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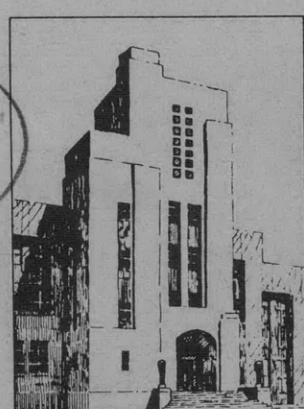
AUTOMATIC REGULATIONS OF DIVING AND RISING OF SUBMARINES

by

Adolf G. Strandhagen and Francis M. Kobayashi



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NOTATION

X, Y, Z	"Wind axes" of submarine
θ	Angle of pitch
α	Angle of attack
η	Path angle
G	Center of gravity
W	Weight of submarine
CB	Center of buoyancy
B	Buoyant force
L	Hydrodynamic lift force
D	Hydrodynamic drag force
M	Hydrodynamic moment
U	Velocity of center of gravity
r	Instantaneous radius of curvature of path of center of gravity
$\overline{m}_1$	Virtual mass referred to X-axis
$\overline{m}_2$	Virtual mass referred to Z-axis
J	Polar moment of inertia including effect of apparent mass about Y-axis
ω	Angular velocity of submarine
ξ	Distance from G to CB
δ	Instantaneous stern plane deflection
δ_0	Initial stern plane deflection
$\Delta\delta$	Difference between instantaneous and initial stern plane deflection
ρ	Mass density of water
A	Reference area for L, D, M
l	Reference length for M
C_L	Lift coefficient
C_D	Drag coefficient
C_M	Moment coefficient

b_x	Drag coefficient due to stern plane deflection
c_x	Drag coefficient due to angular velocity
b_z	Lift coefficient due to stern plane deflection
c_z	Lift coefficient due to angular velocity
b_θ	Moment coefficient due to stern plane deflection
c_θ	Moment coefficient due to angular velocity
$\bar{l}_1$	Perpendicular distance from G to line of action of resultant hydrodynamic force on stern plane
Δh	Deviation from ordered depth
h_0	Initial depth
h_1	Ordered depth
$\left.\begin{matrix} K_1 \\ K_2 \end{matrix}\right\}$	Parameters in control equation

INTRODUCTION

GENERAL REMARKS

The development of efficient detecting devices and various antisubmarine weapons during the last war necessitated effective countermeasures for the submarine to survive in its struggle with pursuing surface ships. The most effective countermeasures were obviously increased submerged speed and range, increased maneuverability, and greater silence in operation. It is now a well-known fact that by the end of the war the German Navy had found the means of increasing the submerged speed of submarines to about three times the then prevailing speed, and had at least one class of high-speed submarines under construction. This lead has been followed up since then and, with atomic power plants within the realm of probability, it is a fair surmise that the gap between the underwater speed of future submarines and the speed of pursuing surface vessels will be a comparatively narrow one.

This increase in submerged speed puts greater stress than has been necessary heretofore on the problems of efficient control of the submarine and of its rising and diving motion; in particular, it makes the use of automatic control devices highly desirable. Such devices are of course not new, and have been brought to a high state of perfection in the torpedo, the fire control of antiaircraft guns, etc. In order to make effective use of such devices, knowledge of the interaction of the various forces controlling the motion of a submarine is indispensable; it is believed that this knowledge can be gained in the most direct way by a theoretical analysis of the problem. Bearing these considerations in mind, the authors set themselves the task of investigating the motion of a submarine under various conditions of control. The solution of this task is given in the present paper in two parts.

In Part I, the equations of motion are set up, simplified, solved, and discussed. Two types of equations of control were incorporated into the analysis of motion and these are sufficient as a beginning to show the effect of nonlinear terms in the equation of motion. Six cases of rising to predetermined depths are investigated. In each case the important items, such as angle of trim, vertical travel versus horizontal travel, angular velocity, and angle of attack are shown in graphical form. From these six cases, trends in the maximum angle of trim, the time required to attain ordered depth, and maximum angular velocity are established. A designer has only to examine the trends presented by these graphs to find the necessary guides to design a suitable control system for this vessel.

In Part II, stability of motion is discussed and it is based on the premise that the equations of motion are nonlinear. Stability of motion in diving and rising implies that any disturbed motion of the submarine, such as one caused by a nearby explosion of a depth charge, can be kept within arbitrary controlled limits, provided that the initial disturbance is kept sufficiently small. It is to be noted that an interesting and effective theorem has been employed to determine whether a motion that is derivable from a system of nonlinear equations is stable; in fact, only within recent times has it been possible to carry out an analysis of this kind. The theorem was applied to the rising maneuver of a submarine in which a specific type of mathematical equation of control or its equivalent automatic device was employed.

LIMITATIONS OF THE THEORY

The development of the hydrodynamic forces and moment present several limitations, and it is pertinent to discuss these before proceeding with the formal development.

The first and most important of these limitations is imposed by the acceptance of the hypothesis that static tests are valid for the determination of hydrodynamic forces and moment. This limitation arises from the assumption that time-dependent variations in hydrodynamic forces and moment are independent of time, or of the previous history of motion.

The second limitation is imposed by the fact that in the absence of definite knowledge of the hydrodynamic phenomena, certain higher order terms in Taylor's expansion of forces and moment are omitted.

This paper proposes to discuss the present state of the theory and to advance the problem within the above limitations. No attempt was made to give an exhaustive treatment either of the analysis of stable motion, or of control devices, or of their equivalent mathematical equations of control; rather it is our hope that this preliminary study will be expanded by discussions and further investigations.

PART I

THE RISE AND DIVE OF A SUBMARINE UNDER CONTROLLED CONDITIONS

EQUATIONS OF MOTION

In the present analysis, the submarine is assumed to execute two-dimensional accelerated motion in a vertical plane. The equations of this motion are derived with reference to the so-called "wind axes," which have their origin at the center of gravity of the submarine. This choice of axes has the advantage of facilitating the practical application of the theory because the experimental constants necessary for a numerical solution may be obtained by relatively simple tests, as will be pointed out later. The rectangular coordinates form a right-handed system of axes, in which the X-axis is tangent to the path of the center of gravity with the positive direction forward; the Y-axis is transverse to the ship with the positive direction to starboard; and the Z-axis is along the normal to the path with the positive direction downward.

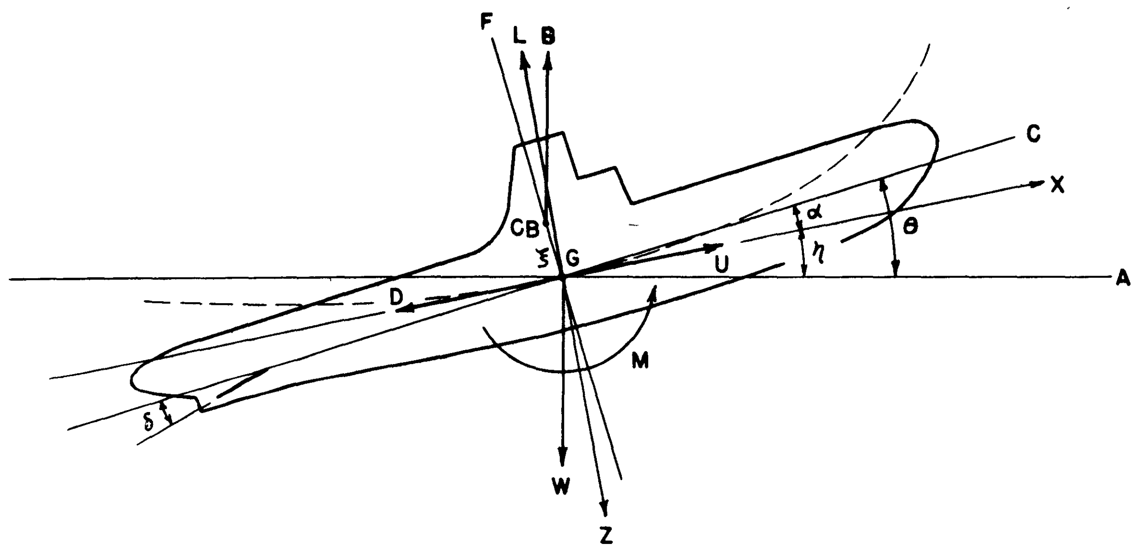

Figure 1 - Forces and Angles

The sign convention and nomenclature adopted for the forces, moments, and angles are shown in Figure 1. In this figure, the A-axis is a horizontal axis through the center of gravity of the ship; the C-axis is an axis of zero lift which passes through the center of gravity; and the X-axis is a tangent to the path of the center of gravity. The angle between the axes A and C is the angle of pitch, denoted by θ, and the angle between the A and X-axes is the path angle, denoted by η. These angles can vary independently and are considered positive when they lie above the A-axis. The angle $\alpha = \theta - \eta$ is the angle of attack; this angle is considered positive when $\theta > \eta$.

The weight of the ship, acting vertically downward at the center of gravity G, is denoted by W. The buoyant force, acting vertically upward through the center of buoyancy CB of the submerged ship, is denoted by B; for simplicity, it is assumed that the ship is in neutral equilibrium when upright, i.e., that CB lies vertically above G on the upright F-axis. Lift and drag forces are denoted by L and D respectively. These forces are the resultant of all the hydrodynamic forces exerted on the hull and diving planes, including the propeller thrust. By definition, the lift is normal to, and the drag is parallel to the X-axis; they are considered positive when directed along the negative directions of the X and Z axes as previously defined.

In general, the system of forces enumerated in the previous paragraph do not all pass through the center of gravity of the ship. The moments of these forces about the center of gravity can be represented by a single moment M, as shown in Figure 1. This moment is considered positive when directed as shown in the figure, in accordance with the usual convention for a right-handed coordinate system.

Since the motion has been assumed to be accelerated motion, Newton's second law applies, whereby the sum of the forces acting in a certain direction is equal to the mass times the acceleration in that direction, and the moment is equal to the product of angular acceleration and the moment of inertia. For the chosen system of coordinates, the only accelerations that need to be considered are the angular acceleration, the linear acceleration tangent to the path, and the normal or centripetal acceleration. Figure 2 shows the positive

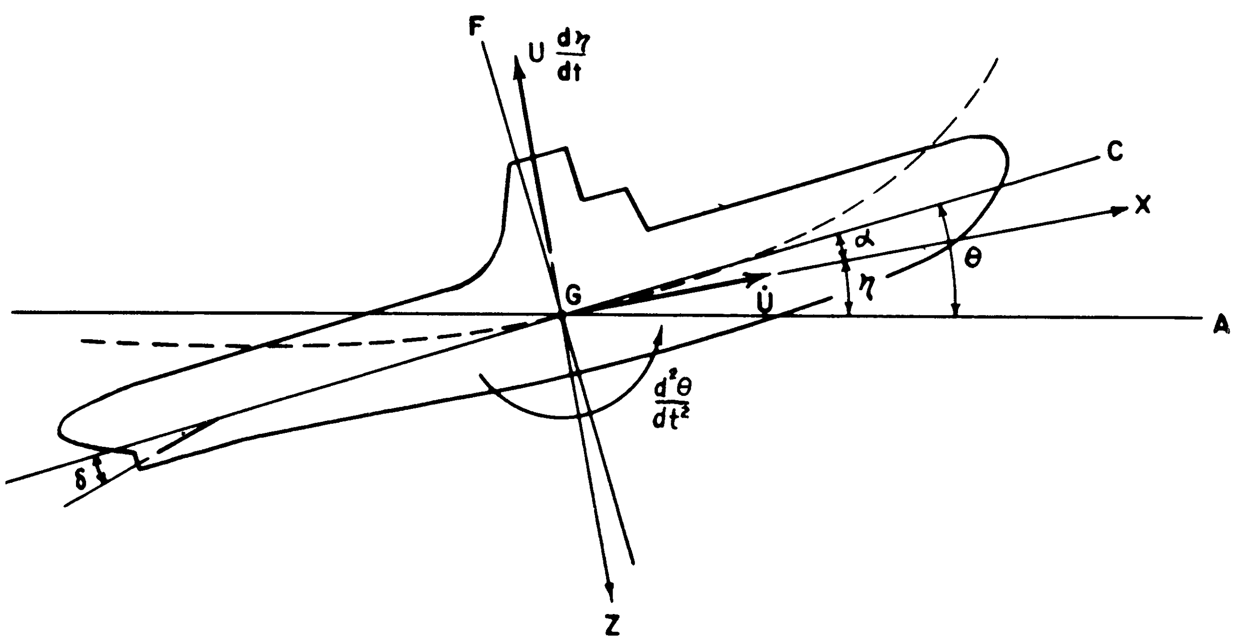

Figure 2 - Components of Acceleration

directions of these accelerations. The angular acceleration is given by $d^2\theta/dt^2$. The component of the linear acceleration along the X-axis is expressed by dU/dt, or simply $\dot{U}$. The normal or centripetal acceleration is equal to U^2/r, or $r(d\theta/dt)^2$, where r is the instantaneous radius of curvature of the path of the center of gravity. Referring to Figure 3, the velocity U may be expressed as $r(d\theta/dt)$. Hence U^2/r may be expressed in the more convenient form $U(d\eta/dt)$, or since $\eta = \theta - \alpha$, as $U(d\theta/dt - d\alpha/dt)$.

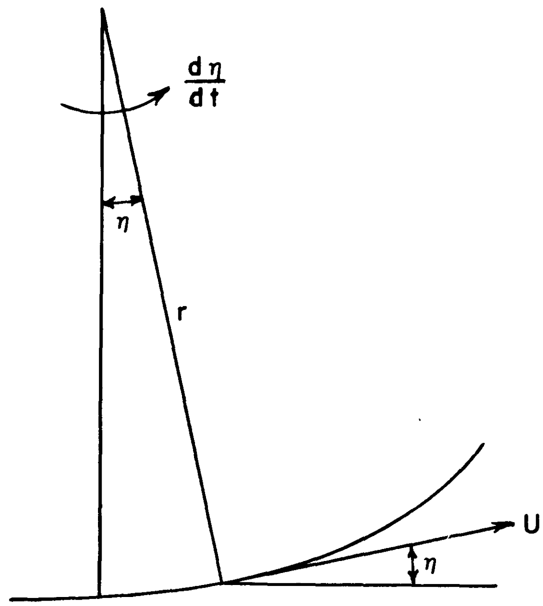

Figure 3 - Tangential Velocity $r(d\eta/dt)$

By combining the considerations and definitions already given, the equations of motion can now be written. Setting the forces in the X and Z directions equal to mass times acceleration in these directions, and the moment about the Y-axis equal to the product of the polar moment of inertia and angular acceleration, we have the following:

$$\bar{m}_1 \frac{dU}{dt} = -(W - B)\sin\eta - D(U, \alpha, \delta, \omega),$$

$$-U\left(\bar{m}_1 \frac{d\theta}{dt} - \bar{m}_2 \frac{d\alpha}{dt}\right) = (W - B)\cos\eta - L(U, \alpha, \delta, \omega), \qquad [1]$$

$$J\frac{d^2\theta}{dt^2} = -\xi \cdot B \cdot \sin\theta + M(U, \alpha, \delta, \omega),$$

where, in addition to the previously defined quantities,

- $\bar{m}_1$ is the sum of the actual and the apparent mass of the submarine along the X-axis,
- $\bar{m}_2$ is the sum of the actual and the apparent mass along the Z-axis,
- J is the polar moment of inertia about the Y-axis including the effect of apparent mass,
- ω is the angular velocity $d\theta/dt$ of the submarine in the X–Z plane about G,
- ξ is the distance from the center of gravity to the center of buoyancy, and
- δ denotes the angular deflection of the stern planes.

The buoyant force B is assumed to be constant or independent of hull-volume changes arising from hydrostatic effects of submerged depth. The drag $D(U, \alpha, \delta, \omega)$, the lift $L(U, \alpha, \delta, \omega)$, and the moment $M(U, \alpha, \delta, \omega)$ signify that these are functions of the variables in the parentheses.

DISCUSSION OF THE COEFFICIENTS OF LIFT, DRAG, AND MOMENT

The exact determination of D, L, and M as functions of U, α, δ, and ω is difficult. This is due to the nonuniform motion and the lack of symmetry of the body about the X-Y planes. Moreover, the effect of surface waves would be important if the submarine were close to the free water surface, but this effect need not be considered here since deep submergence is presupposed in the subsequent discussion.

In view of these difficulties, approximations and reasonable assumptions must be introduced in order to obtain a practical solution. Thus, it will be assumed that the hydrodynamic forces and moments vary as the square of the velocity U and the density ρ; introducing further a characteristic area A and length l, we can write

$$\begin{aligned} D(U, \alpha, \delta, \omega) &= \frac{\rho}{2} A U^2 C_D(\alpha, \delta, \omega), \\ L(U, \alpha, \delta, \omega) &= \frac{\rho}{2} A U^2 C_L(\alpha, \delta, \omega), \\ M(U, \alpha, \delta, \omega) &= \frac{\rho}{2} A l U^2 C_M(\alpha, \delta, \omega), \end{aligned} \qquad [2]$$

where C_D, C_L, and C_M are variable coefficients, usually denoted as the drag, lift, and moment coefficients.

In Equation [2] the functions D, L, and M must possess the following properties which are easily verified:

$$D(0, \alpha, \delta, \omega) = L(0, \alpha, \delta, \omega) = M(0, \alpha, \delta, \omega) = 0,$$

$$\frac{\partial D(0, \alpha, \delta, \omega)}{\partial U} = \frac{\partial L(0, \alpha, \delta, \omega)}{\partial U} = \frac{\partial M(0, \alpha, \delta, \omega)}{\partial U} = 0,$$

while the quantities

$$\frac{\partial^3 D(U, \alpha, \delta, \omega)}{\partial U^3}, \quad \frac{\partial^3 L(U, \alpha, \delta, \omega)}{\partial U^3}, \quad \frac{\partial^3 M(U, \alpha, \delta, \omega)}{\partial U^3}$$

are independent of velocity.

A rigorous solution of the problem would require that C_D, C_L, and C_M be known determinable functions of α, δ, and ω. In general this requirement cannot be met, so that a rigorous solution is not possible; however, an approximate solution can be found based on the mathematical fact that any analytic function in the neighborhood of a point can be expanded in a

Taylor' series. Thus if we expand the general function $C(\alpha, \delta, \omega)$ in the neighborhood of the point $(\alpha = 0, \delta = 0, \omega = 0)$ we get:

$$C(\alpha, \delta, \omega) = (a_0 + a_1\alpha + a_2\alpha^2 + \ldots) + (b_1\delta + b_2\delta^2 + \ldots)$$
$$+ (c_1\omega + c_2\omega^2 + \ldots) + d_1\alpha\delta + d_2\alpha\omega + d_3\delta\omega + \ldots,$$

where

$$a_0 = C(\alpha = 0, \delta = 0, \omega = 0), \quad a_1 = \frac{\partial C(\alpha = 0, \delta = 0, \omega = 0)}{\partial\alpha}, \quad a_2 = \frac{1}{2}\frac{\partial^2 C(\alpha = 0, \delta = 0, \omega = 0)}{\partial\alpha^2}$$

$$b_1 = \frac{\partial C(\alpha = 0, \delta = 0, \omega = 0)}{\partial\delta}, \quad b_2 = \frac{1}{2}\frac{\partial^2 C(\alpha = 0, \delta = 0, \omega = 0)}{\partial\delta^2}, \text{ etc.,}$$

$$c_1 = \frac{\partial C(\alpha = 0, \delta = 0, \omega = 0)}{\partial\omega}, \quad c_2 = \frac{1}{2}\frac{\partial^2 C(\alpha = 0, \delta = 0, \omega = 0)}{\partial\omega^2}, \text{ etc.,}$$

$$d_1 = \frac{\partial^2 C(\alpha = 0, \delta = 0, \omega = 0)}{\partial\alpha\partial\omega}, \quad d_2 = \frac{\partial^2 C(\alpha = 0, \delta = 0, \omega = 0)}{\partial\alpha\partial\omega}, \quad d_3 = \frac{\partial^2 C(\alpha = 0, \delta = 0, \omega = 0)}{\partial\omega\partial\delta}.$$

The assumption is now made that $C(\alpha, \delta, \omega)$ is linearly dependent on δ and ω. With this assumption, Taylor's expansion becomes simpler, since the quantities $b_2, b_3, \ldots, c_2, c_3, \ldots$ as well as $d_1, d_2, d_3, \ldots$ are all equal to zero. The resulting expression becomes

$$C(\alpha, \delta, \omega) = (a_0 + a_1\alpha + a_2\alpha^2 + \ldots) + b_1\delta + c_1\omega$$

However, the series $(a_0 + a_1\alpha + a_2\alpha^2 + \ldots)$ is by definition the function $C(\alpha, \delta = 0, \omega = 0)$. Thus the above expression becomes

$$C(\alpha, \delta, \omega) = C(\alpha, \delta = 0, \omega = 0) + b_1\delta + c_1\omega \qquad [3]$$

The function $C(\alpha, \delta = 0, \omega = 0)$ is determined experimentally by varying this angle α, but keeping the stern-plane angle at zero. In addition, since the angular velocity is equal to zero, the effects of velocity are precluded. Thus it follows that this experimental procedure is usually designated as a static test.

The quantity b_1, which is defined as the partial derivative $\partial C(\alpha = 0, \delta = 0, \omega = 0)/\partial\delta$, is likewise determined by a static-test procedure. The partial derivative is evaluated by determining the slope of the $C(\alpha = 0, \delta, \omega = 0)$ versus δ curve at $\delta = 0$. The model in this experimental test is held fixed $(\omega = 0, t > 0)$ at the angle α equal to zero, while the stern-plane angle δ is varied.

The quantity c_1, which is defined as $\partial C(\alpha = 0, \delta = 0, \omega = 0)/\partial\omega$, is also obtained by experimental methods. In this case, the slope of the $C(\alpha = 0, \delta = 0, \omega)$ versus ω curve at the point $\omega = 0$ is the required quantity.

It has been stated in the preceding discussion that C was any analytic function. If we now return to Equation [2] and stipulate that the drag, lift, and moment coefficients, C_D, C_L, and C_M are analytic functions of α, δ, and ω, the conclusions apply to these functions directly; thus, by analogy, we can write, in a notation that is indicative of the directions and rotation of the moving axes, the following equations:

$$
\begin{aligned}
C_D(\alpha, \delta, \omega) &= C_D(\alpha, \delta = 0, \omega = 0) + b_x\delta + c_x\omega, \\
C_L(\alpha, \delta, \omega) &= C_L(\alpha, \delta = 0, \omega = 0) + b_z\delta + c_z\omega, \\
C_M(\alpha, \delta, \omega) &= C_M(\alpha, \delta = 0, \omega = 0) + b_\theta\delta + c_\theta\omega.
\end{aligned}
\qquad [4]
$$

Equation [4] can be further simplified by the following considerations. First, it should be noted that in the first of the set of three equations the drag of the hull, represented by the first term on the right, is many times greater than either the drag of the stern planes, or the drag due to angular velocity denoted by the second and third terms respectively. In a first-approximation theory, the last two terms in the equation can therefore be dropped.

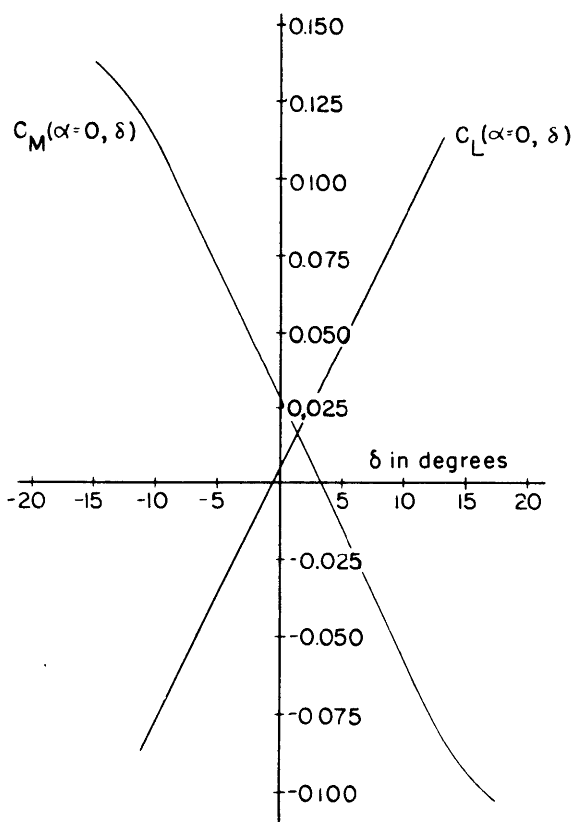

Figure 4 - C_L ($\alpha = 0, \delta$) and C_M ($\alpha = 0, \delta$) versus δ

The partial derivatives b_z and b_θ are obtained by evaluating the slopes of the curves C_L ($\alpha = 0, \delta, \omega = 0$) and C_M ($\alpha = 0, \delta, \omega = 0$) at $\delta = 0$. The range of the stern-plane angle δ, shown in Figure 4, is approximately between minus and plus 16 degrees.[1] To go beyond this range would cause unfavorably large angles of trim. To avoid such unfavorable angles of trim, the stern planes are compelled by a mechanical device to stay within these limits.

The partial derivative c_θ is evaluated from observed data which indicates a dependency upon the angle α. In general the pitch-damping moment may be expressed as follows:

$$\frac{\partial M(\alpha, \delta = 0)}{\partial \omega} = -\rho U A l^2 (e_0 + e_1\alpha + e_2\alpha^2 + \dots) \qquad [5]$$

[1]References are listed on page 44.

where A is a reference cross-sectional area;

ρ is the density of the fluid;

U is the velocity of the fluid with respect to the hull; and

α is the angle of attack.

The constants e_0, e_1, ... are coefficients of the polynomial which can be found by methods of curve-fitting. Expressing [5] in nondimensional form, one obtains

$$\frac{\partial C_M}{\partial \omega} = -\frac{2l}{U}(e_0 + e_1\alpha + \dots) \qquad [6]$$

Since c_θ is the value of the expression shown in Equation [6] when $\alpha = 0$, then

$$c_\theta = \frac{\partial C_M(\alpha = 0,\ \delta = 0)}{\partial \omega} = -\frac{2l}{U} e_0 \qquad [7]$$

At this time it is of interest to discuss the evaluation of one of the cross-derivatives. The cross-derivative $\partial^2 C_M(\alpha = 0,\ \delta = 0,\ \omega = 0)/\partial\alpha\partial\omega$ was omitted from the general expansior for $C_M(\alpha,\ \delta,\ \omega)$ because its contribution was in general negligible, as will now be shown. Differentiating Equation [6] with respect to α gives the following expression:

$$\frac{\partial^2 C_M(\alpha = 0,\ \delta = 0,\ \omega = 0)}{\partial\alpha\,\partial\omega} = -\frac{2l}{U} e_1 = d_\theta \qquad [8]$$

Recalling Taylor's expansion for the coefficient of moment,

$$C_M(\alpha,\ \delta,\ \omega) = C_M(\alpha,\ \delta = 0,\ \omega = 0) + b_\theta\delta + c_\theta\omega + d_\theta\alpha\omega + \dots, \qquad [9]$$

and substituting in this expression the values for d_θ and c_θ, we obtain

$$C_M(\alpha,\ \delta,\ \omega) = C_M(\alpha,\ \delta = 0,\ \omega = 0) + b_\theta\delta + \left(-\frac{2l}{U}\right)(e_0 + e_1\alpha)\,\omega + \dots \qquad [10]$$

Test data[2] indicates that e_1 is extremely small. Since the variation of α is not large, the product $e_1\alpha$ is small in comparison to e_0. Thus the term $e_1\alpha$ may be omitted in the analysis. However, the omission of this term may not be valid for other types of hulls and operating conditions. In some cases, therefore, the cross-derivative may be neglected; in cases where its magnitude is appreciable it must be retained.

The quantity c_z or $\partial C_L(\alpha = 0,\ \delta = 0,\ \omega = 0)/\partial\omega$ is found by assuming that the damping moment is caused by lift forces on the stern planes.[1]. Referring to Figure 5, we write

$$-M = \bar{l}_1 (L \cos\alpha), \qquad [11]$$

where $\bar{l}_1$ is the perpendicular distance from the center of gravity of the hull to the line of action of the resultant hydrodynamic force. Differentiating with respect to ω, Equation [11] becomes

$$\frac{\partial L}{\partial \omega} = -\frac{\partial M}{\partial \omega}\left(\frac{1}{\bar{l}_1 \cos \alpha}\right) \qquad [12]$$

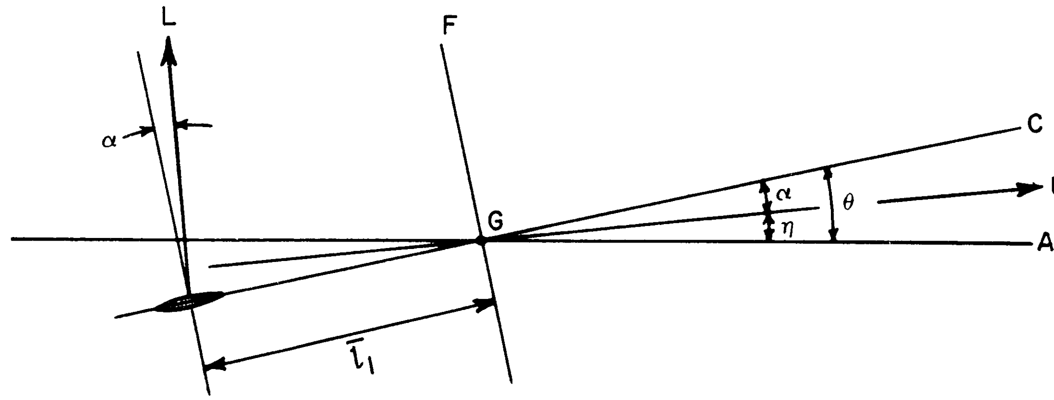

Figure 5 - Lift on Stern Planes

Equation [12] is now expressed in nondimensional form, and with the aid of Equation [6], the following expression is obtained:

$$\frac{\partial C_L}{\partial \omega} = \frac{2l}{U}\frac{(e_0 + e_1\alpha + \dots)}{\bar{l}_1 \cos \alpha} \qquad [13]$$

As mentioned earlier, e_1, e_2, ... are very small quantities, and they may be neglected in the computation. Consequently, Equation [13] becomes

$$\frac{\partial C_L}{\partial \omega} = \frac{2l}{U}\frac{e_0}{\bar{l}_1 \cos \alpha} = c_z \qquad [14]$$

We could now evaluate Expression [14] when $\alpha = 0$, and we could also compute the cross-derivative $\partial^2 C_L/\partial\alpha\partial\omega$; but both of these computations were not carried out, because the entire damping, resulting from the action of the lift forces as developed by the stern planes is only an approximation, and because further refinements do not appear justifiable.

The functions $C_D(\alpha, \delta = 0)$, $C_{L'}(\alpha, \delta = 0)$, $C_M(\alpha, \delta = 0)$, as mentioned earlier, are obtained from experimental static tests. The curves shown in Figure 6 indicate that the functions are nonlinear in character. These curves are expressed as polynomials in the dependent variable α; for example:

$$C_L(\alpha, \delta = 0) = l_2\alpha^2 + l_1\alpha + l_0,$$
$$C_D(\alpha, \delta = 0) = d_2\alpha^2 + d_1\alpha, \qquad [15]$$
$$C_M(\alpha, \delta = 0) = m_4\alpha^4 + m_3\alpha^3 + m_2\alpha^2 + m_1\alpha + m_0,$$

where the l's, d's, and m's are constants that are found by the usual procedure of curve fitting.

Since b_x and c_x are approximately zero, the hydrodynamic coefficients, as functions of α, δ, and ω, are taken to be as follows:

$$C_D(\alpha, \delta, \omega) \cong C_D(\alpha, \delta = 0),$$
$$C_L(\alpha, \delta, \omega) \cong C_L(\alpha, \delta = 0) + b_z\delta + c_z\omega, \qquad [16]$$
$$C_M(\alpha, \delta, \omega) \cong C_M(\alpha, \delta = 0) + b_\theta\delta + c_\theta\omega,$$

where $C_D(\alpha, \delta = 0)$, $C_L(\alpha, \delta = 0)$, and $C_M(\alpha, \delta = 0)$ are defined as polynomials in α, Equation [15]; where b_z and c_z are the slopes of the curves shown in Figure 6; and where b_θ and c_θ are defined by Equations [7] and [14] respectively. We conclude therefore that the $C_D(\alpha, \delta, \omega)$, $C_L(\alpha, \delta, \omega)$, and $C_M(\alpha, \delta, \omega)$ are nonlinear in α but linear in δ and ω; while $C_D(\alpha, \delta, \omega)$ was assumed to be independent of δ and ω. Although other nonlinear effects are present, they may be considered as negligible as far as practical numerical calculation are concerned. For an example, the term $e_1\alpha$ in Equation [10] was omitted, since it was of no practical numerical value.

Substituting the expression shown in Equation [16] in the equations of motion [1], and writing η as $\theta - \alpha$, we obtain the following set of differential equations of motion:

$$\bar{m}_1\dot{U} = -(W - B)\sin(\theta - \alpha) - \frac{\rho}{2}AU^2C_D(\alpha, \delta = 0),$$
$$U(\bar{m}_1\dot{\theta} - \bar{m}_2\dot{\alpha}) = -(W - B)\cos(\theta - \alpha) + \frac{\rho}{2}AU^2[C_L(\alpha, \delta = 0, \omega = 0) + b_z\delta + c_z\omega], \qquad [17]$$
$$J\ddot{\theta} = -B\cdot\xi\cdot\sin\theta + \frac{\rho}{2}AlU^2[C_M(\alpha, \delta = 0) + b_\theta\delta + c_\theta\omega].$$

From these equations it can be seen that there are four dependent unknown variables, i. e., U, θ, α, and δ, while there are only three equations. At least one more equation is necessary to determine completely the motion of the submarine in the vertical plane. If, as is the question in this paper, the problem is that of manuevering, then the additional equation is, for general purposes, a known prescribed variation of the stern-plane angles with time t designated as $\delta(t)$. For example, where there is manual control, the helmsman governs the movement of the stern planes; where there is automatic control, the stern planes are actuated by a mechanism that receives its signals from devices that sense the movement and attitude of the vessel. In either case the function $\delta(t)$ is known, and δ no longer plays the role of an

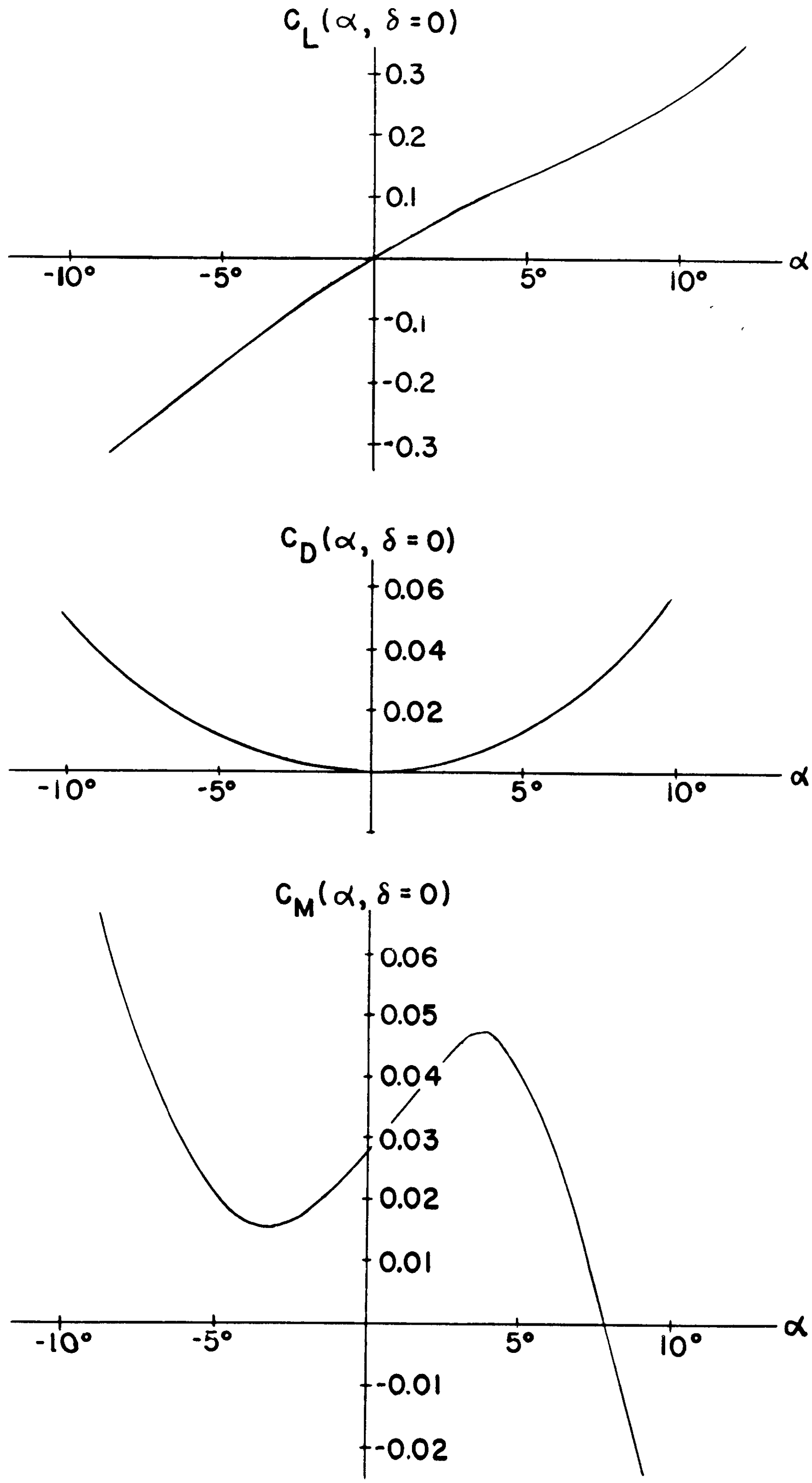

Figure 6 - $C_L(\alpha, \delta = 0)$, $C_D(\alpha, \delta = 0)$, and $C_M(\alpha, \delta = 0)$ versus α

unknown variable in Equations [17]. If the performance of the stern planes is given by $\delta = \delta(t)$, then there are four equations and four unknowns. This holds true both for manual and automatic control. Thus Equations [17] with an additional equation, designated as $\delta = \delta(t)$, form a set of equations which are sufficient in number to predict the motion and direction of the submarine on the vertical plane.

In connection with the problem of this paper, let us assume that the performance of the stern-plane angles is given by $\delta = \delta(\theta, \Delta h)$; where Δh is the deviation from the required depth and where θ has been defined earlier. Since $\delta = \delta(\theta, \Delta h)$ introduces a new variable Δh, then an additional equation is required in which Δh is defined in terms of the dependent variables of Equations [17]. This additional equation together with $\delta = \delta(\theta, \Delta h)$ and Equations [17] form a set of five equations with five unknowns. With this set of equations, it is possible to calculate the solution of motion.

EQUATIONS OF CONTROL

Suppose that among the requirements of an automatic controlling device the following are considered important: To maneuver a vessel from the given depth to a new ordered depth with the greatest efficiency, and to insure that an arbitrary but plausible angle of trim is not exceeded. We will confine ourselves here to these two requirements and study the problem in several stages. First, we will study the case of a vessel governed by a specific type of equation of control with fixed parameters; then we would repeat the test for each variation of parameter. The final pattern that emerges from these calculations will indicate the possibilities and limitations of this type of automatic control. This procedure is advisable because we are dealing here with equations of motion that are nonlinear.

To study the particular operation of the controlling device we visualize a maneuver from a given initial depth to a new ordered depth. In such a maneuver the submarine will undergo a continuous change in angle of trim and a continuous change in relative depth. At the ordered depth the submarine will follow a steady course parallel to the initial course, provided that the initial speed and angle of trim are maintained. It is to be expected that the composition of an equivalent mathematical control equation will contain terms signifying instantaneous changes in relative depth and changes in angle of trim. To choose such an equation, we have the following:

$$\Delta\delta = \delta - \delta_0 = K_1(\theta - \theta_0) + K_2 \Delta h \qquad [18]$$

where $\Delta\delta$ is defined as $\delta - \delta_0$, or the difference between the instantaneous and the initial stern-plane angle;

K_1, K_2 are parameters;

$\theta - \theta_0$ is the difference between the instantaneous and initial angle of trim; and

Δh is the difference between the ordered and instantaneous depth.

Furthermore, Δh is measured so that it becomes zero when the ordered depth is attained as in Figure 7.

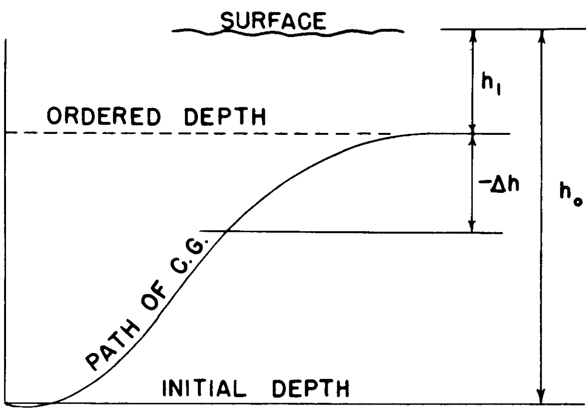

Figure 7 - Definition of Δh

When the ordered depth is reached, Δh becomes zero and θ tends to θ_0 ($\theta_0 = \alpha_0$), so that $\delta = \delta_0$. Thus the stern planes return to their initial value and the submarine maintains a steady, straight course that is parallel to the original straight course.

Other control devices or their equivalent analytical behaviors might be proposed; for example, controls that are sensitive to the rate of change of angle of trim and rate of change of Δh, e.g., $d\dot{\theta}/dt$, and $d(\Delta h)/dt$ respectively. In this report the following equation of control was used:

$$\Delta\delta = K_1(\theta - \theta_0 + \dot{U}/g) + K_2\Delta h \qquad [19]$$

Equation [19] is an analytical representation of a mechanical system shown in Appendix 1. This system was proposed in a previous paper[1] and the same equation is derived again in Appendix 1, using a slightly different method of analysis for purposes of clarity. Equation [19] indicates that the movement of the stern planes is proportional to the rate of change of linear velocity $\dot{U}$; it also shows that this movement is proportional to the angle of trim and to the difference between the ordered and the instantaneous depth.

If one assumes that K_1 and K_2 are positive parameters, then it is necessary to attach a sign convention to Δh, so that the stern planes will assume proper directions for either diving or rising. To show that this sign convention is necessary, one has to picture the state of motion a short time after the command is given for the vessel either to rise or to dive. After a small increment of time, the instantaneous angle of trim is approximately equal to the initial angle of trim, and U is approximately zero, so that Equation [19] becomes

$$\Delta\delta = K_2\,\Delta h \qquad [20]$$

Assuming $K_2 > 0$, then $\Delta\delta$ will take on either positive or negative values as Δh takes on plus or minus values. If we assign a negative value to Δh, then the stern planes will adopt appropriate directions of angle necessary for rising and *vice versa* for diving.

Equations [18] and [19] introduce a new variable Δh, and an additional equation is needed before the solution of the set of differential equations is possible. This additional equation is obtained by considering the kinematics of the center of gravity of the submarine and the relationship of the center of gravity to the ordered depth, Figure 8. The result of the limiting process gives the following:

$$\frac{d(-\Delta h)}{dt} = -U \sin \eta \qquad [21]$$

or

$$\frac{d(\Delta h)}{dt} = U \sin \eta = U \sin(\theta - \alpha) \qquad [22]$$

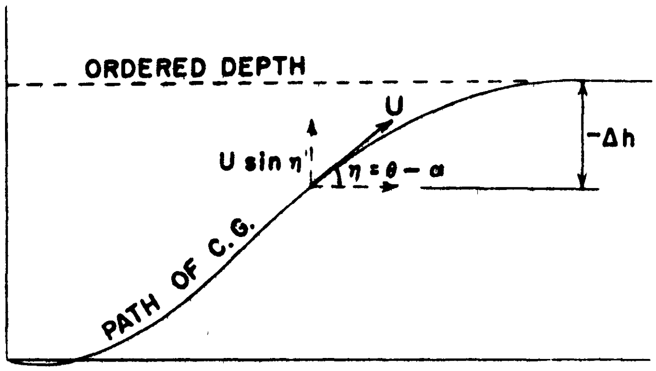

Figure 8 - Rate of Change of $-\Delta h$ with Respect to Time

EQUATIONS OF MOTION WITH AUTOMATIC CONTROL EQUATION

Equations [17], together with Equations [19] and [20], define the motion of the submarine. With a view to numerical integration, it is advisable to express Equations [17] as a system of first order differential equations. The following equations express the change of the equations of motion to first order;

$$\dot{U} = -\frac{(W-B)\sin(\theta-\alpha)}{\bar{m}_1} - \frac{\rho A U^2}{2\bar{m}_1} C_D(\alpha, \delta = 0),$$

$$\dot{\alpha} = \frac{(W-B)\cos(\theta-\alpha)}{\bar{m}_2 U} - \frac{\rho A U}{2\bar{m}_2}[C_L(\alpha, \delta = 0) + b_z\delta + c_z\omega] + \left(\frac{\bar{m}_1}{\bar{m}_2}\right)\omega,$$

$$\dot{\omega} = \frac{\rho A l U^2}{2J}[C_M(\alpha, \delta = 0) + b_\theta\delta + c_\theta\omega] - \frac{\xi B \sin\theta}{J} \qquad [23]$$

$$\theta = \omega$$

$$\Delta\delta = K_1(\theta - \theta_0 + \dot{U}/g) + K_2\overline{\Delta h}$$

$$\dot{\overline{\Delta h}} = U\sin(\theta - \alpha)$$

NUMERICAL INTEGRATION

The Runge-Kutta method was used to integrate the set of nonlinear differential Equations [23]. It is a convenient method of step-by-step integration because the time interval may be varied at any time throughout the calculations. A sample calculation is given in Appendix 3. This example shows the first three steps ($t = 0.5$, $t = 1.0$, $t = 1.5$) of the calculations for a rising maneuver of 10 meters. The parameters of the control equation are $K_1 = 4.36$ and $K_2 = 0.0815$. The other conditions are $B - W = 860$ *kg*, $U = 10$ meters per second, and $\theta_0 = \alpha_0 = -1.117$ degrees. The result of a complete calculation is shown graphically in Figure 13.

NUMERICAL RESULTS FOR PARTICULAR CASES OF DIVING AND RISING

By varying the values of K_1, K_2, and $B - W$, numerical calculations were carried out for six different cases. The cases are summarized in the table below and the results appear In Figures 9 to 14, pages 19 to 24.

Figure	B – W	K_1	K_2	Type of Adjustment
9	0	3.00	0.0815	single
10	0	5.00	0.0815	single
11	0	4.36	0.0815	single
12	0	4.36	0.100	single
13	860 kg	4.36	0.0815	single
14	0	4.36	0.0815	continuous, and single

Only two trim conditions were investigated: a trim condition $B - W = 0$, and $B - W = 860$ *kg*. For each trim condition there is a separate initial condition. The initial values, $U = 10$ meters per second and $\Delta h = -10$ meters, as well as the trim condition $B - W = 860$ *kg*, are all based on the data given in a previous paper.[1] It is to be noted that the parameters K_1 and K_2 do not affect the initial conditions because they are assumed as functions only of the dimensions and physical constants of the control.

With reference to the previously proposed pendulum control[1] given in Appendix 1, K_1 and K_2 are related in such a manner that an increment in one will change the value of the other. Aside from the pendulum control, this paper is concerned mainly with the effect of changes in each parameter K_1, K_2, and $B - W$ separately.

The phrases "single adjustment" and "continuous adjustment" refer to the way the control is set at the initial depth. For single adjustment, this setting of the control is such that it is sensitive to Δh, which is the difference between the instantaneous depth and ordered depth. In order to ensure continuous adjustment, the setting at the initial depth is such that the control automatically and continuously adjusts itself to maintain a constant Δh, no matter what the attitude or depth of the submarine.

From these relatively few cases, it is not possible to obtain a complete picture of each variation of parameter. However, important trends are established within the range of values considered. In practical applications, the values of K_1 and K_2 would probably be limited to relatively narrow intervals, and this fact precludes the necessity of a complete investigation of all possible variations of parameters.

The results of the numerical integration for the various cases are shown in the form of curves. Figures 9 through 14 show the variation of vertical travel with respect to horizontal travel (path of the center of gravity), and the variation with respect to time of the following: the trim angle θ, the angle of attack α, the angular velocity ω, and the stern plane deflection $\Delta\delta$. The path angle η may be obtained by taking the difference of the angle of trim and the angle of attack $\eta = \theta - \alpha$. In plotting the path of the center of gravity and computing the horizontal travel it was found that the velocity decreased slightly. Since this decrease was negligible, it was not taken into account in plotting the graphs.

In all cases it is to be noted that the submarine executes a counter movement in the first few moments. When it is rising, the counter movement is downward. This phenomenon occurs because θ and α change at about the same rate in the first few seconds, and because the downward thrust on the stern planes decreases the lift. When the submarine is diving, the counter movement is upward. A similar phenomenon occurs when a surface ship starts to make a turn.

The curve for the path followed by the center of gravity shows that the submarine has a tendency to go beyond (overshoot) the ordered depth when $B - W > 0$. For the case investigated, the amount of overshoot is not very serious, amounting to less than one-tenth of the initial Δh when $B - W = 860\,kg$. However, for the trim condition $B - W = 0$, the approach to the ordered depth is gradual and there is little tendency to overshoot.

The overshoot tendency may be explained as follows. As the submarine maneuvers from one depth to another, the hydrodynamic forces acting on it are continually changing, excepting the quantity $B - W$, which remains constant. In approaching the desired depth, the other hydrodynamic forces acting on the vessel do not change quickly enough to reduce the vertical velocity to zero, and the result is an overshoot.

Comparing all cases, the rate of climb is greater when $B - W$ and K_2 increase, and when K_1 decreases. When $B - W = 0$, the vertical acceleration results from the hydrodynamic forces. When $B - W > 0$, the vertical acceleration is due to both the hydrodynamic forces and the force $B - W$. Hence the vertical acceleration for the cases when $B - W > 0$ is greater than when $B - W = 0$. Thus the rate of rise increases as $B - W$ increases.

The parameters K_1 and K_2 affect the rate of rise in the following manner. In the first few seconds the stern planes are deflected until they hit a stop. With a decrease in K_1 and an increase in K_2, the length of time during which the stern planes are against the stop increases. Because of these variations in K_1 and K_2, the stern planes are deflected for a period of time long enough to increase the path angle; and as the path angle increases there is a higher rate of rise.

It must be cautioned here that in speaking of various effects due to changes in the parameters K_1, K_2, and $B - W$, the effect is due to changes in one parameter alone, other quantities being held constant. It would be extremely difficult to predict the effect on the rate of rise, or on any other variable, when the values of two parameters are changed at one and the same time.

When a constant value for Δh is maintained, the result is a steady rise. The path of the center of gravity is approximately a straight line, Figure 14. After the first few seconds the angle of trim, the angle of attack, and the stern-plane deflection acquire constant values, and the angular velocity becomes zero. Unless the automatic control is disturbed, the submarine will steadily rise along an approximately straight path. At any point on this straight path the automatic control can be set for single adjustment, so that the submarine will level off at a new depth in an interval Δh. The value chosen for Δh in Figure 14 is -10 meters.

The possibility of levelling off at a different depth is shown in the curves for the path of the center of gravity. The point at which the automatic control is set at single adjustment is marked by a vertical line. It indicates when the submarine starts to level off. The example of continuous adjustment in Figure 14 should be extremely useful when the interval between the initial and final depth is large.

The most important variables to be considered are likely to be these: the time to reach the ordered depth; the maximum pitch angle; and the maximum angular velocity. As K_1 is increased with K_2 held constant, the time to reach ordered depth increases, the maximum pitch angle decreases, and the maximum angular velocity decreases. Variations in K_2 with K_1 held constant have opposite effects. However, this does not mean that an effect may be cancelled by changing K_1 and K_2 an appropriate amount. It must be cautioned again that the trends are the result of varying only one parameter, and the others being constant.

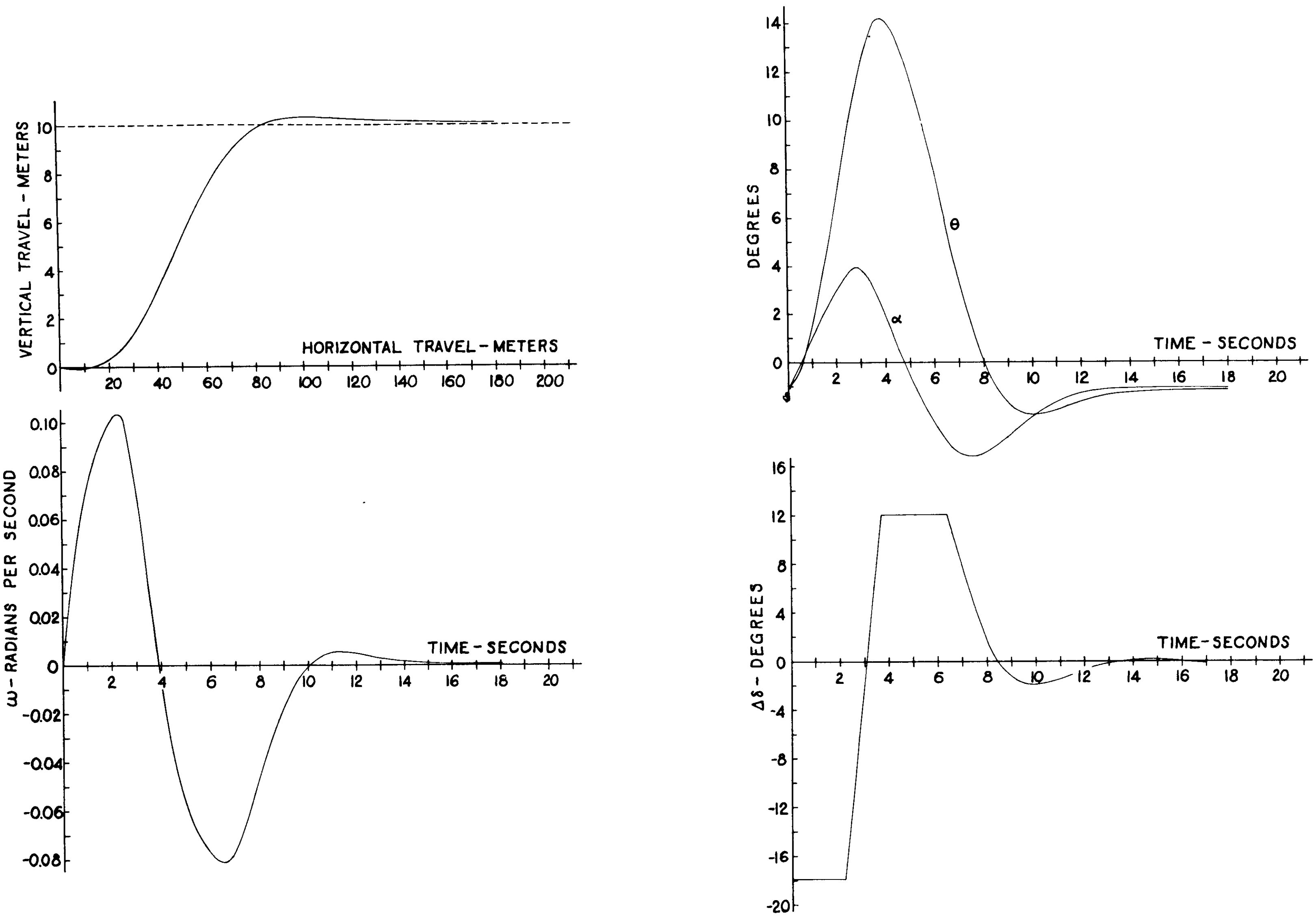

Figure 9 - Surfacing, Single Adjustment $\Delta\delta = 3.00\,(\theta - \theta_0 + \dot{U}/g) + 0.0815\,\Delta h$

$B - W = 0$

CONFIDENTIAL

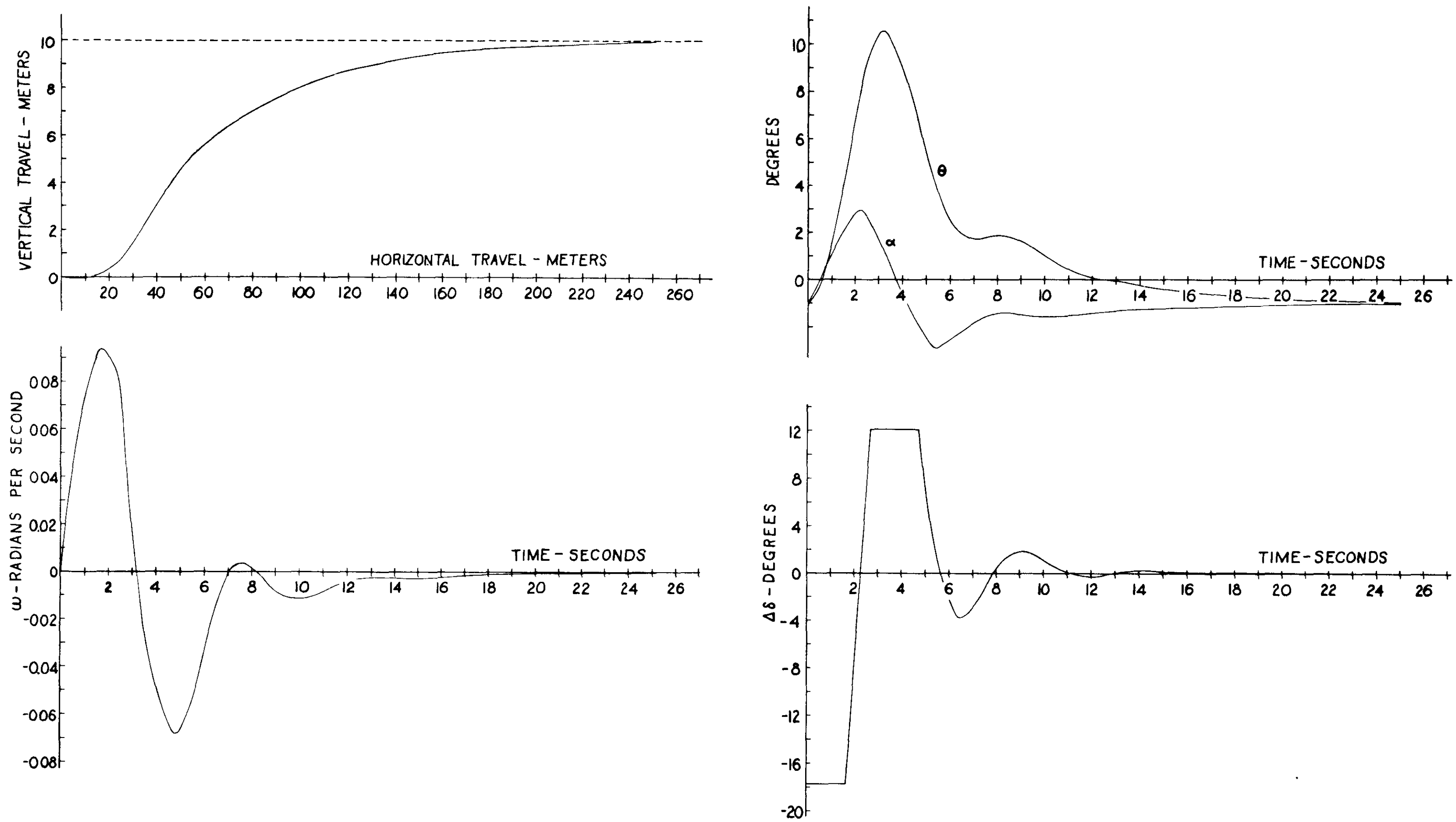

Figure 10 - Surfacing, Single Adjustment $\Delta\delta = 5.00\ (\theta - \theta_0 + \dot{U}/g) + 0.0815\ \Delta h$

$B - W = 0$

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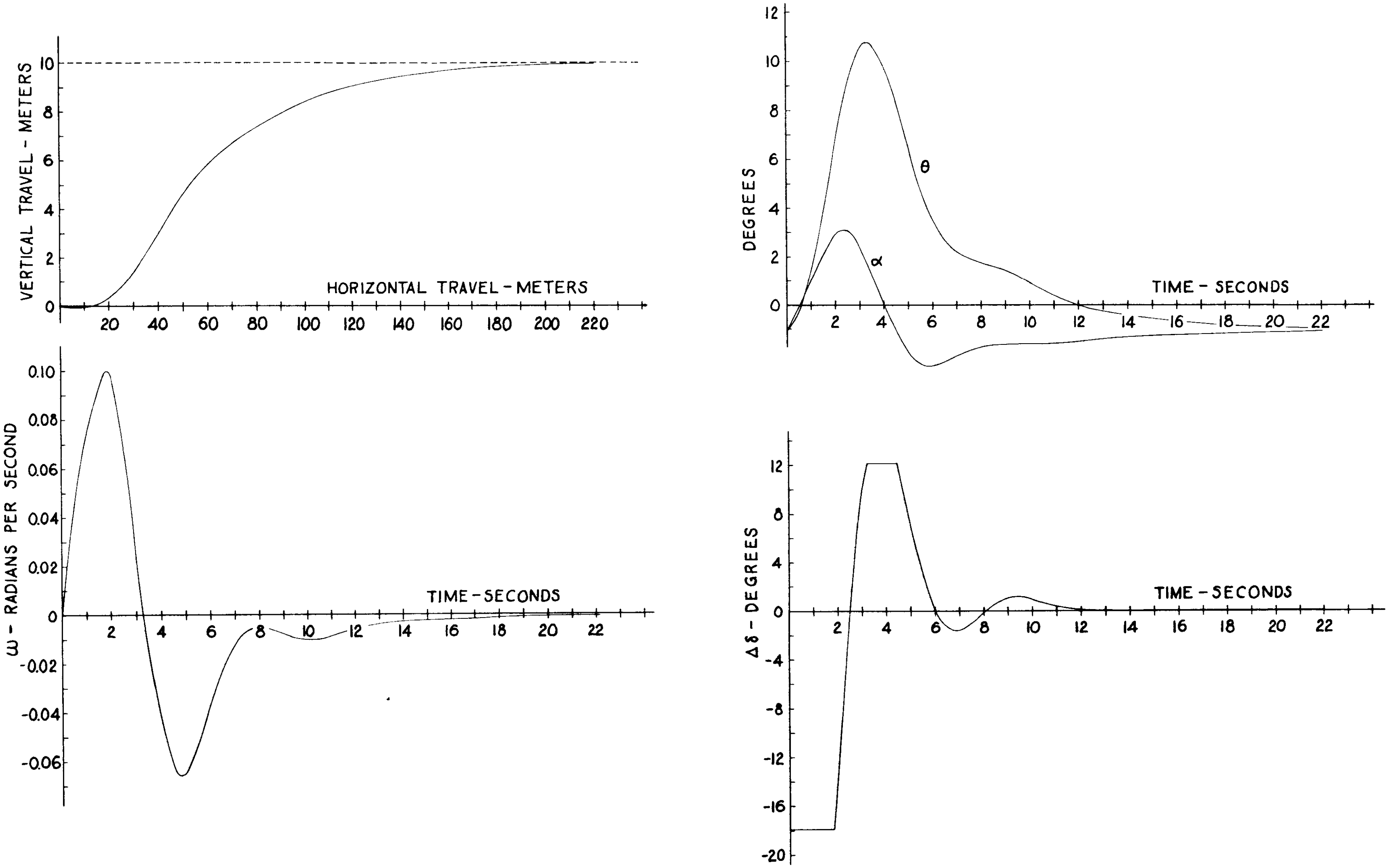

Figure 11 - Surfacing, Single Adjustment $\Delta\delta = 4.36\ (\theta - \theta_0 + \dot{U}/g) + 0.0815\ \Delta h$

$B - W = 0$

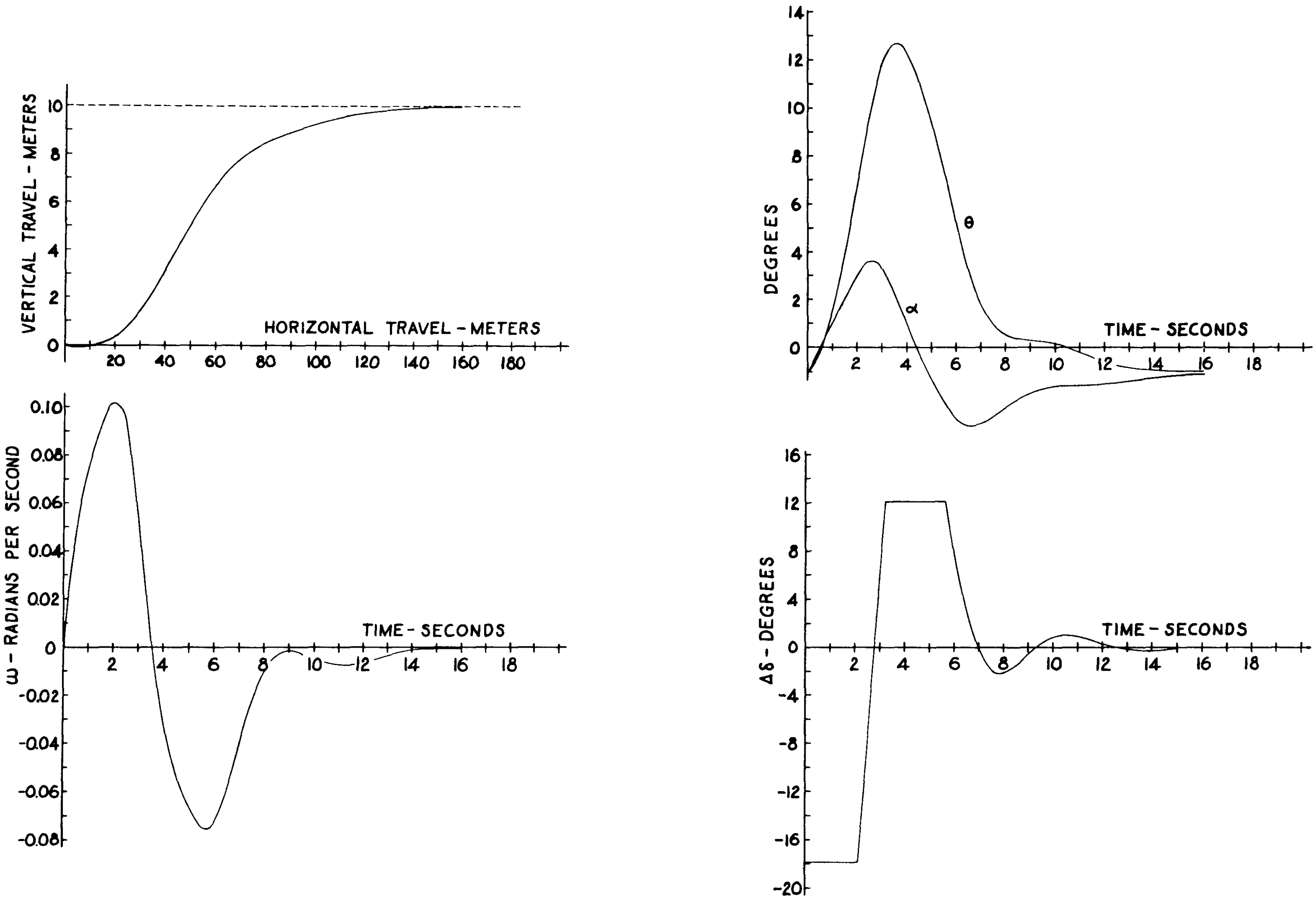

Figure 12 - Surfacing, Single Adjustment $\Delta\delta = 4.36\,(\theta - \theta_0 + \dot{U}/g) + 0.100\,\Delta h$

$B - W = 0$

CONFIDENTIAL

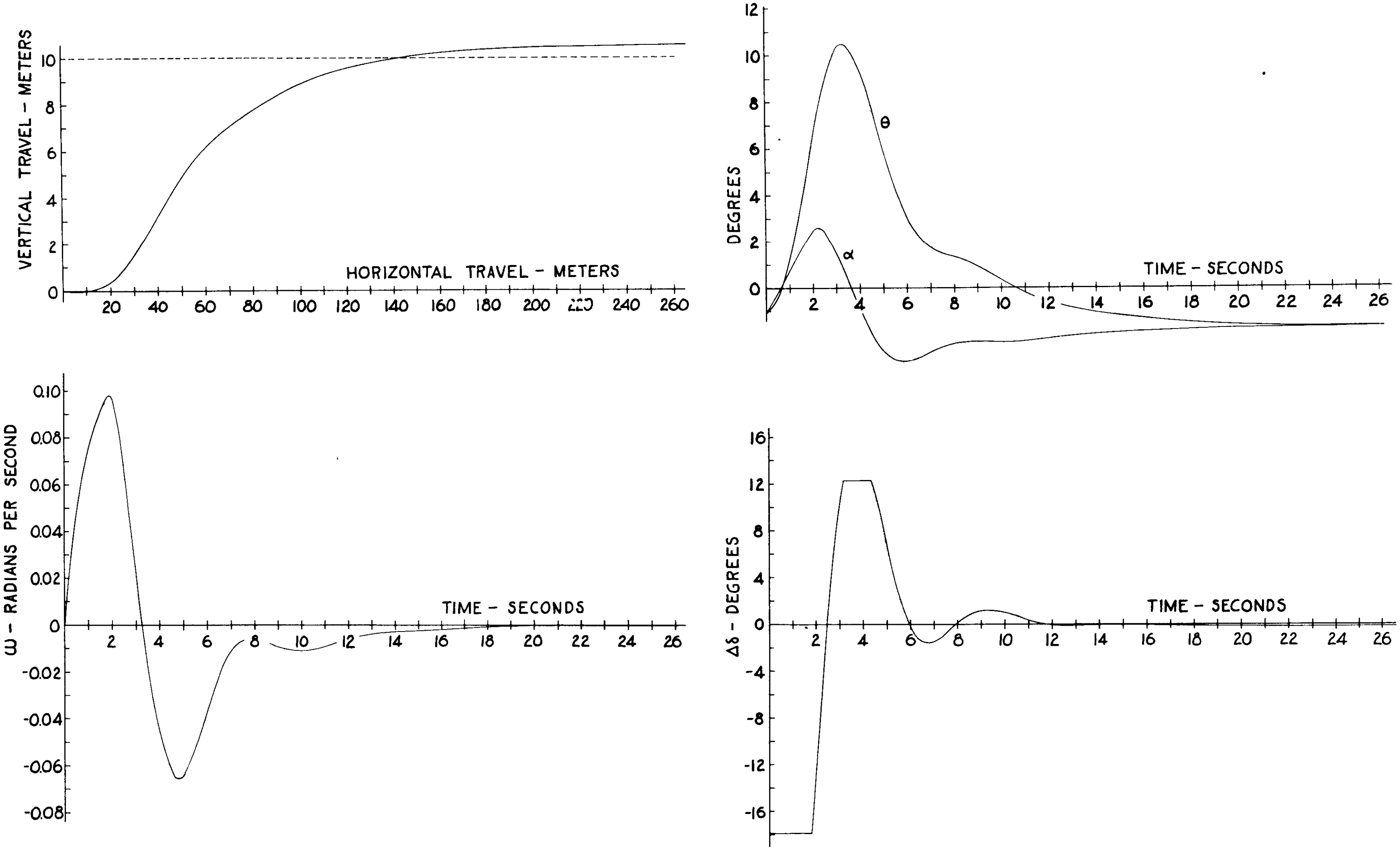

Figure 13 - Surfacing, Single Adjustment $\Delta\delta = 4.36\,(\theta - \theta_0 + \dot{U}/g) + 0.0815\,\Delta h$
$B - W = 860\ kg$

CONFIDENTIAL

CONFIDENTIAL

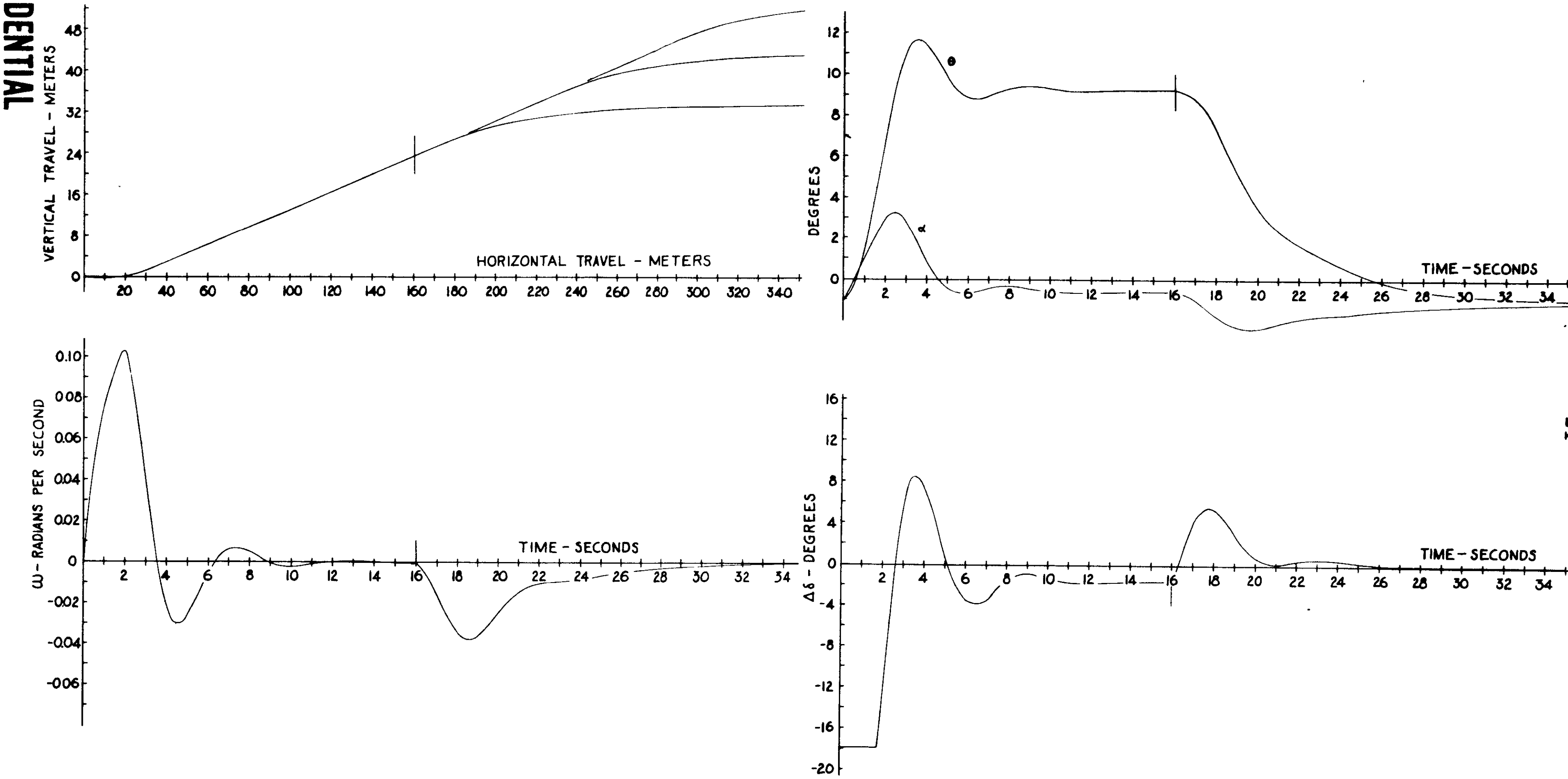

Figure 14 - Steady Climb and Level Off $\Delta\delta = 4.36\ (\theta - \theta_0 + \dot{U}/g) + 0.0815\ \Delta h$

$B - W = 0$

PART II

STABILITY OF MOTION

In this section we are concerned with the analysis and calculations of stability of motion of the submarine as it dives or rises. Specifically, the analysis is concerned with predicting the stability of motion on the basis of nonlinear equations or motion. A perusal of the foregoing graphs shows that angles and distances are no longer small quantities. This is as much as to say that a system of linearized differential equations would not be valid over the entire range of variables. Extensive developments have recently occured in the mathematics of nonlinear differential equations which are applicable to, and give further insight into recurring nonlinear phenomena in nature. Basic theorems have been developed[3,4] and one of these will now be used in our investigation. This theorem will be discussed here and its application to Equations [23] will be outlined later.

STATEMENT OF THE THEOREM

Suppose that the differential equations of motion are in this vector-matrix form,

$$\frac{dz}{dt} = Az + f(z) \qquad [24]$$

where A is a matrix that will be constant in nearly every instance; $f(z)$ is a function of z; and $f(0) = 0$. Equation [24] is to be taken as merely a convenient abbreviation for a system of differential equations.

A particular solution $y(t)$ or [24] is said to be stable, if any other solution $z(t)$ that satisfies the condition

$$\| z(0) - y(0) \| \leq c_1$$

where c_1 is sufficiently small, satisfies the condition

$$\| z(t) - y(t) \| < c_2 \quad \text{for } 0 < t < \infty$$

where c_2 can be made as small as possible by taking c_1 sufficiently small. This means that, given a neighborhood of the vector $y(t)$, the solution $z(t)$ will belong to this neighborhood for all values of $t > 0$, if we take the initial value $z(0)$ sufficiently close to $y(0)$.

To elaborate on this theorem, let $y(t)$ be taken as a solution of Equations [23]. For an example of a solution in graphical form, take any one set of the foregoing graphs showing θ, α, δ, and showing vertical travel versus horizontal travel for a particular case of diving or rising. $z(t)$ is another solution of Equation [24] and represents the disturbed state of motion. Let us suppose that an explosion of a depth charge produces a change in all variables; let this initial disturbed state be denoted by $z(0)$. Here we measure time from the instant of explosion. If the norm of the difference between the disturbed state of motion and the original state of motion is sufficiently small for $t > 0$, then when the curves of motion are plotted, any point on the disturbed path will remain in the close neighborhood of the corresponding point on the original path. Moreover, the difference between the original state and the disturbed state can be made as small as possible by making sufficiently small the norm of the difference between the instantaneous disturbed state and the original state at time, $t = 0$ (the instant of disturbance).

The same theorem can be viewed from a different point, and explained solely in terms of $z(t)$. We may say that a motion is stable if the norm of the disturbed state, denoted by $\|z(t)\|$, is equal to or less than a preassigned small positive constant c for all values of time, providing the initial value of the disturbance $\|z(0)\|$ is sufficiently small. A symbolic representation of a motion that is stable may be, written as follows: given any constant $c > 0$, and if $\|z(t)\|$ is a solution of Equation [24], then we will have $\|z(t)\| < c$ for all values of $t > 0$, provided the initial value of $\|z(0)\|$ is sufficiently small.

The theorem requires that the disturbed state of motion be a solution of Equation [24], or the original set of differential equations. In general $z(t)$ will be a solution of Equation [24] if, after the initial disturbance, the equations of motion are not altered. There is, however, another possibility which we may mention and dismiss, since we assume that it does not arise here. This is the possibility that the initial disturbance might either alter the shape of the vessel, or create unusual hydrodynamic effects (to mention only two cases). In these instances $z(t)$ would no longer be a solution of Equation [24].

In order that a motion may be stable, the following conditions[4] must be satisfied:

1. A must be a constant matrix and $f(0) = 0$;
2. All the characteristic roots of matrix A must have negative real parts;
3. $(\|f(z)\| / \|z\|) \to 0$ as $\|z\| \to 0$.

If, on the other hand, all the characteristic roots of the matrix A have positive real parts, and if

$$\frac{\|f(z)\|}{\|z\|} \to 0 \text{ as } \|z\| \to 0$$

then $z(t) = 0$ is a completely unstable solution of the vector-matrix equation

$$\frac{dz}{dt} = Az + f(z)$$

It is convenient at this time to define the norm of a function $g(t)$. Suppose that $g(t)$ is a vector in the n-dimensional space. Let $g_1(t), g_2(t), \ldots, g_n(t)$ be the components of $g(t)$. Then by definition, $\|g(t)\| = |g_1(t)| + |g_2(t)| + \ldots + |g_n(t)|$, which is to say that the norm of a vector $g(t)$ is equal to the sum of the absolute values of the components of the vector.

APPLICATION OF THEOREM

The preceeding theorem is now applied to the state of motion of the submarine in which a single-adjustment type of control is employed. Before proceeding with the application of this theorem, we note that the term $\dot{U}$ can be shown to be a small quantity of negligible importance in so far as practical calculations are concerned. Considerable simplifications are possible if the term $\dot{U}$ is omitted. With this in mind, the former set of differential equations [23] becomes:

$$\dot{\alpha} = -\frac{(B-W)\cos(\theta-\alpha)}{\bar{m}_2 U} - \frac{\rho A U}{2\bar{m}_2}\left[C_L(\alpha, \delta=0) + b_z\delta + c_z\omega\right] + \left(\frac{\bar{m}_1}{\bar{m}_2}\right)\omega$$

$$\dot{\omega} = \frac{\rho A l U^2}{2J}\left[C_M(\alpha, \delta=0) + b_\theta\delta + c_\theta\omega\right] - \frac{\xi \cdot B \cdot \sin\theta}{J} \qquad [25]$$

$$\dot{\theta} = \omega$$

$$\dot{\delta} = K_1\omega + K_2 U \sin(\theta-\alpha)$$

in which the last two equations of set [23] were combined by first taking the derivative with respect to time of the second equation from the bottom. Equations [25] define the motion of a submarine employing a single-adjustment type of control, and these equations have been found to be more conducive to a solution of stability than the original form (Equations [23]).

The steps in procedure to be followed in order to establish the stability of motion of a submarine in a rising or diving maneuver are these:

a. Establish that $f(0) = 0$. If Equations [25] do not show that $f(0) = 0$, then a suitable transformation of coordinates must be made.

b. Show that matrix A is constant.

c. Prove that the characteristic roots of the matrix A have negative real parts.

d. Finally show that $\|f(z)\|/\|z\| \to 0$ *as* $\|z\| \to 0$

An inspection of Equations [25] will show that these equations do not vanish when the variables θ, α, η, and δ are set equal to zero. For this reason we conclude that $f(0)$ is not zero, and that a suitable transformation is needed to bring Equations [25] into their proper form. Rather than work with the transformation of coordinates, we find it more convenient at this time to establish several relationships that will be useful in connection with the transformation. These relationships will now be discussed; they involve the differential equations of motion when the submarine is on a steady-straight course prior to a diving or rising maneuver. Let us assume that the properties of this motion are as follows: $\theta = \alpha = \alpha_0 = \theta_0$, $\delta = \delta_0$, $U = U_0$, and $\omega = 0$; where the subscript zero indicates the steady-state conditions of equilibrium. Substituting the above steady-state conditions into the equations of motion [25], we obtain:

$$
\begin{aligned}
0 &= s_1 + s_2 \,[C_L(\alpha_0, \delta = 0) + b_z\delta_0] \\
0 &= s_3 \,[C_M(\alpha_0, \delta = 0) + b_\theta\delta_0] + s_4 \sin\theta_0 \\
0 &= 0 \\
0 &= 0
\end{aligned}
\qquad [26]
$$

where $s_1 = -(B - W)/\bar{m}_2 U_0$, $s_2 = -\rho A U_0/2\bar{m}_2$,

$s_3 = \rho A l U_0^2/2J$, and $s_4 = -\xi B/J$.

Expressing $C_L(\alpha_0, \delta = 0)$ and $C_M(\alpha_0, \delta = 0)$ as polynomials in α, as shown by Equation [15], Equations [26] reduce to the following form:

$$
\begin{aligned}
0 &= s_1 + s_2 \,[l_2\alpha_0^2 + l_1\alpha_0 + l_0 + b_z\delta_0] \\
0 &= s_3 \,[m_4\alpha_0^4 + m_3\alpha_0^3 + m_2\alpha_0^2 + m_1\alpha_0 + m_0 + b_\theta\delta_0] + s_4 \sin\theta_0 \\
0 &= 0 \\
0 &= 0
\end{aligned}
\qquad [27]
$$

Returning now to the transformation of coordinates, the conditions of the theorem demand that the following linear transformation be chosen:

$$\theta = \theta' + \theta, \quad \alpha = \alpha' + \alpha_0, \quad \delta = \delta' + \delta_0, \quad \omega = \omega' + 0 \qquad [28]$$

in which θ_0, α_0, δ_0, and $\omega = 0$ are constants that correspond to the steady-state conditions. Substituting the relations shown in [28] in Equations [25], the following set of equations is obtained:

$$
\begin{aligned}
\dot{\alpha}' &= s_1 \cos(\theta' - \alpha') + s_2 \,[C_L(\alpha' + \alpha_0) + b_z(\delta' + \delta_0) + c_z\omega] + s_7\omega' \\
\dot{\omega}' &= s_3 \,[C_M(\alpha' + \alpha_0) + b_\theta(\delta' + \delta_0) + c_\theta\omega'] + s_4 \sin(\theta' + \theta_0), \\
\dot{\theta}' &= \omega' \\
\dot{\delta}' &= s_5\omega' + s_6 \sin(\theta' - \alpha')
\end{aligned}
\qquad [29]
$$

where $s_5 = K_1$, $s_6 = K_2 U_0$, and $s_7 = (\bar{m}_1/\bar{m}_2)$

The functions $C_L(\alpha' + \alpha)$ and $C_M(\alpha' + \alpha_0)$ are expanded according to the relationships indicated in [15]. After this expansion is made, the equations are rearranged so that the relationships shown in [27] are utilized. The results of these substitutions and rearrangements yield the following differential equations:

$$
\begin{aligned}
\dot{\alpha}' &= g_{11}\alpha' + g_{12}\omega' + g_{13}\delta' + 0 \qquad + g_{15}(\alpha')^2 + g_{16}[\cos(\theta' - \alpha') - 1] \\
\dot{\omega}' &= g_{21}\alpha' + g_{22}\omega' + g_{23}\delta' + g_{24}\theta' + g_{25}(\alpha')^4 + g_{26}(\alpha')^3 + g_{27}(\alpha')^2 \\
&\qquad + g_{28}[(\sin\theta' - \theta')\cos\theta_0 + (\cos\theta' - 1)\sin\theta_0] \\
\dot{\delta}' &= g_{31}\alpha' + g_{32}\omega' + 0 + g_{34}\theta' \qquad + g_{35}[\sin(\theta' - \alpha') - (\theta' - \alpha')] \qquad [30] \\
\dot{\theta}' &= 0 + \omega' + 0 + 0 + 0
\end{aligned}
$$

where Nonlinear Contribution

$$g_{11} = s_2(2\alpha_0 l_2 + l_1),\ g_{12} = (s_z c_z + s_7),\ g_{13} = s_2 b_z,\ g_{15} = s_2 l_2$$

$$g_{16} = s_1,\ g_{21} = s_3(4m_4\alpha_0^3 + 3m_3\alpha_0^2 + 2m_2\alpha_0 + m_1),\ g_{22} = s_3 c_\theta$$

$$g_{23} = s_3 b_\theta,\ g_{24} = s_4\cos\theta_0,\ g_{25} = s_5 m_4,\ g_{26} = s_3(4m_4\alpha_0 + m_3)$$

$$g_{27} = s_3(6m_4\alpha_0^2 + 3m_3\alpha_0 + m_2),\ g_{28} = s_4,\ g_{31} = -s_6 = -g_{34} = -g_{35},\ g_{32} = s_5$$

An inspection of Equations [30] shows that when $\alpha' = \omega' = \delta' = \theta' \equiv 0$, these equations vanish. Hence Equations [30] satisfy the requirement of the theorem, namely $f(0) = 0$, where $f(z)$ is the nonlinear contribution shown above. It is also clear that the matrix of the coefficients attached to the variables α, ω, δ, θ is a constant.

The next step in the procedure is to show that the characteristic roots of the matrix have negative real parts. This can be done by computing the determinant $|A - \lambda I|$ where A is the matrix attached to linear powers of α, ω, δ, and θ; where I is the unit matrix, and where λ represents the characteristic roots.[5] The determinant $|A - \lambda I|$, where A is the matrix attached to linear powers of α, ω, δ, and θ; where I is the unit matrix, and where λ represents the characteristic roots.[5] The determinant $|A - \lambda I|$ gives:

$$
\begin{vmatrix}
g_{11} - \lambda & g_{12} & g_{13} & 0 \\
g_{21} & g_{22} - \lambda & g_{23} & g_{24} \\
g_{31} & g_{32} & -\lambda & g_{34} \\
0 & 1 & 0 & -\lambda
\end{vmatrix} \qquad [31]
$$

On expansion this determinant may be arranged as:

$$\lambda^4 + A\lambda^3 + B\lambda^2 + C\lambda + E = 0 \qquad [32]$$

where

$$A = -[s_2(2\alpha_0 l_2 + l_1) + s_3 c_\theta]$$

$$B = s_2 s_3 (2\alpha_0 l_2 + l_1) c_\theta - s_3 s_5 b_\theta - s_4 \cos\theta_0$$

$$- s_3 (s_2 c_z + s_7)(4m_4 \alpha_0^3 + 3m_3 \alpha_0^2 + 2m_2 \alpha_0 + m_1) + s_2 s_6 b_z$$

$$C = s_2 (s_3 s_5 b_\theta + s_4 \cos\theta_0)(2\alpha_0 l_2 + l_1)$$

$$- s_2 s_3 s_5 b_z (4m_4 \alpha_0^3 + 3m_3 \alpha_0^2 + 2m_2 \alpha_0 + m_1)$$

$$+ s_6(- s_2 s_3 b_z c_\theta + s_2 s_3 c_z b_\theta + s_3 s_7 b_\theta - s_3 b_\theta)$$

$$E = s_2 s_3 s_6 b_\theta (2\alpha_0 l_2 + l_1) - s_2 s_4 s_6 b_z \cos\theta_0$$

$$- s_2 s_3 s_6 b_z (4m_4 \alpha_0^3 + 3m_3 \alpha_0^2 + 2m_2 \alpha_0 + m_1)$$

The biquadratic equation [32] has four characteristic roots. These roots will have negative real parts if the following conditions are satisfied:

$$A > 0, \; B > 0, \; C > 0, E > 0 \qquad [33]$$

and

$$A^2 E < ABC - C^2 \qquad [34]$$

The theory is now applied to a submarine rising to a depth of 10 meters above the initial depth, using an automatic control. The constants in the equation of control were assumed to be $K_1 = 6.36$ and $K_2 = 0.0815$. The values of other constants can be found in Appendix 2. The graph of the undisturbed motion is shown in Figure 13.

The biquadratic equation for this case becomes:

$$\lambda^4 + 1.5852\,\lambda^3 + 2.918\,\lambda^2 + 1.4427\,\lambda + 0.15647 = 0$$

It is now evident that the coefficients of this equation are all positive, and that condition [33] is satisfied. It can easily be shown that condition [34] is also satisfied. Hence we conclude that the characteristic roots of the matrix have negative real parts.

There remains only one more item to be proved, i.e., $\|f(z)\|/\|z\| \to 0$ as $\|z\| \to 0$. It can be seen that, if $f(z)$ is a power series with constant coefficients beginning with terms of second order, this condition is satisfied. There remains to show that the condition is satisfied for terms, in Equations [30], of the type:

$$f(z) = \sin(\theta' - \alpha') - (\theta' - \alpha') \qquad [35]$$

We put $f(z)$ in the following form:

$$\lim_{\|z\| \to 0} \frac{\|f(z)\|}{\|z\|} = \lim_{\|z\| \to 0} \frac{|\sin(\theta' - \alpha') - (\theta' - \alpha')|}{|\theta'| + |\alpha'|}$$

Applying the rule of L'Hospital to the above indeterminate fraction, the following expression is obtained:

$$\lim_{\|z\| \to 0} \frac{\|f(z)\|}{\|z\|} = \lim_{\|z\| \to 0} \frac{\cos(\theta' - \alpha') - 1}{|\theta'| + |\alpha'|} \to 0$$

Thus expression [35] satisfies the condition of the theorem. In a similar manner, the other types of expressions, shown in Equations [30], can be proved to satisfy this condition. The four conditions advanced in the outline of procedure having been satisfied, it can now be said that in this case the motion of the submarine in a rising maneuver is a stable motion.

CONCLUSION

There are two points of theoretical importance arising from the present study which deserve further investigation. The first of these concerns the evaluation of cross-derivatives in Taylor's expansion of the hydrodynamic forces and moment. Though these cross-derivatives were found in our present study to be small enough to be neglected, the same may not hold true in experiments with other vessels; hence further investigations along this line with the support of experimental data might well be worth while.

The second point concerns the prediction of stable or unstable motion deriving from a system of nonlinear equations. Heretofore studies in stability of motion of vessels confined themselves to small displacements. The present study went beyond these limits and investigated problems in connection with large displacements; consequently it is significant in that it demonstrates that problems in stability where large displacements occur are not beyond solution.

ACKNOWLEDGMENT

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APPENDIX 1

A MECHANICAL TYPE OF CONTROL

A possible pendulum control that gives the control equation

$$\Delta\delta = K_1(\theta - \theta + \dot{U}/g) + K_2\Delta h \qquad [36]$$

is shown in Figure 15. The constants K_1 and K_2 are functions of the dimensions and physical constants of the control.

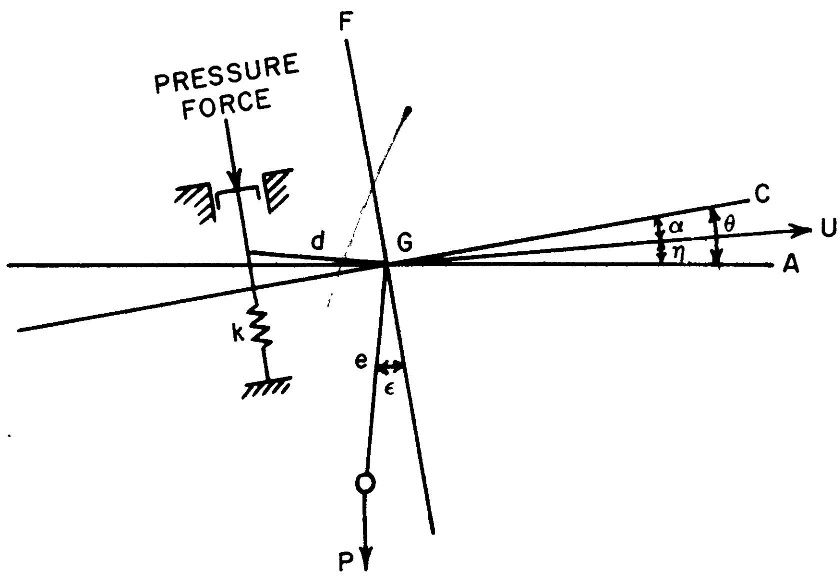

Figure 15 - Pendulum Control

The control consists of four main parts: a lever, a pressure piston, a spring, and a weight. The orientation of these parts in relation to the axes of the submarine are shown in Figure 15. The lever has two arms, of length d and e, which form a right angle. A horizontal, transverse axis fixed in the submarine is located at the right angle of the lever, and the lever can pivot about this axis. The weight P is at one end of the lever, and the pressure piston and spring are connected to the other end.

Since the control is placed at the center of gravity of the submarine, the pendulum is sensitive to changes in acceleration of the submarine. The acceleration due to the motion of the pendulum itself is assumed to be negligible in comparison to the acceleration of the submarine. The weight makes the control sensitive to the attitude of the submarine, and the pressure piston and spring introduce the depth as a control variable.

It is assumed that a linkage exists which will transform the position of the pendulum into a corresponding, stern-plane setting, without the stern planes reacting on the pendulum. Furthermore, it is assumed that the opening for obtaining the static water pressure is at a point where the motion or attitude of the submarine will have no influence.

While the submarine is traveling steadily along a horizontal course at a depth h_0, the pendulum is in the zero position, i.e., with $\epsilon = 0$. For various trim conditions on a steady-horizontal course, the spring will have to be adjusted to maintain the pendulum in its zero position.

To rise to a depth h_1 ($h_1 < h_0$), the force of the spring on the lever is decreased by adjusting the deformation of the spring. The unbalance thus produced causes the pendulum to seek an equilibrium position by rotating counterclockwise. The motion of the control produces a corresponding motion of the stern planes, which in turn causes the submarine to start its maneuver. As the submarine rises, the changes in pressure and attitude act on the control to level out the submarine as it approaches the desired depth. When the submarine has attained the desired depth, the control will be in equilibrium and will occupy the initial zero position. The attitude of trim at both depths will thus be the same.

To dive to a predetermined depth, the procedure is similar. The spring force is increased, and the ensuing motion of the control will cause the submarine to dive and level out at the desired depth.

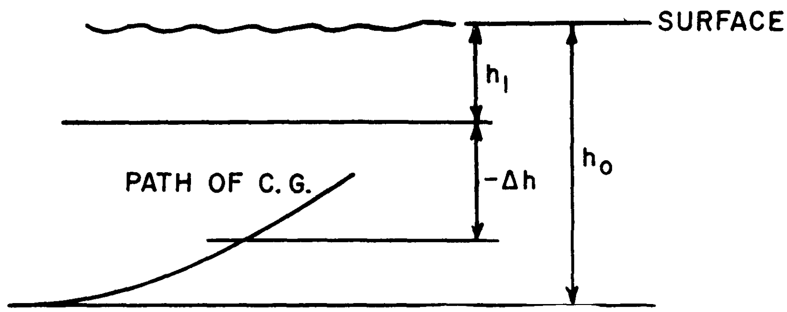

Figure 16 - Relationship between h_0, h_1, Δh

Suppose now that the submarine is trimmed on a steady, straight course at depth h_0. It will be assumed that the angle of attack is positive, i.e., the bow is high (Figure 17).

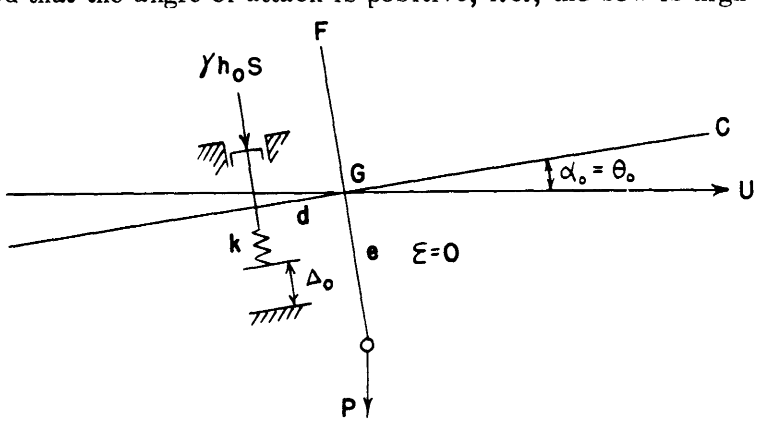

Figure 17 - Position of Control on Steady Straight Course

To maintain $\epsilon = 0$, and to keep the control in equilibrium, the spring will have to be compressed an amount Δ_0. The equilibrium equation about a transverse axis through the center of gravity is

$$\gamma h_0 Sd - k\Delta_0 d - Pe \sin \alpha_0 = 0 \qquad [37]$$

where γ is the specific weight of water; S is the area of the piston; and k is the spring modulus. Since the angle of attack on a steady, straight course is usually very small, Equation [37] may be written as

$$\gamma h_0 Sd - k\Delta_0 d - Pe\alpha_0 = 0 \qquad [38]$$

At the depth h_1 (Figure 17), assuming that the submarine has the same attitude of trim, the equilibrium equation for the control will be

$$\gamma h_1 Sd - k\Delta_1 d - Pe\alpha_1 = 0 \qquad [39]$$

where $\alpha_0 = \alpha_1 = \theta_0$

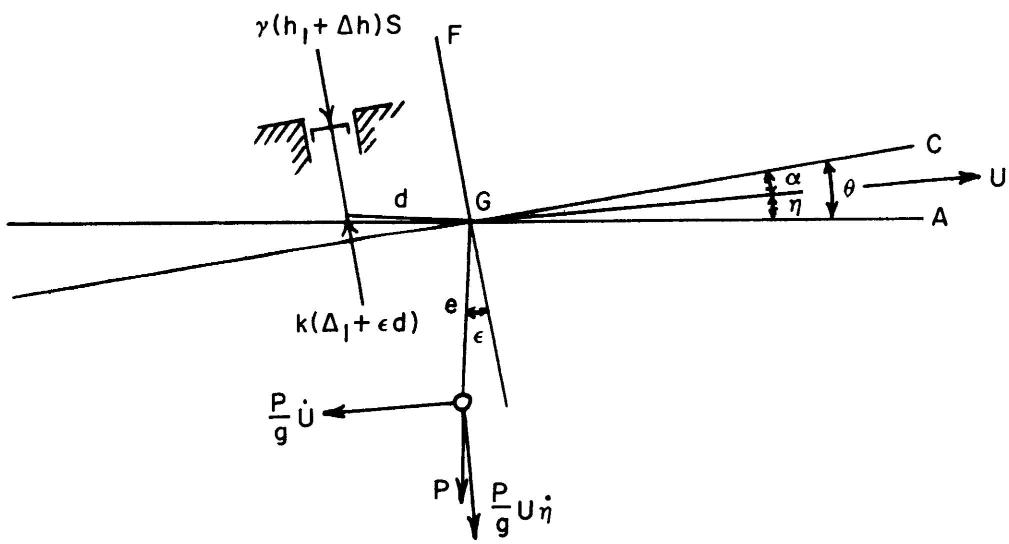

Figure 18 - Position of Control at Depth $h_1 + \Delta h$

Figure 18 shows the external and inertia forces acting on the control when the submarine is at some intermediate depth ($h_1 + \Delta h$) between h_0 and h_1. Assuming that the absolute motion of the pendulum is very slow, the acceleration due to the motion of the pendulum itself is neglected.

Writing the equation of motion about the center of gravity, and using the condition

$$h_1 = \frac{Pe\alpha_1 + k\Delta_1 d}{\gamma Sd} \qquad [40]$$

from Equation [39], and the condition that α_0, α_1, and θ_0, we have

$$\epsilon = \frac{Pe(\theta - \theta_0 + \dot{U}/g - \gamma Sdh}{Pe - kd} \qquad [41]$$

The angles θ, α, and ϵ have been assumed to be small so that the usual approximations for small angles could be made. Small quantities of second order have also been neglected.

The changes in the angle ϵ are transmitted to the stern planes, so that

$$\Delta\delta = C\epsilon$$

where $\Delta\delta = \delta - \delta_0$ is the increment in the stern plane angle, and C is a magnification factor. Equation [41] may be written as

$$\Delta\delta = K_1(\theta - \theta_0 + \dot{U}/g) + K_2\Delta h \qquad [42]$$

where

$$K_1 = \frac{CPe}{Pe - kd}, \text{ and } K_2 = \frac{-CSd}{Pe - kd}.$$

APPENDIX 2

DATA

W (weight of ship) = 88,000 kg,

J = 465,430 $kg.m$ sec^2,

$\overline{m}_1$ = 9540 kg sec$^2/m$,

$\overline{m}_2$ = 14,750 kg sec$^2/m$,

l = reference length of ship = 11.06 m,

$\xi(B - W = 0) = 0.406$ m,

$\xi(B - W = 860) = 0.376$ m,

U_0 = 10 meters per second,

$\overline{l}_1$ = 9.0 meters,

$\alpha_0 = -1.1175°$ for $B - W = 860$,

$m_0 = 0.02903$,

$m_1 = 0.006470$,

$m_2 = 0.0002799$,

$m_3 = -0.00169$,

$m_4 = -0.0000068$

$l_0 = 0.0111$,

$l_1 = 0.03202$,

$l_2 = -0.00111$,

$C_D(\alpha, \delta = 0) = d_2\alpha^2 + d_1\alpha$, where $d_2 = 0.0005$, $d_1 = 0.003$;

$b_z = 0.470$,

$b = -0.4930$,

$c_z = 1.767$,

$c = -1.438$,

$s_1 = -0.000583$,

$s_2 = -0.2259$,

$s_3 = 0.7918$,

$s_4 = -0.07179$,

$s_5 = 6.360$,

$s_6 = 0.8150$,

$s_7 = 0.6468$.

APPENDIX 3

SAMPLE CALCULATIONS

Surfacing - Single Adjustment $B - W = 860\ kg$

Control Equation: $4.36\ (\theta - \theta_0 + \dot{U}/g) + 0.0815\ \Delta h$

Initial Conditions:

$\theta_0 = \alpha_0 = -0.0195$ radians $= -1.117°$

$U_0 = 10$ meters per sec

$\omega_0 = 0$ radians per sec

$\Delta h_0 = -10$ meters

$\Delta\delta$ lies between -0.31120 radians and $+0.21320$ radians

Note: The quantities k, l, m, n, and p used in the Runge-Kutta method of integration are numerical constants which refer only to these sample calculations. They should not be confused with physical constants of the submarine or hydrodynamic forces and moment which have a similar notation.

	t	0.5	1.0	1.5
1	$\sin\eta$	0	− 0.00292	0.01137
2	$0.0903 \times (1)$	0	− 0.00026	0.00103
3	U^2	100.00000	99.99440	99.99200
4	C_D	0.00030	0	0.00038
5	$(3) \times (4)$	0.03000	0	0.03799
6	$-0.0349 \times (5)$	− 0.00105	0	− 0.00133
7	$(2) + (6)$	− 0.00105	− 0.00026	− 0.00030
8	$(7)/g$	− 0.00010	− 0.00003	− 0.00003
9	$\theta - \theta_0 + (8)$	− 0.00010	0.01100	0.04374
10	$4.36 \times (9)$	− 0.00044	0.08420	0.18400
11	$0.0815 \times \Delta h$	− 0.81500	− 0.81600	− 0.81600
12	$\Delta\delta = (10) + (11)$	− 0.31120	− 0.31120	− 0.31120
13	$\delta = (12) + 0.049$	− 0.26220	− 0.26220	− 0.26220
14	$k_1 = (7) \times \Delta t$	− 0.00053	− 0.00013	− 0.00015
15	$l_1 = \omega \times \Delta t$	0	0.02342	0.03728
16	$\cos\eta$	1.00000	1.00000	0.99990
17	$(16)/U$	0.10000	0.10000	0.10000
18	$-0.0583 \times (17)$	− 0.00583	− 0.00583	− 0.00583
19	$17.67 \times \omega$	0	0.82766	1.31747
20	$(19)/U$	0	0.08276	0.13175
21	$\cos\alpha$	0.99980	1.00000	0.99910
22	$(20)/(21)$	0	0.08276	0.13175
23	$0.47 \times \delta$	− 0.12323	− 0.12323	− 0.12323

	t	0.5	1.0	1.5
24	C_L	− 0.02587	0.00300	0.03325
25	(22) + (23) + (24)	− 0.14910	− 0.03747	0.04177
26	$-0.0226 \times U$	− 0.22600	− 0.22599	− 0.22599
27	(25) × (26)	0.03370	0.00847	− 0.00944
28	$0.647 \times \omega$	0	0.03030	0.04824
29	(18) + (27) + (28)	0.02787	0.03294	0.03297
30	$m_1 = (29) \times \Delta t$	0.01393	0.01647	0.01648
31	C_M	0.02239	0.02720	0.03375
32	$-0.493 \times \delta$	0.12926	0.12926	0.12926
33	(31) + (32)	0.15165	0.15646	0.16301
34	$-14.37/U$	− 1.43700	− 1.43704	− 1.43705
35	$(34) \times \omega$	0	− 0.06731	− 0.10715
36	(33) + (35)	0.15165	0.08915	0.05586
37	0.0079 × (3)	0.79000	0.78996	0.78994
38	(36) × (37)	0.11980	0.07042	0.04413
39	$\sin\theta$	− 0.01950	− 0.00670	0.02427
40	−0.0718 × (39)	0.0014	0.00048	− 0.00174
41	(38) + (40)	0.12120	0.07090	0.04239
42	$n_1 = (41) \times \Delta t$	0.06060	0.03545	0.02119
43	$p_1 = U \times (1) \times \Delta t$	0	− 0.01460	0.05680
44	$U = U_0 + k_1/2$	9.99974	9.99965	9.99952
45	$\theta = \theta_0 + l_1/2$	− 0.01950	0.00501	0.04291
46	$\alpha = \alpha_0 + m_1/2$	− 0.01254	0.00445	0.02114
47	$\omega = \omega_0 + n_1/2$	0.03030	0.06456	0.08515
48	$\eta = \theta - \alpha$	− 0.00696	0.00056	0.02177
49	$\Delta h = \Delta h_0 + p_1/2$	− 10.00000	− 10.02350	− 9.97258
50	$0.0815 \times \Delta h$	− 0.81500	− 0.81500	− 0.81276
51	$\sin\eta$	− 0.00696	0.00056	0.02177
52	0.0903 × (51)	− 0.00063	0.00005	0.00197
53	U^2	99.99480	99.99300	99.99040
54	C_D	0.00005	0.00010	0.00111
55	(53) × (54)	0.00500	0.01000	0.11099
56	−0.0349 × (55)	− 0.00017	− 0.00035	− 0.00387
57	(52) + (56)	− 0.00080	− 0.00030	− 0.00190
58	$(57)/g$	− 0.00008	− 0.00003	− 0.00019
59	$\theta - \theta_0 + (58)$	− 0.00080	0.02448	0.06222
60	4.36 × (59)	− 0.00008	0.09400	0.27127
61	$\Delta\delta = (60) + (50)$	− 0.31120	− 0.31120	− 0.31120

	t	0.5	1.0	1.5
62	$\delta = (61) + 0.049$	− 0.26220	− 0.26220	− 0.26220
63	$k_2 = (57) \times \Delta t$	− 0.00040	− 0.00015	− 0.00095
64	$l_2 = \omega \times \Delta t$	0.01515	0.03228	0.04257
65	$\cos \eta$	1.00000	1.00000	0.99980
66	$(65)/U$	0.10000	0.10000	0.10000
67	$-0.0583 \times (66)$	− 0.00583	− 0.00583	− 0.00583
68	$17.67 \times \omega$	0.53540	1.14077	1.50460
69	$(68)/U$	0.05354	0.11408	0.15046
70	$\cos \alpha$	0.99990	1.00000	0.99980
71	$(69)/(70)$	0.05354	0.11408	0.15046
72	$0.47 \times \delta$	− 0.12323	− 0.12323	− 0.12323
73	C_L	− 0.01500	0.1750	0.04900
74	$(71) + (72) + (73)$	− 0.08469	0.00835	0.07623
75	$-0.0226 \times U$	− 0.22599	− 0.22599	− 0.22599
76	$(74) \times (75)$	0.01914	− 0.00189	− 0.01723
77	$0.647 \times \omega$	0.01960	0.04177	0.05509
78	$(67) + (76) + (77)$	0.03291	0.03405	0.03203
79	$m_2 = (78) \times \Delta t$	0.01645	0.01702	0.01601
80	C_M	0.02460	0.03070	0.03700
81	$-0.493 \times \delta$	0.12926	0.12926	0.12926
82	$(80) + (81)$	0.15586	0.15996	0.16626
83	$-14.37/U$	− 1.43703	− 1.43705	− 1.43706
84	$(83) \times \omega$	− 0.04354	− 0.09277	− 0.12236
85	$(82) + (84)$	0.11032	0.06719	0.04390
86	$0.0079 \times (53)$	0.78996	0.78994	0.78992
87	$(85) \times (86)$	0.08715	0.05307	0.03468
88	$\sin \theta$	− 0.01950	0.00501	0.04291
89	$-0.0718 \times (88)$	0.00140	− 0.00036	− 0.00308
90	$(87) + (89)$	0.08855	0.05271	0.03160
91	$n_2 = (90) \times \Delta t$	0.04427	0.02636	0.01580
92	$p_2 = U \times (51) \times \Delta t$	− 0.03480	0.00280	0.10879
93	$U = U_0 + k_2/2$	9.99980	9.99965	9.99913
94	$\theta = \theta_0 + l_2/2$	− 0.01193	0.00944	0.04555
95	$\alpha = \alpha_0 + m_2/2$	− 0.01128	0.00473	0.02090
96	$\omega = \omega_0 + n_2/2$	0.02213	0.06002	0.08246
97	$\eta = \theta - \alpha$	− 0.00065	0.00471	0.02465
98	$\Delta h = \Delta h_0 + p_2/2$	−10.01740	−10.01475	9.94659
99	$0.0815 \times \Delta h$	− 0.81600	− 0.81672	− 0.81451

	t	0.5	1.0	1.5
100	$\sin\eta$	– 0.00065	0.00471	0.02465
101	$0.0903\times(100)$	– 0.00006	0.00042	0.00222
102	U^2	99.99600	99.99300	99.98250
103	C_D	0.00002	0.00011	0.00108
104	$(102)\times(103)$	0.00200	0.01099	0.10798
105	$-0.0349\times(104)$	– 0.00007	– 0.00038	– 0.00377
106	$(101)+(105)$	– 0.00013	0.00004	– 0.00155
107	$(106)/g$	– 0.00001	0.00000	– 0.00016
108	$\theta-\theta_0+(107)$	0.00956	0.02894	0.06489
109	$4.36\times(108)$	0.00284	0.09400	0.28200
110	$\Delta\delta=(109)+(99)$	– 0.31120	– 0.31120	– 0.31120
111	$\delta=(110)+0.049$	– 0.26220	– 0.26220	– 0.26220
112	$k_3=(106)\times\Delta t$	– 0.00007	0.00002	– 0.00078
113	$l_3=\omega\times\Delta t$	0.01106	0.03001	0.04123
114	$\cos\eta$	1.00000	1.00000	0.99970
115	$(114)/U$	0.10000	0.10000	0.09997
116	$-0.0583\times(115)$	– 0.00583	– 0.00583	– 0.00583
117	$17.67\times\omega$	0.39104	1.06055	1.45707
118	$(117)/U$	0.03910	0.10605	0.14571
119	$\cos\alpha$	0.99990	1.00000	0.99980
120	$(118)/(119)$	0.03910	0.10605	0.14571
121	$0.47\times\delta$	– 0.12323	– 0.12323	– 0.12323
122	C_L	– 0.01350	0.01800	0.04750
123	$(120)+(121)+(122)$	– 0.09763	0.00082	0.06998
124	$-0.0226\times U$	– 0.22599	– 0.22599	– 0.22598
125	$(123)\times(124)$	0.02206	– 0.00019	– 0.01581
126	$0.647\times\omega$	0.01432	0.03883	0.05335
127	$(116)+(125)+(126)$	0.03055	0.03281	0.03171
128	$m_3=(127)\times\Delta t$	0.01527	0.01640	0.01585
129	C_M	0.02485	0.03075	0.03690
130	$-0.493\times\delta$	0.12926	0.12926	0.12926
131	$(129)+(130)$	0.15411	0.16001	0.16616
132	$-14.37/U$	– 1.43702	– 1.43705	– 1.43712
133	$(132)\times\omega$	– 0.03180	– 0.08625	– 0.11850
134	$(131)+(133)$	0.12231	0.07376	0.04766
135	$0.0079\times(102)$	0.78997	0.78994	0.78986
136	$(134)\times(135)$	0.09662	0.05826	0.03764
137	$\sin\theta$	– 0.01193	0.00944	0.04555

	t	0.5	1.0	1.5
138	$-0.0718 \times (137)$	0.00086	− 0.00068	− 0.00327
139	$(136) + (138)$	0.09748	0.05758	0.03437
140	$n_3 = (139) \times \Delta t$	0.04874	0.02879	0.01718
141	$p_3 = U \times (100) \times \Delta t$	− 0.00330	0.02349	0.12318
142	$U = U_0 + k_3$	9.99993	9.99974	9.99882
143	$\theta = \theta_0 + l_3$	− 0.00844	0.02331	0.06550
144	$\alpha = \alpha_0 + m_3$	− 0.00423	0.01262	0.02875
145	$\omega = \omega_0 + n_3$	0.04874	0.07563	0.09174
146	$\eta = \theta - \alpha$	− 0.00421	0.01069	0.03675
147	$\Delta h = \Delta h + p_3$	−10.00330	− 9.99271	− 9.87780
148	$0.0815 \times \Delta h$	− 0.81650	− 0.81440	− 0.80504
149	$\sin \eta$	− 0.00421	0.01069	0.03675
150	$0.0903 \times (149)$	− 0.00038	0.00096	0.00332
151	U^2	99.99860	99.99480	99.97640
152	C_D	0	0.00047	0.00185
153	$(151) \times (152)$	0	0.04699	0.18496
154	$-0.0349 \times (153)$	0	− 0.00164	− 0.00645
155	$(150) + (154)$	− 0.00038	− 0.00068	− 0.00313
156	$(155)/g$	− 0.00004	− 0.00007	− 0.00031
157	$\theta - \theta_0 + (156)$	0.01102	0.04274	0.08469
158	$4.36 \times (157)$	0.04000	0.18000	0.36924
159	$\Delta\delta = (148) + (158)$	− 0.31120	− 0.31120	− 0.31120
160	$\eta = (159)\ 0.049$	− 0.26220	− 0.26220	− 0.26220
161	$k_4 = (155) \times \Delta t$	− 0.00019	− 0.00034	− 0.00156
162	$l_4 = \omega \times \Delta t$	0.02437	0.03781	0.04587
163	$\cos \eta$	1.00000	0.99990	0.99930
164	$(163)/U$	0.10000	0.10000	0.09994
165	$-0.0583 \times (164)$	− 0.00583	− 0.00583	− 0.00583
166	$17.67 \times \omega$	0.86123	1.33638	1.62105
167	$(166)/U$	0.08612	0.13364	0.16212
168	$\cos \alpha$	1.00000	0.99990	0.99960
169	$(167)/(168)$	0.08612	0.13364	0.16218
170	$0.47 \times \delta$	− 0.12323	− 0.12323	− 0.12323
171	C_L	0.00115	0.03200	0.06000
172	$(169)+(170)+(171)$	− 0.03596	0.04241	0.09895
173	$-0.0226 \times U$	− 0.22600	− 0.22599	− 0.22597
174	$(172) \times (173)$	0.00813	− 0.00958	− 0.02236
175	$0.647 \times \omega$	0.03153	0.04893	0.05935

	t	0.5	1.0	1.5
176	(165)+(174)+(175)	0.03383	0.03352	0.03116
177	$m_4 = (176) \times \Delta t$	0.01691	0.01676	0.01558
178	C_M	0.02715	0.03370	0.03945
179	$-0.493 \times \delta$	0.12926	0.12926	0.12926
180	(178)+(179)	0.15641	0.16296	0.16871
181	$-14.37/U$	− 1.43701	− 1.43703	− 1.43716
182	$(181) \times \omega$	− 0.07004	− 0.10868	− 0.13184
183	(180)+(182)	0.08637	0.05428	0.03687
184	$0.0079 \times (151)$	0.78999	0.78996	0.78981
185	$(183) \times (184)$	0.06823	0.04288	0.02912
186	$\sin\theta$	− 0.00844	0.02331	0.06550
187	$-0.0718 \times (186)$	0.00061	− 0.00167	− 0.00470
188	(185)+(187)	0.06884	0.04121	0.02442
189	$n_4 = (188) \times \Delta t$	0.03442	0.02060	0.01221
190	$p_4 = U \times (149) \times \Delta t$	− 0.02099	0.05339	0.18376
191	k_1	− 0.00053	− 0.00013	− 0.00015
192	$2k_2$	− 0.00080	− 0.00030	− 0.00190
193	$2k_3$	− 0.00014	0.00004	− 0.00155
194	k_4	− 0.00019	− 0.00034	− 0.00156
195	Σk	− 0.00166	− 0.00073	− 0.00516
196	k	− 0.00028	− 0.00012	− 0.00086
197	l_1	0	0.02342	0.03728
198	$2l_2$	0.03030	0.06456	0.08515
199	$2l_3$	0.02213	0.05002	0.08246
200	l_4	0.02437	0.03781	0.04587
201	Σl	0.07680	0.18581	0.25076
202	l	0.01280	0.03097	0.04179
203	m_1	0.01393	0.01647	0.01648
204	$2m_2$	0.03291	0.03405	0.03203
205	$2m_3$	0.03055	0.03281	0.03171
206	m_4	0.01691	0.01676	0.01558
207	Σm	0.09430	0.10009	0.09580
208	m	0.01572	0.01668	0.01597
209	n_1	0.06060	0.03545	0.02119
210	$2n_2$	0.08855	0.05271	0.03160
211	$2n_3$	0.09748	0.05758	0.03437
212	n_4	0.03442	0.02060	0.01221
213	Σn	0.28105	0.16634	0.09937

	t	0.5	1.0	1.5
214	n	0.04684	0.02772	0.01656
215	p_1	0	− 0.01460	0.05680
216	$2p_2$	− 0.06960	0.00560	0.21758
217	$2p_3$	− 0.00660	0.04698	0.24636
218	p_4	− 0.03099	0.05339	0.18376
219	Σp	− 0.09719	0.09137	0.70450
220	p	− 0.01620	0.01522	0.11742
221	$U = U_0 + k$	9.99972	9.99960	9.99874
222	$\theta = \theta_0 + l$	− 0.00670	0.02427	0.06606
223	$\alpha = \alpha_0 + m$	− 0.00378	0.01290	0.02887
224	$\omega = \omega_0 + n$	0.04684	0.07456	0.09112
225	$\eta = \theta - \alpha$	− 0.00292	0.01137	0.03719
226	$\Delta h = \Delta h_0 + p$	−10.01620	−10.00098	− 9.88356
227	U	9.99972	9.99960	9.99874
228	θ°	− 0.38390	1.39000	3.78520
229	α°	− 0.21659	0.73900	1.65400
230	η°	− 0.16731	0.65100	2.13090
231	Δh	−10.01620	−10.00098	− 9.88356
232	$\Delta\delta$	−17.80000	−17.80000	−17.80000

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