UNITED STATES
EXPERIMENTAL MODEL BASIN

NAVY YARD, WASHINGTON, D.C.

HIGH SEAS TEST TRIP
OSCILLATION AND ACCELERATION MEASUREMENTS

BY F. HORN

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by F. Horn

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I. PURPOSE AND AIM OF THE OSCILLATION, ACCELERATION, AND WAVE MEASUREMENTS.

In addition to the strain and deflection measurements made on the high sea test trip of the M.S. SAN FRANCISCO in the autumn of 1934 which formed the main object of the undertaking and were the particular concern of the director of the general undertaking, Prof. Schnadel, extensive vibration, acceleration and wave measurements were undertaken at my suggestion and under my direction. In particular, Dr. Ing. von den Steinen, Hamburg, undertook the measurement of rolling, pitching, and yawing angles, Dr. Ing. Weinblum the wave measurements, Chief Engineer Weiss the entirely novel measurement of waves along the hull, while I together with my co-worker, Dipl. Ing. Stemmer, took over principally the measurement of accelerations.

The purpose of these measurements was twofold. First an indirect purpose: It was intended to establish by these measurements under what conditions of seaway and position of the ship in the waves the strains, deflections, and stresses ascertained by Schnadel's measurements were set up. These latter measurements should thus be elevated above the range of pure statistics and placed upon a more systematic basis, which obviously must increase their value materially. The second object of the measurements instigated by me was a personal one, namely the most systematic possible investigations of the movements of a ship in a seaway with the aim of furthering research on the subject of the sea-going qualities of a ship.

The chief problem involved herein is as follows: In case we assume a uniform seaway there are extensive theoretical principles already available. These have been established principally by Kriloff (1) and in certain aspects by myself (2) and Kent(3), and recently by Weinblum (4) in a highly systematic and efficient manner. By means of these papers it is actually possible if we content ourselves with certain still somewhat rough approximations with regard to the damping of the ship's oscillations, to derive with practically adequate accuracy from the given ship form, in a uniform seaway and for any desired ratio of ship length to wave length and with any desired angle between the ship's course and the direction of the waves, the movement of translation in up-and-down, forward-and-back, and sidewise direction as well as the rotational movements about the longitudinal, transverse, and vertical axes. Furthermore, again assuming a uniform seaway, the more recent papers of Weinblum supply good, systematic data for the comprehension of the influence of certain values characteristic of the ship, if it has an approximately standard form, as for instance the coefficient of fineness of the CWI and the distribution of displacement over the ship's length. The chief conditions for estimating the sea-going qualities are, as is well known, quite complicated, since in addition to the influences of the ratio of the ship's length to wave length and the angle of the ship's course to the direction of the waves, there is the highly important influence of the ratio of the relative wave period or period of incidence dependent upon the speed of the ship, to each of the three different natural periods (of rolling, pitching, and heaving oscillations), and since in addition it depends not only on the magnitude of the amplitudes of the oscillations arising under these conditions, but also very materially on the momentary position and the direction of motion of the bow with respect to the wave passing the bow at a given moment. The complicated nature of these manifold conditions which are decisive for determining the sea-going qualities have recently been closely simulated by waves artificially propagated in a model basin by Kempf (5), although these experiments, because they are limited to the case in which the course of the ship coincides with the direction of progression of the waves, naturally can encompass only a limited portion of the problem as a whole.

All these theoretical and experimental investigations have the disadvantage, however, that they assume a uniform seaway and thus take into account only forced oscillations, since the natural oscillations initially accompanying them as is known are quickly quenched by damping and then cease to be important. The assumption of a uniform seaway, however, is never completely and seldom approximately fulfilled. Therefore the free oscillations are never extinguished in actual practice, and usually are at least of equal, if not of greater importance than the forced oscillations which, by the way, now differ from wave to wave. Due to this fact the oscillation processes and consequently also the seagoing qualities will be found to be more or less different from those in a uniform seaway, which might be conceived as having resulted, say, by forming averages of wave lengths and heights over a considerable period of time. This difference might possibly be so great that doubt might exist whether the theoretical investigations and data valid for the mean uniform seaway would still apply in any degree. This is conceivably an extraordinarily important problem; indeed it is of decisive importance to continued development. On the manner in which it is answered depends the question whether we may or may not hope to make the data derived on the assumption of a uniform seaway, either by theoretical calculation or by model tests, or perhaps still to be more fully developed in the future, more and more available for a systematic control of the seagoing qualities of a designed ship.

There will be an inclination from the start to answer this question in the affirmative, as a whole. As found by experience, in every ship - and this was also borne out in the case of the SAN FRANCISCO - especially large rolling oscillations set in on the average at a wholly definite angle between the direction of advance of the ship and of the waves, and in the case of larger vessels always with the sea coming at an angle from aft, which in view of the smallness of the wave slope angle existing under these circumstances, can only be explained by comparatively frequent resonances. This is only possible, however, when we can actually count upon an average seaway, at least with respect to the wave length and consequently also with the wave period; in other words, that a mean uniform seaway may be regarded as substituted for the always more or less non-uniform actual seaway as to its effect on the ship as a whole. However, conditions are simplest in rolling oscillations for the reason that in a seaway worth the name with respect to the size of the ship the wave length is always to be taken as great in comparison to the ship's dimension governing rolling oscillations, namely the beam. For heaving and pitching oscillations, of which the latter play a part at least equally important as, if not more so than that of rolling oscillations as far as the seagoing qualities are concerned, the ship's length is the decisive dimension and the ratio of ship's length to wave length is the prime factor in the development of these oscillations. Any greater diversity of wave lengths which might exist in a non-uniform seaway therefore might possibly distort the nature of these oscillations corresponding to an average seaway to the point where they would be unrecognizable.

In this sense the foregoing important question has hitherto remained open, and it was intended to attempt to contribute to a certain extent to its satisfactory analysis by suitable measurements aboard the SAN FRANCISCO. This necessitated above all things, according to the foregoing statements, the making of wave measurements in addition to oscillation measurements, if possible not only in the manner previously familiar, (6), that is, of making several stereophotographic measurements of the waves in the vicinity of the ship from a high position aboard the ship during the oscillation measurements, but beyond this I also considered it advisable and valuable to render it possible to photograph the wave contours along both sides of the ship currently and in synchronism with the oscillation measurements. This thought presented by me for discussion, after the most diverse methods had been considered was finally put into execution in what I considered to be an excellent manner by Oberingenieur Weiss, who will report on this subject himself in a special paper. Furthermore, such a wave measuring device also would appear to be especially desirable and suitable in connection with strain measurements, since it permitted hope of determining with what waves and with what position of the waves with respect to the ship particularly high strain or stress crests were observed.

To permit comparison of the actual movements of the ship in a seaway with those to be expected theoretically in an average uniform seaway - a comparison whose purpose obviously would be a check up of the agreement in tendencies, but not of absolute magnitudes - it was clearly quite essential to attempt, in cases in which wind and seaway were constant, to a certain extent, to record the movements of the ship on as many different courses as possible with respect to the waves; i.e., in such cases it was necessary to run on altered courses or astern. Actually during the return voyage of the ship - the whole outward voyage was rendered useless because of continuous calm weather - four backing tests were made, but unfortunately during these runs wind and waves were not as obliging as the ship's officers, and failed to grant us that which would have been desired for this type of trial run.

Aside from the object of such systematic comparative tests, oscillation and wave measurements are obviously valuable in other directions also. Thus, for example, they may supply statistical material for the important question as to what extent natural oscillations or forced vibrations predominate in the movements of ships under various sea conditions. Of particular significance is obviously also the fixing of maximum values (rolling and pitching angles, heaving and pitching amplitudes or accelerations), such as occur in heavy seas and as such may then yield empirical information important in the designing of ships, especially with respect to strength. Fortunately the high sea test voyage was highly productive in this respect, for the three days of storm granted us by the god of storms in the final week after three months of waiting, brought as heavy weather as could be desired for the purposes named.

Furthermore, in this test voyage extensive measurements of longitudinal, transverse and yawing movement of a ship were made, probably for the first time.

In my present paper I shall confine myself essentially to the instruments, measurements, and data of the oscillation and acceleration measurements, while the data obviously will have to be evaluated in connection with the corresponding wave measurements and the measurements of the ship's speed, force of wind, and direction of wind, the latter of which have been currently undertaken by Ing. Hoppe who participated in the trip as representative of the Hamburgische Schiffbau-Versuchsanstalt. Several measurements of elastic vibrations of the ship's hull will likewise be briefly discussed, for the recording of which some of the instruments were just as suitable as for the oscillations of the ship in a seaway.

The theoretical considerations which governed the selection and installation of the test instruments which at the same time are also important in the choice of methods of evaluating the test data will be elucidated in the following chapter.

From among the great mass of records obtained on the high sea test voyage I have first attempted to select what appeared to me most essential, and have summarized this in the present paper. However, I am fully conscious of the fact that there are still wide gaps in the evaluation, which for reasons to be detailed later, is very troublesome and requires a great deal of time and large personnel. Frequently it has been possible to cite only comparatively short excerpts from the mass of material and it may very well be possible that this or that essential point may crop up subsequently. In that event, I shall report the fact in later papers.

II. THEORETICAL PRINCIPLES FOR THE MEASUREMENT OF OSCILLATIONS AND ACCELERATIONS.

To begin with, it seems appropriate to explain the meaning of the differentiation between oscillation and acceleration measurements.

Fundamentally both are oscillation measurements, in which, however, the records in relation to the oscillations to be measured will be found to differ widely, depending upon the ratio of their frequencies to the natural frequencies of the test apparatus. This is well known from the theory of vibration measuring instruments (7) wherefore it will here be necessary only to summarize briefly the essential predications of this theory to the extent required for our present purpose. I have given the theory briefly in the appendix as adapted to our problem of measuring the movements of a ship in a seaway and the instruments used for this purpose.

All instruments for measuring oscillations are themselves constructed as systems free to oscillate, and therefore have one, and in certain cases two masses in some manner acted upon by a restoring force, either gravity or a spring, whose movement relative to the fixed base and therefore to the position of the ship whose oscillation is to be measured, is suitably recorded. If the mass of the apparatus, considered for the time being as in a state of rest, is displaced from the position of equilibrium by a single impulse and then left to itself, it performs oscillations about this position of equilibrium which are called the natural oscillations of the instrument and which die out more or less rapidly in a so-called free oscillation curve according to the magnitude of the damping present. The frequency in a unit of time (one oscillation per second = 1 Hertz) with which these natural oscillations occur and which is only slightly dependent upon the degree of damping, is one of the chief factors governing the type of measuring instrument.

In order to facilitate discernment of the method of operation of the various apparatus, we will consider first as the fundamental case of oscillation to be measured, a single oscillation with an harmonic curve, that is, a sinusoidal curve. In case the oscillation to be measured results from superposition of several harmonic oscillations of varying frequencies, this can easily be reduced subsequently to the fundamental case of a single harmonic oscillation.

a) OSCILLOMETERS

If the natural frequency of the measuring device is very low compared to the frequency of the oscillation to be measured, the mass of the measuring device retains its position in space practically without change, according to theory, and the curves traced on the recording surface which oscillates with the oscillating body (ship) under these conditions, therefore yields the curve of the oscillation to be measured directly, with the exception of possible reduction or amplification due to the lever arm of the recorder. In this case the instrument functions as an oscillometer. Its records, moreover, are practically independent of the degree of damping.

The gyroscopic instruments for measuring rolling and pitching oscillations operate on the principle of an oscillometer. They are capable of doing so because, owing to the peculiarity of gyroscopic action, the period of the gyro-pendulum can be made extraordinarily large. In the Anschütz gyro it amounts to 12 min., and is thus far above the period of the rolling oscillations which attains a maximum of 30 sec. Aboard the SAN FRANCISCO it amounted to from 12 to 13.5 sec., depending upon the condition of loading. In view of this latter value which is comparatively low for a large ship, even the Petravic pendulum with its 50 sec. natural period is still capable of giving good service.

An oscillometer for heaving oscillations would also have been highly desirable. However, since the amplitudes of heaving oscillations assume orders of magnitude of 10 meters and more, the oscillating weight of the apparatus, in order to maintain its position in space, would have to be able to cover paths of equal magnitude in a vertical direction relative to the ship. Obviously this is impossible of achievement in practice, and therefore direct measurement of heaving oscillations is unfortunately impossible – they must be undertaken by way of acceleration measurements.

b) Accelerometers

If the natural frequency of the measuring device is very high with respect to the frequency of the oscillation to be measured, theory states (see appendix) that

$$a = s\left(\frac{n}{n_0}\right)^2$$

where s signifies the amplitude of oscillation, a the simultaneous record (disregarding a possible magnification due to the writing arm), n the frequency of the oscillation to be measured, n_0 the natural frequency of the measuring device.

For the foregoing equation we may also write $a = s\left(\frac{\omega}{\omega_0}\right)^2$; $a\omega_0^2 = s\omega^2$ with $\omega_0 = 2\pi n_0$, $\omega = 2\pi n$ equal to the so-called circular frequency of the two oscillations. Now in the present limiting case, ($\omega_0 \gg \omega$) the right side represents the acceleration of the oscillation to be measured. This will therefore be measured by the recorded value a, multiplied by the constant of the instrument ω_0^2, thus the designation accelerometer.

If, as hitherto assumed, the oscillation considered is a single sinusoidal oscillation, such an apparatus could, as a matter of fact, also be used as an oscillometer, since in $s = a\left(\frac{n_0}{n}\right)^2$, the frequency n = 1/T can be found from the period T which may be derived from the record. However, since the record a, according to this, gives the oscillation s on a very greatly diminished scale, great care must be exercised in using the instrument for this purpose to obtain a high degree of accuracy in the record so that the result of the evaluation will be as reliable as possible.

Now if the oscillation to be measured no longer consists of a single sinusoidal oscillation, but of several with varying frequencies superimposed, we may write (see appendix):

$$a\,\omega_0^2 = s_1\omega_1^2 + s_2\omega_2^2 + s_3\omega_3^2 + \cdots\cdots$$

under the assumption that in comparison to all existant frequencies the natural frequency of the instrument continues high. Since the terms on the right side represent the accelerations for the individual components of the oscillation, the entire right side represents the total acceleration of the movement, and this is therefore measured throughout by the record $\underline{a}$, multiplied by the constant ω_0^2 of the instrument. This remains true even when, as is almost always the case with ship oscillations in a seaway, the movement is more or less irregular and there is no discernible periodicity; for in this case the entire curve over a certain period of time can be broken up into various sections, which, individually, can be regarded with sufficient accuracy at least, as having come about by superposition of sinusoidal oscillations of varying frequency.

The acceleration of the motion will therefore be practically correctly reproduced by the record even when it is pronouncedly irregular, as long as the condition that ω_0 be large in comparison with the individual ω's is fulfilled. Instruments of which this is true – and as will be seen in detail, the DVL accelerometers and approximately also the Waas instruments come into this category – are therefore applicable inasmuch as the chief requirement is the measurement of the accelerations themselves. This applies to that province of the high sea test voyage which refers to strength measurements, since for these, among other things, the forces of acceleration and consequently also the accelerations of the measurement must be accessible. While for these purposes, therefore, the direct measurement of accelerations is not only desirable but even requisite, it would be much better to measure oscillations directly instead of accelerations, as a matter of fact, to measure the movements of a ship in a seaway. This is successful, as already stated, in the case of rolling and pitching oscillations, but not in the case of heaving oscillations, to find which it will therefore be necessary to proceed by way of acceleration measurements.

The question now is how in the general case of irregular motion the oscillations may be derived from acceleration measurements. In principle this seems a simple matter, since the amplitude curve, in this case the oscillation curve, familiarly results from double integration of the acceleration curve. If we then designate the recorded curve as $\underline{a}$, the acceleration as b, the amplitude as s, and if ω_0 continues large in comparison to all existing ω's, we will have

$$b = \frac{d^2s}{dt^2} = \omega_0^2 \cdot a$$

$$\frac{ds}{dt} = \omega_0^2 \int a\,dt + C_1$$

$$s = \omega_0^2 \int dt \int a\, dt + C_1 t + C_2.$$

It will be possible to determine the two integration constants C_1 and C_2 very generally from the condition that over a long period of time neither the amplitudes s nor the velocities $\frac{ds}{dt}$ may predominate in one direction or the other. A process carried out according to this principle necessarily will be very difficult and tedious. It will be possible to shorten it in cases in which the recorded curve in the neighborhood of the time zone to be investigated contains a portion, even though it be small, in which it covers a path as nearly sinusoidal as possible, in which then there will also be a distinguishable symmetry with respect to a given time axis (Fig. 1). For the time period corresponding to the axis of symmetry we can then write with sufficient accuracy $C_1 = 0$ and $C_2 = \bar{a}\left(\frac{\omega_0}{\omega}\right)^2$ where in $\omega = \frac{2\pi}{T}$ the period T signifies the period corresponding to this sinusoidal path (See Fig. 1). We can then, by writing t = 0 for the time corresponding to the axis of symmetry, for example, write in place of the foregoing equations:

$$\frac{ds}{dt} = \omega_0^2 \int_0^t a\, dt$$

$$s = \omega_0^2 \int_0^t dt \int_0^t a\, dt + \bar{a}\left(\frac{\omega_0}{\omega}\right)^2.$$

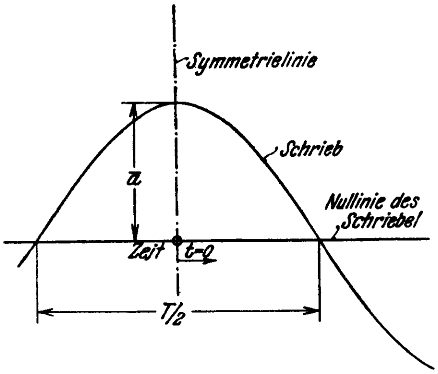

Fig. 1. Schematic example of a case suitable for simplified derivation of the path from the curve recorded by an accelerometer.

Thus, in one or the other of the ways described, the actual derivation of the oscillation curve from the acceleration record would be possible without fundamental difficulties. Cause for uncertainty, however, lies in the fact that because of the scale of $\left(\frac{\omega}{\omega_0}\right)^2$ with which the record is reduced with respect to the oscillation, the former becomes very small and any inaccuracies such as might result from recording arm magnification, for example, or too great thickness of the recorded curve and the like, might lead to greater errors in the data. For this reason it seemed worthy of consideration to use an accelerometer having a lower natural frequency for the heaving oscillations, which up to the present it has not been possible to measure by any means other than by measuring accelerations, in order to obtain the largest and most accurate possible records. Of course as the theory of such a device shows (See Appendix under I 2) this involves acceptance of another disadvantage, for if the natural frequency of the device is only five times as great, for example, as that of an oscillation to be measured and for the time being to be assumed simple harmonic, the amplitudes (overscored) continue in the ratio $\bar{a} = \bar{s}\left(\frac{\omega}{\omega_0}\right)^2$, at least to a good approximation, but the record in view of the required damping (see below) now shows a phase shift with respect to the oscillation which is no longer negligible. This would not be serious as far as the evaluation of an individual oscillation is concerned since the degree of damping is to be taken as known, and the magnitude of the phase shift can thus be determined and the time periods covering the oscillation and the record can be properly arranged. In the case of a compound oscillation, however, especially when irregular, such as oscillations set up by an actual seaway always are, the shift in phase for the various oscillation components varies, and this has the effect that the record can no longer be regarded with sufficient approximation as an acceleration curve as in the case of a higher n_o/n ratio. Therefore it can in general not be used for finding the oscillation curve by double integration in the manner valid for large n_o/n previously indicated. However, as closer investigation shows, (see also the examples in Sec. V) derivation of the oscillations from the records of such an apparatus will always be possible without too great difficulty whenever there is evidence in parts of a certain regularity in the record, especially in the form of symmetry to any time co-ordinate. Moreover, in any case the distortion of the record due to a shift in phase with respect to an accurate acceleration curve (the shift in phase will usually be less than 10^o) will not be sufficiently great to prejudice to any considerable extent analysis of the processes of motion as far as our present aims are concerned. Summing up all the points of view, it seemed advisable to carry out a test for the measurement of heaving oscillations with a special instrument having a relatively low n_o/n ratio for an accelerometer. Such an instrument has been designed at my suggestion by Dr. Ing. Mueller. It is described in detail in Sec. III.

A few more words regarding the damping of oscillometers. The damping of vibrations has, as is well known, two principal effects. First, the natural oscillations are damped down more or less rapidly depending upon the degree of damping. Even this aspect alone requires a high degree of damping while the mass of the measuring apparatus moves. For every deviation of the exciting force, in this case the ship's oscillation, from the harmonic form, sets up natural vibrations of the instrument, whose occurrence means a distortion of the correct record by superposition of pure instrument vibrations, i.e., vibrations not present in the exciting oscillations. Second, damping affects the forced vibrations of the apparatus and when sufficiently strong causes, in particular, the practical avoidance of distortion of the record otherwise occurring in the resonating zone. This is important, although naturally there can be no question of resonance between oscillations caused by a seaway and the natural vibrations of the instruments, for the reason that the measuring instruments also follow the elastic vibrations occurring at the places where they are installed, whether these be set up by the oscillations of the shipform as a whole (flexural or torsional oscillations), or by local vibrations of the part of the ship to which the instrument is attached. In the instruments which act as accelerometers for oscillations in a seaway it is inherently impossible to avoid having such superimposed elastic vibrations appear disproportionately large in the record in comparison with the seaway oscillations, since because $a = s(\frac{\omega}{\omega_0})^2$, the latter (small ω) are reduced on a much larger scale than the former (large ω). However, one thing that can be avoided by sufficiently strong damping is that the elastic vibrations in the record beyond the unavoidable limit mentioned in the foregoing will be further increased by the effect of resonance, which would occur if the natural frequency of the instrument were to coincide either with one of the exciting frequencies of the elastic vibrations (emanating from the engine or propeller) or with one of its natural frequencies (i.e. when the ship is freely vibrating after pitching in a seaway.) Measurement of oscillations due to seaway would suffer greatly from insufficient damping, since in that case these oscillations would be indicated only by the wave shaped course of a broad band bounded by the peak deflections of the elastic vibrations. Furthermore, the determination of the actual magnitude of the deflections of elastic vibrations would be rendered more difficult, in knowing which we are materially interested, as well in connection with the vibrations set up by the action of the engine and propeller as with the free vibration after striking the water.

For all these reasons, therefore, heavy damping of the vibrations of the instruments is requisite. The degree of damping is indicated either by the value $\varkappa = \frac{2W}{\omega_0}$ (see Appendix, Eq. 10) or by the so-called damping ratio γ of two amplitudes following each other in half an oscillation. γ is connected with $\varkappa$ by Eq. 11. The average numerical value for oscillation measuring instruments of the type characterized by the present requirements is $\gamma = 6:1$, corresponding to $\varkappa = 1.14$.

III. DESCRIPTION OF THE MEASURING INSTRUMENTS AND THEIR ARRANGEMENT

The considerations of the foregoing section naturally occupied the foreground in selecting and arranging the measuring instruments, even though, for the rest, as must be freely admitted, for lack of adequate experience we were on the whole somewhat in the dark as to the order of magnitude of the amplitudes of the oscillations and accelerations to be measured and in part also as to the basic suitability of the devices which had never previously been used for ship measurements of this kind, and we had to count upon gaining experience only during the course of the trip. In order to be prepared for this as well as possible it seemed advisable to provide from the start for a certain selection and interchangeability of instruments. Aside from this, a certain reserve had to be provided in case this or the other instrument should be eliminated for some unforeseen reason. Important above all was the following consideration: Due to lack of available personnel, it was impossible to serve every station on the spot, and it was necessary to provide facilities for operating some of the instruments from the central station. On the other hand, it was important always to have several instruments available whose records could be observed directly and would be quickly available for at least temporary evaluation, even though this entailed direct operation. Out of all these considerations the instruments were selected and arranged in the following manner, the arrangement being shown in the plan (Fig. 2):

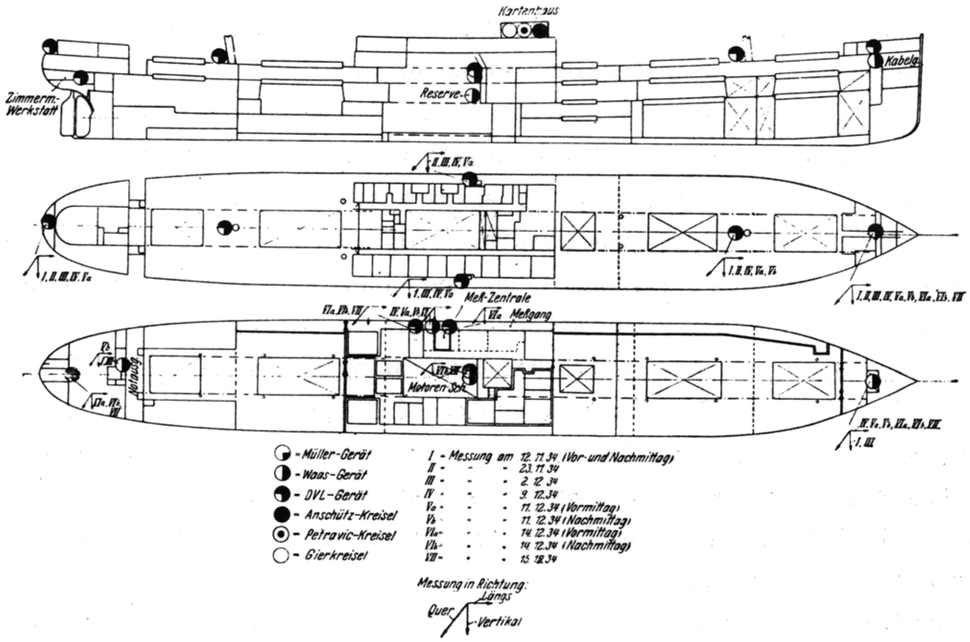

Fig. 2. General plan of the arrangement of instruments for measuring oscillations and accelerations aboard the M.S. SAN FRANCISCO.

1. Through the great kindness of the ship's management, it was possible to clear enough space in the chartroom to arrange a kind of central station for measuring oscillations (Fig. 3). Above all, it was possible there to set up the Anschütz gyroscope for recording rolling and pitching movements, this being a well-known and proven instrument for the purpose. Therefore we may dispense with a detailed description of this device here. The previously mentioned extraordinarily long duration of oscillation of 12 minutes is due

Fig. 3. Oscillometers in the chartroom: Left background Anschütz Gyro, middle foreground Petravic Gyro, on table front special apparatus for measuring rolling accelerations, and to rear case for yaw gyro.

partly to the great angular momentum of the gyroscope ($J\omega$ = 1.13 m sec kg) and on the other hand to the extremely slight distance (2 mm) of the center of gravity of the oscillating system from the axis of suspension. Both, by the way, are not necessary so much for attainment of long duration of oscillation, which in itself would be practically adequate at one fourth its value, as with a view to the greatest possible insensibility to longitudinal and transverse accelerations, which otherwise would result in a certain distortion of the rolling or pitching deflections in consequence of the precession set up by them. In this respect the Petravic gyroscope which was also set up in the charthouse is not so reliable, but is by all means adequate for measurements of more static type. The Anschütz gyroscope was connected with the general time circuit, which was impossible with the Petravic gyroscope. With the latter, it was possible only to achieve an approximate time check by means of occasional marks made by the clock. – Unfortunately I had estimated the magnitude of the maximum pitching angle to be expected wrongly, and due to the recording point adopted on the strength of my estimate, the width of the paper was sufficient only for pitching angles of ± 7½°. At greater angles such as were frequently reached during the days of storm, the gyro struck against something and since the whole gyroscopic movement was disturbed by this, the Anschütz gyroscope was eliminated during these days. Therefore we were glad to have at least the Petravic gyroscope available.

In addition there was set up in the charthouse a gyroscopic apparatus with a free horizontal axis of rotation for measuring yawing movements. For certain periods of time, not too long, such gyros with free axes familiarly maintain whatever axial direction they may have had when started. Mr. von den Steinen who had assumed responsibility for obtaining, setting up, and operating all the apparatus in the chartroom will later himself report on the measurements of yaw with the gyro, as well as on measurements with a special device (for measuring rolling oscillations and accelerations), the principles of which he had already indicated in a previous paper*) and which he had meanwhile been able to construct.

2. In accordance with my statements in II, p. 9, Dr. Ing. O. Mueller of the Versuchsanstalt für Wasserbau und Schiffbau, Berlin, at my request undertook to design a special apparatus for measuring heaving oscillation accelerations. I am deeply obliged to Dr. Mueller for the design and computation of this device, and to Eng. Henschke for its detail plans. Dr. Mueller himself has meanwhile already given a detailed report on this device as to the theories on which it is based and according to which its details were worked out, and therefore, referring to this paper, I am able to express myself very briefly. The principle of design may be seen in the schematic drawing (Fig. 4) taken from the paper mentioned.

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* Werft-Reederei-Hafen 1933, p 199.

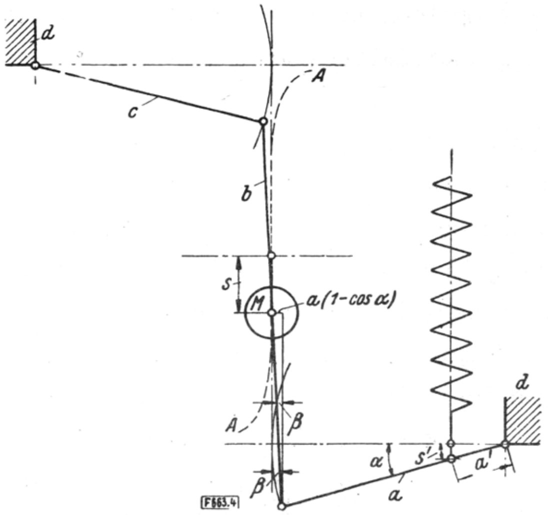

Fig. 4. Structure of the Müller instrument for measuring heaving oscillations (diagrammatic). Lemniscate guide used together with a lever pendulum (widely deflected).

AA path of mass element M restricted to straight line.
a = Lever (30 cm long)
b = Coupling (50 cm long)
e = Link (30 cm long)
d = Frame

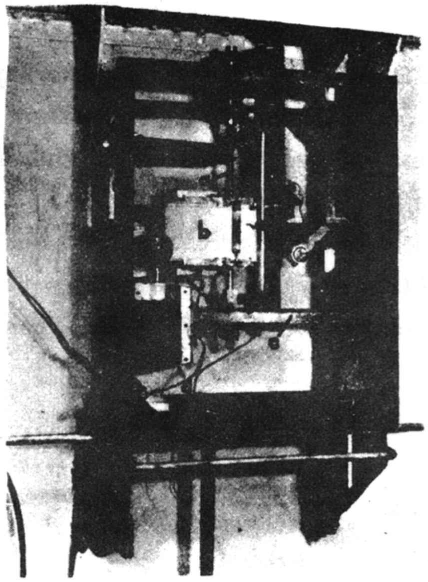

Fig. 5. View of the Müller instrument for measuring heaving installed on the forward engine-room bulkhead.

By the peculiar method of guiding the mass m in an approximately straight line (lemniscate guide) the following advantages were derived:

1. Insensitivity to rotational movements. This is especially important for the present case because in addition to the translation movement to be measured (heaving oscillation) on shipboard, other rotational movements occur in addition, as a rule, which with a measuring device sensitive to rotation would strongly vitiate the values of the translation component to be measured.

2. Direct recording by a scriber connected to the weight, and thereby no distortion of any kind of the record from lever magnification (circular arc guide), secondly because of the extraordinarily high ratio of force of gravity to stilus friction, practically no disturbing influence of the latter.

3. Very simple, solid structure with avoidance of all Coulomb friction.

The natural frequency of the instrument was capable of regulation in two ways: First by dividing the weight in a number of parts, second by changing the lever ratio a'/a (Fig. 4). After it had been found in the first determinative measurements in which a natural frequency of 0.786 Hertz prevailed at the original lever ratio and the original full weight, that for the heaving oscillations to be expected in a heavy seaway the width of the paper would be inadequate, later measurements were made only with an altered lever ratio and a lighter weight, whereupon the number of Hertz was found to be 0.983 by calibration by a free oscillation test. With these it was possible to record the heaving oscillations even in the heaviest sea.

Fig. 5 gives a view of the apparatus installed at the forward edge of the engine room at Frame 74. The weight m seated on the rod a is here hidden by the recorder b. On the extension of the guide c is the copper plate d of the eddy current brake (a large part hidden in the picture by the resistance set in the brake circuit). In a calibration test on board, a free oscillation curve as shown in Fig. 6 was obtained with the damping cut in, i.e. a damping ratio of γ = a/b = 4.25*, from which $\varkappa = 0.93$. From this was found by calculation the so-called amplification curve plotted on the ratio $\lambda = \frac{\omega}{\omega_o} = \frac{n}{n_o}$, i.e. the ratio of the amplitudes of the recorded curve to those of the oscillation to be measured, the latter being assumed to have a simple harmonic form. The location of the instrument at Frame 74 was selected for greatest longitudinal proximity to the center of gravity of the ship. For example on the return voyage across the Atlantic the distance from the latter amounted to 1.2 meters. The records thus reproduced the exact heaving oscillations in sufficiently close approximation.

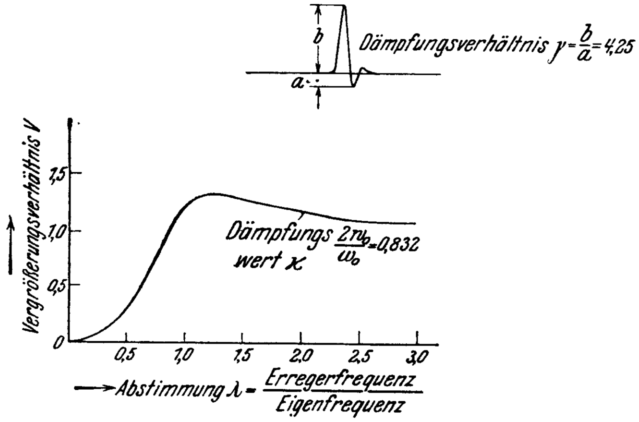

Fig. 6. **Amplification curve of the record of the Müller Instrument at a damping ratio of = 4.25 (see free vibration curve, right, above).**

The instrument lies about 4.5 meters vertically above the center of gravity at the given time. In principle, as Dr. Mueller has explained in greater detail in his aforementioned paper, a location above G is favorable inasmuch as then the unavoidable disturbances in the simultaneous presence of rolling oscillations due to the influence of inclinations on the one hand, and centrifugal forces on the other equalize each other at least in part. Moreover this influence of rolling oscillations becomes important only at greater amplitudes.

3. In addition to the heaving accelerometer which in the nature of things,

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* Stronger damping had been intended by Dr. Müller. On board, however, it was found that because of the horizontal vibrations of the bulkhead to which the instrument was attached, the copper plate struck against the pole faces of the eddy current brake. The distance between the pole faces therefore had to be increased somewhat, which resulted in the decrease in damping to the above value.

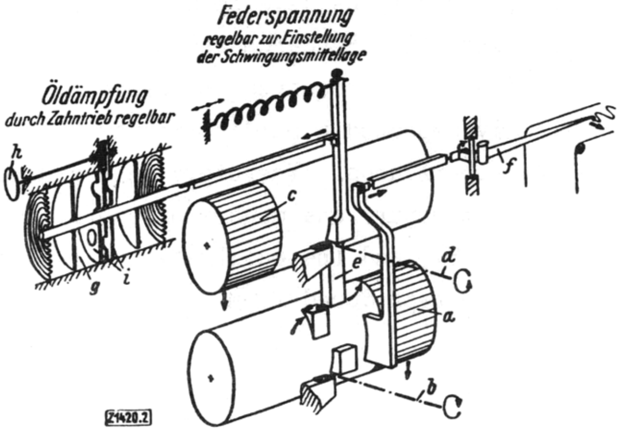

Fig. 7. Structure of the Waas Oscillometer (diagrammatic). The arrows indicate the direction of motion when an impulse from the fulcrum (the movement to be measured) acts upon the system.

Due to the position of the center of gravity, the lower mass a moves about its axis of rotation b in clockwise direction, the upper ones, c and d, in contrary direction. Connection of the two masses by the plate-spring joint e permits only this contrary motion. By shifting a cylindrical part installed inside the two masses the center of gravity can be shifted, thus permitting adjustment of record amplification with respect to the recording lever f, between 0 and 15. Damping is provided by an oil container g, rendered tight by membranes, and is regulated by the set screw h, which actuates the two perforated discs i, which do not oscillate.

and also because of its size and weight had to remain always in one and the same place, it seemed necessary to take along at least two further accelerometers of approved type, the records of which should also be immediately obtainable and comprehensible, but which in contrast to the Mueller device should be as handy as possible and easily movable. The frequency must be sufficiently high on the one hand so that the record for the movements of the ship as a whole in a seaway might be taken as a practically satisfactory acceleration curve (see II, p. 9) and on the other hand low enough so that the recorded curve should not be too small and thus possibly too

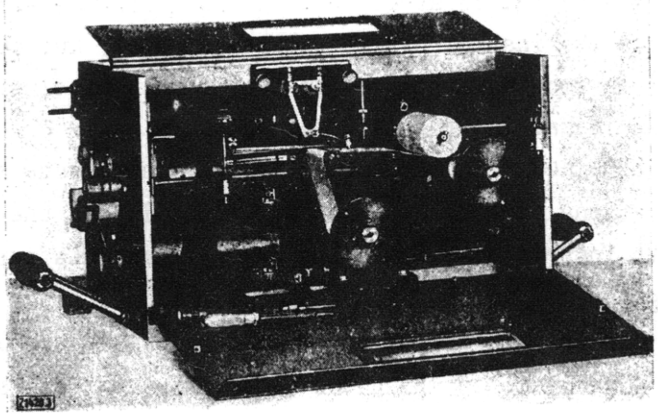

Fig. 8. Waas Oscillometer with selective drive by clockwork or electric motor, open.

inaccurate for evaluation. The apparatus developed by Dip. Ing. Waas for the Heinrich Hertz Institute for Vibration Research and manufactured by the Askania Werke, Berlin, seemed to fulfill these requirements especially well. For this apparatus there has also already appeared a publication by the author, so that a more detailed description is superfluous. Therefore only a schematic diagram with a description appended on the structure and manner of operation of the apparatus (Fig. 7), an outside view (Fig. 8), and the calibration curves for three different damping ratios (Fig. 9) are here reproduced from the paper mentioned. However, these calibration curves which have the same significance as the amplification curve (Fig. 6) of the Mueller apparatus, similarly to this apparatus can be applied directly only in case of simple sinusoidal impulses, and moreover, since the natural frequency of the Waas apparatus amounts to 6.5 Hertz, they apply only for setting up such frequencies which, like those of the elastic vibrations of the ship's hull, lie above or not too far below that of the apparatus. For the evaluation of pitching and heaving oscillations, the calibration curve since $a = s(n/n_o)^2 z$, even at the maximum indicator amplification $z = 15$, yields such a small value that the conversion from the recorded curve to the full scale movement had to be carried out mathematically always, and moreover, according to the above-named formula.

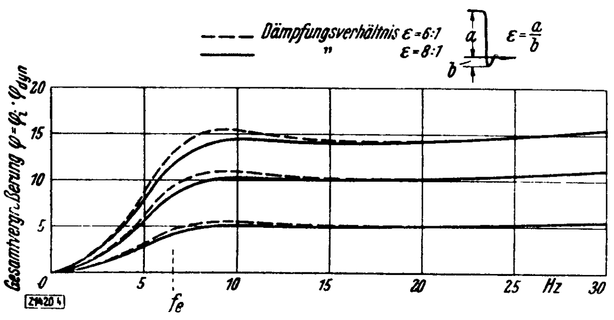

Fig. 9. Calibration curves of the Waas Oscillometer at a natural frequency
The three curves represent various adjustments of the masses.

The two Waas instruments were first used principally for the measurement of vertical accelerations at the ship's ends (cable stage forward and emergency exit aft,abaft the well.) At these places the instruments were likewise used, moreover, on the occasion of progressive runs in smooth water for the measurement of elastic vibrations of the ship's hull. When it had later been found after suitable checks that entirely useful data could be obtained with the automatic DVL-instruments also, the measurement of vertical accelerations at the ship's ends, especially on the days of storm, was turned over to the DVL-instruments, while the Waas instruments were used chiefly for measuring the longitudinal and transverse accelerations, since these instruments may also be used directly in such a way that the weights are deflected in a horizontal direction. The place in which the Waas instrument was set up for measuring the longitudinal acceleration was the cable stage forward, and for measuring transverse acceleration in the test corridor about amidships. A third Waas instrument, somewhat older and placed at our disposal by the Versuchsanstalt fur Wasserbau und Schiffbau, was used on occasion for auxiliary measurements.

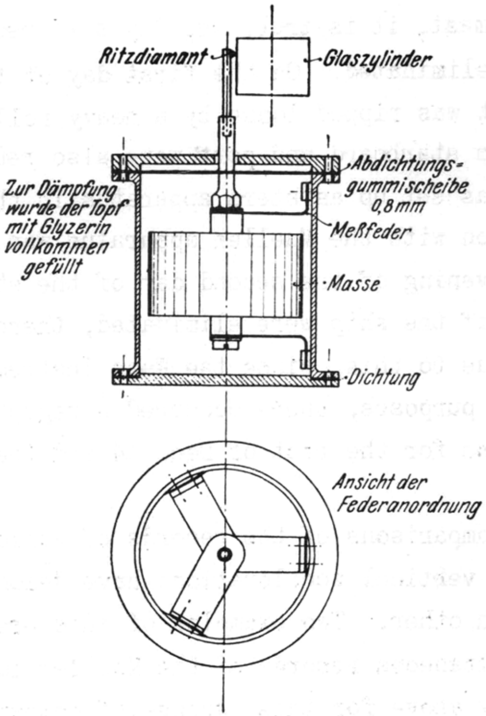

Fig. 10. Diagram of a DVL Accelerometer.

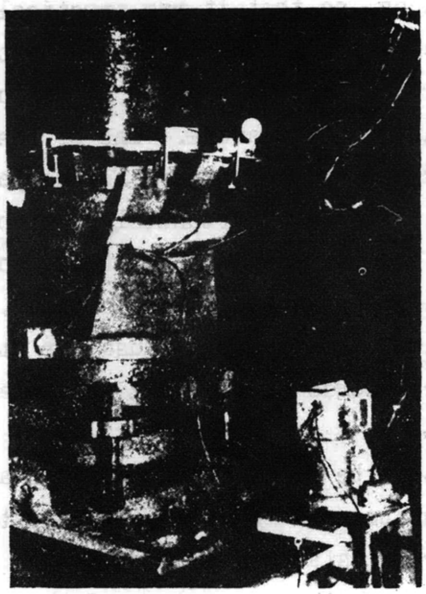

Fig. 11. View of after DVL accelerometer set up in carpenter shop (below, right). (On the rudder post are glass scratching instruments for measuring torsion of the shaft).

4. In addition to the accelerometers described in 2 and 3 operating with scribers and paper and therefore requiring continuous supervision during tests, it was absolutely necessary to have available several additional automatically recording instruments which could be simply set in motion from the central station. By a fortunate coincidence, the DVL had just successfully completed at the time of the test voyage a type of accelerometer operating by the glass scratching process, particularly efficient and equipped with glass cylinder and therefore suitable for long measurements. With very thankworthy helpfulness, the DVL placed six such instruments at our disposition as a loan, having built them especially for the purposes of this test voyage. The structure of the instrument can be plainly noted in Fig. 10, in which the unimportant details, especially the entire drive of the glass cylinder, have been omitted. Fig. 11 shows an external view, taken from a photograph made on board, of the stern apparatus finally set up in the carpenter's shop.

According to the calibrations carried out, the natural frequency of the instruments lies between 20 and 22 Hertz. Since under these circumstances the record is extraordinarily tiny, it must be very highly magnified under a microscope for evaluation, just as in the case of the glass scratching strain gauges. Wherever a more exact analysis was desired, magnification of 300 was used.

The six DVL accelerometers were all set up on the upper deck and at first distributed approximately uniformly over the ship's length (two instruments at the ends, two at the masts, and one each amidships port and starboard, see general plan Fig. 2) The instrument at the after mast, it is true, usually suffered from interferences, so that it was practically eliminated. On the first day of the storm (Dec. 11) the instrument farthest aft was ripped loose by a heavy roller and thereafter the two instruments amidships to starboard and port were also removed as a precautionary measure. One of them was set up as stern apparatus in the carpenter's shop, and the other, for comparison with the Mueller apparatus was set up above the latter. Unfortunately, on the evening of the second day of the storm (Dec. 14) the two instruments at the ends of the ship were eliminated, there being no possibility of noting this in time*. Due to this, since the Waas instruments at that time were already in use for other purposes, there occurred a regrettable gap in the records of vertical accelerations for the rest of Dec. 14 and the whole of Dec. 15.

Let it be said at this point that comparisons of the records of various types of instruments used for the measurement of vertical accelerations have demonstrated altogether satisfactory agreement with each other. Two examples of this are given. The first shows the comparison of the simultaneous records of the Mueller instrument and the DVL instrument set up directly above for this purpose of comparison (Fig. 12) converted to the same scale of acceleration. The agreement is obviously satisfactory. The second example brings a comparison set up under considerably more difficult circumstances (Fig. 13). That is to say, the records of a DVL instrument set up away forward and of a Waas instrument set up aft (in the escape hatch), naturally again converted to the same acceleration scale, were compared with the simultaneous record of the Mueller instrument in such manner that a mean curve proportionate to the correct position of the Mueller instrument was formed from the two former, which curve in ideal conditions must coincide with that of the Mueller instrument. To the uninitiated the two curves may possibly seem somewhat lacking in agreement; to the vibration expert who is accustomed to a great deal of grief in this direction, their agreement is astonishingly good considering in addition to the variety of types used the numerous existing sources of error: Influence of rolling movements, distortion of the Waas record by the guiding of the recording lever, phase shifting

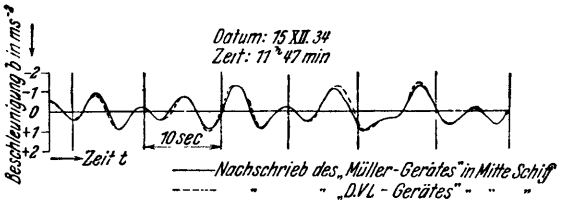

Fig. 12. Simultaneously recorded curves of the Müller and DVL Instruments set up at the same place and converted to the same scale. Date: Dec. 15, 1934. Time: 11:47 A.M.

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* In the bow instrument the cylinder had run out, and in the stern instrument jamming, and in addition, derangement of the time recording marks occurred.

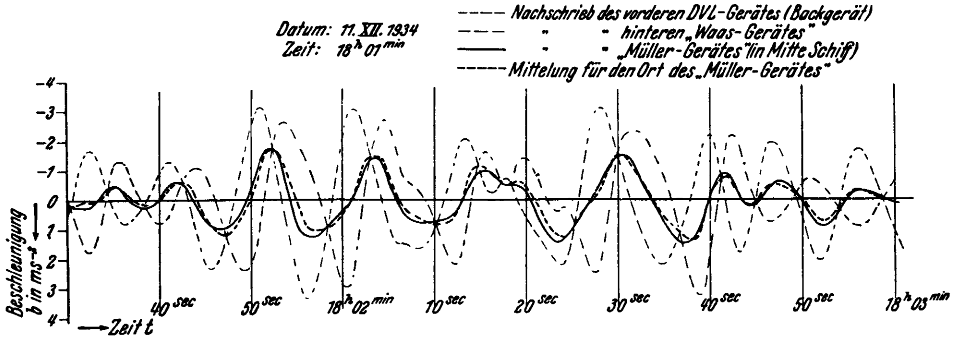


Fig. 13. Comparison of simultaneous records of a forward DVL Instrument and a stern Waas Instrument converted to the same scale, with the Müller Instrument. Date: Dec. 12, 1934; Time: 6:02 P.M.

of the Mueller instrument due to its low natural frequency, deflection of the ship's hull, etc. This comparison demonstrates without doubt that the results of all instruments used are sufficiently reliable for practical purposes. However, for the derivation of the path curve from the acceleration curves by twofold integration it was found advisable for the reasons cited under II, to use the records of the instruments with high natural frequencies as bases.

All instruments were connected with the common time circuit.

IV. SYNOPSIS OF ESSENTIAL MEASUREMENTS MADE.

a) Outward Voyage.

Since on the entire outward voyage with the exception of the first few days after leaving Antwerp during which there still remained much to be done in mounting the instruments, very calm weather prevailed throughout, seaway measurements were out of the question, and we were therefore forced to confine ourselves to such measurements for which calm weather was prerequisite or desirable. There were primarily progressive runs undertaken by Herr Hoppe in behalf of the Hamburg Model Basin with its instruments, and regarding which he is reporting elsewhere; furthermore, measurements of elastic vibrations of the ship's hull during the progressive runs, regarding which a brief account will be given in the following section V under f, together with the corresponding measurements undertaken on the return voyage under heavier loading, and finally, steering maneuvers and turning circles, to which reference will be made on a later occasion.

b) Return Voyage.

During the return voyage on which the ship had rather a full lading in contrast to the voyage out, opportunities repeatedly presented themselves for measurements in seaways, in addition to measurements corresponding to those under a) which it was possible to undertake during the calm weather generally prevailing at the start. Regarding the more important of these the following brief summary together with the accompanying circumstances at the moment might be given. The various methods of analyzing the test data follow in Sec. V. In addition, a considerable number of minor measurements was made, and moreover, chiefly for statistical purposes during a large portion of the return voyage at certain times of the day regular measurements of the rolling and pitching oscillations were made with a gyroscopic device.

1. Backing Test on Nov. 12. This occurred between the mouth of the Columbia River and San Francisco. In the forenoon a pretty regular swell prevailed from ahead at an angle of about 60° to the course (Fig. 14a). The mean wave length was estimated to be about 70 m. The wind blew with a force of 3 (WS) in a direction about 15° (absolute) to the normal course, from port. The ship velocities found from the measurements of Herr Hoppe are noted for the various tacks, as in all subsequent course charts also.

The run had been planned according to the sketch shown in Fig. 14a, according to which an adequate general idea would have resulted from the courses which in general were altered in steps of 15°, but unfortunately it was impossible to carry it out to this extent for various reasons. First, because of diverse interruptions, it was possible to put the greater portion of the testing apparatus in operation only in the third stage (30° to the normal course), and then for the time being the run could be carried out according to schedule only up to and including stage 270°, since because of a shift in wind and tide the ship approached too closely to land. It was possible to resume the run only after several hours in consideration of the .

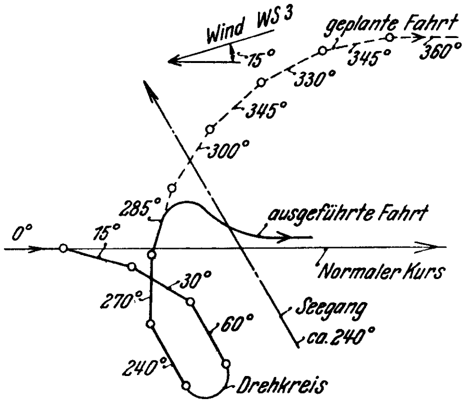


Fig. 14a. Course chart of backing test on the morning of Nov. 12, 1934. The second part of the test shown by broken line had to be given up prematurely.

Kurs gegen Normalkurs	Kurs gegen geschätzte Seegangsrichtung	Zeit	Mittlere Geschwindigkeit
0°	120° gegen See	10^{35} — 10^{40}	11,60 kn
15°	135°	10^{40} — 10^{50}	11,30 kn
30°	150°	10^{50} — 11^{00}	11,00 kn
60°	180°	11^{00} — 11^{20}	10,50 kn
240°	0° mit der See	11^{30} — 11^{40}	12,65 kn
270°	30°	11^{40} — 11^{50}	11,85 kn
285°	45°	11^{50} — 11^{55}	11,60 kn

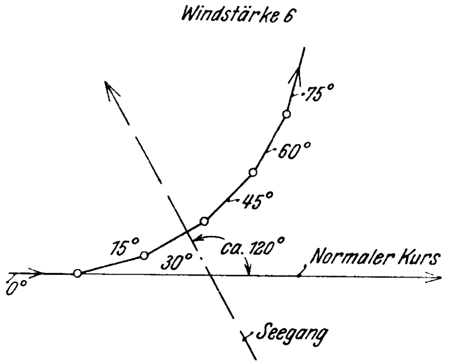


Fig. 14b. Course chart of backing test on the afternoon of Nov. 12, 1934.

Kurs gegen Normalkurs	Kurs gegen geschätzte Seegangsrichtung	Zeit	Mittlere Geschwindigkeit
0°	120°	16^{00} — 16^{10}	10,20 kn
15°	105°	16^{10} — 16^{20}	10,45 kn
30°	90°	16^{20} — 16^{30}	10,80 kn
45°	75°	16^{30} — 16^{40}	10,70 kn
60°	60°	16^{40} — 16^{50}	10,80 kn
75°	45°	16^{50} — 17^{00}	10,80 kn

schedule of the ship command, which was done as shown in Fig. 14b, and which would actually have filled the lack left by the runs which had been unsuccessful. Unfortunately the wind had changed greatly in direction and in force in the meantime, and in consequence the sea was running quite differently and moreover more irregularly than during the forenoon. Naturally the connection between the first and second half of the test suffered therefrom, and the result as a whole was detrimentally affected. Moreover, it must be set down that the time apportioned to the individual tacks is too short for more systematic tests and especially frequency investigations, and the data therefore are incomplete in this direction also. However, since on the entire return voyage time was extremely scant especially on the short runs due to a greatly altered schedule, the ship's officers were unable with the best intentions to place more time at our disposal.

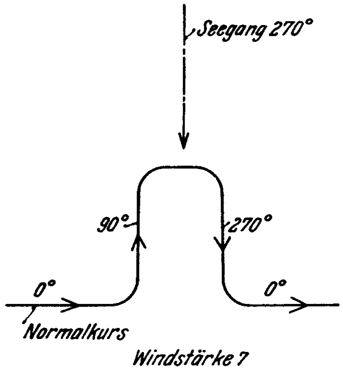


Bild 15.

Fig. 15. Course chart of trial run, Nov. 23, 1934. (Gulf of Tehuantepec).

2. On Nov. 23 the ship crossed the Gulf of Tehuantepec, in which, under the influence of a strong shore wind, a short, steep sea running from the land prevailed. Although in the course of the day a short steering test was run as shown in the sketch, Fig. 15, during which the ship at times ran with and at times against the seas, the chief significance of the measurements made on this day lay in the fact that the normal course of the ship ran practically exactly at right angles to the waves, and that since there were therefore no conditions favoring the setting up of pitching oscillations and since furthermore the wave periods of these short waves were much too small to cause rolling oscillations worth mentioning, practically the only movement of the ship consisted of heaving oscillations, which, however, due to these conditions were of more imposing magnitude. In this connection let me state that opportunity also was offered to observe the same exclusive large heaving oscillations on another vessel running on the same course.

3. Backing Test, December 2. On the forenoon of this day on which moderately long (100 to 120 m) and rather steep (average of about 7 m height) and unfortunately rather irregular seas were running, a backing run as shown in Fig. 16, of course only a brief one because of lack of time, was interposed. To the courses shown in this sketch there correspond the following angles of direction of the waves which were estimated to be running at an angle of 30° to the normal course, from starboard: 0, 15, 30, 45, and 60°. We were able on the whole to complete the test according to schedule. It showed some quite imposing heaving and pitching amplitudes and accelerations, in one case even, as may here be stated in advance, in the forward part of the ship the greatest acceleration ($\sim$ 5.3 m/sec^2)

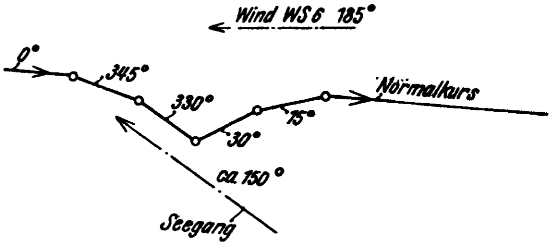


Fig. 16. Course chart of trial run, Dec. 2, 1934.

Kurs gegen Normalkurs	Kuis gegen geschätzte Seegangsrichtung	Zeit	Mittlere Geschwindigkeit
0°	160°	11^{00} — 11^{10}	9,27 kn
345°	165°	11^{10} — 11^{20}	9,42 kn
330°	180°	11^{20} — 11^{30}	9,50 kn
30°	120°	11^{30} — 11^{40}	9,60 kn
15°	135°	11^{40} — 11^{50}	10,00 kn
0°	150)	11^{50} ff.	9,27 kn

observed in general, and which was not even exceeded during the days of the storm.

4. Backing Tests on December 9. Since the voyage was approaching its end without our having been able to encounter a moderately regular and considerable swell or a heavier sea, another backing test was run on Dec. 9 in spite of less favorable conditions, - for there was a sort of cross sea with the main swell across the port bow at an angle of from 60 to 75° to the normal course and having an average wave length of about 150 m, and a new following sea coming up from starboard. This run was made as shown in Fig. 17 with somewhat wider scope than that described in 3. This test also went off according to schedule, on the whole. It is to be emphasized that here for the first time vertical accelerations were measured solely by the DVL instruments with the exception of the Mueller instruments, while of the Waas instruments one was used for the measurement of longitudinal oscillations (surging), and the other for that of transverse oscillations.

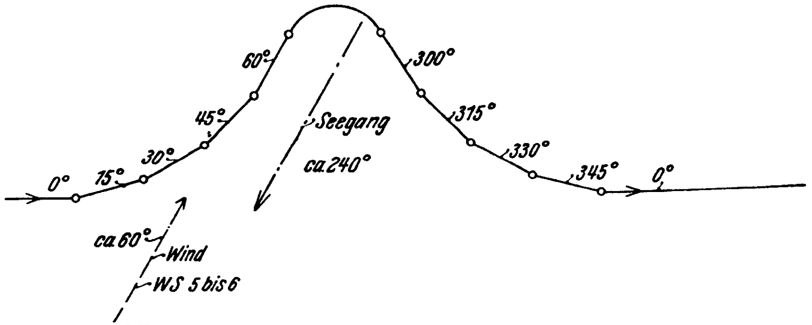


Fig. 17. Course chart of trial run, Dec. 9, 1934.

Kurs gegen Normalkurs	Kurs gegen geschätzte Seegangsrichtung	Zeit	Mittlere Geschwindigkeit
15°	135°	10^{00} — 10^{10}	12,40 kn
30°	150°	10^{10} — 10^{20}	12,15 kn
45°	165°	10^{20} — 10^{30}	11,70 kn
60°	180°	10^{30} — 10^{50}	10,65 kn
300°	60°	11^{10} — 11^{20}	11,40 kn
315°	75°	11^{20} — 11^{30}	12,00 kn
330°	90°	11^{30} — 11^{40}	11,70 kn
345°	105°	11^{40} — 11^{50}	12,05 kn
0°	120°	11^{50} — 12^{00}	12,17 kn

5. Gale on Dec. 11. A violent gale having come up in the night of Dec. 10 to 11, there was a heavy sea on the morning of the 11th, running from aft and on

the port side at an angle of about 45° to the course of the ship. The first measurement of the day was undertaken in this sea. The distribution of the accelerometers was the same as in 4. Unfortunately the after DVL instrument set up on deck aft of the wheel house was struck by a breaker coming from aft, without being noticed at first, so that for this measurement and for the next one also the important acceleration measurement at the stern was eliminated. Since wind and sea continued to increase in force - in the course of the forenoon the force of the wind increased from 8 to 11, and subsequently to 12 at times - the ship was forced to lie to, and so the next measurement, carried out at noon, took place with the ship lying to. During the course of it, several violent shocks were recorded, (see Vd). The accompanying pitching angles were so great that the Anschütz gyro whose recording apparatus was arranged for a maximum pitch angle of only 7½°, frequently bumped and therefore had to be abandoned, unfortunately, as also in the measurements of the 14th and 15th (See under 6). Even though chronological recording was impossible in the other measurements (see III 1), the Petravic gyro offered a substitute inasmuch as it permitted determination of the magnitude of the amplitudes and of the periods. - For an additional general test which took place late in the afternoon, one of the Waas instruments had meanwhile been set up aft, so that a complete acceleration measurement was once more assured. Wind and sea had persisted with undiminished force, and again heavy shocks were observed and recorded. Especially noteworthy in this regard are the records of the Waas instrument aft, inasmuch as the violent shocks in the after portion of the ship occurring on the periodically recurring emersion of the propeller are plainly recognizable from them.

(Fig. 18 missing in original)

Fig. 18. Course chart of trial run, Dec. 14, 1934.

Kurs gegen Normalkurs	Kurs gegen geschätzte Seegangsrichtung	Zeit	Mittlere Geschwindigkeit
0°	0°	bis 9^{40}	11,85 kn
30°	30°	9^{40} — 9^{50}	11,65 kn
45°	45°	9^{50} — 10^{00}	11,18 kn
60°	60°	10^{00} — 10^{10}	10.60 kn
75°	75°	10^{10} — 10^{20}	11,30 kn
90°	90°	10^{20} — 10^{30}	10,60 kn
255°	105°	10^{30} — 11^{00}	6,40 kn
240°	120°	11^{00} — 11^{10}	5,90 kn
210°	150°	11^{10} — 11^{20}	3,40 kn
180°	180°	11^{20} — 11^{30}	0 50 kn

The yawing gyro likewise participated in this test. – Wind and sea having let up somewhat after midnight, the ship resumed its course at about 2:15. Measurements were made here too, but unfortunately the only satisfactory records available as far as acceleration is concerned, are those of the forward DVL accelerometer, which, alone, are incapable of giving an adequate idea of the occurrence as a whole.

6. Runs in Gale on December 14 and 15. The two days following the described days of storm, which were comparatively calm, were taken advantage of to overhaul the entire measuring set-up as far as possible. In addition, in place of the DVL accelerometer aft which had been washed away by the seas, one of these instruments was set up in the carpenter's shop (see plan, Fig. 2), and furthermore, one additional instrument apiece in the measuring corridor, fore and aft and thwartships.

On the morning of Dec. 14, there was a moderate sea coming approximately from aft, with a wind force of about 7. Since, moreover, it was fairly regular, it was decided once more to undertake a more extensive backing test. It was carried out as shown in the accompanying sketch, Fig. 18. It was necessary greatly to curtail the final part of the voyage which had been planned more extended, i.e. with smaller angles in changing course, because during the course of the voyage wind and sea increased extraordinarily in force, so much so, that during the final runs against the seas it was necessary to run at greatly reduced speeds, and the ship's officers finally considered it preferable to continue to lie to after conclusion of the test. Under these circumstances, this run naturally could not yield that which is usually hoped for from a backing test and which can usually be obtained only in approximately uniform conditions of wind and sea. The results must therefore be evaluated independently of this. The location of the oscillation and acceleration measuring instruments, with the exception of the changes in the DVL instruments stated in the foregoing section, was the same as in 4 and 5. The yawing gyro was also set up. After the ship had temporarily resumed its old course at about 2 a.m. when the storm failed to remain at full height, a brief measurement (gyro, Mueller, forward and aft DVL instruments) having been obtained, it was again forced to lie to at about 4 a.m. The storm and the sea continued to heighten and reached their maximum toward midnight. Unfortunately, during the numerous measurements undertaken during this afternoon and evening and then too in the course of the following day, the acceleration measurements, as was subsequently found, were considerably damaged by the fact that the important forward DVL instrument had run out on the 14th at 18^{h}, and that the after instrument showed stoppages and interruptions of the chronographic record. Since these two defects were not noticed in time and since as a result no remedial measures could be taken, the only records available for this time are the oscillation measurements with the Petravic gyro and the acceleration measurements with the Mueller instrument and the DVL instrument installed directly above it – measurements which, although they furnish a good idea of occurrences as a whole, are not wholly adequate for detailed investigation, especially since the measurements with the Petravic gyro can not be arranged chronologically for the reasons given in the foregoing.

All in all, however, there are sufficient oscillation and acceleration measurements from the three days of the storm to furnish completely adequate material for the purposes of the measurements.

As regards the load condition of the SAN FRANCISCO, the data for the days of the storm are contained in Dr. Schnadel's paper. Quoting from this, let it be repeated that the mean draught amounted to 7.26 m, the displacement 13,073 T, and the additional lading 7,624 T. The same condition may with sufficiently close approximation be regarded as having existed for the test runs carried out on the two last days before the storm, the 2nd and 9th of December.

For the two test runs made on Nov. 12 and 23 in the earlier stage of the return voyage, whose results on the whole are not sufficiently important in the scope of the present paper to render publication of further particulars of loading worth while, it will be sufficient to indicate the load condition by the draughts and the accompanying magnitudes of displacement and additional lading:

Nov. 12.: Draught forward 5.13 m, aft 7.59 m, amidships 6.36 m, displacement 11,225 T, additional lading 5,776 T.

Nov. 23.: Draught forward 7.14 m, aft 7.83 m, amidships 7.46 m, displacement 13,484 T, additional lading 8,035 T. To the extent that knowledge of the location of centers of gravity is required, especially the metacentric heights, the data may be found under Vb (p. 31).

V. VARIOUS EVALUATIONS AND THEIR RESULTS.

Inasmuch as the acceleration measurements were planned as a means to an end, that is, as foundations for the strength investigations, they have been placed at Dr. Schnadel's disposal for his investigations and there worked up, for which reason they need not be mentioned here. With the exception of the section on elastic vibrations, I shall speak here only of those evaluations related to the movements of the ship in a seaway.

a) I might preface the investigation with the question to the solution of which, because of its fundamental importance brought out in the introductory section, the high sea test trip was intended to contribute as far as possible, – the question, that is to say, whether it is possible to consider a normal seaway – by this is meant a seaway attended by normal irregularity – as to its effect on a ship with respect to the latter's pitching and heaving movements, as replaced by a seaway of average regularity. It has already been mentioned that for the solution of this question the carrying out of backing tests is especially important, the data from which we will now attempt to evaluate. Of course it must be admitted quite openly from the start that in spite of the highly commendable efforts of the ship's officers to meet our desires in this regard as far as possible, the yield in the mentioned direction is not very imposing. In the first place simply for the reason that in the nature of things such trips can only be successful to an extent when the irregularity of the seaway does not exceed a certain normal limit. This, however, was the case on only two occasions as stated in the summary under IV; first on Nov. 12 in the forenoon, and possibly also on Dec. 2, during the course of a test trip on each of the two days. On the other runs on the afternoon of Nov. 12, and on Dec. 9 and 14, during test runs of about two hours each, as related in greater detail under IV, conditions for systematic evaluation were much less favorable. Under these circumstances the few runs made in the two first named tests, especially since the individual runs covered only 10 minutes each, of which not even the minutes immediately following the change of course are decisive, can not by far be considered adequate to permit drawing general conclusions in the sense of checking up on the practical usefulness of theoretical assumptions (See I, p 3). In making the following first attempt in this direction notwithstanding, this is done less to derive decisive data at this early stage, than to arrive at a conclusion as to the basic applicability of such backing tests in the sense of the purpose indicated, and/or as to what course shall be pursued in future cases in order to obtain satisfactory results.

The investigation is based on the following thought: The theoretical postulates for the conditions of motion of a ship on various courses considered as limited to forced oscillations, in a uniform seaway for various ratios of wave length to ship length have recently been given by Weinblum in a particularly clear manner (4b) by introducing the so-called heaving and pitching functions. These represent the heaving and pitching amplitudes setting in at the unit of half a wave height, divided by certain constants dependent upon the form of the WL, the wave being assumed to be moving past slowly, and are therefore susceptible to calculation in a basically simple way. In Figs. 19 and 20, the heaving and pitching functions ε (k) and ϑ (k) * for the SAN FRANCISCO are plotted as functions of the angle α between the direction of the run and the waves with various wave lengths. Then, under the simplified assumption that the form of the WL does not change materially in the region of the wave zone, the following applies for the forced oscillations:

$$\text{Heaving oscillation } z = r \frac{\varepsilon (k)}{a_w} \mu_2 \cos (\omega t - \varepsilon_2)$$

* In the cited paper by Weinblum, heaving and pitching functions for various parameters k = $\frac{\lambda}{L}$ are plotted. The notations ε (k) and ϑ (k) are retained here also, but since the ship length here involved is wholly definite, the wave lengths are taken directly as parameters in the figures.

Pitching oscillation $$\psi = \frac{r}{L}\,\frac{\vartheta_{(k)}}{2\,c_L}\,\mu_1 \sin\,(\omega t - \varepsilon_1).$$

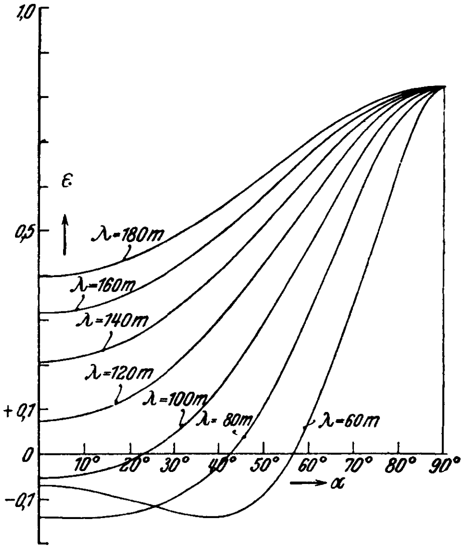


Fig. 19. Theoretical heaving function ε (k) as function of the course angle α to the direction of seaway for various wave lengths λ .
Ship's length L = 131 m.
Fullness at waterline α_{w} = 0.825.
Fining at waterline t = -0.5

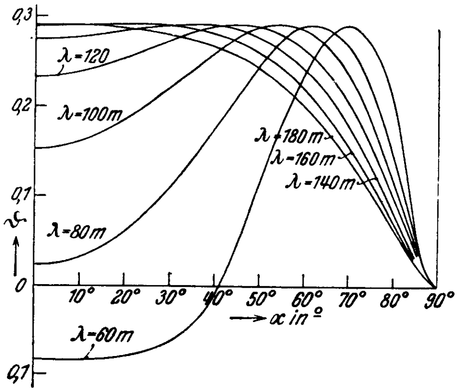


Fig. 20. Theoretical pitching function ε (k) as function of the course angle α to the direction of seaway for various wave lengths λ.
Dimensions as in Fig. 19, and in addition $2c_L$ = 0.0976.

The following notations are used herein:

r = ½ wave height
α_w = coefficient of fineness of the WL
c_L = $\frac{J_L}{BL^3}$ with J_L = moment of inertia of WL area
ω = $\frac{2\pi}{\tau_r}$ frequency of wave excitation
τ_r = relative wave period (period of encounter)
$\varepsilon_1, \varepsilon_2$ = phase angle between oscillation and excitation
μ_1, μ_2 = the so-called amplitude distortions, governed principally by the ratio of the natural frequency ω_1 or ω_2 of the pitching or heaving oscillation to the frequency of excitation , but in addition also by the degree of damping. Actually we have

$$\mu_1 = \frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_1}\right)^2\right]^2+\left(\frac{2w_1}{\omega_1}\right)^2\left(\frac{\omega}{\omega_1}\right)^2}}\,,\quad \mu_2 = \frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_2}\right)^2\right]^2+\left(\frac{2w_2}{\omega_2}\right)^2\left(\frac{\omega}{\omega_2}\right)^2}}\,,$$

wherein w_1 and w_2 signify the damping values. As found in numerous model tests carried out in the Versuchsanstalt für Wasserbau und Schiffbau, Berlin, the values $\frac{2w_1}{\omega_1}$ and $\frac{2w_2}{\omega_2}$ are approximately of equal magnitude, that is of the order of about 0.45. Accordingly the curve of μ_1 or μ_2 on the frequency ratio $\frac{\omega}{\omega_1}$ or $\frac{\omega}{\omega_2}$ is as shown in Fig. 21. The wave dimensions (λ, r) being thus known – by which the absolute wave period $\tau = \sqrt{\frac{2\pi\lambda}{g}}$ and the speed of advance of the wave $c = \sqrt{\frac{g\lambda}{2\pi}}$ and the ship velocity v and the angle α being known, the relative wave period $\tau_r = \frac{\lambda}{c - v\cos\alpha}$ being given also – then the foregoing equations yield in the amplitudes z and ψ hereafter to be found (with sine or cosine term = 1), values with which the oscillation deflections measured in the respective runs can be compared. For determining the wave dimensions, wave measurement was intended for which purpose the primary provision was measurement of wave contours along the hull with the ship running with or against the seas, by means of Weiss' wave measuring apparatus. Wherever this failed to yield satisfactory data, and the stereophotographic method was also inadequate, it was necessary to fall back on the estimates regularly made during the course of the tests.

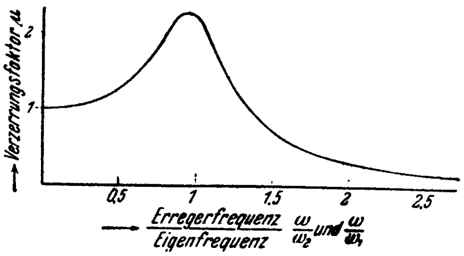
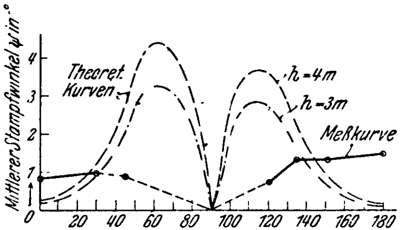


Fig. 21. Amplitude-distortion factor $\mu_1 \cong \mu_2$ at a damping coefficient $\varkappa = \frac{2w_1}{\omega_1} = \frac{2w_2}{\omega_2} = 0.45$

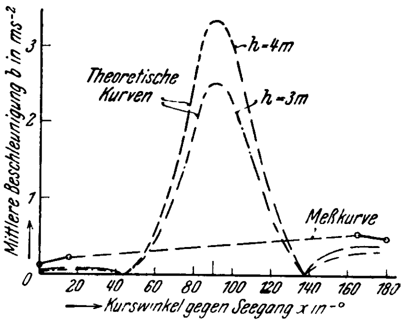
Thus it was possible to consider the ideal comparative values as sufficiently established. The measured values to be compared with these could, because of the far-reaching non-uniformity of the seas and therefore also of the measured oscillation deflections, be regarded only as mean values of the latter, wherefore it was necessary to limit ourselves from the start to the establishment of relative values. The striking of averages can be performed in various ways. We can either simply form the arithmetical mean of all the measured amplitudes for the time of the individual run, or with the planimeter measure separately all the areas above or below the base line bounded by the curve for the regions corresponding to the individual runs, divide the result by the corresponding time and multiply it by $\frac{\pi}{2}$, i.e. by the ratio of the crest ordinate of a sinusoid to a rectangle of equal area. As has been shown by check-ups, the two methods lead to practically adequate agreement.

With the pitching oscillation curves recorded by the Anschütz gyro we can use the values thus obtained directly for comparison. The results are shown in Fig. 22 and 24 for the runs on the forenoon of Nov. 12, and on Dec. 2. In these the black, solid zig-zag curves represent the connecting lines of the measured points obtained for the individual runs. Wherever it was impossible to make separate runs, the probable course is indicated by chain-dotted lines. The broken curves represent the result of theoretical calculation, i.e. on the basis of one wave length as found



Wave length $\lambda \approx$ 80 m.
Wave height $h \approx$ 3-4 m.

Fig. 22. Comparison of the mean amplitudes of pitching angles measured in the trial runs of the morning of Nov. 12, 1934 on various courses with the theoretical values found on the assumption of a uniform seaway.



Wave length $\lambda \approx$ 80 m.
Wave height $h \approx$ 3-4 m.

Fig. 23. Comparison of the mean amplitudes of heaving acceleration measured in the trial runs of the morning of Nov. 12, 1934 on various courses with the theoretical values found on the assumption of a uniform seaway.

on an average from the Weiss wave measurements, or about 80 m on Dec. 12, and about 120 on Dec. 2. The curves are plotted for various assumed wave heights, by which, however, for purposes of comparison with the measured curves, the effective heights should be understood. (See footnote, p. 45)

For the heaving movement, the Mueller instrument yields acceleration curves for whose comparison with the theoretical curves, since these are oscillation curves, a conversion is first necessary, which will best be carried out with the latter. Since we are dealing with pure forced oscillations, that is simple sine oscillations, these are converted into acceleration curves by multiplication by ω^2, wherein ω will be different for every course angle, in accordance with the change

Wave length $\lambda \approx$ 120 m.
Wave height $h \approx$ 3-4 m.

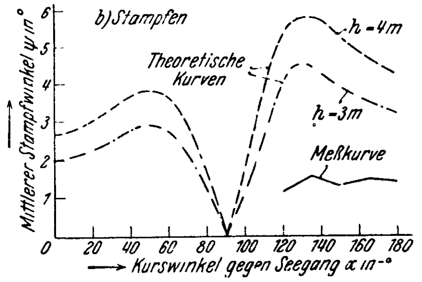


Fig. 24. Plotted as in Fig. 22 for test runs of Dec. 2, 1934

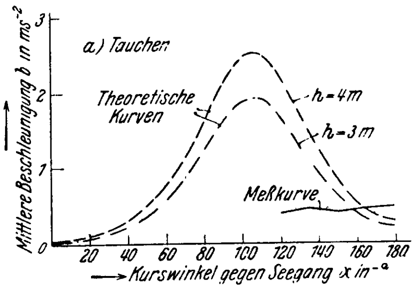
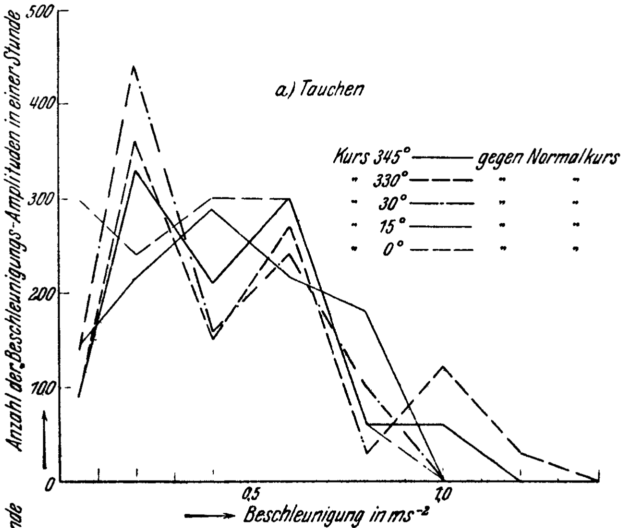


Fig. 25. Plotted as in Fig. 23 for test runs of Dec. 2, 1934

in the relative wave period (Begegnungsperiode) associated with this. In this way, and moreover on the same assumptions for wave lengths and heights as for pitching oscillations, have the comparisons in Figs. 23 and 25 been carried out. Since it is a matter of comparing amplitudes, it will, moreover, be unnecessary to differentiate between positive and negative values of the theoretical curves.

For a comparison with respect to heaving motion, the test data are unfortunately highly incomplete, since for Nov. 12 the entire range of wave directions from athwartship to 45° forward and aft, which is especially important for the analysis of heaving is missing, and for Dec. 2 only the range from 0 to 60° is available. For the available measured points, it is true, a certain agreement in tendency with the theoretical curves can, on the whole, be noted, but in view of the incompleteness of the data, this actually does not mean much. – For pitching oscillations somewhat more complete data are available, since on Nov. 12 it was found possible to take measurements on three additional runs. Here, however, the result of comparison, in both test runs moreover, is obviously completely negative. In the tendency of the courses of the theoretical and the measured curves no common feature is recognizable, rather the tendency is opposite. Of the large amplitudes to be expected on runs at 30° to athwartship there is hardly an indication in the measured data, while on the other hand the amplitudes with and counter to the seas were found to be inordinately large as compared with theory.



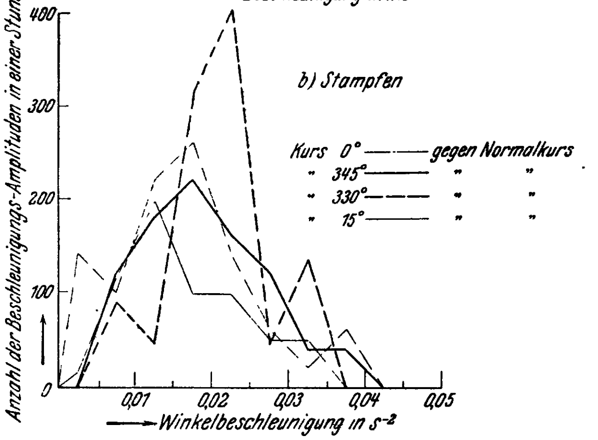


Fig. 26a. Periodicity of heaving acceleration amplitudes for Dec. 2, 1934.

Fig. 26b. Periodicity of angular acceleration amplitudes of pitching, Dec.12,1934.

The cause for this lack of agreement is probably to be sought in the following:

1. On Nov. 12 the seaway as a whole was too slight, and therefore the forced oscillations were insufficient in proportion to the free oscillations, and the latter therefore predominated and rendered a comparison with only the forced oscillations in view, illusory.

2. Although the seaway was higher on Dec. 2, it was not yet considerable and furthermore, it was so much more irregular as to be inadmissible for the stated

purposes of a backing test (cross seas already rising).

In my opinion, therefore, the conclusion need not necessarily be drawn from the lack of a positive result of these two backing tests, that this type of test is altogether unsuitable for the object in view. However, the conclusion will be drawn that only in the presence of considerably higher seas, which, moreover, must be regular to a certain extent, will the carrying out of such tests have prospects of success.

b) Frequency Curves.*

In the case of frequency curves, statistical compilations are involved. These can be made, within the present scope, in two directions:

1. We arrange the amplitudes of oscillations or accelerations measured on a test run consecutively according to size and above each of these set down the number counted. Fig. 26a and 26b will serve as examples, in which for the runs over a changing course on Dec. 2 the amplitudes of heaving acceleration on the one hand and pitch angle acceleration on the other, arranged by individual courses, are plotted. These curves form a certain complement to those of Figs. 24 and 25 in which corresponding mean values were plotted. For the rest, however, because of lack of time and personnel, it has not yet been possible to work up the great mass of material on this subject systematically, and more detailed examination and later reports are reserved for the future. Here, I therefore confine myself to reporting in the following Section d) several noteworthy specific values, especially maximum values.

2. A frequency investigation is made with respect to oscillation periods. This investigation concerns the important and much discussed question as to what proportion of the general movement of a ship at any given moment is comprised of natural oscillations on the one hand, and oscillations induced by the seaway on the other. In this connection, let us call to mind the fact familiar to us from the theory of vibrations that even when excitation is uniform, natural oscillations are set up in addition to the forced oscillations, which are only suppressed by damping more or less quickly, according to the strength of the latter. When the excitation is irregular natural oscillations are constantly set up again and again. It must therefore be assumed from the start that when, as in the case of rolling oscillations, damping is not great, the natural oscillations in an irregular seaway, i.e. practically always, will greatly predominate, which will be evident from the fact that the period of the observed rolling oscillations will move principally in the order of magnitude of the natural oscillation period. In the case of heaving and pitching oscillations this is not so certain from the first because of the much stronger damping prevalent here. When we seek to solve this question by a

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* Evaluations for rolling and pitching angles were very kindly placed at my disposal by Herr von den Steinen for this report.

frequency investigation in the following, we must understand from the start that this method can only be very rough; for in the observed periods the result of superposition of oscillations of various periods becomes affective without the possibility of separating them. Nevertheless we can then arrive at certain conclusions which will be useful in practice, if the charts show marked characteristics.

In order to be able to use these charts in the desired sense, it is inherently necessary to have a rather accurate fore-knowledge of the natural frequencies of the three oscillations. In practice a precise preliminary calculation is essential only for heaving and pitching oscillations, because in the case of rolling oscillations the assumption just expressed and justified is so clearly born out by the respective frequency charts, that we are justified conversely in drawing an inference from these charts as to the rolling period. In any event, this should also be calculated previously, at least approximately. As a basis of calculation we take in one case the load conditions for Nov. 12, and then those for the days of the storm (Dec. 11 to 15), which may with sufficient accuracy be regarded as applying on Dec. 2, also.

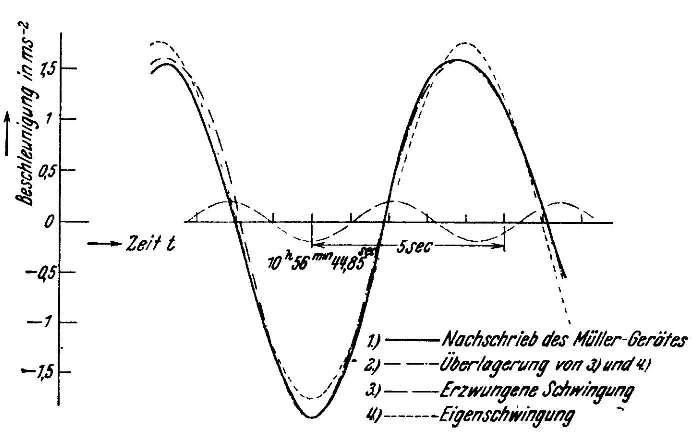

Fig. 27. Experimental division of a suitable acceleration record of the Müller Instrument into accelerations of forced and natural oscillations for Nov. 12, 1934, 10 hr. 56 min.

1. Rolling period (estimated)

$$T = 2\pi \frac{j'_x}{\sqrt{g \cdot MG}}.$$

The radius of gyration j'_x of the mass moment of inertia taken with regard to the centroidal axis may be estimated for standard type cargo ships in loaded condition at about 0.45 B according to the available empirical values, in which the effect of virtual mass is already included. For the SAN FRANCISCO we find, according to this, $j'_x = 8.1$ m, which applies for the days of storm, since the load condition for these days may be taken as pretty normal. For Nov. 12 this value has been converted based on the difference in load between the two conditions, and is found to be 7.85 m. The metacentric height, based on the conclusive dockyard stability test and the special consideration of loading conditions for Nov. 12, was found to be 1.75 m, and for the days of storm 1.61 m. Thus we have rolling periods of

$$T = 2\pi \frac{8{,}1}{\sqrt{9{,}81 \cdot 1{,}75}} = 11{,}9 \text{ sec. on Nov. 12,}$$

and

$$T = 2\pi \frac{7{,}85}{\sqrt{9{,}81 \cdot 1{,}61}} = 12{,}8 \text{ sec. on stormy days.}$$

2. Heaving and pitching periods.

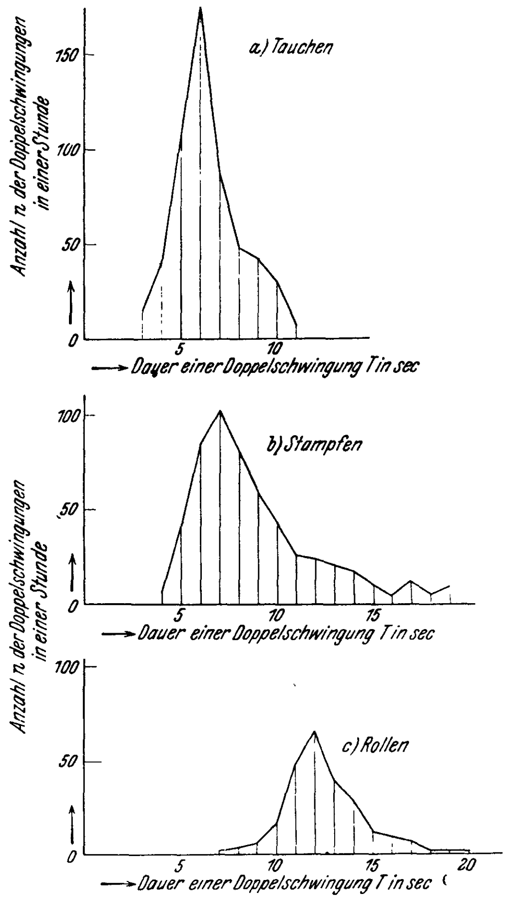

Fig. 28. Frequency of periods on Nov.12, 1934, A.M.

a) Heaving.
b) Pitching.
c) Rolling.

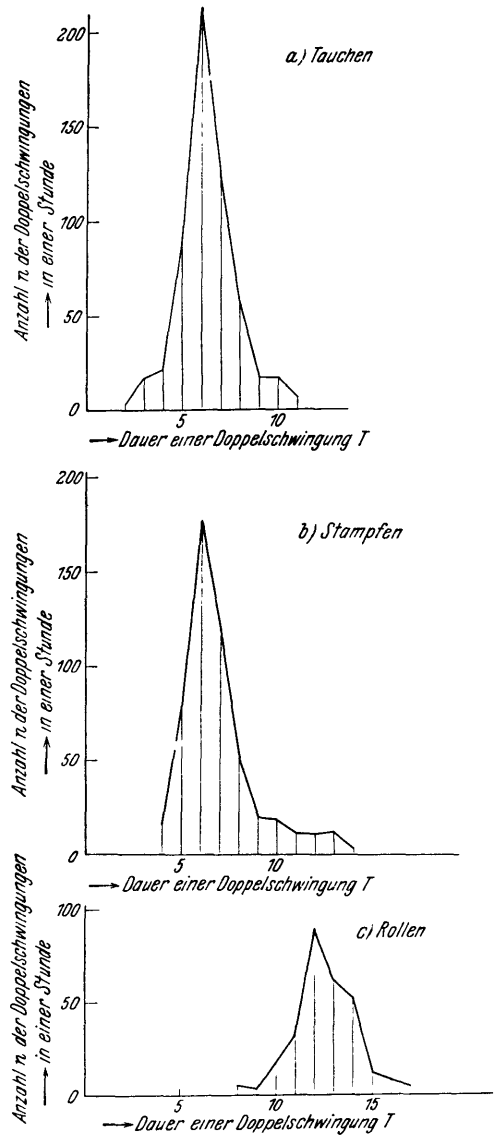

Fig. 29. Plotted as in Fig. 28 for Nov. 12, 1934, P.M.

In connection with these periods, the determination of which would otherwise be extremely simple, the virtual mass plays an extraordinarily important part, as is well known. For the heaving period we have

$$T_2 = 2\pi \sqrt{\frac{V + V''}{g F_w}},$$

and for the pitching period

$$T_1 = 2\pi \sqrt{\frac{J_y + J_y''}{P \cdot M_L G}}.$$

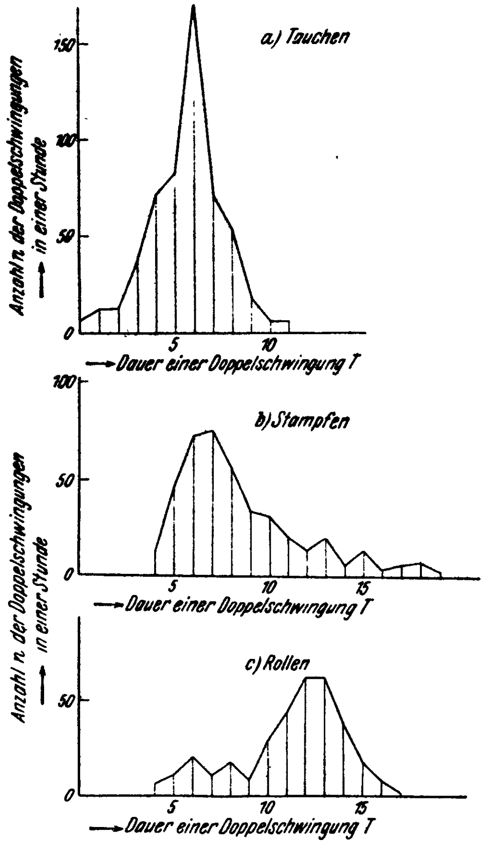

Fig. 30. Plotted as in Fig. 28 for Dec. 2, 1934.

Herein V signifies displacement, F_w area at the waterline, P weight of the ship, $\overline{M_L G}$ the longitudinal metacentric height, J_y mass moment of inertia of the ship taken with respect to the transverse axis passing through the center of gravity (can be found with sufficient accuracy from the curve of weight distribution over the length); further, V" and J" signify additions to V and J_y respectively, which are governed by the influence of virtual mass. These quantities were calculated completely analogously to the method reported by Frank M. Lewis (10) for determining virtual mass in connection with the flexural vibrations of a ship's hull, but with the discrepancy caused by the difference in type of movement between the heaving or pitching oscillations and the elastic vibration of a ship with two or three nodes. The conditions existing in heaving and pitching can be expressed with sufficient accuracy according to Lamb (11) §§ 114/115. The oscillation periods were

on Nov. 12 $T_1 = 6.51$ sec, $T_2 = 7.34$ sec.,
Nov. 23 $T_2 = 7.68$ sec.,
storm days $T_1 = 6.94$ sec, $T_2 = 7.63$ sec.

At this point I wish to add that from one place in the Mueller record for Nov. 12, forenoon, at which a remarkable symmetry was evident over a certain range (Fig. 27, solid curve) I attempted to resolve the resultant oscillation into two sine oscillations. This was completely successful (individual oscillations in broken line, their resultant in dot-dash line), and for the two superposed oscillations yields periods of 7.98 and 4.29 sec. This run was made with a head sea at an angle of about 30° to the course. To the estimated mean wave length of about 70 m and the ship's speed of 11 knots there corresponds a relative wave period of 4.45 sec., which actually is close to the one period found from analysis of Fig. 27. This suggests interpreting the period of 7.98 sec. as a natural oscillation period. In comparison with the foregoing calculated result (7.34 sec.) this, however, seems remarkably high.*

In investigating the periods by frequency analysis several methods may be adopted:

α) The most obvious method would seem to be simply to mark off over the successive periodic intervals the appropriate number. By this method a series of evaluations has been made as follows:

For heaving, pitching, and rolling periods:

Backing test of Nov. 12, forenoon, (Fig. 28a, b, c),
Backing test of Nov. 12, afternoon, (Fig. 29a, b, c),
Backing test of Dec. 2, (Fig. 30a, b, c);

for pitching and rolling periods in addition, the run in the storm of Dec. 11, Fig. 31a, b).

Herein, for the backing tests likewise, it was not attempted to divide the record according to separate courses, owing to its primitive character. In addition to this however, by way of experiment, a corresponding chart shown in Fig. 32a, b, and c was made for the forenoon of Nov. 12, for heaving, pitching, and rolling, but divided according to courses.

In discussing the results we begin with the simplest case, that of rolling oscillations. Here the periods crowd together in a form similar to that of a resonance curve about a quite plainly distinguishable period of maximum frequency, and according to previous statements, there can be no doubt that this period is identical with the natural period. Moreover, in this connection it is immaterial whether the seaway is more or less

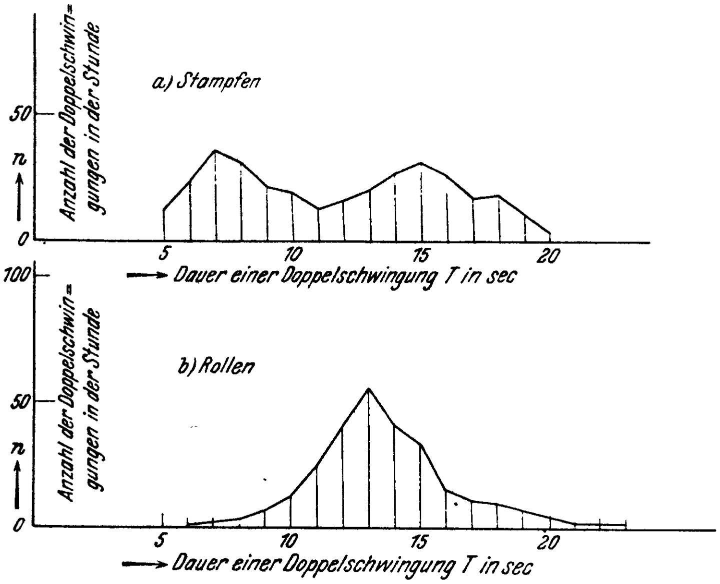

Fig. 31. Frequency of periods for Dec.11,1934
a) Pitching.
b) Rolling.

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* In my opinion, no great importance can be attached to this analysis because the heaving natural oscillation is subject to greater damping, and therefore inherently incapable of approximation by a sine curve over the greater range here involved. At any rate, the solution by this means cannot be regarded as rigorous.

irregular. Neither, apparently, does the direction of the course to the sea play an important part. Thus the two backing tests in the forenoon and afternoon of Nov. 12, although the sea differed rather widely and the courses were run at wholly different angles to the sea likewise, yield practically the same magnitude of period corresponding to the crest of the frequency chart, i.e., 12 seconds. Furthermore, its agreement with the natural period derived from the approximate calculation is also satisfactory. – In consequence of the loading condition having been changed in the interim, a somewhat greater natural period (12.5 – 13 sec.) appears in quite logical sequence for the backing test on Dec. 2 and the run in the storm of Dec. 11, according to the respective frequency charts, likewise in good agreement with the preliminary estimate. That the period of greatest frequency for the run in the storm is somewhat higher than on Dec. 2, must probably be ascribed to the fact that in the former the long waves coming diagonally from astern set up relative wave frequencies which drew the resultant periods somewhat upwards.

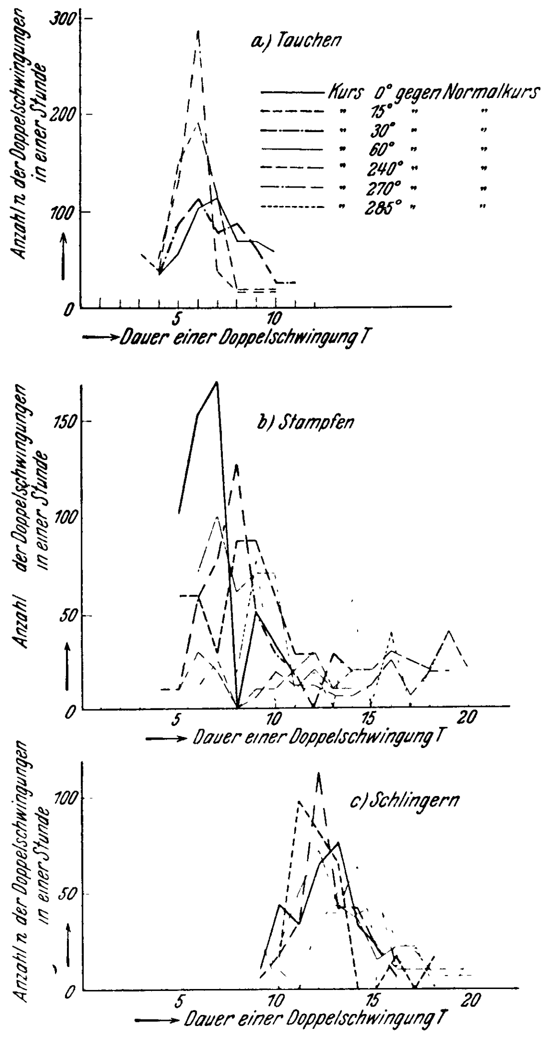

Fig. 32. Frequency of periods on test run of Nov. 12, 1934, A.M., divided by courses according to the list with Fig. 14a.

a) Heaving.
b) Pitching.
c) Rolling.

In the case of the pitching oscillations it is found that if we first identify the place of the crest of the frequency curve with the natural period here too, there results very good agreement of the tests carried out on Dec. 2 under practically the same loading conditions (7 sec.) on the one hand with each other and on the other hand with the theoretically calculated natural period (6.94 sec.). Next to the crest corresponding to a period of 7 sec. in the chart for Dec. 11, there is a second very markedly pronounced crest at 15 sec. This is likewise easy to explain. During this measurement the ship ran at about a 45° angle to a following sea at about 30 knots, and with an estimated wave length of from 180 to 200 meters, the wave period corresponding to this would be about 15 sec. With this period of excitation, already quite high with respect to the natural period, the influence of the natural oscillations due to irregularities will no more be so great as to prevent the predominant influence of the wave excitation from becoming noticeable more frequently in the period likewise. This, for example, is plainly distinguishable in the curve of the Mueller instrument reproduced in Fig. 33, for this time period, and it is evident that under

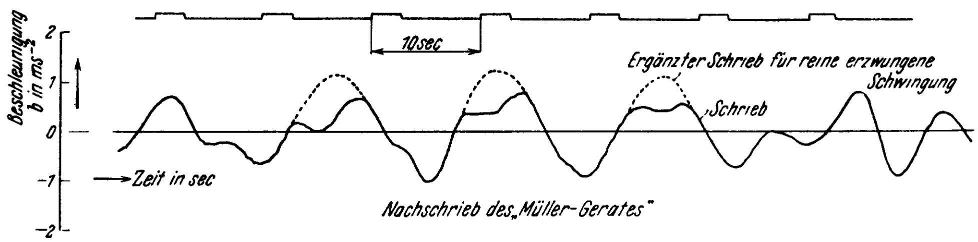

Fig. 33. Derived curve of the Müller Instrument for Dec.11, 1934, 9 hr. 38 min. Example of preponderance of forced oscillation in a following sea.

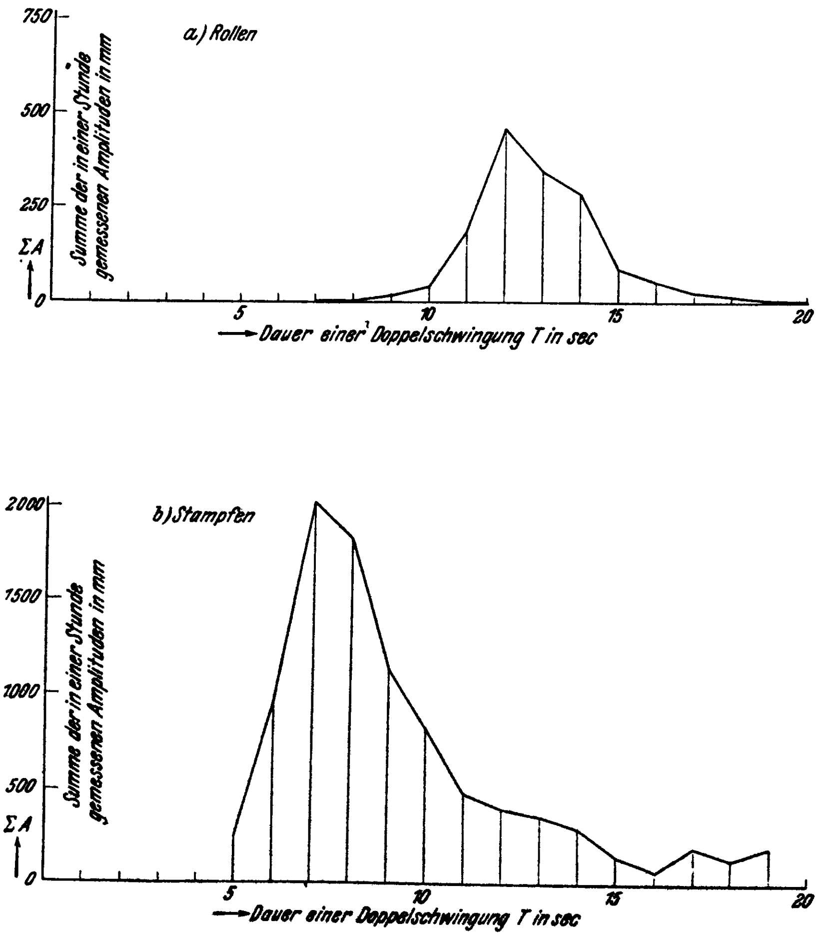

Fig. 34. Sum of amplitudes measured hourly, plotted over the appropriate periods, for Nov. 12, 1934, A.M.
a) Rolling.
b) Pitching.

these conditions the relative wave period will in many cases be able to make itself felt in the frequency investigations of the heaving and pitching oscillations. Not so clear with respect to the pitching oscillation period are the results of the test runs on Nov. 12, in the forenoon and afternoon. The chart of the former run shows the greatest frequency at about 7 sec., that of the second run at 6 sec. The calculated value amounted to 6.51 sec.

For the heaving periods the greatest frequency is found for Nov. 12, forenoon and afternoon, and for Dec. 2, in agreement, at about 6 sec. Between this period and the corresponding calculated natural heaving periods of 7.34 or 7.63 sec., there are considerable differences. I hesitate to draw further conclusions from this, until the important tests (of Nov. 9 and 11, and Dec. 14), as yet lacking, have also been evaluated.

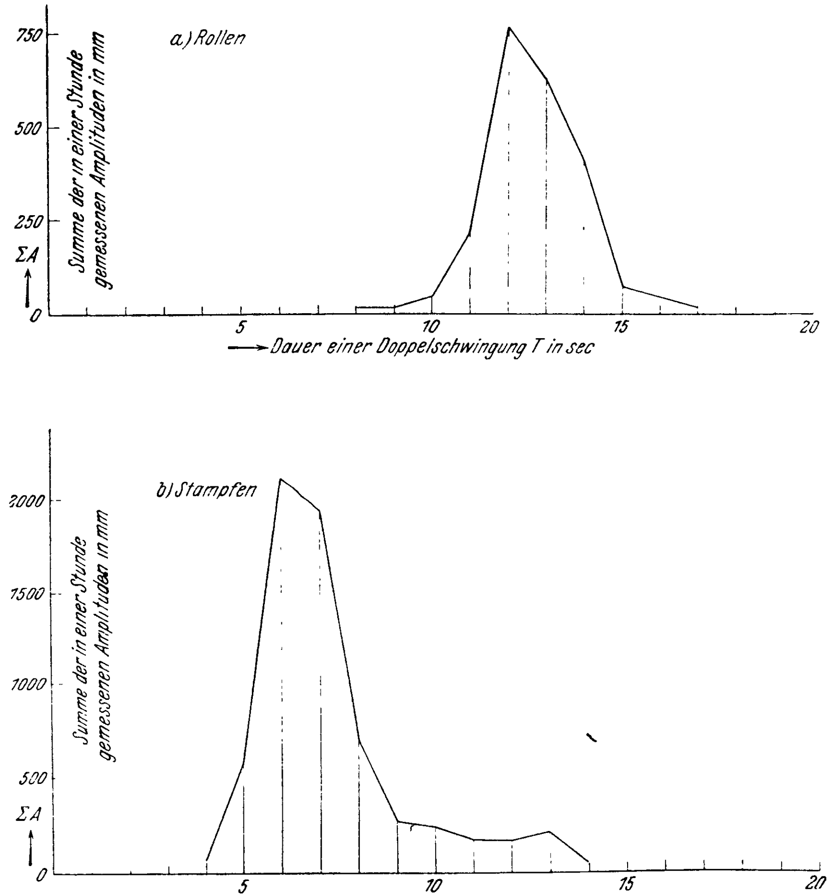

Fig. 35. Plotted as in Fig.34, for Nov. 12, 1934, P.M.

In a chart of period frequencies plotted by courses (Figs. 32a, b, and c for Nov. 12, forenoon) the greatest uniformity is again found for rolling, the natural oscillation period (~12 sec.) preponderating, with the exception of the case of a following sea, in which the period of greatest frequency amounts to 14 sec.

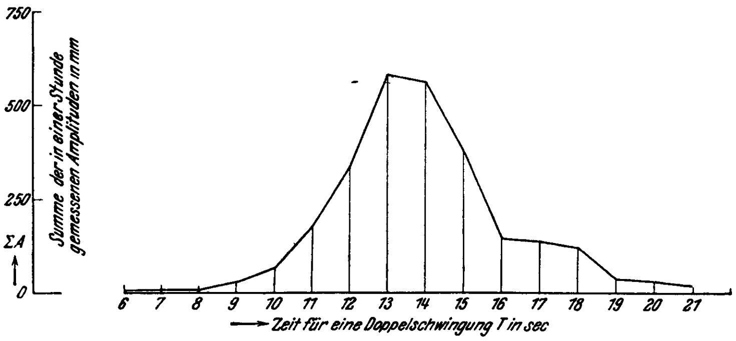

Fig. 36. Plotted as in Fig. 34, for rolling oscillation, Dec. 11, 1934.

But neither in heaving nor pitching are the periods of greatest frequency far removed from the natural oscillation period, in general, with the exception that in pitching, the relative wave period becomes noticeable with a following sea by a relatively high frequency of large periods (12 to 16 sec.).

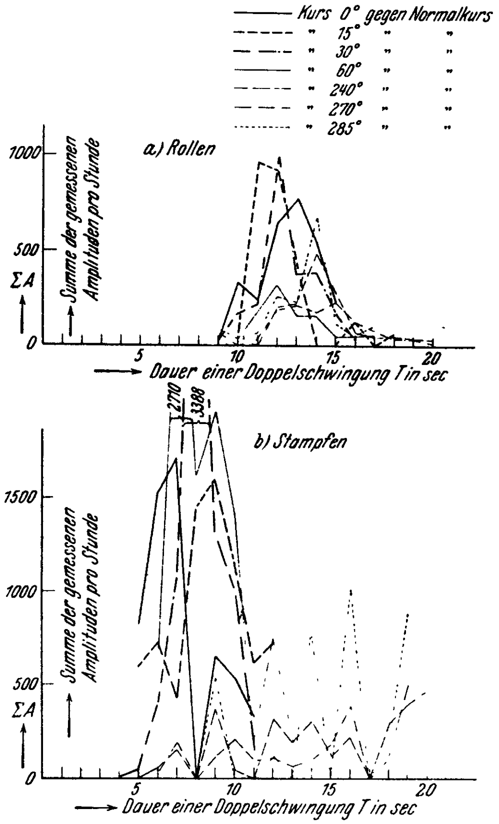

Fig. 37. Plotted as in Fig.34 for test runs of Nov.12, 1934, divided by courses according to list with Fig.14a. Date: Nov.12, 1934, forenoon.

β) There is obviously inherent in the evaluation of the hitherto treated type of frequency curves the defect that the periods are judged solely according to number and entirely regardless of whether they occur with a large or small amplitude. On the other hand it is apparently an improvement when the sum of all amplitudes belonging to each period interval is plotted on these (Σ A). This sum may also be interpreted as the product of the mean amplitude and the number of periods, and it may be found thereby that actually when this method of plotting is used both values involved for evaluating the period interval are considered. By this method the following evaluations have been carried out:

For rolling oscillations: The test runs on Nov. 12, forenoon and afternoon (Figs. 34a and 35a), and the run in the storm of the morning of Dec. 11 (Fig. 36).

For pitching oscillations:
Test run on Nov. 12, forenoon, (Fig. 34b) and afternoon (Fig. 35b).

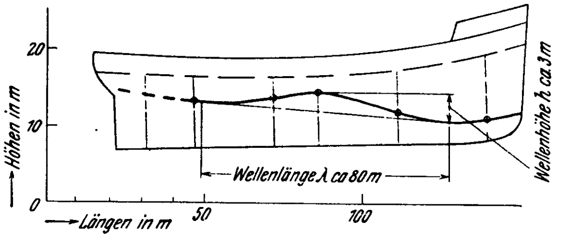

Fig. 38. Result of evaluation of the records of the Weiss wave measuring instrument for Nov. 12, 1934, 11 hr. 07 min (running exactly head-on to the seas).

The information obtained from these charts as to the predominant periods agrees essentially with that on the corresponding pure period frequency curves; also inasmuch as, for example, there is a difference of about 1 sec. between the predominant pitching oscillation periods for the forenoon and afternoon backing tests of Nov. 12, as had already been noted in Fig. 28b and 29b. Even though according to this the plotting of Σ A over T against the pure period frequency supplied no especially new information, I nevertheless consider it basically advisable.

This suggests the idea of undertaking classification by courses in view of the improved method of plotting. At all events, such an attempt has been made in Figs. 37a and 37b, for the pitching and rolling oscillations of Nov. 12, in the forenoon. Here too nothing essentially new results as compared with the charts of pure period frequency. Nevertheless it is noteworthy that with a following 45° sea a pronounced maximum value of ΣA, high in an absolute sense also, sets in at 14 sec. for rolling oscillations. We might incline toward the conclusion that there is resonance, but doubtless the natural period amounts to only about 12 sec. and the relative wave period likewise must under present conditions lie below 14 sec.

The frequency investigations described in this section are still especially deficient in comparison with the total amount of material at hand, above all with respect to the backing test of Dec. 9 and the tests on the days of the storm. I hope soon to have an opportunity to complete it, the necessity of which I realize.

c) DETERMINATION OF WAVE PROFILES.

The Weiss wave measuring apparatus supplies records in the shape of films, which Herr Weiss will describe and show in greater detail in his report, and which in conjunction with the heaving and pitching measurements, permit approximate determination of the wave profiles and the consecutive positions of the ship in the wave.

Through the records of the wave measuring instrument the position of the wave contour at any given moment, with respect to the ship is known. As long as the mean wave length remains within the limits governed by the distance of the extreme test stations in the bow and at the stern from each other (108 m), this in conjunction with heaving and pitching measurements will be sufficient to determine the wave curve, since then at least one complete wave will be outlined on the side of the ship, and we will thus be able to read off directly the magnitudes of chief interest, wave length and wave height. Thus, for instance, Fig. 38 based on a measurement of Nov. 12, forenoon, on a course contrary to the sea, shows a wave length of about 80 m (noted in this order of magnitude several times), and a wave height of about 3 m.

Essentially more difficult, however, will be evaluation in the particularly interesting cases of heavy seas. Due to the longer waves which prevail in that case, it will obviously no longer be possible to obtain directly a complete image of the seaway. But even in this case there is the possibility of obtaining at least an approximate profile, which may be conceived as being obtained from the intersection of a vertical plane in the mean direction of the run (run without yawing) with the surface of the water for a greater range than that of only one ship length, and taken with respect to an absolute system independent of the ship.

It will be most expedient to consider two relative systems. The first is firmly connected with the ship and therefore will accompany it in all its movements. In this system we know the wave contour at any moment for the ship's length. The second is firmly connected with the waves, and therefore is also a moveable system. With respect to the mean position of rest of the individual particles of water it carries out a pure translation with the wave velocity. For this system the wave contour is to be found.

This problem can obviously be solved by setting up equations relating the two relative systems to each other.

1. Movement in the vertical (heaving): The ship, and thus also system 1 carries out the heaving oscillations $S = f(t)$, which can be found by integrating the measured heaving accelerations. This movement is already relative to the mean water surface, and therefore also to the second relative system.

2. Rotation about the horizontal centroidal axis at right angles to the ship (pitching). The ship performs the pitching oscillations $\psi = f(t)$, which are known from the records of the gyroscopic instrument.

3. Rotation about the horizontal longitudinal axis through the center of gravity (rolling). This movement can be neglected in the present case, since it is obviously merely a question of the mean wave contour regarded as in the longitudinal plane of the ship, which is obtained by averaging the wave contours on both sides of the ship.

4 and 5. Rotation about the vertical axis through the center of gravity (yawing), and movement in the horizontal plane at right angles to the ship. This movement will also be neglected, since its consideration would hardly change the final result.

6. Horizontal movement in the direction of the longitudinal axis of the ship. For investigation of this, both systems must first be taken with respect to quiet water.

a) The ship system moves forward through the quiet water in the direction of its course at the mean velocity v found by means of the speed measuring instruments.

b) The ship oscillates to and fro about this direction of advance (surging). This is neglected, like 4 and 5.

c) The wave system moves forward with respect to the quiet water with a speed of advance c. Here, however, we are forced to assume that the wave contour within the time period considered does not change with respect to the second system, and that a uniform wave speed of advance c does actually exist, which, naturally will not usually be the case. The figures show, however, that in the cases considered of Dec. 2 and 11, this assumption may be regarded as to a certain extent true. If α designates the angle of the ship's course to the seaway then the speed of advance of the waves in the direction of the ship's course, i.e. in our second relative plane, will amount to $c/\cos\alpha$ with respect to still water. By combining a) and c) we get the equation

$$\text{relative velocity, } c_r = v - \frac{c}{\cos\alpha}$$

connecting the ship and the wave system.

Thus, by points 1 to 6 the relations connecting the first system in which we know the wave contour with the second system in which we desire to find the true wave contour, are established on the assumption that the unknown still present in the latter equation, either c or c_r, can be solved. This is relatively simple in cases in which the relative wave period, as with a following sea and with the ship lying to, is so great in comparison with the natural periods of the heaving and pitching oscillations that it can be derived quite satisfactorily from the oscillation or acceleration records (see Sec. dδ1.) Such an instance is presented, for example, in Fig. 40. In cases in which this does not apply, the first obvious step is to find the value c_r by attempting to determine the relative advance of an especially conspicuous region of the wave, say the crest, along the side of the ship by means of measurements following each other as closely as possible in time. This was actually tried, but failed to produce results for the reason that in actual wave structure such points, even slightly differentiated, are hardly ever found. Even the crest is hardly to be recognized as such, and can hardly be followed. Consequently another method is found to be more suitable in practice, i.e., first to estimate the wave length and use the corresponding wave speed of advance as a basis, from which, since v is measured, c_r also follows. Then it will be expedient to plot

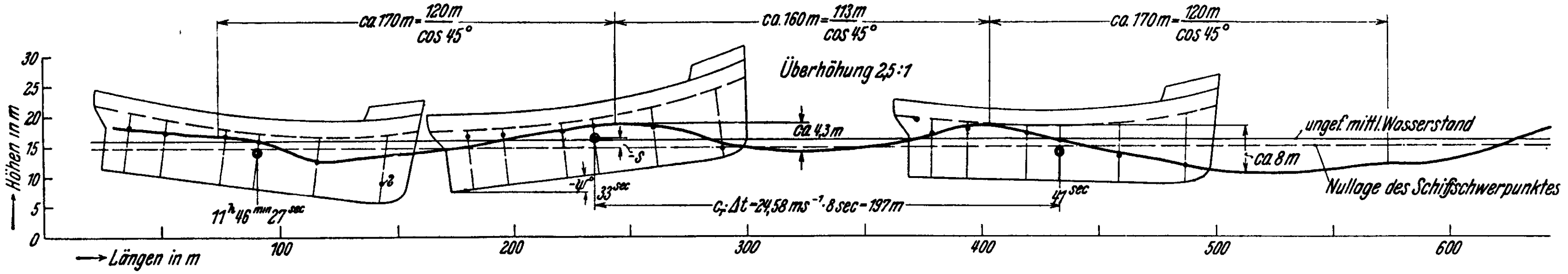

Fig. 39. Determination of a continuous wave train from the records of the Weiss wave measuring instrument and evaluations in Fig. 42 for Dec.2, 1934, 11 hr. 46 min.

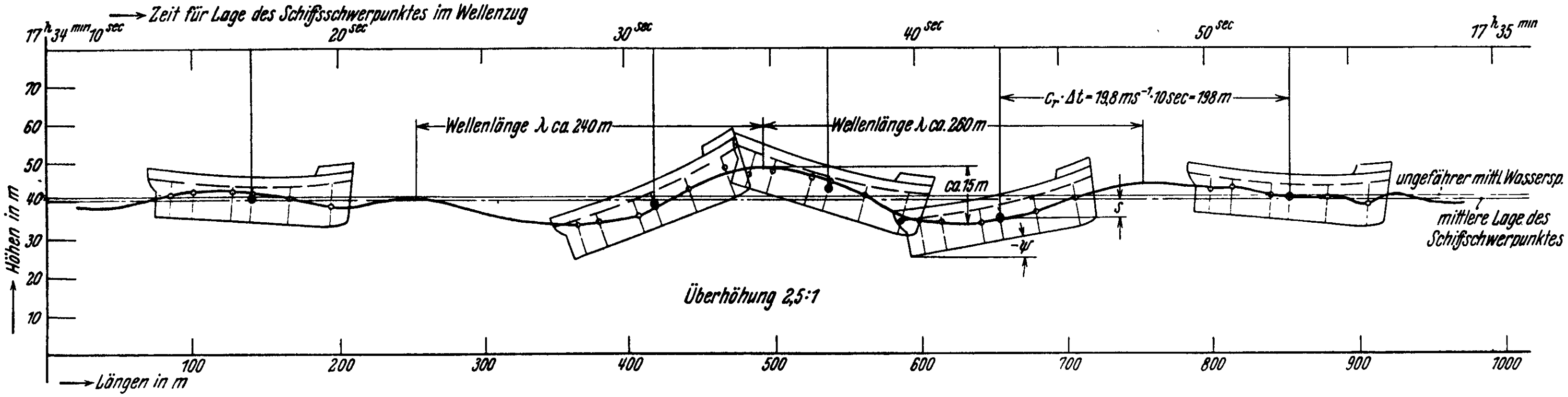

Fig. 40. Plotted as in Fig.39, using evaluations in Fig.49 for Dec. 12, 1934, 17 hr. 34 min. as basis.

the result in the following form as shown in Figs. 39 and 40, which represent the second relative plane. After establishing a horizontal base line we mark in by means of the curve of the heaving paths and pitching angles the position of the ship and the Weiss wave contour for a time period. After the timeΔt has elapsed, the relative ship system has shifted with respect to the absolute wave system by the distance $-c_r \Delta t$. It can therefore be drawn in anew together with the wave contour. By repeating this process and using a uniform seaway as a basis, the wave contours should mutually overlap and give the exact image of the desired true wave contour, whereby it will be possible by means of the wave length to be measured on the chart to check the wave length and velocity originally postulated, and if necessary to correct them by repeating the process. Certain deviations of the individual parts of the contours from each other can be explained by the alteration of the seaway with time.

d) NOTEWORTHY SPECIAL MEASUREMENTS, SPECIAL VALUES, PARTICULARLY MAXIMUM VALUES.

Heaving and pitching oscillations will be treated in common in this section for the reason that, as will be demonstrated, frequent reference to common charts is made in the evaluations, and both types of oscillations must be referred to simultaneously, especially in determining the position of the ship in the wave.

In contrast, the rolling oscillations in most cases play a separate part, more or less, which, moreover, was as a rule not of equal importance for the essential aims of the measurements. The little to be said in this section about rolling oscillations will therefore be added in conclusion.

1. Heaving and Pitching Movements.

α) On the test run of Nov. 12 in steaming against the seas there were rather imposing heaving movements although the swell was only moderate (wave length about 70 to 80 m, height about 4 m). The most violent that occurred on a course at about 30° to a head sea has already been shown in Fig. 27. Evaluation showed a maximum acceleration of about 1.4 m/sec^2, and a maximum difference in height between the highest and lowest position of about 4.1 m. There is no wave contour measurement for this time period. Similarly great deflections and accelerations occurred several times in the range of runs made in the forenoon at 0° to 45° to the seaway. – The fact that in the continuation of the test run in the afternoon the heaving accelerations, even in running at right angles to the sea when they should actually have been greatest, were found on the whole to be even smaller than in the forenoon, is without doubt to be attributed to a new seaway having been blown up in another direction by a change in wind which had set in in the interval.

The greatest observed pitching angle was 5° and occurred on a run directly against the sea. On the same course very high vertical accelerations, up to 2.75 m/sec² were also observed at the forward station. The absolute maximum acceleration of the order of 2.95 m/sec² occurred in the afternoon with the seas following at about 15°, obviously again due to the altered conditions of the sea just indicated. Furthermore, this last-named moment was also characterized by violent impact of the forward body of the ship and correspondingly strong elastic vibrations (see Fig. 54).

β) On Nov. 23 on the run across the Bay of Tehuantepec, heaving oscillations formed the greatly preponderant part of the movement of the ship because of the conditions cited in IV 2. There are no direct wave measurements available for this day, but it is evident from the evaluation of several records of the Mueller instrument, portions of whose curve are quite regular, that the wave period lay somewhere about 7 sec, and that therefore the wave length will be of the order of magnitude of about 75 m. Since the waves appeared to be very steep, we will probably have to reckon with an average wave height of 6 m, as was done in the estimate.

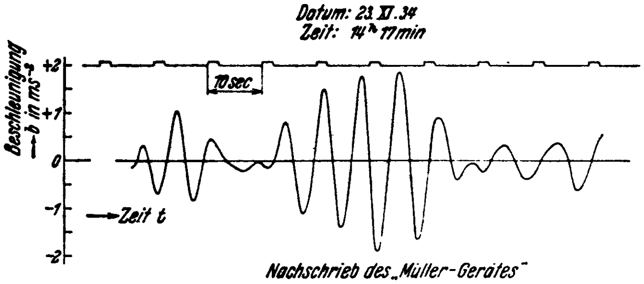

Fig. 41. Derived curve of the Müller Instrument for Nov. 23, 1934, 14 hr. 17 min. Large heaving oscillations in a thwartship sea in the Gulf of Tehuantepec. (See Fig. 15).

Especially early in the afternoon while the ship was about in the middle of the bay, several remarkably large deflections were measured. Fig. 41 shows a part of such a record. Evaluation shows a maximum acceleration of about 1.9 m/sec² and a maximum oscillation deflection (from top to bottom) of 5 m in round numbers. It may appear surprising that at a wave height of about 6 m and in a condition not far from resonance the deflections were not even greater. However, it must be considered, first that for the effect on the ship, the effective wave height must be taken into account, and not the geometrical wave height. With an approximative formula given by myself (2)* this leads, in the present case, to a diminution of the wave height from 6 to 3.3 m. Without the distortion factor μ (see Fig. 21), this wave height, under the conditions existing in the present instance of a sea running exactly athwartships, would also be about equal to the total deflection of the ship. The fact that the distortion factor here is about equal to $\frac{5}{3.3} = 1.5$ instead of 2 as in normal damping

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* $r_{eff} = r_0 e^{-\frac{2\pi H'}{\lambda}}$ with λ = wave length, $r_0 = \frac{1}{2}$ geometrical wave height, r_{eff} = effective wave height, $H' = \frac{V}{F_w}$ = mean draught of the ship (V = displacement, F_w = area at waterline).

conditions (see Fig. 21), must probably be ascribed primarily to the irregularity of the seaway, which to a large extent suppresses the effects of resonance. Since in the present case, the degree of irregularity of the sea might be considered as absolutely normal, the fact just established seems to express a result of fundamental importance. – For the rest, the short steering test carried out as shown in Fig. 15, as far as we have been able to learn hitherto, has yielded no especially noteworthy results.

γ) Among the test runs on Dec. 2 and Dec. 9, only one special case is worthy of mention aside from the statistical data given under a) and b) for the run on Dec. 2, which occurred on the latter day. In view of the not at all considerable seaway, this resulted in highly remarkable values for the heaving and pitching oscillations. As evaluation of the contour measurements showed, a seaway with wave lengths of about 120 m and heights of 4 to 6 m was running during the test on Dec. 2. For the special instance mentioned, which occurred while the ship was on a course about 45° to a head sea, the entire process of movement including the run of the waves, has been evaluated in detail, (Fig. 42). The curves of the pitching oscillations were recorded directly by the record of the Anschütz Gyro (Fig. 43) and showed the maximum amplitude of about 5½°, which, for this seaway, is highly remarkable (total deflection about 11°). Since the Mueller instrument by chance had failed to function during the period involved,

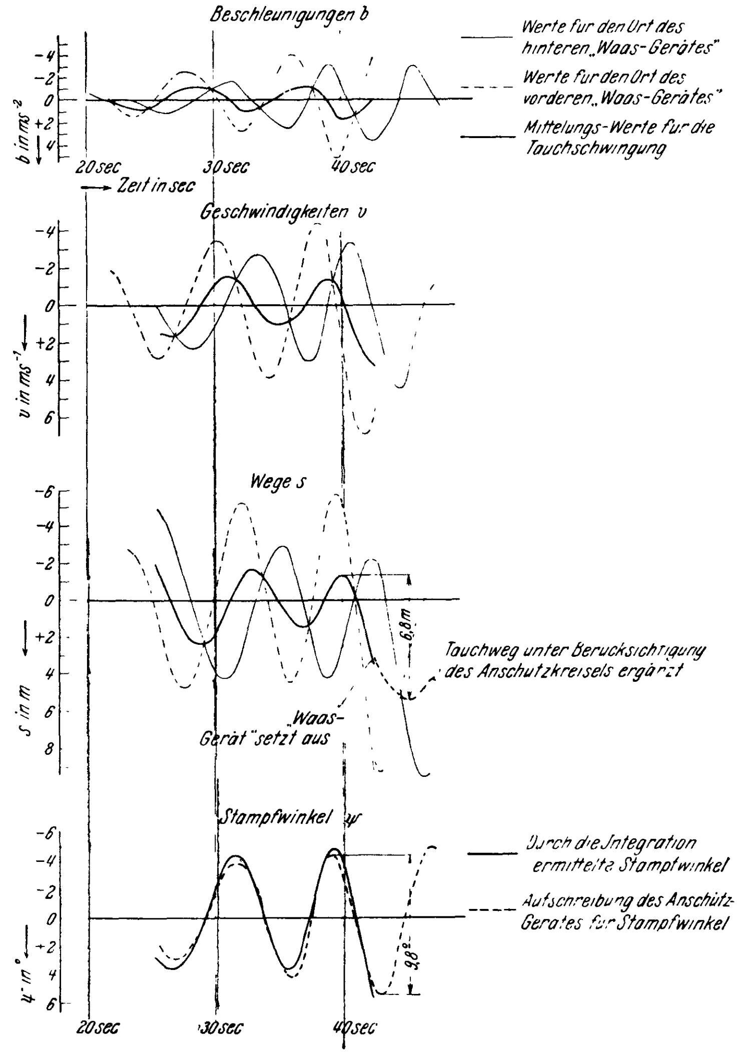

Fig. 42. Complete evaluation of the movements of the ship on Dec. 2, 1934 at 11 hr. 46 min. by integration of acceleration measurements. Especially great acceleration due to deep wave hollow, see Fig.39. Note regarding signs: Paths, velocities and accelerations positive when directed downward. Pitching angle positive when the deflection of the bow is downward.

the heaving oscillation was derived from the records of the two Waas instruments set up in the bow and stern respectively, the mean curve in these records (solid line in chart) obviously representing the heaving acceleration, from which it was then possible by double integration to derive the heaving oscillation curve. The heaving and pitching curves as in c), together with the wave contour measurements then yielded the consecutive positions of the ship in the wave (Fig. 39), from which the entire process leading up to the unusual conditions of movement of the ship is plainly to be recognized. According to this, the ship, so to say, fell into a hole, which, probably as a result of the cross sea already prevailing to a greater or less extent on this day, had formed at the point of intersection of two wave hollows crossing each other. – Among the noteworthy data of this exceptional case, the following are worthy of emphasis: Maximum heaving acceleration 2.1 m/sec^2, maximum heaving displacement 6.8 m; maximum angular acceleration of pitching movement 0.065 $\frac{1}{sec^2}$, maximum total vertical acceleration at the forward station 5.32 m/sec^2, maximum vertical path at this station 15 m.

It is to be noted that no angular acceleration of the pitching movement and of the total vertical acceleration of such magnitude were again attained even on the days of storm which soon followed, which in itself is not astonishing since during the latter days the ship either ran before a following sea or lay to; for which reasons therefore the relative wave period was very much greater than on Dec. 2, and consequently the movements of the ship were found to be very much gentler.

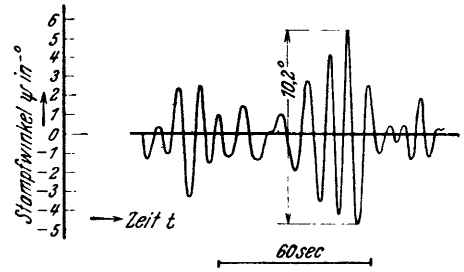

Fig. 43. Curve derived from the pitching angles recorded by the Anschütz Gyro for the time period of Fig. 42. Date: Dec. 2, 1934; Time: 11 hr. 46 min.

Although every seaman is familiar with these things, it nevertheless seems of considerable interest to become acquainted quantitatively through the foregoing positive figures with the enormous influence of running against the sea even with a seaway of such moderate force. Finally the following:

1. From the difference between the records of the forward* and after accelerometers divided by the distance between the two instruments, we obviously get the angular acceleration of the pitching movement, and by double integration of this curve, the pitching angle curve. The latter is shown at the bottom of Fig. 42 in comparison with the curve recorded directly by the Anschütz Gyro. The good agreement shows how reliably the instruments function, and that it is possible to rely upon the results of evaluation not only in this case but also in those to follow.

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* The record of the forward Waas instrument does not cover the entire range here involved because it was prematurely interrupted by the violent shock in striking the water.

2. For comparison a photograph of the simultaneous record of the forward DVL instrument is shown in Fig. 44, which instrument occupied practically the same position as the forward Waas instrument. The DVL record shows that violent oscillations were set up when the bow of the ship struck the water in falling into the "hole". The fact that these are much less noticeable in the Waas records, is due to the circumstance that obviously oscillations of high frequency were involved, which the Waas instrument functioning as a vibrometer, was able to reproduce only in natural size, i.e., very small, while on the other hand, the DVL instrument as accelerometer recorded the accelerations which were comparatively large in spite of the smallness of the oscillations. Further particulars regarding evaluation of such superposed elastic vibrations will be found in Sec. f). For evaluation of the movements of the ship as a whole, such distortions through elastic vibrations obviously are not favorable, for which reason it is better in such cases to make use of records of instruments with lower natural frequency, as was done in this case also.

δ) During the stormy days a whole series of measurements are noteworthy, and have been evaluated in various directions:

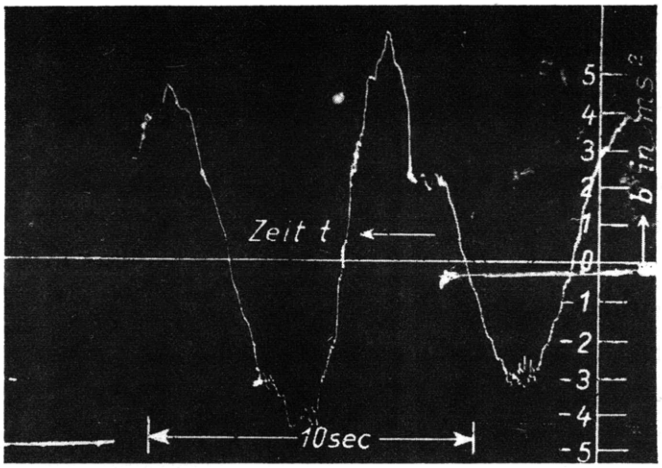

Fig. 44. Micro-photograph of the record of the DVL Instrument for the time period of Fig. 42.

1. As described under IV 5., the storm tests started with a run at full speed with a following sea at 45° to the course. In this part of the test, although larger rolling oscillations occurred frequently (see below under 3), there were no particularly great heaving and pitching oscillations since the sea had not yet attained its full height. In this condition however, the movements of the ship are noteworthy inasmuch as the relative wave period was rather high on the average, viz., of the order of magnitude of twice the heaving and pitching periods, and consequently, entirely in accord with the statements of the theory of vibrations, the position of the ship in the waves at any given moment began to approach the position of static floating. Under these circumstances, therefore, the heaving and pitching oscillations necessarily gave an approximate idea of the wave excitation itself, since the natural oscillations occurring here also, caused by the irregular contour of the seas, in general are quite distinct from the forced oscillations due to the great difference in periods already mentioned, and conse-

quently usually permit these forced oscillations to stand out recognizably. In this way indirect wave measurement actually becomes possible, at least measurement of wave lengths, so long as the mean angle of the direction of advance of the waves to the course of the ship has been estimated with any degree of accuracy, which is entirely possible.

Fig. 33, in reference to a heaving acceleration measurement, has already supplied a typical example of this, and three additional ones may be found in Fig. 45 (heaving acceleration), and Fig. 46a and 46b (pitching angle from Anschütz measurement). From all these instances, which might be multiplied to any desired extent, the resulting average in round numbers is 15 sec. From

$$\tau_r = \frac{\lambda}{c - v \cdot \cos \alpha} = \frac{2\pi \frac{c^2}{g}}{c - v \cdot \cos \alpha} = 15$$

we get, with v = 13 knots = 6.7 m/sec and $\alpha = 45°$, the wave velocity c = 16.8 m/sec, and the wave length λ = 180 m, which may be regarded as a highly probable value.

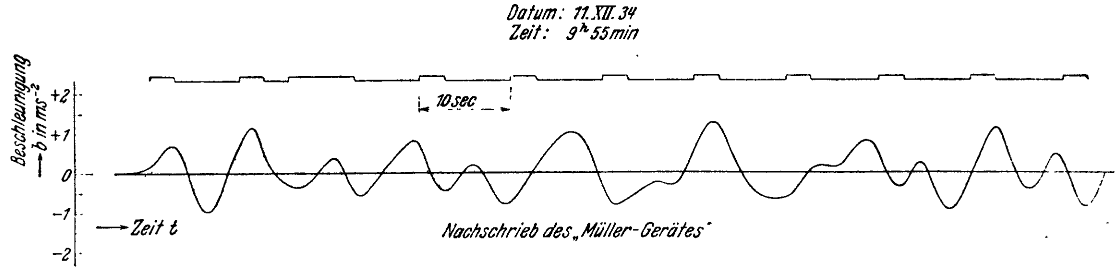

Fig. 45. Derived curve of the Müller Instrument for Dec.11, 1934, 9 hr. 55 min. Long periods of forced oscillations in a following sea.

What has just been stated regarding the behavior of the ship in a following sea also applies in approximately the same way to the ship when lying to, assuming that the wave lengths are very great, for then the absolute wave periods, which in the case of the ship lying to are identical with the relative wave periods, will

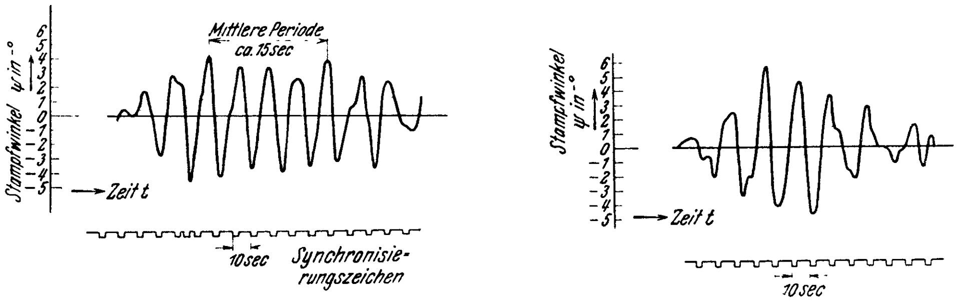

Bild 46a.

Abb. 46b.

Fig. 46a and 46b. Derived curves of the pitching angles of the Anschütz Gyro for Dec. 11, 1934.
a) 10 hr. 04 min.
b) 10 hr. 55 min.

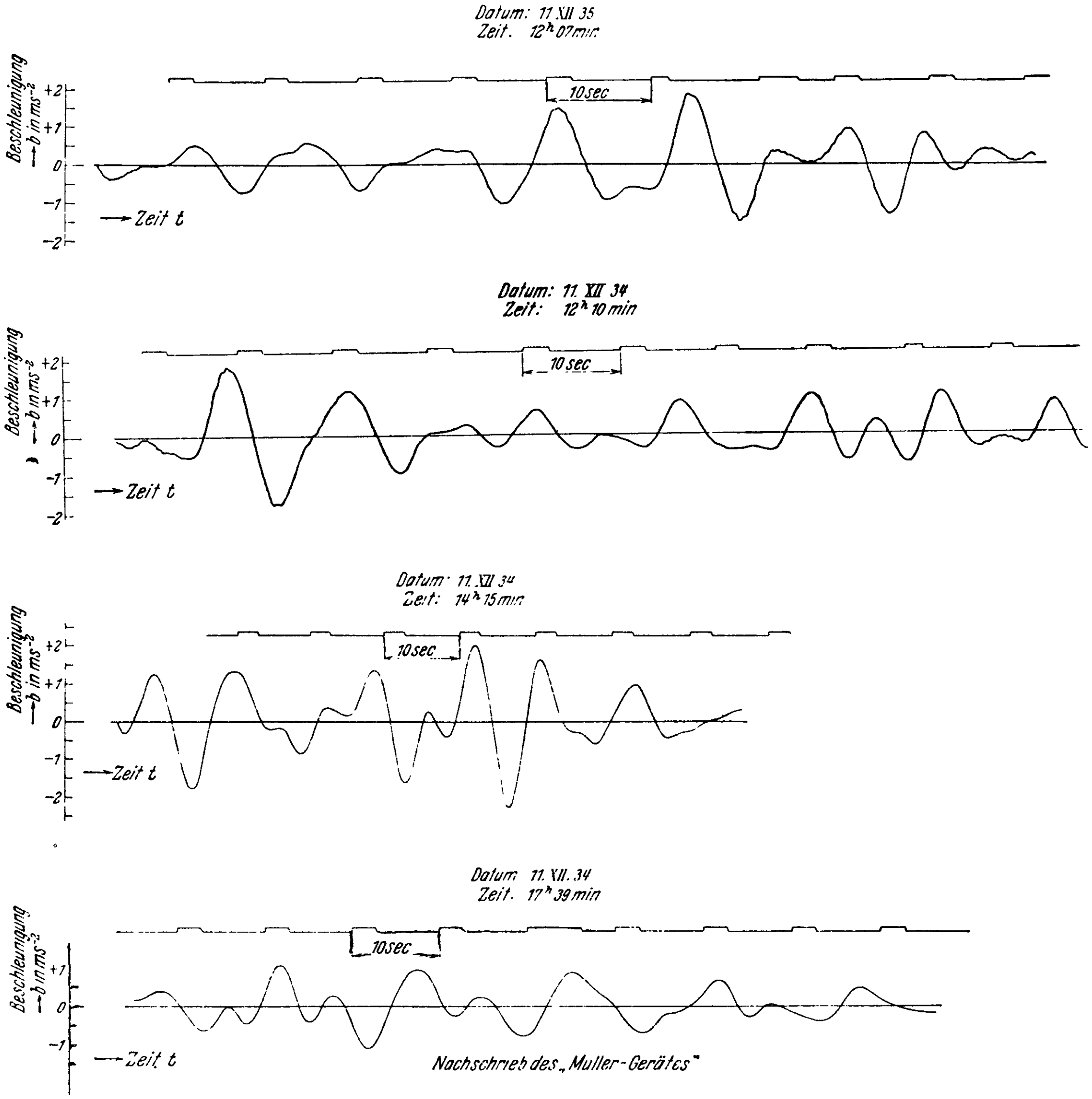

Fig. 47. Derived curves of the Müller Instrument for Dec.11, 1934, a) 12 hr. 07 min., b) 12 hr. 10 min., c) 14 hr. 15 min., d) 17 hr. 39 min. Ship lying to in a heavy sea.

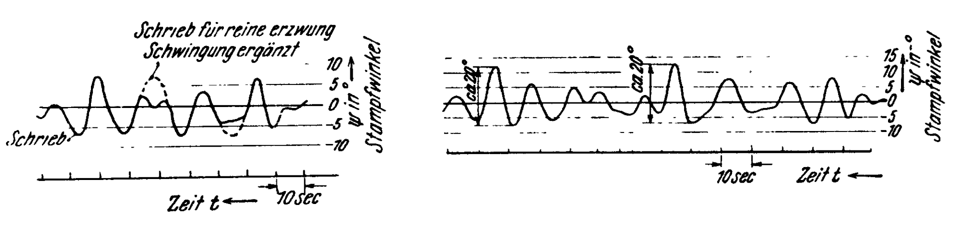

Fig. 48. Derived curves of the Petravic Gyro.
a) For Dec.11, 1934, 17 hr. 58 min.
b) For Dec.14, 1934, 18 hr. 16 min.

be so large that essentially the same typical condition of movement of the ship will set in which was previously indicated in the case of a following sea. Indeed, among the numerous measurements made with the ship lying to, we find the same records recurring frequently. Fig. 47 gives examples of this recurrence in heaving, Fig. 48 in pitching. From this we derive absolute wave periods with an average of 12 to 15

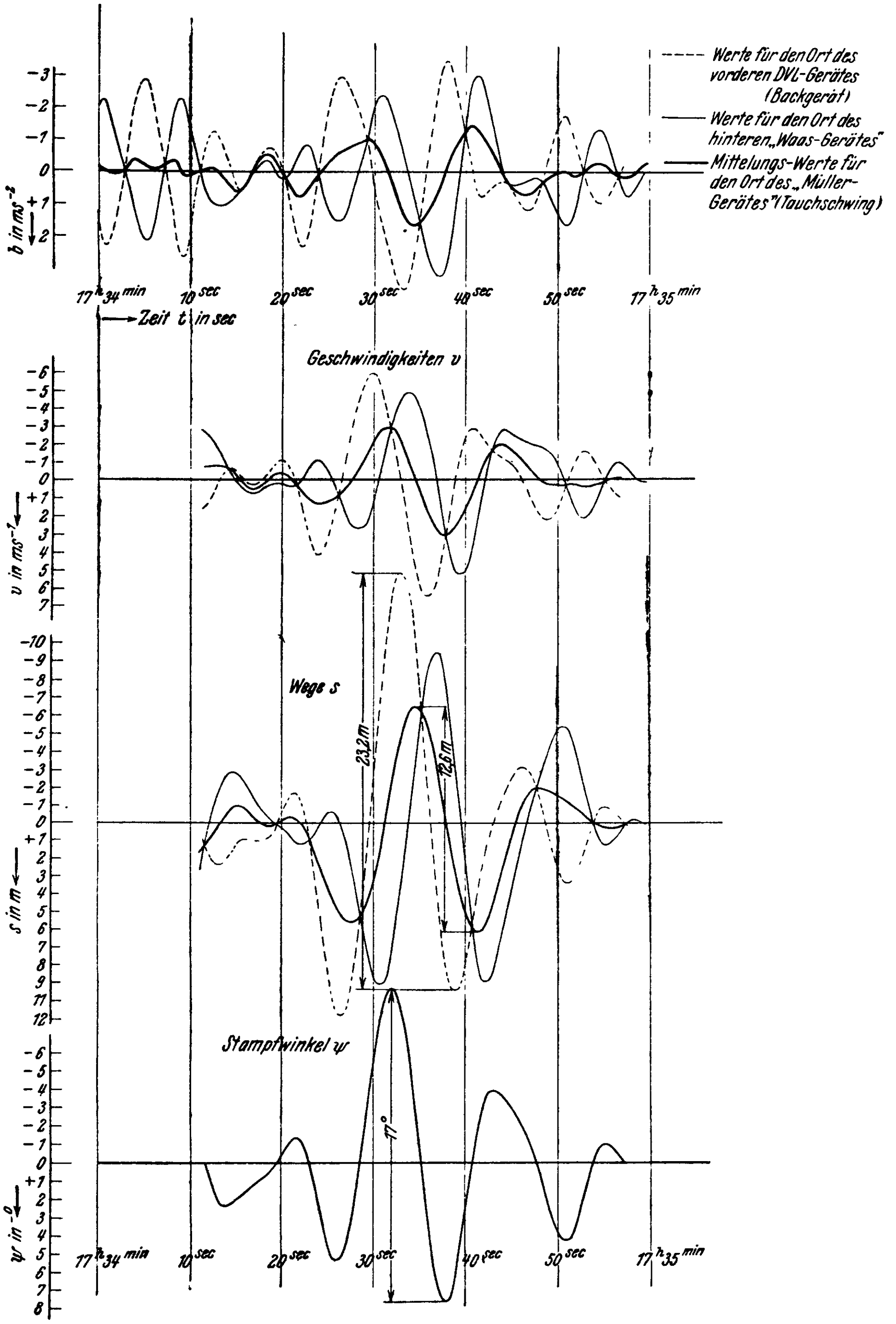

Fig. 49. Complete evaluation as in Fig. 42, for Dec. 11, 1934, 17 hr. 34 min.

sec, in some cases up to 18 sec, which correspond to wave lengths of 225 to 500 m. Not only visual evidence, but also frequent stoppages of the time spacing between the passage of consecutive wave crests proved that such wave lengths really coincide with actual conditions.

2. Among numerous cases in which especially great movements took place with the ship lying to, I am able for the present to cite only one example of full evaluation, i.e. inclusive of the corresponding wave measurements. Additional examples, if any, will be reserved for a supplementary report. However, it is to be assumed that the example here given (Fig. 49 and 40) describes a typical case, and that therefore it supplies a good idea of what occurred in numerous similar cases on these days of stormy weather. This case is rendered more valuable by the fact that it has been discussed in greater detail by Prof. Schnadel because of the heavy loading connected with it.

Otherwise the method of evaluation corresponds essentially with that of the detailed example already given for Dec. 2, (Fig. 42). The records of the forward DVL instrument (port) and the Waas instrument located aft in the escape hatch were used for evaluation of oscillations. Although a simultaneous Petravic record was also at hand, no material use could be made of it due to the lack of a time record, which naturally rendered the evaluation more difficult. Nevertheless, the results of the evaluation, as was found by conscientious checking, can be regarded as sufficiently reliable for practical purposes (Fig. 49). There was obtained a maximum pitching angle of 17°, a maximum heaving path of 12.6 m, a maximum vertical path at the forward test station of 25 m, a maximum heaving acceleration of 1.5 m/sec^2 a maximum resultant vertical acceleration at the forward test station of 3.5 m/sec^2. Thus the latter value is considerably below the corresponding maximum value of that applying for Dec. 2 (Fig. 42). The wave length is found by this evaluation to be of the order of magnitude of 250 m, the wave height about 15 m.

2. Maximum Values.

The greatest pitching angle measured throughout amounted to 24° (total angle). It occurred on the forenoon of Dec. 11, about half an hour after the ship hove to. Fig. 50 shows the record of the Petravic gyro for the period involved. In themselves, such great pitching angles are not as surprising as they might appear at first glance, since as has just been explained under δ1, as the positions assumed by the ship when lying to on the very large waves prevailing in this instance approach the static position, the pitching angles take

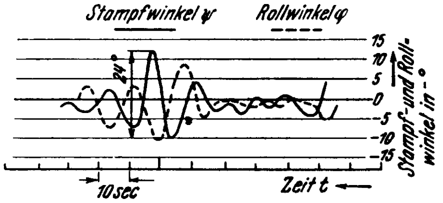

Fig.50. Derived record of the Petravic Gyro for Dec.11,1934, 11 hr. 40 min. (time of the greatest pitching angle).

on the order of magnitude of the wave slope angle. Now, even when the ratio of wave height to wave length is quite normal = 1:20 using a sinusoid as a base, the greatest wave slope angle = $\frac{\pi h}{\lambda}$ = $\frac{\pi}{20}$ = 0.157 = 9°. Actually, as is evident from the wave measurements made by Weinblum, considerably greater angles occur. Moreover, in combination with the appearance of this maximum pitching angle there occurred a particularly violent impact followed by great free oscillation of the ship's hull, as may be seen from the record of the forward DVL instrument (Fig. 51). Unfortunately this instrument is the only one of the accelerometers that functioned over this period of time, and furthermore no wave measurement is available. However, Fig. 51 supplies an impressive idea of the more particular circumstances accompanying this occurrence. The maximum acceleration in this case amounted to about 4.5 m/sec².

Pitching angles up to 20° were measured frequently during the stormy days. Moreover, the acceleration in heaving as well as the resultant vertical movement at the bow often attained the orders of magnitude indicated by Figs. 49 and 51; however, as far as we have been able to determine up to the present, did not exceed these materially.

Extraordinarily long wave periods with correspondingly great wave lengths have been recorded on several occasions. Thus, for instance, from one of the Mueller records reproduced in Fig. 47 (Dec. 11, $17^{hr}39$) is obtained a whole series of long periods, averaging 16.8 sec. The accompanying wave length would be approximately 440 m. Regarding the heights involved in waves of such great size, no evaluation of the Weiss wave measurements is available as yet, but in the stereophotographic exposures of Weinblum height differences up to 18 m have been noted.

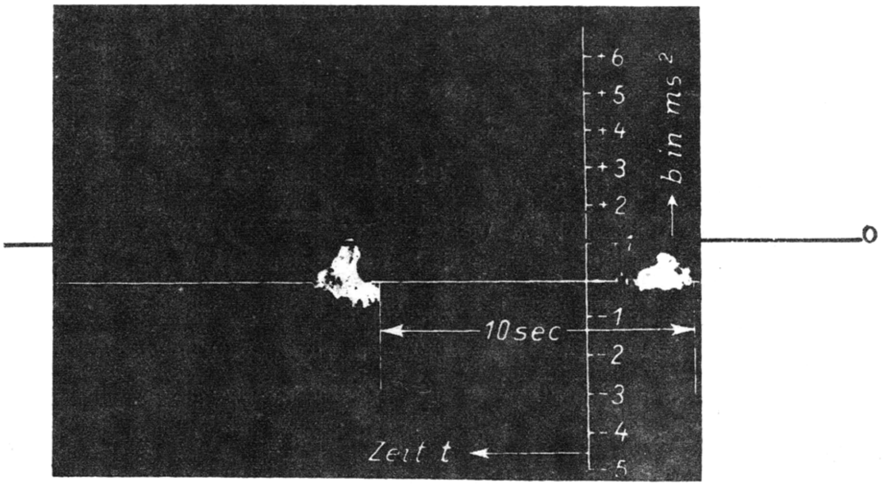

Fig. 51. Micro-photograph of the record of the forward DVL Instrument for the time period of Fig. 50.

3. Rolling Oscillations.

Regarding these only a few words will be said here. The measurements of rolling oscillations served an essentially statistical purpose, and so far as appropriate evaluations have been made they have been treated in Sec. Vb. There was hardly any opportunity for noteworthy other measurements since no really great rolling oscillations occurred. The experience previously encountered throughout (see Sec. I) was borne out, to the effect that in rolling oscillations we may speak of a pronounced condition of resonance, and consider the actual, i.e. irregular, seaway in its effect on the ship as replaced, on the whole, by an average regular seaway. Of course no actual state of resonance was ever reached, however, even when such a state was approached it was possible to note wider deflections. Thus, for example, on the morning of the first day of storm the ship first ran with the sea following at about 45°, which according to p 49 corresponded to a relative wave period of about 15 sec, while the natural period amounted to about 13 sec. On this occasion rolling oscillations up to 16° were several times measured, in one instance of 20°, and the mean amplitude proper to the period stage of about 13 sec was found to be 6.4°.

On the afternoon of Dec. 14 in changing from the regular course to hove to position especially large rolling angles occurred, for which no records are available, however. The greatest angle was about 22°.

e) Measurements of Longitudinal, Transverse, and Yawing Oscillations.

For investigation of this typical group of oscillations which is distinguished from the group of rolling, pitching and heaving through the fact that no restoring force and therefore no natural oscillation exists, rather extensive measurements were made. For yawing oscillations certain evaluations made by von den Steinen are already available. However, it has been impossible to take them up in this report, and likewise the report of measurement of longitudinal and transverse oscillations has had to be reserved for future publication.

f) Elastic Vibrations of the Hull.

In this connection the following questions are of interest:

1. Do vibrations of the shipform set in due to operation of the engine and propeller? Is a state of resonance discernible? Of what order are the vibrations and what order of magnitude have their amplitude?

On the trip out as well as on the return voyage no elastic vibrations whatever in horizontal direction, and vertically only very trifling ones were discernible. Therefore measurements were made only at the ends of the ship, viz., with the Waas instrument. In making the measurements, the engine RPM was altered by small stages over a wide range, running at each stage for a sufficiently long time. Since no new type of measurement is involved and since no unexpected results were obtained they will be touched upon only briefly.

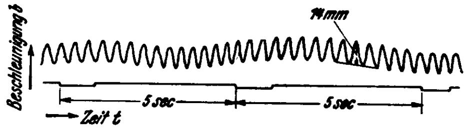

Fig. 52. Derived curve of the after Waas Instrument for Oct.18, 1934, 15 hr. 34 min. Elastic vibrations at the stern. Calculation of frequency and amplitude:
28.5 vibrations in 10 sec.; $n = \frac{28.5}{10} = 2.85$ Hertz; total amplification for this by calibration see Fig.9, 3.16; measured acceleration amplitude $\bar{a} = \frac{14}{2} = 7$ mm, and thus an amplitude of vibration of $\bar{s} = \frac{7}{3.16} = 2.22$ mm.

On the outward trip on a day on which the following load condition existed: Draught forward 4.20 m, aft 5.41 m, mean 4.80 m, Δ = 8,230 T, additional load = 2,780 T, a maximum of amplitudes plainly marked, of about 2.2 mm aft (Fig. 52) and 1 mm forward was noted. The frequency of vibration amounted to twice the number of revolutions, thus indicating a vibration of the second order and obviously, moreover, one of the second harmonic (with three nodal points), which was confirmed by the fact that the vibration forward was exactly of opposite phase to that aft. The natural frequency of this vibration consequently amounted to 2 x 85.7 = 171.4. The cause of this vibration probably is to be sought only in the effect of the finite length of the connecting rod, since only in this way can an impulse of the second order come about. When the RPM was raised to 88 there set in between n > 87 and 88 high frequency transient vibrations (Unterschwingungen).

On the return trip on which the ship had another load condition (on the day of measurement, forward draught 7.10 m, aft 7.77, mean 7.43 m, Δ = 13,484 T, additional load = 8,035 T) two conditions of resonance were recorded, one at $n \cong 83$/min, appropriate to a vibration of the first harmonic of the first order, and another at about n = 77,* which as on the trip out corresponded to a vibration of the second harmonic of the second order. In the former state of resonance the amplitudes were of the magnitude of 2.2 mm aft, 1.3 mm forward, and in the latter state .85 mm aft and 0.1 mm forward. The smallness of the vibrations of the first harmonic of the first order gives proof that, first, the engine (5-cylinder Diesel motor + compressor cylinder is very well balanced, and second, that the propeller blades turned out very uniformly both as to construction and more particularly as to pitch.

For the vibrations of the first harmonic we would get by Schlick's formula

$$n/\text{min} = C\sqrt{\frac{J_0}{\Delta L^3}}$$

with J_0 = moment of inertia, midship section = 28.75 m^4
Δ = displacement 13,073 T, condition on return trip

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* (Translator's Note: This apparently should be 177).

L = overall length = 136 m
$C \approxeq 2{,}800{,}000$ for freighters

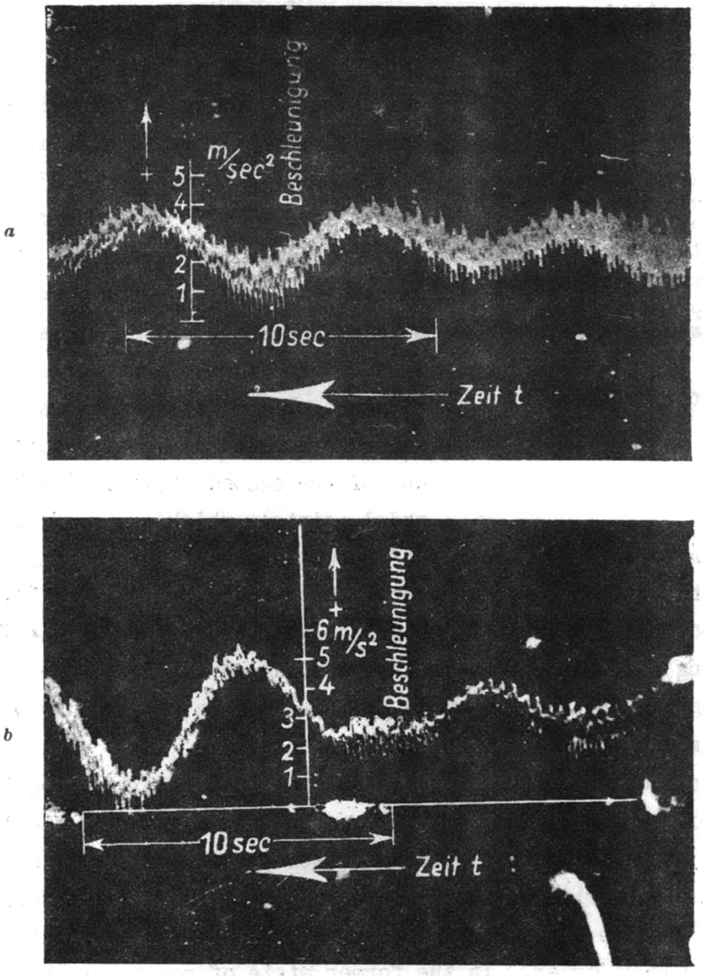

Fig. 53. Micro-photograph of record of after DVL Instrument. Elastic vibrations at stern:
a) For Dec.3, 1934, 16 hr. 58 min.
b) For Dec.5, 1934, 16 hr. 31 min.

a frequency of $n = 82.8$. A systematic vibration calculation taking into consideration the virtual mass according to Lewis (10) and the shear deflection according to Lockwood Taylor (12) yields a frequency of $n = 83.1$.

I desire to mention in addition that after the cargo of 3,024 T of wheat had been delivered in Liverpool on the return voyage, the vibration in the midship range was noticeably greater than before. The explanation is simple, for, because of the elimination of weights which on the whole had been lying more amidships, the base line of the vibrations necessarily moved farther from the crest of the vibration curve, which caused increase of the amplitudes in the middle and decreases at the ends. It was no longer possible to make measurements here since the instruments had already been dismounted and packed.

Fig. 53 shows a section of a record of the DVL instrument set up at the extreme end of the stern behind the after edge of the wheel house, made just a few days after the measurement made on the return trip, previously mentioned. According to the evaluation undertaken we have in the principal vibration one of double the frequency of the RPM, i.e., once more a vibration of the second harmonic of the second order; in addition there appears a vibration of the fourth order, obviously due to the fact that the propeller is fourbladed. Even though the vibration appears prominently in the form of a rather wide band on the record, it is nevertheless only the accelerations corresponding to the high frequencies which cause the elastic vibrations to stand out so prominently, especially even with respect to the oscillations due to seaway, while the amplitudes, as shown by evaluation, are tiny. This is an illustration of how greatly distorted the vibration processes may become in accelerometers.

2. Frequently elastic vibrations of the shipform in a seaway were recorded, which arose from the violent impact of the foreward body on the water. A typical example of this is shown in Fig. 54, excerpt from a measurement of vertical vibrations made by a Waas instrument set up in the cable tier forward, on Nov. 12 in the afternoon when a very moderate sea was running. At first a very high frequency vibration of about 10 Hertz is set up. The evaluation, which by the way, it was possible to carry out only very roughly, shows a tiny magnitude for the amplitude of vibration (~0.3 mm), but already a considerable magnitude of the acceleration amplitude of ~1.4 m/sec². Occurrences of such an impact nature are, as shown by Dr. Schnadel in detail, of considerable influence, hitherto not well known and not taken into account, in the loading of the hull. As is to be noted from the continued curve of the record, the high frequency elastic vibrations, as is the nature of things, fade out very rapidly, and there then remain only the slow vibrations, especially of first harmonics, in which the entire ship vibrates for the subsequent seconds, which, moreover are very plainly recorded by the Mueller instrument amidships as well as by the instrument at the stern. This gradual fading out of vibrations is very plainly discernible in the example given in Fig. 55, (taken on Dec. 2). Here too belongs the example of Fig. 44, already shown previously, of the violent shock observed on Dec. 2.

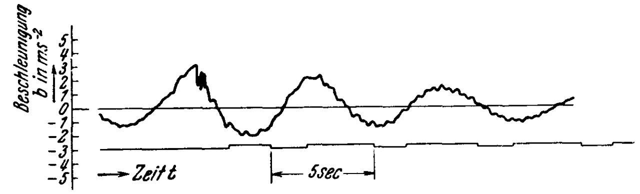

Fig. 54. Derived curve of forward Waas Instrument for Nov.12, 1934, 16 hr. 35 min. Strong vibration after bow struck water.

For the days of stormy weather an example of the setting up of such violent vibrations by the impact of the foreward body on the water has already been presented in Fig. 51. An additional example, likewise highly characteristic, is given by Fig. 56.

3. With the large trim angles which the ship assumed while lying to on the days of storm (see p. 50), it was unavoidable that frequently in quite regular time intervals the screw emerged partly from the water. On these occasions very pronounced vibration phenomena set in, very good records of which were obtained on the afternoon of Dec. 11 in measuring the vertical oscillations aft with a Waas instrument set up in the escape hatch. Figs. 57a and 57b give examples of these. A checkup of the frequencies of these vibrations shows at once that they are of the order of magnitude of four times the shaft RPM, and that consequently the impulses emanate from the four blades of the propeller driving through the water in succession. That the impulses must be especially violent when the propeller is partly emersed is also obvious. The high frequency amounting to about 6.4 Hertz might seem strikingly high, since the corresponding propeller RPM will be $\frac{6.4}{4}$ x 60 = 96. Obviously, however, the RPM was held to this limit by the regulator during emersion of the screw. The amplitude of vibration in this case is likewise small, about 0.6 mm; the maximum acceleration has the order of magnitude of 1 m/sec².

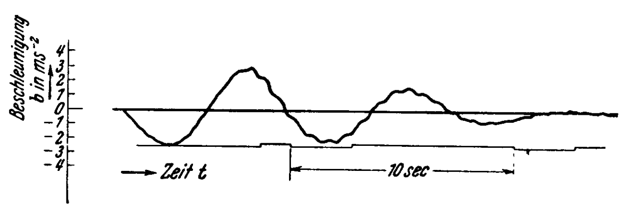

Fig. 55. Derived curve of forward Waas Instrument for Dec.2, 1934, 11 hr. 35 min. Elastic free vibration after bow struck water. Date: Dec.2, 1934; Time: 11.35 AM.

The example presented in Fig. 57b concerns the same case studied in greater detail in Fig. 13.

VI. CONCLUDING REMARKS

It is intended here briefly to summarize the essential experiences gained with the measuring apparatus used aboard the SAN FRANCISCO on her high sea test voyage and the data subjected to a brief analysis, above all with respect to further development.

a) Behavior of the Test Set-up

α) The Anschutz Gyro proved its value without question and is indispensible for measurements aboard ship with the object of ascertaining the behavior of ships in a seaway.

For recording pitching angles a clearance of 15° on all sides should be provided for in the future. For rolling angles also, even though no particularly great angles occurred on this occasion (max. 22°), the 30° clearance on all sides allowed in the present instance ought to be further increased as much as possible.

β) The Petravic Gyro is particularly useful for static measurements in

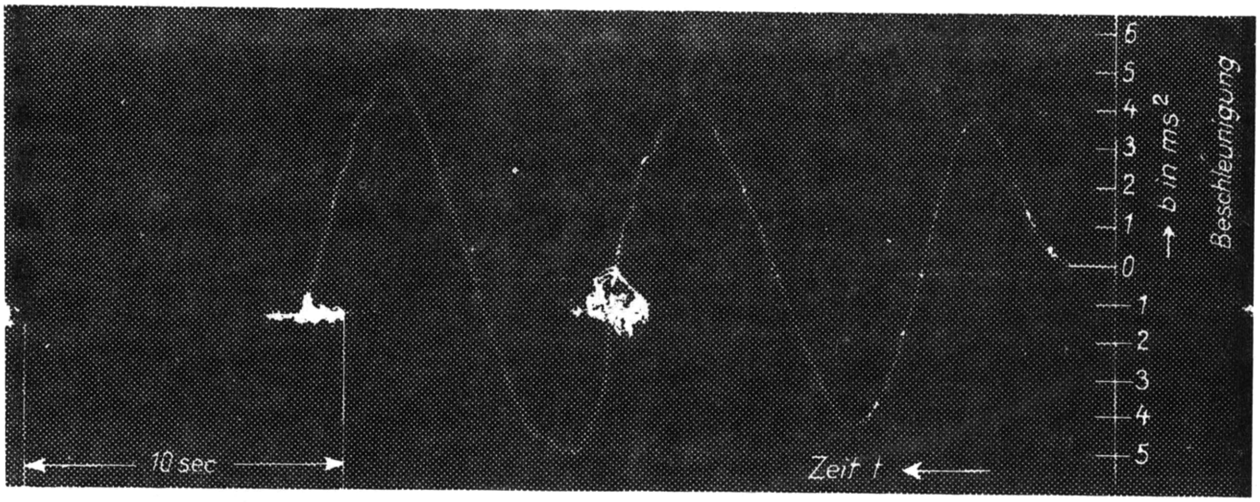

Fig. 56. Micro-photograph of record of forward DVL Instrument for Dec. 11, 1934, 11 hr. 15 min. Strong vibration when bow strikes water.

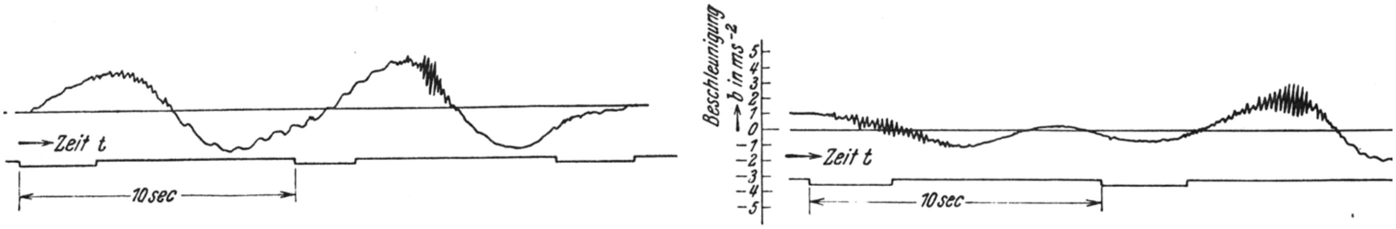

Fig. 57a and 57b. Derived curves of after Waas Instrument for Dec.11, 1934, P.M. Strong vibrations at stern with propeller partly emerged.

which a high degree of accuracy and chronological recording are not required.

γ) Regarding the success of the yawing gyros, I should not yet care to volunteer an opinion, since I have not been able to devote any time to the evaluation of measurements.

2. Accelerometers.

α) General. Accelerometers are indispensible particularly for two reasons: First, for important purposes, especially for strength investigations, accelerations are absolutely required to be known in order to determine the forces of acceleration; secondly there has hitherto been no possibility and there is now no possibility in view, of measuring heaving oscillations by any other means than through the accelerations.

β) The Mueller instrument for measuring heaving functioned very well and reliably throughout. At a frequency of about 1 Hertz to which it was adjusted in the principal measurements, it furnished on the average probably the most convenient sized record.

γ) The Waas Instruments, although they had been developed for quite different purposes, proved surprisingly suitable particularly for seaway measurements. In spite of the narrow width of the paper, for example, the size of the record is adequate even for more exact evaluation when the instruments are set up at the ends of the ship to measure vertical accelerations. It might perhaps be of advantage to design an otherwise similar instrument with a lower Hertz number (about 2 — 3) for measuring vertical accelerations amidship. Furthermore, for seaway measurements the speed of the paper which is calculated for the measurement of elastic vibrations would have to permit of considerable reduction. — A great advantage of the Waas instrument, in addition to its handiness, is also the possibility of using it for horizontal vibrations as well as vertical ones, which is an important factor in making measurements aboard ship.

δ) The DVL Instruments likewise gave very good service. However, they require highly experienced and conscientious attendance, and then too, evaluation of the measurements, above all when chronological records are requisite, is very difficult and consumes a great deal of time. In his report, Prof. Schnadel has already mentioned a further development of the automatic instruments used for his measurements, in the sense of distant recording of the curves of instruments belonging to a group of the same type, possibly on a common strip of film with a common synchronous baseline. The same would also be desirable for automatic accelerometers, and would greatly facilitate evaluation and increase their accuracy.

ε) General Arrangement and Interaction of the Instruments. In measurements aboard ship, not only the excellence and proven qualities of the individual instruments are decisive factors, but also very materially, suitable arrangement, good interaction, and good mutually complementary results. For future cases, i.e. those in which seaway measurements and not vibration measurements occupy the foreground, it might be sufficient, to judge from the experience of the high sea test voyage, to have three self-recording instruments, operating as nearly as possible in the described distant transmission manner, one each in the bow, the stern, and amidships, for the measurement of vertical movements. In my opinion, there will be required in addition at least two instruments arranged for recording (Waas Instruments), which might be quickly set up here or there for determining conditions at any given moment, and which can be used for any measurements appearing worth while over longer or shorter periods of time. Aside from these, moreover, I consider an instrument, destined for constant measurement of heaving and giving a directly visible record, such as the Mueller instrument or a Waas instrument operating with a lower Hertz number, to be desirable.

3. Wave Measuring Set-up.

This proved very valuable on the one hand, for the strength investigations of Prof. Schnadel (see his report), and on the other it seems suitable in practice, as shown in Section Vc of my report, in any event to supply useful data for the determination of wave measurements and the consecutive positions of the ship on the waves. Evaluation of the measurements made aboard the SAN FRANCISCO will be further completed in this direction. Regarding the further development of the set-up undertaken by Weiss himself, he will report in his own paper.

b) Evaluation of Data.

1. The described measurements have yielded a considerable mass of experimental values and empirical facts which seem important for studying the movements of a ship in a seaway and their effects on the ship. The following data are worthy of emphasis:

α) In the case of rolling oscillations, the irregularity of the seaway results in rendering the influence of the natural oscillations absolutely predominant, compared to which oscillations with the period of encounter (i.e. the period of wave excitation) are much diminished. From this, a fact already essentially known by its tendency which, furthermore, is of decided importance in the estimation of the principles of damping means and damping methods, has found confirmation based on statistics.

β) Likewise, in heaving and pitching movements, even if they are evaluated according to the magnitude of the concomitant amplitudes, do not generally vary greatly from the natural oscillation periods; only with large relative periods, i.e. for example with the ship lying to in a following sea with large waves, does the relative period become noticeable to a highly pronounced and characteristic extent.

γ) For the orders of magnitude of the oscillation and acceleration amplitudes under various conditions of seaway the measurements have supplied extensive material, the statistical evaluation of which is yet to be completed. Furthermore, a number of noteworthy maximum values have been obtained.

δ) In view of the magnitudes of amplitudes, especially of heaving oscillation, the difference between actual and effective wave heights, governed by the well-known Smith Effect, was well borne out.

ε) By combining the results of the Weiss wave measurements with those of the measurements of the ship's movements in a seaway, it is possible to find the wave dimensions and wave shapes with sufficient approximation. Furthermore, the wave period and with it also the wave length in the case of a following sea and with the ship lying to can be derived approximately also from the oscillation, or as the case may be, acceleration measurements.

ζ) Based on measurements, several examples are given of the occurrence of elastic vibrations of the shipform on the one hand on impact of the foreward body of the ship on the water, and on the other on emersion of the propeller out of the water.

2. Toward solution of the question whether and to what extent systematic investigations already made or to be made in the future by theoretical means or by model tests based on uniform waves permit conclusions as to the behavior of a ship in an actual seaway, i.e. more or less irregular (see Sec. I), the tests carried out to this end (backing tests), were incapable of giving any material contribution. It was shown that such tests can have prospects of success only when, first, the seaway is so high as to afford the ship opportunity to display its seagoing qualities, and second, when it is not irregular beyond normal limits. These prerequisites wére never fulfilled to a sufficient extent in the various backing tests. In my opinion, however, this need by no means be considered as cause for doubting the possibility of success of such tests or to deny it. Rather should precisely this method be emphasized in future trials, for we are here faced with a serious task, whose ultimate aim it is to control the seagoing qualities of a ship in advance. For this theoretical and model investigations will supply the systematic fundamentals, and occasional shipboard measurements the tests.

(Next two paragraphs acknowledge co-operation of organizations and persons actively or financially participating in the test voyage.)

APPENDIX.

Brief Summary of the Theory of Vibration and Acceleration Measuring Instruments*.

I. Let the vibration to be measured be a simple harmonic vibration

$$s = \bar{s} \sin (\omega t+\delta) \qquad (1)$$

On the mass m of the measuring instrument, when thrown out of equilibrium, let act a so-called restoring force, exerted, say, by a spring, and proportional to the travel of the spring, and in addition a damping force, which let be proportional to the velocity of the mass relative to the vibrating body (ship) the vibrations of which are to be measured. Let the path of the mass during vibration be

absolute: $f = s + a$ (2)

relative to the test station: a (record).**

The equation for the movement of the mass will then be

$$m\frac{d^2f}{dt^2} + 2W\frac{da}{dt} + c \cdot a = 0 \qquad (3)$$

and according to (2)

$$\frac{d^2s}{dt^2} + \frac{d^2a}{dt^2} + 2w\frac{da}{dt} + \omega_0^2 \cdot a = 0 \qquad (4)$$

where

with $\omega_0^2 = \frac{c}{m}, \quad w = \frac{2W}{m}.$

From (1) we get

$$\frac{d^2s}{dt^2} = -\bar{s} \cdot \omega^2 \sin (\omega t + \delta). \qquad (5)$$

Introduced into (4):

$$\frac{d^2a}{dt^2} + 2w\frac{da}{dt} + \omega_0^2 \cdot a = \bar{s} \cdot \omega^2 \sin (\omega t + \delta). \qquad (6)$$

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* The notations used are tabulated at the end.

**Direct recording of the path of the mass is postulated, and accordingly the record a is identified with the path of the mass.

Particular solution (forced vibration):

$$a = \bar{a} \sin (\omega t + \delta - \varepsilon), \qquad (7) \quad (7)$$

in which

$$\bar{a} = \frac{\bar{s}}{\sqrt{\left[\left(\frac{\omega_0}{\omega}\right)^2 - 1\right]^2 + \varkappa^2 \left(\frac{\omega_0}{\omega}\right)^2}} = \frac{\bar{s}}{\sqrt{\left[\left(\frac{n_0}{n}\right)^2 - 1\right]^2 + \varkappa^2 \left(\frac{n_0}{n}\right)^2}} \qquad (8) \quad (8)$$

$= \text{Amplitude}$

$$\operatorname{tg} \varepsilon = \frac{\varkappa \frac{\omega_0}{\omega}}{\left(\frac{\omega_0}{\omega}\right)^2 - 1} = \frac{\varkappa \frac{n_0}{n}}{\left(\frac{n_0}{n}\right)^2 - 1} \qquad (9)$$

= Tangent of the phase angle between vibration and excitation

$$\varkappa = \frac{2w}{\omega_0} = \text{damping coefficient} \qquad (10)$$

Between and the so-called damping ratio γ , i.e. the ratio of two amplitudes following one another during half of a vibration (see free vibration curves in Figs. 6 and 9) the following relation exists:

$$\gamma = e^{\frac{\pi w}{\omega_0}} \quad \Big| \quad \frac{\pi w}{\omega_0} = \text{logarithmic decrement}$$

$$\varkappa = \frac{2w}{\omega_0} = \frac{2}{\pi} \frac{\log \gamma}{\log e} = 1{,}465 \log \gamma. \qquad (11)$$

For instance, with the term $\gamma = 6:1$, frequently obtained with more recent testing instruments

$$\varkappa = 1{,}14.$$

Limit Cases.

1. n_0 very small as compared to n, $\left(\frac{n_0}{n}\right)^2 \rightarrow 0$.

Herewith, according to (9) and (8):

$$\operatorname{tg} \varepsilon = 0, \ \varepsilon = \pi^{3)}, \quad \text{and therefore } \bar{a} = -s \quad a = -s. \qquad (12)$$

The absolute path of the mass in space, according to (2): f = 0.

Limit case L represents the case of an instrument functioning as oscillometer.

2. n_0 very large as compared to n,

From (8), by extending the right side with $\left(\frac{\omega}{\omega_0}\right)^2$ is obtained:

$$\bar{a} = \frac{\bar{s}\left(\frac{\omega}{\omega_0}\right)^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_0}\right)^2\right]^2+\varkappa^2\left(\frac{\omega}{\omega_0}\right)^2}} \rightarrow \bar{s}\left(\frac{\omega}{\omega_0}\right)^2 \tag{13}$$

$$\bar{a}\cdot\omega_0^2 = \bar{s}\cdot\omega^2 \tag{13a}$$

Moreover

$$\operatorname{tg}\varepsilon = \frac{\varkappa\frac{n}{n_0}}{1-\left(\frac{n}{n_0}\right)^2} \rightarrow \varkappa\frac{n}{n_0}. \tag{14}$$

Now if n_0 is so large in comparison with n, that even with the usual and requisite high damping values $\varkappa$ (for example $\varkappa = 1.14$), the right side of (14), in which n/n_0 appears only in the first power becomes approximately = 0, then the phase angle becomes $\varepsilon \cong \pi$, and thus

$$\sin(\omega t + \delta - \varepsilon) = -\sin(\omega t + \delta) \tag{15}$$

From (5) we get, according to this

$$a\cdot\omega_0^2 = -\bar{a}\,\omega_0^2\sin(\omega t + \delta)$$

and from this, according to (13a)

$$a\cdot\omega_0^2 = -\bar{s}\cdot\omega^2\sin(\omega t + \delta) = -s\cdot\omega^2. \tag{16}$$

Herein the right side generally represents the acceleration of the vibration to be measured in a period of time t, which accordingly in this limit case is reproduced by the ordinate α of the record multiplied by the instrument constant ω_0^2. Thus the instrument functions as an accelerometer.

If n_0 is not as large in comparison with n as would be necessary in order to let the right side of (14) differ slightly from zero with the requisite high damping values, the amplitude equation (13a) will practically apply in this case also, but a noticeable phase displacement will occur and (16) will therefore apply only if this phase displacement is taken into account. For example, when $n_0/n = 5$, and $\gamma = 6$, i.e. $\varkappa = 1.14$ (see above), a phase displacement of about 13° will occur.*

3. $n_0 = n$, resonance.

* The Muller instrument (see Sec. III, 2) in later measurements had a frequency of 1 Hertz, in round numbers, and a damping ratio of $\gamma = 4.25$. With reference to the natural heaving oscillations of the ship, the period of which was about 7.5 sec., n/n_0 being equal to 0.133, recalculation gives the relatively unimportant phase displacement of about 7%.

$$\bar{a} = \frac{\bar{s}}{\varkappa}$$
$$\operatorname{tg} \varepsilon = \infty, \quad \varepsilon = 90^0. \tag{17}$$

According to this, with damping at the usual magnitude at which $\varkappa$ does not differ materially from unity, the amplitude distortion in the record is not very considerable. For $\gamma = 6$ and $\varkappa = 1.14$, $\bar{a}$ will be equal to 0.877 s.

II. Let the vibration to be measured represent two or more superposed harmonic vibrations:

$$s = s_1 \sin(\omega_1 t + \delta_1) + s_2 \sin(\omega_2 t + \delta_2) + \dots \tag{18}$$

Equations (2), (3), and (4) obviously remain valid unchanged. In place of (5) we will have

$$\frac{d^2 s}{dt^2} = -\bar{s}_1 \omega_1^2 \sin(\omega_1 t + \delta_1) - s_2 \omega_2^2 \sin(\omega_2 t + \delta_2) - \dots \tag{19}$$

and in place of (6)

$$\frac{d^2 a}{dt^2} + 2w \frac{da}{dt} + \omega_0^2 \cdot a = \bar{s}_1 \omega_1^2 \sin(\omega_1 t + \delta_1) + s_2 \omega_2^2 \sin(\omega_2 t + \delta_2) + \dots \tag{20}$$

Particular solution:

$$a = \bar{a}_1 \sin(\omega_1 t + \delta_1 - \varepsilon_1) + \bar{a}_2 \sin(\omega_2 t + \delta_2 - \varepsilon_2) + \dots \tag{21}$$

with

$$\bar{a}_1 = \frac{\bar{s}}{\sqrt{\left[\left(\frac{n_0}{n_1}\right)^2 - 1\right]^2 + \varkappa^2 \left(\frac{n_0}{n_1}\right)^2}} \quad \text{etc.} \tag{22}$$

Limit Cases.

1. When n_0 is small in comparison with all n's, then obviously Eq. (12)

$$a = -s$$

continues to be valid unchanged. The instrument functioning as a vibrometer thus records the resultant vibration.

2. n_0 very large in comparison with all ω's.

According to (16), we get

$$a_1 \omega_0^2 = -s_1 \omega_1^2$$
$$a_2 \omega_0^2 = -s_2 \omega_2^2$$
$$\dots\dots\dots\dots$$

Now since the ordinate of the entire record is $a = a_1 + a_2 + \ldots$, we have

$$a \cdot \omega_0^2 = -s_1 \omega_1^2 - s_2 \omega_1^2 - s_2 \omega_2^2 - \ldots \qquad (23)$$

The right side represents the sum of the accelerations of the various vibrations and thus also the total acceleration effective at the test station. This will thus be reproduced in this case also, by the ordinate of the record multiplied by the instrument constant ω_0^2.

If n_0 is not sufficiently large in comparison with n, with reference to (14), for one of the various vibrations of which the resultant vibration is composed, there will be, according to the final expression of I, 2), a phase displacement between the record and the acceleration for this vibration component, the result of which will be that the recorded curve will no longer present an accurate acceleration curve.

Strictly speaking, all the preceding equations and rules are valid only under the assumption that the vibration process to be measured has already gone on for a certain time. At the beginning of the process, that is to say, natural vibrations of the mass of the instrument still occur, which must first be suppressed by damping before only the forced vibrations here alone considered will remain. This is the one of the two reasons why all modern instruments for measuring vibrations have strong damping, which is the more necessary, the more irregular the process to be measured, like the vibrations of a ship in a seaway in the present instance. The second reason is that in respect of (17) greater distortions in the resonating state are to be avoided, and that therefore the value of $\varkappa$ may not differ very widely from unity. However, as is evident from the relationship between $\varkappa$ and γ (Eq. 11), this calls for strong damping.

Notation.

s	path of vibrations to be measured.
α	ordinate of the record of the instruments.
$\bar{s}$, $\bar{\alpha}$	corresponding amplitudes.
n, n_1, n_2,	frequencies per second, preferably of the exciter vibration.
n_0	natural frequency of the instrument.
$\omega = 2\pi n, \ldots\ldots \omega_0 = 2\pi n_0$	corresponding circular frequencies.
$T = \frac{1}{n} = \frac{2\pi}{\omega}, \ldots\ldots T_0 = \frac{1}{n_0} = \frac{2\pi}{\omega_0}$	" vibration periods.
δ, ε	phase angles
m	mass
c	elastic constant
$2W$	damping terms of movement equation
$2w = \frac{2W}{m}$	relative damping factors.
$\varkappa = \frac{2w}{\omega_0}$	damping coefficient

For the rest, the notations used in the report agree essentially with the recommendations of the STG - Committee for Stability and Vibration Research, see periodical Schiffbau 1934, pp 7 and 8.

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