

During completion of the turning circle, the individual points of the ship move in concentric circles about the center of curvature of the turning circle. It is to be investigated in what direction the individual points – and above all the rudder – move with respect to the water at rest. First, every point has a velocity in the direction of the keel line of the ship; this velocity is uniform for all points. In addition every point has a transverse velocity perpendicular to the velocity in the direction of the keel line; this velocity varies for each point, both as to direction and magnitude. For a single point of the ship this transverse velocity equals zero, this being the basal point of the perpendicular dropped from the center of curvature of the path of the center of gravity to the keel line. The parts of the ship forward of this point have a velocity running into the circle, those abaft this point have a transverse velocity directed outward. The magnitude of the transverse velocity depends on the distance of the point considered from the base point of the perpendicular. The ship appears to carry out a turning motion about this basal point. In order to distinguish it from the center of the path, this point will hereinafter be known as the "turning point" of the ship.

Thus the transverse velocity v_p of each individual point of the ship may be regarded as the peripheral velocity on a circle, whose radius (r) is equal to the distance of the point in consideration from the turning point. Then $v_q = r\ \omega$, when ω is the angular turning velocity of the ship.

Thus both the velocity components of each point of the ship are known. By vectorial addition of these two values there results the relative velocity between the water at rest and the point under consideration. The angle δ between the keel line and the resultant direction of motion is obtained from the relation

$$\mathrm{tg}\ \delta = v_q/v.$$

The angle of incidence α' to the rudder is $\alpha' = \alpha \pm \delta$ (see Fig. 1), depending upon what kind of maneuver is being executed, – a turning circle or meeting the helm.

For determining the angle of incidence, then, the values v, ω, α, r and the diameter of path curvature are required.

These values change rapidly during the course of a steering test. Since in addition they are highly dependent upon each other, unequivocal conclusions from the knowledge of their values at any given moment are possible only when it is assured by the test method that values corresponding in time are measured simultaneously. The most important prerequisite then, is synchronism of the measured values.

Since the phenomena in which we are interested take place in a few seconds, all values are photographed, that is to say, by using transcription systems all the measured data are recorded on a common chart which is photographed at definite time intervals. This method has such great advantages over recording instruments or even readings and notes that it was used for the values here investigated in every instance where quick chronological changes were involved.

Without going into great detail as to the test methods, let us state that the ship velocity through the water was determined by means of a mercury differential manometer (see Fig. 2). The apparatus was calibrated on the measured mile. From suitable tests, moreover, its test accuracy, even when altering the course, was also known. At the same time the propeller R.P.M. had been measured.

To determine the diameter of the turning circle the path of the ship during the steering test was measured in the familiar manner from an auxiliary vessel at anchor.

The course of the ship at a given rudder angle was photographed with the other values in the chart at definite intervals in the test. From these values it was possible to obtain the angular velocity of the turning movement.

Since in addition absolute synchronism between the measurements aboard ship and aboard the auxiliary vessel was assured by suitable transmission of orders from the ship to the tender, it was possible to determine the turning point of the ship from the measured path and the course of the ship, which was known for all times from the photographs. As a matter of fact, the turning point of the ship is the basal point of the perpendicular dropped from the center of curvature of the path at any moment to the keel line. The turning point, as has also been shown by similar calculations carried out by another method, is comparatively far forward, i.e. about 1/6 to 1/4 of a shiplength from the bow. According to more recent tests carried out more accurately, it might possibly lie even forward of the bow. In the comparative calculations here carried out the distance of the rudder from the turning point of the ship is about: r = 100 m or 130 m.

C. Calculation and Measurement of the Rudder Torque

Before taking up the determination of the rudder torque and the calculation of the loads in the steering apparatus based upon the knowledge of the former, the most important viewpoints that find application in the designing of steering mechanism will be presented.

Decisive in determining the value of a steering mechanism, above all, is its reliability in operation. If in addition to this it is possible to achieve an increase in economy, this is a valuable gain, but not the essential property of the design, for after all, a single collision caused by failure of the steering mechanism can destroy values which not even the most economical mechanism can make good in years.

This high degree of reliability can be attained only when the loads possible under the most unfavorable circumstances are known. If these values are available there are no difficulties in attaining adequate economy of operation in addition to reliability in operation.

Since the most reliable and satisfactory method of determining these loads is still by measurement, it will be understood what importance attaches to the development of suitable test methods. Beyond this there is the need so to analyze the present test data that an adequately accurate and reliable method of calculating in advance the loads to be expected in newly designed mechanisms can be given.

In the following an investigation will be made as to the extent to which it is possible to achieve most favorable loads by suitable design of the individual parts of a steering mechanism.

1. a) THE RUDDER.

It is customary to determine the rudder area from the ratio of the long.sec. area to the rudder area for the various classes of vessels by empirical values. In changing course two sections in chronological order must be distinguished, the first beginning when the helm is put over and ending when uniform turning velocity of the ship ceases, while the second comprehends the section of uniform turning velocity. The time curve $\omega = f(t)$ and the absolute value of the turning velocity attained with a given rudder depends in the first section essentially upon the mass moment of inertia of the ship taken with respect to the turning axis at the moment, and in the second section upon the underwater form of the ship. The rudder pressure exerted by the rudder for the starting and maintenance of the turn depends on the velocity of the ship and the magnitude of the rudder area. The shape of the rudder, whether narrow or wide, is determined by the general design.

In dimensioning the steering equipment and the steering engine, it is not the rudder pressure so much as the torque on the rudder post that is decisive. This torque is governed essentially by the distribution of the rudder area with respect to the rudder post, i.e. the degree of balance.

Decision as to the balance most desirable for an apparatus can be arrived at only by considering the special conditions at the rudder. The difficulties of deciding upon the balance arise from the fact that large rudder forces are required in consideration of maneuverability not only in running forward but also in backing. A rudder well balanced for running forward, i.e. operating with small torques, will have considerably larger torques in backing than in running forward, and as a matter of fact the maximum loads will occur the sooner in backing and will be the greater in absolute quantities, the more the mechanism has been designed only from the point of view of obtaining an apparatus which will operate economically running forward. Nevertheless, in dimensioning the steering equipment and steering motor, the maximum loads occurring in backing are decisive.

Therefore when the greatest possible load on the mechanism is to be reduced, it will be necessary under certain conditions to discard the most favorable balance for running forward. This leads to the design of "overbalanced" rudders, i.e. rudders in which the center of pressure lies forward of the rudder post. Separated from the steering gear, such rudders would of themselves assume a certain steering angle under the influence of flow.

It depends upon the special characteristics of a design which viewpoints shall govern in individual cases. However, in order to render it possible to decide with adequate reliability, the position of the center of pressure on the rudder must be known. To what extent model test data and rudder moment measurements on ships supply needed information will be investigated later in this paper.

b) THE STEERING GEAR.

The design of the steering gear is determined by the two conditions under which the rudder must operate with complete reliability. This is the normal mode of operation: The rudder and the steering gear are driven by the steering motor, and the reserve drive, rudder and steering gear, is actuated by a handwheel. When the apparatus is geared to the handwheel, the steering motor having been incapacitated, the members of the crew working the handwheel must be protected against unintentional automatic shifting of the rudder. The steering gear must therefore be so constructed that the rudder, without continuous exertion of force either by the steering motor or the handwheel, will maintain any position once assumed. This can be achieved either by installing a brake or by designing the rudder post with an automatic lock.

Arguments in favor of the self-locking post are the experiences gained during many years of operation and the reliability obtained in operation.

In favor of the brake is the increase in economy attainable by its installation; for the choice between a post with or without an automatic lock simultaneously governs the efficiency of the apparatus; self-locking drives operate with efficiencies of $\eta > 0.5$, and non self-locking drives with efficiencies of $\eta < 0.5$. The efficiency is determined as

$$\eta = \mathrm{tg}\,\alpha'/\mathrm{tg}\,(\alpha' + \varrho),$$

when α' is the pitch angle and ϱ the angle of friction of the post. To determine the pitch angle α' therefore, the friction angle ϱ must be known. In general, this is the case to a certain degree of approximation: with constant RPM, normal load, and proper lubrication, ϱ lies between 4° and 5°. Then, in order to obtain automatic braking, α' must be less than ϱ. For reasons of safety, the pitch angle is made too small rather than too large. Through this, the losses in the post, already inherently large, are increased still further, and may, by inadmissible heating, give rise to interruptions of operation.

For this reason it is desirable to determine the characteristics of the friction angle in operation from tests with the steering engine and the steering post.

c) THE STEERING MOTOR.

Most modern mechanisms operate with electric motors. The following considerations therefore apply essentially to steering motors. In addition to knowing the torque curve at the rudder post and the frictional losses in the steering gear, it is required for the calculation of the steering motor to know the special working conditions of a steering mechanism.

Characteristic of the method of operation of the steering motor is the starting, repeated frequently at short intervals. The relatively high starting currents occur for only short periods, as do the maximum values in putting the helm over. Therefore it is permissible to make use of the short-time overload capacity of the electric motor. If, in addition, the maximum torque attainable in the mechanism is known from suitable balancing and córrect selection of the pitch angle of the spindle, it will be possible under certain condition to reduce the dimensions of the steering motor materially and still retain the requisite operating safety. Thus the centrifugal moment of the motor armature is decreased, and smaller currents will suffice to accelerate the masses in starting. The heating of the armature will fall even lower. In following a course these circumstances have an especially beneficial effect in the sense of improved economy, since on such runs the rudder is put over at small angles and thus the required output depends essentially only on the current consumption in starting.

The data required for a calculation of the steering motor which will take these conditions into consideration can best be obtained by tests of full-scale mechanisms. The results of such measurements will be given farther on in this paper.

2. DESCRIPTION AND INVESTIGATION OF TEST METHODS FOR DETERMINING THE TORQUE ON THE RUDDER POST M_{Sch} AND OF THE TORQUE ON THE MOTOR M_M.

Reference has already been made to the necessity of determining the loads in steering mechanisms by test. It is possible to measure the torque delivered by the steering motor and for mechanisms having a rudder post and connecting rods, the forces in the latter. The various test methods are described in the following, and investigated as to their reliability and test accuracy.

a) DETERMINATION OF THE TORQUE ON THE STEERING MOTOR.

Measurement of the torque in the steering motor would be carried out in the simplest way by determining the twist of a calibrated test shaft. Up to the present, this possibility has not been put into use, since space conditions do not permit of the installation of a suitable test apparatus.

Determination of the torque is effected by flow measurements. In detail, the tests are based upon the following wellknown relations between the currents and the torque of an electric motor:

The torque of an electric motor M_M depends upon the armature (J_a) and on the field current (J_f) and on the magnetic conduction value Λ of the path of the lines of force.

We have:

$$M_M = \text{Const.} \cdot \Phi \cdot J_a$$

$$\Phi = \Lambda \cdot (J_f \cdot z)$$

therefore

$$M_M = \text{Const.} \cdot J_a \cdot J_f \cdot \Lambda$$

and Φ signifies magnetic flux, and z the number of windings in the pole coils.

By direct measurement it is possible to determine only the currents (J_a and J_f). The magnetic conduction value Λ can not be measured in operation. Since it is necessary to know it in order to compute the torque, and since in addition the value is variable and depends on the currents at any given moment and the magnetic saturation of the path of the lines of force, the dependence of the torque on the currents, the so called torque characteristic of the motor is determined already on the test field. This being available, it accounts for the variable behavior of the magnetic conduction value, and it will suffice to measure the currents (J_a and J_f) to determine the torque.

This measurement of current is rendered difficult from a technical standpoint by the fact that with the helm over the current assumes rapidly varying values. These rapid changes, which are essential in obtaining an idea of conditions with the helm over, can no longer be measured by time recording instruments. Self recording instruments are inadequate because of their excessive inertia. Experience has shown that only the oscillographic measuring bands, which are nearly free of inertia, yield satisfactory results. The various current values are photographed in the wellknown manner on sensitized paper.

By completing all preliminary work before the tests, it is necessary only to switch the oscillograph on at a given signal and off again after conclusion of the test in order to photograph the test data.

It is made possible to synchronise the measured values with the values of the other test stations by measuring the time on one roll and the rudder angle on another simultaneously with the steering motor currents.

The accuracy of the torques determined by this method can be rendered relatively high by taking suitable precautions in determining the torque characteristics and analyzing the data. Experience has shown that even in unfavorable instances the calculated values may be expected to be correct within ± 5%.

Fig. 3 contains the plan of the hook-up for the steering apparatus with the test stations indicated. Fig. 4 represents the torque characteristics of a steering motor, giving the torque as a function of the ampere windings for running right or left. Fig. 5 reproduces original oscillograms photographed during steering tests at three velocity stages. The torques on the steering motor depending on the rudder angles and time calculated from the oscillograms using the torque characteristics have been plotted in Fig. 6. Further analysis will be made in a subsequent part of this paper.

b) DETERMINATION OF THE TORQUE ON THE RUDDER POST FROM THE DEFLECTIONS OF CALIBRATED TEST BOLTS.

From the fact that in spite of many trials it has not been possible to this date to substitute a better test method for the test bolt procedure notwithstanding the various disadvantages of the latter, the difficulty of measuring the torque on the post becomes evident.

Direct measurement of this torque is possible only with a certain design of post. This method will be described later in this paper.

In the greater number of mechanisms the torque can be determined only indirectly by means of the measured forces in the connecting rods. The first measurements carried out by Wellenkamp determined the forces from the elongations of the connecting rods. Since this method was too inaccurate and too complicated, Wellenkamp developed the test bolt method, by which the forces in the connecting rods are determined from the deflections of calibrated test bolts.

Detailed descriptions of this process occur in the papers of Schöneich and Schwarz. Since there have been no fundamental changes in the present day test set-up the description here will be brief. The structure and arrangement of a test bolt is shown in Fig. 7.

The deflection of the rods amounts to about 0.5 mm at most, so that it is necessary to interpose a system of lever rods with 1:50 magnification in order to record the values.

The force (Q) itself acting on the rod could be calculated with sufficient accuracy from the magnitude of the deflection (f); we have

$$f = \frac{Q \cdot l^3}{24 \cdot E \cdot J}$$

and

l = length between the bearing centers of the core bolt,

E = modulus of elasticity,

J = moment of inertia of the greatest cross section.

The values obtained by calculation are not unequivocal since the deflection is affected by elastic hysteresis. The bolts and their recording instruments are therefore calibrated in a press before being installed aboard.

The installation of the testing apparatus – substitution of a test bolt with recording instrument for the rod connecting the movable nuts and the connecting rods – requires several hours and during this time throws the steering mechanism out of commission. The tests are carried out in the steering gear compartment;

the measured values are recorded so that in this method also subjective errors of observation are excluded, and it is necessary only to throw the test apparatus in or out of gear to make the measurements.

In addition to the deflection of the test bolts - as a matter of fact the deflections of the starboard and port bolts are measured simultaneously - the rudder angle and time are recorded on the same roll of paper.

It is possible only with difficulty to give information as to the accuracy attainable with the test bolt process. Since all deflections of the bolts are very small, the effect of errors at small loads naturally is accentuated out of all proportion. Furthermore, the installation of the testing apparatus in the steering gear compartment, which, because of its location immediately above the propellers is exposed to violent vibrations, has a highly deleterious effect. Experience has proven that accuracy for small loads is small, and that for medium and large loads it is adequate. It is not possible to give the degree of accuracy of a value obtained by the test bolt method numerically, since no satisfactory values are available for comparison. With several steering mechanisms it was possible in addition to determine the torque in the post simultaneously by another method. Comparison of the values obtained by the two methods at all events permits certain conclusions a posteriori as to accuracy. This will be taken up later.

In order at least to have a certain basis for judging the values obtained by the test bolt method, one and the same steering maneuver was repeated four times and the values thus obtained were compared. The results have been plotted in Fig. 8.

The first three maneuvers were carried out one immediately after the other, the fourth about a half hour later. The third chart best shows the probable force curve for the connecting rods. The fourth chart yields an important insight as to how false analysis of the indicated values may result. It can be seen from the chart how with the rudder fore and aft, i.e. there being no load upon it, forces arise in the connecting rods and therefore deflections in the bolts. This is explained by the fact that when the steering motor runs down at the end of a maneuver the spindle and nut come to rest in such a position that the bolts above the side pieces exert a force on the connecting rods which is too small to overcome the static friction in the various bearings, but large enough to effect a noticeable deflection in the bolts. Since all the parts, - bolt, side pieces, connecting rod, are fitted exactly and have no play, and since furthermore the static friction is so great that a considerable torque is required either in the steering motor or the rudder post in order to overcome it, such stresses in steering mechanisms can not equalize each other.

These conditions are particularly unfavorable to measurement since they render satisfactory adjustment on the zero line difficult and thus may lead to false analysis of the records especially at small loads.

To discard the test bolt method because of these deficiencies would be a mistake. As will be shown below in connection with the recorded values, it is possible to obtain very good results with suitable care in the manufacture and installation of the testing instruments. Aside from the fact that up to now no indisputably better test method is available, the high stresses are recorded with a very high accuracy by means of the test bolt method, and in the last analysis it is exactly these maximum stresses that are of interest in judging of the reliability in operation of the steering mechanism of a new ship. In order to measure smaller loads also with adequate accuracy, it will be necessary to insert smaller bolts for short runs and the smaller stresses to be expected from them.

In conclusion it is to be noted that the test bolt method is suitable only conditionally to measure the chronological curve of forces in the connecting rods, but on the other hand it indicates with sufficient accuracy the maximum values. Therefore the test bolt method today is still indispensable for judging finished mechanisms.

In recent years rudder torque measurements have repeatedly been carried out with carbon piles. On the connecting rods between shallow lathe cuts carbon piles are set which alter their electrical resistances under the action of the forces in the connecting rods. If the resistance is fed by a source of current with constant voltage, the current flowing in a circuit connected with the carbon pile is a gauge of the resistance and thus also of the force acting in the connecting rod. In the course of the steering maneuver the currents of the carbon piles installed on the two connecting rods are photographed by an oscillograph. It is impossible to supply information regarding the actual accuracy of this test method. By way of comparison there are plotted in addition to the data given in Fig. 19, 22, 28, 30 the torques determined by this method. The torques have been designated as M_{KD}.

c) DETERMINATION OF THE TORQUE FROM THE ANGLE OF TWIST OF THE RUDDER POST.

The determination of torque is carried out most simply from the angle of twist of a calibrated piece of shafting. The obvious thing to do is to apply this test method, successfully used in mechanical engineering, to steering mechanisms also for the determination of the torque in the post. This has been done with several of the mechanisms here investigated. That this test method had not found application in earlier mechanisms or even quite generally is due to the fact that for this type of tests a suitable design of post is prerequisite.

In the present instance, the arrangement and design of the rudder post for Ship I and II were suitable for this type of test (see Fig. 9). The torque acting on the rudder is taken up at the lower end on a length of the post which is short in comparison with the total length. With this combination of rudder and rudder post, where in all cases the transmission of the torque occurs over the same length of post, the angle of twist is not affected by the height at which the resultant rudder pressure attacks the rudder. In every case the torque will be transmitted to the post at the same point. Since in addition the rudder support absorbs all bending stresses and the post therefore is only subjected to twisting, the angle of twist β is always a conclusive gauge of the torque acting on the post.

The relation between the turning moment M_d and the angle of twist β is given by the formula

$$\vartheta = \frac{M_d}{G \cdot J_p} .$$

where G signifies the shear modulus and J_p the polar moment of inertia of the post cross section.

Here ϑ is the angle of twist of two cross sections separated by a distance of 1 cm. When the two cross sections are separated by a distance L, the angle of twist will be $\beta = L\,\vartheta$.

The arrangement of the test instruments is shown in Fig. 10. A tube is inserted into the hollow rudder post and welded to the lower end of the post. The upper end of this tube emerges from the rudder post through a loosely fitting stuffing box, and to it is fastened the lever transmission with scriber. The scriber writes on a strip of paper which together with the recording device is fastened to the yoke. The recording device thus accompanies the yoke and therefore also the upper end of the post which is rigidly joined to the yoke in all its movements, so that, since the scriber is rigidly connected with the lower end of the post by the tube, the travel of the pencil on the paper is a gauge of the twist of the post and thus of the torque.

The recording instrument used was the same pantograph recorder as that developed for the test bolt method, but with a smaller magnification ratio (1:20).

The arc of the angle of torsion at any given moment is measured and transmitted to the recording arm by suitable transmission devices. The magnification ratio in the testing apparatus is adapted to the prevailing load by suitable adjustment of the distance a (see Fig. 10) so that easily readable records are obtained at all loads.

In order to permit analysis of the curves obtained on the recording instrument, torques of various magnitudes are exerted by hydraulic press on the post of the ship while docked, and the deflections of the pencil as functions of the known torques are determined. While under load, the rudder and post are subjected to shocks by ramming with a wooden beam in order to eliminate the influence of the frictional forces in the bearings of the post. Fig. 11 shows the calibration curves with and without shock.

How closely the measured values coincide with the actual ones it is impossible to show in figures, since we lack satisfactory figures for comparison. However, since no testing device such as the test bolt is involved, the post itself being used for measurements, it may be assumed that the values given by the recording instrument correspond very accurately to the actual loads. In order to obtain at least an idea of how to judge the values obtained by this method, one and the same steering maneuver was repeated three times in close succession. The values plotted in Fig. 12 agree extraordinarily well. This, however, merely proves that this test method records small and medium loads also with certainty. Regarding the accuracy of the records, information is still lacking.

The obvious thing to do is to compare the torques obtained from the deflection of the test bolt, the torsion of the post, and the carbon pile measurements with each other. In Figs. 19, 22, 28 and 30 (WRH No. 8) corresponding values have been plotted. The torque curves agree chronologically, but not as to their absolute values. In any event these data permit conclusions as to the order of magnitude of the torque in the post.

In judging methods of measuring the torque in rudder posts with respect to the test accuracy obtainable by their means, it is necessary to consider the peculiar difficulties of these tests. Small and medium torques are measured most accurately from the post torsion, the design of the post permitting. Large loads are best measured by means of the test bolt method. If it is impossible to measure the torsion of the post and if it is desired to make comparatively accurate measurements for small loads also with the test bolt method, special thinner bolts must be used.

C3. CALCULATION OF LOADS IN THE STEERING GEAR.

The calculation of the loads in the individual parts of the apparatus from measured values is given in the following:

P = Force in the circumference of the spindles (kg)
Q = Force in the connecting rods (kg)
R = Lever arm on the yoke = distance from center of rudder post to center of connecting rod bolt (m)
r = mean radius of the spindles (m)
α = rudder angle
α' = pitch angle of the spindles
ρ = friction angle
$\ddot{u}$ = ratio of the transmission between the spindles and the motor.

The basic equation for calculating the torque in the spindle (M_{Sp}) is

$$\frac{P}{Q} = \mathrm{tg}\,(\alpha' \pm \rho)$$

$\tan(\alpha' + \rho)$ applying while lifting a load, i.e. for positive torques on the post, and $\tan(\alpha' - \rho)$ while lowering a load, or for negative torques.

Then

$$M_{Sp} = P \cdot r = Q \cdot r \cdot \mathrm{tg}\,(\alpha' \pm \rho).$$

Now since

$$Q = \frac{M_{Sch}}{R \cdot \cos \alpha}$$

we have

$$M_{Sp} = \frac{r}{R} \cdot \frac{1}{\cos \alpha} \cdot \operatorname{tg}(\alpha' \pm \varrho) \cdot M_{Sch}.$$

Then the torque on the motor amounts to

$$M_M = \frac{1}{\ddot{u}} \cdot M_{Sp}.$$

Taking into consideration the losses due to friction in the rods of the steering gear, in the spindle bearings and in the transmission with

$$\eta_{gest.} = 0{,}95$$
$$\eta_{getr.} = 0{,}95,$$

we get the following equations for calculating the individual torques:

$$M_{Sp} = \frac{r}{R} \cdot \frac{1}{\cos \alpha} \cdot \frac{1}{\eta_{gest.}} \cdot \operatorname{tg}(\alpha' \pm \varrho) \cdot M_{Sch};$$

$$M_M = \frac{1}{\ddot{u}} \cdot \frac{1}{\eta_{getr.}} \cdot M_{Sp}.$$

The torque on the post, M_{Sch} is the resultant of three components: On the assumption that the rudder lies in a flow which is affected neither by the propeller race nor the turning of the ship, there would be a torque acting on the rudder which hereinafter will be designated as M_{Str}. Its calculation will be discussed farther on. When the rudder enters the propeller race an additional torque is set up which shall be known as M_{Schr}. Finally there is set up in the bearings of the rudder post a frictional moment M_R. The direction of turning and therefore also the sign of the torques is determined by the conditions prevailing at the given time. The torque on the post is found as a resultant moment by vectorial addition.

$$M_{Sch} = M_{Str} \dashrightarrow M_{Schr} \dashrightarrow M_R.$$

C4. CALCULATION OF THE TORQUE M_{Sch} ON THE RUDDER POST.

To determine the resultant torque on the rudder post the individual components must be calculated separately. Therefore the calculation of M_{Str}, M_{Schr} and M_R is given.

C4 a) CALCULATION OF M_{Str}.

In calculating the rudder force and the torque of this force, the rudder is regarded as a section in flow to which the principles of the theory of flow apply. The force R acting upon the rudder is the resultant of the lift A and the drag W. The position of the point of attack is known from model tests, so that the torque with respect to any given turning axis can be calculated also.

In order to calculate the rudder as a section in flow there must be known: The section, the rudder area F, its width B and height t, the aspect ratio F/b^2 and further, the velocity of flow and the angle of incidence α' of the rudder with the flow.

The coefficients of flow c_a, c_w, c_m for the given section are obtained from similar sections whose coefficients have been determined by model tests. The justification for applying the principles of the theory of similarity even when the coefficients of model and full scale structure have different values is given by experiments. The only essential requirement is that there be no critical conditions between the two coefficients. This requirement is satisfied.

The model tests carried out in the Göttingen Wind Tunnel apply to areas with the aspect ratio $F/b^2 = 0.2$. For conversion to other aspect ratios the following formulas exist:

$$1.\ c_{a_2} = c_{a_1} = c_a;$$

$$2.\ c_{w_2} = c_{w_1} + \frac{c_a^2}{\pi} \cdot \left(\frac{F_2}{b_2^2} - \frac{F_1}{b_1^2}\right)$$

$$3.\ \alpha_2 = \alpha_1 + 57.3^\circ \cdot \frac{c_a}{\pi} \cdot \left(\frac{F_2}{b_2^2} - \frac{F_1}{b_1^2}\right).$$

These conversion formulas strictly speaking are valid only for oblong areas. It has been proven, however, by means of experiments (see bibl. 2, p 37 and 50) that they also apply to square areas. The rudders here investigated have aspect ratios of $F/b^2 > 1$. The good agreement between calculated values and test values confirms the fact that for the aspect ratios here investigated the conversion formulas continue to be valid. Doubtless model tests of sections similar to the rudder as to form and aspect ratio yield more accurate data for calculation.

Since no such values were known, it was necessary to use as a basis the values calculated for sections with an aspect ratio of $F/b^2 = 0.2$. These values then, taking into consideration the aspect ratio F_2/b_2^2, were calculated for the rudder.

If in addition the velocity of flow and the angle of attack of the rudder are known, the lift and drag and the torque M, with respect to the leading edge, can be computed.

We have

$$A = c_a \, F \, q$$

$$W = c_w \, F \, q$$
$$M = c_m \, F \, q \, t$$

$q = \frac{\rho}{2} v^2$; ρ signifies the density of the water (kg sec^2/m^4); v the velocity of the water (m/sec); t the depth of the rudder in the direction of flow (m).

The coefficients of flow are unknown quantities; the forces are calculated in kg and the torques in mkg.

For calculating the torque on the rudder post it is required to know the position of the point of attack of the resultant force $R = A \leftrightarrow W$. The distance e of this point from the leading edge of the rudder can be determined from the coefficients of flow.

We have

$$e/t = c_m (c_a \leftrightarrow c_w).$$

In determining the torque, not R, but really the force N perpendicular to the longitudinal axis of the rudder should be used as a basis. The difference between R and N, however, is only slight, so that by using R, only small errors result which can be accepted in the interest of simplification.

Since $R = A \leftrightarrow W = F \, q(c_a \leftrightarrow c_w)$, the coefficients of flow converted for the aspect ratio of the rudder have been plotted not singly but as $c_a \leftrightarrow c_w = \sqrt{c_a^2 + c_w^2}$.

For calculating the torque acting on the rudder post there must be known in addition to the position of the point of attack of the force R, the distance y between this point and the rudder post. If the distance of the rudder post from the leading edge is designated as a, and if $e = t \; e/t$ is the distance of the point of attack of the resultant force R from the leading edge, then the lever arm of the force R, with respect to the rudder post will be $y = a - e$ for an overbalanced rudder and $y = e - a$ for a normally balanced rudder.

With respect to the designation of rudder torques as positive or negative a torque will be regarded as positive when e is greater than a in putting the helm over from the fore and aft position, and as negative when a is greater than e. Beyond this, rudder torques will then be designated as positive in this paper, when, regardless of the direction of movement of the rudder, they act upon the rudder spindle as loads to be lifted; torques having the effect of a load to be lowered are designated as negative moments.

Determination of the angle of incidence α' of the flow against the rudder has already been taken up in the first part of this paper. As long as the ship has not yet begun a turning movement and $\omega = 0$, the angle of incidence is equal to the rudder angle α. If the ship has a definite turning speed and if the distance r of the rudder from the ship's center of turning is known, the tangential velocity v_q of the rudder can be calculated as $v_q = r\omega$. Then the angle of drift at the rudder will amount to $\tan \delta = v_q/v$, where v is the speed of the ship in the direction of the keel line. The angle of incidence α' is then $\alpha' = \alpha \pm \delta$.

For this angle of incidence the flow coefficients will then be derived from the converted curves.

Determination of the velocity is based upon the following considerations: In correspondence with the varying distances of the different rudder sections from the shell plating, the velocity of the water at the rudder will vary. The parts in direct proximity to the shell plating will be under the influence of the wake, while the lower portions will be affected little or not at all thereby. It is impossible to state according to which function the velocity will be distributed over the rudder area. For purposes of calculation a mean velocity is selected. A guide for the choice of this mean velocity is supplied by the rudder torques measured on Ship I and II at the beginning of the steering maneuver, i.e. where there is as yet no influence of the propeller race and of the turning of the ship. The calculations for determining the torque carried out for comparison best agreed with these measured values when the velocity measured by the log was introduced as the relative velocity of the rudder and the water. The calculations show further that this relative velocity is rather somewhat smaller than the speed of the ship than greater. In order to demonstrate this the comparative calculations were carried out in part with velocity values equal to the ship's speed, but in part also with velocity values lying between the ship's velocity and the translational velocity of the propellers.

C4b). CALCULATION OF M_{Schr}.

The tests and comparative calculations given in this paper were carried out only with twin-screw vessels.

It is difficult to determine the additional torque set up when portions of the rudder area immerge into the propeller race, and probably this can not be done at all by calculation alone. Tests of rudder models, which like the rudder itself, are impinged upon simultaneously by two flows differing both as to direction and magnitude, will supply reliable data for the calculation. There were no such model data available. If, notwithstanding, this paper supplies a sufficiently accurate method of determining the additional torque set up by the propeller race, it nevertheless acknowledges the weaknesses of calculation in spite of the good agreement between the measured and calculated values. In any event, the reported method of calculation for the first time makes it possible to gain an idea of the influence of the propeller race in the designing of new steering apparatus.

Development of this method of calculation without the use of test data obtained with full scale mechanisms would have been impossible, for without knowledge of the measured torques it would have been impossible to give data on the velocity and direction of the propeller race, the magnitude of the immersed area and the conditions of flow upon which to base the calculation.

For calculating the additional torques set up by the propeller race the rudder is considered as a flat plate part of which is impinged upon by a jet of water.

According to the formula given in Hütte I, 25th edition, p 383, this may be relied upon as an approximate calculation, even though strictly speaking the assumptions upon which the formula was set up are not applicable to the present case. As shown by the results, the error involved is slight considering the peculiar circumstances. In addition the calculation has the advantage of simplicity and clearness.

If P is the additional force set up by the propeller race, then

$$P = \rho \; Q \; v'' \sin(\alpha - \gamma).$$

Details of the calculation are shown in Fig. 16.

The rudder area F' impinged upon by the propeller race is found graphically from a chart giving the arrangement of the rudder and the propellers.

The distance y' of the point of attack P from the rudder post is likewise found graphically. The additional torque set up by the propeller race is then

$$M_{Schr} = P \; y' \quad \text{(see Fig. 16).}$$

To determine F and y' the angle at which the propeller race impinges upon the rudder must be known. Determination of this angle is taken up below.

The velocity at which the propeller race hits the rudder can only be estimated. Calculation for all maneuvers here investigated gave good agreement with measured values when the velocity of the race was taken as $v'' = 0.7\ v$ (v = velocity of the ship).

In itself this calculation may not be carried out with the ship velocity v, but must be performed with the translational velocity of the propellers n_s H, since even with the ship at rest forces are exerted on the rudder by the propeller race. In place of the coefficient 0.7, one of about 0.63 to 0.64 would then have to be introduced.

The determination of the angle at which the propeller race strikes the rudder is based upon the following considerations: It may be concluded from the comparison of the calculated and measured values and the good agreement found here for that part of the steering test in which the rudder does not yet get into the range of the propeller race, that the individual factors determining the moment M_{Str} are correctly postulated. Therefore it may be assumed that during the continued progress of the steering maneuver M_{Str} will be derived correctly. If, then, in the further course of the maneuver material differences occur between the calculated and measured torques, these differences, disregarding the friction of the post which will be dealt with separately, will be caused by the propeller race. The difference between the calculated and the measured torque values thus is a gauge of the influence of the propeller race, and the problem consists in determining the regularity of this torque.

The difference between calculated and measured values begins even before reaching the rudder angles at which, when the propeller race is parallel to the longitudinal axis of the ship, the rudder would really first project into the propeller race. This was observed in all the ships investigated. It leads to the conclusion that the propeller race diverges at a certain angle. The magnitude of this angle of divergence, so far as the test data permit us to form an idea, depends upon the shape of the after body of the ship and the arrangement of the propellers and upon the distance of the various sections of the propeller race from the propeller. For the ships here investigated the angle of divergence is between 2° and 10°, the large angles applying to the greatest distance between the propellers and the rudder. The angle of divergence for the ship on its course is designated as γ_0 .

Under the influence of the turning of the ship the direction of the propeller race changes. Taking into consideration that the water in the propeller is accelerated in longitudinal direction, the influence of the turning of the ship on the deflection of the propeller race is less than that upon the rest of the flow. The angle δ found from the velocity in longitudinal direction and from the transverse velocity is therefore taken at only about half its value in determining the direction of the propeller race. The resultant direction of the propeller race with respect to the keel line is then found to be

The angle at which the propeller race impinges upon the rudder is ($\alpha \pm \gamma$), ($\alpha + \gamma$) being valid so long as the propeller race diverges toward the midship plane and ($\alpha - \gamma$) when the propeller race diverges from the midship plane.

The comparative calculations thus carried out with all the ship here investigated agreed very well with measured values, considering the peculiar conditions.

C4c) CALCULATION OF M_R.

The frictional moment in the post bearings depends upon the type of bearing, upon the dimensions of the bearing points and on the rudder pressure. If the post has two bearings and if P_1 and P_2 are the bearing forces and 2 r_1 and 2 r_2 the diameters of the bearings, then

$$M_R = P_1$$

The bearing friction coefficient μ is 1.27μ. Details of its calculation are given in Fig. 15. The type of shaft bearings has a marked influence on **the magnitude of the frictional moment, the bearing forces being influence thereby.**

The rudders of Ships III and IV have bearings different from those of I and II. In consequence the resulting frictional moments are considerably greater in Ships III and IV.

The moment of friction is always in opposite direction to the direction of movement. That is, it acts upon the rudder spindle in the sense of a load to be raised and is therefore called a positive moment.

This completes the description of the calculation of the three components of the resultant rudder post moment, and M_{Sch} can be calculated as the sum of these three moments.

D. ANALYSIS.

D.1.

With the aid of operating conditions known from measurement and using the calculations reported in the foregoing sections, the torques on the propeller post were computed for four ships at various speeds. These values and those known at the same time from rudder torque tests are plotted in Fig. 17, 19, 22, 24, 29.

For Ship I the calculation has been given in detail. For the other ships only the initial values and the results are given. It seemed advisable to cite a relatively large number of data in order to show that in spite of the different arrangements of the various steering mechanisms the calculation always yielded correct results.

There belongs to one calculation:

For Ship I at 24 knts.:	Fig. 13 – 17	and	Table	1 – 5
For Ship 2 at 32 " :	Fig. 18 – 19	"	"	6 – 7
For Ship 3 at 23 " :	Fig. 20 – 22	"	"	8 – 9
For Ship 3 at 29 " :	Fig. 23 – 24	"	"	10 – 11
For Ship 4 at 19.5 " :	Fig. 25 – 28	"	"	12
For Ship 4 at 25 " :	Fig. 29 – 30	"	"	13 – 14.

The values for the rudder torques resulting from the various test methods differ from each other according to the test accuracy of the individual methods. The value obtained by calculation generally is between those resulting from measurement, a proof of the practical usefulness of calculation.

To render the calculation given in Fig. 13 to 28 and Tables 1 to 14 understandable, several explanations are given in the following:

As has already been stated in the foregoing the resultant torque on the rudder post M_{Sch} is divided into three components, M_{Str}, M_{Schr}, M_R, the sums of which, in each instance considering the direction of turning of the torque give the resultant torque. Each of these torques is calculated separately.

1. For the present comparative calculation the magnitude and aspect ratio

of the rudder area are known (see Fig. 13). For a new design it would first have to be assumed in connection with the ratio of the lateral plan*) to the rudder area for completed structures. The division of the rudder area according to height and depth is determined to a large extent by the total design – draught of the ship.

2. Without going into the question as to whether thin or thick rudder sections are more practical – which in the present instance had already been answered by the given sections – it may be assumed that a section will be chosen either from towing tests or known structures. Since the aspect ratio and the section of the rudder are thus known, the flow coefficients of the rudder can be calculated as functions of the angles of incidence by using the familiar conversion formulas. Fig. 13 contains the values for the comparative calculation here carried out.

3. If, in addition, a definite degree of balance is selected for the first design, the location of the rudder post is fixed. Since the position of the point of attack of the resultant rudder force R is known from the section values, the torque on the post can be determined.

4. Prerequisite to further calculation is a knowledge of the flow about the rudder as to magnitude and direction. In the present case the chronological curves of the operating values were known, and have been plotted in Fig. 14. In setting down the velocity of the water at the rudder it is necessary to take into consideration the ship form and the position of the rudder with respect to the propeller race. The data obtained in this paper all have their origin in twin-screw vessels, with which the rudder when in fore and aft position or at small angles did not project into the propeller race, which, however, does not preclude restriction of the basic information obtained from generalization. The velocity in the individual depths on the rudder varies; in the upper layers there will be lower velocities due to the influence of wake than in the lower ones some of which are as much as 4 m from the shell plating. The velocity distribution on the rudder as a function of the distance from the shell plating was not measured. For practical calculation the knowledge obtained from the comparative calculation that the velocity at the rudder lies between the ship velocity and the speed of advance of the inside propeller is sufficient, it being of no decisive importance to the result whether the speed selected is more nearly equal to the speed of the ship or to the speed of advance of the propeller. For rudders lying in the propeller race it is best to choose the speed of advance of the propeller.

5. The angle of incidence α' which varies under the influence of the turning of the ship is calculated by taking into consideration that $\omega = f(t)$ and the known distance of the rudder from the center of turning of the ship. Fig. 14 contains the values obtained for the comparative calculation here carried out. For a new design, for which no information is available from previous structures, i.e. for which $\omega = f(t)$ and the center of turning are not known, it is difficult to determine the angle of incidence. This will be taken up farther on.

*) **Area of longitudinal section.**

6. In order to be able to calculate M_R simultaneously with M_{Str} the values required for this have been reported in Fig. 15, which, taking into account the special arrangement of the post bearings (see also Fig. 21) are available for every design. Furthermore, Table 3 of Series 1 to 13 contains the calculation of M_{Str} and M_R. The values for the angle of incidence α', the mean velocity v' and the coefficients of flow $C_a \nrightarrow C_w$ and e/t have been derived from the corresponding figures as results of the calculations carried out previously. Fig. 14 contains rudder post torques which were determined from torsion measurements. Series 15 contains the difference between calculated and measured values. These values served first as a basis of the determination of the fundamental curve of the additional torque set up by the propeller race.

7. In Fig. 16, the determination of M_{Schr} has been carried out partly by graphic means and partly by calculation. For a new design the speed of advance of the propellers and the angle of divergence of the propeller race determined by the relative position of the rudder to the propeller race must be known. The values given in Fig. 31 can be used as a starting point for determination of the angle of divergence γ_0. The speed of the propeller race at the rudder must be set down as $v'' = 0.6\ n_s H$.

8. Fig. 17 and Table 5 give the values for Ship No. 1 at 24 knots.

In the same way as the results of measurements and calculations for Ship No. 1 in a steering test at 24 knots are given in Fig. 13 to 17 and Table 1 to 5, data for several other ships are reported in the following. Here, however, the method of calculation has not been shown in detail as it was for Ship No. 1.

Fig. 18 gives the initial values for Ship No. 2, and Fig. 19 and Table 7 the results of the measurements and calculations. Table 6 gives the calculation of the additional moment set up by the projection of the rudder into the propeller race.

For Ship No. 3 at about 23 knots Fig. 20 and Table 8 contain the conversion of the flow coefficients for the rudder. Fig. 21 contains the starting values and Fig. 22 and Table 9 the results of the calculation and measurements.

For Ship No. 3 at about 29 knots, Fig. 23 and Table 10 give the starting values for the calculation. Fig. 24 and Table 11 give the results.

CORRECTION

In the paper "Investigations of Ships on Steering Maneuvers" by Marinebaurat Dr.-Eng. K. Fischer, WRH, No. 7 of Apr. 1, 1936, p 83, the three last lines of the first paragraph, top, left, should read: Self-locking gears operate at efficiencies $\eta < 0.5$, not self-locking gears with efficiences $\eta > 0.5$.

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For Ship No. 4 at about 19.5 knots, Fig. 25 to 28 and Table 12 give the most important values of the calculation and measurements.

D.2. MAXIMUM LOADS IN STEERING MECHANISMS.

For determining the dimensions of the various parts of steering apparatus the possible maximum loads must be known. The prerequisites for large values of the rudder torque are given with: 1. In turning maneuvers with the rudder hard over and high speed, since then the speed reaches its maximum values; 2. In meeting the helm, since then, although because of the preceding turn the velocity has already decreased considerably, the angles of incidence and consequently the flow coefficients attain high values; at the same time, the point of attack of the resultant rudder force shifts at large angles, so that the lever arm likewise increases; 3. In backing tests, since then, in spite of lower speeds the lever arm of the resultant rudder force with respect to the rudder post is considerably greater than while running forward.

In which of the three cases the highest loads occur only a check calculation taking into consideration the special conditions of the design can show.

To give an idea of the conditions prevailing in turning and meeting the helm, the measured values for two different ships have been given in Fig. 31. No comparative calculation was carried out, since the running values had not been measured, and furthermore the flow coefficients for angles greater than 30° were not known. The data show that in meeting the helm the torques attained are only approximately as large as those in turning, since the ship velocity in turning decreases very rapidly and therefore the succeeding maneuver of meeting the helm starts with considerably lower velocities.

To determine the maximum load, therefore, a comparison between the greatest torque of a turning test and a steering maneuver while backing will suffice.

It has been attempted to calculate the maximum torques while backing. The calculation is only conditionally valid since, first, no flow coefficients for sections impinged upon from the rear are available and furthermore only the propeller RPM, but not the velocity of the ship through the water is known. To find the lift and drag forces acting on the rudder, the flow coefficients for slender sections were selected.

The results of calculation and measurement have been plotted in Fig. 32 and 33. They show on the assumption that approximately the correct flow coefficients have been selected, that the best agreement between calculation and measurement is achieved when the velocity of the water at the rudder is assumed to be about 75% of the speed of advance of the propellers.

For a new design, then, the loads in backing can be calculated in advance in good approximation when the RPM to be expected in backing is known approximately, and when in addition the flow coefficients of the rudder impinged upon from the rear are available.

CONCLUSION

D.3. CALCULATION OF STEERING MECHANISM FOR NEW SHIPS.

Investigation and study of the kind here carried out are worth while only if the knowledge obtained through them facilitates advance calculation of new structures for the engineer, or at least show him how he can obtain the necessary data.

It has been shown that it is possible to make a calculation when the following values are known: 1. $\alpha = f(t)$; 2. $\omega = f(t)$; 3. $v = f(t)$; 4. the approximate location of the center of turning of the ship; 5. the position of the rudder relative to the propeller race; 6. rudder area, section, and aspect ratio. Among these values, the speed in putting the helm over $\alpha = f(t)$ is known. Furthermore, at least approximately, the velocity curve $v = f(t)$ is known from comparison with similar ships. The position of the rudder with respect to the propeller race is known; the angle of divergence of the propeller race can be obtained from the values plotted in Fig. 34. In this figure the various angles of divergence obtained by analysis have been plotted. Unknown are the position of the center of turning of the ship and the time curve of the angular velocity of the turn, $\omega = f(t)$, unless here too it is possible to make use of comparison with similar ships. The center of turning of the ship can be assumed to be in the vicinity of the bow for a preliminary design.

It is more nearly correct, however, to attempt to determine the behavior in time of these values, important in the calculation of rudders from model tests. The time curve of the angular velocity of turning, $\omega = f(t)$, depends upon the mass moment of inertia of the ship and the masses of water set into motion, taken with respect to the turning axis, upon the lateral plan and the rudder force. Basic knowledge can be obtained only by model tests. With what accuracy it is possible to apply model data to full scale structures can be determined by comparative calculations.

It is also more nearly correct to determine the flow coefficients with a model rudder operating under similar conditions as the full scale rudder; i.e. a velocity corresponding to the speed of advance of the propellers will be superimposed upon the uniform velocity of the water corresponding to the ship velocity for that part of the rudder area which projects into the propeller race. Investigations in this sense by the various model basins would yield important information for rudder apparatus.

As long as no such model data are available, the calculation will have to be carried out as here demonstrated using the values of similar ships. The results of calculation prove the reliability of this process, above all because the maximum stresses can be determined with adequate accuracy.

D.4. CALCULATION OF THE FRICTION ANGLE OF THE RUDDER SPINDLES.

In Tables 18 to 21 the friction angle ρ in the rudder spindle has been determined from simultaneously measured torques on the rudder post M_{Sch} and on the steering motor M_M.

We have

$$M_M = \frac{1}{\ddot{u}} \frac{1}{\eta_{getr}} \frac{1}{\eta_{gest}} \frac{1}{\cos\alpha} \frac{r}{R} \tan(\alpha \pm \rho) M_{Sch}$$

We write

$$\ddot{u}\ \eta_{getr}\ \eta_{gest}\ \frac{R}{r} = constant$$

Then ρ can be calculated from the relation

$$\mathrm{Tan}(\alpha' \pm \rho) = C \cos \alpha \frac{M_M}{M_{sch}}$$

The meanings of the various symbols have already been indicated in the foregoing.

Whether $\tan(\alpha' + \rho)$ or $\tan(\alpha' - \rho)$ will be used in the calculation will depend upon the sign of the torques M_{Sch} measured on the post. In case $\rho > \alpha'$, $(\alpha' - \rho)$ will be negative. However, since $\tan(\alpha' - \rho) = \tan(\rho - \alpha')$, the course of the currents in the motor armature will decide whether $\rho > \alpha'$ or $\rho < \alpha'$ As long as the steering motor functions as a motor, $\rho > \alpha'$; when it functions as a generator $\rho < \alpha'$. According to the current values obtained for the mechanisms here investigated, ρ is always greater than α'.

In regard to the calculations carried out, the following details are to be noted:

The torques M_M measured at the start of a steering test contain in addition to the torque required to overcome the post torque, a torque for the acceleration of the parts set in motion. This accelerating moment fades very rapidly (see also Fig. 5 and 6). For calculating the friction angle the torques M_M can be used only during the uniform velocity while the rudder is being put over. The no-load torque M_{Mo} included in electrical measurements even at uniform speeds is subtracted from the measured torque M_M, so that the friction angle ρ is calculated from the moment $M_M' = M_M - M_{Mo}$.

It was possible to obtain the most accurate values of the friction angle from measurements of the steering apparatus of Ship No. 1 and No. 2, since the post torques always are unequivocally negative torques of relatively high value. The steering apparatus of both ships is similar down to the pitch angles of the spindles and the gear ratios. It is to be noted that nearly equal friction angles were found in the two cases.

The determination of the friction angle from the tests on Ship 3 and 4 is more inaccurate as long as the post torques are small and not fixed as to direction.

It becomes more accurate to the same extent as the torques increase with increasing rudder angles. In this calculation it is evident that the accuracy of the tests for determining the torque on the rudder post is inadequate for small values. To judge these measurements, therefore, the torque on the steering motor can be used. For Ship 3, then, taking into consideration the torques on the steering motor and assuming a friction angle of about 5° at the beginning of a steering maneuver, we would then have to conclude that the torques on the post are not positive, as measured, but negative.

This gives rise to the question whether the measurement of torques on the steering motor might suffice to determine the loads in steering mechanisms. Since the friction angle is not constant in the course of a steering test, the question must be answered in the negative in principle.

On the other hand, however, the changes in the friction angle are relatively slight, so that for ships in which the installation of test bolts would be too expensive and would consume too much time, determination of the torque on the steering motor would be wholly adequate. For determining the torque on the post, ρ can be taken as about 5°. If, to get an idea of the torques thus determined, in addition the torques are simultaneously calculated according to the viewpoints given in the preceding sections of this paper and the two values compared, it may be possible to determine the torques on the post with sufficient accuracy.

The calculations carried out in Tables 18 to 21 yield a friction angle of about 5° to 7°, and consequently a friction coefficient $\mu = \tan \rho$ = about 0.1. For the design of rudder spindles we obtain the following information from this and from several empirical values:

The requirement of absolute operating reliability leads to self-locking spindles, which, however, in order to attain good efficiencies and still fulfill this requirement are designed with pitch angles only slightly smaller than the friction angle. According to experiences with completed mechanisms the product should be $pv\mu = 2 \div 3$. p is the pressure per unit of area in the spindle in kg/cm²; v is the gliding speed in m/sec, and $\mu = \tan \rho$. The pressure per unit of area should not rise above 50 to 70 kg/cm², if possible. The gliding speed is governed by the speed prescribed for putting the rudder over. This completes all the data necessary for the design of the rudder spindles.

D.5. SPECIAL OBSERVATIONS.

Attention will here be directed to several observations made during the tests and in analyzing the data.

a) Separation of flow from the rudder.

The torque curve M_{Str} of Ships 1 and 2 after attaining a maximum value about 11 seconds after the rudder is put over decreases very abruptly at a rudder angle of about 30° and then increases once more (see Fig. 19 and 35).

This behavior is caused by the separation of flow from the rudder when the angle of incidence becomes equal to the critical angle of incidence. The flow coefficients converted in 13b for the aspect ratio F/b^2 of the rudder prove that the critical angle of incidence for this aspect ratio is about 30°.

In order to investigate conditions even more exhaustively two steering tests were carried out at the same ship velocity, but putting the rudder over at different speeds. The results have been plotted in Fig. 35 and show that when the rudder is put over slowly the sudden decrease in torques indicating the separation of flow from the rudder does not set in. The decrease in torque values observed after about 15 seconds is explained by the increase in the drift angle caused by the increase in turning speed, and the consequent decrease in the angle of incidence.

In putting the rudder over slowly, therefore, the critical angle of incidence is not attained, since when the rudder angle (α) is about equal to the critical angle of incidence (α'), the ship already has such a high turning velocity that a sufficiently large angle of drift (δ) exists to prevent attainment of the critical angle of incidence ($\alpha' = \alpha - \delta$).

The results show that it is impractical to put the rudder over too quickly, since when there is separation of flow from the rudder violent shocks may occur and with them undesirable additional loads.

The speed with which the rudder is put over, therefore, should be adapted as closely as possible to the turning speed of the ship ($\omega = f(t)$. In principle it is desirable to put the rudder over quickly up to about 25° in order to change the course of the ship quickly. The rudder may be put over from 25° to hard aport or hard astarboard considerably slower in order, by taking advantage of the angle of drift, to avoid separation of flow from the rudder and consequently also undesirable sudden loads on the apparatus.

b) Torsional oscillations in the rudder post.

In measuring the torsions of the rudder post, periodic fluctuations of the test values were noted when the rudder immerged into the propeller race. In Fig. 36 and 37 excerpts from several test series are shown.

It was natural to conclude that torsional vibrations in the rudder post were involved. Therefore it was attempted to compute the natural frequency of the rudder post and rudder and to determine the period of all possible impulses.

For the calculation of the natural frequency the rudder post with the rudder blade was regarded as a bar fixed at one end, whose other end carries a large mass (rudder blade). The fixation occurs in the yoke. The natural frequency of such a system amounts to

$$T = 2\pi \sqrt{\frac{\theta}{GJ} L}$$

T = period of one vibration

L = length of post

J = polar area moment of inertia of the post

G = shear modulus

Θ = mass moment of inertia of the rudder blade with respect to the rudder post.

The various factors in the present instance have the following values:

$L = 400$ cm; $J = 12.10^4$ cm²; $G = 800{,}000$ kg/cm²;

$\theta = 2680$ m kg sec².

The period then amounts to

$T = 0.21$ sec

and the frequency

$\nu = 4.75$ sec^{-1}

To get an idea of the influence of the virtual mass a model test was carried out with a system similar to the rudder post and the rudder blade, which consisted of a thin wire (rudder post) with its upper end fixed, to the lower end of which a steel plate (rudder blade) was fastened. The natural frequency of this system was determined by means of an oscillograph, first in air and then in water. In air it amounted to $T_L = 0.087$ seconds, and in water $T_W = 0.099$ seconds. The increase in the period under the influence of the virtual mass amounts to about 15%.

This figure, naturally, can be taken only as a rough idea. The natural frequency of the rudder post and rudder can therefore probably be assumed to be about 5 sec^{-1}.

The impulses exerted upon the rudder in the propeller race emanate first from the propeller race impinging periodically (depending upon the RPM and the number of blades) upon the rudder, and secondly from the shocks caused by the separation of flow when the critical angle of incidence is reached.

The greatest fluctuations in test values were noted in steering tests at about 30 knots initial velocity. The propeller RPM at the beginning of these tests then amounts to about 6 sec^{-1} and falls very rapidly to about 5.3 sec^{-1}, thus attaining values approaching the natural frequency of the rudder post very closely.

The fluctuations in measured values noted not only in tests at 30 knots but also at 24 knots with a period of 16 sec^{-1} probably must be attributed to natural frequencies of the test instruments. It is surmised that bending oscillations of the measuring tube installed in the rudder post are involved. The amplitude of the deflections was materially reduced after the rudder post was filled with oil to dampen the bending oscillations (see Fig. 37).

The number of the present tests is not adequate to give conclusive information on torsional vibrations in the rudder post; nor is it possible at present to clear up the still unsolved questions by systematic tests.

It is important, however, that the tests and calculations have shown that torsional vibrations can arise and that therefore it is well, with respect to the reliability of the mechanisms to investigate these conditions already in drawing up the design.

The cross section of the post has the greatest effect on the natural frequency of the rudder, and then too the moment of inertia of the actual rudder body, as distinguished from the post. By changing the balance, this moment of inertia and with it the natural frequency can be affected. In general, however, the balance will not be able to allow for the natural frequency of the rudder resulting from it. Thus the only remaining recourse is to enlarge the cross section of the post. If it is thus possible even by structural measures to bring the natural frequency above zones of resonance with the propeller impulses, precautions must at the same time be taken so that the rudder and post will not be set into vibration by sudden loads. As has been shown in the foregoing this sort of impulse occurs when, on reaching critical angles of incidence, the flow separates from the rudder. This occurs when the rudder is put over with such speed that the angle of drift formed when the ship turns can have no effect on the decrease of the angle of attack. The speed with which the rudder is put over must therefore be adapted to the increase in the turning speed of a ship.

E. SUMMARY.

1. The factors determining the rudder force and the torque on the post are given.
2. The most important considerations in designing steering mechanisms are given.
3. The test methods for determining the loads in steering mechanisms are given and investigated as to test accuracy.
4. The calculation to determine the loads in steering mechanisms is given.
5. Comparisons are made between measured and calculated values.
6. The conditions under which maximum loads may occur in steering mechanisms are studied.
7. The availability of data obtained from this paper for the calculation of steering mechanisms for new structures is investigated.
8. The friction angle of the rudder spindles in operation is determined.
9. The assumed conditions under which torsional vibrations in the rudder post may occur are given.

Sources.

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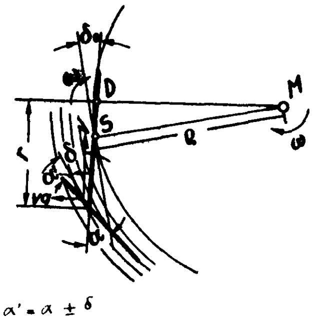
S = Center of gravity of ship
D = Turning point of ship
M = Center of curvature of path of center of gravity
α = Rudder angle
δ = Angle between CL and flow
α' = Angle of incidence with rudder
δ_0 = Angle of drift = angle between path tangent to path of c. of g. and CL
v = Speed of ship through water in longitudinal direction
vq = transverse velocity of individual points on the ship
ω = Turning speed of ship
ϱ = Radius of curvature
2ϱ = Diamter of turning circle
r = Distance of points on ship from turning point of ship



I. $vq = \omega r$ II. $\tan \delta = \frac{vq}{v}$ III. $\alpha' = \alpha \pm \delta$

Fig.1. Symbols for Operating Values.

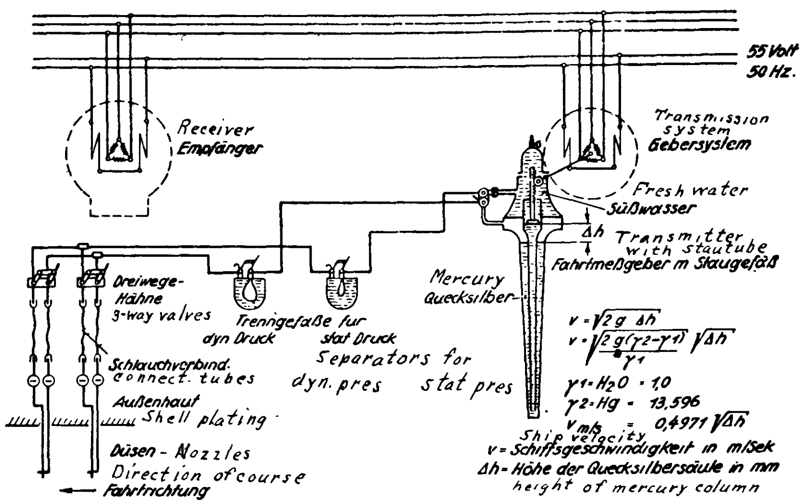
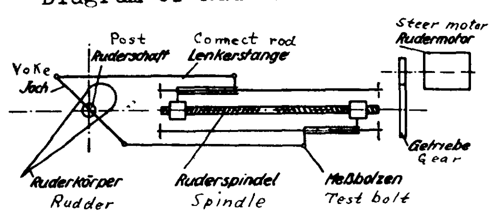
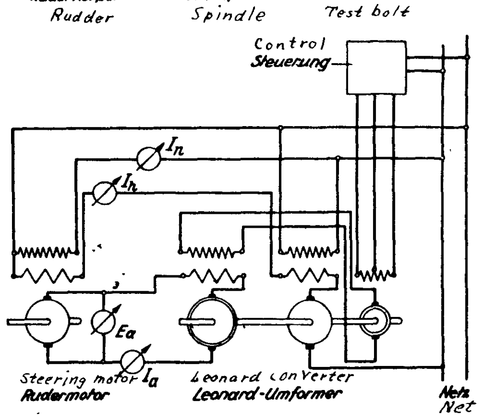


Fig.2. Principles of Test Layout.

Diagram of Rudder Mechanism.





Hook-up of Rudder Mechanism.

Fig.3.

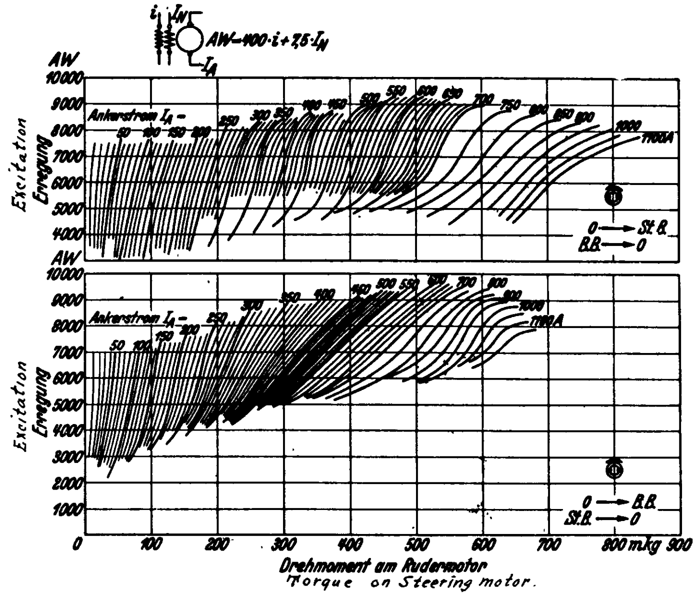


Fig.4. Torque on Steering Motor.

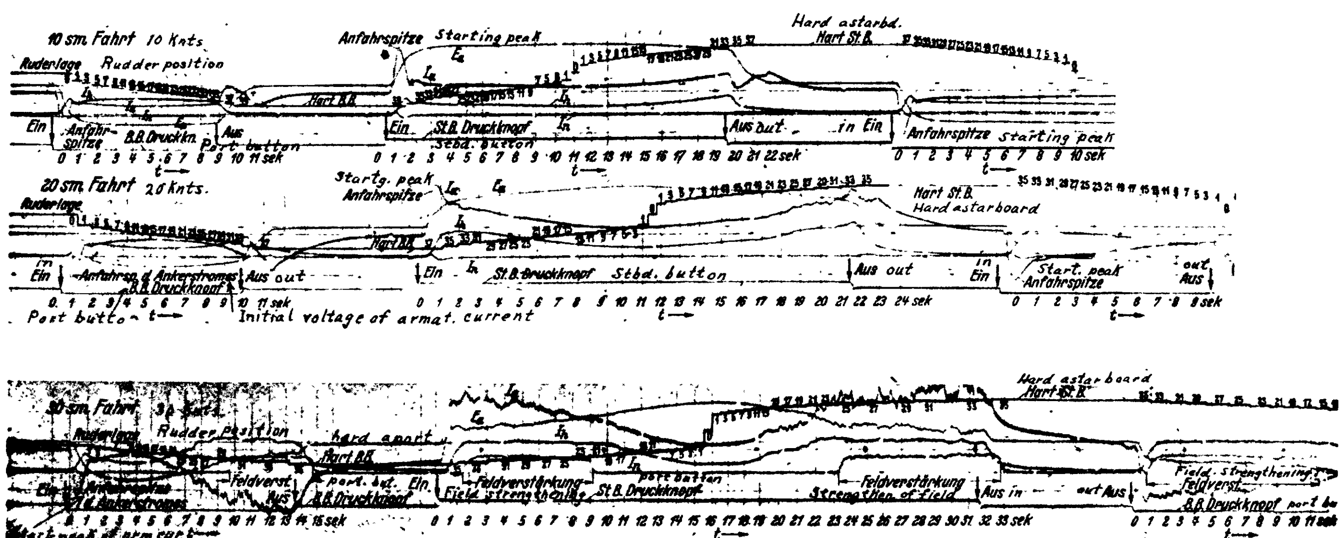


Fig.5. Cruiser LEIPZIG. Measurement of Rudder Torques with Helm over, by means of Oscillograph.

Scales E_a 1 mm = 8,26 Volt, I_a 1 mm = 6,94 Amp., I_h 1 mm = 9,34 Amp., I_n 1 mm = 0,332 Amp.

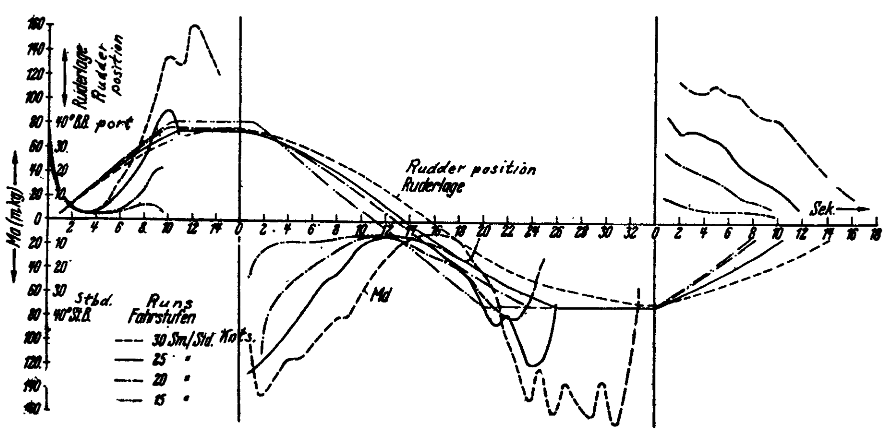


Fig.6. Meeting the Helm. Torque on Steering Motor.

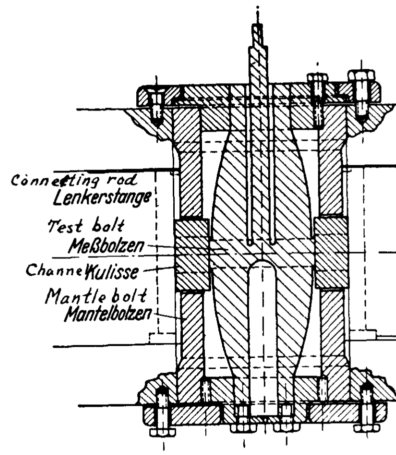


Fig.7. Rudder Test Bolt.

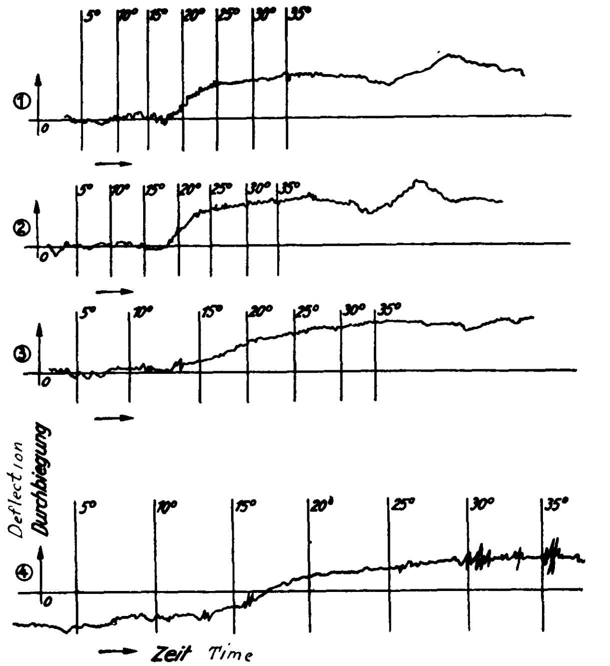


Fig.8. Test Bolt Chart of a Maneuver repeated four Times.

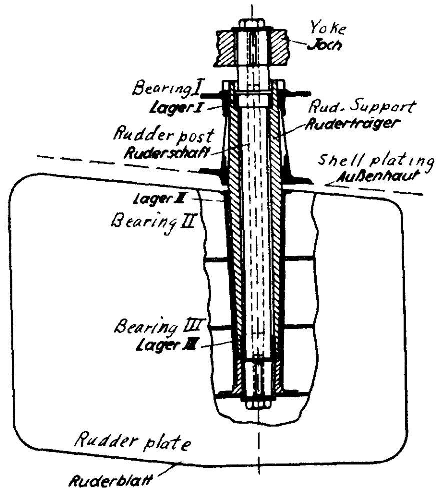


Fig.9. Diagram of Rudder Post Bearings for Ship 1 and 2.

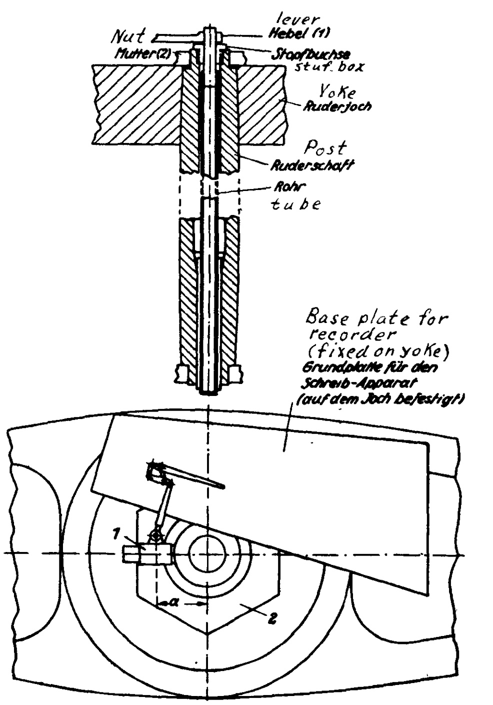


Fig.10. Arrangement of Instruments for Torsion Measurement.

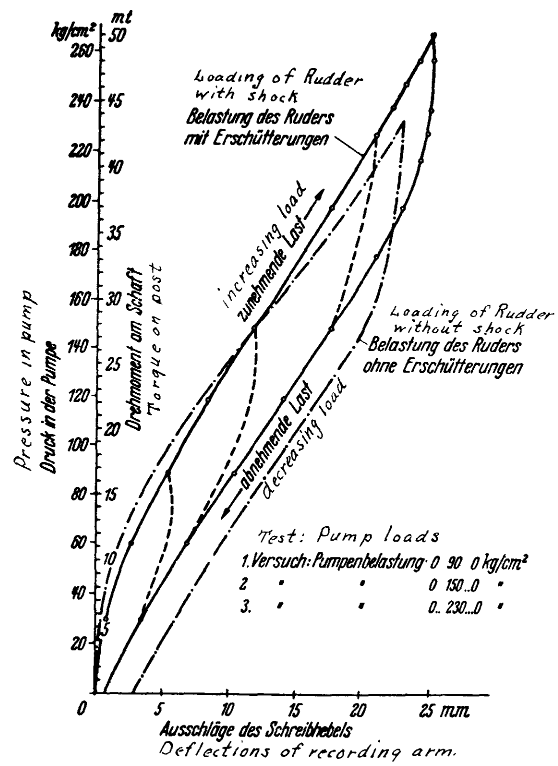
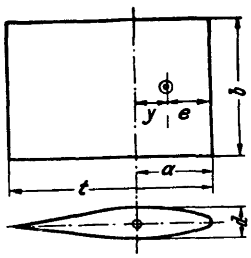


Fig.11. Calibration Curves for Torsion Tests.



Ship length: L = 169 m
Area of longit. sect.: F_L = 675 m²

$\frac{\text{Area of L.S.}}{\text{Rudder Area}} = \frac{F_L}{F} = 38$

1. Rudder values.
   - t = 5100 mm
   - b = 3500 mm
   - a = 1900 mm
   - y = a - e
   - d = 710 mm
   - F = ~ 17.8 m²
   - F/b² = 1.46
   - t/d = 7.2 about like Göttingen Section No.411

Fig.13a

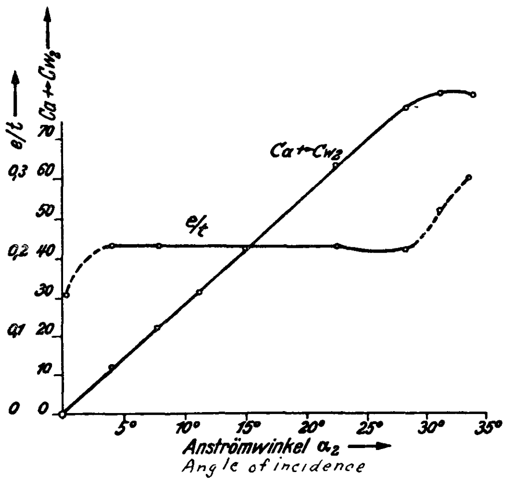


Fig.13b

2. Conversion formulas:

1. $Ca_2 = Ca_1 = Ca$;
2. $Cw_2 = Cw_1 + \frac{Ca^2}{\pi} \times \left(\frac{F_2}{b_2^2} - \frac{F_1}{b_2^1}\right)$
3. $\alpha_2 = \alpha_1 + 57{,}3^\circ \times \frac{Ca}{\pi}\left(\frac{F_2^2}{b_2} - \frac{F_1}{b_1^2}\right)$

$\frac{F_1}{b_1^2} = 0{,}2;\quad \frac{F_2}{b_2^2} = 1{,}46$

$\left(\frac{F_2}{b_2^2} - \frac{F^1}{b_1^2}\right) = 1{,}26$

$e/t = \frac{Cm}{Ca \dashrightarrow Cw_2}$

Fig.13. Ship No.1. Flow Values for the Rudder.

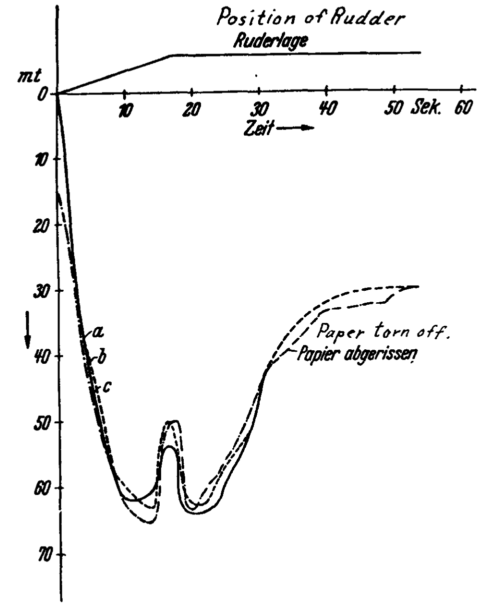


Fig.12. Three Comparable Maneuvers according to Torsion Tests.

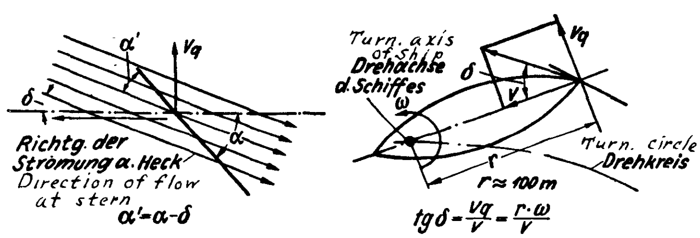


Fig.14a.

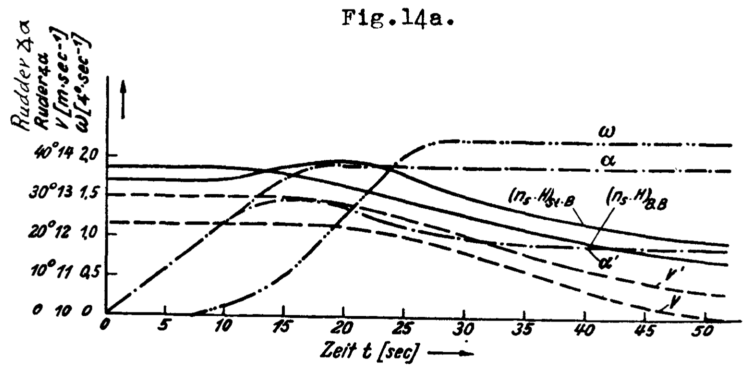


Fig.14b.

Fig.14. Ship No.1. Determination of Angle of Attack α' on the Rudder.

Measured values:
1. Time: t(sec)
2. Rudder angle: α [∡°]
3. Ship velocity: v(m sec⁻¹)
4. Propeller RPM: n_s(sec⁻¹)
5. Ang. Veloc. of turn: ω (∡° sec⁻¹)
6. Distance of stern from ship's turning point

Calculated values:
1. Angle between axis of ship and direction of flow at stern:
2. Direction of flow at rudder: α' [∡°]

Translational velocity of propellers: n_s H (m/sec)
H = Propeller pitch = 3.4 m.

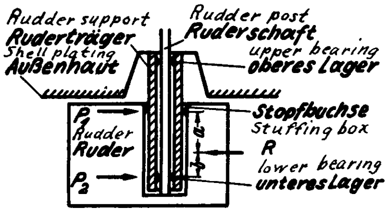


Abb. 15.

1. Calculation of Friction Moment M_R.

1. $R = P_1 + P_2$ 2. $P_1 a = P_2 b$ $a = 1.50$ m $b = 0.34$ m

Friction moment in stuffing box bearings $= P_1 \mu_1 r_1$ 1.27
Friction moment in lower bearing $= P_2 \mu_2 r_2$ 1.27

$$M_R = P_1 \mu_1 r_1 \, 1.27 + P_2 \mu_2 r_2 \, 1.27$$

$2r_1$ = Diam. of stuffing box bearing; $r_1 = 0.350$ m
$2r_2$ = Diam. of lower bearing; $r_2 = 0.215$ m
= = Friction coefficient = 0.20
$P_1 = 0.2$; $P_2 = 0.8$ R

$M_R = 0.06$ R R = Resultant force on rudder

Fig.15. Ship No.1. Calculation of M_{Str} and M_R and Determination of the Difference between the resultant calculated Torque $M_{Str} +\!\!\!\to M_R$ and the Torque M_{VD} measured on the Rudder Post.

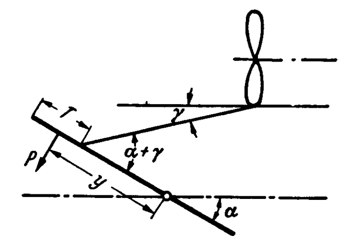


Fig.16a. Determination of Rudder Area F' impinged upon by the Propeller Race.

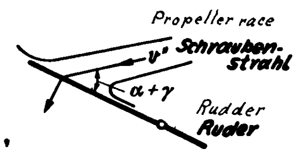


Fig.16b

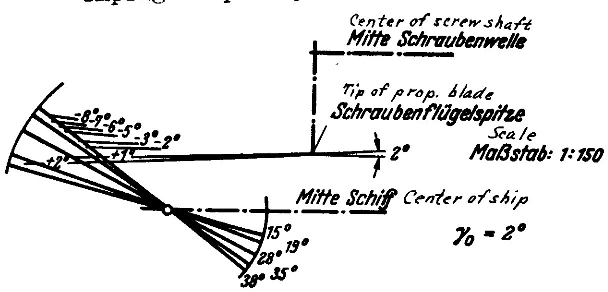


Fig.16c. Graphic Method of Determining T and y'

T = Depth of rudder area impinged upon by the propeller race
F' = T b; b = width of rudder area
y' = Distance of rudder post from resultant point of attack of the force P produced by the propeller race
γ = Angle of inclination of propeller race to CL in turning test
γ_0 = Angle of inclination of propeller race to CL on her course
$\gamma = \gamma_0 - \frac{\delta}{2}$
$\alpha + \gamma$ = Angle of incidence of race on prop.
P = ρ Q v" sin($\alpha + \gamma$) (see Hütte, Ed.25, p 383)
ρ = Density of water = 102(kg sec²/m⁴)
Q = F' v". v" = 0.7 v; 0.7 = empir. Coeff. Better: v" = 0.63 n_s H; 0.63 = empirical coefficient; n_s H = translational velocity of the propellers
v = ship velocity in m/sec
v" = velocity of race at rudder
M_{Schr} = P y = ρ F' 0.7² v² sin($\alpha + \gamma$) y'
M_{Schr} = 50 y' F' v² sin($\alpha + \gamma$) (mt)

Fig.16. Ship No.1. Determination of Additional Torque M_{Schr} set up by the Propeller Race.

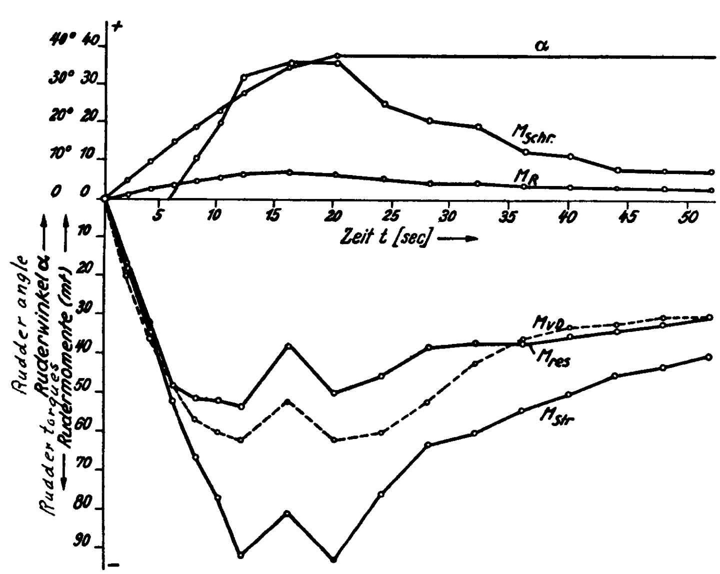
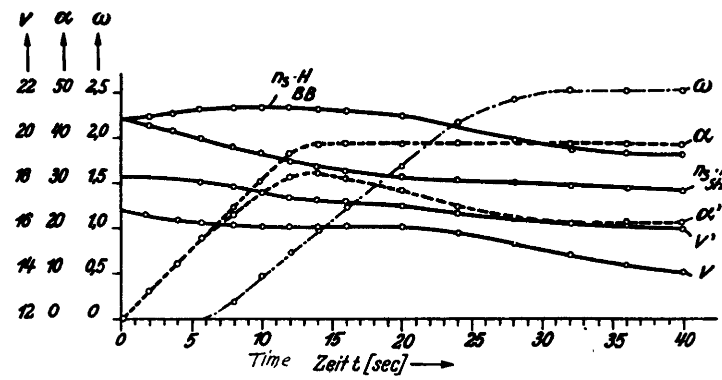
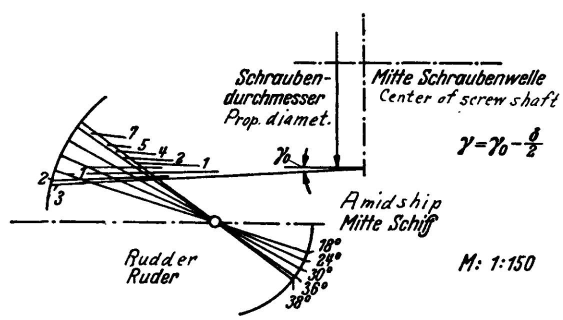


Fig.17. Ship No.1. Measured and Calculated Torques on the Rudder Post.

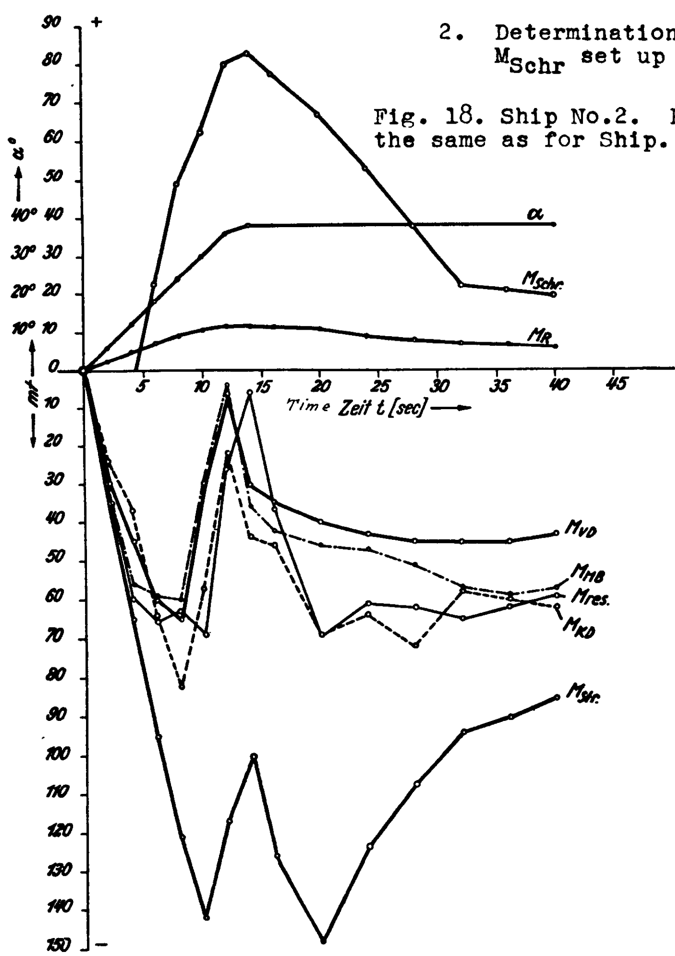


1. Operating values and Angles of Incidence α'.



2. Determination of the Additional Torque M_{Schr} set up by the Propeller Race.

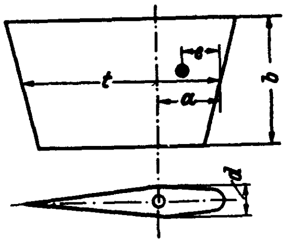
Fig. 18. Ship No.2. Rudder and Flow Coefficients are the same as for Ship. No.1.



M_{MB}: Torque by Test Bolt Measurement
M_{VD}: Torque by measuring torsion of Post
M_{KD}: Torque by Carbon Pile Measurement.

Fig.19. Ship No.2. Measured and Calculated Torques on Rudder Post.

Rudder values:
About like Göttingen section 429 1:8.8
Ship length L = 166 m Area of longitudinal section F_L = 719 m² F_L: F = 45



t = 4.90 m
b = 3.20 m
F = 16 m²
= 1.54 m
e = t e/t
d/t = 1:9
F/b² = 1.57

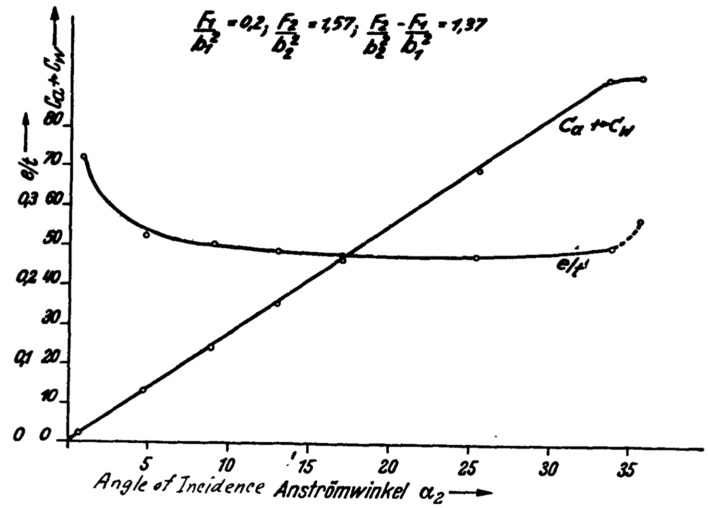
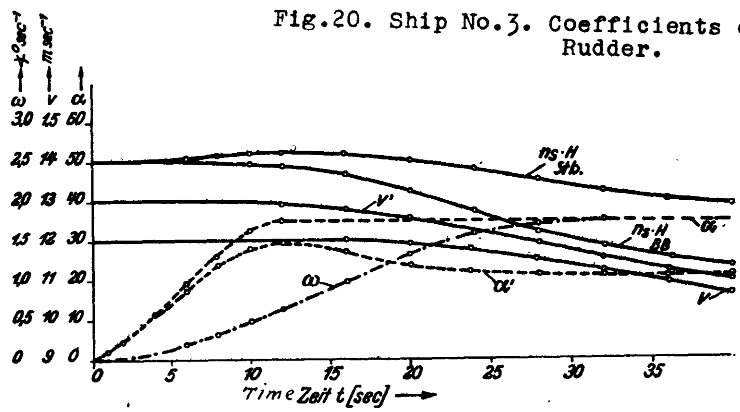
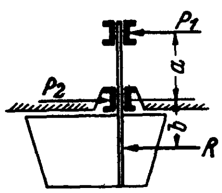


Fig.20. Ship No.3. Coefficients of Flow for the Rudder.



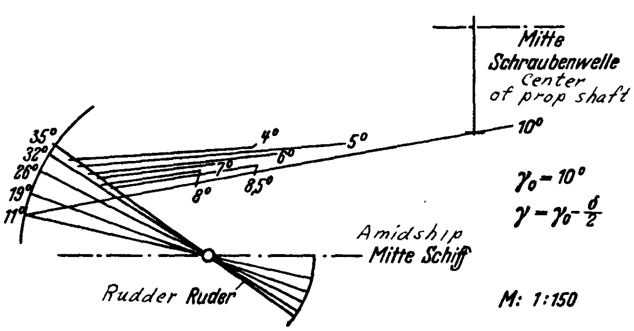
1. Operating Values and Angles of Incidence α'.



I. $P_2 = R + P_1$ a = 1800 mm
II. $P_2\, b = P_1(a + b)$ b = 1800 mm
$2r_1$ = diameter of upper bearing = 500 mm
$2r_2$ = diameter of lower bearing = 680 mm
$\mu_1 = \mu_2$ = friction coefficient = 0.20
$M_R = P_1 \mu_1 r_1 1.27 + P_2 \mu_2 r_2 1.27$

$M_R = 0.23\ R$

2. Calculation of the Moment of Friction M_R.



3. Determination of the Additional Torque (M_{Schr}) set up by the Propeller Race. The same designations apply as for Ship No. 1.

Fig. 21. Ship. No. 3.

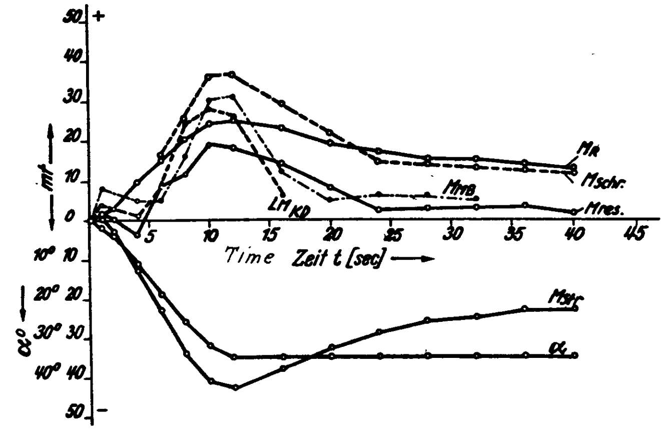
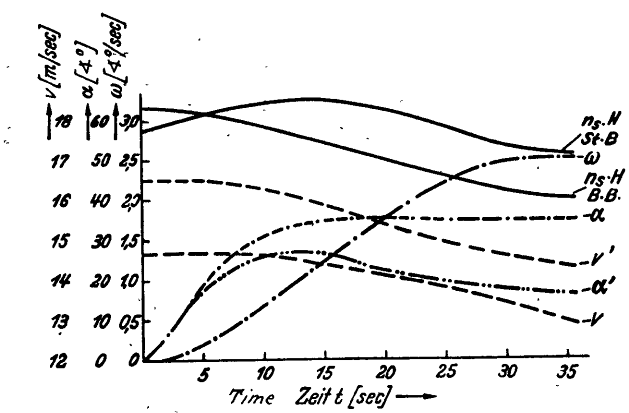
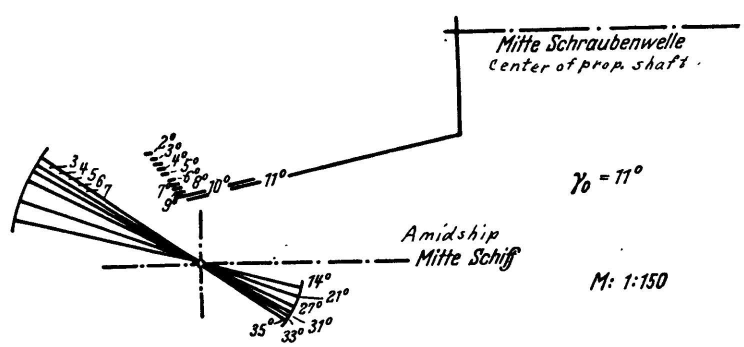


Fig. 22. Ship No.3. Measured and calculated Values on the Rudder Post.



1. Operating Values and Angles of Incidence α'.



2. Determination of Additional Torque M_{Schr} set up by the Propeller Race. The same designations apply as for Ship. No. 1.

Fig. 23. Ship No. 3.

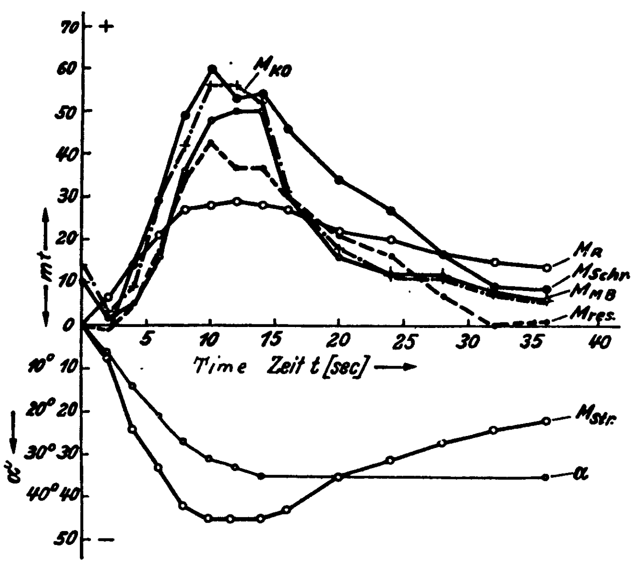


Fig. 24. Ship No.3. Measured and calculated torques on the rudder post.

TABLE 1.

α_1	100·Ca	100·Cw	100·Cm	100·Ca/π	100·1,26 $\frac{Ca}{\pi}$	100·1,26 $\frac{Ca^2}{\pi}$	100·Cw_2	100 (Ca +→ Cw_2)	$\frac{Cm}{Ca +\to Cw_2}$	57,3·1,26·ca/π	α_2
0,0°	2,0	0,81	0,3	0,64	0 81	0,016	0,83	2,0	0,15	0,46	0,5°
1,5°	11,7	0,94	2,5	3,7	4,[illegible]5	0,545	1,49	11,8	0,212	2,65	4,16°
2,9°	21,2	1,22	4,5	6,7	8,4[illegible]	1,79	3,01	21,4	0,210	4,85	7,75°
4,4°	30,2	1,85	6,5	9,6	12,1	3,65	5,50	30,7	0,212	6,9	11,3°
5,8°	40,5	2,51	9,2	12,9	16,3	6,6	9,11	41,5	0,222	9,35	15,15°
8,8°	59,2	4,54	13,7	18,9	23,8	14,0	18,54	62,0	0,212	13,6	22,4°
11,7°	71,4	7,30	15,6	22,8	28,8	20,3	27,9	77,0	0,203	16,5	28,2°
14,7°	71,2	15,9	20,6	23,0	29,0	28,8	36,7	80,5	0,256	16,6	31,3°
17,8°	68,4	22,7	23,6	21,8	27,5	18,8	41,5	80,5	0,293	15,7	33,5°

TABLE 2.

t	α	V	$n_s \cdot H$ St. B	$n_s \cdot H$ B.B	ω	$r \cdot \frac{\pi}{180} \cdot \omega$	tg δ	δ	α'
0	0	12,3	13,4	13,7	0	0	0	0	0
1	2	12,3	13,4	13,7	0	0	0	0	2°
2	5	12 3	13 4	13 7	0	0	0	0	5°
4	10	12,3	13,4	13,7	0	0	0	0	10°
6	15	12,3	13,4	13,7	0	0	0	0	15°
8	19	12,3	13,4	13,7	0	0	0	0	19°
10	23	12,3	13,4	13,7	0,1	0,175	0,0142	~ 1°	22°
12	28	12,3	13,5	13,7	0,2	0,35	0 028	1,5°	265°
14	32	12,3	13,6	13,6	0,35	0,61	0,0495	3°	29°
16	35	12,3	13,7	13,5	0,60	1,05	0,0855	5°	30°
20	38	12,2	13,9	13,3	12,5	2,2	0,180	10°	28°
24	38	12,0	13,7	13,0	1,9	3,3	0,275	15°	23°
28	38	11,8	13,3	12,7	2,2	3,8	0,320	18°	20°
32	38	11,5	13,0	12,5	2,2	3,8	0,33	18°	20°
36	38	11,2	12,8	12,2	2,2	3,8	0,34	19°	19°
40	38	10,7	12,6	11,9	2,2	3,8	0,355	19°	19°
44	38	10,4	12,3	11,7	2,2	3,8	0,365	20°	18°
48	38	10,2	12,1	11,5	2,2	3,8	0,37	20°	18°
52	38	10,0	11,9	11,4	2,2	3,8	0,38	21°	17°

TABLE 3.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
t	a	α'	$C_a +\!\!\rightarrow C_w$	e/t	e	y	v′	$F \cdot q \cdot 10^3$	R	M_R	M_{Str}	$M_{Str} +\!\!\rightarrow M_R$	M_{VD}	M_{Schr}
0	0	0	0	0	—	—	13,0	154	0	0	0	0	0	0
1	2	2	0,05	0,21	1,07	0,83	13,0	154	7,7	+ 0,46	— 6,4	— 6,0	— 7,5	— 1,5
2	5	5	0,14	0,21	1,07	0,83	13,0	154	21,6	+ 1,3	— 18,0	— 16,7	— 20	— 3,3
4	10	10	0,27	0,21	1,07	0,83	13,0	154	41,5	+ 2,5	— 34,5	— 32	— 36	— 4,0
6	15	15	0,41	0,21	1,07	0,83	13,0	154	63	+ 3,8	— 52,0	— 48	— 48	± 0
8	19	19	0,52	0,21	1,07	0,83	13,0	154	80	+ 4,8	— 66,5	— 62	— 57	+ 5
10	23	22	0,60	0,21	1,07	0,83	13,0	154	92,5	+ 5,5	— 77	— 71	— 60	+ 11
12	28	26,5	0,72	0,21	1,07	0,83	13,0	154	111	+ 6,6	— 92	— 85	— 62	+ 23
14	32	29	0,77	0,225	1,15	0,75	13,0	154	119	+ 7,1	— 89	— 82	— 63	+ 19
16	35	30	0,79	0,24	1,22	0,68	12,9	151	119	+ 7,1	— 81	— 74	— 52	+ 22
20	38	28	0,76	0,21	1,07	0,83	12,7	147	112	+ 6,7	— 93	— 86	— 62	+ 24
24	38	23	0,63	0,21	1,07	0,83	12,6	145	92	+ 5,5	— 76	— 70	— 60	+ 10
28	38	20	0,55	0,21	1,07	0,83	12,3	138	76	+ 4,5	— 63	— 58	— 52	+ 6
32	38	20	0,55	0,21	1,07	0,83	12,0	131	72	+ 4,3	— 60	— 56	— 42	+ 14
36	38	19	0,52	0,21	1,07	0,83	11,7	125	65	+ 3,9	— 54	— 50	— 36	+ 14
40	38	19	0,52	0,21	1,07	0,83	11,3	116	60	+ 3,6	— 50	— 46	— 33	+ 13
44	38	18	0,49	0,21	1,07	0,83	11,0	110	54	+ 3,2	— 45	— 42	— 32	+ 10
48	38	18	0,49	0,21	1,07	0,83	10,8	105	51,5	+ 3,1	— 43	— 40	— 30	+ 10
52	38	17	0,47	0,21	1,07	0,83	10,6	102	48	+ 2,9	— 40	— 37	— 30	+ 7

$F \cdot q = F \cdot \varrho/2 \cdot v'^2$; $\varrho = 101{,}9\,[\mathrm{kg} \cdot \mathrm{sec}^2 \cdot \mathrm{m}^{-4}]$; $F = 17{,}8\ \mathrm{m}^2$; $F \cdot q = 910\ V^2$; $R = F \cdot q \cdot (C_a +\!\!\rightarrow C_w)$; $M_R = 0{,}06\ R$; $M_{Str} = R \cdot y\ [\mathrm{mt}]$; M_{VD} = gemessene Werte; $M_{Schr} = M_{VD} - (M_{Str} +\!\!\rightarrow M_R)$.

TABLE 4.

t	α	δ	γ	$\alpha \pm \gamma$	T	F′	y′	v	v^2	$\sin(\alpha \pm \gamma)$	$50 \cdot y' \cdot F'$	$v^2 \sin(\alpha \pm \gamma)$	$M_{Schr.}$ [m t]
6	15	—	+ 2	17	—	—	—	12,3	150	0,29	—	—	
8	19	—	+ 2	21	0,4	1,35	2,9	12,3	150	0,36	196	54	+10,6
12	28	1,5	+ 1	29	1,1	3,7	2,4	12,3	150	0,48	445	72	+32
16	35	5	— 2	33	1,1	3,7	2,4	12,3	150	0,54	445	81	+36
20	38	10	— 3	35	1,0	3,4	2,5	12,2	150	0,57	425	85	+36
24	38	15	— 5	33	0,7	2,4	2,7	12,0	144	0,54	325	78	+25
28	38	18	— 6	32	0,6	2,0	2,8	11,8	140	0,53	280	74	+21
32	38	18	— 6	32	0,6	2,0	2,8	11,5	132	0,53	280	70	+19
36	38	19	— 7	31	0,4	1,35	2,9	11,2	126	0,52	196	65	+12,7
40	38	19	— 7	31	0,4	1,35	2,9	10,7	114	0,52	196	59	+11,5
44	38	20	— 8	30	0,3	1,0	3,0	10,4	108	0,50	150	54	+ 8,1
48	38	20	— 8	30	0,3	1,0	3,0	10,2	104	0,50	150	52	+ 7,8
52	38	21	— 8	30	0,3	1,0	3,0	10,0	100	0,50	150	50	+ 7,5

TABLE 5.

t	α	$M_{Str.}$	M_R	$M_{Schr.}$	M_{res}	M_{VD}
0	0	0	0	0	0	0
2	5	— 18	+ 1,3	—	— 16,7	— 20
4	10	— 34,5	+ 2,5	—	— 32	— 36
6	15	— 52	+ 36,	—	— 48	— 48
8	19	— 66,5	+ 4,8	+ 10,6	— 51,2	— 57
10	23	— 77	+ 5,6	+ 20	— 52	— 60
12	28	— 92	+ 6,6	+ 32	— 53,4	— 62
16	35	— 81	+ 7,1	+ 36	— 38	— 52
20	38	— 93	+ 6,7	+ 36	— 50	— 62
24	38	— 76	+ 5,5	+ 25	— 45,5	— 60
28	38	— 63	+ 4,5	+ 21	— 38	— 52
32	38	— 60	+ 4,3	+ 19	— 37	— 42
36	38	— 54	+ 3,9	+ 12,7	— 37	— 36
40	38	— 50	+ 3,6	+ 11,5	— 35	— 33
44	38	— 45	+ 3,2	+ 8,1	— 34	— 32
48	38	— 43	+ 3,1	+ 7,8	— 32	— 30
52	38	— 40	+ 2,9	+ 7,5	— 30	— 30

TABLE 6.

t	α	δ	γ	$\alpha \pm \gamma$	T	F′	y′	v	v^2	sin $(\alpha \pm \gamma)$	$50 \cdot y'F'$	$v^2 \cdot \sin (\alpha \pm \gamma)$	M_{Schr}
6	18	—	+ 3	21	0,5	1,7	2,8	16,1	260	0,36	240	93,5	+ 22,4
8	24	1	+ 2	26	1,0	3,4	2,5	16,1	260	0,44	425	115	+ 49
10	30	3	+ 1	31	1,2	4,1	2,3	16,0	256	0,52	470	133	+ 62,5
12	36	4,5	+ 1	37	1,4	4,75	2,2	16,0	256	0,60	520	153	+ 80
14	38	6	0	38	1,4	4,75	2,2	16,0	256	0,62	520	159	+ 83
16	38	7,5	— 1	37	1,3	4,4	2,3	16,0	256	0,60	505	153	+ 77,5
20	38	10	— 2	36	1,1	3,7	2,4	16,0	256	0,59	445	151	+ 67
24	38	13,5	— 4	34	0,9	3,0	2,6	15,6	243	0,56	390	136	+ 53
28	38	15,5	— 5	33	0,7	2,4	2,6	15,2	230	0,54	310	124	+ 38
32	38	17	— 7	31	0,4	1,35	2,9	14,7	216	0,52	195	112	+ 22
36	38	17	— 7	31	0,4	1,35	2,9	14,3	205	0,52	195	107	+ 21
40	38	17	— 7	31	0,4	1,35	2,9	13,9	194	0,52	195	100	+ 19,5

TABLE 7.

t	α	$M_{Str.}$	M_R	$M_{Schr.}$	M_{res}	M_{MB}	M_{VD}	M_{KD}
2	6	— 35	+ 2,5	—	— 32,5	— 36	— 30	— 24
4	12	— 65	+ 4,7	—	— 60	— 56	— 45	— 37
6	18	— 95	+ 6,9	+ 22,4	— 66	— 59	— 60	— 64
8	24	— 121	+ 8,8	+ 49	— 63	— 60	— 65	— 82
10	30	— 142	+ 10,3	+ 62,5	— 69	— 30	— 30	— 57
12	36	— 117	+ 11,2	+ 80	— 26	— 4	— 6	— 22
14	38	— 100	+ 11,3	+ 83	— 6	— 36	— 30	— 44
16	38	— 126	+ 11,2	+ 77,5	— 37	— 42	— 35	— 46
20	38	— 148	+ 10,7	+ 67	— 70	— 46	— 40	— 69
24	38	— 123	+ 8,9	+ 53	— 61	— 47	— 43	— 64
28	38	— 108	+ 7,8	+ 38	— 62	— 51	— 45	— 72
32	38	— 94	+ 6,8	+ 22	— 65	— 57	— 45	— 58
36	38	— 90	+ 6,5	+ 21	— 62	— 59	— 45	— 60
40	38	— 85	+ 6,1	+ 19,5	— 59	— 57	— 43	— 62

TABLE 8.

α	$100\,C_a$	$100\,C_w$	$100\,C_m$	$100\,C_{a/\pi}$	$100 \cdot 1{,}37 \cdot \frac{C_a}{\pi}$	$100 \cdot 1{,}37 \cdot \frac{C_a^2}{\pi}$	$100\,C_{w_s}$	$100\,C_a \leftrightarrow C_{w_s}$	$\frac{C_m}{C_a \leftrightarrow C_{w_s}}$	$57{,}3^\circ \cdot 1{,}37 \cdot \frac{C_a}{\pi}$	α_s
0	2,8	0,84	1,0	0,89	1,22	0,034	0,87	2,8	0,356	0,70	0,7°
1,5°	12,9	0,98	3,4	4,1	5,6	0,72	1,70	13,0	0,262	3,2	4,7°
2,9°	23,6	1,26	6,0	7,55	10,3	2,44	3,70	23,8	0,252	5,9	8,8°
4,4°	34,0	1,74	8,4	10,8	14,8	5,0	6,74	34,7	0,242	8,5	12,9°
5,8°	44,4	2,27	10,9	14,1	19,3	8,6	10,87	46,0	0,238	11,1	16,9°
8,8°	65,8	4,26	16,4	21,0	27,4	18,0	22,26	69,0	0,238	16,5	25,3°
11,7°	84,9	6,66	21,7	27,0	37,0	31,4	38,06	92,5	0,235	21,2	32,9°
14,7°	83,0	14,9	26,7	26,4	33,4	28,0	42,9	93,0	0,287	20,8	35,5°

TABLE 10.

t	α	δ	γ	$\alpha \pm \gamma$	T	F′	y′	v	v^2	$\sin \cdot (\alpha \pm \gamma)$	$\frac{50}{y' \cdot F'}$	$v^2 \cdot \sin \cdot (\alpha \pm \gamma)$	$M_{Schr.}$
4	14	0,6	+ 11	25	0,8	1,0	3,2	14,7	215	0,42	160	90	+ 14
6	21	2	+ 10	31	0,8	1,8	2,9	14,7	215	0,52	260	112	+ 29
8	27	3	+ 10	31	1,3	3,4	2,6	14,7	215	0,52	440	112	+ 49
10	31	5	+ 9	40	1,3	3,4	2,6	14,7	215	0,64	440	137	+ 60
12	33	6	+ 8	41	1,1	2,8	2,8	14,5	210	0,65	390	136	+ 53
14	35	8	+ 7	42	1,1	2,8	2,8	14,4	208	0,67	390	139	+ 54
16	35	9	+ 6,5	41,5	1,0	2,4	2,8	14,3	205	0,67	335	137	+ 46
20	35	13	+ 6	41	0,9	1,8	2,9	14,1	200	0,65	260	130	+ 34
24	35	15	+ 5	40	0,7	1,4	3,1	13,9	194	0,64	220	124	+ 27
28	35	17	+ 4	39	0,5	0,9	3,2	13,6	185	0,63	144	117	+ 17
32	35	18	+ 3	38	0,3	0,5	3,4	13,2	175	0,63	85	110	+ 9,4
36	35	19	+ 3	38	0,3	0,5	3,4	12,9	166	0,63	85	105	+ 8,9

TABLE 11. Compilation of calculated and measured torques.

t	α	$M_{Str.}$	$M_{R.}$	$M_{Schr.}$	$M_{res.}$	M_{MB}	M_{KD}
0	0	0	0	—	—	+ 10	+ 14
2	6	— 7,3	+ 6,5	—	— 0,8	+ 2	+ 3
4	14	— 24	+ 15	+ 14	+ 5	+ 5	+ 9
6	21	— 33	+ 21	+ 29	+ 17	+ 16	+ 30
8	27	— 42	+ 27	+ 49	+ 34	+ 36	+ 42
10	31	— 45	+ 28	+ 60	+ 43	+ 48	+ 56
12	33	— 45	+ 29	+ 53	+ 37	+ 50	+ 56
14	35	— 45	+ 28	+ 54	+ 37	+ 50	+ 52
16	35	— 43	+ 27	+ 46	+ 30	+ 30	+ 31
20	35	— 35	+ 22	+ 34	+ 21	+ 16	+ 18
24	35	— 31	+ 20	+ 27	+ 16	+ 12	+ 12
28	35	— 27	+ 17	+ 17	+ 7	+ 12	+ 12
32	35	— 24	+ 15	+ 9,4	± 0	+ 8	+ 8
36	35	— 22	+ 14	+ 9,8	+ 1	+ 6	+ 6

TABLE 9. Compilation of calculated and measured torques

t	α	$M_{Str.}$	M_R	$M_{Schr.}$	M_{res}	M_{MB}	M_{KD}
1	2	+ 0,2	+ 1,6	—	+ 1,8	+ 8	+ 4
2	4	— 3	+ 3,5	—	+ 0,5	+ 7	+ 3
4	11	— 13	+ 9,5	—	— 3,5	+ 5	+ 1
6	19	— 23	+ 15	+ 16,5	+ 8,5	+ 5	+ 8
8	26	— 34	+ 20	+ 25,5	+ 11,5	+ 16	+ 24
10	32	— 41	+ 24	+ 36	+ 19	+ 30	+ 28
12	35	— 43	+ 25	+ 36,4	+ 18	+ 31	+ 26
16	35	— 38	+ 23	+ 29	+ 14	+ 12	+ 6
20	35	— 33	+ 19	+ 22	+ 8	+ 5	—
24	35	— 29	+ 17	+ 14,5	+ 2,5	+ 6	—
28	35	— 26	+ 15	+ 13,7	+ 2,7	+ 6	—
32	35	— 25	+ 15	+ 12,9	+ 2,9	+ 5	—
36	35	— 23	+ 14	+ 12,2	+ 3,2	—	—
40	35	— 23	+ 13	+ 11,5	+ 1,5	—	—