NAVY DEPARTMENT
DAVID TAYLOR MODEL BASIN
WASHINGTON, D.C.

RESISTANCE-STRAIN CHARACTERISTICS OF STRETCHED FINE WIRES

by

W.J. Sette, L.D. Anderson, and J.G. McGinley

[redacted]

September 1945

Report R-212

PERSONNEL

This report is the work of W.J. Sette, L.D. Anderson, and J.G. McGinley. B. Germann, H. Shoub, and Miss K. Tilghman assisted with the test computations. Valuable suggestions were made by Dr. M. Greenfield, Dr. V. McBrien, Ens. E. Wenk, Jr., USNR, and especially by Dr. B.L. Miller. The report was checked by W.F. Curtis and H. Shoub.

TABLE OF CONTENTS

RESISTANCE-STRAIN CHARACTERISTICS OF STRETCHED FINE WIRES

ABSTRACT

The change in electrical resistance with elongation was measured in fine wires of several alloys and one pure metal. In the plastic region the percentage change in resistance was found to approximate twice the axial strain. The test methods are described and some of the results are presented and discussed.

INTRODUCTION

Wire-resistance strain gages, making use of the change in resistance of a fine wire when stretched or compressed, are now being extensively employed for static and dynamic strain measurements. At the David Taylor Model Basin they have been used for the usual strain determinations and for the determination of strains generated by impact, for the measurement of strains far beyond the elastic range and into the plastic region, and for the measurement of small strains of a few parts per million in ship propeller shafts. Several reports (1)*(2) describing the development and application of these gages have been published but, as far as is known, no systematic and extended investigation of their properties has been undertaken. Because of their wide use under a variety of conditions and because of questions arising concerning their performance, the Taylor Model Basin has conducted a number of basic tests, some of which are reported here.** A digest of some of the results was published in another Taylor Model Basin report in June 1944 (4).

In essence, a wire strain gage consists of a grid of fine wire cemented to a strip of thin paper which is in turn cemented to a structural member. When the structure under the gage is deformed, a strain equal to the strain of the surface to which it is attached occurs in the wire and the electrical resistance of the wire changes in consequence. The strain in the surface of the structure is computed from the measured change in resistance on the basis of a calibration figure previously obtained for the gage.

The material of the wire plays a basic part in the action and a knowledge of its resistance-strain characteristics is fundamental. This report deals with the resistance-strain relationships for wires stretched freely, that is, without the support of cement and paper.

---

* Numbers in parentheses indicate references on page 26 of this report.

** The influence of drying time of the mounting cement on the performance of Baldwin-Southwark SR-4 wire gages is reported in Reference (3).

THEORY

For a homogeneous wire the resistance is given by the expression

$$R = \frac{\rho L}{A} \qquad [1]$$

where R is the resistance,

ρ is the specific resistance,

L is the length, and

A is the cross-sectional area.

When a free wire is stretched, the cross section is contracted by an amount proportional to its elongation, represented by Poisson's ratio for the material. The longitudinal extension and the accompanying transverse contraction both act to increase the resistance of the wire. Specific resistance has also been found by several observers to change with applied strain (5) (6) (7). Published data on these phenomena are surprisingly scant.*

The change of resistance of a wire with strain may be expressed as a differential ratio called the resistance-strain coefficient or the sensitivity factor. This ratio, denoted here by F, is defined as

$$F = \frac{\frac{dR}{R}}{\frac{dL}{L}} \qquad [2]$$

For commercial wire gages the sensitivity factor ** is usually stated by the supplier. Strains are computed by dividing this factor into the proportional change of resistance.

Differentiation of Equation [1] with all terms considered variable, and division of the result by R, gives

$$\frac{dR}{R} = \frac{dL}{L} - \frac{dA}{A} + \frac{d\rho}{\rho}$$

It follows that

$$F = \frac{\frac{dR}{R}}{\frac{dL}{L}} = 1 + 2\nu + \frac{\frac{d\rho}{\rho}}{\frac{dL}{L}} \qquad [3]$$

where ν, representing Poisson's ratio referred to the instantaneous length rather than to the initial length, has been substituted for $-\frac{1}{2}\frac{dA}{A}\Big/\frac{dL}{L}$.

---

* Since the text of this report was written, considerably more data have been published in a Naval Research Laboratory report (8). Reference (8) and an unpublished memorandum by Lt. Comdr. R.W. Goranson, USNR, Bureau of Ships, Navy Department, describe results in the plastic range similar to those presented herein.

** See the Appendix.

Equation [3] implies that the change of resistance of a wire with strain is proportional to the changes in length and cross section and also to whatever change in specific resistance may occur. Since Poisson's ratio approximates 0.35 in the elastic range for many materials, the first two terms add up to about 1.7. It has been found that the third term is positive for most materials, negative for a few, and generally less than 10 in absolute magnitude.

In the purely plastic region strains occur at constant volume; the changes which occur in the wire are principally changes in form. The value of ν accordingly becomes 0.5. What the specific resistance does is a question best answered by experimental evidence. If elastic and plastic strains produce similar effects, then in the plastic region the third term of Equation [3] should remain constant and the resistance-strain coefficient should increase by 0.3 since 2ν increases from 0.70 to 1.0. If the material merely flows without additional internal stress, the specific resistance may remain constant, and its differential change will then be zero, making the third term zero. In this event the sensitivity factor in the plastic range should be 2 regardless of the material and of the value in the elastic range. If the material is originally soft, and if flow results in work-hardening with an accompanying change in specific resistance, the factor will differ from 2. The data to be reported show that in the plastic region the resistance-strain coefficient approximates 2 or a slightly higher value.

TEST METHODS

The original test method adopted at the Taylor Model Basin for determining the resistance-strain coefficients of stretched wires underwent major changes in the manner of mounting and loading the specimens as unexpected difficulties arose. Consequently, the tests are divided into two groups, depending on the type of mounting; they will be described separately.

TESTS WITH WIRE MOUNTED VERTICALLY

In the first tests the wire specimen, about 5 feet in length, was mounted vertically and soldered at the upper end to a rigid support overhead, as shown in Figure 1. The lower end was soldered to a hook from which hung a light bucket. A small piece of paper was cemented to the hook and marked with a fine horizontal line to serve as an index or target. Load was applied gradually by placing known increments of sand in the bucket by the use of a small glass tube with markings for 1/2-gram increments. Changes in length were measured by following the index with a telescope of a Gaertner Model M 903 cathetometer. Simultaneously, corresponding resistance changes were measured by a Wheatstone bridge circuit of which the specimen formed one arm.

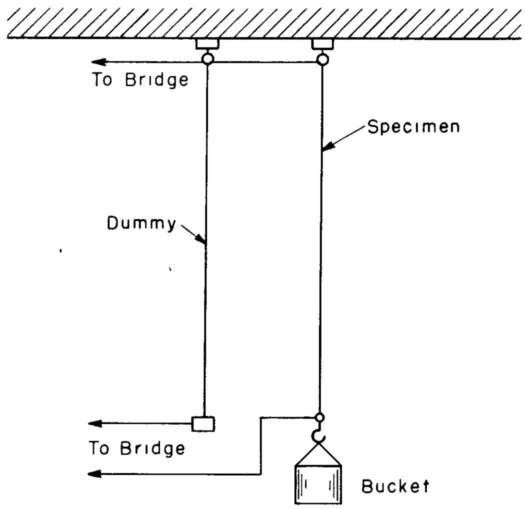


Figure 1 - Vertical Loading Setup

Most of these first tests were made with Advance wire 0.001 inch in diameter, which is frequently used in strain gages.

The suspended specimen was unavoidably subjected to a slight longitudinal restraint by a short length of Number 38 annealed copper wire, having a diameter of 0.0039 inch, leading from the lower terminal to the bridge.

After some preliminary experiments, a dummy or unloaded wire was incorporated into the resistance bridge to compensate for temperature effects. Of the same material as the specimen and approximately equal to it in length, the dummy was suspended a few inches to one side and was tautened to reduce random movement by attaching a small weight to its lower end. A wooden screen reaching from the floor to the ceiling of the laboratory protected the specimen and the dummy from air currents. A small opening in the front of the screen provided access for loading and exposed the target to view for taking measurements.

Loads were applied gradually to avoid impact disturbance as much as possible. Successive load increments of 1.0 or 1.5 gram were applied until yielding was indicated, whereupon the increments were reduced to 1/2 gram. Bridge readings were taken after each increment had been added, and several readings were taken at intervals between successive loadings in the region of yield.

The results of the first few tests were inconsistent and not reproducible, and efforts to improve the technique met with slight success. Parallax in the cathetometer telescope caused a high least count in the strain readings; the least count was about 0.0008 inch in a wire length of about 55 inches. In spite of great care in handling specimens, kinks and bends could not be entirely avoided. Building vibration occasionally seemed to affect the results. One of the most undesirable effects was the impact and disturbance encountered incident to application of the loads to a freely suspended wire of such a small diameter. Disturbance of the specimen was intensified when repeated runs were attempted to check the consistency of results. It was necessary to remove all the sand from the bucket at one time and start over from the beginning because individual increments of load could not be removed from the container.

The early runs with 1-mil* Advance wire indicated the probability of a small change in the resistance-strain coefficient in the region of plastic yield. This indication was not in accord with previously published information. To verify its existence more accurate data were necessary. It was decided therefore that a major change in testing technique was desirable. A new method was adopted in which load was applied to a horizontally mounted wire by a micrometer screw, as described in the following section.

TESTS WITH WIRE MOUNTED HORIZONTALLY

With the horizontal or micrometer-screw method of testing, known extensions could be produced in the wire with a high degree of accuracy by a micrometer slide of 2-inch range. However, the magnitude of the corresponding loads was not readily determinable so that when this method was adopted, the secondary objective of collecting stress-strain data was abandoned in favor of a more accurate determination of the resistance-strain characteristics of the wire. A knowledge of the resistance-strain relationship is essential since the use of the wire strain gage depends on the accurate conversion of small measured changes in resistance to the corresponding changes in strain.

The setup for horizontal testing was mounted on a bench top of sufficient length to accommodate wires of about 55 inches span. At one end of the bench top, a bakelite block, rigidly clamped in a small vise, provided a common support for the specimen and for the dummy, which were suspended horizontally side by side. One end of each wire was hard-soldered to a terminal clip on the bakelite block. The other end of the specimen was soldered to a fitting on the micrometer slide while the corresponding end of the dummy was similarly attached to an insulated clip on a steel bracket secured to the bench top beside the micrometer slide. Sufficient tension was applied to the dummy wire to make it taut, in the belief that compensation for the effects of temperature changes would be improved. This tension was not adjusted during the test. When desired, a traveling microscope was placed at midspan on the bench top to measure the sag in the specimen.

During the course of the tests it became necessary to move the setup to new quarters. Advantage was taken of the move to improve the equipment further. A photograph of the improved setup is shown in Figure 2. In place of the bench top, a mahogany platform 1.7 inch thick was used. Mahogany was selected for its stability under atmospheric changes. The platform was set on thick felt pads on a bench. The vise was replaced by a wooden block screwed to the platform. A strip of bakelite, and terminals for the specimen and

---

* Wire having a diameter of 0.001 inch is called 1-mil wire.

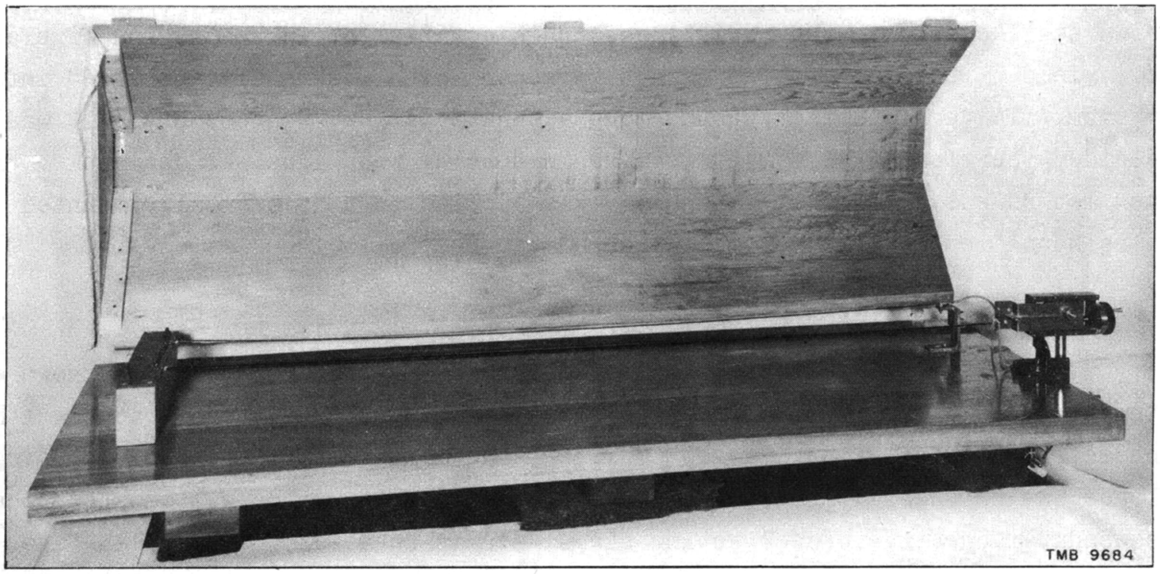


Figure 2 - Photograph of Micrometer-Screw Loading Arrangement

The hinged hood which shields the wires from air currents is swung back in this view. The dummy wire stretches from the wood block on the left to the steel bracket on the right. The specimen wire is attached to a fitting on the micrometer slide, which is moved to the right or left by appropriate rotation of the crank of the micrometer screw. The mahogany slab on which the setup is mounted is placed on felt pads to minimize the effects of building vibration.

dummy, were mounted on this block. This altered arrangement was used for the tests on Nichrome and Iso-Elastic wire.

In the tests which utilized the micrometer screw other precautions were taken to minimize errors. A wooden hood was placed over the stretched wires to protect them from random air currents. Rubber pads were placed under the bench to isolate it from vibration of the building. It was found that more consistent results were obtained when the current flowing to the bridge arms was reduced. The horizontal disposition of the setup facilitated examination of the specimen under a microscope; by this means it was found that the Advance wire being tested was in poor condition owing to mishandling of the spool at some previous time. Accordingly a new spool was procured. It is believed to be important in tests of this kind to have as nearly perfect a sample as possible to insure even stress distribution throughout the length of the wire. Kinks in the wire cause early failure; they were avoided by unwinding only a sufficient length of wire to permit soldering it to a terminal, and then carefully unwinding the rest of the desired length. Gradual bends in the wire due to tight coiling on the spool could not be avoided.

The use of the micrometer-screw loading method improved the accuracy since it permitted measurements of length to closer than plus or minus 0.0001 inch, as compared to plus or minus 0.0008 inch with the vertical method. It was also possible to repeat runs readily.

Originally a Carey-Foster bridge was used for the resistance measurements, with a Leeds and Northrup Kohlrausch-type slide wire. Later a shunt-type Wheatstone-bridge method was adopted, as shown in Figure 3a. This proved more satisfactory because it increased the range obtainable.

For most of the tests reported herein a table-type galvanometer with telescope-and-scale combination was used as a null indicator to obtain bridge balance. The galvanometer was a Leeds and Northrup Type 45, List Number 2284-b. The tests on Nichrome and Iso-Elastic wire employed an a-c bridge with a cathode-ray oscilloscope for the null indicator. This arrangement permitted more rapid balance, which is desirable in the plastic region of a material, and it reduced troublesome thermoelectric effects.

The a-c bridge was supplied with a signal of 200 cycles per second from an external oscillator through a variable-ratio, line-to-line transformer with an electrostatic shield. A line-to-line grid transformer coupled the bridge to a preamplifier. Hum picked up by the exposed specimen and the dummy was filtered out by a shunt resonant circuit tuned to 200 cycles per second, which was placed between the preamplifier output and the cathode-ray oscilloscope. Variable air-capacitors placed as shown in Figure 3b served to balance residual capacitances to ground.

TEST PROCEDURE, HORIZONTAL LOADING

When a specimen had been installed and checked under the microscope, the test runs began. With the wire in a slack condition the bridge was balanced by adjusting the shunt decade resistance box S shown in Figure 3. The micrometer was advanced by successive 10-mil increments and the bridge was rebalanced after each increment to determine the accompanying change in resistance. This process was continued until the wire had acquired a given elongation. The tension was then relaxed and the resistance was again measured with the wire in its slack condition. A second run terminating at some selected greater extension was then made and the tension was relaxed again. This cycle was repeated until the specimen failed. During the later runs on a specimen the micrometer increments were usually increased to 20 or more mils.

Nominally, a suspended specimen assumes the form of a catenary, as in Figure 4. Consequently the increases in the micrometer setting do not produce corresponding extensions of the wire until the wire is fairly taut. It was originally planned to measure the sag of the specimen and from it to compute the true values of the extension and also the true unstretched length.

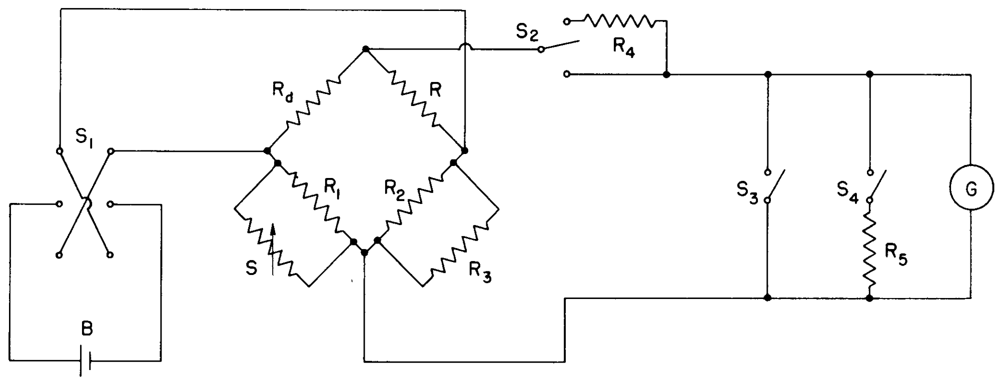


Figure 3a - Direct-Current Shunt Bridge

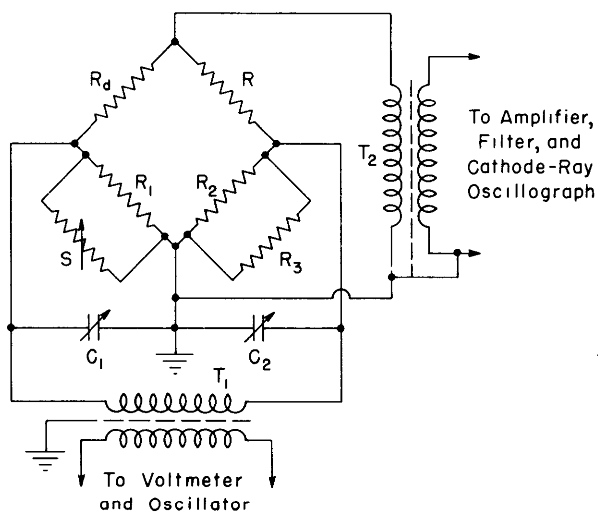


Figure 3b - Alternating-Current Bridge

Figure 3 - Diagrams of Test Bridges

Parts List

R - Specimen

R_d - Dummy

R_1, R_2 - Precision wire-wound resistors, 100 or 1000 ohms each.

S - Precision decade resistance box

R_3 - Precision wire-wound resistor or decade resistance box. Value selected to obtain initial balance at convenient setting of S.

R_4 - Carbon resistor, 10,000 ohms

R_5 - Wire-wound resistor, 10 ohms

S_1 - Double-pole, double-throw switch

S_2, S_3, S_4 - Single pole, single-throw switches

G - Table-type galvanometer with telescope and scale

B - 6-volt battery

T_1 - Line-to-line transformer

T_2 - Line-to-grid transformer

C_1, C_2 - Variable air capacitors, 0 to 250 $\mu\mu f$

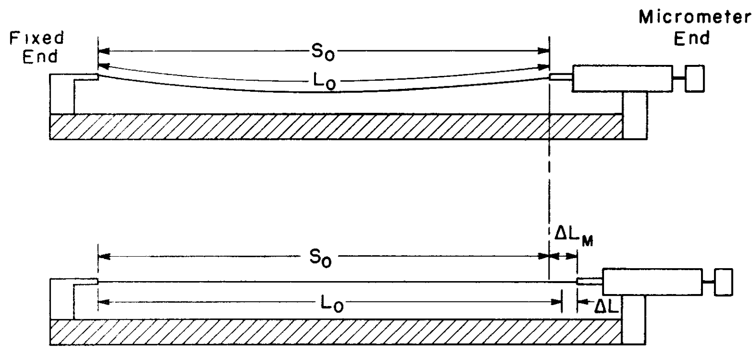


Figure 4 - Illustration of Method of Determining the True Length and Extension of a Suspended Wire

Here S_0 is the initial span,
L_0 is the unstretched or free length of the wire,
ΔL_M is the micrometer extension,
$L_0 + \Delta L$ is the length of the stretched wire and is measured with a steel tape, and
ΔL is the wire extension.
The extension ΔL of the wire is calculated from

$$\Delta L = \frac{1}{F} \frac{\Delta R}{R_0 + \Delta R} (L_0 + \Delta L)$$

where R_0 is the initial resistance of the wire and ΔR is the change in resistance when the wire is stretched.

For extensions beyond those to be used in this computation, the sag has slight effect, and further elongations of the wire equal the micrometer increments well within the experimental error.

However, the sag data were found to be inconsistent, presumably owing to variable compliance introduced by bends in the wire. The sag measurements were therefore discontinued. Instead the length was measured with a steel tape at a value of extension at which the observed value of the resistance-strain coefficient had settled down. The change in resistance relative to the slack condition was also measured. By use of the resistance-strain sensitivity factor found for the material in the elastic range, the change in length of the wire at the selected extension was calculated from the change in resistance. By subtracting the calculated change in length from the steel-tape reading the true initial length was obtained. Figure 4 may assist the reader in visualizing the procedure. The correction to the length was found to be small. However the procedure was necessary for converting the micrometer readings into true total extensions and strains.

The first trials of the micrometer method gave consistent results which could be checked at will. The existence of a reduction in the resistance-strain coefficient in the plastic range as found for 1-mil Advance wire with the vertical setup was repeatedly verified. To determine whether or not this reduction of the coefficient in the plastic range was peculiar to Advance wire, it was decided to test two other materials not commonly employed in wire-resistance strain gages. Manganin and platinum were selected, the former because it has been reported to have a low coefficient, about 0.47, and the latter because of the reported high value of 5.1 for iridium-platinum (2). The only available manganin wire was insulated with bakelite but the insulation was considered not to be a serious disturbing element.* The platinum wire was furnished by E.F. Mueller of the National Bureau of Standards. As has been mentioned previously, two other materials used in strain gages, Iso-Elastic and Nichrome, were also tested.

At times in the course of a run with the specimen under considerable applied tension, the bridge balance drifted with time. As will be explained later, this was taken as a sign of yield. The drift made it necessary to take the bridge readings at regular intervals, varying from a few seconds to several minutes, for each extension. When no regular drift was observed, four readings were taken for each extension. The scatter in the readings varied with the thermoelectric properties of the material and was usually less with moderate tension on the specimens.

TEST RESULTS

The results of the tests described in the preceding sections are presented in the form of curves plotted from data obtained by measurement and by computation.

The measured quantities were the setting of the shunt resistance box at balance, the setting of the micrometer, and the length of the wire at some taut condition. The resistance-box readings were converted by appropriate computations to values of $\Delta R/R$, that is, the change in wire resistance caused by an extension ΔL, divided by the average resistance R of the wire in the interval. This ratio, divided by the strain, yields for practical purposes the instantaneous sensitivity factor defined on page 2. In Figures 5, 6, 7, 8, 9, 10, and 11 the resistance-strain coefficients derived from the data for the test specimens are plotted against micrometer setting and conventional strain.

Also computed from the test data for specimens of Advance, platinum, and Iso-Elastic wire was the quantity $(R_X - R_0)/R_0$, where R_0 represents the initial resistance of the wire and R_X represents the resistance after a given extension. This quantity is plotted against strain in Figures 11, 12, and 13. The slope of these curves is the sensitivity factor as normally defined.

Details on the data will be found in the legends below the figures. Items common to many of the figures require some explanation.

---

* Bare manganin wire 0.001 inch in diameter has since been tested with results similar to those found for the manganin specimen discussed in this report.

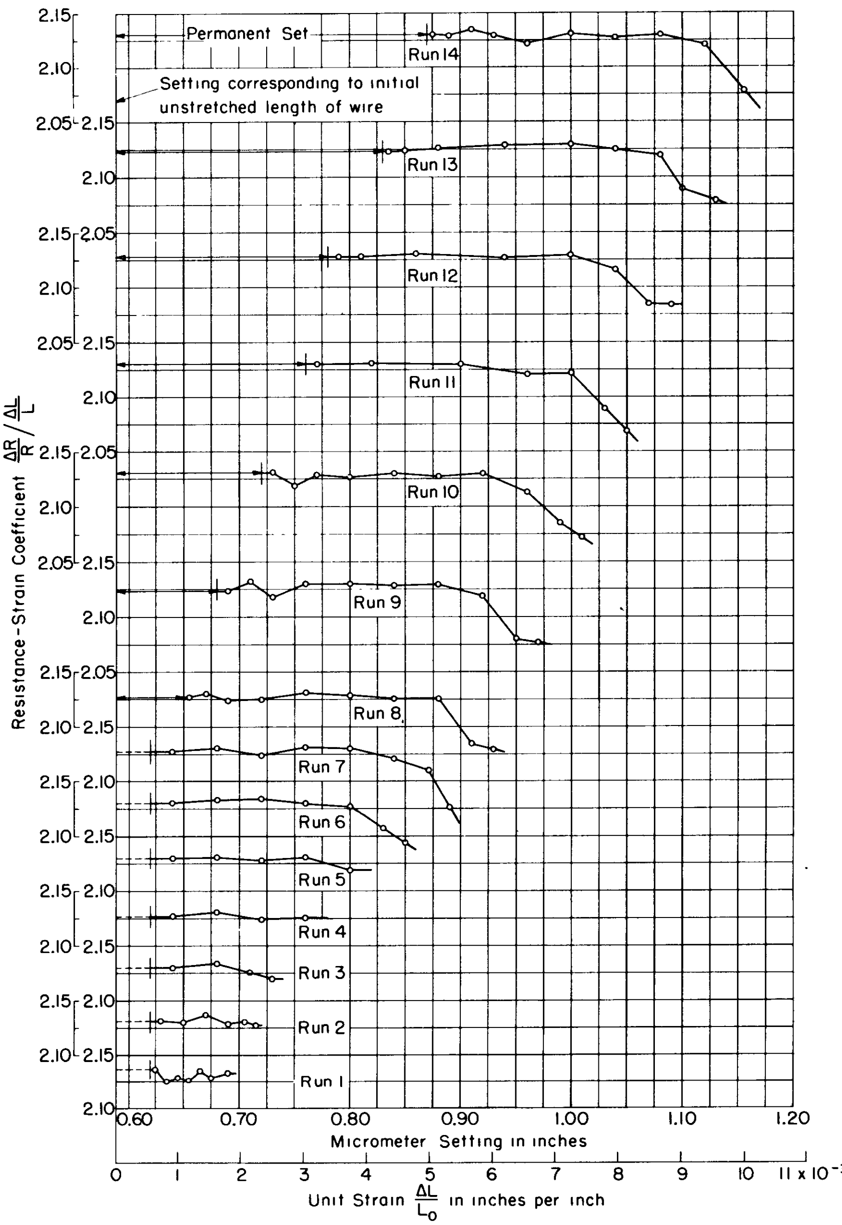


Figure 5 - Resistance-Strain Coefficient Plotted on Micrometer Setting and Strain for 14 Consecutive Runs on 1 Specimen of Advance Wire 0.001 Inch in Diameter

Because of the expanded vertical scale used, initial low values of coefficient caused by the gradual application of load could not be plotted. For each run the first point plotted includes the interval between it and the short vertical line preceding it. The initial horizontal line in each run, beginning with Run 8, indicates accumulated permanent set. When these tests were made, interest was centered on checking the consistency of the results. The averages for the different runs agree within plus or minus 0.4 per cent in the elastic range.

Note the drop at the end of the runs beginning with the curve for Run 6. It is probable that permanent set occurred during this run, but not enough data were recorded to permit calculation.

The bridge balance did not drift with the specimen under tension nor was any recovery effect observed. However, it will be noted that no one run was carried into the region of pure plastic flow and that the relatively sluggish d-c bridge was used.

See also Figures 6 and 11.

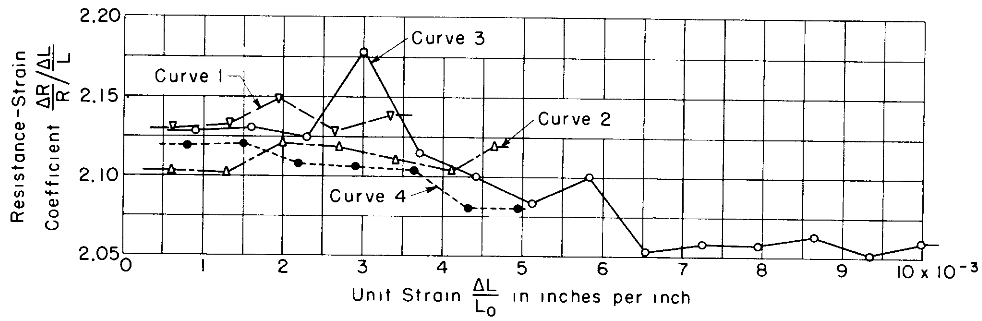


Figure 6 - Resistance-Strain Coefficient Plotted on Strain for 4 Specimens of Advance Wire 0.001 Inch in Diameter

These runs were made to check the consistency of the coefficient for different specimens. For each specimen the curve is a composite of a number of consecutive runs, including only the readings obtained the first time a given micrometer setting was reached. Each specimen was stretched to failure. Curve 3 is for the specimen of Figure 5.

Note the expanded vertical scale. The values for the different specimens in the elastic range generally agree within plus or minus 1 per cent. The average value for elastic strains is 2.12. The high point at a strain of 0.003 in Curve 3 involves readings made on consecutive days, as do several other points on this and the other curves shown. The high value was caused by a change occuring between runs, as is evidenced by the relative smoothness of the associated curves of Figure 5.

Curve 3 extends some distance into the purely plastic range, where the coefficient averages 2.06.

In all cases the runs are numbered consecutively and proceed from bottom to top of the plots. Since the purpose of the early runs was to obtain preliminary information and to work out any waves and kinks in the wire, if possible, the specimen was subjected to only small elongations in these runs. However, the condition of the wire did not improve greatly even after moderate permanent extension.

The indicated permanent sets between runs were obtained from the wire lengths before and after a run. The computation of the "free" lengths, as explained on page 9, involves the assumption that the change in resistance under elastic extension is a linear function of the strain. This assumption and the method of calculation seem to be justified by the data except in the case of manganin, which had little or no elastic range.

It had been thought that permanent set could be computed from the resistance of a specimen before and after a run. If the changes in resistance were due to purely geometrical changes, the sensitivity factor for permanent strain should be 2. However, the tests showed that the factors in the plastic range were not exactly 2 and therefore this method of computing set was rejected except for checking. Instead, the resistance-strain coefficient for permanent set, discussed on page 23, was computed from the resistance changes and from the permanent-set values derived from the free lengths.

(Text continued on page 20)

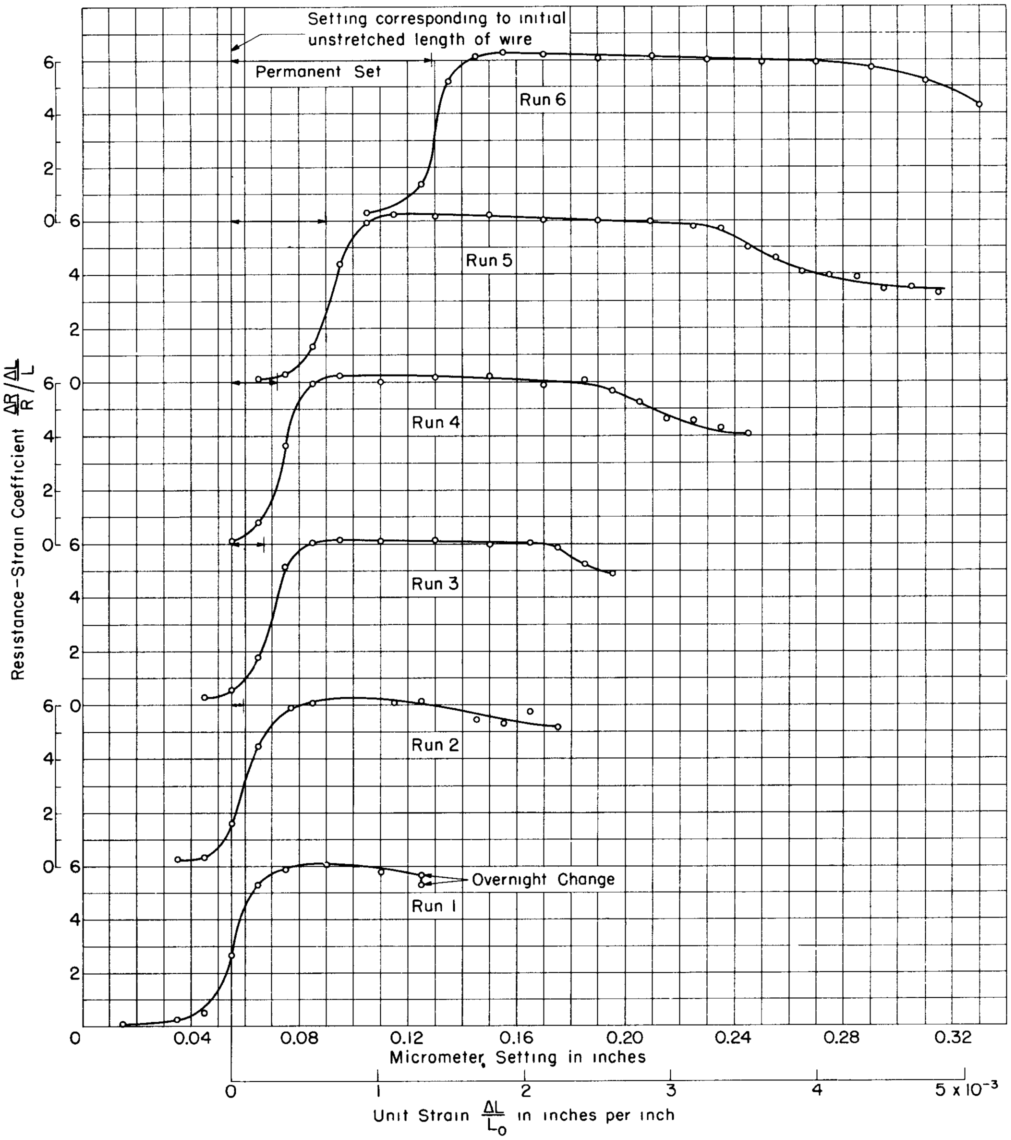


Figure 7 - Resistance-Strain Coefficient Plotted on Micrometer Setting and Strain for 6 Consecutive Runs on 1 Specimen of Platinum Wire 0.002 Inch in Diameter.

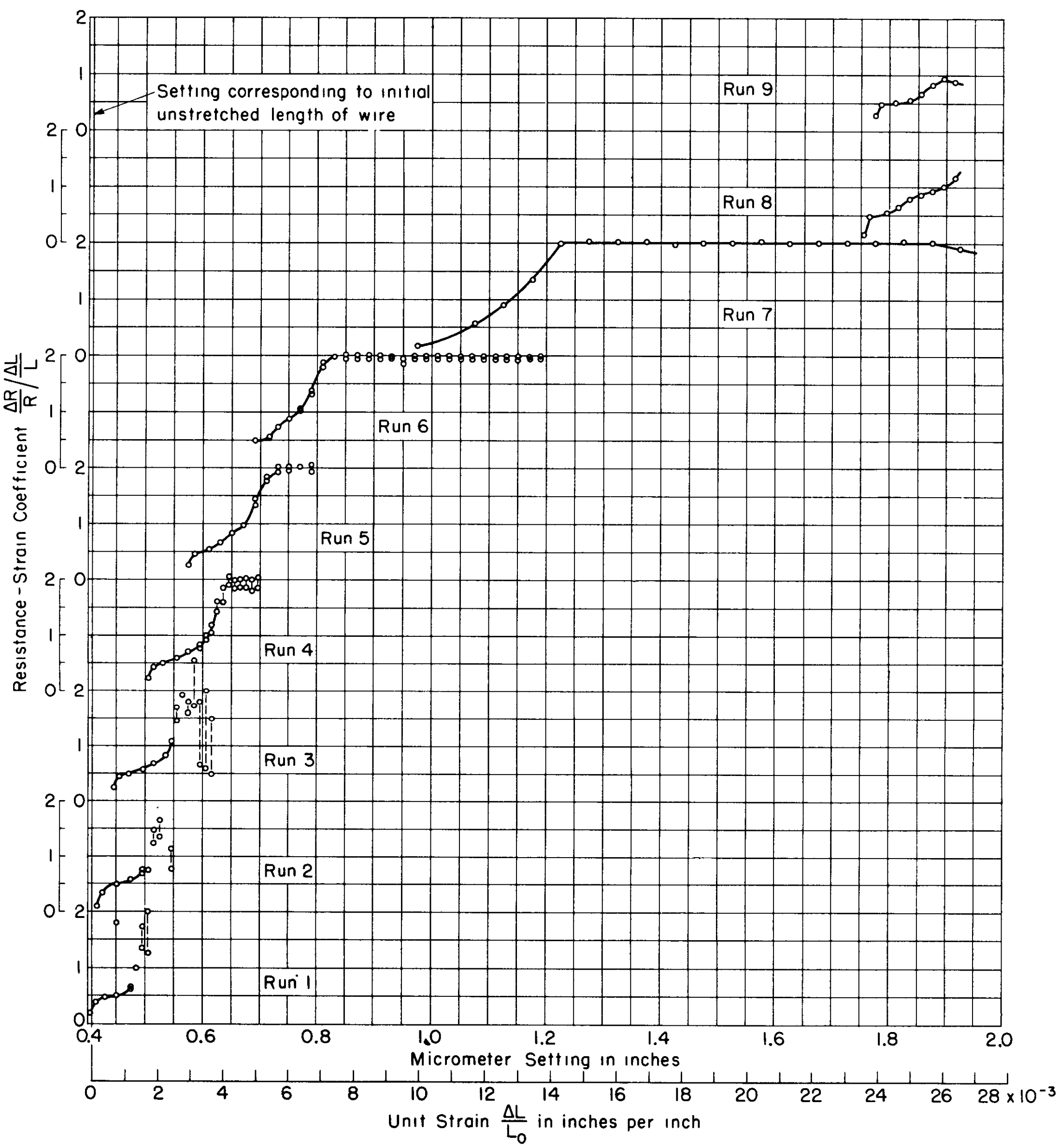
The initial low values of coefficient are caused by the catenary effect. Until the wire is taut, the strain and resistance do not change proportionally with micrometer increments. The region of constant factor increases after each run, indicating that work-hardening may have occurred. For Runs 5 and 6 there is a slight downward trend in what is considered to be the elastic range; see the Appendix. There appears also to be a slight progressive increase in the factor between runs. The average value in the elastic range is 6.10 with extreme values of about 6.25 and 5.90.

Platinum has a large temperature coefficient of resistance and there was considerable random scatter in the data.

At large strains, bridge drift occurred. The micrometer setting was increased by 0.010 inch every 5 minutes.

The minimum value of the coefficient, 3.2, occurred at the end of Run 5. The factor for permanent set from Figure 12 is 2.01; see page 23.

Two other specimens of this material gave similar values for the coefficient in the elastic range. They failed at relatively low elongations because of kinks, although one specimen had begun to show a reduction in the resistance-strain factor.



Setting corresponding to initial unstretched length of wire
Run 9
Run 8
Run 7
Run 6
Run 5
Run 4
Run 3
Run 2
Run 1
Resistance - Strain Coefficient $\frac{\Delta R}{R} / \frac{\Delta L}{L}$
Micrometer Setting in inches
Unit Strain $\frac{\Delta L}{L_0}$ in inches per inch
28 x 10⁻³

Figure 8 - Resistance-Strain Coefficient Plotted on Micrometer Setting and Strain for 9 Successive Tests on 1 Specimen of Manganin Wire 0.0063 Inch in Diameter, Bakelite-Insulated

In this case the gradual rise in the coefficient for the initial extensions caused by the catenary effect is shown on the curves by the first one or two points only. The later gradual rise is ascribed to increasing proportion of permanent set. The coefficient for the elastic range is about 0.50. However, there is practically no early range of linearity and therefore the purely elastic range is almost negligible. This material is different from other materials tested in that the coefficient for the elastic range is less than 2.

With the specimen under tension the resistance values were found to change markedly with time. The change was ascribed to conversion of elastic strain to permanent set, the phenomenon commonly known as creep. The magnitude of the effect depended on the amount of strain and the time interval between successive extensions. At each of the larger extensions two or more resistance observations were made and thus two or more values of the coefficient were obtained. In computing the coefficient for a given extension and time the resistance was compared with the last resistance reading made at the previous extension. During the early runs time was recorded but a definite schedule for the observations was not maintained, with the result that many irregularities are present in the curves. Beginning with Run 4 readings and extensions were usually made at fixed time intervals and more uniform data resulted. In some cases the curves have not been faired in where two values of the coefficients are plotted at one extension.

The lower groups of points in Curves 4, 5, and 6 represent values observed 3 minutes after the strains were applied. The upper points are for 5-minute intervals. For the 3-minute points the resistance is referred to the resistance value recorded at the previous extension after a 5-minute wait. If the 3-minute readings had been compared with 3-minute readings at previous extensions the values of the coefficient would have agreed more nearly with the values for the 5-minute intervals.

The horizontal portion of Curve 7 presents values for the coefficient obtained for equal micrometer increments in the plastic region. These points were derived from data recorded 4 minutes after the extension was applied. The micrometer was taken up to the limit of its travel, 1.93 inch, without failure of the specimen. This was the longest run on any specimen. It will be seen that in the purely plastic range the coefficient is 2.00.

Run 8 shows a more gradual rise than its predecessors, perhaps because of work-hardening during Run 7.

After Run 8 the specimen was relaxed and extended 60 times to the maximum micrometer setting, 1.93 inch, in an attempt to harden the wire. Run 9 followed. Note from comparison with previous runs that the initial linear portion is somewhat greater, indicating that hardening may have occurred.

Subsequent to the foregoing tests, a sample of bare manganin wire 0.001 inch in diameter was tested with results quite similar to those for the large insulated specimen discussed here, showing that the insulation and the relatively large size of the subject specimen did not exert a controlling influence.

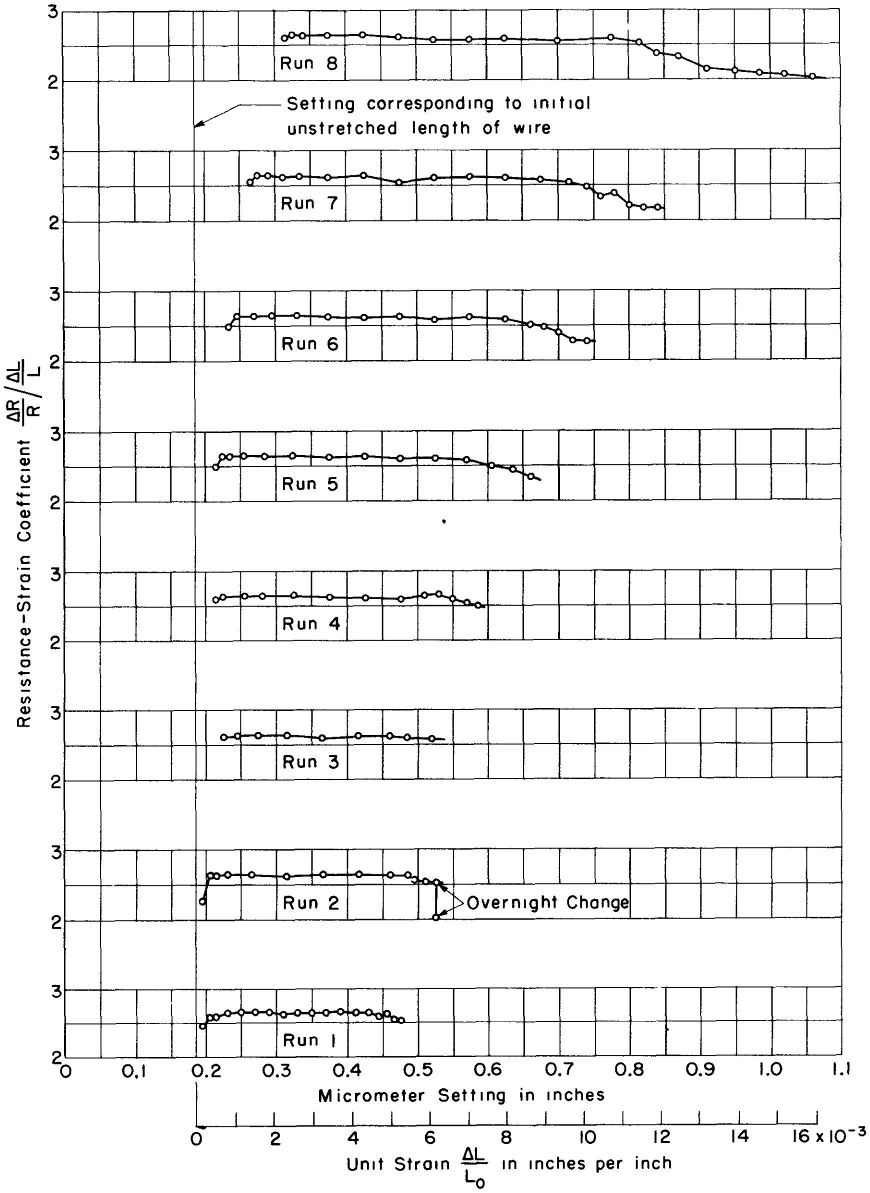


Figure 9 - Resistance-Strain Coefficient Plotted on Micrometer Setting and Strain for 8 Consecutive Runs on 1 Specimen of Nichrome Wire 0.001 Inch in Diameter

This specimen had a relatively large elastic range, especially during the later runs after the material had undergone some work-hardening. The coefficients in the elastic range average 2.63, with a spread of 0.01 between runs. In the region of yield the lowest observed value of coefficient was 2.02, which occurred just prior to failure at the end of Run 8.

At large extensions the resistance values changed appreciably with time. Accordingly, successive extensions and readings were made at 2-minute intervals. The data were taken at micrometer increments of 10 mils. For simplicity in plotting, the data have been grouped into larger intervals. A result of the grouping is that scatter of the coefficients, caused perhaps by random errors, is reduced.

A slight recovery was noted after most of the runs; i.e., after the tension was relaxed, the resistance of the specimen decreased gradually by a slight amount.

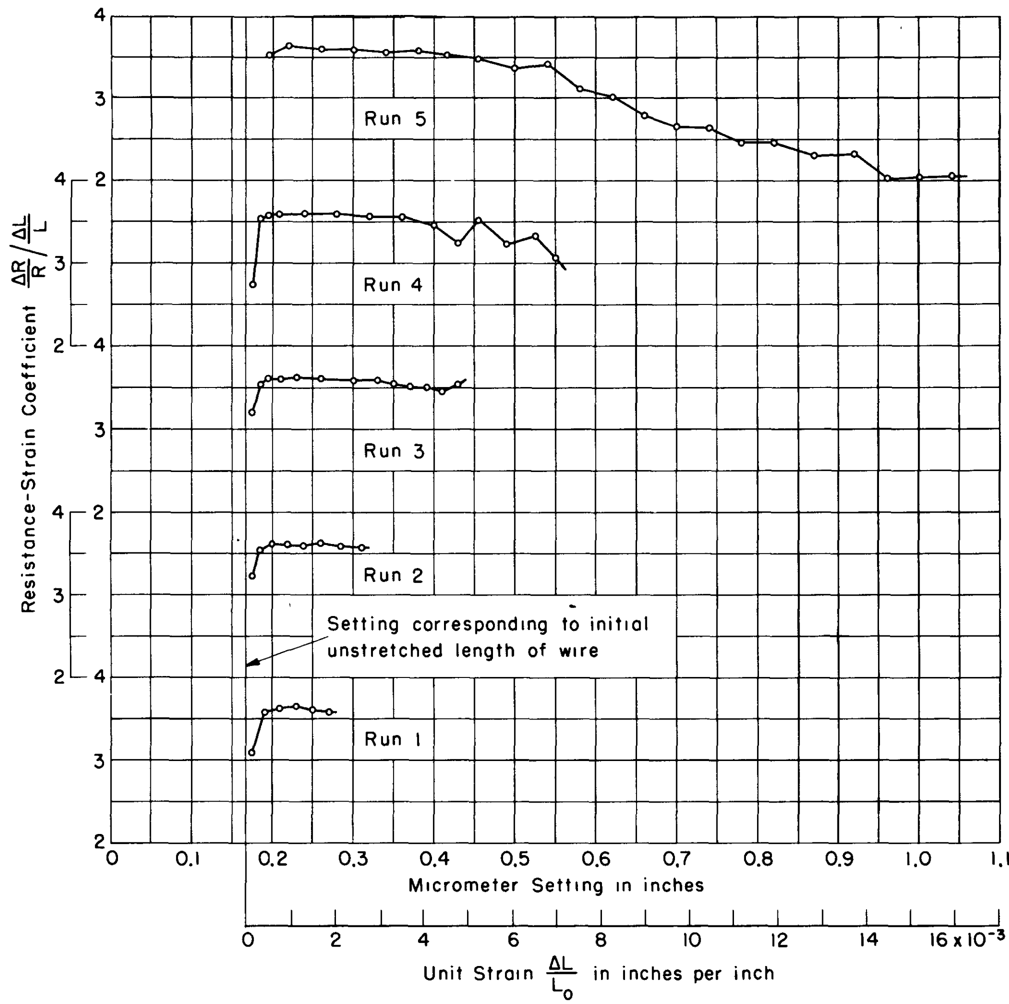


Figure 10 - Resistance-Strain Coefficients Plotted on Micrometer Setting and Strain for 5 Consecutive Runs on 1 Specimen of Iso-Elastic Wire 0.003 Inch in Diameter

The gradual rise in the coefficients caused by catenary effect and the permanent set prior to a run are shown at the beginning of each run. In the elastic range the coefficients average 3.60; the variations between runs were of the order of 0.01. In the purely plastic region just before failure at the end of Run 5, the coefficient averages 2.05.

At large extensions successive micrometer increments and resistance readings were made at 1-minute intervals. In Run 5 the micrometer was advanced by 10-mil increments up to a micrometer setting of 0.900 inch, after which the increments were increased to 40 mils and applied at 1-minute intervals. The fact that the coefficient remains relatively high at a micrometer setting of 0.920 and drops suddenly at higher micrometer settings may be related to the change in the strain-time conditions.

Little permanent set occurred before Run 5 so that the data on permanent set are not reliable. In Run 5 this specimen showed the longest transition region between the purely elastic and purely plastic ranges of any material tested, possibly because most of the work-hardening occurred in one run, rather than during several runs. The inference is that the material is capable of considerable work-hardening.

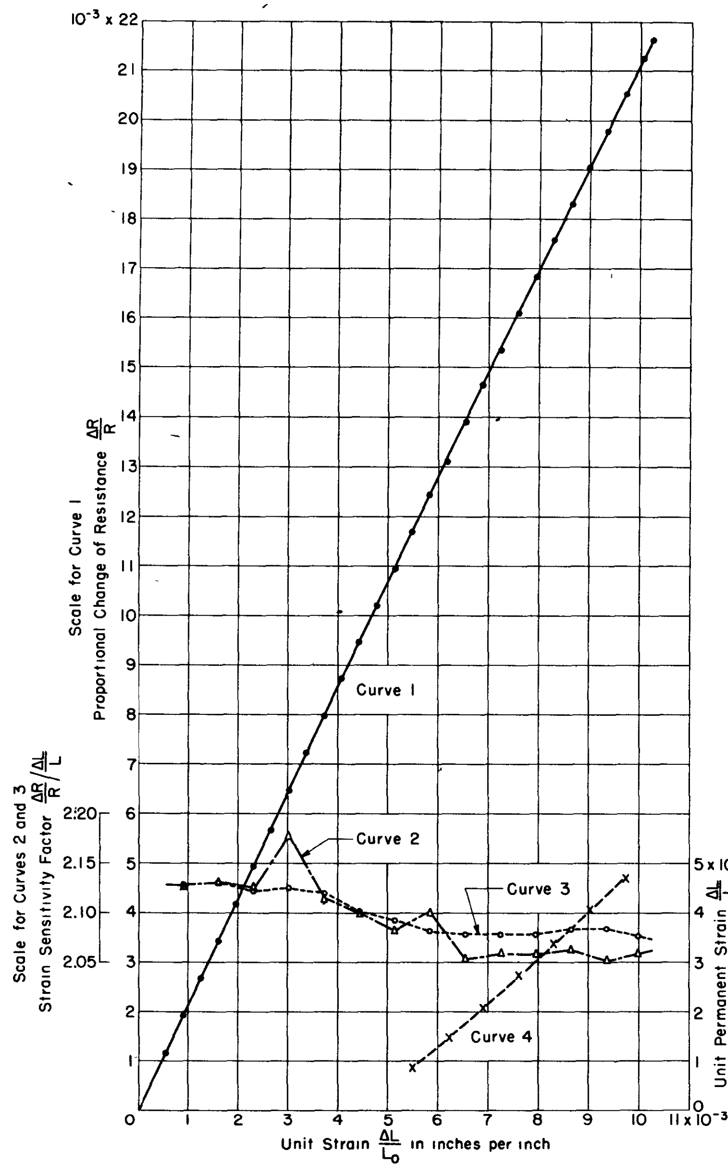


Figure 11 - Composite Curves for the Advance Wire Specimen of Figure 5

The 14 runs of Figure 3 are combined in these curves.

Curve 1 is the cumulative proportional change in resistance corresponding to the indicated strains. The plot is somewhat analogous to the envelope of a family of stress-strain curves where increasing load has been applied and relaxed repeatedly. The points correspond to bridge readings obtained when each micrometer setting was first reached during the 14 runs. Obscured by the scale are irregularities which are reflected in magnified form by Curve 2.

The slope of Curve 1 is by definition the resistance-strain coefficient or sensitivity factor. The slope calculated from the readings obtained the first time each micrometer setting was reached is shown in Curve 2. There is one particularly large departure from the general trend caused by an unexplained over-night change in bridge balance. Note the decrease in factor at large extensions where permanent set occurs. The average value in this range is 2.06. The factor estimated for permanent set is 2.06, see page 23.

Curve 3 is a composite of Run 1 and the new data of the succeeding runs in Figure 5. It thus involves the change in resistance between the second time a micrometer setting was reached and the first time the succeeding setting was reached. The factors at high strains are accordingly greater than for Curve 2, indicating that additional permanent set occurred when a setting was attained a second time, causing less set in the succeeding interval.

Curve 4 shows the amount of permanent strain introduced as the total strain was increased. The slope of the curve at the largest strains approximates unity, indicating that the strain increments are purely plastic.

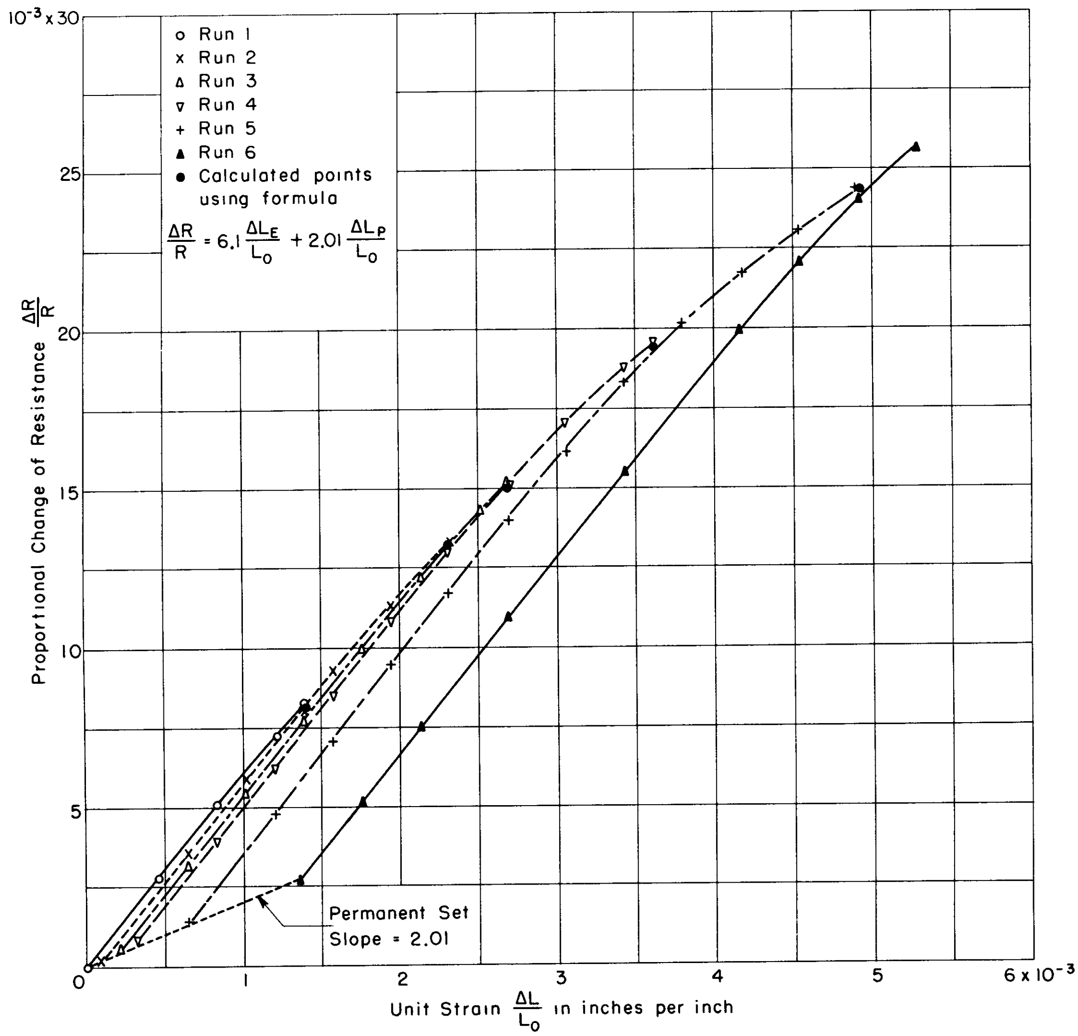


Figure 12 - Proportional Change in Resistance Plotted on Strain for 6 Consecutive Runs on Platinum Wire 0.002 Inch in Diameter

These curves are for the specimen of Figure 7. The change in resistance and the strain referred to the initial resistance and length of the specimen are plotted. The slope of these curves corresponds to the resistance-strain coefficients of Figure 7.

The curves are analogous to a family of stress-strain curves taken on a single specimen with the load repeatedly relaxed and reapplied. The failure of the curves to coincide is due to permanent set with the accompanying permanent increase in resistance of the specimen. The change in resistance and the total permanent strain prior to each run constitute the first points in the successive curves. If a line is drawn through these bottom points, the slope will be the coefficient for permanent set, 2.01 in this case.

Note that the points on the curves lie below the last points of preceding curves, indicating that further permanent strain is developed when a strain is attained a second time.

There are also plotted, as solid circles, 5 points computed from the equation

$$\frac{\Delta R}{R_0} = 6.1\,\frac{\Delta L_e}{L_0} + 2.01\,\frac{\Delta L_p}{L_0}$$

This is an extension to the complete strain range of Equation [4] discussed on page 23 of this report. The computation can be carried out only for the last point of each run since a knowledge of the proportion of elastic and plastic strains is necessary. It will be seen that the agreement between calculated and observed values is good. It follows that, given the total strain and change in resistance, the elastic and plastic components of the strain can be estimated if the coefficients are known.

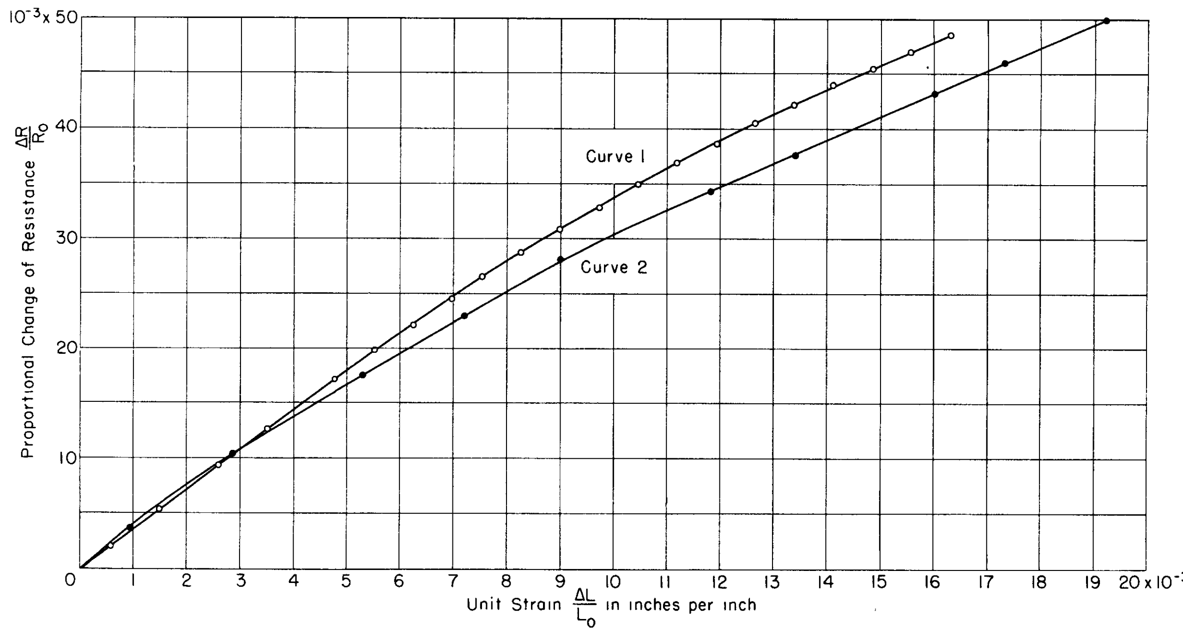


Figure 13 - Proportional Change in Resistance Plotted on Strain for Iso-Elastic Wire

Curve 1 is for the Iso-Elastic specimen of Figure 10. The values of proportional change in resistance plotted are those obtained when a strain value was attained the first time. The lack of linearity at large strain is quite noticeable.

Curve 2 shows the same relation for a strain gage using Iso-Elastic wire 0.001 inch in diameter. This curve has been replotted from Reference 9.

The two curves are similar in form although Curve 2 deviates from linearity more markedly.

It will be noted from some of the graphs, an especially good example of which will be found in Figure 7, that the computed coefficients start off with low values. This is a manifestation of the gradual application of tension due to the catenary effect in the suspended wires. The low values result from taking successive elongations as equal to the micrometer increments. This can be done without error except at the beginning of a run. When corrections computed from the sag measurements were applied to the initial sensitivity factors, better but not close agreement with the later values was obtained. Attempts to apply a correction based on a compliance factor allowing for effect of bends in a wire also failed. The sag measurements were therefore discontinued.

With all specimens except Advance wire it was observed that when the wire was in the plastic region, the bridge balance would drift fairly rapidly at first, and then more slowly. The drift was taken to indicate that plastic flow was occurring. The change in the bridge setting required for balance was always such as to make the sensitivity factor approach 2 for the interval. Thus the resistance of platinum decreased and that of manganin increased, even though the micrometer setting remained unaltered. The possibility of drift made desirable the introduction of an a-c bridge to expedite readings in the later tests.

Some of the specimens exhibited a recovery effect; after a wire had been stretched well into the plastic region and then returned to a slack condition, the bridge balance was found initially to drift slowly. The change was in the direction of a decreasing resistance. In all cases the change was small. Advance wire showed no such effect, and no data were recorded for platinum. For both of these materials the d-c galvanometer bridge was used.

DISCUSSION OF RESULTS

A study of the curves of Figures 5 to 11 suggests that as a rule the resistance-strain coefficient of a material is characterized by three regions: an initial range of strain in which the coefficient remains constant at some characteristic value, an intermediate or transition zone in which the coefficient has values between the initial value and 2, and a final region in which the coefficient approximates 2 as the strain is increased indefinitely. Curve 3 of Figure 6, Curve 8 of Figure 9, and Curve 5 of Figure 10, are examples. The platinum specimen failed before the third region was reached, but Curve 5 of Figure 7 shows the first two regions. For manganin, Figure 8, the first region was found to be very short, but the third region is the longest of any observed. The division adopted excludes, as extraneous, initial low values resulting from the catenary effect mentioned on page 20.

Evidence indicates that the three regions can be identified with the elastic range, with a region of yield, and with a region in which pure plastic flow occurs. In the first range the resistance of a specimen remains constant with time at a fixed strain. When the tension is relaxed and reapplied, the resistance returns to the value characteristic of the tension. For strain values in this range there is no evidence of permanent set.

In the intermediate range of strains, the resistance of a specimen changes with time in such a direction that the resistance-strain coefficient for the interval approaches 2. When the tension is relaxed it is found that permanent set has occurred. When a strain in this range is applied a second time it is found that the resistance of the specimen differs from the value existing when the strain was first applied. When the strain is relaxed a second time, additional permanent set is observed. In this intermediate zone and in the third region it is desirable to apply strain increments according to a fixed time schedule. When this is done, the coefficients in the second region alter progressively with strain from the elastic-range values to a value of approximately 2 at high strains.

In the region of pure plastic flow increments of strain produce nearly equal increments of permanent set; the resistance-strain coefficient remains constant at a value of approximately 2 if the measurements are made

TABLE 1

Comparison of Resistance-Strain Coefficients for the 5 Test Materials

Material	Diameter Inches	Resistance-Strain Coefficient in Elastic Range	Resistance-Strain Coefficient in Plastic Range	Resistance-Strain Coefficient for Permanent Strain
Advance	0.001	2.12	2.06	2.06
Platinum	0.002	6.1	*	2.01
Manganin	0.0063	0.5	2.00	2.02
Nichrome	0.001	2.63	2.04	2.06
Iso-Elastic	0.001	3.60	2.05	*

* Data are not available.

at equal intervals of time. The length of the time interval does not seem to matter, so long as, once chosen, it is maintained between observations.

In the elastic range the coefficients are found to vary from 0.5 for manganin to 6.1 for platinum. Repeated runs on a given specimen are found to give very closely equal values of the coefficient. The largest differences observed in the course of repeated runs occur in the case of platinum, Figure 7. This material shows an increase in coefficient from about 6.0 to 6.2 between the first and last runs. The difference is believed to be greater than the experimental error and may have been caused by strain-hardening of the wire.

The phenomenon of strain-hardening appears to enter into the reported results. One result of strain-hardening is the accompanying increase in elastic range and yield point. Such a result is evident in Figures 5, 7, 8, 9, and 10, where the initial range of constant values increases for the later runs. The case of platinum, Figure 7, is a good example. As explained under Figure 8, it was attempted to strain-harden manganin by repeatedly applying a large strain. As evident from Curve 9 of Figure 8, the attempt met with some success.

It was pointed out on page 3 that if a material undergoes changes in form only, in the purely plastic range, the resistance-strain coefficient should be exactly 2. Of the materials tested only manganin exhibited a coefficient of exactly 2, within the experimental error, at large strains. Advance, Nichrome, and Iso-Elastic wires had coefficients slightly greater than 2 in the plastic region. For the present the reasons for the deviations may only be conjectured. One possibility is that cold flow results in strain-hardening which in turn may increase the specific resistance of a material as it is strained. Data for copper, steel, and other metals indicate large increases in resistivity of hard-drawn wire over annealed wire. The amount of hardening and the associated increase in resistivity required to account for the deviation of the coefficient from 2 would be slight. Another possibility is necking of the specimens. Local reduction of cross section could increase the resistance beyond expected values. Whatever the cause, it should be observed that the deviations from 2 are slight and that resistance changes in the plastic region, for the materials tested, are largely ascribable to dimensional change alone.

As mentioned on page 12, resistance-strain coefficients may be computed for permanent strain. The data were not taken with this aim in mind and consequently are somewhat scant. The computation has been carried out for the available data with results as in Table 1. For comparison there are also listed in Table 1 average values of the coefficients for the elastic and purely plastic range. There is relatively good agreement between the permanent-strain and plastic-range coefficients. This result is to be expected. The changes in shape of the specimen are the same, and strain-hardening, if a factor, should be present in both cases. One possible source of difference is that a wire under tension and subjected to a further strain in what has been termed the purely plastic region may still be capable of supporting some of the increased strain elastically. Accordingly, the coefficient even in the far plastic region may have an elastic component.

Between the elastic and purely plastic ranges, as already mentioned, the resistance-strain coefficients assume values intermediate between 2 and the value in the elastic range. As presented in Figures 5 to 10, where relatively long intervals have been plotted, the points form comparatively smooth curves. The original data were taken at smaller intervals in most cases and showed more variation. It is believed that the deviations are largely attributable to difficulties in maintaining a rigid time schedule for the tests.

The fact that in the intermediate region the coefficients assume intermediate values suggests that in this region the change in resistance may be considered as having two components, one associated with the elastic component of strain and the other associated with the permanent strain. This breakdown may be expressed mathematically by the following relation:

$$\frac{\Delta R}{R} = \frac{F_e \, \Delta L_e + F_p \, \Delta L_p}{L' + \frac{\Delta L}{2}} \qquad [4]$$

where $\frac{\Delta R}{R}$ is the proportional change in resistance,

F_e is the resistance-strain coefficient in the elastic range,

F_p is the resistance-strain coefficient for permanent strain,

ΔL is the total extension beyond the elastic limit,

ΔL_p is the permanent elongation,

ΔL_e is the elastic component of ΔL and equals $\Delta L - \Delta L_p$, and

L' is the length of the specimen at the elastic limit.

To apply the formula to an actual case the division of the total strain into its two components must be known. For the data given in this report the division is known for points at the end of individual runs. The permanent strain may be determined after the tension is relaxed, and the elastic strain is the remainder of the total. Computations by the use of Equation [4] have been made for platinum and the results are plotted in Figure 12 on page 19. It will be seen that the agreement with the curves is satisfactory. Similar computations have been made for Nichrome with satisfactory results. The data for Advance, manganin, and Iso-Elastic wires are not suitable for checking this formula.

Generalization of the results presented for the plastic range for application to all metals seems not to be immediately warranted. Many more data are needed. However, it is reasonable to expect that many other metals and alloys will exhibit similar dependence of resistance on dimensional changes when subjected to plastic strains. It is hoped to return to this investigation when the opportunity arises. Equipment for the simultaneous study of stress, strain, and resistance has been built. It is planned to use this equipment to determine the correspondence between the stress-strain and resistance-strain characteristics of metals, especially in regard to yield and strain-hardening.

Of the materials tested, Advance wire exhibited the least change in coefficient at large strains, presumably because the initial value was not greatly different from 2. Because of the relative constancy of its coefficient, this material appears to be highly suitable for strain gages to measure plastic strains. Baldwin-Southwark Type A gages use Advance wire. Baldwin-Southwark Type C gages use Iso-Elastic wire which initially has greater sensitivity but which shows marked non-linearity as the plastic range is approached.

Prof. A.V. de Forest has noted the behavior of Iso-Elastic gages in an unpublished report (9). His data have been plotted in Figure 13 for comparison with Taylor Model Basin data. It will be noted that the Taylor Model Basin data refer to a specimen of Iso-Elastic wire free of the constraints confining the wire in a gage, whereas de Forest's data refer to an actual gage using Iso-Elastic wire. It will be seen from Figure 13 that the gage exhibits more non-linearity at high strains than does the free specimen. Whether the difference is due to variation between specimens of Iso-Elastic wire or is caused by the different mounting conditions is open to question. More work should be done with gages beyond the elastic region to determine to what extent the results for free wire hold for wire made into gages.

In Reference (9) data are given for Advance wire, in the form of a curve relating proportional change in resistance and conventional strain. The curve is slightly concave upward at large strains. It is not clear from Reference (9) whether the data are for a free wire or for a gage. The upward slope is to be expected if Equation [3] is valid, as is explained in the Appendix.

It should be observed that the fact that a wire yields does not preclude its use in a gage. The important elements are that the wire be constrained to follow the strains of the structure to which it is attached and that it undergo proportional changes in resistance. A very ductile material with practically no elastic range would presumably be satisfactory if its sensitivity factor remained constant throughout the strain range.

CONCLUSIONS

1. The resistance-strain coefficients of Advance, platinum, Nichrome, and Iso-Elastic wire are essentially constant in their elastic ranges. The specimen of manganin wire tested had practically no elastic range.

2. In the truly plastic range of these materials the resistance-strain coefficient is 2 or slightly greater.

3. It is inferred that, for many metals, the sensitivity factor in the plastic range is determined almost completely by dimensional changes, although other changes with strain may have a slight effect.

4. For measurement of strains which carry the gage wire into its plastic region, a wire with a sensitivity factor of about 2 is essential if a constant gage calibration factor is desired. However, as explained in the Appendix, it is the instantaneous sensitivity factor that remains constant, not the factor referred to the initial resistance and length.

5. Of the materials tested, Advance wire, with a sensitivity factor of 2.12 in the elastic range, appears to be the most suitable for measurements extending into the plastic region.

6. The data indicate a decrease of about 3 per cent in sensitivity factor for Advance wire in the plastic region. Tests with actual gages should be made before any such correction is applied to measurements with gages using Advance wire.

REFERENCES

(1) "The Development of Electrical Strain Gages," by A.V. de Forest and H. Leaderman, Technical Note 744, National Advisory Committee for Aeronautics, January 1940.

(2) "Characteristics and Aircraft Applications of Wire Resistance Strain Gages," by A.V. de Forest, Instruments, Vol. 15, Number 4, April 1942, pp. 112-114 and 136-137.

(3) "The Performance of Wire-Resistance Strain Gages as Influenced by the Drying Time of Mounting Cements" by B.L. Miller, Ph.D, L.D. Anderson, and H. Shoub, TMB Report R-213, to be published.

(4) Taylor Model Basin CONFIDENTIAL Report R-254, Part 17, by W.J. Sette, June 1944.

(5) "Effect of Tension on the Thermal and Electrical Conductivity of Metals," by P.W. Bridgman, Proceedings of the American Academy of Arts and Sciences, Vol. 59, Number 6, November 1923, pp. 117-137.

(6) "On the Increase in Resistance to the Passage of an Electric Current Produced on Certain Wires by Stretching," by Herbert Tomlinson, Proceedings of the Royal Society of London, Vol. 26, March 1, 1877 to December 20, 1877, pp. 401-410.

(7) "Untersuchungen über die Widerstandsänderung von Drähten durch Zug" (Investigation of the Change of Resistance in Wires Produced by Tension), by E. Czerlinsky, Jahrbuch der deutschen Luftfahrtforschung, 1938, pp. 377-380. TMB Translation 183, to be published. Translated by F.A. Raven, Ph.D.

(8) "Electrical Resistance Changes in Fine Wires Subject to Plastic and Elastic Elongation," by T.E. Pardue, E. Kammer, and I. Vigness, Naval Research Laboratory Report 0-2377, 28 September 1944.

(9) "Report on Impact Research," by A.V. de Forest, A.R. Anderson and R. Fanning, Massachusetts Institute of Technology, Research Laboratory of Dynamics of Materials, May 1941.

APPENDIX

MATHEMATICAL COMPARISON OF THE CONVENTIONAL SENSITIVITY FACTOR AND THE INSTANTANEOUS SENSITIVITY FACTOR

The sensitivity factor as defined in this report differs slightly from that customarily used. Normally the changes in resistance and in length are referred to the original resistance and length. If we denote by F_0 the factor so determined, then

$$F_0 = \frac{\frac{\Delta R}{R_0}}{\frac{\Delta L}{L_0}} \qquad [5]$$

where R_0 is the initial resistance and L_0 is the initial length. The factor used in this report, denoted by F, refers to the resistance and length in the stretched condition. Since R and L are both variable, the factor so defined has been called the *instantaneous* sensitivity factor.

Elastic strains are usually so small that there is no significant difference between the two factors. For large strains in the plastic region, the instantaneous factor remains constant at approximately 2, whereas F_0 varies with strain in accordance with Equation [9], derived below. As defined by Equation [2]:

$$F = \frac{\frac{dR}{R}}{\frac{dL}{L}} \qquad [6]$$

or

$$\frac{dR}{R} = F\frac{dL}{L} \qquad [7]$$

Taking F as constant, integration of Equation [7] leads to

$$R = CL^F \qquad [8]$$

where C is a constant of integration.

Assume the wire to be extended so that $R = R_0 + \Delta R$ and $L = L_0 + \Delta L$. Substituting in Equation [8] and applying the binomial expansion, it follows that

$$F_0 = \frac{\frac{\Delta R}{R_0}}{\frac{\Delta L}{L_0}} = F\left(1 + \frac{F-1}{2}\,\frac{\Delta L}{L_0}\right) \qquad [9]$$

It is apparent that if F is constant, F_0 increases with strain, although the variation is small except for very large strains. On page 149 of Reference (9), $\Delta R/R_0$ for Advance wire is plotted against strain. An upward tilt to the curve is present at a strain of 0.04, in agreement with Equation [9].

For the purely elastic region it is probable that F_0 remains constant in analogy with Hooke's Law. If it does, it may be shown that the instantaneous factor varies with strain as follows:

$$F = F_0 \frac{1 + \frac{\Delta L}{L_0}}{1 + F_0 \frac{\Delta L}{L_0}} = F_0 \left[1 - (F_0 - 1) \frac{\Delta L}{L_0} \right] \qquad [10]$$

Equation [10] indicates that a decrease in F for platinum, amounting to 1.5 per cent, should be expected at a strain of 0.003. A decrease of about 4 per cent was observed.