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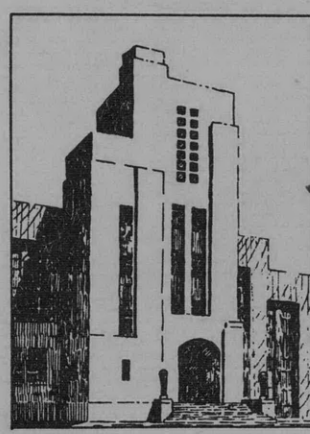
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STRUCTURAL VIBRATION PROBLEMS OF SHIPS —  
A STUDY OF THE DD692 CLASS OF DESTROYERS

by

NORMAN H. JASPER



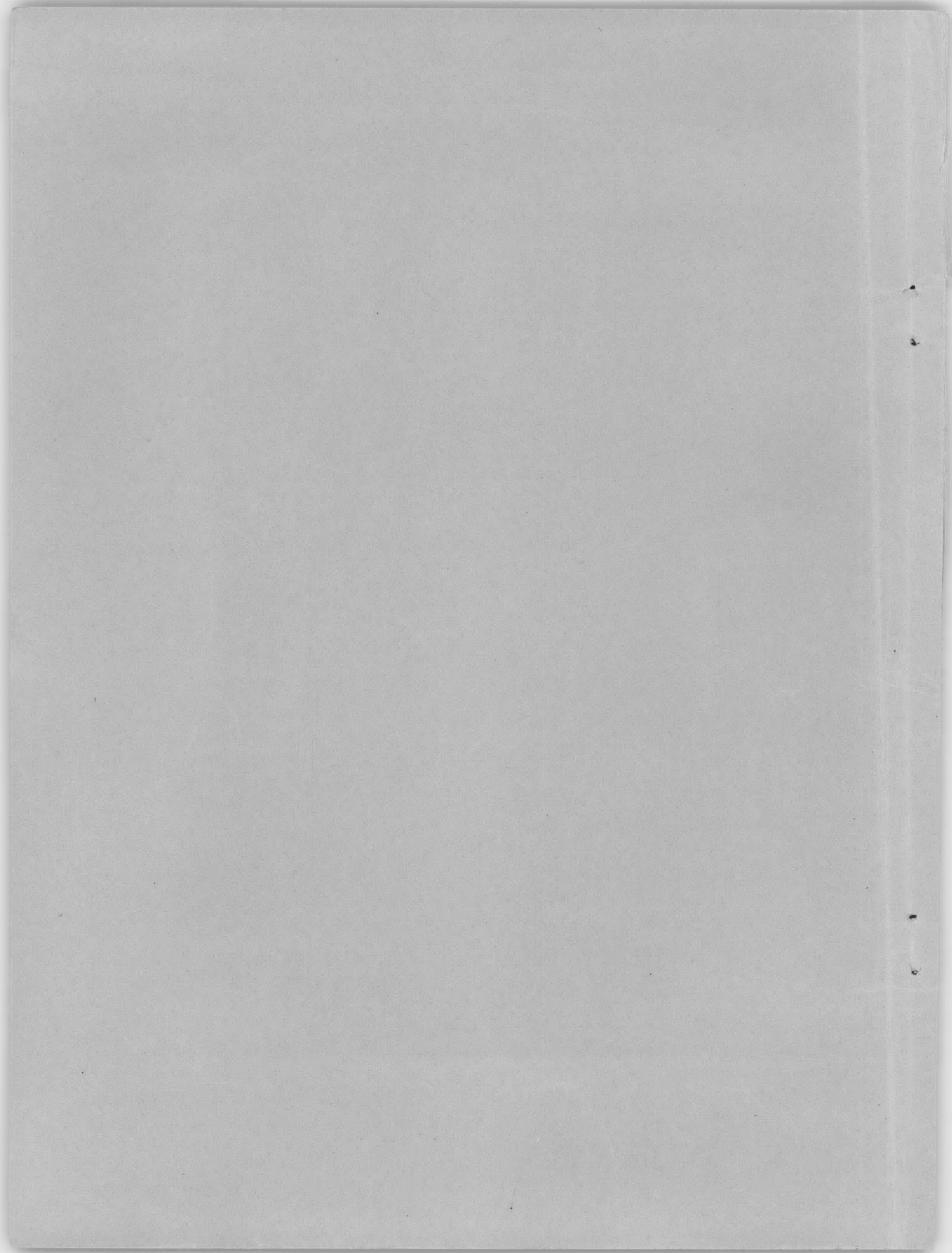
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STRUCTURAL VIBRATION PROBLEMS OF SHIPS -  
A STUDY OF THE DD692-CLASS DESTROYERS

by

Norman H. Jasper

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February 1950

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USS CHARLES R. WARE, DD865

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ABSTRACT

This report discusses the problem of the structural vibration that has been found to exist to an objectionable extent in the DD692 Class of destroyers. The major aim of the investigation reported herein was to determine the cause of the unusual sensitivity of these ships to first order unbalanced forces. It was hoped that means could be found to alleviate the existing situation.

The type of vibration was identified as an athwartship three-noded flexural vibration at a shaft speed near 240 rpm and a one-noded torsional mode of vibration at a shaft speed near 310 rpm. No significant differences in behavior were found to exist between the "long hull" and "short hull" destroyers of this class. The cause of the excessive vibration was found to be due to a lack of torsional and flexural rigidity of the hull girder together with the action of unbalanced forces in the propulsion system, which, although acceptable in more rigidly built ships, are of sufficient magnitude to cause excessive vibration on lightly built, high-speed ships, such as the DD692 Class destroyers. The principal remedial measures proposed are:

(a) Adoption of revised specifications for straightness and balance of propulsion shafting as well as for the maximum permissible runout at the propeller taper.

(b) Increasing the torsional and flexural rigidity of the hull girder by means of diagonal stiffeners.

The specifications referred to under (a) are suggested for all naval vessels.

The natural modes of vibration of the hull of the DD865 were determined by using a vibration generator. The amplitudes and modes of vibration of the submerged tail-shaft propeller system as well as the stresses in the strut arms were measured during a vibration-generator test. A simple method is given, on the basis of these measurements, for calculating the vibrational response of the hull girder at any resonance due to any combination of steady-state vibratory forces acting on the hull. The effects of ship design on the vibration characteristics of the hull are considered. Several special topics, such as the torsion of the ship girder and the rigidity of struts and strut bearings, are also discussed.

1. INTRODUCTION

The investigation of the vibration problem of the DD692-Class destroyers presented in this report was authorized by the Bureau of Ships<sup>1</sup> in 1948.

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<sup>1</sup>References are listed on page 112.

The unusual sensitivity of DD692-Class (long and short hull) destroyers to exciting forces at propeller-shaft frequency had been recognized for some time by all activities concerned. The result of vibration tests conducted previously on vessels of this class, which were experiencing severe vibrations, had been reviewed. Corrective measures taken had, in general, alleviated conditions somewhat. There had been no evidence furnished by these various tests as to the cause of this unusual vibratory response or as to the means which, if applied, could be relied upon to reduce the objectionable vibration to predetermined levels.

The necessity for a systematic investigation was evident. The David W. Taylor Model Basin proposed a test program<sup>2</sup> including a vibration-generator survey of a long-hull destroyer of this class. Tests based on this proposed program were carried out jointly by representatives of the Taylor Model Basin and the Material Laboratory of the New York Naval Shipyard during June and July 1948. These tests form the basis of the present report. The major aim of the investigation was to determine the cause of the unusual sensitivity of these ships to first-order unbalanced forces. It was hoped that some alterations might be accomplished which would reduce the sensitivity to unbalance or, if this were not practicable, to determine tolerances for balancing of shafting and propellers. A knowledge of the modes of vibration is necessary in analyzing the motion of a ship when subjected to external loads. It was the intention, during this series of experiments, to define as many modes of vibration of the hull girder as possible. The data obtained from the vibration generator tests were also to be used to check the validity of the several methods available for calculating the natural modes of vibration of ships.

The material reported herein can be grouped logically in several major subdivisions. The introduction and pertinent background are presented respectively in the first two sections. The third section gives the general theory applicable to the problem, and the test program is outlined in some detail in the fourth. The fifth section describes the instrumentation used in the experiments, the sixth the results derived from the test data and the seventh is an analysis of these results. The last two sections (eighth and ninth) summarize respectively the conclusions and recommendations.

## 2. HISTORY AND BACKGROUND

The DD692 (SUMNER) Class of destroyers has a history of unusually severe vibration. Consequently a number of vibration tests have been made on vessels of this class by various naval activities. The more important measurements--taken from 44 such tests--are tabulated in Table 1.



TABLE 1  
Hull Vibration Summary  
DD692 Class Destroyers (Short Hull)

Activity Making Test	DD Number Name and Date Tested	Propeller Blade Frequency Vibration						First Order Vibration						Remarks
		Location	Direction	Order	Prop. RPM	VPM	Ampl. mils	Location	Direction	Prop. RPM	Ampl. mils	Units	Port	
New York Naval Shipyard	692 SUMNER 12 Jun 44	Director	Athwart-ships	3	200	600	15	No First Order Criticals Observed						1st Hull Tested
New York Naval Shipyard	708 DICKSON 6 Aug 45	Bridge	Athwart-ships	3	200	600	8	Bridge	Athwart-ships	270	10			
		Fantail	Athwart-ships	3	200	600	4	Fantail	Athwart-ships	270	8			
Mare Island Naval Shipyard	777 ZELLERS 17 Nov 44	Director	Athwart-ships	3	200	600	10	Director	Athwart-ships	250	27			
									Athwart-ships	330	38			
Puget Sound Naval Shipyard	777 ZELLERS 3 Jan 45	Director	Athwart-ships	3	110	330	4	Director	Athwart-ships	240	11			Pre-Repair. A deck survey showed the existence of a 3-noded transverse mode of vibration at about 250 CPM.
				3	200	600	6		Athwart-ships	320	23			
		Note: The vessel was equipped with double-arm intermediate struts												
Puget Sound Naval Shipyard	777 ZELLERS 22 Jan 45	Director	Athwart-ships	3	180	540	7	Director	Athwart-ships	240	6			Post-Repair
									Athwart-ships	330	18			
		Note: During overhaul new tailshafts were installed with runout less than 8 mils and new propellers were installed having an unbalance less than 70 in-ounces. The auxiliary intermediate strut arms were removed, the main arms were reinforced.												
Mare Island Naval Shipyard	778 MASSEY 7 Jan 45	Director	Athwart-ships	3	200	600	6	Director	Athwart-ships	240	20			
									Athwart-ships	320	32			
Puget Sound Naval Shipyard	778 MASSEY 11 Feb 45			3				Director	Athwart-ships	240	6			Post-Repair
									Athwart-ships	320	37			
		Note: The auxiliary intermediate strut arms were removed, main arms reinforced. Propellers were balanced to within about 80 in-ounces. Runout at propeller taper was less than 4 mils.												
New York Naval Shipyard	778 MASSEY 29 Apr 46	Bridge	Athwart-ships	3	82	246	4	Bridge	Athwart-ships	255	42	7	35	Pre-Repair
		Fantail	Athwart-ships	3	82	246	6	Fantail	Athwart-ships	255	45	8	37	
New York Naval Shipyard	778 MASSEY 28 Jun 46							Bridge	Athwart-ships	255	22	5	17	Post Repair
		Fantail	Athwart-ships	3	215	645	6	Fantail	Athwart-ships	255	14	2	12	
New York Naval Shipyard	780 STORMES 29 Jun 46	Bridge	Athwart-ships	3	200	600	6	Bridge	Athwart-ships	255	22	20	6	Drafts:- Fwd. 11'-6" Aft. 14'-0"
		Fantail	Athwart-ships	3	110	330	6	Fantail	Athwart-ships	260	13		13	Displacement 2944 Tons
			Athwart-ships	3	200	600	6							
Puget Sound Naval Shipyard	745 BRUSH 16 Jan 46	Director	Athwart-ships	3	194	582	7	Director	Athwart-ships	240	5			
			Athwart-ships	3	240	720	2.5		Athwart-ships	320	18			
			Athwart-ships	3	320	960	1.5							

TABLE 1 (continued)  
Hull Vibration Summary - DD692 Class Destroyers (Long Hull)

Activity Making Test	DD Number Name and Date Tested	Propeller Blade Frequency Vibration						First Order Vibration						Remarks	
		Location	Direction	Order	Prop. RPM	VPM	Ampl. -mils	Location	Direction	Prop RPM	Ampl Both Units	Port Only	Stbd. Only		
New York Naval Shipyard	711 GREENE 26 Jun 45	Bridge	Athwart-ships	3	190	570	4	Bridge	Athwart-ships	305	11				
New York Naval Shipyard	712 GYATT 23 Jul 45	Bridge	Athwart-ships	3	200	600	5	Bridge	Athwart-ships	305	10				
New York Naval Shipyard	742 KNOX Dec 44 and Feb 45	Director	Athwart-ships Athwart-ships	3 4	200 150	600 600	14 2	Director 3-bladed Str. Eng Rm. 4-bladed	Athwart-ships Athwart-ships Athwart-ships	300 to 315 235 300	very large 14 12				
New York Naval Shipyard	817 CORRY 24 May 47	Bridge	Athwart-ships Athwart-ships Athwart-ships	3 3 3	80 100 200	240 300 600	8 6 7	Bridge	Athwart-ships Athwart-ships Vertical	240 310 260	20 19 10	15	9	Pre-Repair Displacement 3100 Tons	
New York Naval Shipyard	817 CORRY 27 Aug 47	Bridge	Athwart-ships Athwart-ships Athwart-ships	3 3 3	80 100 200	240 300 600	8 4 6	Bridge	Athwart-ships	240	15			Post-Repair	
Note: Propellers were balanced during repair period															
New York Naval Shipyard	821 JOHNSTON 26 Aug 47	Bridge	Athwart-ships Athwart-ships Athwart-ships	3 3 3	85 110 200	255 330 600	2 4 5	Bridge	Athwart-ships	250	18	17		Drafts Fwd. 12'-0" Aft. 13'-9" Displacement 3000 Tons	
David Taylor Model Basin	882 FURSE 31 Aug 45	Fantail Bridge	Athwart-ships Athwart-ships	3 3	180 220	540 660	7 13	Fantail Bridge	Athwart-ships Athwart-ships	240 240	28 28				
Mare Island Naval Shipyard	882 FURSE 28 Oct 48	Small Amount of 4th Order Vibration						Director	Athwart-ships Athwart-ships	240 310	6.5 8.5				Displacement 3150 Tons 4-bladed pro-pellers
New York Naval Shipyard	832 HAMSON 25 Apr 47	Bridge Fantail	Athwart-ships Athwart-ships Athwart-ships Athwart-ships	3 3 3 3	80 200 80 200	240 600 240 600	8 6 7 5	Bridge Fantail	Athwart-ships Athwart-ships	240 240	20 20	18 15	4 4	Drafts:- Fwd. 13.5' Aft. 13.8'	
New York Naval Shipyard	833 THOMAS 25 Apr 47	Bridge	Athwart-ships Athwart-ships Athwart-ships	3 3 3	80 120 200	240 360 600	4 3 5	Bridge	Athwart-ships	240	13	10	7	Draft: Fwd. 13'-0" Aft. 13'-6" Displacement 3070 Tons	
New York Naval Shipyard	841 NOA 24 May 47	Bridge	Athwart-ships Athwart-ships Athwart-ships	3 3 3	84 110 200	252 330 600	7 7 5	Bridge	Athwart-ships Athwart-ships Vertical	250 315 260	18 21 10	14	5	Displacement 3350 Tons	
Boston Naval Shipyard	841 NOA 16 Dec 48	Not Determined						Director	Athwart-ships Athwart-ships	235 310	7.5 11				Displacement 3200 Tons 4-bladed pro-pellers
New York Naval Shipyard	843 WARRINGTON 24 May 47	Bridge	Athwart-ships Athwart-ships	4 4	85 150	340 600	1 3	Bridge	Athwart-ships Athwart-ships Vertical	240 310 260	24 18 8	12	17	Pre-Repair Displacement 3140 Tons	
New York Naval Shipyard	843 WARRINGTON 10 Sep 47	Blade Frequency Vibration Not Investigated for Post-Repair Trial Note: During repair period, propellers were balanced and starboard tail shaft straightened						Bridge	Athwart-ships	235	18	7	15	Post-Repair Draft Fwd. 12'-7" Aft. 14'-7" Displacement 3210 Tons	

TABLE 1 (continued)  
Hull Vibration Summary  
DD692 Class Destroyers (Long Hull)

Activity Making Test	DD Number Name and Date Tested	Propeller Blade Frequency Vibration						First Order Vibration						Remarks
		Location	Direction	Order	Prop. RPM	VFM	Ampl. $\pm$ mils	Location	Direction	Prop RPM	Ampl. $\pm$ mils	Both Units Only	Port Only	
New York Naval Shipyard	844 PERRY 24 May 47	Bridge	Athwart-ships	3	80	240	7	Bridge	Athwart-ships	244	13	12	8	Displacement 3000 Tons
			Athwart-ships	3	110	330	5		Athwart-ships	325	10			
			Athwart-ships	3	200	600	6		Vertical	260	5			
Mare Island Naval Shipyard	883 NEWMAN K PERRY 26 Oct 48	Small Amount of 4th Order Vibration						Director	Athwart-ships	230	6			Displacement 3235 Tons 4-bladed pro-pellers
							Athwart-ships	310	14					
New York Naval Shipyard	865 WARE 9 Oct 45	Bridge	Athwart-ships	3	80	240	3	Bridge	Athwart-ships	310	16	11	5	
			Athwart-ships	3	200	600	5		Athwart-ships	240	6			
New York Naval Shipyard	865 WARE 22 Sep 47	Bridge  No. 3 Gun Mt.	Athwart-ships	3	80	240	7	Bridge	Athwart-ships	240	27			Displacement 3272 Tons
			Athwart-ships	3	110	330	5		Athwart-ships	310	28			
			Athwart-ships	3	200	600	8	No. 3 Gun Mt.	Athwart-ships	240	7			
			Athwart-ships	3	190	570	8		Athwart-ships	310	17			
			Athwart-ships	3	260	780	6							
			Vertical	3	140	420	6							
			Vertical	3	260	780	6							
David Taylor Model Basin	865 WARE 21 Jun 48	Fantail Director	Athwart-ships	4	140	560	2	Fantail	Athwart-ships	230	24	16	12	Displacement 3415 Tons Pre-Repair Sea Calm
			Athwart-ships	4	130	520	2		Athwart-ships	310	24	16	14	
							Director	Athwart-ships	230	20	12	10		
								Athwart-ships	310	24	12	12		
David Taylor Model Basin	865 WARE 22 Sep 48	Small Amount of Blade Frequency Vibration						Fr. 188	Athwart-ships	230	9			Displacement  Post-Repair Sea Varied From Calm to Rough
								Athwart-ships	330	10				
							Director	Athwart-ships	230	10				
								Athwart-ships	330	12				
New York Naval Shipyard	866 CONE 7 Sep 45	Bridge	Athwart-ships	4	60	240	2	Bridge	Athwart-ships	240	7			
			Athwart-ships	4	150	600	4		Athwart-ships	300	9			
New York Naval Shipyard	878 VESOLE 30 Jan 47	Bridge	Athwart-ships	3	80	240	3	Resonance at 240 RPM not Reached Due to Fog						Average Draft 13.9'
			Athwart-ships	3	200	600	6							
New York Naval Shipyard	886 ORLECK 10 Dec 45	Bridge  Str. Eng. Rm.	Athwart-ships	3	80	240	4	Bridge	Athwart-ships	240	19	16	2	
			Athwart-ships	3	190	570	6		Str. Eng. Rm.	Athwart-ships	240	23	22	
			Athwart-ships	3	80	240	5							
			Athwart-ships	3	190	570	6							
New York Naval Shipyard	887 BASS 14 Dec 45	Bridge  Str. Eng. Rm.	Athwart-ships	3	80	240	5	Bridge	Athwart-ships	240	6	5	4	
			Athwart-ships	3	190	570	6		Str. Eng. Rm.	Athwart-ships	240	7	6	
			Athwart-ships	3	80	240	7							
			Athwart-ships	3	190	570	5							
New York Naval Shipyard	890 MEREDITH 13 Dec 46	Bridge	Athwart-ships	3	90	270	10	No "240" RPM Critical Present						Draft Fwd. 11'-3" Aft. 11'-10" Displacement 2525 Tons
			Athwart-ships	3	200	600	6							

TABLE 1 (continued)  
 Hull Vibration Summary  
 DD692 Class Destroyers (Long Hull)

Activity Making Test	DD Number Name and Date Tested	Propeller Blade Frequency Vibration						First Order Vibration					Remarks	
		Location	Direction	Order	Prop. RPM	VFM	Ampl. $\frac{1}{2}$ mils	Location	Direction	Prop. RPM	Ampl. $\frac{1}{2}$ mils	Ampl. $\frac{1}{2}$ mils		
										Both Units	Port Only	Starboard Only		
Mare Island Naval Shipyard	715 WOOD 31 Jan 49	Frame 198 Main Deck	Athwart-ships	3	220	660	27	Director at Frame 72	Athwart-ships	240	20			Displacement 3121 Tons Draft Fwd. 13'-3" Aft. 13'-6" 3-bladed propellers 5 - 8 foot waves
			Athwart-ships					Frame 198 Main Deck	Athwart-ships					
			Athwart-ships					1st Flat.	Athwart-ships					
			Athwart-ships					1st Flat.	Athwart-ships					
Puget Sound Naval Shipyard	785 HENDERSON 1 Oct 45	Propellers were Badly Unbalanced Runout at Propeller Taper was 8.5 mils at the Port Shaft, 3 mils at the Starboard Shaft.		3				Frame 200	Athwart-ships	240	19	17	9	Pre-Repair 3-bladed propellers
			Director					Athwart-ships						
Puget Sound Naval Shipyard	785 HENDERSON 18 Oct 45	Note: The port tailshaft was replaced with one having a propeller taper runout of 3.5 mils. The starboard shaft was balanced.	Director	3	210	630	8	Director	Athwart-ships	240	12			Post-Repair 1 3-bladed propellers
								Athwart-ships						
Puget Sound Naval Shipyard	785 HENDERSON 30 Oct 45	Note: A four-bladed propeller had been installed. This materially improved the ability to range and reduced machinery vibration.	Director	4	150	600	2.5	Director	Athwart-ships	240	8			Post-Repair 2 4-bladed propellers
								Athwart-ships						
Boston Naval Shipyard	820 RICH 4 Jan 49	Small Amount of 4th Order Vibration						Director	Athwart-ships	240	6			Displacement 3100 Tons 4-bladed propellers
Mare Island Naval Shipyard	834 TURNER 29 Oct 48	Pronounced Degree of 4th Order Vibration						Director	Athwart-ships	240	9			Displacement 3000 Tons 4-bladed propellers
								Athwart-ships						
Mare Island Naval Shipyard	835 CECIL 30 Oct 48	4th Order Vibration Present to an Appreciable Degree						Director	Athwart-ships	235	7			Displacement 3000 Tons 4-bladed propellers
								Athwart-ships						

TABLE 1 (continued)  
Hull Vibration Summary  
Other Classes of Destroyers

Activity Making Test	Class and Ship Tested	Propeller Blade Frequency Vibration						First Order Vibration					Remarks	
		Location	Direction	Order	Prop. RPM	VPM	Ampl. <del>in</del> mils	Location	Direction	Prop. RPM = VPM	Ampl. <del>in</del> mils	Both Units Only		Port Only
New York Naval Shipyard	DD75-185 147							Fantail	Athwart-ships	220	10			
New York Naval Shipyard	DD186-347 221							Bridge	Athwart-ships	230	2			
New York Naval Shipyard	DD380 382							Bridge	Athwart-ships	225	24			
New York Naval Shipyard	DD380 400							Bridge Fantail	Athwart-ships Athwart-ships	225 225	15 8			
New York Naval Shipyard	DD409 420							Bridge	Athwart-ships	220	4			
New York Naval Shipyard	DD421 425	Director	F and A	3	150	450	24	Director 4-bladed	Athwart-ships	210	14			
			Athwart-ships	4	112	450	9							
				3	140	420	14							
				4	105	420	3	Str. Eng. Rm. 4-bladed	Athwart-ships	210	12			
	427	Fantail	Vertical	3	170	510	12	Bridge	Athwart-ships	230	17			
				3	250	750	25			288	14			
				3	288	860	20	Fantail	Athwart-ships	230	16			
	604							Bridge	Athwart-ships	225	12			
	615							Bridge	Athwart-ships	220	7			
New York Naval Shipyard	DD445 502							Number 5 Gun	Athwart-ships Athwart-ships	180 210	20 19			
Mare Island Naval Shipyard	DD445 540	Fantail	Athwart-ships Vertical	3	175	525	4							
				3	245	735	8							
New York Naval Shipyard	DD445 470							Bridge	Athwart-ships	335	6			

TABLE 1 (continued)  
Hull Vibration Summary  
Other Classes of Destroyers

Activity Making Test	Class and Ship Tested	Propeller Blade Frequency Vibration						First Order Vibration						Remarks
		Location	Direction	Order	Prop. RPM	VPM	Ampl. <del>in</del> g's	Location	Direction	Prop. RPM =VPM	Ampl. Both Units	Port Only	Stbd Only	
New York Naval Shipyard	DD453 496	Bridge	Athwart-ships	3	200	600	5	Fantail	Vertical	280	4			
			Athwart-ships	3	200	600	5							
		Vertical	3	130	390	4								
		Vertical	3	200	600	3								
	DD453 619	Bridge	Athwart-ships	3	170	510	3	Bridge	Athwart-ships	230	5			
		Fantail	Athwart-ships	3	170	510	2	Fantail	Athwart-ships	230	5			
	DD453 624							Bridge	Athwart-ships	235	35			
								Str. Eng. Rm.	Athwart-ships	290	12			
Athwart-ships									235	30				
Athwart-ships									290	17				
DD453 634							Bridge	Athwart-ships	220	20				
							Str. Eng. Rm.	Athwart-ships	280	12				
Athwart-ships	220	25												
DD453 639	Bridge	Athwart-ships	3	150	450	10	Bridge	Athwart-ships	215	16				
DD453 637							2nd Deck Fr. 183	Athwart-ships	280	13				
DD453 645	Bridge	Athwart-ships	3	150	450	8	Bridge	Athwart-ships	230	28				
	Fantail	Athwart-ships	3	150	450	3	Fantail	Athwart-ships	230	18				
New York Naval Shipyard	DD459 460	Director Base	F and A Athwart-ships	3	155	465	12	Director Base	Athwart-ships	225	39			
			Athwart-ships	3	150	450	7							
			Vertical Athwart-ships	3	155	465	8							
			Athwart-ships	3	150	450	5							
Mare Island Naval Shipyard	DD459 460	Director	F and A	3	151	453	15	Director	Athwart-ships	232		50	28	
								Bridge	Athwart-ships	225	12			
New York Naval Shipyard	DD459 601	Bridge	Athwart-ships	3	150	450	5	Bridge	Athwart-ships	230	13			
		Fantail	Athwart-ships	3	150	450	4	Fantail	Athwart-ships	230	13			



The destroyers in this class are of two types, the so called "Short Hulls" and the "Long Hulls." The principal characteristics of the two types of hulls are indicated in Table 2.

TABLE 2

## Comparison of the Principal Characteristics of Destroyers

	DD692 - Long Hull	DD692 - Short Hull
Length between perpendiculars	383' - 0"	369' - 0"
Length overall	390' - 6"	376' - 6"
Extreme breadth Molded	40' - 10 9/16"	40' - 10 9/16"
Draft above Bottom of Keel Amidships, Designers	13' - 0 1/2"	13' - 0 1/2"
Standard Displacement	2425 Tons	2200 Tons
Hogging Condition (full load)		
Max Bending Moment, Frame F	51,170 Ft-tons	-
Max Shear Forward, Frame 74	526.6 Tons	-
Max Shear Aft, Frame 148	534.5 Tons	
3-Bladed Propellers		
Tip clearance	2' - 6"	2' - 6"
Diameter	12' - 2"	12' - 2"
Pitch	12' - 7"	12' - 7"
4-Bladed Propellers		
Tip clearance	2' - 6"	2' - 6"
Diameter	12' - 2"	12' - 2"
Pitch	12' - 2"	12' - 2"
Propeller Thrust (Each propeller)		
350 rpm	191,000 lb	Allowance for normal fouling included
310 rpm	159,000 lb	
246 rpm	88,000 lb	

The long hull is identical with the short hull except for the addition of a 14-foot section amidships. The DD445 (FLETCHER) Class destroyers closely approximate the molded dimensions of the short hulls, and exhibit a pattern of vibration which appears to be somewhat similar to that found on the DD692 Class.

The DD692, the first ship of the class under investigation, evidenced excessive third-order vibration of the hull and of machinery items when tested in June 1944, but no vibrations of propeller-shaft frequency were encountered. However, subsequent vibration surveys of other ships of this class have indicated the presence of large-amplitude first-order athwartships vibration, such as could be excited by mass unbalance of the propulsion

shafting. The first-order vibration exhibits a definite resonance pattern with criticals at about 240 and 310 propeller rpm for the short hulls. This first-order vibration has been particularly objectionable at the bridge, at the fantail and at the foremast. The foremast has a natural frequency near 320 cpm and therefore amplifies the motion to which its base is subjected--resulting in fairly large accelerations of the radar antenna which is situated atop this mast. Several structural failures of these antennas have occurred because of excessive vibration. The large first-order vibrations exist even though the shaft propeller balance and the propeller pitch are within specification allowances. Both the short and long hulls are subject to this first-order vibration to about the same degree of severity. There is no known previous history of severe first-order vibration on turbine or electric-drive naval vessels. The indication is therefore that this class of ships is unusually sensitive to the type and manner of application of the first-order exciting forces acting on the vessel.

The propeller-blade frequency vibration which principally affected the machinery items and the ability to range with the main gun directors, was reduced to an acceptable magnitude by the installation of four-bladed NACABS propellers. This reduction in severity of vibration is due to (a) the reduction in exciting force per blade at a given rpm by about 30 percent and (b) the lowering of the critical speed (since the thrust per propeller blade varies about as the square of the propeller rpm, the lowering of the critical speed by 25 percent will reduce the exciting force by about 44 percent).

The first-order vibration was not greatly affected by the installation of four-bladed propellers, although some improvement was generally noted as illustrated by a comparison of the DD785 tests of 18 and 30 October 1945, see Table 1. In addition to the first-order vibration, a large-amplitude low-frequency vibration was reported on several tests. This vibration was at a constant frequency, about 140 cpm, independent of speed. It will be shown later in this report that the latter vibration corresponds to the two-noded mode of transverse flexural vibration excited by wave forces or other hydrodynamic excitations.

The 692-Class destroyers were originally fitted with single-arm cantilever-type intermediate struts. A considerable number of failures of these struts occurred due to cracks in the strut near the point of attachment to the hull. Strain measurements made on single-arm struts of the USS WALKE (DD723)<sup>3,4</sup> showed the presence of large vibratory stresses during sharp turns. The intermediate struts were provided with an additional strut arm<sup>5</sup> and stress measurements were made on this modified strut arm as applied to the SUMNER (DD692).<sup>6</sup> These measurements showed a considerable reduction in stress.

Similar measurements were made some time later in 1944 by the California Institute of Technology.<sup>7,8</sup> There have been reports stating that the level of vibration increased after installation of the double-arm strut; however no actual data are available which would substantiate a conclusion to that effect. It was suggested<sup>9</sup> that the tailshaft might have a natural whirling frequency near the upper hull critical (320 cpm), and such a condition would readily explain the sensitivity of these ships to first-order vibration. In order for this to be true it would be necessary for the intermediate strut to vibrate in such a way that it would offer little restraint to the shaft.

The cause of the first-order vibration was generally believed to be due to mass unbalance of the propeller-shafting system. In order to determine the relative contribution of each shaft it was usual practice to operate at the critical rpm (about 240 and 320 rpm) with both shafts simultaneously and then with one shaft dragging. If severe vibration was indicated the propellers were removed, checked for uniformity in pitch and then balanced. Generally this treatment would result in reducing the first-order vibration by about 20 percent.

It is to be noted that a runout at the propeller is equivalent to a mass unbalance of the propeller. In a few cases, for example Puget Sound Naval Shipyard tests of the DD785 in October 1945<sup>10</sup> and the DD777 in January 1945<sup>11</sup>, both the propellers and the tailshafts were balanced and checked for runout. Unfortunately the intermediate struts were altered at the same time, making it impossible to separate out the contribution of the several changes. In the DD785 tests<sup>10</sup> an attempt was made to determine the effect of a known unbalance of the line shafting on the first order vibration. A 50-lb weight was strapped to each shaft producing about 500 inch-pounds added unbalance on each shaft. Records of the vibration amplitude in the main fire-control director at 235 rpm showed an increase of 50 percent above those measured without the added unbalance. Since the amount and phase of the inherent shaft and propeller unbalance relative to the added unbalance was unknown, these data were inconclusive. It was apparent from the various tests, however, that the magnitude of the first-order vibration may be reduced by reducing the effective unbalance of the propellers and propulsion shafting.

The two hull criticals of about 240 cpm and 320 cpm frequency referred to previously, have generally been associated with transverse vibration, but only the lower critical mode was identified,<sup>10, 11</sup> prior to the present tests as the three-noded mode of athwartships flexural vibration. The higher mode, occurring at about 320 cpm, was erroneously thought to be a multinoded mode of athwartships vibration.

To summarize this section - it has been found that the DD692-Class destroyers are highly sensitive to exciting forces of propeller-rpm frequency such as are produced by effective mass unbalance of the propulsion system. The vessels tested had first-order criticals of athwartship vibration near 240 and 320 propeller rpm. This vibration was predominant even though shafting and propellers met Bureau of Ships specifications for straightness and balance. The excessive propeller-blade frequency vibration originally present in these ships was reduced to an acceptable value by replacing the original three-bladed propellers with four-bladed NACAB propellers. The structural failures of the single-arm intermediate struts were stopped by fitting an auxiliary strut arm inboard of the strut.

### 3. THEORETICAL CONSIDERATIONS

A relatively small harmonic exciting force, when applied to a massive elastic structure such as a ship, can produce appreciable vibratory motion of the structure. The amplitude of this vibratory motion depends principally on the magnitude of the exciting force, the point and frequency of application, and the magnitude and character of the damping forces.

The vibratory motion of the ship may, in general, be resolved into its normal modes of vibration. A force acting at a node cannot excite the mode corresponding to that node but can and does excite other modes. The effectiveness in exciting vibration in a given mode is proportional to the ordinate of the amplitude profile at the point of application of the force. For example, if the propeller were vibrating at a much larger amplitude than the after strut barrel, for a particular mode, then a force applied at the propeller would produce correspondingly much larger hull vibration than the same force applied to the after strut barrel. If, as has been suggested, the intermediate strut does not offer any appreciable restraint to transverse motion of the tailshaft, then the fundamental mode of shaft whirling would probably fall close to 320 cpm and cause large vibratory amplitudes at the corresponding shaft speed. This in turn would mean that the effectiveness of any unbalance forces in the tailshaft in exciting vibration would be high.

The most effective position of load application for purposes of vibration-generator tests are at the fore and aft extremities of the vessel; here maximum effectiveness is obtained for all modes of vibration. It is necessary to provide a stiff connection between the vibration generator and the main structure of the ship girder in order to minimize bothersome local resonance effects.

Experience has shown the necessity of having the depth of water equal to or greater than six times the draft of the vessel in order practically to eliminate the effect of the sea bottom on the vibration characteristics associated with vertical flexural vibrations. An electrical analogue of this bottom effect on the virtual mass has been described by J.J. Koch in an article entitled "Eine Experimentelle Methode zur Bestimmung der Reduzierten Masse des Mitschwingenden Wassers bei Schiffsschwingungen" Ingenieur Archiv, IV Band, 2 Heft 1933.\*

Vibration of the hull or of its component parts may be excited by impulsive forces such as caused by the action of gunblast; the resulting vibratory motion is a transient vibration. The more usual type of vibration encountered is excited by periodic forces which, in the great majority of cases, emanate from the ship's machinery or its propellers, although external excitation is also possible. The latter motions are of the forced steady-state type. Forced vibrations on shipboard are seldom noticeable unless the frequency of the exciting force is near a resonance frequency of the ship or of a component part of the ship. Since a ship and its structures may have many modes of vibration, the opportunity for such resonances are manifold.

Revolving propellers and propulsion shafting are the major causes of ship vibration in modern turbine-driven vessels. These vibrations may be due to effective unbalance of the propellers and propulsion shafting or due to hydrodynamic interaction between the propeller blades and the water and hull. The predominant forces causing hull vibration are generally due to the action of the propeller blade suction forces on the hull and its appendages. The latter forces are known as hydrodynamic surface forces. Objectionable vibrations due to mass unbalance of the propeller or shafting are seldom found on modern ships.

An unbalanced propeller or propulsion shaft exerts a revolving force on the ship which has a frequency of one per revolution. A runout at the propeller shaft taper is equivalent to a mass unbalance of the propeller, that is, the center of rotation and the center of mass of the propeller do not coincide. Similarly, if there are variations in pitch between the several propeller blades, a rotating force having a frequency of one per revolution will exist.

The possible methods of reducing or avoiding excessive vibration in general are as follows:

1. Elimination or reduction of exciting forces

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\*Recently published in English as TMB Translation 225.

ii. Avoidance of resonance between the periodic exciting forces and the frequencies corresponding to the natural modes of vibration of the structures.

iii. Use of vibration neutralizers or dampers.

Of the above methods the first one is by far the most effective. Because it is often impracticable to eliminate the exciting forces, however, it would then be advisable to look into the possibility of avoiding resonance. The third course is the least desirable, but is sometimes the only solution practicable. Damping forces, which are forces opposing the motion of the system, have a decisive influence on the response of the system near the resonance frequency. It is this damping force alone which keeps the motion of the vibrating body to a finite value. It is well to keep the latter point in mind.

An analysis will be made of the motion of the propeller-tailshaft system in order to:

(a) Determine the extent to which the elastic properties of the system cause it to behave differently from a rigid rotor. In effect this requires the determination of the deflection curve of the shaft as a function of the rpm. It is necessary to adjust the boundary conditions so that the calculated and experimentally determined deflection curves agree.

(b) Find a method which will be suitable for calculating the natural whirling speed of the propeller-tailshaft system.

The effect of the elastic properties of the tailshaft system can be reduced by assuring that the whirling frequency of the system is sufficiently removed from the operating range of propeller rpm. If the whirling speed occurs within the running range then, assuming zero damping, the amplitudes will become infinite.

A schematic sketch of the propeller tailshaft system is shown in Figure 1a. The shaft can be considered as a beam of flexural rigidity EI subjected to bending loads.

The differential equation governing the motion of beams is

$$\frac{\partial^2 M}{\partial x^2} = p \quad \text{where} \quad \begin{array}{l} x \text{ is the horizontal coordinate} \\ M \text{ is the bending moment at } x \\ p \text{ is the transverse load per} \\ \text{unit length} \end{array} \quad [1]$$

this can be written as

$$\frac{\partial^2}{\partial x^2} \left[ EI \frac{\partial^2 y}{\partial x^2} + P(y' + e) \right] = p = -\mu \frac{\partial^2 e}{\partial t^2} - \mu \frac{\partial^2 \gamma}{\partial t^2} - \mu \frac{\partial^2 y}{\partial t^2} \quad [2]$$

where P is the propeller thrust

$\mu(x)$  is the mass per unit length at x

$y(x)$  is the absolute deflection of the geometric shaft center at x,  $y = Y + y'$



- $Y$  is the absolute motion of the bearings,  
 $y'(x)$  is the elastic motion of the geometric shaft center relative to the bearings,  
 $e(x)$  is the crookedness of the shaft at  $x$ , i.e., deviation from a straight shaft,  
 $\gamma(x)$  is the mass eccentricity relative to the geometric center at any point  $x$ , and  
 $t$  is the time

An attempt was made to solve Equation [2] by means of an iteration process, under the simplifying assumption that  $Y$  and  $\gamma$  were constants and that  $e$  could be represented by a sine function. The attempt proved unsuccessful because of the complexity of the expression (ii) and due to the lack of knowledge as to the boundary conditions.

Next an attempt was made to determine the relative importance of the propeller thrust  $P$  and the propeller weight  $W$  in determining the natural whirling frequency and the mode of vibration, under the assumption of several different boundary conditions. The results of this analysis are given in Table 3.

There are several methods available for determining approximate values of frequencies of mass elastic systems. These several methods can be applied directly if the boundary conditions are known. In the case under consideration the boundary conditions are not definitely known but the experimental value of the frequency of the fundamental mode of flexural vibration and the approximate mode shape are known, see Figure 35 on page 62.

In the "Rayleigh Method" the total maximum kinetic energy of the system is equated to the maximum potential energy of the system. The resulting equality can be solved for frequency directly if the assumed analytical mode configuration satisfies the boundary conditions. If the assumed analytical expression for the mode configuration involves unknown terms which are functions of the restraints, then it is necessary to minimize the difference between the kinetic and potential energies in order to determine the unknown terms. The solution then proceeds as before. It should be noted that the latter method gives the optimum mode shape out of a family of possible shapes. The last method has been termed the "Rayleigh-Ritz Method." An excellent method for determining the fundamental mode shape as well as the associated frequency is the Stodola Method<sup>12, 13</sup> provided that the boundary conditions are known.

The analysis in Table 3 is based on the assumption that the boundary conditions applicable to the physical propeller-tailshaft installation, illustrated by Figure 1a, are approximately as shown in Figure 1b. The sine

TABLE 3.

Frequency Equations for the Fixed-Pinned Beam Illustrated in Figure 1b

Case	Overhang d	Propeller Thrust	Propeller Mass	Assumed Mode Shape y From Figure 1	CPM Natural Frequency - Rayleigh Method based on Assumed Mode Shape
1	0	0	0	Theoretically exact Shape	$f = 148 \sqrt{\frac{EI}{\mu L^4}} = 920$ (Theoretical value)
2	0	0	0	$y = 2 \sin \frac{\pi x}{L} + \sin \frac{2\pi x}{L}$	$f = 60\pi \sqrt{\frac{EI}{\mu L^4}} = 1170$ (Rayleigh Method)
3	0	0	0	$y = 2 \sin \frac{\pi x}{L} + \sin \frac{2\pi x}{L}$	$f = \frac{30}{\pi} \sqrt{\frac{6EI\pi^4}{\mu L^4}} = 1440$
4	0	P 159,000 lb	0	do	$f = \frac{30}{\pi} \sqrt{\frac{6EI\pi^4}{\mu L^4} - \frac{2P\pi^2}{\mu L^2}} = 1438$
5a	d	0	m	do	$\omega^2 = \frac{4EI\pi^4}{L^4} \left\{ \frac{\frac{5\pi}{2}(1+\alpha) + 2\sin\pi\alpha + \frac{1}{4}\sin 2\pi\alpha + \frac{2}{3}\sin 3\pi\alpha - \frac{1}{2}\sin 4\pi\alpha}{\mu \left[ \frac{5\pi}{2}(1+\alpha) + 2\sin\pi\alpha - \sin 2\pi\alpha - \frac{2}{3}\sin 3\pi\alpha - \frac{1}{8}\sin 4\pi\alpha \right] + \frac{m\pi}{L} [2\sin\pi\alpha + \sin 2\pi\alpha]^2} \right\}$ $\omega^2 = \frac{4\pi^2 r^2}{60^2}, \quad \alpha = \frac{d}{2} \quad \text{(Rayleigh Method)}$ <p>f = 750 CPM</p>
5b	0	0	0	do	f = 1170 (Case 5 reduces to Case 2)

By substituting the assumed mode shape into the differential equation

- Cases 1 through 5 are illustrated by Figure 1.
- The numerical calculations in the table are carried out for the following values of the parameters.
 

$m = 40 \frac{\text{lb sec}^2}{\text{in.}}$	$d = 64 \text{ in.}$	$L = 400 \text{ in.}$	$\mu = 0.084 \frac{\text{lb sec}^2}{\text{in.}^2}$
$I = 2740 \text{ in.}^4$	$E = 30 \times 10^6 \frac{\text{lb}}{\text{in}^2}$	The shaft is assumed to be straight and balanced	
- The experimental value of the frequency associated with the fundamental mode of shaft whipping in air was 660 cpm.

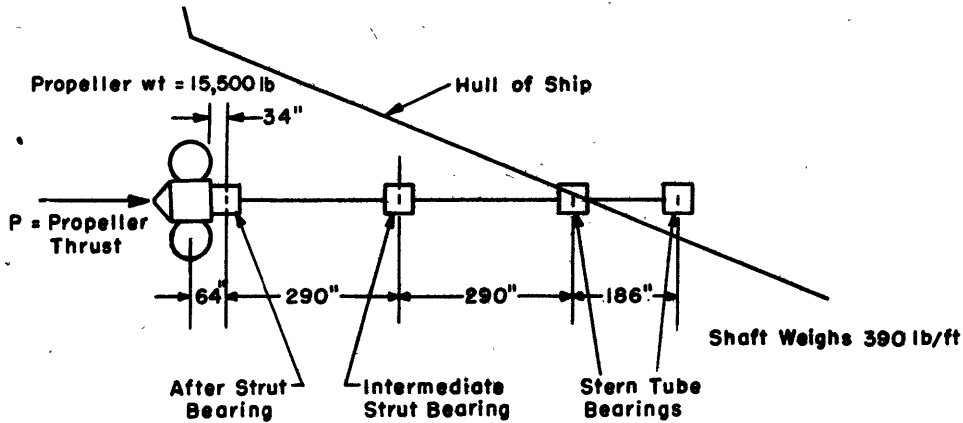


Figure 1a - Schematic Arrangement of Propeller and Propeller Shaft

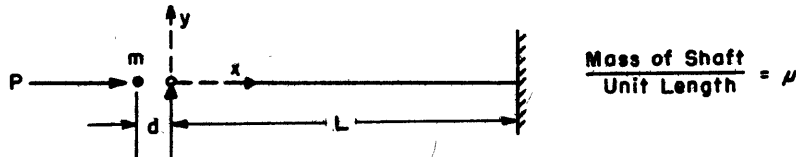


Figure 1b - Pinned-Fixed Beam with Overhanging Mass

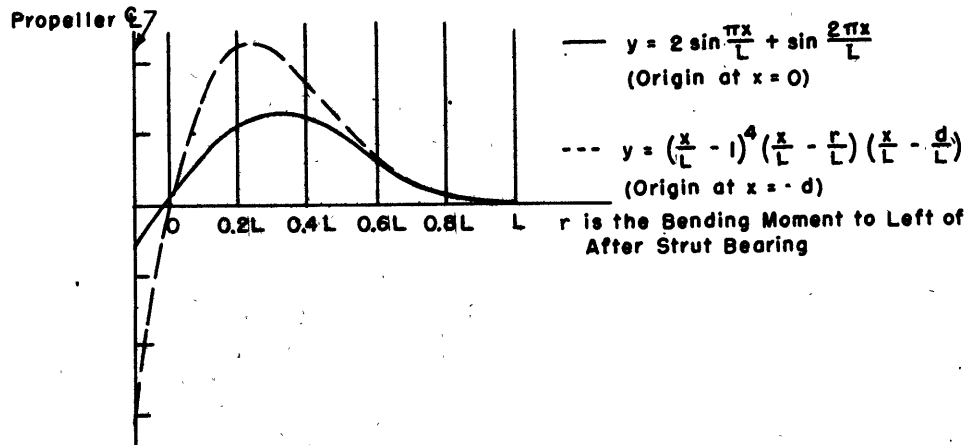


Figure 1c - Assumed Deflection Curves

Figure 1 - Propeller-Tailshaft System

function, Figure 1c, satisfies the boundary conditions and was used in applying the Rayleigh method to the problem. The value of the length L was taken as 400 inches on the basis of an inspection of the experimental deflection curve. Case 1 of Table 3 gives the theoretically exact value of the natural frequency. Case 2 gives the value based on the Rayleigh analysis; it is seen that this value is 27 percent higher than the theoretical value. Comparison of Cases 3 and 4 indicates that the propeller thrust does not have an

appreciable effect on the resonance frequency. Case 5 is derived in order to evaluate the effect of an overhanging mass such as the propeller. Case 5a reduces to Case 2 when  $d$  is taken as zero. It is seen that the overhanging propeller does have a very significant effect on the resonant whirling frequency of the shaft; the overhanging mass reduces the frequency from 1170 to 750 cpm. Inasmuch as the frequency values given by the Rayleigh approximation are about 27 percent too high, viz Case 1 and 2, the corrected values for Case 5 would be about 590 cpm (Case 5a) and 920 cpm (Case 5b) respectively. It is believed that Case 5, together with the indicated corrections, gives a fairly realistic approach to the calculation of the whirling frequency.

Considerable effort has been made to determine analytically a set of boundary conditions or restraints which would satisfy the experimental deflection curves and resonant frequencies. These attempts were not successful. It is believed that the actual restraints can be determined on the basis of a comparison of model and full-scale experiments. Great care must be taken in model experiments to include sufficient similarity in the model.

It may be concluded that it is, at this time, impractical to calculate the deflection pattern of the shaft, primarily because the extent of the bearing restraints is unknown. Also, it has been shown that the propeller mass and propeller overhang have an appreciable effect on the whirling frequency whereas the effect of the propeller thrust is quite small. An approximate method for calculating the natural whirling frequency of tailshaft systems similar to those on the DD692-Class destroyer has been given--see Case 5a, Table 3.

#### 4. TEST PROGRAM

The vessel selected for this test was a long-hull destroyer, the DD865 (CHARLES R. WARE), which had a history of excessive first-order vibration and which was equipped with four-bladed NACABS propellers. A long-hull destroyer was selected because more reports of excessive vibration had been received from the long-hull than from short-hull ships. Since the entire DD692 class will eventually be fitted with four-bladed NACABS propellers it was desirable to test a ship so equipped. The tests conducted were as follows:

(a) A vibration survey of the vessel was conducted on 21 June 1948, in water of depth exceeding 100 feet, enroute from Norfolk to New York in order to determine the vertical, athwartships and torsional motion of the hull, particularly near the bridge and on the fantail. Special attention was given to the identification of objectionable first-order vibration as

torsional or flexural vibration. At each resonant frequency a traverse of the main deck was made in order to determine the corresponding mode of vibration of the hull. The runout of the main propulsion shafting at the several inboard bearings was measured, at a low shaft rpm, with a dial indicator. During this survey the vessel displaced 3415 tons and the sea was calm.

(b) The TMB Medium Vibration Generator was installed on the fan-tail, Main Deck, Frame 200, upon arrival at the New York Naval Shipyard, see Figure 2.

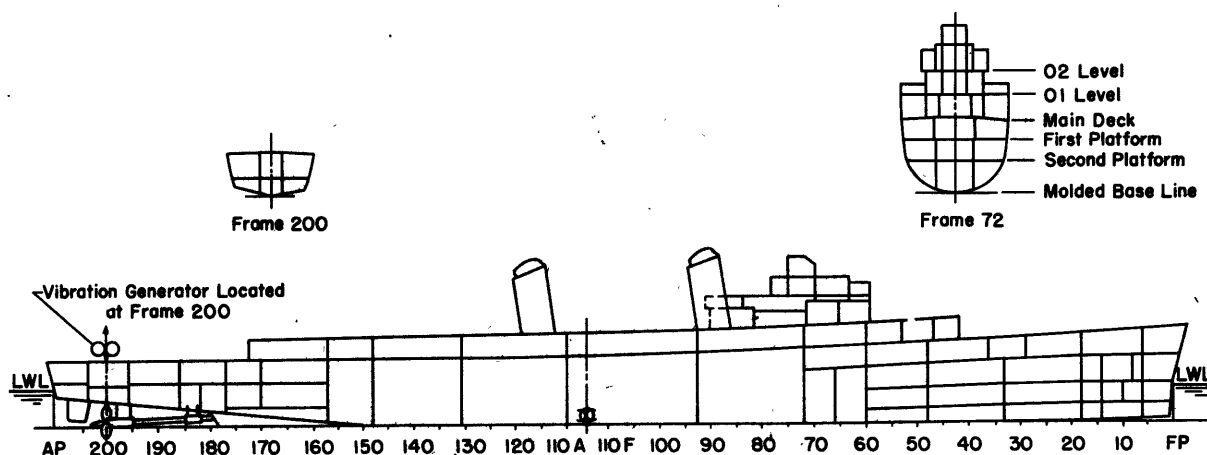


Figure 2 - Inboard Profile and Sections of USS WARE (DD865)  
Showing Location of Vibration Generator

(c) The vessel was then taken to a location in Long Island Sound-- where the depth of water was about 110 feet--to conduct the vibration-generator tests. The latter consisted of a series of forced-vibration tests which were made in order to determine as many of the vertical, transverse and torsional vibration resonances and modes of vibration of the hull, as possible. By means of releasing the anchor and snubbing it after a short length of chain had run out, the free response of the hull and the associated damping were determined.

(d) The vessel was returned to the New York Naval Shipyard and dry-docked. A vibration generator test was then conducted on the propeller-shaft strut system in order to determine its natural modes of vibration in air. The Lazan vibration generator and the "440-Pound Losenhausen" vibration generator<sup>14</sup> were installed for this purpose.

(e) Vibration pickups were installed on the strut barrels and on the tailshaft on the starboard side for subsequent submerged tests. The small





shafts, (2) determine the load carried by the intermediate strut bearing and (3) calibrate the strain gages previously installed on the strut in terms of the load acting at the strut bearing.

(g) The propellers and tailshafts were then removed. The propellers were checked for uniformity of pitch and were then dynamically balanced. The shafting was checked for crookedness, the runout was measured at the coupling faces, at the propeller taper and at each journal. The shafting was then straightened and dynamically balanced in an Akimoff balancing machine.<sup>15</sup> Records were kept of all work done so that the condition of the propellers and tailshafts as received from the ship and as reinstalled could be determined. The machine shop was also instructed to report the minimum runout to which a shaft of this size could be straightened and to report the degree of accuracy to which shafts and the propellers could be balanced. Before removal of the tailshafts the bearing clearances were checked at all struts and stern tubes. All bearing clearances were restored to design values.

(h) The condition of the propellers and tailshaft at the completion of the shipyard work can be considered to be the best practicably attainable. A final vibration survey, similar to that outlined under Item (a) was conducted at the end of the shipyard availability. The reduction of vibration amplitudes measured during this test can be considered the best practicably attainable without altering the structure of the ship or balancing the line shafting. The displacement of the vessel during these final tests was 3100 tons.

The program just outlined has been based on the following considerations: The initial survey, Item (a), was made in order to obtain a picture of the initial vibratory condition of the vessel for comparison with other ships of this class and for comparison with the final survey outlined in Item (e). The vibration-generator tests (b), (c), (e) were made in order to establish the normal modes of hull vibration and the response of the hull to steady-state vibratory forces. These data will also serve as a check on theoretical calculations now being made at the Taylor Model Basin. The tests outlined under Items (d) and (f) were made to determine whether there are any unusual conditions existing in the shaft strut system which are responsible for the peculiar sensitivity of these ships to first order vibration. The work described under Item (g) was done partially to determine the effect on the severity of vibration of balancing the propellers and tailshafts and partially to obtain data for setting up specifications for balancing and straightening propellers and shafting.

## 5. INSTRUMENTATION

The instrumentation employed in this series of tests will be described in a general way only.

The operation and construction of the TMB 5000-pound vibration generator, which was installed on the fantail so as to provide the sinusoidal exciting forces desired for determining the natural modes of hull vibration is described in detail in Reference 16. The power for the vibration generator was supplied by a General Motors diesel generator set, Model 6-71. The vibration generator was installed on a rigid foundation<sup>17</sup> at the center line of Frame 200. Photographs of the installation are shown in Figures 4a and 4b. A cross section at Frame 200 is shown in Figure 2 on page 19.

During the vibration-generator tests, vibration measurements were made by means of a Shure crystal accelerometer, the output of which was fed into a General Radio vibration meter which indicated displacement directly. In many cases the output of the General Radio vibration meter was again amplified by a Brush amplifier and recorded by a direct-inking Brush oscillograph. The measurements obtained with these electrical instruments were checked by means of a TMB pallograph, which is a mechanical vibrometer of the seismic type. Another instrument which was used to measure displacement amplitudes was the well known Geiger vibrograph. A tabulation of the characteristics of most of the instruments listed above as well as of many others is given in Reference 18.

A Lazan vibration generator was used to excite the natural modes of the tailshaft. This oscillator utilizes the centrifugal force produced by eccentrically supported weights. The eccentrics are so arranged as to give a sinusoidally varying exciting force. The speed of the oscillator is regulated within one percent by a General Electric THY-MO-TROL speed control. The complete unit is described in Reference 19.

The 440-pound Losenhausen generator, which operates on the same general principle as the Lazan oscillator, was used to vibrate the intermediate strut. Its construction and operation are given in detail in Reference 14.

The strain-gage installation on the strut arms utilized SR-4 wire strain gages, Type A. The output of the gages was amplified by a TMB Type "1A" carrier-type strain indicator and was recorded on a Hathaway oscillograph. Reference 20 describes this strain indicator in detail.

Two Statham accelerometers<sup>18</sup> were mounted in waterproof boxes, one each on the starboard intermediate- and main-strut barrels. The output of these accelerometers was amplified by a special carrier-type strain indicator and recorded on the Hathaway oscillograph.



Figure 4a - Stern View Showing Temporary Shack

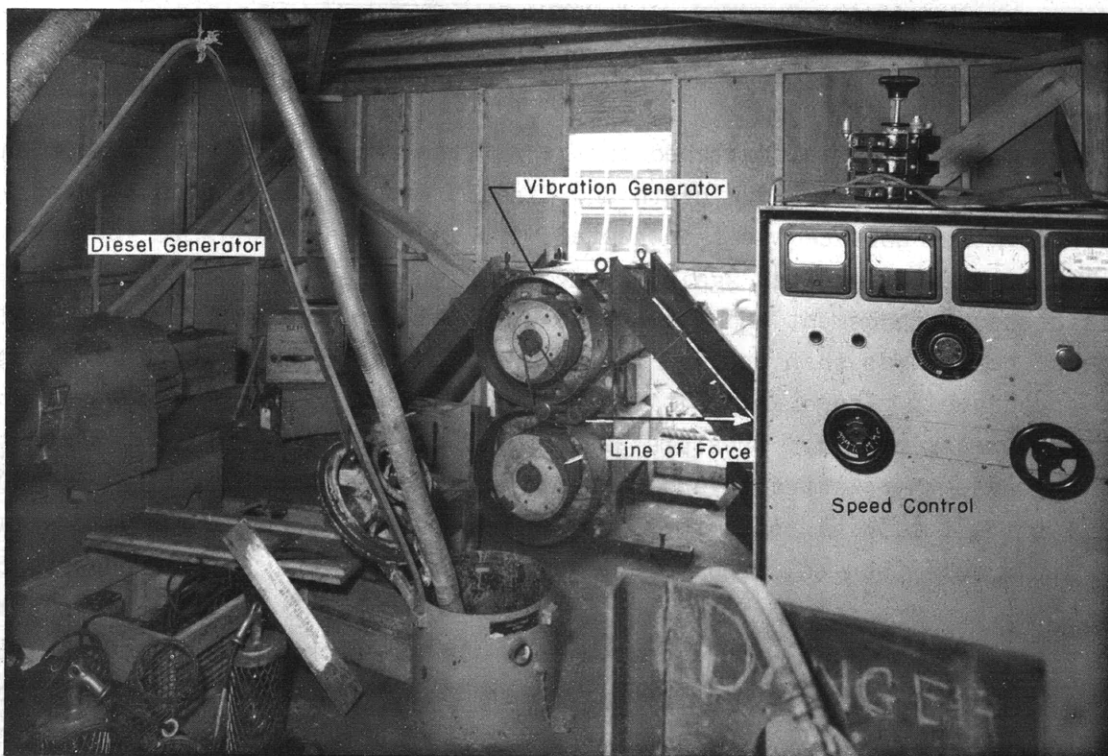


Figure 4b - Interior of Vibration-Generator Shack on Fantail

Figure 4 - DD865 as Rigged for Vibration Tests

The following procedure was used for mounting and waterproofing the SR-4 strain gages attached to the outside of the struts. The surface to which the gage was to be attached was cleaned and ground smooth. A housing, see Figure 5, was then spot welded over the location of the gage. A line for

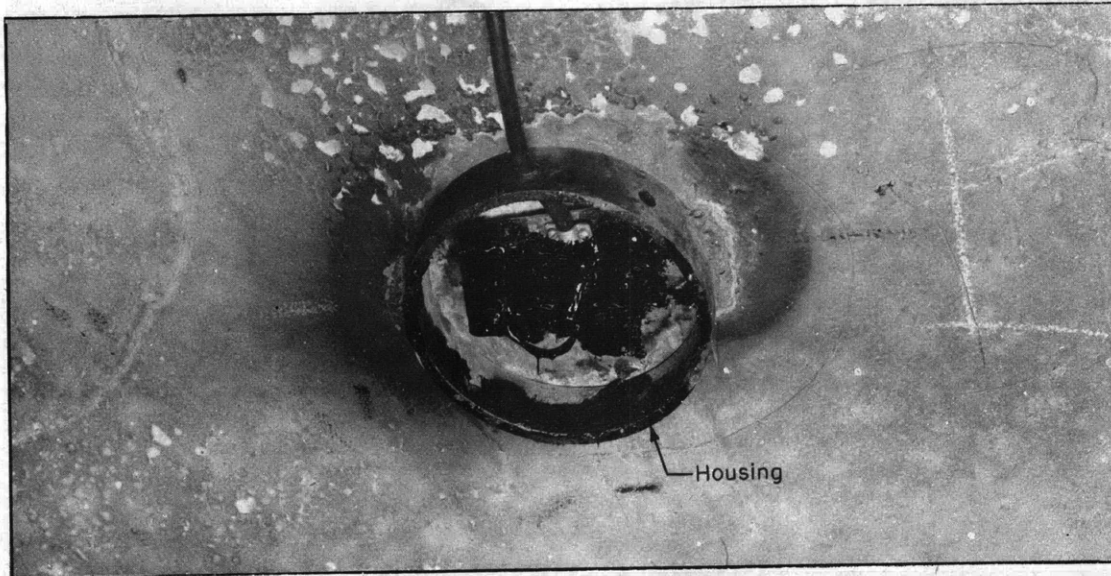


Figure 5 - Detail of SR-4 Strain Gage Installation

centering the gage was scratched on the surface and the surface cleaned and smoothed with a carborundum cloth and finally with acetone or carbon tetrachloride. The surface was then heated with infra-red heat lamps for about 20 minutes. The surface was again cleaned with acetone. Then both area and gage were coated with cement and allowed to dry until no longer tacky; a second coat of cement was then applied to the surface and to the gage and the gage was immediately placed in position and pressed down so as to remove air pockets and excess cement. Pressure was applied to the gage for about 5 minutes. The gage was then heated by an infra-red lamp to a temperature between 100° and 120° F. for 30 minutes, the lamp being held about one inch from the gage. The leads were then soldered to the gage. Both gage and lead wires were covered with a thin coat of Petrocene "A" wax. Upon hardening of the wax the waxed gage was covered with a coating of Ozite B extending out over the plate for at least 1/2 inch beyond the wax. Care was taken to keep the Ozite hot and in the liquid state. As soon as the Ozite had cooled the steel ring surrounding the gage was filled with asbestile and then covered with a sheet-metal plate. It was necessary to provide the submerged portion of the wires

leading from the gage to the amplifiers with sufficient mechanical and electrical insulation to withstand penetration of the salt water--which would reduce the resistance of the insulation. The experience during the tests was that microphone cable was not suitable but that a heavy rubber cable would stand up during several hours of immersion.

## 6. TESTS AND TEST RESULTS

### 6.1 Vibration-Generator Survey of the DD865 made in Long Island Sound

The purpose of these tests was the determination of the normal modes of vibration of the hull of the DD692-Class long-hull destroyers as well as to obtain the hull-girder response to known exciting forces. The displacement of DD865 during this test varied from 3265 tons to 3220 tons and the maximum trim was 9 inches by the stern, see Table 4. The weather was clear and the sea was calm throughout the tests. The depth of water at the test site was 110 feet.

Initially the TMB 5000-pound vibration generator was installed on the center line of the main deck at Frame 200. The installation necessitated stiffening of the main deck. The temporary structure covering the installation is shown on BuShips Plan DD865-S2905-348594. Photographs showing the test setup are shown in Figures 4a and 4b.

Prior to the tests the vibration generator was oriented to excite only vertical flexural hull vibrations. Amplitudes of vibration were measured at several stations over a range of frequencies from 60 to about 800 cpm. The

TABLE 4

Displacement of DD865 During Test Period

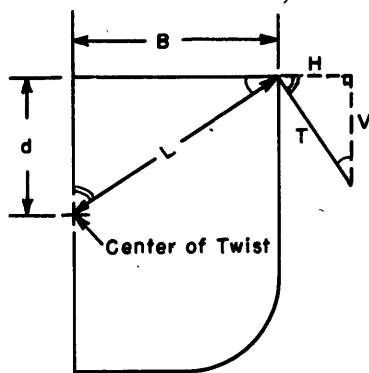
Date 1948	Location	Displacement	Draft	
			Forward	Aft
21 Jun	Leaving Norfolk	3415	13' - 8"	14' - 8"
28 Jun	Long Island Sound	3265	13' - 6"	14' - 0"
30 Jun	Long Island Sound	3220	13' - 3"	14' - 0"
3 Jul	New York Naval Shipyard	2925	12' - 3"	13' - 3"
7 Jul	New York Naval Shipyard	2720	11' - 8"	12' - 6"
19 Jul	New York Naval Shipyard (Wallabout Bay)	2486	10' - 4"	12' - 4"
20 Jul	New York Naval Shipyard	2486	10' - 4"	12' - 4"
22 Sep	Enroute New York to Norfolk	3100	Mean Draft 13' - 6"	

Total miles steamed as of 3 July 1948 since commissioning were 107,886 with original set of strut- and stern-tube bearings.

frequencies at which the various vibration resonances occurred were determined as accurately as practicable and the amplitudes at each one of these resonances were measured along the length of the vessel by means of a General Radio vibration meter. Check measurements were made with a TMB pallograph. Care was taken to assure that the pattern of vibration was constant while the amplitude measurements were being made.

The vibration generator was then oriented to excite athwartships forces which were applied at a distance of about 7.6 feet from the center of twist. These forces therefore are equivalent to the application of an athwartships force at the center of shear plus a couple. With this arrangement of the vibration generator both torsional and transverse vibrations are excited and for purposes of the present tests it was necessary to separate the effects of the two types of vibration. For the DD865, the effect of torsion on the amplitudes of the 2- and 3-noded flexural resonance is less than two percent for the first and second mode of athwartships flexural vibration as determined from an analysis of the data in Figures 11, 12, and 15. The torsional amplitudes, free from the effect of athwartships vibration, can be determined as follows: Inspection of Figure 6 shows that the torsional amplitude at any section is equal to the ratio of the vertical component of torsional vibration at any point to the horizontal distance of that point from the center line of the ship, provided no other vibration is present.

The ship tested is shown in Figure 2. Its main characteristics are given in Table 2 on page 9. The modes of vertical vibration up to and including the five-noded mode are plotted in Figures 7 to 10. The first two modes



Section at any Frame

Triangle dBL is similar to triangle TVH

$$\therefore \frac{T}{L} = \frac{V}{B} = \frac{H}{d} = \phi \quad [1]$$

$\phi$  is the torsional amplitude, and

$$d = B \frac{H}{V} \quad [2]$$

$$H = \phi d \quad [3]$$

$$V = \phi B \quad [4]$$

Figure 6 - Useful Relationships in Analyzing Torsional Vibrations

The value of the horizontal component H of a torsional vibration T is constant over any deck as shown by Equation [3].

The position of the center of twist can be determined by Equation [2].

of athwartships flexural vibration are shown in Figures 11 and 12. A resonance representing coupled torsional and athwartships vibration was measured at 480 cpm. The torsional component of this mode is plotted in Figure 13, and the measured athwartships vibration at 480 cpm, which includes relatively large torsional components is plotted in Figure 14. The fundamental mode of torsional vibration is illustrated in Figure 15. The exciting force as well as its point of application is indicated on all the figures. Figure 16 shows a plot of amplitudes of vertical vibration measured at Frame 203. Figure 17 is a similar plot of amplitudes of transverse vibration measured at the outboard edge of the main deck at Frame 196. For purposes of comparison a plot of the natural modes of flexural vibration of a free-free uniform bar is presented in Figure 18.

Due to the low damping forces present in hull vibration it was practically impossible to maintain the vibration exactly at the critical value during each survey. The mode shapes have been plotted as actually measured; the corresponding amplitudes are less than would obtain at the exact critical frequency. Additional runs were made, however, to determine as nearly as possible the maximum resonance amplitudes at selected locations. The values of probable resonance amplitudes can be calculated on the basis of these peak measurements by multiplying the plotted values by the ratio of the probable peak amplitude to the plotted amplitude for the particular location selected. This ratio will be called the "probable off-resonance factor." To illustrate

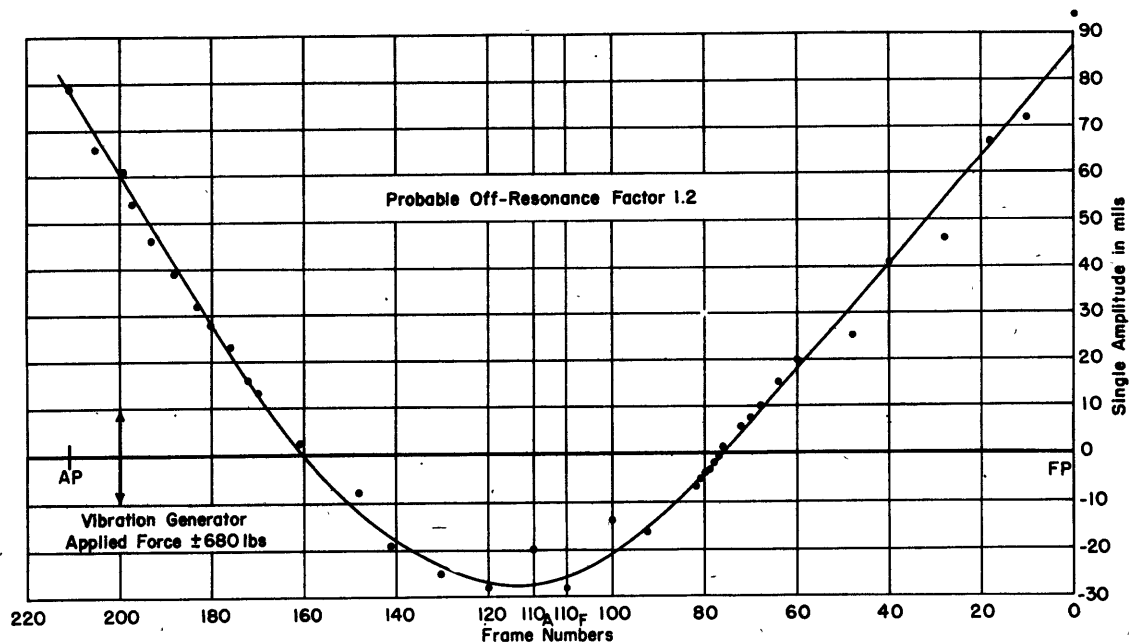


Figure 7 - Two-Noded Mode of Vertical Flexural Vibration of the DD865 at a Frequency of 79 cpm

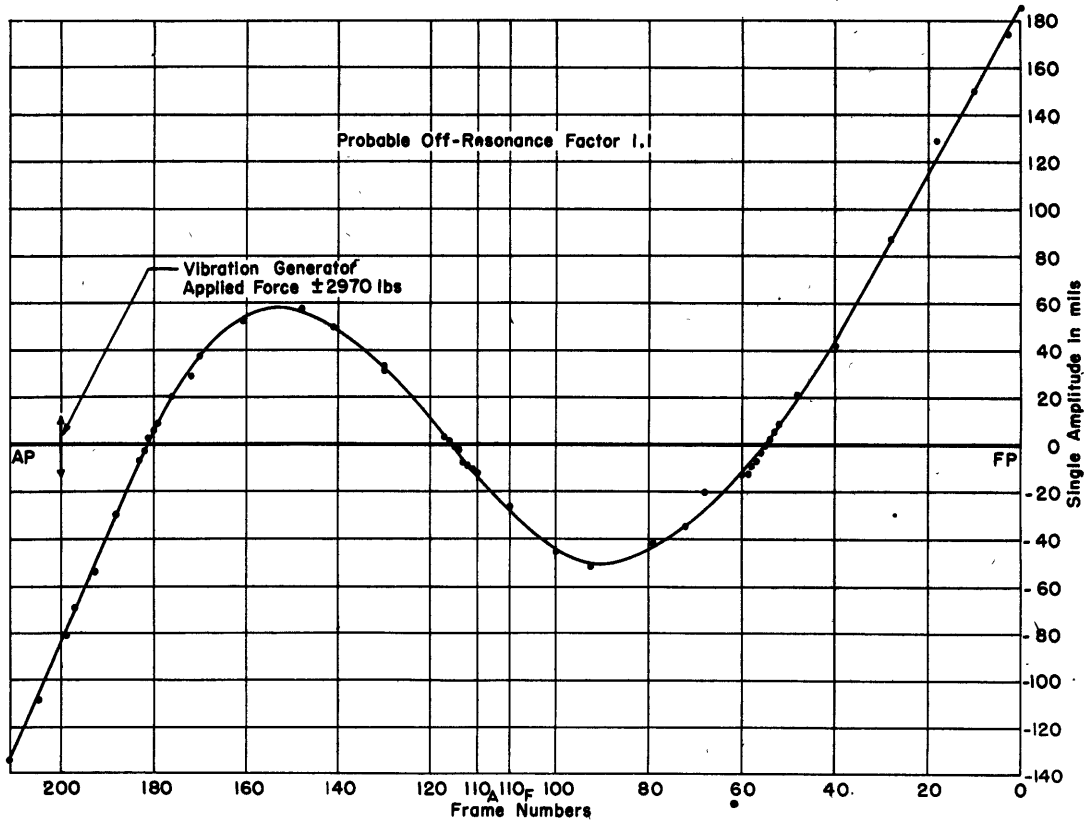


Figure 8 - Three-Noded Mode of Vertical Flexural Vibration of the DD865 at a Frequency of 165 cpm

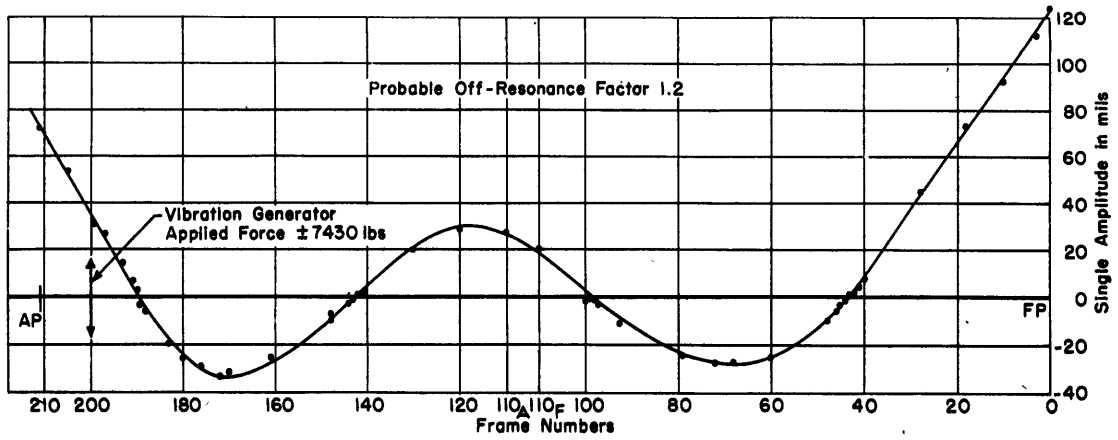


Figure 9 - Four-Noded Mode of Vertical Flexural Vibration of the DD865 at a Frequency of 261 cpm



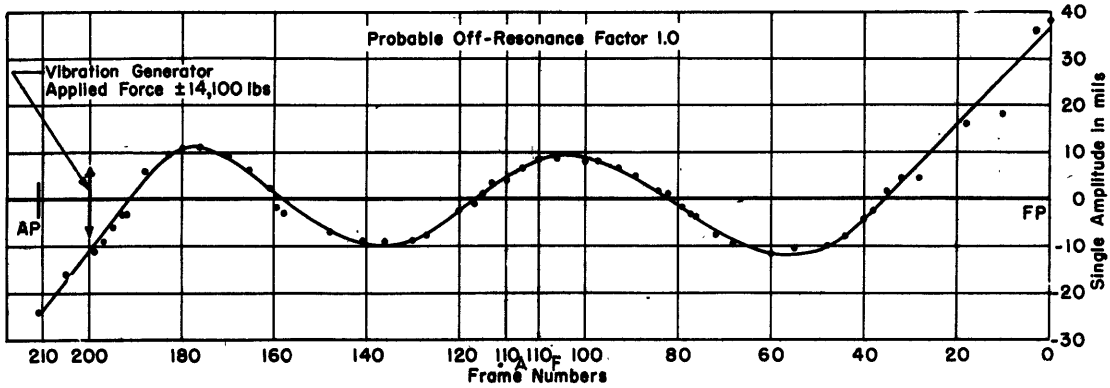


Figure 10 - Five-Noded Mode of Vertical Flexural Vibration of the DD865 at a Frequency of 360 cpm

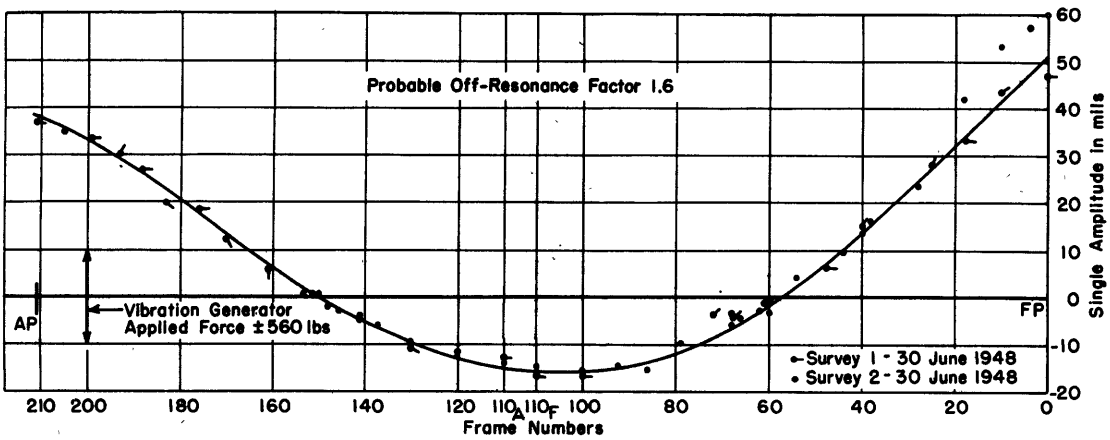


Figure 11 - Two-Noded Mode of Athwartships Flexural Vibration of the DD865 at a Frequency of 132 cpm

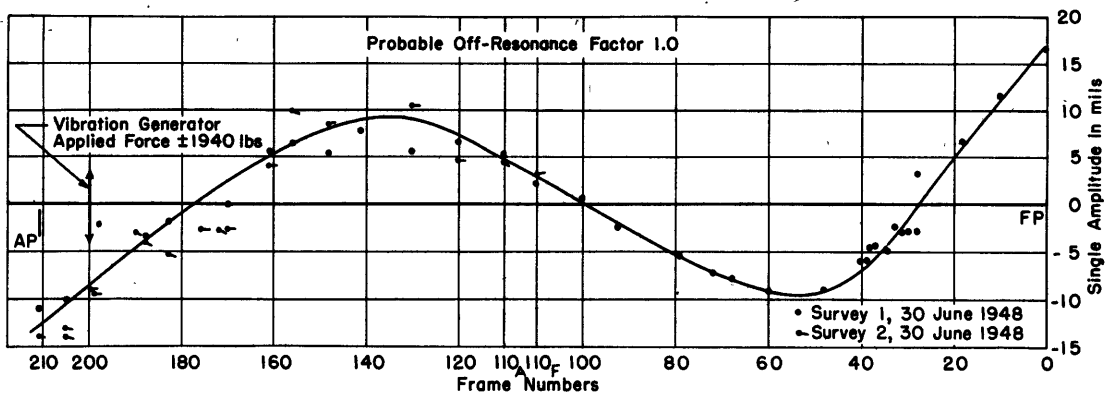


Figure 12 - Three-Noded Mode of Athwartships Flexural Vibration of the DD865 at a Frequency of 246 cpm

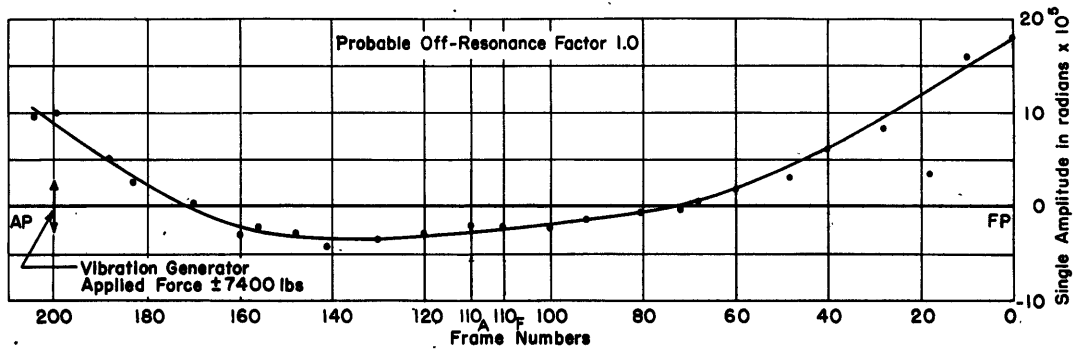


Figure 13 - Amplitudes of Torsional Vibration Measured on the DD865 at a Frequency of 480 cpm

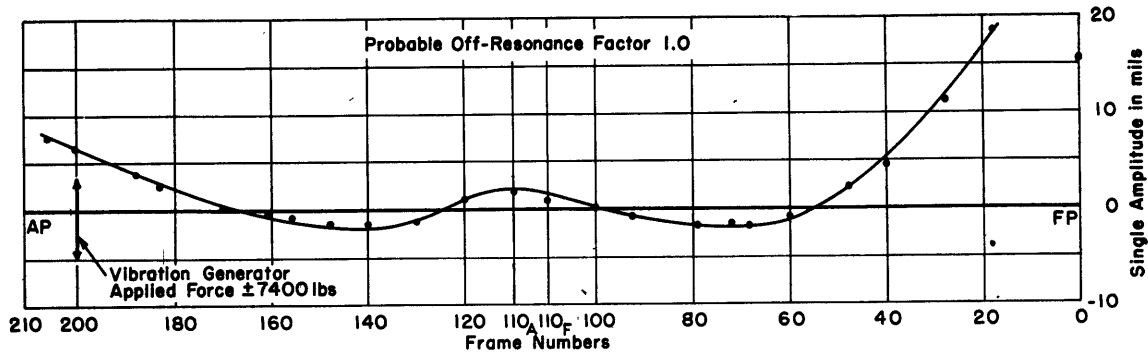


Figure 14 - Amplitudes of Athwartships Vibration Measured Along Starboard Side of the DD865 at a Frequency of 480 cpm

The test was made in Long Island Sound on 1 July 1948. A GR Pickup at the bow and at Frame 196 were Recorded on a Brush Oscillograph.

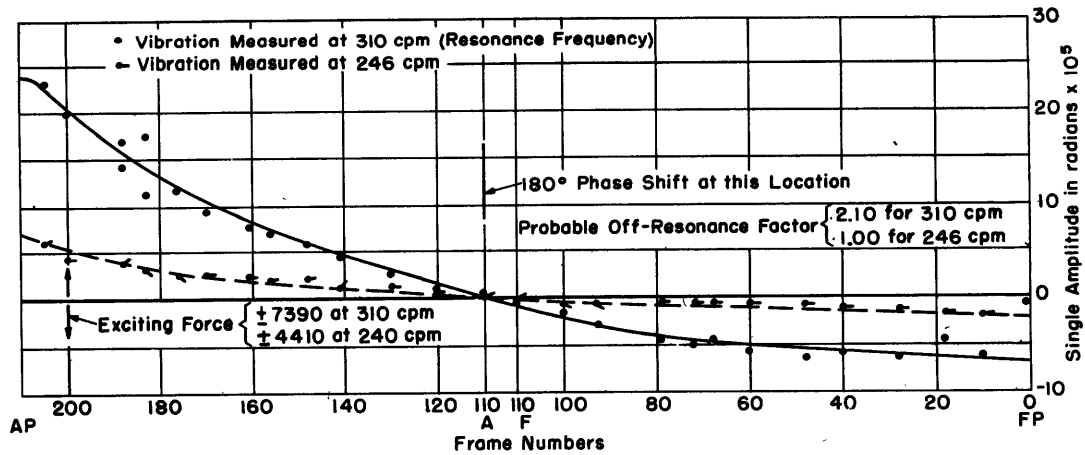


Figure 15 - Amplitudes of Torsional Vibration Measured on the DD865 at Frequencies of 310 and 246 cpm

Athwartship amplitudes of vibration which were measured at the edge of the main deck near the midships section are less than 1 mil single amplitude.

The measurements at 246 rpm were made in order to determine the extent of coupling between the one-noded torsional and the three-noded flexural modes of vibration.

These data were measured with a crystal accelerometer and recorded on a Brush Oscillograph. The displacement of the vessel was 3220 tons.

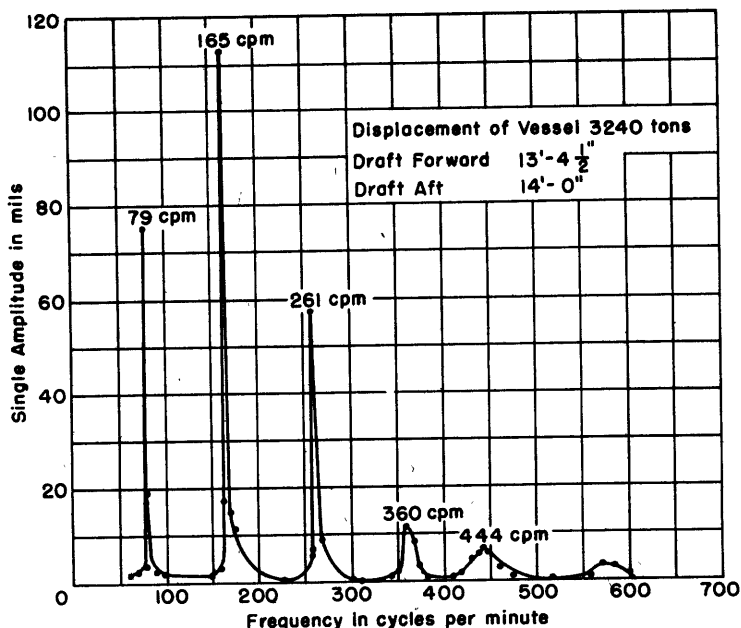


Figure 16 - Resonance Curve for Vertical Vibration on the DD865

The eccentricity setting of the vibrator was 90°; the exciting force was 0.109 (cpm)<sup>2</sup> in pounds. Measurements were made at Frame 203 with a crystal accelerometer recording on a Brush Oscillograph.

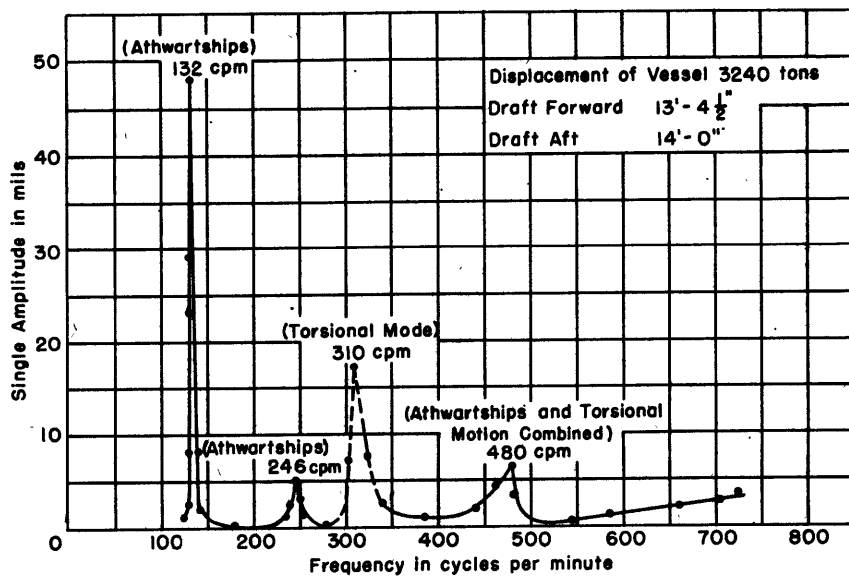


Figure 17 - Resonance Curve for Torsional and Athwartships Vibration Measured at Frame 196

The eccentricity setting of the vibrator was 24°; the exciting force was 0.032 (cpm)<sup>2</sup> in pounds. Measurements were made at Frame 196 with a crystal accelerometer recording on a Brush Oscillograph.

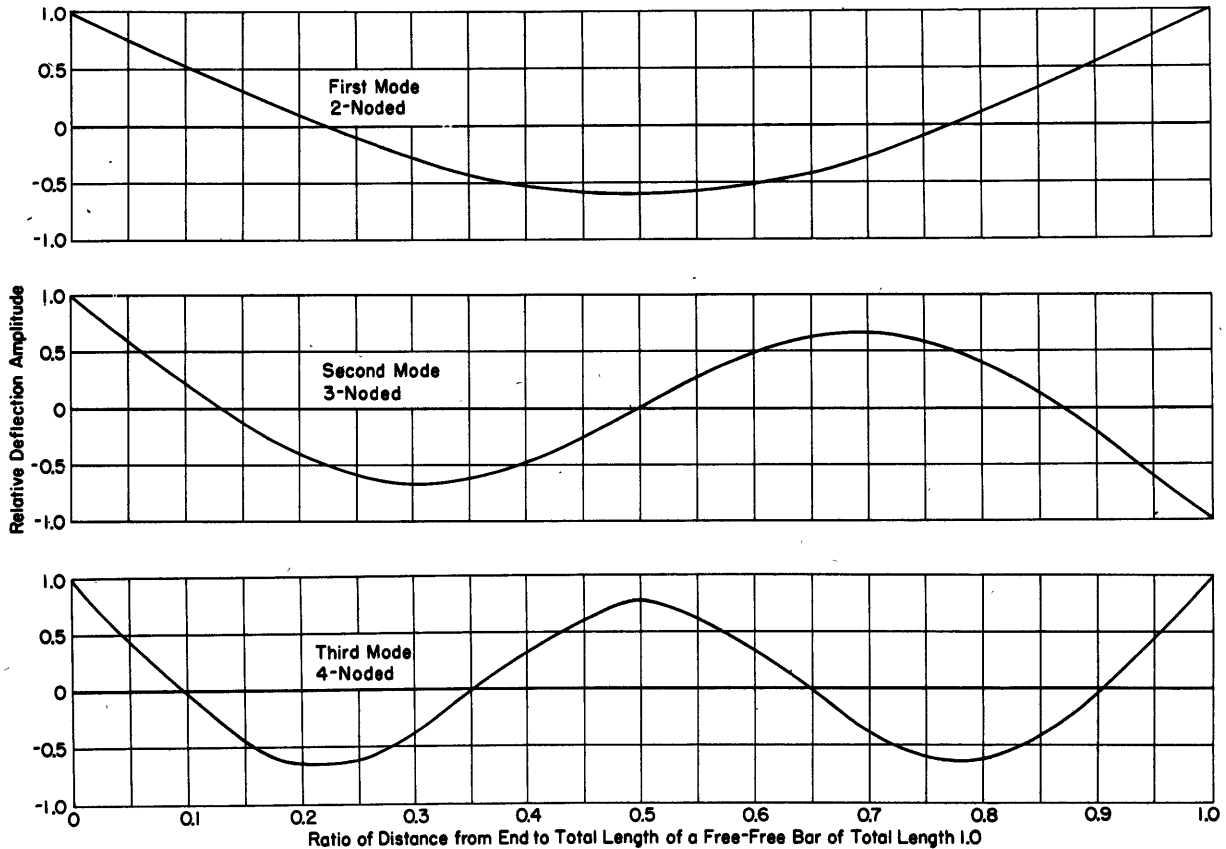


Figure 18 - Natural Modes of Flexural Vibration of a Free-Free Uniform Bar

the application of this factor: The plotted value of the torsional amplitude at Frame 188, for the first torsional resonance, is  $16 \times 10^{-5}$  radians. A separate survey gave a torsional single amplitude of  $34 \times 10^{-5}$  radians. Therefore the "probable off-resonance factor" is 34 divided by 16 or 2.1. All values plotted in Figure 15 should be multiplied by 2.1 in order to obtain the probable amplitudes at the 310 cpm torsional resonance.

## 6.2 Damping

An estimate was made of the damping associated with flexural hull vibration, based on the decay of a free vibration. To obtain a damped free vibration, the anchor was dropped several fathoms and the fall then suddenly arrested. The resultant vertical vibration of the hull was measured simultaneously at both the bow and at the stern. The actual record obtained is reproduced in Figure 19. The decay of the free vibration indicated that the first mode of vibration was subjected to about 0.35 percent of critical damping which corresponds to a decrement of 0.022. The value of the logarithmic decrement (0.022) may be compared with that measured on the USS HAMILTON (DD141), 0.023, and that measured on the USS ODAX (SS484), 0.033.

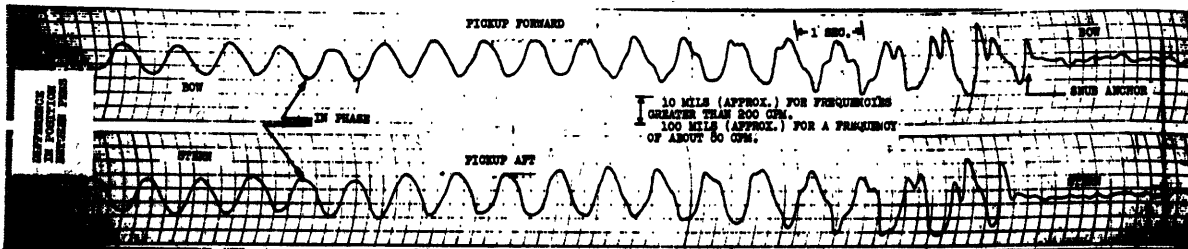


Figure 19 - Damped Free Vertical Vibrations Due to a Sudden Load Applied at the Bow

This record illustrates the propagation of flexural shock motion through the ship.

A calculation of the logarithmic decrement and the percent of critical damping associated with the several modes was made on the basis of the following considerations:

$$\text{Kinetic Energy of the ship, at resonance} = \frac{1}{2} \int_0^l \mu \omega_N^2 y^2 dx \quad [3]$$

$$\text{Kinetic Energy of an equivalent mass located at the vibration generator, at resonance} = \frac{1}{2} m_0 \omega_N^2 y_0^2 \quad [4]$$

$$\therefore m_0 = \frac{\int_0^l \mu y^2 dx}{y_0^2} \quad [5]$$

where  $\mu$  is the mass per unit length  $f(x)$ ,

$y$  is the amplitude in the given mode  $f(x)$ ,

$y_0$  is the value of  $y$  at the vibration generator,

$x$  is the horizontal space coordinate,

$m_0$  is an equivalent mass located at  $x_0$  which would possess the same kinetic energy as the ship, and

$\omega_N$  is the resonance frequency.

$$c_0 = \frac{P}{\omega_N y_0} \text{ at resonance, see Reference 12 page 68} \quad [6]$$

$P$  is the maximum value of the alternating force applied by the vibration generator

$$\text{the critical damping, } c_{c0} = 2m_0 \omega_N \text{ (at } x_0) \quad [7]$$

$$\text{the "logarithmic decrement" } \delta \approx 2\pi c/c_c \quad [8]$$

$$\frac{c}{c_c} = \frac{c_0}{c_{c0}} \quad [9]$$

A similar analysis can be made for the torsional modes of vibration. The values of  $P$ ,  $y$ ,  $\mu$  were obtained from the vibration-generator test data and from calculation. The values of the logarithmic decrements and of  $c/c_0$  are given in Table 5, for the several modes.

TABLE 5

Damping Characteristics Calculated On the Basis of the  
Vibration Generator Data

Type of Vibration	Number of Nodes	Logarithmic Decrement	Percent of Critical Damping
Athwartships	2	0.024	0.37
Flexural	3	0.053	0.84
Vertical	2	0.055	0.88
Flexural	3	0.028	0.45
	4	0.040	0.63
	5	0.15	2.38
Torsional	1	0.11	1.75
These calculations are made under the assumption that the damping is viscous in character.			

### 6.3 Propagation of a Stress Wave due to an Impulse Applied at the Bow

An impulse was applied to the bow--again by dropping and then arresting the drop of an anchor. The resultant vibration was measured simultaneously at both bow and stern. An actual record, Figure 19, shows that about 0.4 second elapses before the stress, resulting from the impulse, reaches the stern. Furthermore, when the stern starts to deflect it moves in a direction opposite to that in which the bow moved when it started to deflect under the applied load; that is, initially the bow and stern move in opposite directions although a time lag exists between these initial motions. According to a recent point of view,<sup>21</sup> it is believed that when an impulse is applied to the bow all the natural modes of vertical flexural vibration are excited and all modes are initially in phase at the point of application of the impulse. At the opposite end of the ship girder, on the other hand, the even modes are out of phase with the odd modes of vibration, thus tending toward a maximum amplitude at the point of load application--the bow--and toward a minimum at the stern. Similarly, the vertical amplitude due to the rigid-body pitching motion is in phase with the applied force at the point of application of the force but in opposite phase at the other end of the ship.

### 6.4 Pattern of Vibration at Frame 72

Most of the vibration measurements made on this class of destroyers by the several agencies were made at Frame 72, 02 deck level. Inasmuch as the vibration here has been objectionable it was decided to make a series of

thorough measurements of the hull response at Frame 72. Some questions had been raised as to whether or not the superstructure was bending transversely like a cantilever beam.

The significant instruments used in this test were the Shure crystal accelerometers. The outputs of these pickups were integrated in a General Radio vibration meter to give a voltage proportional to displacement which was recorded by a direct-inking Brush oscillograph. Two pickups were used and simultaneous measurements were made in order to permit the determination of phase relationships between the several motions. Two sets of measurements were made. The first measurements were made on 7 July, with DD692 alongside a shipyard dock, at a displacement of 2720 tons; the second test was made on 19 July in Wallabout Bay at a displacement of 2486 tons. The results were very nearly the same in both cases. The resonance frequencies were near 265 cpm and 332 cpm for the 2486-ton displacement and 257 cpm and 330 cpm for the 2720-ton displacement. The data given in this report will be those obtained on 19 July 1948. Check measurements with a Geiger vibrograph were made during the latter test and are reported in Enclosures (F) and (G) of the Material Laboratory report<sup>15</sup>. The two sets of data showed good agreement. The position of the ship and the data applicable to the test of 19 July are shown in Figure 3 on page 20. Figure 20 shows a cross section at Frame 72.

Measurements of athwartships vibration amplitudes at the resonance frequency of 310 cpm, made during the vibration-generator surveys in Long Island Sound, had shown that the athwartships component of vibration was less than 1 mil in the portion of the ship between Frames 79 and 148. The only flexural modes of vibration which could have an appreciable component at this frequency would be the three- and four-noded modes. Since very little athwartships motion was measured near the amidships position, which corresponds to an antinode for the four-noded flexural vibration, and near a node of the one-noded torsional mode, it is evident that the resonance near 310 cpm corresponds to a relatively pure torsional mode of vibration.

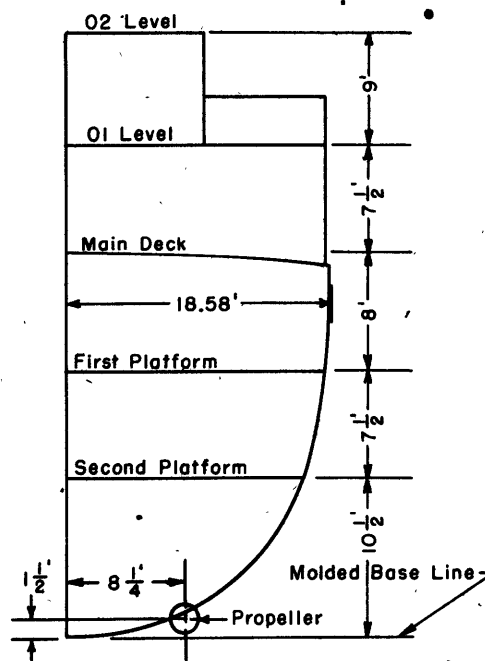


Figure 20 - Cross Section at Frame 72 (DD865) Looking Forward

Inspection of the phase relationships indicated in Figure 24 on page 39 shows that the force applied at the stern is in phase with the flexural motion at the point of application of the load up to a frequency of about 250 cpm. A phase reversal then takes place over a frequency change of about 15 cpm. In these tests the exciting force was applied to the hull at a point two feet above the main deck. Thus when the force is a maximum in the starboard direction the applied moment is a maximum in the clockwise direction, looking forward. In actual operation the exciting force is transmitted through the strut bearings to the hull; this will reverse the relative phase of force and moment, that is, when the force is a maximum in the starboard direction the applied moment is a maximum in the counterclockwise direction looking forward. Furthermore since the lever arm is 17.0 feet for a force applied at the propeller and 7.6 feet for the force applied by the vibration generator (Figure 21) it is evident that, in practice, any forces applied to the main strut bearing will be 2.24 times as effective in exciting torsional

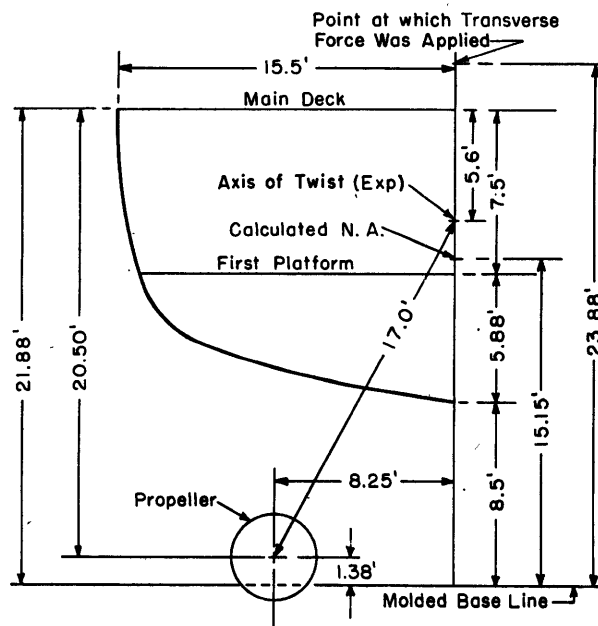


Figure 21 - Cross Section at Frame 200

hull vibration as is the same force applied with the vibration generator installation used in this test.

The pattern of vibration at Frame 72, resulting from a concentrated force applied in the plane of the propellers by the vibration generator (Figure 21) is shown in the graphs Figures 22 through 25. A separation of the torsional and the athwartships vibration components is effected in Figure 24.



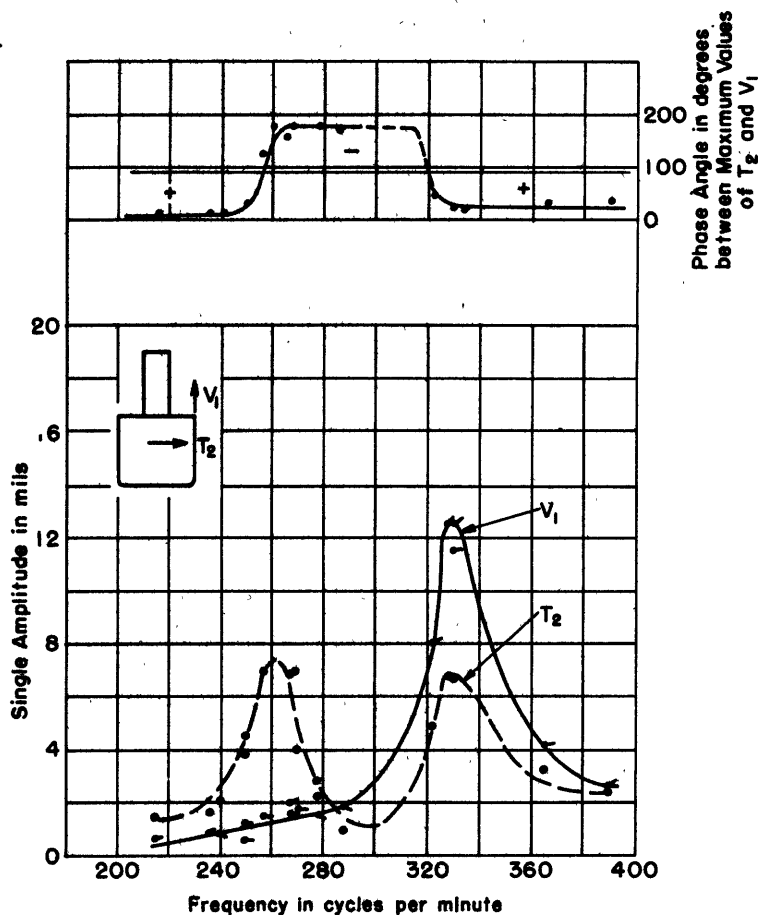


Figure 22 - Vibration Amplitude and Phase Relationships Measured at Frame 72

The eccentricity setting of the vibrator was  $60^\circ$ ; the exciting force was  $0.077 \text{ (cpm)}^2$ .  
 The measurements were made with a crystal accelerometer recording on a Bruah Oscillograph.  
 The displacement of the ship was 2486 tons.  
 The water depth at the test site was 35 feet.  
 Date of Test: 19 July 1948.

The data in Figure 25 show quite definitely that the superstructure does not bend as a cantilever beam but does act as an integral part of the hull in the torsional mode. The mode at 265 cpm contains a small amount of torsional component, see Figures 24 and 15.

If the force generated by the TMB vibration generator had been applied to the main strut bearing, a measurement of about 50 mils athwartships vibration on the 02 level at the upper critical would have been expected, due to greater value of the applicable moment arm. The amplitude at the lower critical would have increased but slightly since here the major contribution is due to flexural vibration.

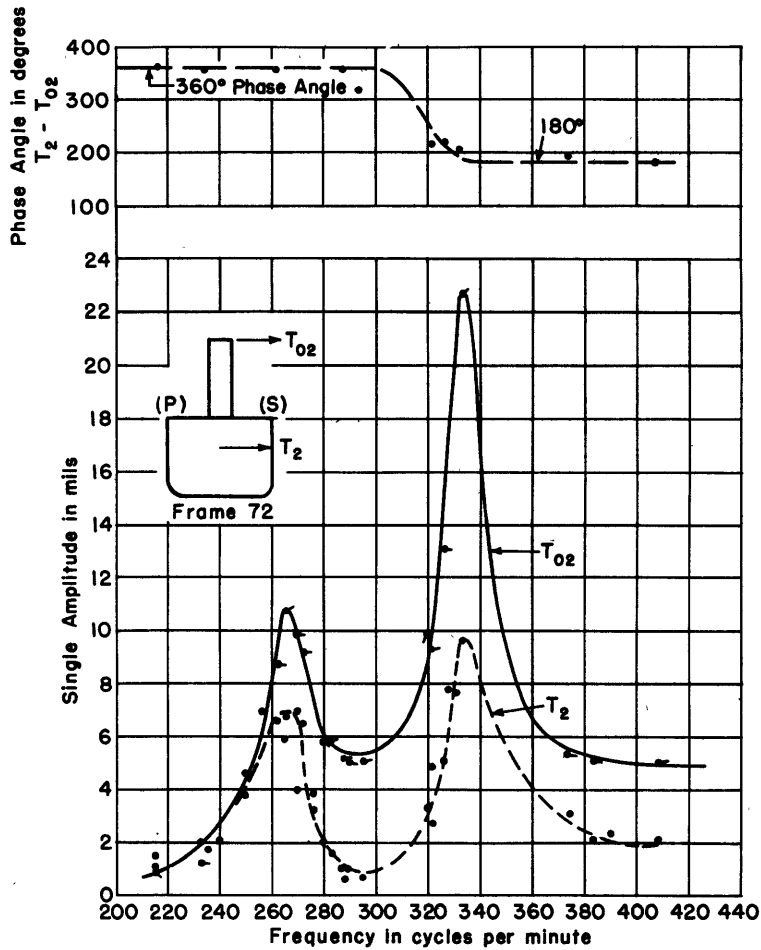


Figure 23 - Vibration Amplitude and Phase Relationships Measured at Frame 72 on the DD865

The eccentricity setting of the vibrator was 60°; the exciting force was 0.077 (cpm)<sup>2</sup>.  
 The measurements were made with a crystal accelerometer recording on a Brush Oscillograph.  
 The displacement of the ship was 2486 tons.  
 The water depth at the test site was 35 feet.  
 Date of Test: 19 July 1948.

Analysis of the data plotted in Figures 22 through 25 leads to the important conclusion that if in actual service a vibration pattern is encountered at Frame 72, 02 level, that shows considerable inequality of the heights of the two peaks on the amplitude-frequency plot, which generally occur near 240 and 310 shaft rpm, and the larger peak occurs at the higher shaft speed, then the exciting force is very probably a concentrated one due to effective unbalance of the propeller tailshaft system. Vice versa, if this pattern does not obtain, then balancing the propeller alone will not result in an appreciable benefit. The vibration tests so far conducted on vessels of this class, see Table 1, show a predominance of patterns in which the second critical (near 310 cpm) produces the larger vibration. However, the inequality of the resonance peaks is generally much smaller than that which

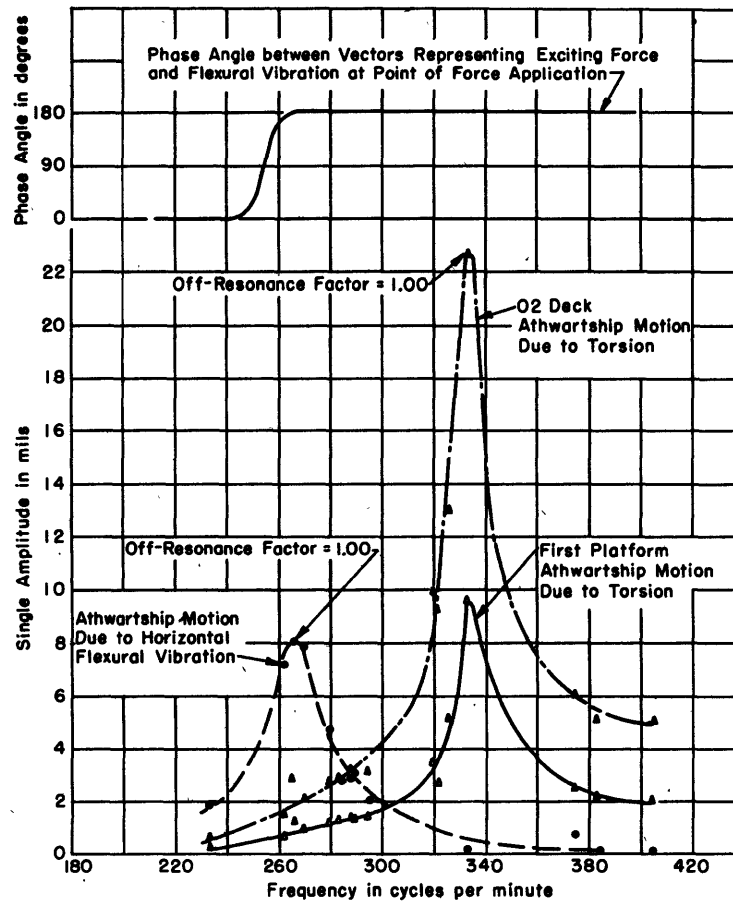


Figure 24 - Torsional and Athwartships Motion at Frame 72 on the DD865 Calculated on the Basis of the Data in Figures 22 and 23

The exciting force is  $0.077 \text{ (cpm)}^2$ .

would obtain for a concentrated load at the main strut bearing. The indication is, therefore, that the actual unbalance condition encountered on these ships is a combination of propeller and distributed shaft unbalance.

It is to be noted that in making vibration-generator tests it is very difficult, in practice, to maintain a vibration exactly at the critical for an appreciable length of time. It is therefore possible that the peak amplitude plotted for any particular resonance is not the largest attainable at that frequency, and different values of amplitudes may be measured during different test runs.

### 6.5 Pattern of Vibration at Frame 188

Generally, the more important exciting forces are applied at the stern. Also vibration measurements are often made at the stern. For these reasons it was decided to make a fairly detailed analysis of the hull response

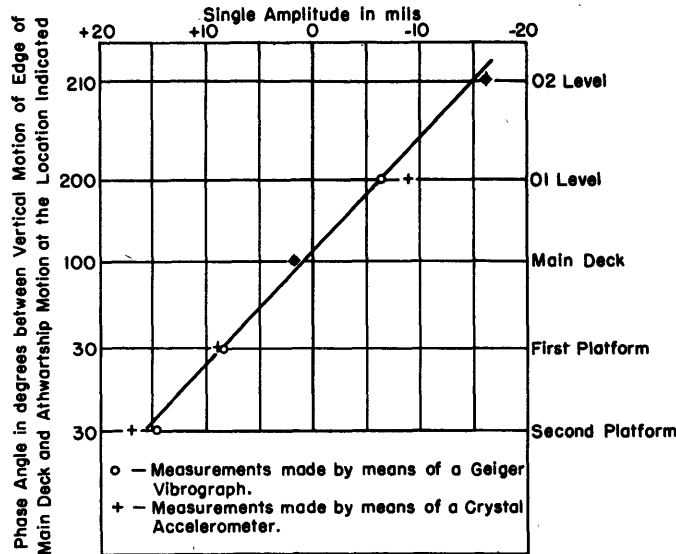


Figure 25 - Vibration Amplitudes and Phase Relationships Measured at Frame 72 on the DD865 at 332 cpm

All measurements made with the crystal accelerometer have been corrected such as to represent equivalent simultaneous measurements.

The eccentricity setting of the vibrator was 60°; the exciting force was 0.077 (cpm)<sup>2</sup>.

The displacement of the ship was 2486 tons.

The depth of water at the test site was 35 feet.

Date of test: 19 July 1948.

at the fantail, Frame 188, the measurements being made at the same time as those reported in the preceding subsection. Geiger vibrographs were used to measure the amplitudes and phase relationship. The data obtained are given in Enclosures (D) and (E) of the Material Laboratory report<sup>15</sup> and are replotted in Figure 26 for convenience. The measurements of athwartships vibration made at the upper critical substantiate the data thus far presented, supporting the conclusion that this critical is a torsional mode of vibration. Analysis of the data given in Figure 26 shows that the axis of twist lies approximately 5.6 feet below the main deck.

If the exciting force had been applied through the main strut bearings rather than at the position of the vibration generator, the athwartships amplitude at the upper-critical speed for the main deck and for the second deck would have been about 2.24 as large as the ones plotted, because of the greater applicable moment arm (see the pertinent discussion in the preceding subsection). The amplitude of transverse vibration at the lower-critical speed

would also have increased appreciably. This is due primarily to the fact that Frame 188 is located near a node of the three-noded mode of athwartships vibration; thus the torsional contribution is appreciable at this location.

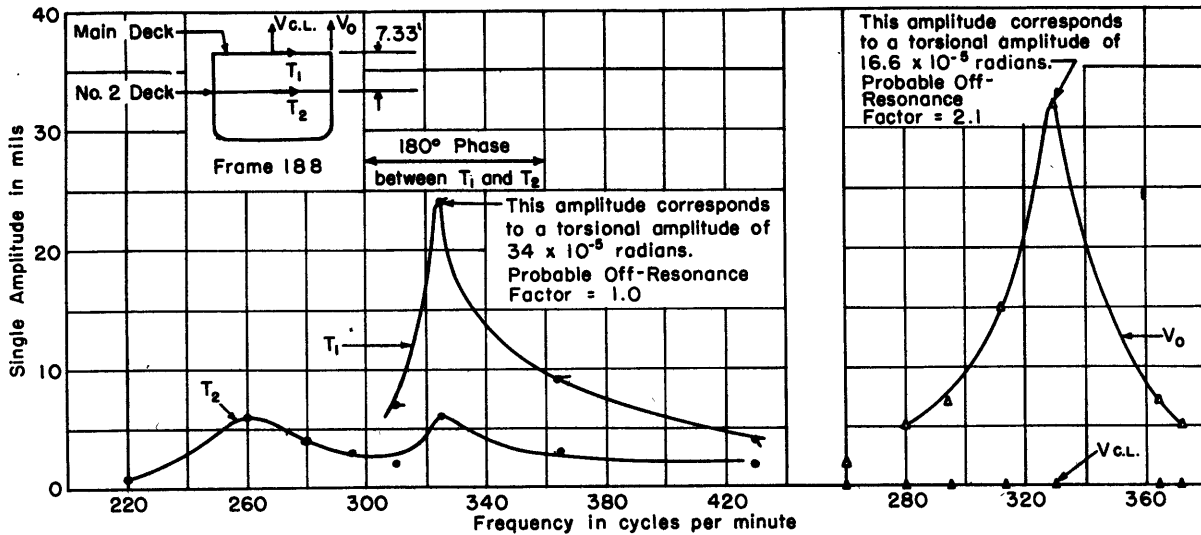


Figure 26 - Vibration Amplitudes and Phase Relationships Measured at Frame 188

The eccentricity setting of the vibrator was  $60^\circ$ ; the exciting force was  $0.077 \text{ (cpm)}^2$ . The measurements were made with Geiger vibrographs on 19 July 1948.

### 6.6 Torsional Rigidity of the Hull Girder

When it became evident that the excessive vibration usually experienced near 310 cpm resulted from the first torsional mode of vibration, some thought was given to possible methods of stiffening the hull girder so as to make it less susceptible to such vibration.

If the strains were a maximum near the node, as one would expect on the basis of simple torsion theory, then the most advantageous place at which to increase the torsional rigidity would appear to be near the node. Such a stiffening might be effectively applied by means of diagonal stiffeners welded to the shell plating.

Figure 27 shows the distribution of the mass moment of inertia as estimated for the load condition of 3400 tons displacement. The amplitudes of torsional vibration at any section of the ship are plotted in Figure 15; it is to be noted that the plotted values in Figure 15 should be multiplied by the "probable off-resonance factor" 2.1. The increment of torque contributed by any element  $\Delta x$  in length, is equal to

$$\Delta T = I_{\mu} \varphi \omega^2 \Delta x \quad [10]$$

and the torque at any section is

$$T_x = \int_0^x I_{\mu} \varphi \omega^2 dx \quad [11]$$

where  $I_{\mu}$  is the mass moment of inertia per unit length,  
 $\varphi$  is the angular deflection at any point  $x$ ,  
 $\omega$  is the angular frequency of vibration, and  
 $x$  is the fore and aft coordinate.

The values of  $I_{\mu}$  and  $T_x$  are plotted in Figure 27 for the torsional vibration in the fundamental normal mode. The torque diagram closes; this

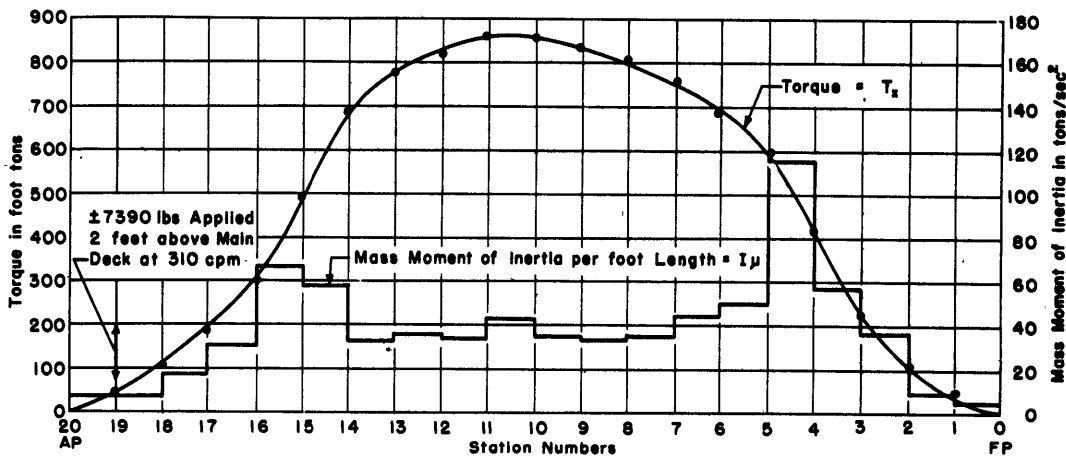


Figure 27 - Torque and Mass Moment of Inertia Distribution for One-Noded Mode of Torsional Vibration from Vibration-Generator Test at 310 rpm

The probable off-resonance factor has been applied to the amplitude values in Figure 15 in order to obtain the torque plotted in this figure.

fact indicates that the experimentally determined mode shape as well as the estimated distribution of mass moment of inertia are reasonably correct.

Torsion of thin-walled girders has been discussed by Bredt<sup>22</sup>, Vedeler<sup>23</sup>, Hovgaard<sup>24</sup>, Timoshenko<sup>25</sup>, Wagner and Kimm<sup>26</sup>, Goodier<sup>27</sup>, Bruhn<sup>28</sup> and others. The fundamental theory was given by Bredt and the hydrodynamic analogy of two-dimensional fluid flow and stress distribution in torsion was established by Prandtl.

According to this theory, the product of the torsion stress and the thickness of the plating is constant all around the contour of the section of a single-deck ship. This is not accurate when applied to structures

containing sharp corners or other discontinuities. It is furthermore assumed that only the plating is effective in resisting torsion. In accordance with the above theory, let  $q$  be the mean torsional stress at any point of the section's contour,  $t$  the thickness of the plating at that point,  $T$  the torque, and  $A$  the total area enclosed by the contour.

$$qt = \frac{T}{2A} \quad [12]$$

Now if  $L$  be the length of the total girth and  $dL$  the length of an element of the boundary and if  $\theta$  be the angle of torsion per unit length along the fore and aft axis and  $G$  the modulus of rigidity in shear, then it can be shown (Timoshenko<sup>25</sup>, pp 269-273) that

$$\theta = \frac{qt}{2AG} \int_0^L \frac{dL}{t} = \frac{T}{4A^2G} \int_0^L \frac{dL}{t} \quad [13]$$

The torsional flexibility can be defined as

$$\frac{1}{4A^2G} \int_0^L \frac{dL}{t} = \frac{\theta}{T} \quad [14]$$

and the torsional rigidity is the reciprocal of the torsional flexibility.

The torsional rigidity of circular bars is equal to  $JG$ , where  $J$  is the polar moment of inertia of the section. An equivalent  $J_e$  for single-hull ship sections may be defined as follows:

$$\frac{T}{\theta} = \frac{4A^2G}{\int_0^L \frac{dL}{t}} = J_e G \quad \therefore J_e = \frac{4A^2}{\int_0^L \frac{dL}{t}} \quad [15]$$

If there are additional decks the above formulas must be modified along the lines indicated by Hovgaard<sup>24</sup>. Applying the above theory to the midship section and to the section at Frame 189 we find, for the conditions given in Figure 27:

For the midships section of the DD692, which has no inner decks and neglecting the effect of the superstructure

$$qt = \frac{T}{2A} = \frac{840 \times 2240 \times 12}{2 \times 720 \times 144} = 109 \text{ lb/in.}$$

$$\theta = \frac{qt}{2AG} \int_0^L \frac{dL}{t} = \frac{109 \times 3046}{1440 \times 144 \times 11 \times 10^6} = 0.015 \times 10^{-5} \text{ radians/in.}$$

$$\frac{T}{\theta} = 15,000 \times 10^{10} \frac{\text{in}^2\text{-lb}}{\text{radian}}$$

The torsional stress in the main-deck plating = 218 psi.  
 Repeating this calculation for the case in which the effect of the super-structure is taken into consideration according to the procedure given on page 101 of Hovgaard<sup>24</sup> we find that

$$qt = 104 \text{ lb/in. for the shell plating}$$

$$qt = 79 \text{ lb/in. for the main-deck plating inside the deck house}$$

$$qt = 25 \text{ lb/in. for the deck-house plating}$$

$$\theta = 0.14 \times 10^{-5} \text{ radians, } \frac{T}{\theta} = 16,000 \times 10^{10} \frac{\text{in}^2\text{-lb}}{\text{radian}}$$

For the section at Frame 189

$$qt = \frac{T}{2A} = \frac{106 \times 2240 \times 12}{2 \times 452 \times 144} = 21.5 \text{ lb/in}$$

$$\theta = \frac{qt}{2AG} \int_0^L \frac{dL}{t} = \frac{21.5 \times 4180}{2 \times 452 \times 144 \times 11 \times 10^6} = 0.006 \times 10^{-5} \text{ radians/in.}$$

$$\frac{T}{\theta} = 4500 \times 10^{10} \frac{\text{in}^2\text{-lb}}{\text{radian}}$$

The torsional stress in the main-deck plating is 98 psi, and the value of T is taken from Figure 25.

As a check on the validity of the application of this theory the experimental value of  $\theta$  was obtained from Figure 15 by determining the slope of the torsional amplitude curve for 310 cpm at Frame 110 aft, and at Frame 189. The values obtained in this manner are approximately  $0.015 \times 10^{-5} \text{ in.}^{-1}$  for Frame 110 aft and  $0.035 \times 10^{-5} \text{ in.}^{-1}$  for Frame 189, corresponding to the calculated values of  $0.015 \times 10^{-5}$  and  $0.006 \times 10^{-5} \text{ in.}^{-1}$  respectively. The agreement at Frame 189 is poor, partially due to relatively large inaccuracies in the calculated torque near the fantail and to the presence of an intermediate deck.

It is seen on inspection that  $\theta$ , the angle of torsion per unit length, varies inversely as  $A^2$ , directly as T and directly as the line integral, see Equation [13]. Since A decreases rather rapidly towards the stern it is evident that it is quite possible for  $\theta$  to be larger nearer the stern than amidships. For a uniform bar, of course,  $\theta$  would be a maximum at the node. Test results as well as the calculations show that the DD692 has a stern that is torsionally relatively flexible, due primarily to the large decrease in A--the cutting away of the after portion of the ship starting at



Frame 157. On the other hand, forward of amidships the torsional rigidity has been increased somewhat due to the action of the superstructure. The torsional rigidity of the hull, with the effect of the superstructure and of the longitudinals neglected, is about  $4500 \times 10^{10}$  in.<sup>2</sup>-lb or less at Frame 189,  $15,000 \times 10^{10}$  in.<sup>2</sup>-lb amidships and  $17,700 \times 10^{10}$  in.<sup>2</sup>-lb at Frame 72. Test data indicate that the effect of the superstructure should be included. The large concentration of mass moment of inertia near the quarter points, Figure 25, relatively far away from the node, makes for a low natural frequency resulting in a relatively flexible girder torsionally.

The torsion of a ship girder is actually more complicated than indicated by the preceding theory. The sides, decks and bottom of the ship theoretically partially resist the torque through bending; this action provides an increase in the torsional stiffness, the extent of which is unknown. An excellent discussion of combined torsion and bending is given by Prof. J. N. Goodier formerly of Cornell University.<sup>27</sup>

The variation of torque, shown in Figure 27, will obtain whenever the ship vibrates in its fundamental torsional mode, whether this vibration is excited by a seaway, vibration generator, or any other force.

An attempt was made to measure the torsional strains on the main deck. The location of the stations at which the measurements were made are indicated in Figure 28. The measured strains are given in Table 6. Since

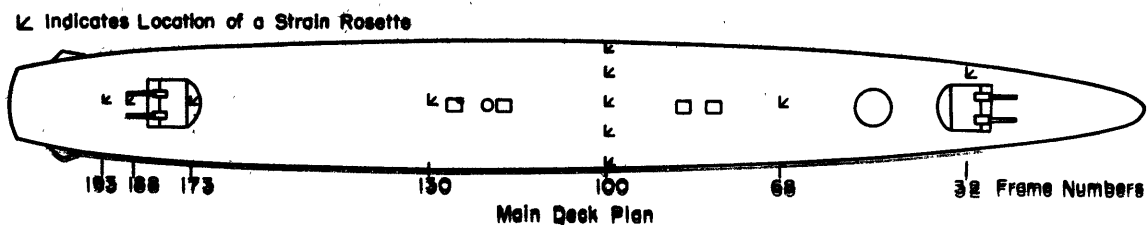


Figure 28 - Location of Strain-Gage Stations on Main-Deck

the strains were measured on the top side of the deck plating only, there is some question as to the value of the measurements. However the following analysis can be made. No measurable strains in any direction were obtained at the three-noded athwartships flexural resonance. At the one-noded torsional resonance no measurable strains were obtained in either the fore-and-aft or the athwartships direction. The strains measured near the midships section were a maximum near the deck edges and a minimum near the center line. The theory of shear flow indicates a uniform shear stress over the main-deck plating under the assumption that the superstructure deck is ineffective in

TABLE 6

Strain Measurements on the Main Deck of the DD865 During Vibration-Generator Test

The strain is measured along an axis making an angle of 45° and 135° with the fore and aft center line.

Station	Location on Main Deck	Frequency of Vibration - cpm											
		320	215	240	260	270	280	295	310	330	365	390	410
		Strain in microin/in.											
1	Frame 100 Starboard Edge	7.5	0	0	0	0	-	0	1.0	7.0	1.0	-	-
2	Frame 100 10 feet Starboard of Center Line	6.0	0	0	nil	2.0	-	0	1.0	4.0	-	-	-
3	Frame 100 Center Line	1.0	0	0	0	0	-	0	0.5	1.0	0	-	-
4	Frame 100 10 feet Port of Center Line	7.0	0	0	0	0	-	0	0.5	-	-	-	-
5	Frame 100 Port Edge	10.0	0	0	nil	0	-	0	3.0	15.0	0	-	-
6	Frame 130 Center Line	10.0	0	0	0	0	-	-	2.0	8.0	2.0	-	-
7	Frame 68 Center Line	2.0	0	0	-	1.0	-	-	0	1.0	-	-	-
8	Frame 173 Center Line	20.0	0	0	-	0	0	0	9.0	25.0	2.0	1.0	0
9	Frame 188 Center Line	16.0	0	0	-	-	-	2.0	11.0	20.0	-	-	-
10	Frame 32 Port	22.0	0	0	-	0	-	1.0	6.0	18.0	3.5	-	-
11	Frame 193 Center Line	-	-	-	-	-	-	-	-	-	-	-	-

The three-noded mode of athwartships flexural vibration occurred near 260 cpm; measurements at this frequency showed no measurable strains in any direction.

The measurements were made on 19 July 1948 with a 10 in. Whittemore strain gage.

The exciting force was 0.077 (cpm)<sup>2</sup> applied 2 feet above the main deck, Frame 200.

resisting torsion. It is believed that the twisting of the hull girder caused diagonal wrinkles in the deck plating, i.e. buckling of the plating due to initial unfairness of the plating--thus producing larger strain readings than would obtain due to shear alone. The relatively large strains in way of the forward and after gun mounts are probably due to local buckling of the deck caused by the bending moments applied to the deck structure by the heavy,

cantilevered 5-in. gun mounts. It is concluded that the shear stresses at the lower hull critical are very small and that a comparison of the calculated and measured stresses would not be valid.

It is to be noted that the shear stresses due to vibration are small but the resulting angular vibratory displacements can be considered excessive.

#### 6.7 Shear Stress Distribution in Beams and the Centers of Twist and Shear

The problem of locating the center of shear has not been answered with general satisfaction. An excellent analysis of combined torsion and bending in beams of any cross-section has been given by Goodier.<sup>27</sup> His analysis takes account of the distortion within the cross section of the beam due to the effect of Poisson's ratio. The discussion in this section will be based to some extent on Professor Goodier's paper. It is also of interest to be able to determine the axis about which the transverse section of a hull girder rotates when it vibrates torsionally or when it is subjected to a static twisting moment.

In order to analyze the motions of any body such as a ship, the total motion may be resolved into a rigid-body motion and into the elastic or plastic displacements within the body. The rigid-body motion can always be described by stating the linear motion of the center of gravity and the rotation of the body about the center of gravity. The displacements accompanying strain can be described by giving the displacements of the component particles of the body relative to each other. In considering the vibrational behavior of a body, the latter displacements need to be found.

Goodier defines the center of shear as that point of the cross section through which the resultant shear must act in order that the rotation along the centroidal axis of a straight beam of uniform cross section shall be uniform. This definition of the center of shear reduces to that usually given in the literature on strength of materials if Poisson's ratio is given a value of zero. Goodier's definition of the center of shear appears to be a rational one. It should be emphasized that the cross sections of the beam are identical and the line of centroids is straight. Forces which act through the center of shear do not produce torsion; they cause flexure. Forces which do not pass through this center give a twisting moment equal to the moment of the force about the center of shear plus a shearing force. The center of shear is determined by the shape of the cross section and is not affected by the distribution of mass. It should be noted that in accelerated motion the mass distribution does affect the load on the beam.

Following are a set of relationships which will make it possible to calculate in a practical manner the shear-stress distribution due to a combination of bending and torsion in a thin-walled shell, such as a single-hull ship, and at the same time to calculate the position of the center of shear according to Goodier. The method is perfectly general for any type of cross section and can be extended to multicellular structures. The approach will be to determine first the shear-flow distribution due to flexure alone, next the center of shear is determined, and then the moment of the external forces about the center of shear gives the twisting moment. Once the twisting moment is known the shear-flow distribution due to this torque can be evaluated. The total shear-stress distribution is obtained by superposition of the shear stresses due to flexure and due to torsion.

Equations [16], [17], and [18] below are taken from Reference 27.

$$\oint \tau_b ds = -\frac{\mu A_c}{1+\mu} \left( \frac{P_x \bar{y}_c}{I_y} - \frac{P_y \bar{x}_c}{I_x} \right) \quad [16]$$

$$\oint \tau_t ds = 2 G A_c \theta \quad [17]$$

$$\oint \tau_b ds + \oint \tau_t ds = \oint \tau ds = 2 G A_c \theta - \frac{\mu A_c}{1+\mu} \left( \frac{P_x \bar{y}_c}{I_y} - \frac{P_y \bar{x}_c}{I_x} \right) \quad [18]$$

where  $\oint$  denotes a line integral around a closed curve,  
 $\tau_b$  is the shearing stress tangent to  $s$  due to flexure,  
 $\tau_t$  is the shearing stress tangent to  $s$  due to torsion,  
 $\tau$  is the combined shear flow due to flexure and torsion,  
 $s$  is the closed path around which the integration takes place,  
 $\mu$  is Poisson's ratio,  
 $A_c$  is the area enclosed by  $s$ ,  
 $P_x, P_y$  are the shear components at the section in the positive direction of the  $x$  axis or  $y$  axis, respectively. ( $x$  and  $y$  axes are principal axes through the centroid of the section),  
 $I_x, I_y$  are the sectional moments of inertia taken about the  $x$  and  $y$  axes, respectively,  
 $\bar{x}_c, \bar{y}_c$  are the coordinates of the centroid of the area  $A_c$ ,  
 $G$  is the modulus of rigidity in shear, and  
 $\theta$  is the angle of twist per unit length.

Assuming that the shear at the section due to the external and d'Alembert's inertia forces is known, the integrals can be evaluated as soon as the shear-stress distribution due to flexure and torsion is known. The shear-stress distribution due to flexure can be calculated by a method such

as is developed in Figure 29a. The shear-stress distribution due to torsion can be calculated once the torque is known. The torque calculation requires the determination of the center of shear, which may be found as follows:

Assume that the force  $P_y$  is applied through the center of shear as shown in Figure 29b, i.e., we have flexure only. Then the moment of the internal forces about the center of shear is zero, and the moment of the external and internal forces about any point is zero.

The distribution of shear stress can be determined by assuming initially that the shear stress is zero at an arbitrary point such as A in Figure 29b. The actual shear flow due to flexure is obtained by adding a constant shear flow  $q_{b2}$  to the assumed shear flow  $q_{b1}$  such that the moment of the combined shear flow  $q_b = q_{b1} + q_{b2}$  about the center of shear is zero. The term shear flow denotes the product of the shear stress times the thickness over which the shear stress is considered uniform; it represents therefore a force per unit length.

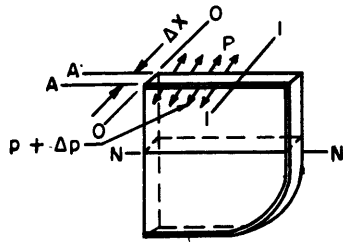
Take moments about the center of shear (refer to Figure 29b); then

$$P_y \times 0 = \oint \tau_b \times dA \times \bar{\rho} = \left[ -\sum q_{b1_x} (y - \epsilon_y) \Delta s + \sum q_{b1_y} (x - \epsilon_x) \Delta s \right] \\ + \left[ -\sum q_{b2_x} (y - \epsilon_y) \Delta s + \sum q_{b2_y} (x - \epsilon_x) \Delta s \right] = 0 \quad [19]$$

where  $q_{b1_x}$  denotes the component of an increment of the shear flow in the positive direction of the x-axis. The meaning of  $\bar{\rho}$ ,  $dA$ ,  $s$  is indicated in Figure 29b. The first term on the right-hand side of Equation [19] is the contribution of the assumed determinate shear flow distribution  $q_{b1}$  and the second term is the contribution of the constant shear flow  $q_{b2}$ . Equation [19] is in general written for two axes normal to each other in order to determine the center of shear  $\epsilon_x$ ,  $\epsilon_y$ , for sections without symmetry. There are two equations of the form of Equation [19] containing the three unknowns  $\epsilon_x$ ,  $\epsilon_y$ , and  $q_{b2}$ . A third equation is needed to solve for the unknowns. This third relationship is given by Equation [16], repeated here,

$$\oint \tau_b ds = -\frac{\mu A_c}{1 + \mu} \left[ \frac{P_x \bar{y}_c}{I_y} - \frac{P_y \bar{x}_c}{I_x} \right] \quad [16]$$

In the case of thin-walled closed shells such as a ship section, the right-hand side of [16] is small because  $\bar{y}_c$  and  $\bar{x}_c$  are small. If  $\mu$  is taken as zero the right hand side of [16] is equal to zero. Equations [16] and [19] permit the determination of the center of shear and the shear-stress distribution. If the loads are not applied through the center of shear they can be resolved into a force through this center plus a couple. The shear stresses



Half Section of a Ship

Formula For the Shear Stress in a Ship Section Due to Flexure

Sections A and A' are parallel and a distance  $\Delta x$  apart.

$\tau = 0$  at Section 0-0, by symmetry

$\Delta F = \tau \times t \times \Delta x$  for Section 0-0 to 1-1

(1)  $\tau = \frac{1}{t} \frac{dF}{dx}$  but  $p = \frac{My}{I}$

$F = \int_{0-0}^{1-1} p t ds = \int_{0-0}^{1-1} \frac{My}{I} t ds$

Let  $\int_{0-0}^{1-1} y t ds = m =$  moment of area about the neutral axis N-N

then

$F = \frac{M}{I} m$

$\frac{dF}{dx} = \frac{m}{I} V + M \frac{d}{dx} \left( \frac{m}{I} \right)$

If the ship is fairly uniform  $\frac{d}{dx} \left( \frac{m}{I} \right) \rightarrow 0$

and  $\frac{dF}{dx} \approx \frac{mV}{I}$  ∴

(2)  $\tau \approx \frac{mV}{It}$

where  $\tau$  is the shear stress,

$t$  is the plating thickness,

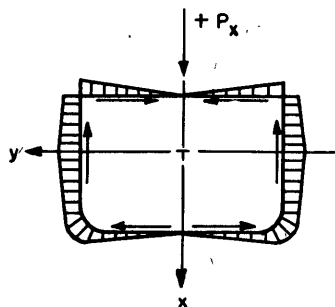
$F$  is the force on that portion of the shell contained between the center line 0-0 and the section 1-1 under consideration,

$M$  is the bending moment at any transverse section,

$I$  is the area moment of inertia of the whole section about the neutral axis N N

$ds$  is an increment of length along the shell, and

$P$  is the total shear at the section



X X and Y Y are Principal Axes

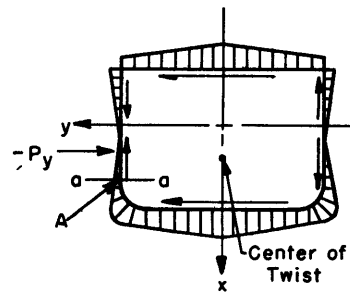


Figure 29a - Shear Stresses in a Transverse Section of a Ship

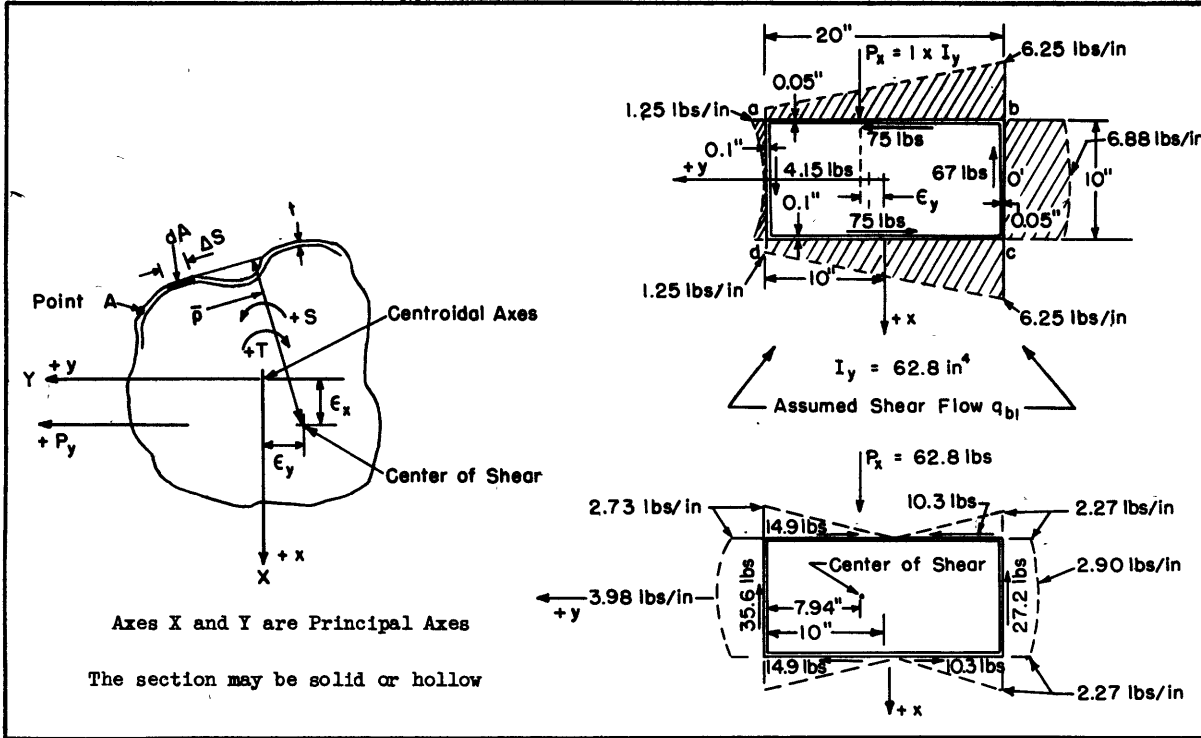


Figure 29b

Figure 29c - Shear Flow Distribution  $q_b = q_{b1} + q_{b2}$  Due to Load  $P_x$  Applied Through Center of Shear

Figure 29 - Method of Calculating Shear Stress and Shear-flow Distribution of a Single-Hull Ship

due to this couple can then be superimposed on the shear stresses due to bending. For thin-walled shells the shear-flow theory of Bredt for torsion<sup>22</sup> can be used to determine the stress distribution due to torsion alone. It will be helpful to remember that for any section having two or more axes of symmetry the center of shear is located at the intersection of these axes. For any section having a point of symmetry such as an equilateral triangle the center of shear is located at that point. For any section having but one axis of symmetry the center of shear is located on that axis but not in general at the centroid of the section.

As a simple illustration of the method outlined, the center of shear and the shear stress distribution will be calculated for the box girder illustrated in Figure 29c. Assume the shear stress is zero at  $x = 0, y = 10$  although obviously this is not the case. The center of shear will lie on the  $y$ -axis by symmetry. Assume  $\mu$  equal to zero and choose the axes as shown; then

$$\begin{aligned}
 q_{b_1} &= t \times \tau_{b_1} \text{ at } a = 5 (0.1) (2.5) = 1.25 \text{ lb/in.} \\
 &= t \times \tau_{b_1} \text{ at } b = 1.25 + 20 (0.05) (5) = 6.25 \text{ lb/in.} \\
 &= t \times \tau_{b_1} \text{ at } 0' = 6.25 + 5 (0.05) (2.5) = 6.875 \text{ lb/in.} \\
 &= t \times \tau_{b_1} \text{ at } c = &= 6.25 \text{ lb/in.} \\
 &= t \times \tau_{b_1} \text{ at } d = &= 1.25 \text{ lb/in.}
 \end{aligned}$$

The assumed shear flow  $q_{b_1}$  is indicated in Figure 29c.

From [19]

$$1461.5 = 62.85 \epsilon_y + 400 q_{b_2}$$

From [16]

$$\oint \tau_b ds = \oint \left( \frac{q_{b_1}}{t} + \frac{q_{b_2}}{t} \right) ds = 0 = - \left[ \frac{4.15}{0.10} + \frac{2(75)}{0.05} + \frac{67}{0.05} \right] + 1100 q_{b_2}$$

$$q_{b_2} = \frac{4389}{1100} = +3.98 \text{ lb/in.}$$

Solving for  $\epsilon_y$  gives

$$62.85 \epsilon_y = 1461.5 - 400(3.98) \quad \epsilon_y = + 2.06 \text{ in.}$$

The distribution of the shear flow (flexure only) is shown in Figure 29c.

If the load is not applied through the center of shear ( $\epsilon_x = 0$ ,  $\epsilon_y = + 2.06$  in.) then the twisting moment is equal to the moment of the loads about the center of shear. In general, whether the beam is subjected to bending alone or to a combination of bending and torsion, application of Equations [18] and [19] will permit the determination of the stress distribution.

The confusion that exists in the literature as to the meaning of the terms "elastic axis," "center of shear," "center of twist," "flexural center" is discussed in NACA Technical Note No. 562 "Remarks on the Elastic Axis of Shell Wings" by P. Kuhn.<sup>29</sup>

The point about which the section of a straight beam subjected to twisting rotates is called the center of twist. The rotation will be measured relative to that point in the section which has a uniform rotation about the z axis. In the case of the full-scale tests, which are discussed in this report, it was necessary to find the center of twist for at least one section of the vessel in order to make possible the calculation of the translatory component of motion, at any point, resulting from a given amount of torsional vibration.





are present that are not associated with the torsional mode, then it is necessary to separate out the torsional motion before applying the preceding formula [20] for the experimental location of the axis of twist.

Any contribution to vertical and athwartships components of vibratory motion that is not due to pure torsional vibration and is erroneously assumed as being due to torsion, will introduce an error. The test data show that the athwartships amplitudes due to flexure were very small at the one-noded torsional resonance; also since no vertical exciting forces were present there should be no extraneous vertical components of flexural vibration present. Another error may be introduced due to the rigid-body translation and rolling motion of the ship. The angular amplitude due to rolling is equal to  $T/I\omega^2$ , where  $T$  is the torque applied by the vibration generator to the ship,  $I$  is the polar mass moment of inertia of the ship about its center of gravity, and  $\omega$  is the angular frequency of vibration.

$$\theta = \frac{T}{I\omega^2} = \frac{7390 \times 7.6}{50 \times 400 \times 2240 \times (5^2 \times 40)} = 0.13 \times 10^{-5} \text{ radians} \quad [21]$$

The torsional amplitude at the fantail is about  $50 \times 10^{-5}$  radians in the first torsional mode. It is seen that the effect of rolling may be neglected. However the rigid-body translatory motion, about 0.4 mils single amplitude, cannot be neglected in determining experimentally the axis of twist for the section of the ship extending between Frames 50 and 170. Sufficient data for determining the axis of twist with a fair degree of accuracy were obtained for the section at Frame 188 only; see Figure 26. Very little thought had been given, at the time of these tests, to the center of twist.

To summarize:

- (a) A method has been suggested for calculating the center of shear and the torque acting at any section of a straight beam.
- (b) The center of shear is a function only of the shape of cross section and the elastic characteristics of the beam.
- (c) The center of twist may be measured during a vibration-generator test provided that it is possible to separate out the pure torsional contribution.
- (d) Theoretical calculations made to determine the natural frequencies and modes of torsional vibration require a knowledge of the position of the center of shear. It does not appear that the literature has taken account

of this fact to date. In general, if the center of shear and the center of mass do not coincide then coupling between flexural and torsional modes of vibration will take place.

### 6.8 Rigidity of Tailshafts and of Intermediate Strut Arms As Determined by Tests

In order to determine the rigidity of the intermediate strut arm, the shafts and the rubber bearing, a number of load-deflection tests were made as illustrated in Figures 31 and 32; the figures show the load-deflection curves as well as the manner in which the loads were applied.

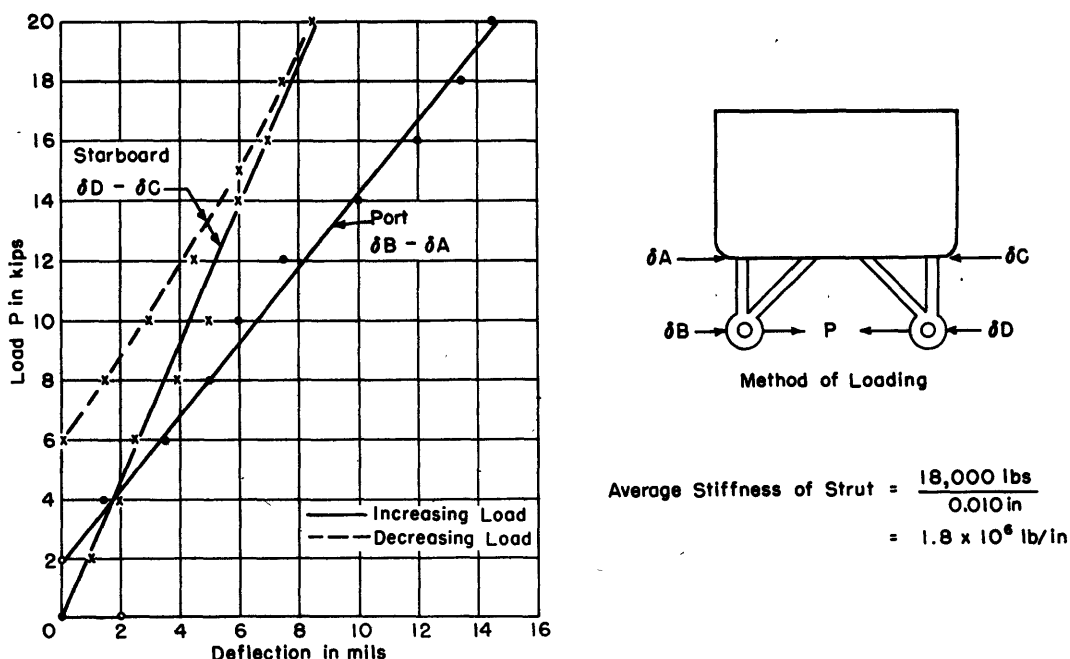


Figure 31 - Stiffness Data for Intermediate Strut Arm (DD865)

The starboard strut arm is longer than the port arm. The shafting was removed for this test.  
Date of test: 30 July 1948.

Inspection of Figure 31 shows that the average stiffness of the strut itself is about  $1.8 \times 10^6 \text{ lb/in.}$  in the horizontal direction. The slopes of the load deflection curves in Figure 32a give the stiffness of the rubber bearing as  $0.53 \times 10^6$  and  $0.66 \times 10^6 \text{ lb/in.}$  for the port and starboard bearings, respectively. The average combined stiffness of the strut bearing and strut is, therefore about

$$\frac{1}{1.8} + \frac{2}{0.66 + 0.53} = 0.45 \times 10^6 \text{ lb/in.}$$

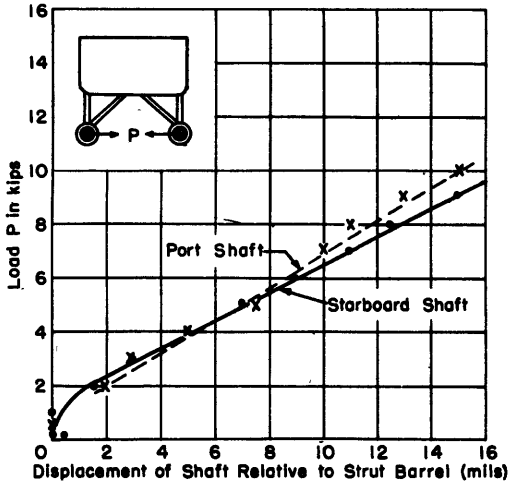


Figure 32a

Stiffness of Rubber Bearing

Port Strut  $0.525 \times 10^6$  lb/in.

Starboard Strut  $0.660 \times 10^6$  lb/in.

(determined from the slope of the curve above)

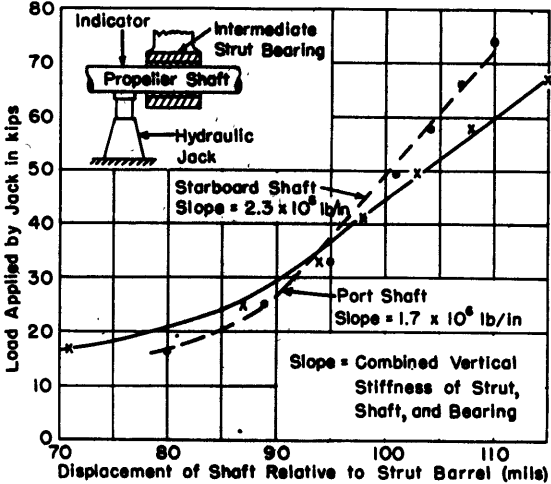


Figure 32b

Force required to lift shaft off the rubber bearing was 9.1 kips for the port shaft and 5.0 kips for the starboard shaft.

Figure 32 - Stiffness Data for Rubber Bearing and Shaft at Intermediate Strut

Date of test: 26 July 1948

The stiffness of the propeller shaft at the intermediate strut bearing is calculated to be  $0.035 \times 10^6$  lb/in. The strut-bearing combination is therefore, about 13 times as stiff as the shaft at that point. The force required to lift the shaft clear of the bearing was 5000 pounds for the starboard shaft and 9100 pounds for the port shaft. As long as the effective vertical vibratory force acting at the intermediate strut is less than the dead load carried by the bearing, the bearing will offer an effective restraint to the vertical whipping motion of the shaft.

Since the clearance between the propeller shaft and the intermediate strut bearing is appreciable, about 60 mils, and since there is no obvious steady transverse force holding the shaft against the bearing surface, it is seen that the only restraint to athwartships whipping motion is due to friction and other forces occasioned by local deformation of the rubber bearing. A test with the vibration generator showed that an athwartships alternating force of about 110 lb--applied through the shaft at the bearing--was successfully resisted by these restraining forces; the limiting force which can be resisted is unknown. In this test however, the shaft was stationary and the bearing was dry; in actual operation it is possible that less restraint is offered. It is evident that if the vibratory forces are large enough, a point is reached at which the restraint is overcome and the shaft will whip in an athwartships plane as though the intermediate strut bearing were nonexistent. If the vibration amplitudes exceed the available clearance, impact

between shaft and bearing occurs. Since the bearing is loaded with an average dead load of  $1/2(5.0 + 9.1)$  kips the average deformation of the rubber bearing will be about 7050 lb divided by  $0.45 \times 10^6$  lb/in. or 15.6 mils.

Measurements of the vibratory motion of the shaft and strut bearings, made during the afloat test in Wallabout Bay, have been plotted in Figure 36 on page 63. These measurements indicate that the restraint offered by the intermediate strut bearing is sufficient to prevent the occurrence of resonance whipping motions at the bearing for the magnitude of vibration amplitudes normally experienced by the DD692-Class destroyers. The latter tests were made with nonrotating shafts, however, and it is probable that unusually severe vibratory forces acting on the propeller-shaft system (such as during a high-speed turn) will disturb the equilibrium of the shaft in its bearings, thus inducing temporary whipping vibrations of the shaft section supported between the stern-tube bearing and the after strut.

Measurements of strain at the root of the outboard arm of the starboard intermediate strut (see Enclosure (S) of Reference 15) indicated a principal stress of the order of 750 psi for an athwartship load of 14,000 pounds applied at the center of the strut bearing. The error in measuring the stress for the particular test condition may have been as large as 300 psi, so that the actual stress might have been anywhere from 450 to 1050 psi.

In order to compare the stiffness of the double-arm strut with that of the original single-arm strut, the stiffness of the latter was calculated by considering the strut cantilevered from the hull with a concentrated load P applied at the center of the strut bearing. The value obtained was  $0.13 \times 10^6$  pounds per inch as compared to an experimental value of  $1.8 \times 10^6$  pounds per inch for the double-arm strut. This great increase in rigidity indicates the effectiveness of designing members so as to carry loads in tension or compression rather than in bending. The combined rigidity of the strut and rubber-bearing combination was about  $0.11 \times 10^6$  pounds per inch for the single-arm strut as against  $0.45 \times 10^6$  pounds per inch for the double-arm strut which is now generally installed on these ships. The rigidity of the shaft support has, therefore, been increased by a factor of four. Calculations show that the maximum stresses at the root of the outboard starboard strut arm, due to a transverse static load of 14,000 pounds applied to the bearing, have been reduced from a calculated value of about 13,500 psi for the single-arm strut to a measured value of somewhere between 450 and 1050 psi for the double-arm strut. The maximum stresses ever measured at the root of the single arm strut during operation at sea were of the order of 25,000 psi--as mentioned in the beginning of this report. It is, therefore, reasonable to expect that the stresses in the double-arm strut will be very low, say not more than 2000 psi. Actually no failures of double-arm struts have

been heard of, although a considerable number of failures of single-arm struts have occurred.

Strain measurements made during high-speed turns on the DD723,<sup>4</sup> DD771,<sup>7</sup> and the DD772,<sup>8</sup> all of which were equipped with the original single-arm intermediate strut, showed a maximum stress of the order of 25,000 psi at the root of the strut arm--occurring at rather low frequencies, probably near 300 cpm. Similar measurements made on the DD692,<sup>6</sup> which was fitted with a double-arm strut, gave a maximum stress of about 1500 psi--also at a frequency near 300 cpm. This indicates a reduction in stress by a factor of about 16, a reduction that fits in rather well with the preceding theory.

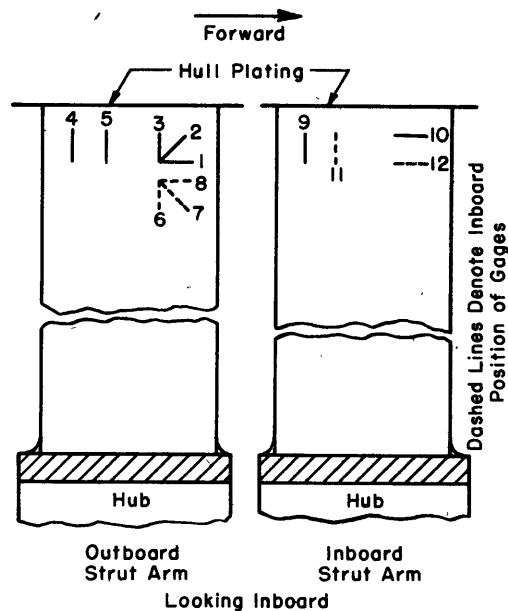
The double-arm strut is therefore to be preferred from considerations of both structural strength of the strut and hull vibration.

### 6.9 Stresses in Struts During Vibration-Generator Tests

The SUMNER-Class destroyers were originally fitted with single-arm intermediate struts. Several such struts failed due to cracking near the point of attachment to the hull. Measurements of stresses near the juncture of the strut to the hull showed that stresses of the order of 25,000 psi were experienced.<sup>7,8,9</sup>

The intermediate struts were redesigned as double-arm struts.<sup>5</sup> In order to get an idea of the stress in these redesigned struts the following tests were made. SR-4 strain gages were installed on both the inboard and outboard arms of the intermediate and after strut arms. The gages were located about 4 inches from the juncture of the hull plating and the strut arm. The gage installation was waterproofed, as described under INSTRUMENTATION, and the vessel was then floated and anchored in Wallabout Bay (see Figure 3 on page 20). A vibration-generator test was made during which the amplitudes of vibration of the hull, the shafting, and the strut barrels were measured. At the same time measurements of dynamic strains at the locations indicated in Figure 33 were made. The vibration amplitudes measured at Frames 188 and 72 have been plotted in Figures 22 through 26. The single amplitudes of athwartships vibration corresponding to the torsional resonance at 330 cpm and recorded simultaneously with the strains, were 24 mils and 19 mils for the main and intermediate struts respectively. The only strain gages which were usable were Gages 2 and 8 on the main strut and Gage 10 on the intermediate strut. The other gages evidenced an excessive shift in zero due to insufficient waterproofing. Time did not permit rebalancing of the gage circuits. The strains, as well as the amplitudes, were a maximum at the resonance near 330 cpm. The strains were too small to give measurable data at other than the 330 cpm resonance; that is, the stresses were less than about

Intermediate Strut		Main Strut	
Lead No.	Gage No.	Lead No.	Gage No.
1	1	13	1
2	2	14	2
3	3	15	3
4	4	16	4
5	5	17	5
6	6	18	6
7	7	19	7
8	8	20	8
9	9	21	9
10	10	22	10
11	11	23	11
12	12	24	12



Identification of Cable Leads

Figure 33 - Arrangement of SR-4 Strain Gages on Main and Intermediate Strut Arms of the DD865

Accelerometers were located on the barrels of the strut arms in order to provide simultaneous measurements of strains and athwartships accelerations.

30 psi. The principal stresses indicated in the main strut were about 170 psi along the axis of the strut and 67 psi at right angles to the strut axis. The principal stresses in the intermediate strut were too small to measure. The values of stress given above are accurate to within 20 percent.

It is evident that the stresses in the intermediate and main struts occurring during ordinary operation will be negligible. During high-speed turns somewhat higher stresses may be encountered.

#### 6.10 Mode of Vibration of Tailshaft-Propeller System and the Vibratory Response of the Ship to Forces Applied Through the Propeller-Shaft System

The exciting forces which are applied to the ship through the propeller-shaft system are of primary importance in analyzing the behavior of the DD692-Class destroyers. The objectionable vibration is caused by effective first-order unbalance forces, and it is therefore important to determine the magnitude of these forces, as well as to determine the effect on vibration of the elastic characteristics of the structures through which the forces are applied to the hull girder.

It was suggested<sup>9</sup> that the tailshaft might have a natural whirling or whipping frequency near the upper hull critical (320 cpm). The fundamental natural frequency of whipping is calculated to be about 300 cpm in water if the restraint of the intermediate strut is removed. This vibration could also occur if the intermediate strut vibrated so as to offer but little restraint to the shaft. In Section 6.8, entitled "Rigidity of Tailshafts and of Intermediate Strut Arms as Determined by Tests," it is shown that the intermediate strut does restrain the motion of the shaft except during severe transient vibration such as may occur during turns.

The mode of vibration of the propeller-tailshaft system, in air, was determined by means of a vibration generator installed as shown in the photograph, Figure 34. The results of this test are presented in Figure 35. It is seen that the fundamental natural frequency of vibration of this system occurs at 660 cpm and that the largest amplitude of the propeller shaft occurs aft of the intermediate strut bearing. A second resonance is approached near 1200 cpm.

The behavior of the propeller-tailshaft system was again examined when the entire vessel was vibrated during the afloat tests in Wallabout Bay (see Figure 3 on page 20 for the test conditions). The results of this test made possible a tie-in between the motion of the underwater appendages and the motion of the hull proper. The motion of the hull at Frames 72 and 188 was measured at the same time that the tailshaft measurements were made. The vibratory amplitude of several points on the propeller-tailshaft system have been plotted in Figure 36 for a number of different frequencies. It is seen from an inspection of this figure that the hull resonances are reflected in the motions of the tailshaft. The mode of vibration of the starboard tailshaft has been plotted in Figure 37 on the basis of the data given in Figures 35 and 36. Looking at Figure 37 it is apparent that the amplitudes of vibration of the shaft relative to the bearings are nearly the same for both the 270- and the 330-cpm vibration. This fact indicates that the natural frequency of whipping vibration in water is higher than 330 cpm because, as resonance is approached, the amplitude of vibration will vary rapidly with frequency, see Figure 35.

A method of determining the natural whirling frequency of tailshafts is given in Bureau of Ships Design Data Sheet DDS43-1 Part 1, dated December 1944. The whipping-frequency formula used therein gives the frequency of a shaft of length equal to the longest span of the tailshaft, with simple supports at both ends. A correction is made if the spans of the tailshaft are not equal. A calculation of the whirling frequency of the tailshaft of the DD865 by the formula just referred to gives a fundamental whipping resonance



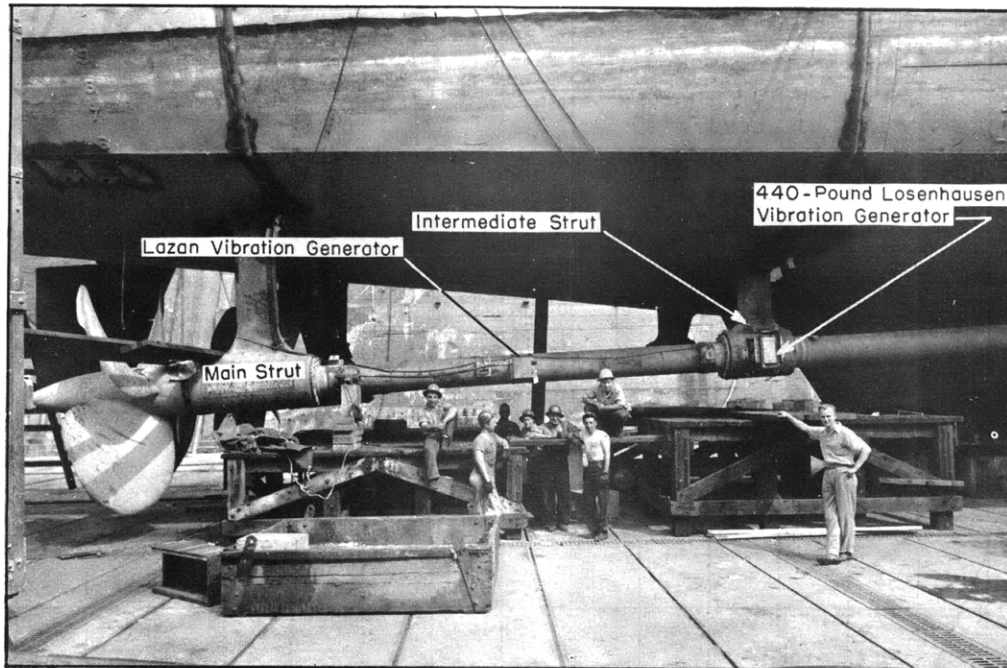


Figure 34a - Installation of Vibration Generator on Tailshaft and Strut

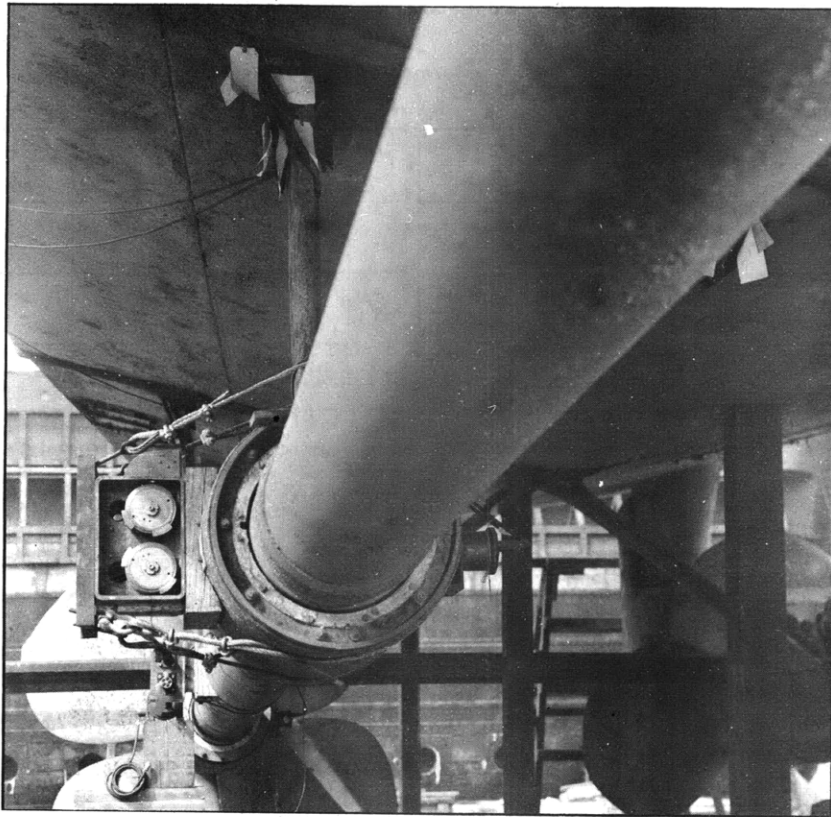


Figure 34b - Installation of Vibration Generator on Starboard Shaft - Looking Aft

Figure 34 - Shafting and Propellers of DD865 Instrumented for Tests

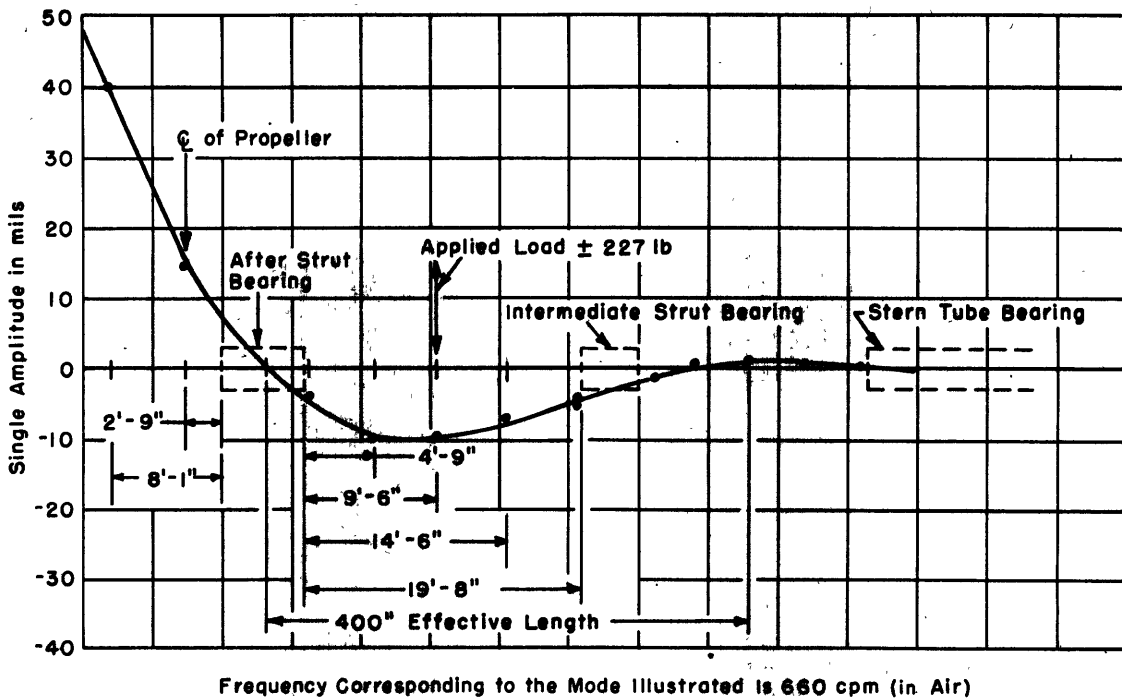
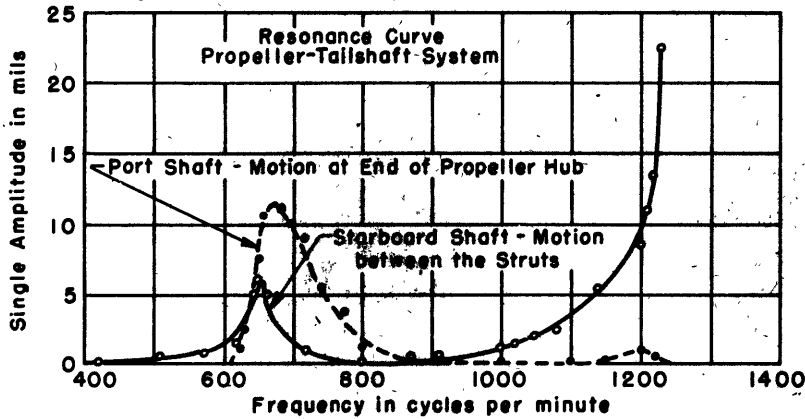


Figure 35 - Fundamental Mode of Whirling Vibration of the Propeller-Tailshaft System as Determined by Means of a Vibration-Generator Test

A Lazan vibration generator was used with a setting of 140°. The relative phase of the motion at the several stations was recorded.

near 1140 cpm, whereas the experimental value obtained in the manner described in this report is 660 cpm. It is apparent that a more realistic formula is required. The disagreement between these values of 1140 cpm and 660 cpm is considerably greater than that found in model tests carried out at the Massachusetts Institute of Technology in 1944.<sup>30, 31</sup> Actually, the model tests

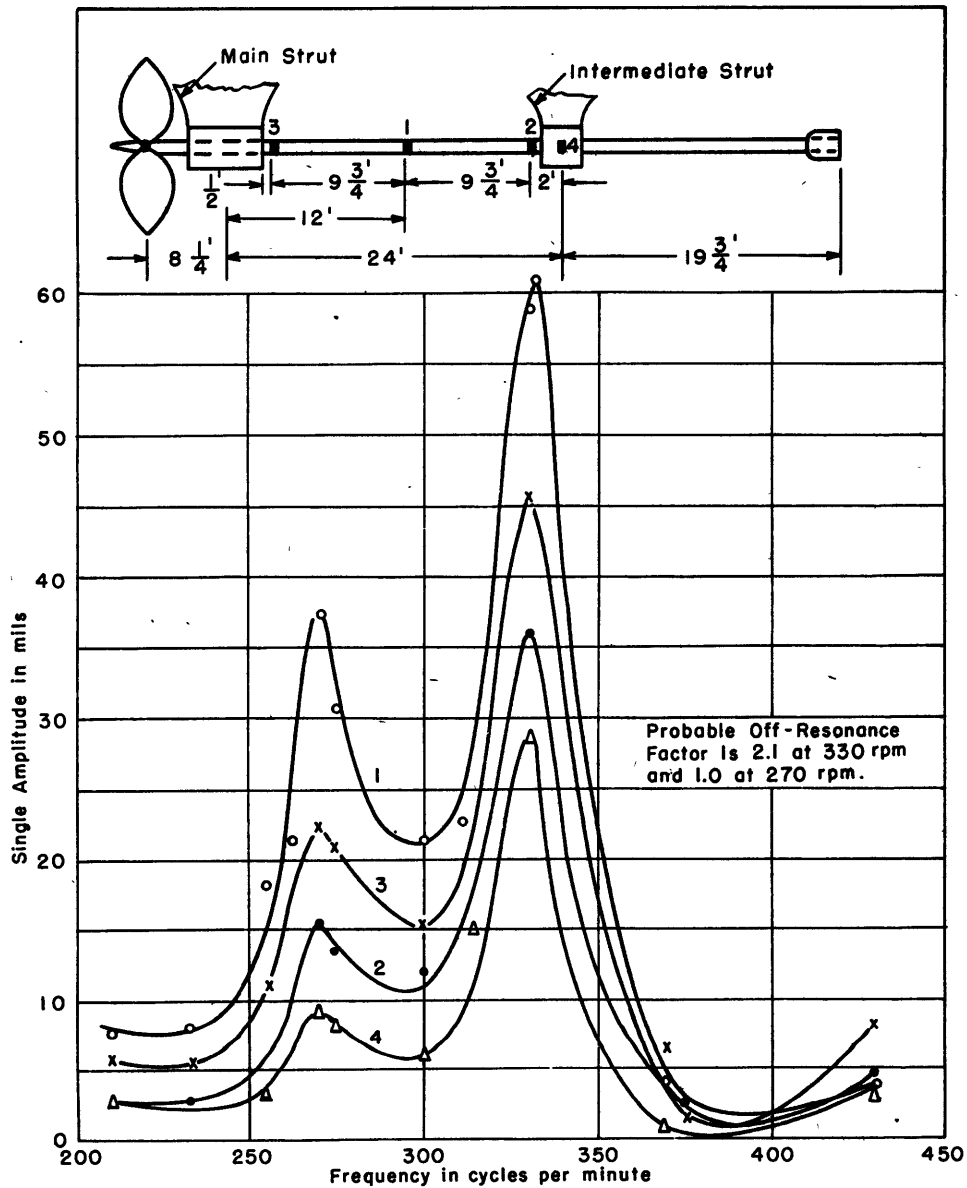


Figure 36 - Athwartships Vibration Amplitudes of Submerged Propeller-Tailshaft System Measured During Vibration-Generator Test in Wallabout Bay

Athwartships excitation was by means of the TMB medium vibration generator located on the Main Deck at Frame 200. The eccentricity setting of the generator was  $60^\circ$ .

showed that the bearing restraints in the model approached simple supports and the same restraints were assumed in the MIT calculations, made to check the agreement between theory and experiment. The DD865 tests show that the restraints are far different from those of the model tests. It is necessary

to base the calculation of whipping frequencies on a more equitable basis. It is suggested that the frequency calculation be based on a shaft-deflection curve of the shape illustrated by the test results, Figures 35 and 37, applying the Rayleigh Method of equating the potential to the kinetic energy. Since the potential energy is a function only of the deflection curve, a reasonable value for the frequency should be obtained--see, for example, Case 5 of Table 3.

The plot, Figure 37, has been used to determine the influence coefficients for forces applied to any point on the tailshaft. That is, the deflection of the shaft, at a location x, in response to an application of a

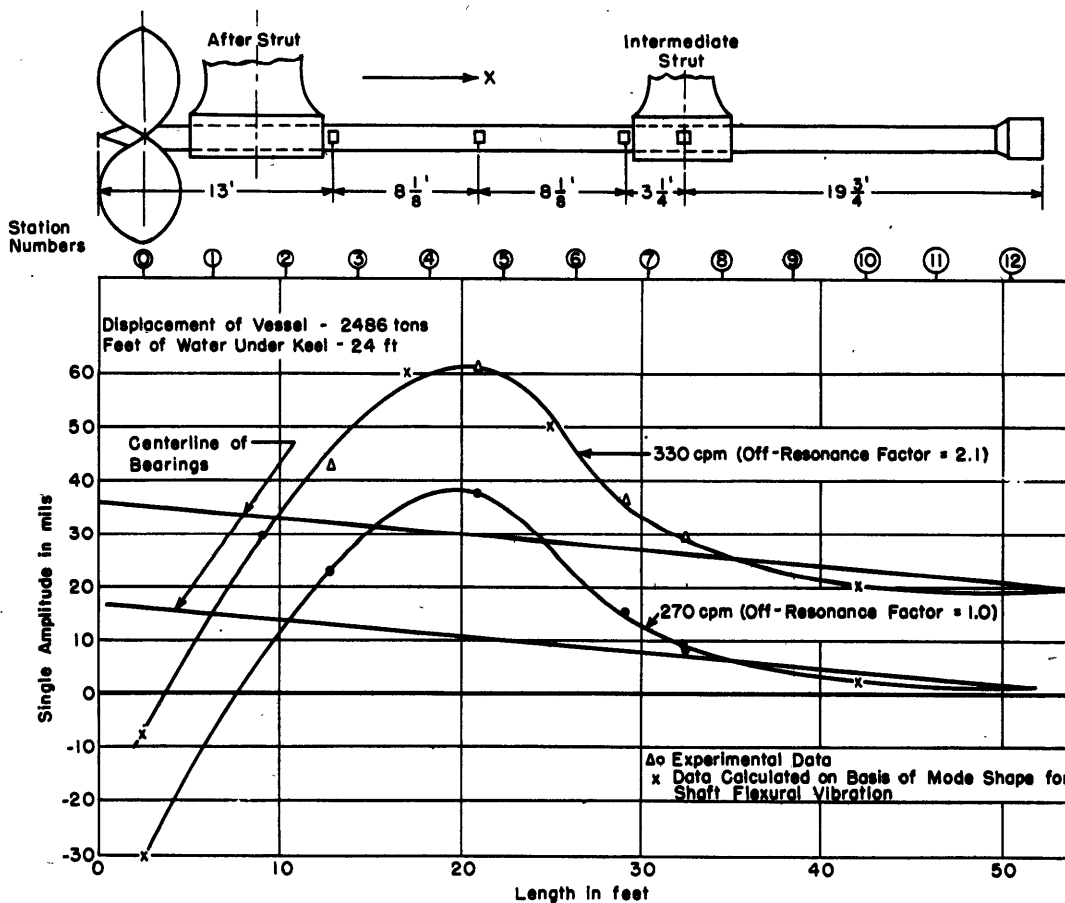


Figure 37 - Transverse Vibration Amplitudes of Starboard Shaft During Vibration Generator Survey

The TMB vibration generator was located on center line of Main Deck at Frame 200 and oriented so as to apply forces in the transverse direction. The applied force was  $\pm 8400$  lb at 330 cpm and  $\pm 5600$  lb at 270 cpm.

The displacement of the ship was 2486 tons.  
Depth of water under keel was 24 feet.

known exciting force at a point  $x = \epsilon$ , was determined for any location  $x$  on the tailshaft. By using the principle of reciprocity, the deflection of the point  $\epsilon$  due to the application of a given force at the point  $x$ , can be determined, since  $\delta_{\epsilon x} = \delta_{x\epsilon}$  where the influence coefficient  $\delta_{\epsilon x}$  is the deflection at  $\epsilon$  due to a unit load applied at point  $x$ . It can now be seen that the data determined in this test, together with a knowledge of the natural modes of vibration of the hull, will permit the determination of the motion of the hull in response to any forces applied to the tailshaft propeller system, provided that the characteristics of that system remain unchanged.

It is readily seen, from an inspection of Figure 37 and from a knowledge of the natural modes of vibration of the hull, that exciting forces applied to the tailshaft section of the main propulsion shafting are of more significance than an application of the same exciting force to the line shaft. The absolute amplitudes of vibration plotted in Figure 37 are directly proportional to the influence factor applicable to the point under consideration and for that reason it is obvious that propeller unbalance is of considerably more significance in exciting the three-noded flexural vibration than in exciting the one-noded torsional mode of vibration.

Table 7a shows the approximate effect on the first-order hull criticals for a given amount of mass unbalance of the propeller, tailshaft, and lineshaft respectively. These figures are based on influence factors calculated from the forced-vibration data presented in this and in prior sections. It is assumed that the shaft is restrained to the extent that was present during the tests. During turns this restraint may be lost due to the effect of the hydrodynamic lifting forces which tend to relieve the load on the intermediate strut. In the latter case the influence factors and the consequent vibration will be greatly increased. Calculation indicates that the intermediate strut bearing may be relieved of its dead load, which was found to be about 6000 lb, during full rudder turns at speeds as low as 20 knots. For a discussion of hydrodynamic loads see Section 7.2. The importance of keeping the intermediate strut bearing loaded should be noted.

The influence factors applicable to the tailshaft-propeller system could be reduced by installing extra supports. The maximum reduction possible is obtained by assuming continuous support of the shaft. In this case the vibration contributed by a uniform distributed unbalance of the tailshaft would be reduced by 58 percent for the three-noded flexural mode and by 24 percent for the one-noded torsional mode. The contribution of forces applied to the propeller would be reduced by 94 percent for the three-noded flexural mode and increased 440 percent for the one-noded torsional mode of vibration.

TABLE 7

Hull Vibration Due to Known Exciting Forces Applied to the Propeller Shaft System at Resonance Frequency Indicated.

(a) Vibratory Motions Due to a Mass Unbalance Equivalent to 0.010 Inch Mass Eccentricity of Propellers and Shafting.

(b) The Influence Coefficient  $\delta_{ex}$ \* for Forces Applied to the Propeller Tailshaft System

Item	One-Noded Torsional Mode	Three-Noded Flexural Athwartship Mode
	Based on Experimental Data	
Each Propeller Weight 1.2 x 15,500 lb	-1.03 x 10 <sup>-3</sup> in.	-1.77 x 10 <sup>-3</sup> in.
	-1.14 x 10 <sup>-5</sup> rad.	-1.74 x 10 <sup>-3</sup> in.
Each Tailshaft	+4.63 x 10 <sup>-3</sup> in.	+0.78 x 10 <sup>-3</sup> in.
	+5.1 x 10 <sup>-5</sup> rad.	0.76 x 10 <sup>-3</sup> in.
Lineshaft Port	+0.60 x 10 <sup>-3</sup> in.	+1.70 x 10 <sup>-3</sup> in.
	+0.66 x 10 <sup>-5</sup> rad.	+1.66 x 10 <sup>-3</sup> in.
Lineshaft Starboard	+1.0 x 10 <sup>-3</sup> in.	+5.40 x 10 <sup>-3</sup> in.
	+1.12 x 10 <sup>-5</sup> rad.	+5.3 x 10 <sup>-3</sup> in.

The upper figure in each box is the athwartships amplitude that would be measured at a point 2 feet above the Main Deck at Frame 200. The lower figure is the vibratory amplitude in the given mode that would be generated at Frame 200.

The positive sign indicates that the motions will add if the unbalance forces are in phase.

Station†	$\delta_{ex}$ (in./lb)		$\delta_{ex}$ (in./lb)	
	One-Noded Torsional Mode		Three-Noded Flexural Athwartship Mode	
	Experimental Value††	Continuous Support Value†††	Experimental Value††	Continuous Support Value†††
0	-2.02 x 10 <sup>-6</sup>	+8.8 x 10 <sup>-6</sup>	-5.53 x 10 <sup>-6</sup>	+2.85 x 10 <sup>-6</sup>
1	+3.73	+8.6	-1.78	+2.6
2	+9.0	+8.2	+2.1	+2.4
3	+12.8	+8.0	+5.2	+2.2
4	+14.9	+7.55	+6.7	+2.0
5	+14.9	+7.4	+6.4	+1.8
6	+11.3	+6.9	+3.9	+1.6
7	+8.2	+6.7	+2.1	+1.3
8	+6.5	+6.3	+1.2	+1.1
9	+5.7	+6.1	+0.7	+0.9
10	+5.0	+5.9	+0.5	+0.7
11	+4.9	+5.5	+0.4	+0.5
12	+4.8	+5.3	+0.2	+0.3

†The station numbers refer to those given in Figure 37.

††This value is obtained from the experimental data plotted in Figure 37.

†††This value assumes continuous support of the tailshaft.

\* $\delta_{ex}$  is the deflection 2 feet above the Main Deck at Frame 200 due to a load of one pound applied at the given station or location on the tailshaft system.

No great benefit can therefore accrue from increasing the number, or stiffness, of the bearings except that every effort should be made to assure that the natural whipping resonance occurs appreciably outside the operating rpm of the vessel. The cure for the excessive first-order vibration must be sought primarily by increasing the stiffness of the hull or by decreasing the exciting forces.

The condition of mass unbalance of tail shafting and propellers, existing during the pre-repair trials of the DD865, is indicated in Figures 38 through 40, and the crookedness of the tailshaft at that time is shown in Figure 41. The propeller-tailshaft systems were balanced by the New York Naval Shipyard during the overhaul period to within 90 oz-in. in each of the two transverse planes passing through the after and the intermediate strut bearing respectively. This residual unbalance corresponds to a mass eccentricity of about 0.0003 in., which can be considered as perfect. The shaft was also straightened before commencing balancing. For details of the balancing and straightening work, see Reference 15.

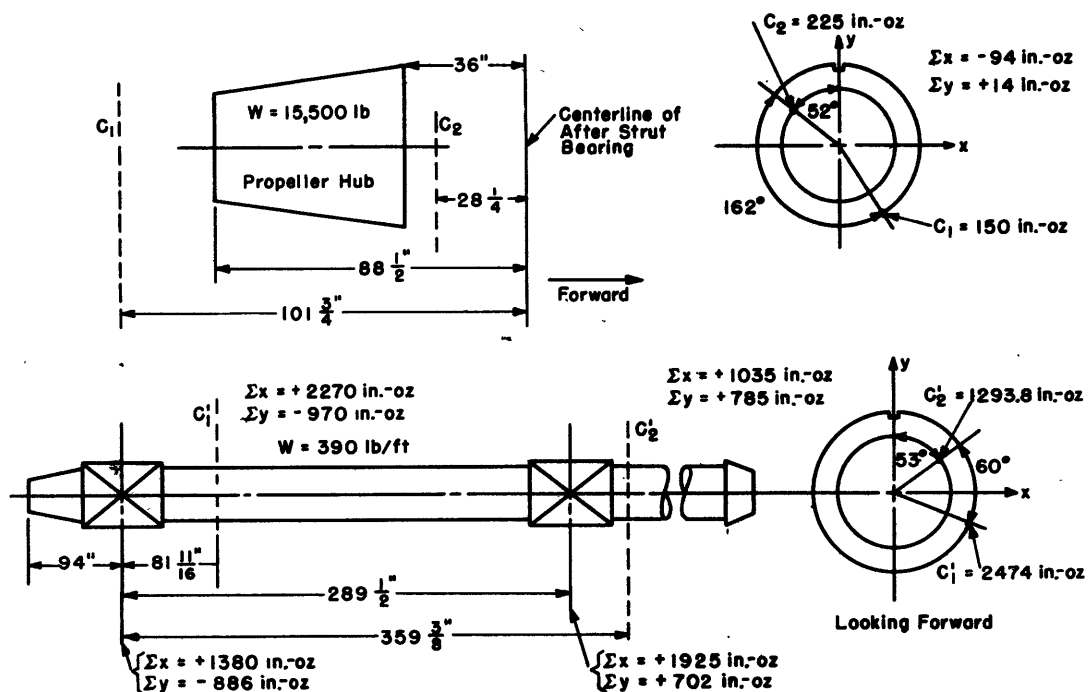


Figure 38 - Balancing of the Starboard Tailshaft and Propeller on the DD865

$C_1C_2C_1'C_2'$  are the planes in which correction weights are applied.

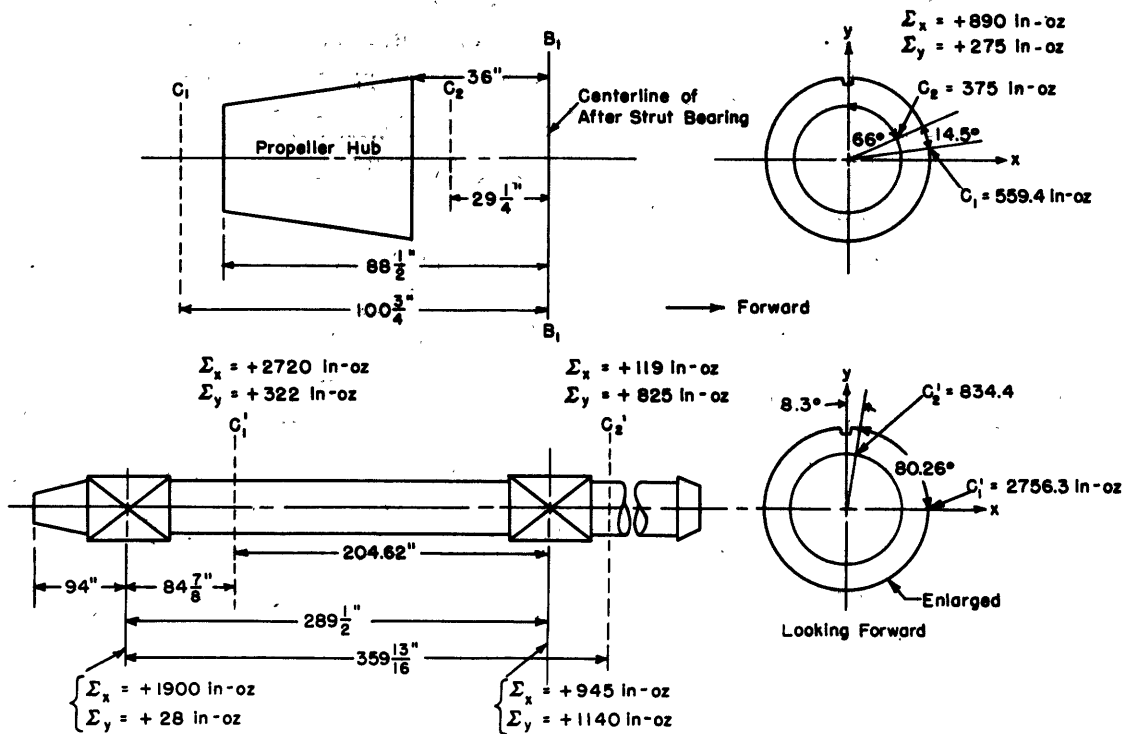


Figure 39 - Balancing of the Port Tailshaft and Propeller on the DD865

$C_1C_2C_1'C_2'$  are the planes in which correction weights are applied.

A calculation was made to determine the reduction in vibratory motion which could be expected due to this balancing and straightening work. The procedure was as follows:

- (a) The shaft and propeller were assumed to have been balanced and straightened perfectly. This assumption was nearly valid.
- (b) The centrifugal forces due to the initial unbalances and eccentricities (Figures 38 to 41) were calculated taking into consideration the proper phase relationships.
- (c) The deflection at Frame 200, due to the action of the forces determined in (b), was calculated by making use of the influence factors in Table 7. (The axis of twist is assumed to be 7.6 feet below the center of the vibration generator)
- (d) By making use of the mode configuration, Figures 12 and 15, it is possible to determine the motion at any point of the ship, if the motion at one point is known and if the vibration is resonant in the respective mode.



Since the motion at Frame 200 had been calculated in (c) preceding, the motion at Frame 72, 02 deck level, was easily determined.\*

(e) These calculated amplitudes were increased by 30 percent for the one-noded torsional mode and by 15 percent for the three-noded flexural mode of athwartships vibration. These percentage increases were based on a rough calculation of the effect of alternating lift forces which act on the tailshaft due to the vibratory motion of the shaft, see Section 7.2.

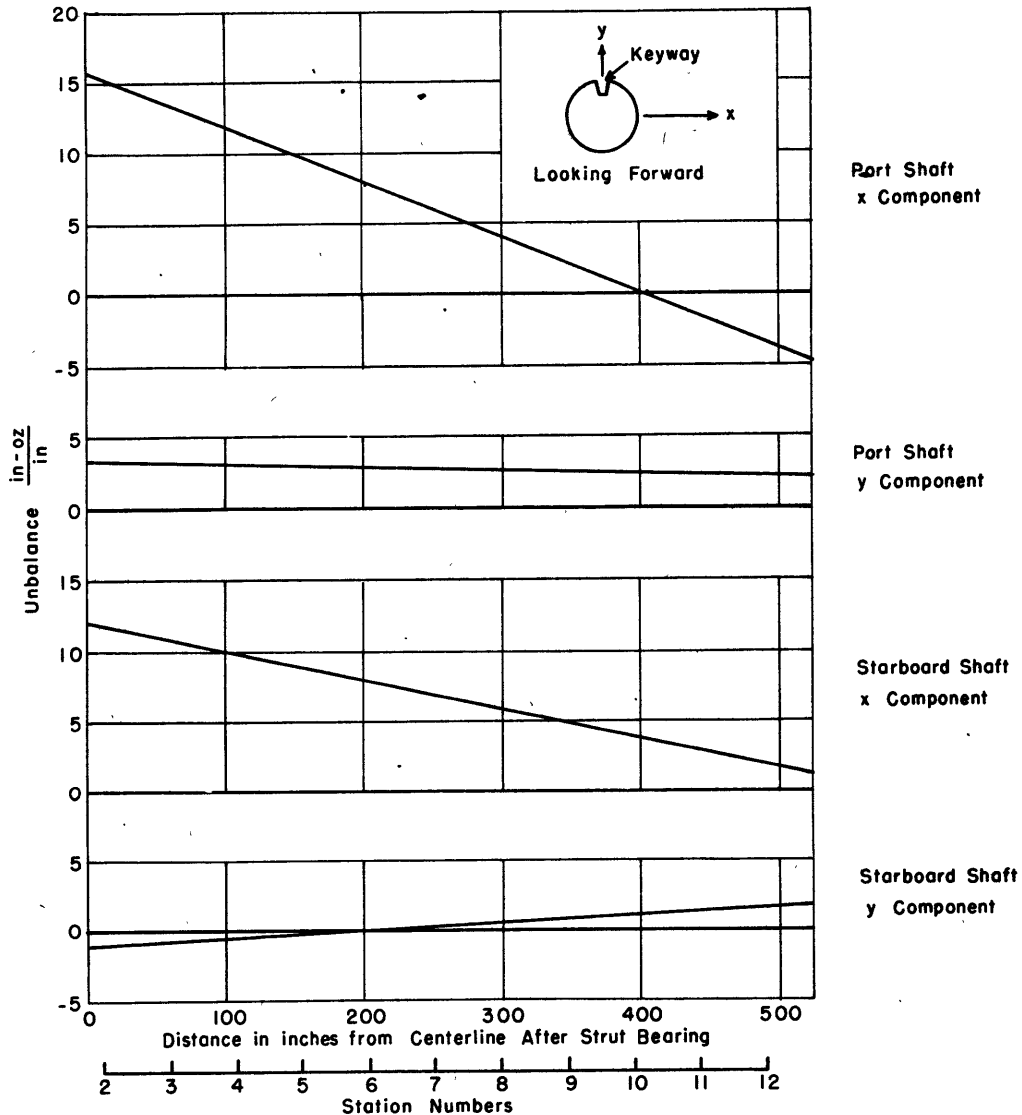


Figure 40 - Assumed Linear Distribution of Unbalance in Tailshafts of DD865 as of 21 June 1948 Based on Data of Figures 38 and 39

Mass eccentricity per  $\frac{\text{inch - oz}}{\text{inch length}}$  unbalance is 0.00192 inch.

\*Underway measurements have usually been made at Frame 72, 02 deck.

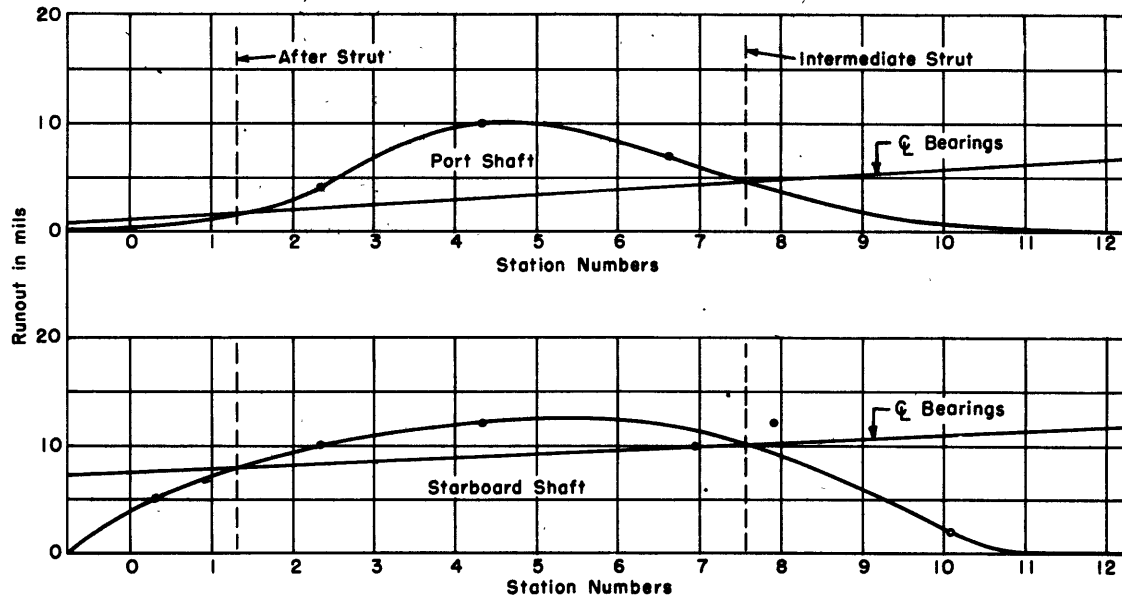


Figure 41 - DD865 - Runout of Shaft as Received (as of 21 June 1948)

The runout was predominantly in the x direction.

The location at which vibration measurements were made, both prior to and subsequent to the shipyard overhaul, was at Frame 72, 02 deck. Here the single amplitudes of athwartships vibration were reduced from 24 mils to about 10 mils at the torsional hull critical (310 rpm) and from 20 mils to about 10 mils at the three-noded athwartship flexural critical (230 rpm). This is a reduction of 14 mils and 10 mils for the torsional and the flexural critical respectively. Table 8 gives a breakdown of the amplitudes of vibration generated by the several exciting forces. Item (a) in this table gives the major part of the reduction in vibration due to the balancing and straightening work that was done at the shipyard during the overhaul period.

The amplitudes listed opposite Item (b) are due to the effect of the athwartships component of propeller thrust; this component is an alternating force of frequency of the propeller rpm.

The amplitudes resulting from the alternating lift forces were approximated and found to be about 15 percent of the amplitudes due to unbalance forces in the propeller-tailshaft system for the three-noded flexural mode of athwartship vibration, and 30 percent for the one-noded torsional mode of vibration. The contribution of the forces described in Item (c) of the table can be determined more accurately by a series of approximations in which the amplitudes of the propeller-shaft system are calculated, all forces except the lift forces being considered. The lift forces are then calculated as indicated in Section 7.2, and a second approximation to the

TABLE 8

Relative Contribution of the Various Exciting Forces, Based on DD865 Test Data

Item	Three-Noded Mode of Athwartships Flexural Vibration			One-Noded Mode of Torsional Vibration		
	Frame 200* mils	Frame 72** mils	Percent of Total	Frame 200* mils	Frame 72** mils	Percent of Total
(a) Propeller Tailshaft System. Effective Mass Unbalance	3.71	3.0	14.6	15.6	10.8	56.1
(b) Component of Propeller Thrust, Normal to Propeller Shaft	0.53	0.43	2.1	1.93	1.34	7.0
(c) Lift Forces Acting on Tailshaft	0.56	0.45	2.2	4.7	3.25	16.9
(d)*** Lineshaft Mass Unbalance 0.060" eccentricity (port) 0.030" eccentricity (stbd)	20.6	16.6	81.1	5.6	3.85	20.0
Total	25.4	20.5	100	27.8	19.2	100
Amplitude Measured on DD692 During Pre-Repair Trials		20			24	
Amplitude Measured on DD692 During Post-Repair Trials		10			10	
*At a location 2 feet above main deck. **At the O2 deck level. ***Estimated value.						

tailshaft amplitudes is made by adding the contribution of the lift forces. A fair estimate can be made, however, on the first attempt. It may be assumed that during the post-repair trials the reduction in vibratory forces and amplitudes was as follows: All of Item (a) Table 8 and one-half of Items (b) and (c), Table 8. This corresponds to a reduction of athwartship amplitude at Frame 72 equal to  $3.44 = (3.0 + \frac{1}{2} \times 0.43 + \frac{1}{2} \times 0.45)$  mils for the three-noded flexural mode and  $13.1 = (10.8 + \frac{1}{2} \times 1.34 + \frac{1}{2} \times 3.25)$  mils for the one-noded torsional mode. The difference between this calculated reduction and the amplitudes measured during the pre-repair trials should equal the amplitudes measured during the post-repair trials and would be ascribed to unbalance in the line shafting and to other undetermined causes. Amplitudes of  $16.6 = (20 - 3.4)$  and  $10.9 = (24 - 13.1)$  would have been

expected during the post-repair trials instead of the actually measured single amplitudes of about 10 mils for the two hull criticals. The lack of better agreement for the flexural critical may be ascribed to experimental errors. (It is probable that the measurements during the two sets of trials were not made exactly at resonance, since it is difficult to maintain resonance for an appreciable length of time.) Considering the various sources of possible error, the check between calculation and measurement is considered quite good.

Inspection of Table 7a will, at a glance, show the relative importance of the several types of excitation. It is seen, for example, that effective mass unbalance in the propeller-tailshaft system is very effective in exciting torsional hull vibration but not nearly as effective in exciting the flexural hull critical. Line-shaft unbalance, on the other hand, is much more effective in exciting flexural vibration than torsional hull vibration. The mass unbalance forces, Items (a) and (d) of Table 8, are the most important excitations giving rise to first-order vibration.

It is evident from the above and from a study of Tables 7 and 8, that, if it is desired to decrease the first-order vibration at both hull criticals without increasing the stiffness of the hull, the effective mass unbalance of both the line shaft and the propeller-tailshaft system must be reduced to a value considerably below the values permitted by the present specifications.

The vibration amplitudes measured during the post-repair trials of the DD865 should be considered as the lowest that can be practicably attained on the DD865 by means of straightening and balancing of tailshafts and propellers. These values can be reduced further by balancing of the line shafting. Balancing of shafting and propellers will be discussed in a later section.

To recapitulate, the main conclusions arrived at in this section are:

(a) The cure for the excessive first-order vibration must be sought primarily by increasing the stiffness of the hull or in decreasing the first-order exciting forces caused by effective mass unbalance of the propeller-shafting system.

(b) It is important to maintain a load on the intermediate strut bearing, otherwise the natural whirling frequency of the tailshaft will fall within the running speed of the vessel.

(c) The formula now given in Bureau of Ships Design Data Sheet DDS43-1 is not sufficiently accurate for shaft installations of this type, and should be revised, possibly along the lines indicated in Section 3.

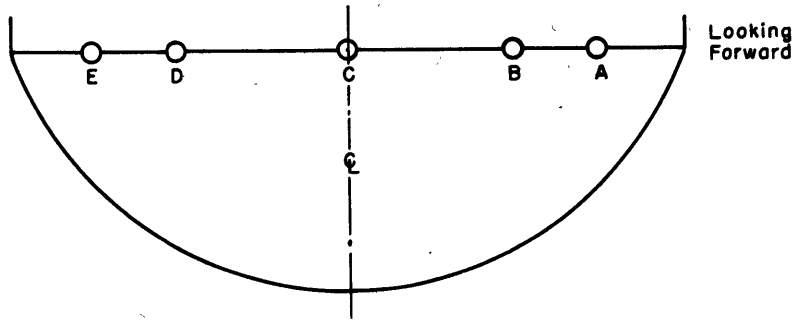
### 6.11 Underway Vibration Surveys of DD865 Before and After Overhaul at New York Naval Shipyard

The pre-repair and post-repair underway vibration surveys were made in order to determine the effect on the hull vibration of straightening and balancing of propellers and propeller shafting accomplished during the shipyard overhaul. Secondary aims were the determination of the modes of vibration in which the ship vibrates at the two hull criticals occurring near 240 and 310 shaft rpm and the measurement of the major vibrations predominating throughout the vessel's speed range. It is to be noted that the vibrations of the ship, when at sea, wax and wane and that the plotted values must be considered as approximations only.

The displacement of the vessel was 3415 tons during the pre-repair trials and 3100 tons during the post-repair trials. The difference in displacement should not affect the vibration amplitudes to an appreciable extent.

6.11.1 Pre-Repair Trials: During the pre-repair trials of 21 June 1948, measurements were made of athwartships vibration amplitudes at the base of the forward gun director and in the steering-engine room near Frame 205. Vertical measurements were made at the sides of the main deck at Frame 72 and at the midship section. In addition, measurements were made of the runout of the line shafting at various bearings. The depth of water throughout these trials was from 95 to 210 feet. The runout at all line-shaft bearings was about 1 mil except at the starboard shaft bearing near Frame 134 which gave a runout of 3 mils. Hull criticals were noted near 230 and 310 rpm. At these propeller speeds large first-order athwartships vibrations were encountered, especially at the ends of the ship and in the bridge structure. In order to establish the character of these vibrations the following procedure was followed.

At the 230 rpm speed, athwartships vibration measurements were made at many stations along the length of the vessel. These measurements showed that the hull was vibrating in the three-noded mode of transverse flexural vibration. At the 310-rpm critical both vertical- and athwartships-vibration measurements were made along the length of the vessel. The vertical-vibration amplitudes were a maximum at the ends of the vessel decreasing in magnitude towards the midships section, thus indicating a one-noded torsional mode of vibration. This is substantiated by the measurements plotted in Figure 42. It is to be noted that although vibration measurements on this class of ships have been made for several years, it had never been suggested that the hull resonance near 310 cpm was a torsional hull vibration; it had been generally assumed that this critical corresponded to a multi-noded



Station	E	D	C	B	A
± mils	13	6	0	10	14.

Figure 42 - Vertical Vibration Amplitudes Measured in Athwartships Passageway of Wardroom, Frame 72 During Pre-Repair Trials of the DD865

Frequency 310 vpm (1st order). From pattern of amplitudes it can be seen that the 310 vpm mode which gives large athwartship amplitudes in bridge, is associated with possible torsional motion of the hull.

This figure is reproduced from Enclosure (C) of New York Material Laboratory Report No. 4985-8.

athwartships flexural vibration. This conclusion was reached primarily because measurements were made at the bridge and at the fantail only; no overall vibration survey of these vessels had ever been made, as far as is known. It becomes evident that careful, comprehensive vibration surveys or vibration-generator surveys should be made whenever excessive vibration is encountered.

Throughout the trial runs athwartships vibrations at a frequency of about 120 cpm, independent of ship speed were encountered, see Figure 43. These vibrations were a maximum near the ends and the center of the ship and correspond to vibrations in the fundamental athwartships flexural mode. Similarly, vertical vibrations at 80 cpm--corresponding to the two-noded vertical flexural mode--were encountered independent of speed. These vibrations are probably excited by the wave motions, and although the amplitudes are appreciable no physical discomfort is encountered--probably because the corresponding accelerations are very low. The vibrations of 120-cpm frequency have been mentioned in several reports on vibration surveys of this class of ships, without explanation. The vibration-generator tests definitely establish that the vibrations are those corresponding to the fundamental mode of flexural vibration.

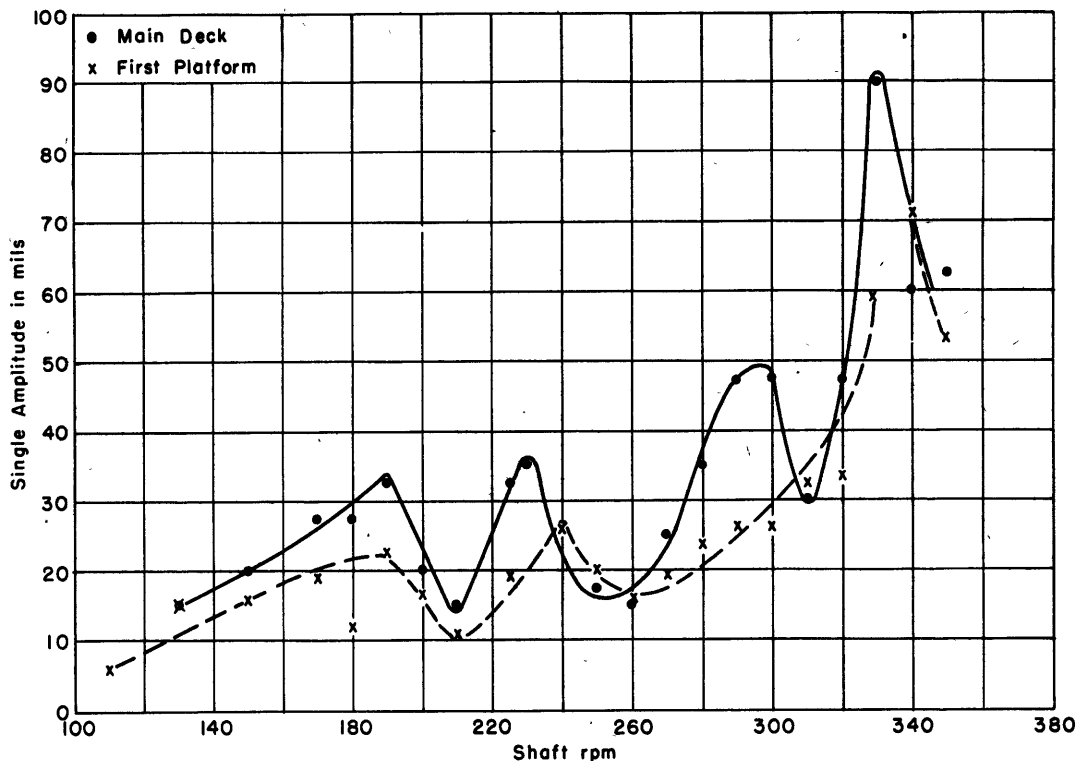


Figure 43 - Athwartships Vibration Amplitudes of 120-cpm Frequency at Frame 188 Main Deck and 1st Platform Measured During Post-Repair Trials of DD865

The athwartships vibration at 120 cpm was in the two-noded mode. The amplitudes at the main deck and at the first platform were in phase.

6.11.2 Post-Repair Trials: The measurements made during the post-repair trials were of the same type as those made during the pre-repair survey. Figure 44 is a graph of the athwartships amplitudes measured at the forward gun director at the various steady shaft-rpm. Inspection of this figure shows that the balancing work accomplished during the overhaul has reduced the peak vibrations near 230 and 310 rpm by a factor of about two. It is also evident, as shown by the discussion of the data presented in Section 6.4, that the vibratory exciting force was not a concentrated one during either trial, inasmuch as the peak amplitudes at both the 240-cpm and the 310-cpm peak are of the same magnitude. At the higher shaft-rpm random vibrations at a frequency of 240 cpm were measured. It should be noted that the sea became fairly rough during the later part of the trials. Figure 45 shows a plot of the athwartships first-order vibration measured near the fantail; considerable improvement over the pre-repair vibration level is noted. It is evident that the

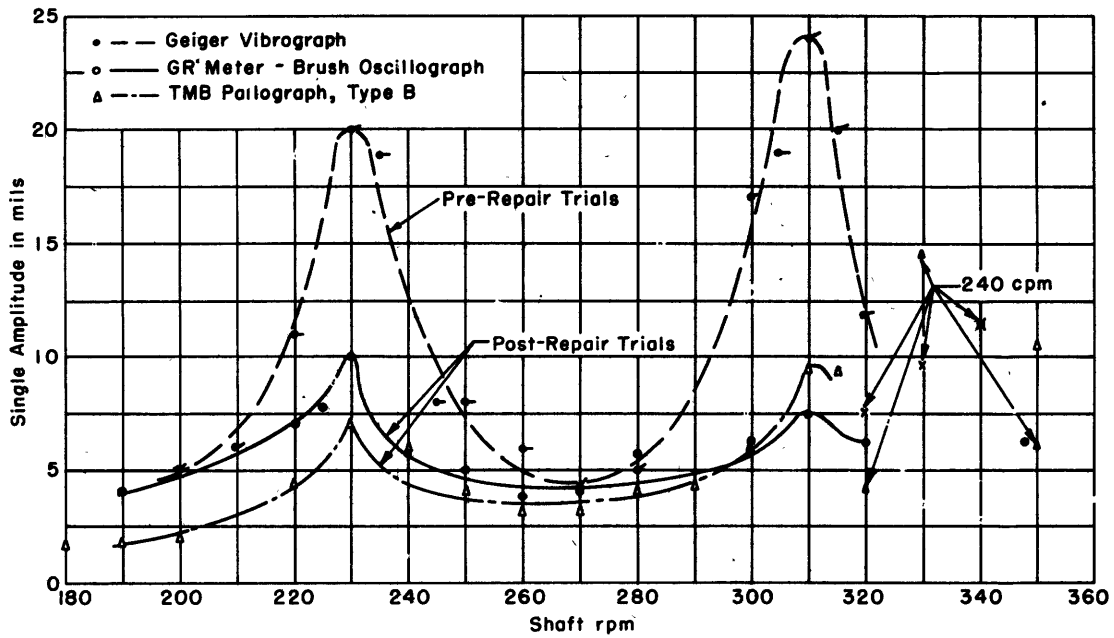


Figure 44 - Athwartships Vibration of DD865 Measured at the Base of the Main Gun Director Frame 72, 02 Level

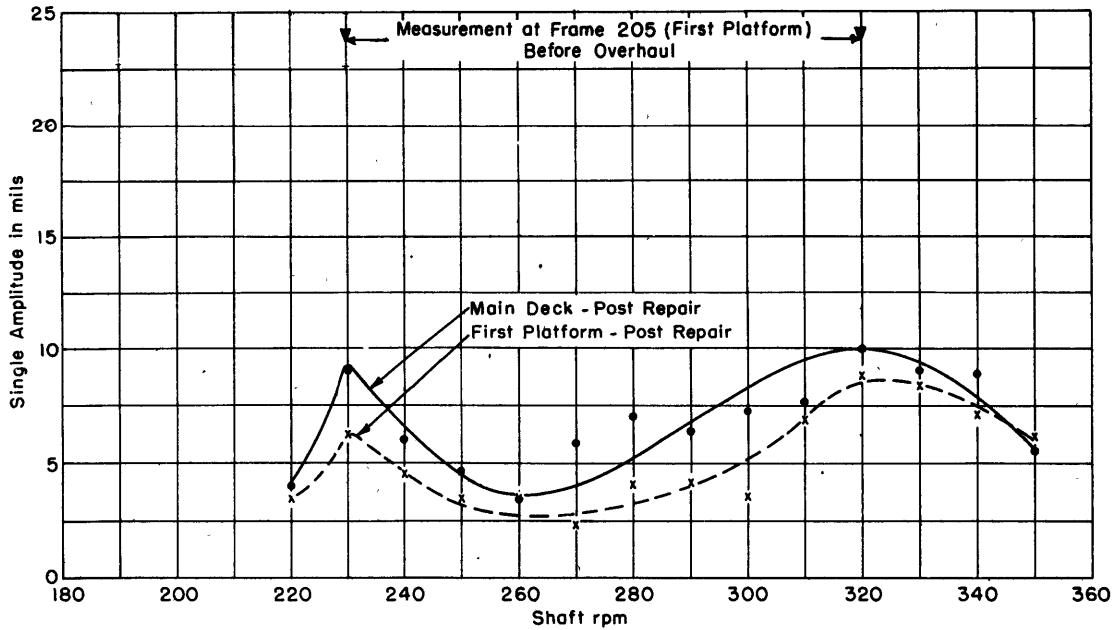


Figure 45 - Amplitudes of Athwartships Vibration Measured at Frame 188 Main Deck and First Platform

The athwartships vibration was of propeller shaft frequency.  
 The amplitudes at the Main Deck and at the First Platform were in phase.  
 Measurements were made with a General Radio Meter and Brush Oscillograph.



amplitudes at the main deck are larger than those at the first-platform level; this is in accordance with the discussion of Section 6.5 where it was noted that appreciable torsional vibration would occur near Frame 188 at the 240-rpm resonance. Figure 46 is a plot of the torsional vibration amplitudes measured along the vessel at 330 cpm, showing definitely that the vibration

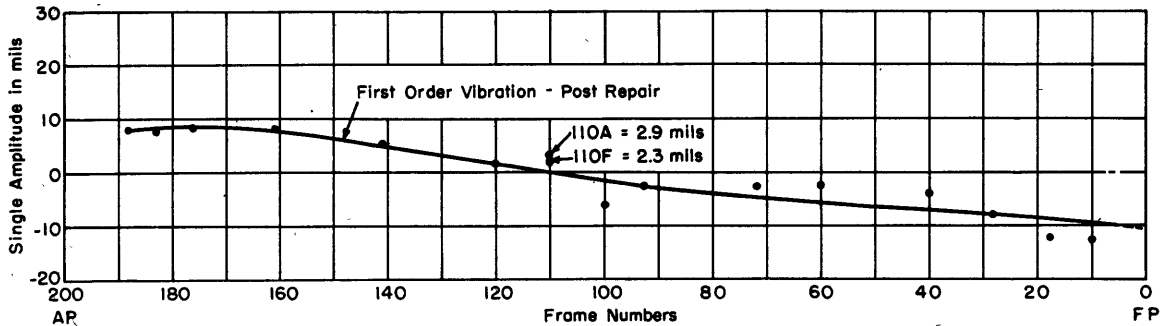


Figure 46 - Torsional Hull Vibration at 330 cpm (22 Sept. 1948)

The measurements were made with a General Radio Meter and Brush Oscillograph.

measured is a torsional mode of hull vibration. Figure 47 shows a plot of athwartships vibration measured at points along the ship at 235 propeller-rpm.

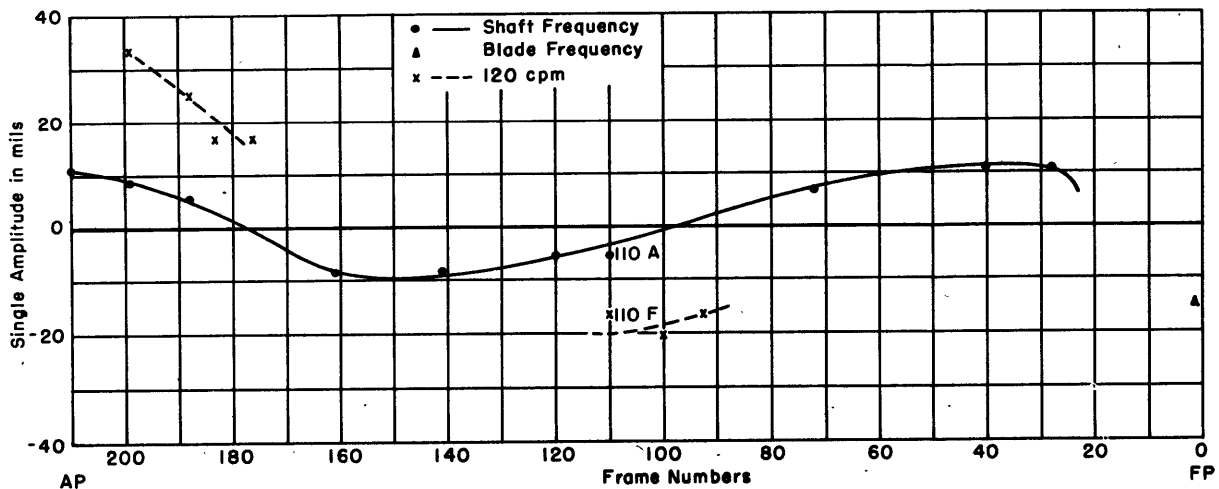


Figure 47 - Main Deck Survey - Athwartships Vibration Amplitudes Measured at 235 rpm During Post-Repair Trials of 22 Sept. 1948

The measurements were made with a General Radio Meter and a Brush Oscillograph.  
The phase relationships are as indicated.

The first-order vibration corresponds to the three-noded flexural mode of athwartships vibration; the random vibration at 120-cpm frequency, also plotted in Figure 47, corresponds to the fundamental flexural vibration.

Figure 48 is a plot of athwartship vibration measured at several deck levels at a speed of 240 and 330 propeller-rpm. Inspection of this figure indicates that the vibration at 240 rpm is a predominantly athwartship flexural vibration, whereas the vibration near the higher critical is predominantly a torsional hull vibration. The commanding officer of the DD865 expressed the opinion that the vibration had improved appreciably as compared to the pre-repair trials. This opinion is probably based on sensory perception of vibration.

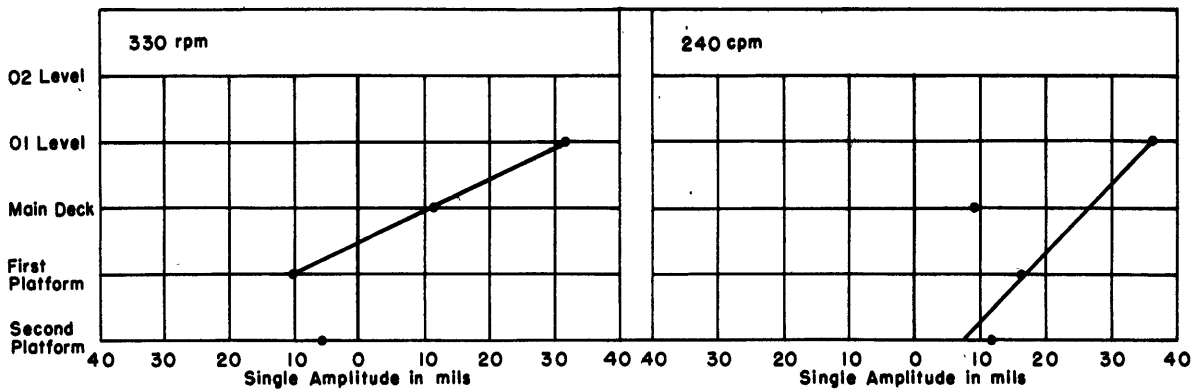


Figure 48a - Torsional Survey at Frame 58

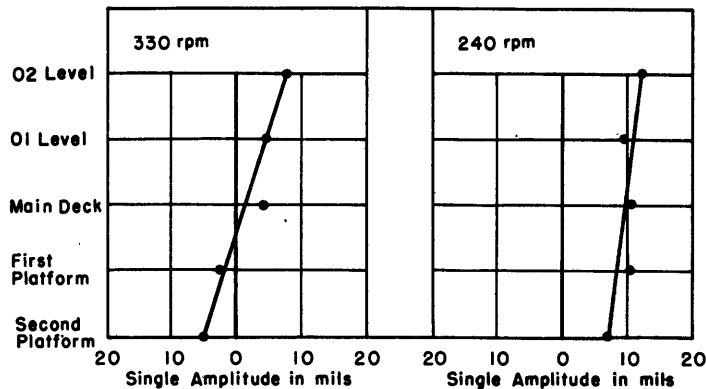


Figure 48b - First Order Athwartships Vibrations at Frame 72

Figure 48 - Vibration Amplitudes Measured at Frames 58 and 72 During Post-Repair Trials

6.11.3 Conclusion: The level of vibration existing during the post-repair trials is approximately the optimum attainable for the DD865, by means of balancing and straightening of tailshafts and propellers. Further improvement would have to come through structural alteration of the vessel, redistribution of dead load or by reducing the first-order unbalance forces by means of a complete balance of all propulsion shafting. The maximum athwartship first-order-vibration single amplitude, measured at either the fantail (Frame 188) or at the base of the forward gun director, was about 10 mils for the condition of the propulsion system obtaining during the post-repair trials. The bothersome hull criticals near 240 and 310 shaft rpm have been identified as the three-noded mode of flexural athwartship vibration and the one-noded mode of torsional hull vibration respectively. Underway vibration surveys should be made in calm seas, if comparative data are to be obtained.

6.12 Vibration Surveys Made on DD692-Class Destroyers Subsequent to the DD865 Tests of 1948.

The USS CHARLES R. WARE (DD865) was selected as a representative vessel of the "Long Hull" destroyers. The torsional vibration observed on this ship had not been reported on any previous trials of these ships. In order to ascertain whether or not the conditions found on the DD865 were typical of the class, it was decided to make vibration surveys on a number of other vessels of the class. These measurements were requested by a directive from the Chief of the Bureau of Ships to the several Naval Shipyards, see Appendix. The directive included a list of suggested vibration measurements. Reports of nine trials, made in accordance with the directive have been received to date, see References 32 through 40. The results of these trials show that the pattern of vibration existing on the DD865 is typical for this class of destroyers. Some of the more salient results of these trials are shown in Figures 49 through 58. It is to be noted that only the data obtained by the Boston Naval Shipyard, Figures 49 through 52, are based on simultaneous values of phase and amplitudes. The Boston data show very clearly that the first order vibration near 310 rpm corresponds to the one-noded mode of torsional hull vibration. Table 1, which is a tabulation of comparative vibration data obtained from a number of vibration surveys of both long- and short-hull destroyers includes the latter tests. Comparison of results given for long- and short-hull vessels does not disclose any salient difference in behavior. The flexural resonances of the short hulls occur at slightly higher shaft-rpm but the pattern of vibration is the same for both types of hulls.

(Text continued on page 85.)

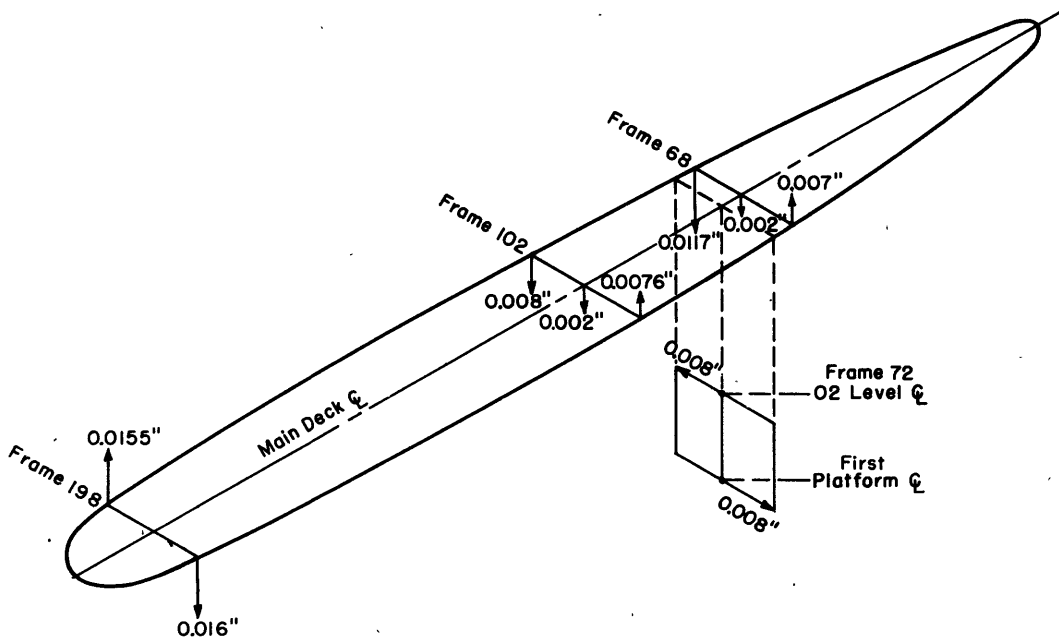


Figure 49 - USS NOA (DD841) - Amplitudes of First Order Vibration Measured at Upper Hull Critical (310 rpm) During Underway Trials

All amplitudes are single amplitudes.  
Data are taken from Boston Naval Shipyard Report 63920.

The sea was moderate to calm.  
The phase relationships are as indicated.  
The vessel displaced 3200 tons.

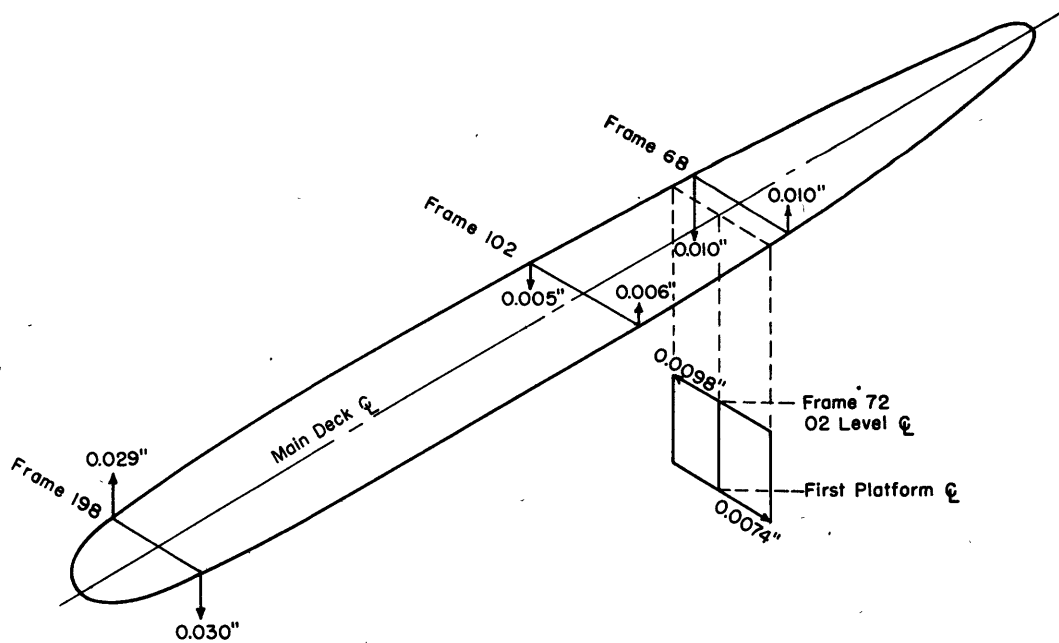


Figure 50 - USS RICH (DD820) - Amplitudes of First Order Vibration Measured at Upper Hull Critical (315 rpm) During Underway Trials

All amplitudes are single amplitudes.  
Data are taken from Boston Naval Shipyard Report 63936.

The phase relationships are as indicated.  
The sea was calm.  
The vessel displaced 3100 tons.

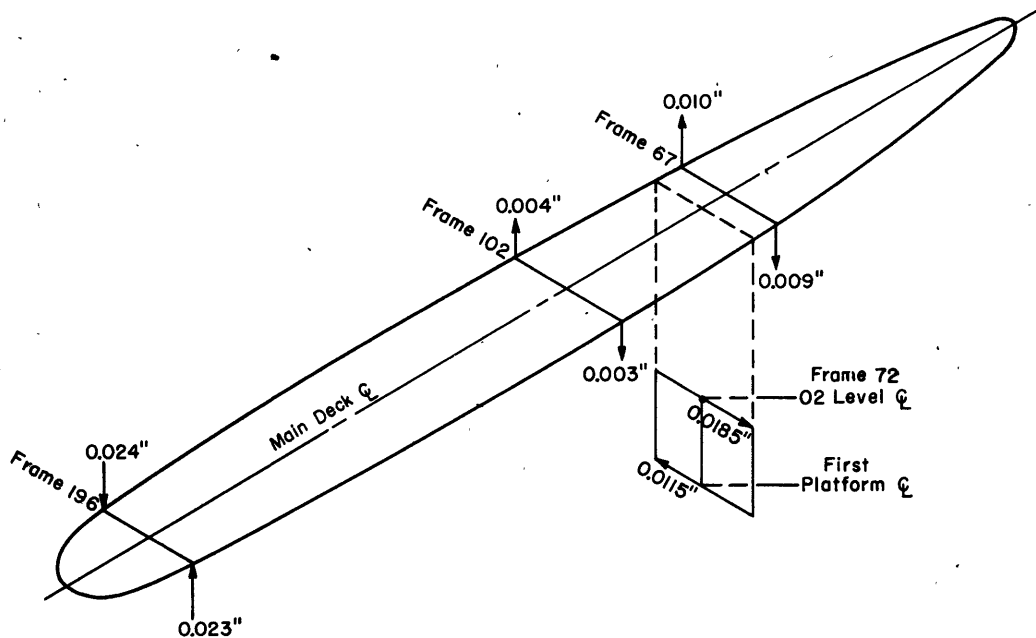


Figure 51 - USS POWER (DD839) - Amplitudes of First Order Vibration Measured at Upper Hull Critical (320 rpm) During Underway Trials

The amplitudes given are single amplitudes.  
The data are taken from Boston Naval Shipyard Report 64141.

The phase relationships are as indicated.  
The sea was calm with slight swells.  
The vessel displaced 3190 tons.

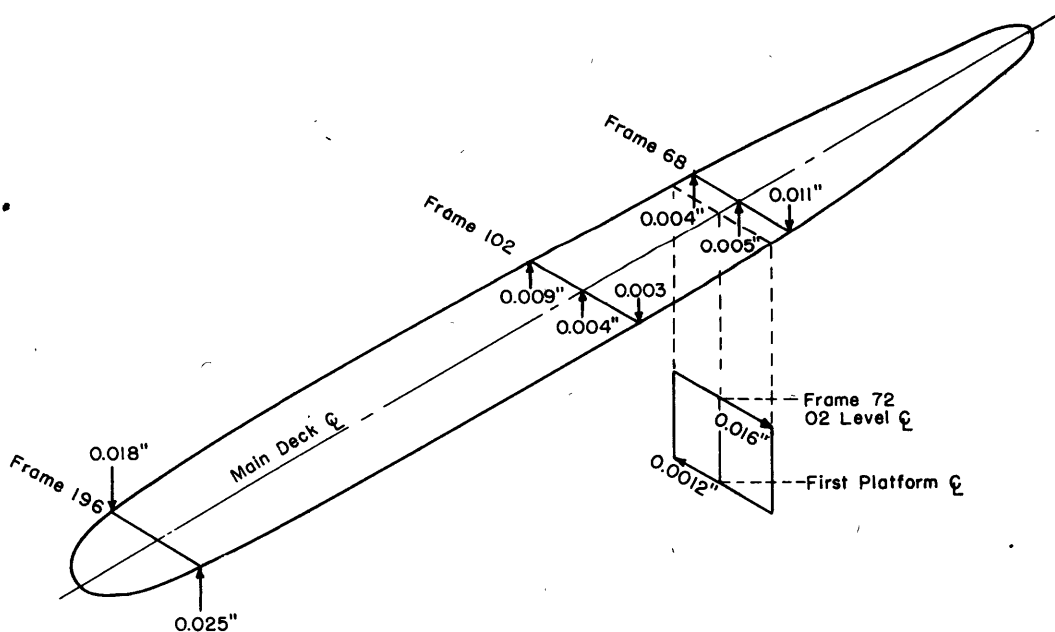


Figure 52 - USS SMALL (DD838) - Amplitudes of First Order Vibration Measured at Upper Hull Critical (315 rpm) During Underway Trials

The amplitudes given are single amplitudes.  
The data are taken from Boston Naval Shipyard Report 64194.

The phase relationships are as indicated.  
The sea was moderate.  
The vessel displaced 3000 tons.

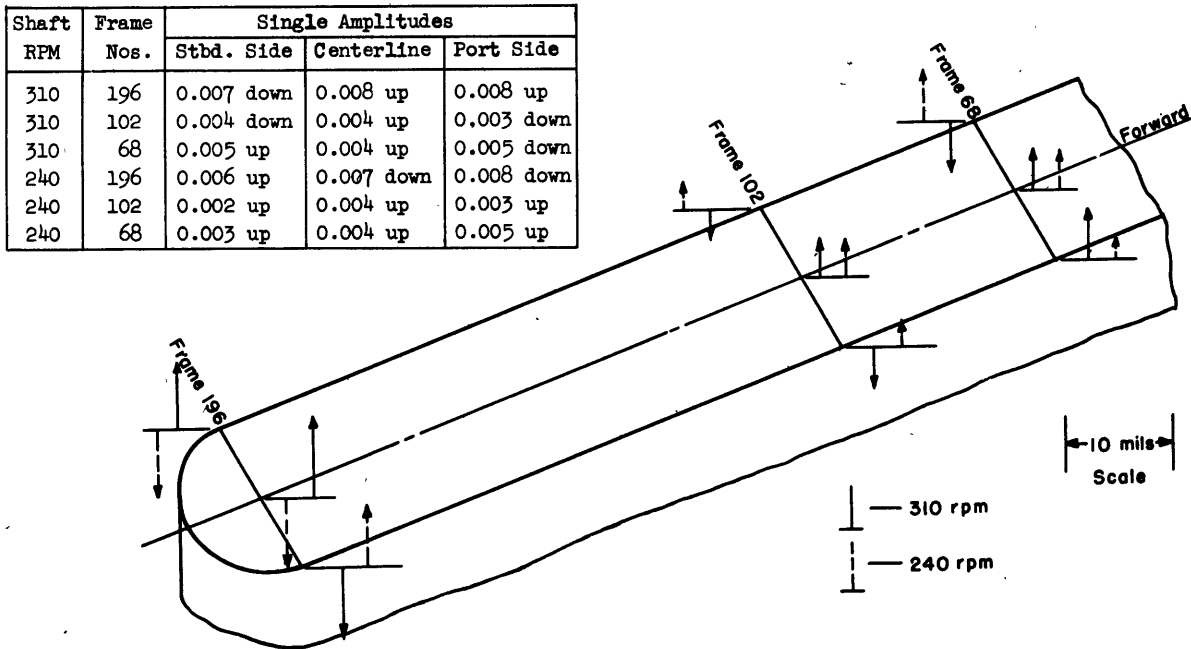


Figure 53 - USS TURNER (DD834) - Amplitudes of First Order Vibration Measured During Underway Trials

Phase relationships between measurements at several frames were not obtained.  
The sea was rough, waves were 5 to 8 feet high.

Data are taken from Mare Island Industrial Laboratory Report 9956-48.  
The vessel displaced 3000 tons.

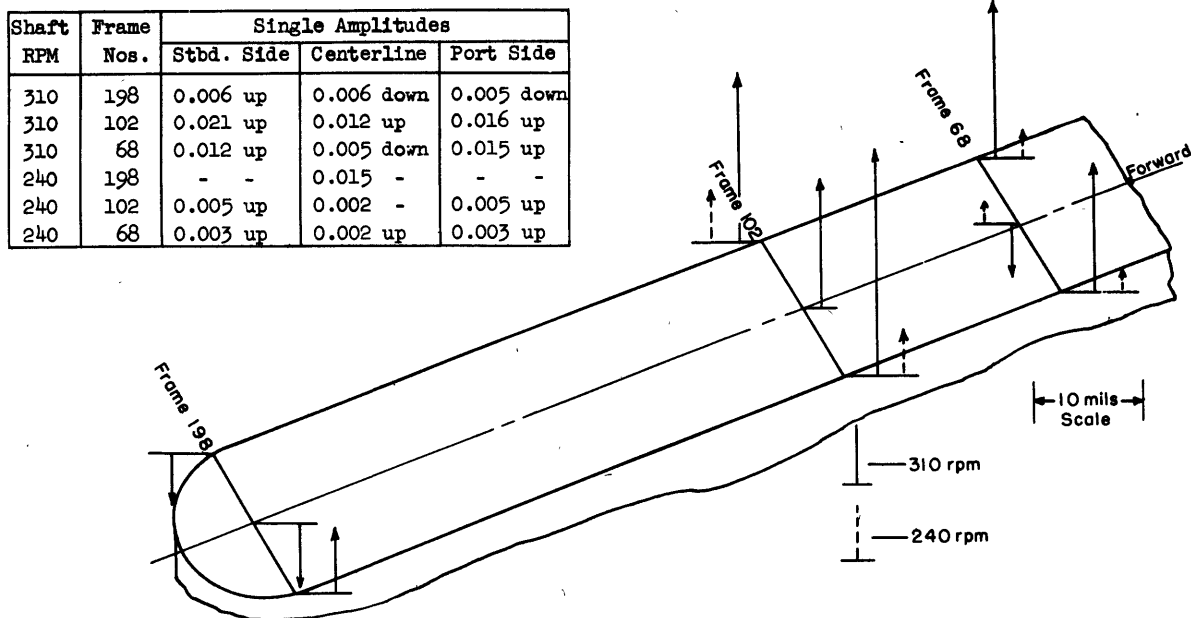


Figure 54 - USS PERRY (DD883) - Amplitudes of First-Order Vibration Measured During Underway Trials

Phase relationships between measurements at several frames were not obtained.  
The sea was moderate.

Data are taken from Mare Island Industrial Laboratory Report 9954-48.  
The vessel displaced 3230 tons

Shaft RPM	Frame Nos.	Single Amplitudes		
		Stbd. Side	Centerline	Port Side
310	198	0.007 up	0.004 down	0.006 up
310	102	0.004 up	0.006 up	0.005 up
310	68	0.004 up	0.002 up	0.004 down
240	198	0.003 up	0.003 down	0.003 up
240	102	0.002 up	0.001 up	0.001 up
240	68	0.003 up	0.001 up	0.003 up

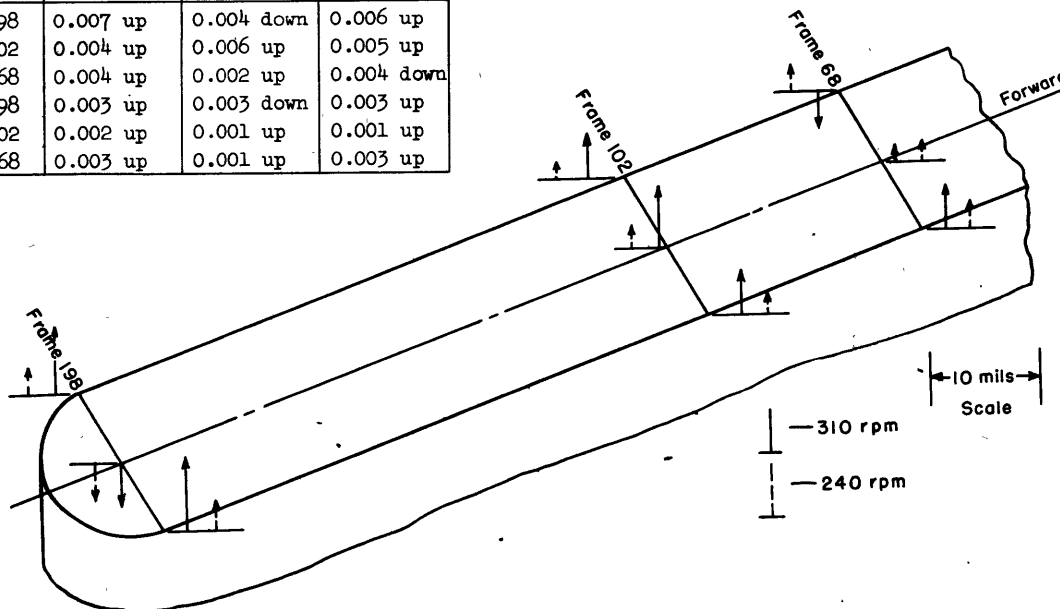


Figure 55 - USS FURSE (DD882) - Amplitudes of First Order Vibration Measured During Underway Trials

Phase relationships between measurements at several frames were not obtained.  
The sea was moderate with swells.

Data are taken from Mare Island Industrial Laboratory Report 9955-48.  
The vessel displaced 3150 tons.

Shaft RPM	Frame Nos.	Single Amplitudes		
		Stbd. Side	Centerline	Port Side
310	198	0.010 down	0.003 up	0.008 up
310	102	0.005 down	0.005 up	0.006 up
310	68	0.005 down	0.003 down	0.005 down
240	198	0.006 down	0.004 up	0.004 down
240	102	0.003 down	0.002 up	0.002 down
240	68	0.002 down	0.004 down	0.003 down

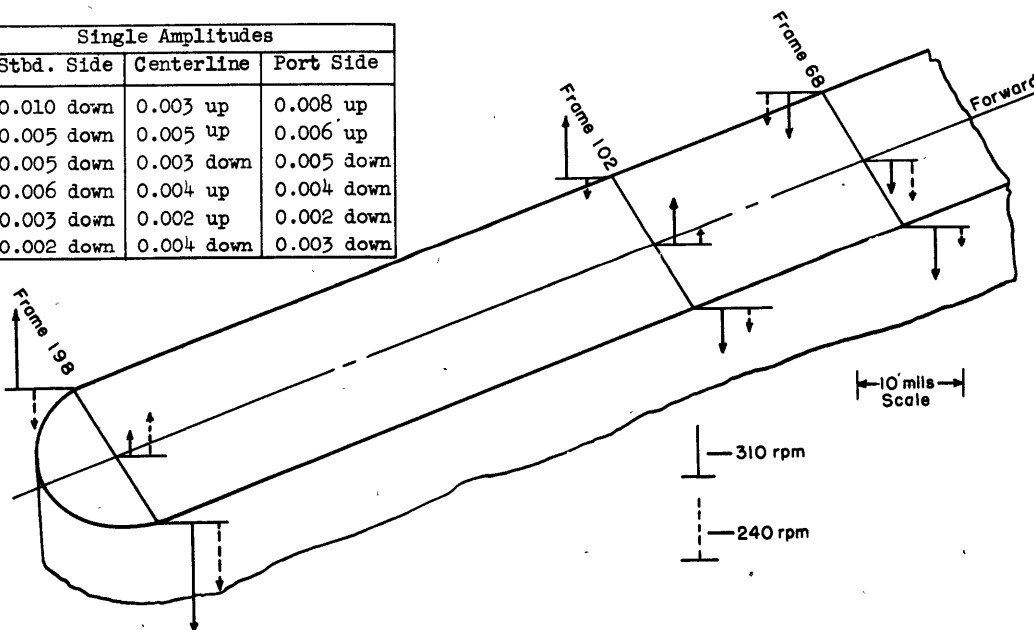


Figure 56 - USS CECIL (DD835) - Amplitudes of First Order Vibration Measured During Underway Trials

Phase relationships between measurements at several frames were not obtained.  
The sea was moderately rough.

Data are taken from Mare Island Industrial Laboratory Report 9957-48.  
The vessel displaced 3000 tons.

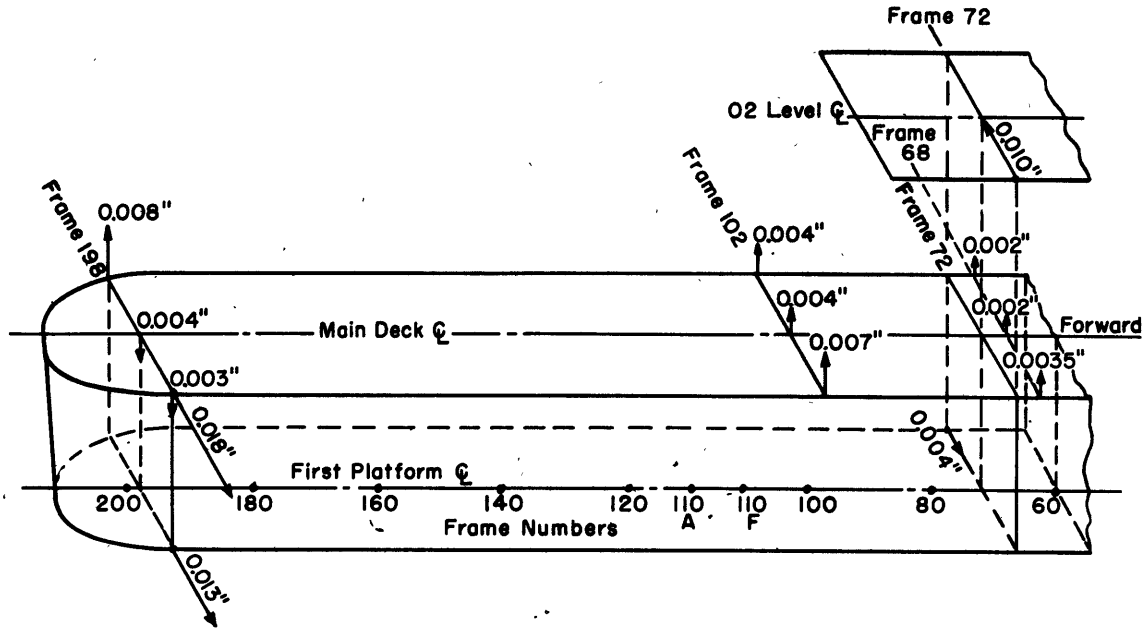


Figure 57 - USS WOOD (DD715) - Amplitudes of First Order Vibration Measured During Underway Trials at 240 RPM

Amplitudes given are single amplitudes.  
 Phase relationships between measurements at several frames were not obtained.  
 The sea was rough, waves were 5 to 8 feet high.

Data are taken from Mare Island Industrial Laboratory Report 1290-49.  
 The vessel displaced 3120 tons.

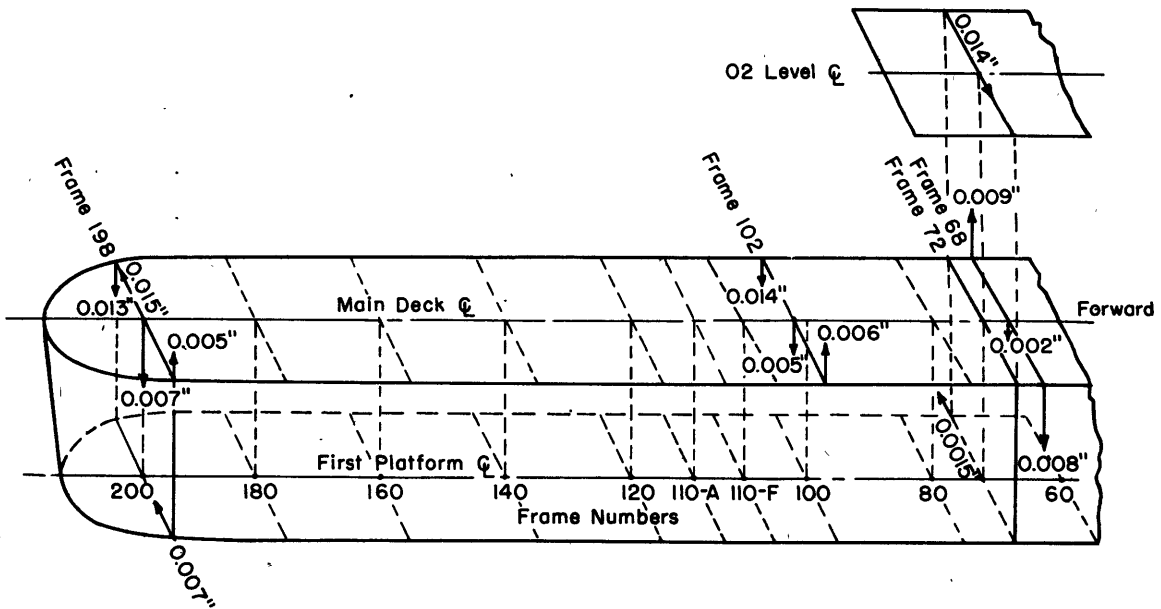


Figure 58 - USS WOOD (DD715) - Amplitudes of First Order Vibration Measured During Underway Trials at 310 RPM

Amplitudes given are single amplitudes.  
 Phase relationships between measurements at several frames were not obtained.  
 The sea was rough, waves were 5 to 8 feet high.

Data are taken from Mare Island Industrial Laboratory Report 1290-49  
 The vessel displaced 3120 tons.



It can therefore be concluded that the vibration characteristics which were determined from tests conducted on the DD865 apply in general to the entire class

## 7. ANALYSIS OF DATA

Some of the data presented and discussed in the preceding section will be analyzed in detail. For convenience this analysis is broken up into logical subdivisions.

### 7.1 Natural Modes of Vibration

The normal or natural modes of vibration, as determined with the TMB medium vibration generator, have been presented in Section 6.1. The frequencies associated with the vertical modes of vibration are in the ratio of 1, 2.1, 3.3 for the first three modes. Only the two- and three-noded modes of athwartships flexural vibration were defined by the tests, the higher modes were masked, if present, by strong torsional hull vibrations. The data substantiate a general pattern, noted on several ships, namely, that the frequencies associated with the first two or three modes of flexural vibrations vary approximately as the numbers 1, 2, and 3.

It became evident from an inspection of the test results, that the torsional and athwartships modes of vibration were coupled to some extent. Such coupling will obviously occur in view of the nonuniform variation of mass distribution and cross section along the ship. In the case of the DD692-Class destroyers, coupling was present at the modes defined in Figures 12 to 14--indicating a lack of symmetry in weight distribution. Such asymmetry probably is due primarily to the concentrated masses of guns, directors and ammunition. The one-noded torsional mode was relatively pure, however, as indicated by the discussion in Section 6.4.

The shape of the one-noded mode of torsional vibration, Figure 15, is worthy of some attention. For a free-free bar this mode configuration would be a cosine curve symmetrical about the midships section. In the case of the destroyer the amplitudes of torsional vibration are much larger towards the stern than towards the bow. It was shown in Section 6.6 that this situation possibly results from the effect that the cutting back of the hull towards the stern has on the torsional flexibility of the ship. The flexibility varies inversely as the square of the cross-sectional area of the ship. Since this area is relatively small at the stern the flexibility is large and, vice versa, the rigidity is low relative to the midship section.

Some interesting data regarding the "virtual mass" of water which is considered to move with the ship were obtained from the vibration-generator tests. Comparison of the measured values of natural frequencies associated with the one-noded torsional and the three-noded athwartships flexural vibration, obtained during the deep-water tests in Long Island Sound as compared to the shallow water tests in Wallabout Bay (Figure 3), show that the proximity of the sea bottom has no appreciable effect on the resonance frequencies. This indicates that for athwartship vibration, the vibrational energy stored in the water is mainly confined to the water extending horizontally beyond the sides of the ship. For torsional vibration the energy stored in the water is probably rather small as can be seen by the following illustration: A cylinder vibrating torsionally in a frictionless fluid would not impart any energy to the fluid, consequently the entrained mass of water is zero in this case. Actually ship sections lie somewhere between a cylinder and a rectangular prism. Thus some energy will be imparted to the water.

The damping associated with the fundamental mode of flexural vibration was about 0.35 percent of the critical value. This value is of the same order as found in similar tests. The vibration resonances are rather sharp due to the low magnitude of the damping forces. It is difficult to measure accurately the vibration amplitudes at resonance because the speed and power controls for the TMB medium vibration generator are not sufficiently stable to maintain a given power dissipation of the ship for a time interval long enough to permit the necessary measurements. Near resonance, the variation of power dissipation and vibratory motion with variation in frequency of vibration is very high for systems with low damping. The amplitude profiles presented in Section 6.1 may be subject to some error in the absolute value of the amplitudes although an attempt has been made to keep this error to a minimum by the use of a so called "probable off-resonance factor" as discussed in Section 6.1.

Table 9 gives the values of the natural frequencies of hull vibration as calculated by various methods; see Reference 41. An inspection of this table shows that the relatively simple formula proposed by Burrill gives the best agreement with the experimental values. However, such formulas do not enable one to determine the mode of vibration. The modes of vibration calculated by the Prohl-Myklestad method,<sup>41</sup> with consideration given to the effect of energy associated with deflections due to bending, shear, and rotary inertia, did show rather good agreement with the experimentally determined modes of vibration.

TABLE 9

Comparison of Calculated and Experimental Values of Natural Frequencies of Hull Vibration of the DD865

Mode of Vibration	Natural Frequency of Vibration - cycles per minute						
	Experi- mental Frequency	Rayleigh- Ritz Method Bending Only	Prohl-Myklestad Method		Schlick Formula	Burrill's Formula	Horn's Formula
			Bending Only	Bending and Shear			
Two-noded Vertical	79	70	65	64	94	81	
Three-noded Vertical	165	136	133	133			
Four-noded Vertical	264	317	208	191			
Five-noded Vertical	360		312	265			
Two-noded Athwartships	132	115	115	107			
Three-noded Athwartships	246	211	231	207			
One-noded Torsional	310						386

These data are for a displacement of about 3200 tons.

Very little is known today about the torsional vibration of ships or about vibration due to coupled bending and torsion. This problem is intimately connected with the question of torsional rigidity of thin-walled box girders. The latter problem will be discussed in Section 7.3.

Although very little has been said in the literature on torsional vibration of ships, it is quite possible that the problem has been present but has not been recognized as such, since torsional vibration is felt physically either as a vertical or as an athwartships vibratory motion. It is suggested that some effort be devoted to the problem of torsional vibration of ships.

## 7.2 Significant Forces Acting on the Ship

The significant forces acting on the hull appendages are as follows:

- (a) Inertia forces (d'Alembert's forces) due to effective mass unbalance of the propeller shafting.

- (b) Inertia forces due to effective mass unbalance of the propeller.
- (c) Hydrodynamic drag and lifting forces acting on the tailshaft due to runout or whipping motion of the tailshaft.
- (d) Hydrodynamic drag and lifting forces acting on the tailshaft during high-speed turns. These forces are essentially unidirectional.
- (e) Propeller-thrust forces.
- (f) Forces associated with the pressure field surrounding the rotating propeller.

The inertia forces acting on the propeller shafting are due to both actual mass eccentricity of the shaft, such as caused by an eccentric bore or nonhomogeneity of the material, as well as to runout of the shafting. The effect of these forces is especially detrimental when they occur in the tailshaft section because their effectiveness in causing torsional vibration is greatest near the stern.

The inertia forces acting on the propeller are due to mass eccentricity and runout of the propeller. The propeller can be balanced to within a few inch-ounces; however the runout at the propeller taper may give rise to much larger effective unbalance forces than actual mass unbalance. It is important that this runout be kept within acceptable limits.

The hydrodynamic lifting forces acting on the tailshaft due to the runout and whipping motions of the shaft can be determined by use of the relationships shown in Figure 59; also see Reference 42. If the shaft is rotating concentrically while the vessel is proceeding on a straight course there will be no lifting forces acting on the shaft because the velocity of the water normal to the shaft axis,  $V_0$ , is equal to zero. If, however, the shaft is now assumed to rotate with an eccentricity  $\epsilon$ , see Figure 59b, then the velocity of approach is equal to

$$V_0 \approx 2\pi\epsilon N \quad [22]$$

where  $N$  is the rps of the shaft. Substituting this value of  $V_0$  in Formula (ii) of Figure 59 it is seen that,

$$L \approx 8\pi^3 a^2 N^2 \epsilon \rho \text{ lb/ft} \quad [23]$$

This is a periodic force of Frequency  $N$ .

For  $\epsilon$  equal to 0.010 in. and for a shaft diameter of 16 1/2 in., the lift force will be about 5.4 lb per lineal foot at 310 rpm. The direction of the lift force, for speeds less than the resonance whipping frequency, is in

$$\gamma = 2\pi a V_p \quad (i)$$

$$L = \rho \gamma V_0 \quad (ii)$$

where  $\gamma$  is the circulation,

$V_p$  is the peripheral velocity units/sec,

$V_0$  is the velocity of the undisturbed fluid units/sec,

$\rho$  is the mass density of the fluid, and

$L$  is the lift force per unit length.

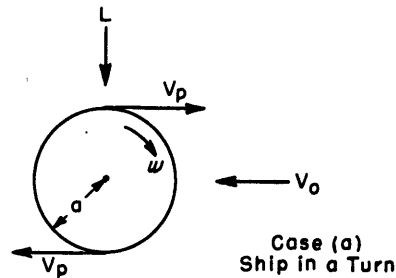


Figure 59a - Ship in a Turn

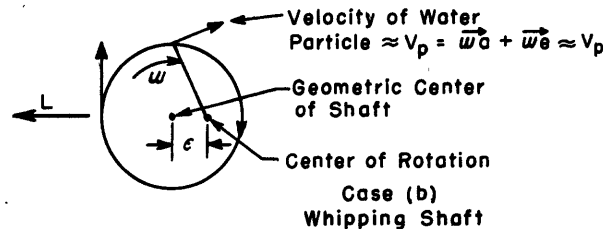


Figure 59b - Whipping Shaft

Figure 59 - Lift Force Acting on a Rotating Cylinder

phase with the inertia force which acts on the shaft due to the eccentricity  $\epsilon$ . For normal eccentric rotation of the tailshaft these lift forces are of the order of 10 to 30 percent of the forces due to effective mass unbalance of the shaft.

When the ship executes high-speed turns large unidirectional lift forces will act on the shafting. For example let the vessel make a hard turn at 32 knots. Assuming that the angle that the direction of water flow makes with the longitudinal shaft axis is 20 degrees, then it follows that the water velocity normal to the shaft,  $V_0$ , is about 18 ft per second. If the 16 1/2-in. diameter propeller shaft is turning at 350 rpm, Equations (i) and (ii) of Figure 59 will give a theoretical lift force of 4500 pounds per lineal foot. Experiments by Dodge and Thomson<sup>42</sup> have shown that the theoretical value is too high. By taking the ratio of the experimental to the theoretical lift coefficient from the reference quoted it is found that a

more equitable value of  $4500 \times 2/7.5 = 1200$  pounds per lineal foot is obtained for the lift force. This force would act downward on one shaft and upward on the other, thus tending to list the ship.

The hydrodynamic drag force acting on the shaft can be calculated from the well known relation

$$D = C_D \frac{\rho V^2 A}{2}$$

where  $D$  is the drag force per unit length,

$C_D$  is the drag coefficient, determined experimentally,

$\rho$  is the mass density of the fluid,

$V$  is the normal velocity of the fluid relative to the shaft, and

$A$  is the projected area of the shaft, per unit length.

The alternating drag force incident to the whipping motions of the shaft is negligible. The unidirectional drag force acting on the shaft during the high-speed turn assumed for the calculation of the lift force will be of the order of 250 pounds per lineal foot.

The propeller thrust per shaft is about 191,000 pounds at full power rpm, allowing for about 6 percent overload due to fouling. For the DD828 conversion the thrust will be increased to 261,000 pounds per shaft. The most severe stresses in the shaft and in the strut arms, due to combined axial and transverse loads, can be expected during high-speed turns at full power.

The periodic forces associated with the rotating pressure field around the propeller do not cause undue vibration in this class of destroyers, when 4-bladed propellers are fitted, and will therefore not be discussed here.

It had been suggested that the exciting force which causes the objectionable vibration might be due to the casting off of a vortex from the propeller hub. Tests made at the Taylor Model Basin in order to check this possibility were reported in TMB ltr C-SS/S24-9 of 14 March 1949 to the Chief of the Bureau of Ships. The tests indicated that the suggested cause was not likely to be a factor in causing vibration.

### 7.3 Some Effects of Ship Design on the Vibration Characteristics of the Hull

The vibratory motion of a ship is a function of the following factors:

(a) The elastic characteristics of the hull and its appendages, which determine the behavior of the structure in response to the applied loads.

(b) The exciting forces acting on the vessel and its appendages, the character of these forces and their point of application.

(c) The manner and magnitude of the energy dissipation, i.e., the type and magnitude of the damping forces.

These factors have been discussed in previous sections of this report. This section will be confined to an analysis of Item (a) above, in order to determine the influence of certain features of design on the vibratory behavior of ships. The data presented in Section 6.0 will be utilized in this analysis.

It has been indicated in Section 6.0 that the excessive torsional vibration of the DD692-Class destroyers is due, to a large extent, to the lack of torsional rigidity of the hull, especially toward the stern. The torsional stiffness of the hull girder is supplied almost entirely by the plating, when either transverse or longitudinal framing systems are employed. It has been shown that in this case the torsional stiffness is given by  $T = \frac{4A^2G}{\theta \int \frac{dL}{t}}$ , the nomenclature being the same as that defined in Section 6.6.

This expression shows that the stiffness varies directly as the square of the area bounded by the plating. Since the stern of high-powered ships, such as destroyers, is cut back to an appreciable extent, in order to accommodate the large-diameter propellers, the rigidity of this portion of the vessel is appreciably reduced. It is desirable to keep the frequency corresponding to the lowest mode of torsional vibration above the operating rpm of the ship. In the case of lightly built vessels operating at high shaft speeds, such as destroyers, this condition is not realized as readily as in slower, more rigidly built ships. For a ship with a uniform cross section and with the center of gravity assumed to coincide with the center of twist, the frequency of torsional vibration is

$$f = \frac{n}{2L} \sqrt{\frac{K_T}{\mu}} \text{ (cps)} \quad [24]$$

where  $n = 1, 2, 3, \dots$ ,

$K_T$  is the torsional stiffness,

$\mu$  is the mass moment of inertia per unit length about the axis of oscillation, and

$L$  is the length of the ship.

It is seen therefore that, for a given ship, the resonance frequency varies as the square root of the torsional stiffness and inversely as the length of the ship. It should be noted also that the resonance frequency varies inversely as the square root of the mass moment of inertia.

Destroyers, being lightly built, possess a relatively low torsional rigidity. The DD692-Class destroyer is longer than any previously constructed and, in addition, has a relatively larger effective mass moment of inertia than the older destroyers due, in part, to the large masses of the twin 5-inch batteries and ammunition which are located at a considerable distance from the center of shear. It is to be noted that the influence of such masses on the amplitude and resonance frequency of torsional vibration is greatest when they are located near the ends of the ship, that is, near points of large torsional vibration. If such masses were added at the location of a node, they would have no effect on that mode of vibration. Theoretically the positions of the nodes will shift with a change in mass distribution. It is therefore to be remembered, in dealing with torsional vibration of ships, that it is generally advantageous to locate the load as close to the waterline and as near to amidships as possible.

In order to increase the torsional stiffness of the hull girder without appreciably increasing the weight, a diagonal or spiral arrangement of the framing is suggested. The benefits of such a system may be obtained to a somewhat lesser degree by the installation of diagonal stiffeners secured to the shell plating. To get some idea of the relative contribution to torsional stiffness of the shell plating itself and of the diagonal stiffeners, an elementary analysis can be made on the assumption (1) that longitudinal distances between transverse sections remain unchanged and (2) that the configuration of the transverse sections do not change.

By referring to Figure 60, it is seen that Point A is displaced to the position A' due to a rotation of  $\theta$  radians per unit length. The original length of the stiffener O-A is

$$L = \frac{s}{\cos \phi} \quad [25]$$

The new length after distortion is

$$L + \Delta L = [s^2 + (s \cdot \tan \phi + \theta sb)^2 + s^4 \theta^2 \tan^2 \phi]^{1/2} \quad [26a]$$

The terms containing  $\theta^2$  may be neglected since  $\theta$  is small, then

$$L + \Delta L = \frac{s}{\cos \phi} [1 + 2 \theta b \sin \phi \cos \phi]^{1/2} \quad [26b]$$

The first term is large compared to the remainder of the bracketed expression, and since

$$(a + \epsilon)^{1/2} \approx a^{1/2} + \frac{1}{2} a^{-1/2} \epsilon \quad [27]$$

where  $\epsilon$  is a small quantity,



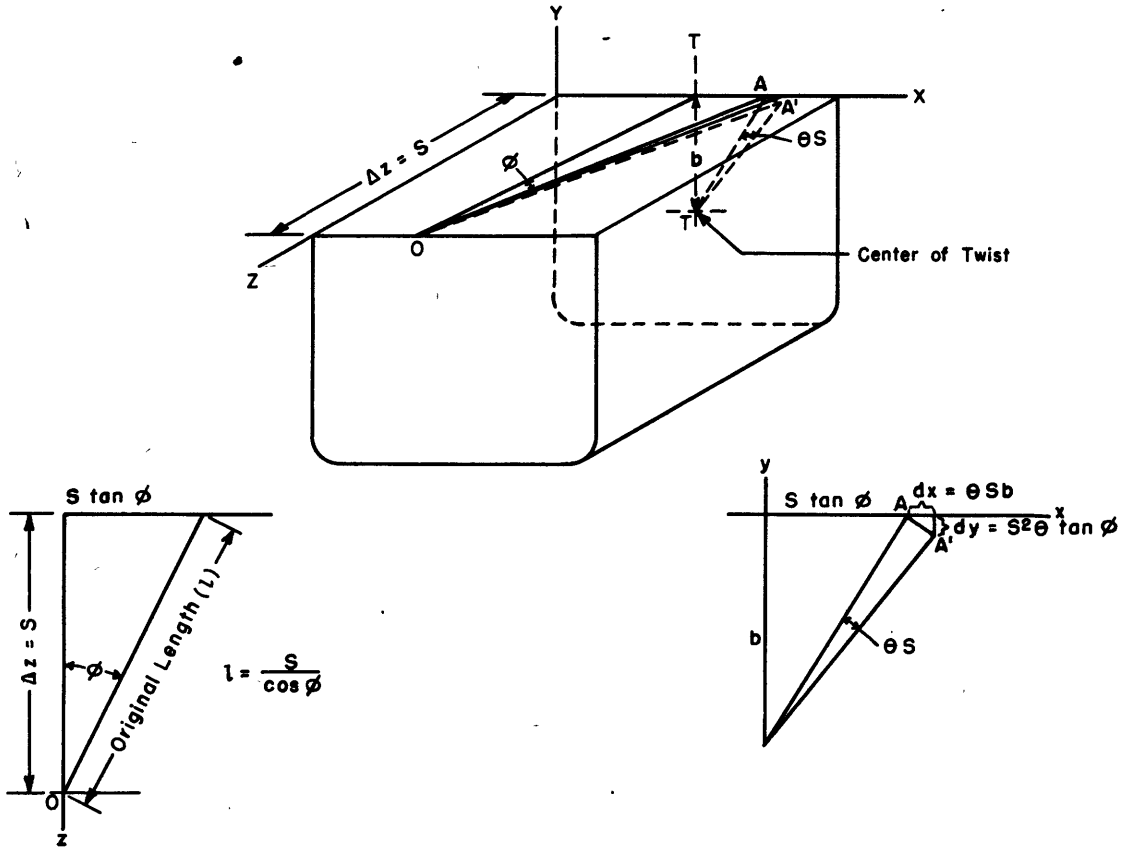


Figure 60 - Illustrations to Determine the Torsional Rigidity of Diagonal Stiffeners

Plane XZ is normal to plane XY and contains the stiffener OA.  
 Line T-T is normal to plane XZ and passes through the center of twist.  
 θ is the twist per unit length.

$$L + \Delta L = \frac{S}{\cos \phi} + s \theta b \sin \phi \quad [26c]$$

The strain is

$$\frac{\Delta L}{L} = \theta b \sin \phi \cos \phi \quad [27]$$

The force acting in the direction of the strut is

$$P = E a \cdot (\text{strain}) = E a \theta b \sin \phi \cos \phi \quad [28]$$

where a is the cross sectional area of the strut, and E the modulus of elasticity. The sum of the moments of the strut forces P about an axis perpendicular to the transverse section under consideration gives the resisting torque contributed by the struts. These formulas apply only if the strains are small and if the struts are restrained from buckling.

The torsional stiffness of the shell plating is, from Section 6.6, Equation [15]

$$\frac{T_{\text{shell}}}{\theta} = \frac{\text{inch}^2\text{-pounds}}{\text{radian}} = \frac{4A^2G}{\int \frac{dL}{t}}$$

The torsional stiffness of the diagonal framing can be determined by using Equation [28] and assuming that the sides of the ship are nearly perpendicular to the transverse section under consideration.

$$\frac{T_{\text{stiffener}}}{\theta} = \frac{\text{inch}^2 \text{ pounds}}{\text{radian}} = E \sin^2 \phi \cos \phi \sum ab^2 \quad [29]$$

The total torsional stiffness of the section is,

$$\frac{T}{\theta} = \frac{T_{\text{shell}}}{\theta} + \frac{T_{\text{stiffener}}}{\theta} = \frac{4A^2G}{\int \frac{dL}{t}} + E \sin^2 \phi \cos \phi \sum ab^2 \quad [30]$$

It is seen that the contribution of the stiffeners is zero when the angle  $\phi$  is zero.

The calculation indicated by Equation [30] will now be carried out for Frame 189, as an example. Using the ips system,

$$\int_0^L \frac{dL}{t} = 4180, \quad A = 652 \times 10^2 \text{ in}^2, \quad G = 11 \times 10^6 \text{ psi};$$

$$E = 30 \times 10^6 \text{ psi}, \quad b = 190 \text{ in}, \quad \phi = 45^\circ,$$

There will be N stiffeners--in the form of 12.3-pound angles, 6 inches by 4 inches by 3/8 inch--attached to the side shell plating. The portion of the shell plating assumed to act with the stiffener is 30 times the plate thickness. Therefore,

$$a = (1/4 \times 1/4 \times 30 + 3.61) = 5.49 \text{ sq. inches}$$

$$\frac{T}{\theta} = \frac{4 \times 42.5 \times 10^{14} \times 11}{4.18 \times 10^3} + 30 \times 10^6 \times 0.304 [N \ 5.49 \times 3.6 \times 10^4]$$

$$\frac{T}{\theta} = \left[ \begin{array}{l} 4470 \times 10^{10} \\ \text{(plating)} \end{array} + \begin{array}{l} N \times 180 \times 10^{10} \\ \text{(stiffener)} \end{array} \right] \frac{\text{inch-pounds}}{\text{radian/inch}}$$

It is readily seen that in order to double the torsional stiffness at this section, it would be necessary to provide stiffeners equivalent to 90 square inches cross-sectional area in the form of 12.3-pound angle irons attached to the side plating at an angle of  $45^\circ$  with the longitudinal axis. There are of course, a number of combinations of  $\phi$ , b and a which will give the same added stiffness.

By doubling the torsional stiffness of the DD692-Class hull, the fundamental mode of torsional vibration would be raised from 310 cpm to  $(310\sqrt{2}) = 440$  cpm. Thus the critical would be considerably out of the range of propeller rpm and the residual torsional vibration would be negligible. This approach is desirable. Inspection of the ship plans indicates that this situation might have been accomplished if the longitudinals had been arranged to run in a diagonal direction rather than in the fore and aft direction.

For example, the torsional stiffness of the hull plating at Frame 72 is calculated to be  $17,700 \times 10^{10}$  inch<sup>2</sup> pounds/radian. If the longitudinals had been installed at an angle of 45° with the fore-and-aft axis an additional torsional stiffness of about  $16,000 \times 10^{10}$  inch<sup>2</sup> pounds/radian would have been obtained, that is, the torsional stiffness would have increased about 90 percent. For reasons of symmetrical bending stiffness the longitudinals would then have been skewed alternately in opposite directions. In a new ship design, which necessitates greater torsional rigidity than can be provided by the normal framing arrangement, such "diagonal" or spiral framing could eliminate both transverse and longitudinal framing. It is believed that a hull girder would thereby be provided which is stiffer in torsion for a given weight of structure, than that of a conventionally framed vessel--although the stresses in the longitudinals due to bending would be increased. The torsional stiffness could be increased similarly, by stiffening the plating. The reinforced plating would then resist the twisting motion by developing compressive and tensile stresses without buckling.

In the case of the DD692-Class of destroyers it is necessary, of course, to deal with the existing situation. It would be possible to provide diagonal stiffening as illustrated by the example. This diagonal stiffening would be most effective if installed on the side shell of the vessel, extending from the fantail to about the forward quarter point. Furthermore, the diagonal stiffeners should be crossed to provide symmetrical bending stiffness. The additional weight of the stiffening would be about 40 tons, but because this weight will be located near the waterline, it is improbable that it would have a detrimental effect on the stability of the ship.

It has been indicated in Section 6.7 that the superstructure does contribute to some extent to the torsional rigidity of the hull girder, although the contribution to flexural rigidity is not nearly as significant. The stresses associated with torsional vibration are very small and it is perhaps due to this fact that the torsional rigidity of the ship girder has received little attention from ship designers. The behavior of the DD692-Class destroyers does show, quite definitely however, that excessive torsional vibration may occur even though the stresses are very small.

The propeller-tailshaft-strut system has an important influence on the vibratory motion of the ship because most of the effective unbalance, lift, and drag forces of propeller-rpm frequency are applied to this system. It has been indicated, in Section 3.0, that in a linear system the effectiveness of a given vibratory exciting force is directly proportional to the amplitude of vibration at the point of application of that force. The amplitudes and mode of vibration of the propeller-tailshaft-strut system are therefore of utmost importance in determining the response of the ship to forces applied to this system. It is desirable, for vessels subject to first-order vibration, to have the lowest natural whirling frequency of the shafting well above the operating rpm--two times the top rpm, if practicable. The strut bearings should be loaded at all times. It is possible that, for a particular location of the intermediate strut bearing, that bearing will carry no load. If this condition exists, the bearing is totally ineffective. Since any runout of shafting or propeller is equivalent to a mass eccentricity, it is obvious that such runouts must be kept to a minimum, especially at the propeller. To that purpose, it is advisable to have as short an overhang of the propeller as possible. The resonance frequency of whipping of the propeller shaft is affected by the stiffness of the shaft supports, i.e., the bearings. It has been found, see Section 6.8, that the stiffness of the bearing material is only a fraction of the stiffness of the strut and therefore the stiffness of the shaft support is influenced predominantly by the rubber bearing. Tests made at M.I.T. with scale-model propeller-tailshaft strut systems<sup>30, 31</sup> showed that, for rubber bearings, the bearing behaved essentially like a point support. This is also shown by the test results plotted in Figures 35 and 36. When one considers the large bearing clearances, the bearing material and the magnitude of the vibratory amplitudes it is reasonable to expect that the strut bearings cannot offer much more restraint than that of a point support. The actual location of this point support is difficult to determine and therefore any calculations made on the basis of plans are rather indeterminate. It would be advisable to use shorter bearing lengths and position the bearings so as to assure that they will be loaded at all times. If it were possible to assure fixity at the strut bearings by means of very small clearances and hard bearing material, the effect of vibratory forces, applied to the shaft could be somewhat reduced; however that does not appear to be an effective or practical solution.

The loads that are applied to strut arms may become very high during full-rudder high-speed turns as shown in Section 7.2. These forces are primarily lift and drag forces generated by the flow of water over the revolving shafting. The situation may be aggravated by the vibratory response

of the shafting if anything occurs which may lower the whipping frequency of the shaft to a value near the propeller rpm, such as might be caused by the unloading of a strut bearing. The original single-arm struts of the DD692-Class destroyers were subjected to high stresses during turns because the loading was resisted by direct and bending stresses in the strut arm. The double-arm strut which was installed later, reduced the stresses materially, although no doubt the same exciting forces are still present, because the applied forces are now resisted by strut action. It is suggested that double-arm struts be used whenever practicable. Excessive lift forces could of course be eliminated by enclosing the outboard shafting in a tunnel.

The thrust acting on the propeller disc acts as a column load on the tailshaft and lowers the critical whipping frequency to some degree--especially if the line of action of the thrust does not coincide with the centerline of the shaft. Here again, the advisability of keeping the propeller overhang to a minimum is indicated.

The balance and runout of the line shafting is not as critical as that of the tailshaft because the influence function for all modes of vibration is greatest near the ends of the vessel.

In conclusion it is well to state that the trend in destroyer design has been toward faster, longer, more heavily loaded ships operating at higher propeller rpm and carrying more off-center weights. In this evolution it is inevitable that a point is reached at which the exciting forces become so large and the rigidity of the hull so low as to cause objectionable vibration of the hull girder and equipment, even though the static strength of the vessel is adequate. This point has been exceeded in the DD692-Class destroyers. The primary factors that determine the vibration of a structure are the rigidity, the damping and mass distribution of that structure and the exciting forces acting on that structure. Therefore in order to keep the vibratory motion on such lightly built, high-powered ships within reason it is necessary to increase the rigidity of the hull or to reduce the exciting forces or both--since the damping and mass distribution cannot readily be controlled. An increase in torsional rigidity is particularly desirable and can probably be accomplished without great difficulty. A reduction in exciting forces can be obtained without appreciable difficulty by raising the standards for balancing and alignment of shafting and for balancing and manufacture of propellers, by increasing propeller-tip clearances and by using a greater number of propeller blades.

#### 7.4 Balancing of Propellers and Propulsion Shafting

The balancing of propulsion shafting has received scant attention to date. The desirability of installing a reasonably well balanced propeller has been appreciated and BuShips General Machinery Specification S44-1 requires that propellers shall be dynamically balanced so that the centrifugal force at rated rpm shall not exceed one percent of the weight of the propeller. This requirement actually is equivalent to a static balance. It has been repeatedly stated in this report that a runout of the propeller is equivalent to a mass unbalance. In order that the propeller balance have any significance it is essential that the permissible runout at the propeller taper be specified.

The critical runout "e" is defined here as that runout which will produce a centrifugal force equal to the weight of the rotor at rated speed. Then

$$F = W = \frac{W}{g} a = \frac{W}{g} (2\pi)^2 r^2 e \qquad e \approx \frac{10}{f^2} \qquad [32]$$

where W is the weight of the rotor,

g is the acceleration of gravity in inches per sec<sup>2</sup>, and

f is the frequency of rotation in cps.

a is the maximum value of radial acceleration of the rotor. It is seen that the critical eccentricity is a function only of the rpm.

If it is desired that the centrifugal force due to runout at the propeller be not more than that specified by the BuShips Specification S44-1 for the propeller itself, then the allowable eccentricity should be  $1/100(10/f^2)$ . The runout is twice the eccentricity. The allowable runout for the DD692 propeller would then be about 0.0056 in. It should be apparent that there is little justification for expenditure of time and funds in securing a high degree of propeller balance unless the effective unbalance due to runout is kept within a proportionate magnitude. It is more difficult to keep the actual propeller runout down to 5 or 6 mils than to balance a propeller to within BuShips specifications.

A situation as follows could easily arise. Large first order-vibration is diagnosed as due to propeller unbalance. The propellers are removed, balanced and reinstalled. The vibration of the ship may now still be the same or worse than before due to the effective unbalance created by propeller runout--because in the original condition the propeller unbalance may have counteracted the effective unbalance force due to propeller runout.

Suggested specifications for propeller runout and propeller balance will be given at the end of this section.

The balancing and straightening of propeller and line shafting is a difficult operation. The relatively long shafts are flexible and the balancing theory applicable to rigid rotors does not apply to flexible rotors. A flexible rotor may be balanced for one particular speed and be unbalanced at other speeds.

Flexibility is a relative term. The suggested balancing procedure given in this section is essentially that for a rigid rotor except that it is made under conditions which tend to minimize any flexibility effects present. The primary unbalance effect of shafting is no doubt due to eccentricity of the shaft bore. It is impractical to check the concentricity of the bore, especially since the greatest eccentricities are liable to occur at the center of the shaft. It does appear that the small savings in weight obtained by boring the tailshaft is not worth the disadvantages incurred by this procedure. It is suggested that some thought be given to the possibility of specifying solid shafting in future installations.

The following procedure for balancing and straightening of shafting is suggested. (From this point on a complete record of all work, from start to finish should be kept so that the condition of the shaft as received and as reinstalled on the ship can be checked at any time. These data should be made available to the Bureau of Ships.)

The shaft is removed to the machine shop and there it is supported at the journals by means of rollers as shown in Figure 61. The shaft should be rotated by means of a torque applied close to the rollers, in order to

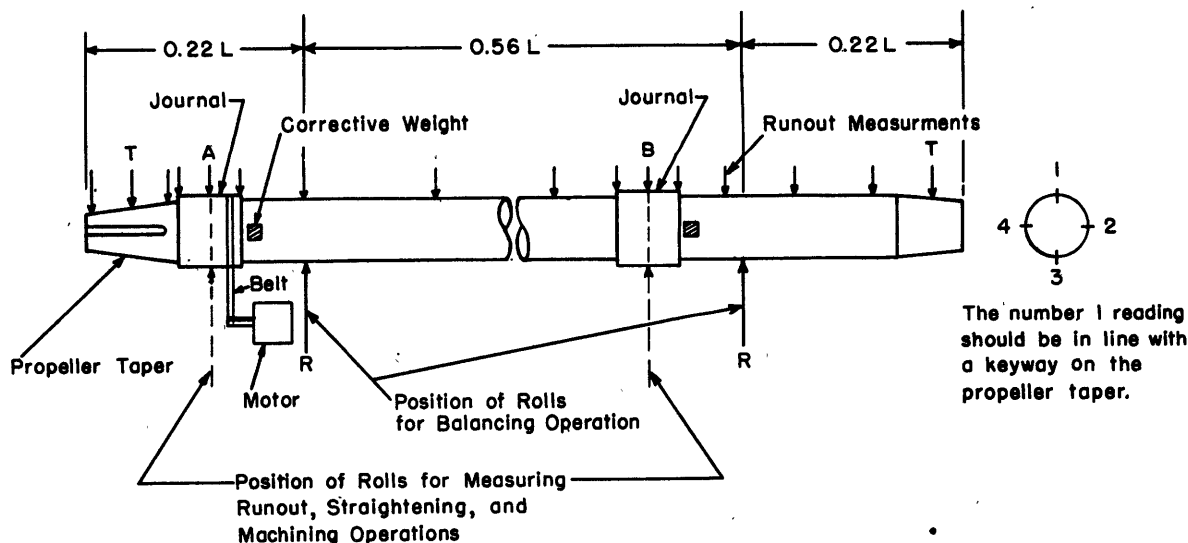


Figure 61 - Setup for Balancing and Straightening Propulsion Shafting

Permissible runout is 0.001 inch at Points A, B, and R and it is 0.005 at Points T.  
Permissible runout at all other points is 0.015 inch.

avoid exerting additional bending forces on the shaft. The first operation on the shaft is the determination of the circularity of the journals and the condition of straightness of the shaft, as received. The circularity of the journals is indicated by the dial readings A and B opposite the rolls when the shaft is slowly rotated on the rolls. This reading should be 0.001 inch or less according to BuShips' Specification S43-1. Additional readings on the propeller taper journals and shafting should be made as indicated by arrows in Figure 61 so as to determine the straightness of the shaft. The runout on any part of the shaft should not exceed the values noted in Figure 61.

If it is found that the journals are not circular cylinders then they should be machined. Next the shafting is straightened to within the specified tolerances, and the shaft is made smooth and concentric at the two locations R at which the shaft will be supported for the balancing operation; see Figure 61. The balancing operation follows after the above work has been completed.

The balancing is done dynamically at a speed which should be as near the highest operating rpm of the shaft as practicable. Care should be taken to ascertain that the balancing speed is not near a whipping resonance. As an approximation, for this purpose, the whipping frequency may be assumed to be equal to that of a uniform fixed-fixed beam of length equal to  $0.56 L$  (where  $L$  is the length of the shaft) and having the same cross section as the shaft. This approximation is based on the assumption that the elastic curve of the shaft, when supported as described below, will approach that of a continuous beam supported on equally spaced supports.

The position of the two sets of rolls R during the balancing operation should be selected so as to make the deflection at the ends of the overhang equal to that at the center of the shaft. For a uniform shaft this position is about  $0.22 L$  from the ends of the shaft, where  $L$  is again the length of the shaft. This position of the supports approaches the condition obtained in the actual shipboard installation. It is necessary as indicated before that the runout at the location of the supports R be kept to a minimum. This can be assured by checking the runout during the straightening operation with the shaft supported at its journals and then taking a light cut at position R (see Figure 61) to make the runout less than 0.001 inch. The standard balancing procedure as outlined by den Hartog<sup>12</sup> and Timoshenko<sup>13</sup> is followed.

The problem of specifying definite tolerances for balance and runout of propulsion shafting and propellers will now be discussed. The accuracy of balancing depends on several factors, primarily on the inherent accuracy



of the balancing equipment, the time available, the method of balancing and on the personnel actually performing the balancing operation.

Reference 43 gives the results of an investigation which was made at Puget Sound Naval Shipyard in order to determine the sensitivity of several commercial balancing machines used to balance rigid rotors dynamically. It was proposed therein to establish the "Maximum Allowable Unbalance," in either of two planes used for the correction of dynamic unbalance, as a fraction of the critical unbalance. From this reference, the following relationship was obtained:

$$U_A = CWe = FW \quad [33]$$

where  $U_A$  is the allowable unbalance in inch-pounds,

$C$  is a numerical factor based on the allowable vibration amplitude,

$W$  is the weight of the rotor in pounds,

$e$  is the critical eccentricity in inches  $\approx 10/f^2$

$F = Ce$ , and

$f$  is the rated speed in revolutions per second.

The recommended value of  $C$  was

$$C = 0.01 + 0.00065 \left( \frac{R}{1000} \right)^2 \quad [34]$$

where  $R$  is the revolutions per minute.

The report of the tests conducted at Puget Sound indicates that most rotors can easily be balanced within the given limits. The suggested permissible unbalance is equivalent to allowing a centrifugal force, at rated speed, equal to one percent of the weight of the rotor, in each balancing plane for rated speeds up to about 1000 rpm.

It is recommended that the suggested specifications represented by Equations [32], [33], and [34] be used for balancing propulsion shafting of ships. The permissible runout at the propeller taper should not be greater than one percent of the critical eccentricity, Equation [32]. The propeller should be balanced dynamically to insure an accurate static balance such that the centrifugal force due to mass unbalance does not exceed one percent of the designed weight of the propeller when it is rotating at its designed rpm.

For the DD692-Class destroyers the suggested specifications would be equivalent to permitting the following maximum unbalances and runouts:

(a) Propeller maximum unbalance about 40 inch-pounds maximum runout (twice the eccentricity) at propeller taper = 0.0055 inch.

(b) Tailshaft, the maximum unbalance in each of the two correction planes, is about 57 inch-pounds.

(c) Lineshaft, the maximum unbalance in each of the correction planes is  $0.0028 W$  inch-pounds, where  $W$  is the weight of the shaft section.

Adherence to these suggested specifications should keep the athwartships single amplitude of first-order vibration, at Frame 72, 02 deck, below a level of 10 mils for both the three-noded flexural and the one-noded torsional modes of vibration which occur near 240 and 310 rpm respectively.

Actual balancing of DD692-Class shafts and propellers at the New York Naval Shipyard<sup>15</sup> and at the Puget Sound Naval Shipyard<sup>10, 11</sup> has shown that a much better balance can be obtained than that suggested above. It appears from these data that it is quite feasible to obtain balances within 10 inch-pounds in each correction plane. Inasmuch as the actual balancing time is small compared to the preparations required incident to balancing it is suggested that, when balancing has been decided upon, an attempt should be made to obtain a balance within 10 inch-pounds. The most difficult requirement to fulfill is expected to be the specified runout at the propeller taper.

The entire shaft-balancing question would probably resolve itself if solid shafting were used. This would permit controlling shaft balance by specifying the permissible runout of shafting.

It is suggested that a DD692-Class destroyer be selected to undergo the recommended balancing procedure and that underway vibration measurements be made both before and after balancing.

It is further suggested that the above balancing tolerances be made immediately applicable to the DD828.

#### 7.5 Response of the Hull Girder to Vibratory Forces Acting on the Propeller and Shafting

The vibratory amplitudes of the hull of the DD692-Class long-hull destroyers, due to forces acting on the propeller and shafting can be calculated by using the data given in Table 7. The amplitude of vibratory motion at Frame 200 is obtained by multiplying the exciting forces by the respective influence coefficients. The amplitude contributions resulting from all the exciting forces are added to give the resultant motion at Frame 200. The motion at any other point on the ship can then be determined by making use of the mode configurations given in Section 6.1. These calculations have been made for the pre-repair and post-repair condition of the DD865 and the results are discussed in Section 6.10. It was shown there that the three-noded

flexural athwartship vibration, existing near 240 propeller rpm, is caused primarily by line-shaft unbalance and that the one-noded torsional hull vibration near 310 propeller rpm is caused in the main, by tailshaft unbalance.

The accuracy of this type of calculation depends on the accuracy of the factors given in Table 7. It is believed that an accuracy of 30 percent can be realized. Anything that would change the mode or frequency of vibration of the ship or of its main propulsion shafting will, of course, change the influence coefficients given in Table 7.

It has been shown by the test data (see Section 6.4) that, if the amplitude of first order vibration measured at Frame 72, 02 deck is considerably larger at the upper hull critical, (near 310 rpm), than at the lower hull critical (near 240 rpm), then the excitation must be a concentrated force probably applied at the propellers. Conversely if the measured amplitudes are of about the same magnitude for the two criticals, then a considerable part of the exciting force must be distributed throughout the propulsion system.

#### 7.6 Response of the Hull Girder to Vibratory Forces Acting on the Hull Girder

The response of the hull girder to exciting forces acting on the hull directly can be determined by utilizing the data presented in Section 6.1 in the form of graphical plots of mode configurations for known applied forces. These graphs represent, in effect, a plot of influence functions. That is, the graphs show at a glance the deflection in a particular mode of vibration, at any point on the hull, for a given force applied at Frame 200. By application of the reciprocity theorem it is possible to determine the motion at any point of the hull, in any given natural mode of vibration, for any distribution of exciting forces acting on the hull. The calculation can be carried out directly by merely adding the contribution of the several exciting forces. Simple algebra is the only necessary mathematical operation. This process is essentially the same as outlined in the preceding section.

### 8. CONCLUSIONS

1. The bothersome hull criticals of the 692-Class destroyers are identified as a three-noded athwartships vibration near 240 rpm and a one-noded torsional vibration near 310 rpm. The data obtained from the DD865 tests is representative of the entire class.

2. The unusual sensitivity of the DD692-Class destroyers to shaft-rpm exciting forces is due primarily to the relatively low torsional and flexural stiffness of the hull girder. The stern of the vessel, especially, possesses

but little torsional rigidity; a condition that greatly increases the vessel's susceptibility to vibratory forces applied at the stern.

3. Diagonal framing or diagonal stiffeners can be used to advantage in increasing the torsional stiffness of ships. A method of computing the effect of diagonal stiffening on the torsional characteristics of the hull is given in Section 7.3.

4. The load distribution on a ship appreciably affects the amplitudes and resonance frequencies of vibration. In general, with some exceptions, weights should be kept as close to the waterline and as near amidships as possible in order to reduce torsional vibration.

5. It is quite probable that the bothersome first-order hull vibration can be reduced to an acceptable value by proper balancing of propellers and propulsion shafting as well as by keeping the runout at the propeller within the recommended tolerances; see Section 7.4.

6. It is probable that the use of hollow propulsion shafting has contributed appreciably to the vibration difficulties experienced on the DD692-Class destroyers and that the disadvantages entailed by the use of hollow shafting may outweigh the advantages.

7. Runout at the propeller taper is more apt to cause vibration than propeller unbalance.

8. The reduction in first-order vibration obtained as a result of the balancing work done in the New York Naval Shipyard is considered consistent with the calculated reduction in vibration based on the use of the influence factors represented by the mode shapes in Section 6.1 and by the data in Table 7. These data may be used to calculate the steady-state response of the ship at resonance due to any forces applied to the vessel.

9. Most unbalance conditions existing on DD692-Class destroyers are a combination of propeller, tailshaft, and lineshaft unbalance.

10. The pattern of athwartships vibration measured on the O2 deck Frame 72 can be used to indicate whether or not the first-order exciting forces are concentrated or distributed in space; see Section 6.4.

11. The formula given in Bureau of Ships Design Data Sheet DDS43-1 for calculating the whipping frequency of the propeller shafting is unsafe since it may give considerably higher frequencies than actually obtain. For example, the lowest resonance frequency of whirling or whipping of the tailshaft-propeller system of the DD865, as obtained experimentally by vibrating the shaft in drydock, is about 660 cpm (in air), whereas the formula just referred to gives about 1140 cpm.

12. It is important that the strut and stern-tube bearings be loaded at all times.

13. During turns it is probable that the intermediate strut bearing becomes intermittently unloaded, resulting in large vibratory amplitudes and loads at the intermediate strut.

14. The propeller overhang should be kept to a minimum.

15. The strut bearings act as simple supports with respect to the struts. The point of support is not necessarily at the middle of the bearing.

16. The stiffness of the strut bearings is determined primarily by the stiffness of the rubber bearings. The stiffness of the intermediate double-arm strut is about  $1.8 \times 10^6$  pounds per inch, the stiffness of the rubber bearing is about  $0.60 \times 10^6$  pounds per inch and the combined stiffness is about  $0.45 \times 10^6$  pounds per inch. The combined stiffness of the double-arm strut is about four times that of the single-arm strut-bearing combination.

17. The vibration generator tests conducted on the DD865 confirm previous observations made on other vessels of this class that the stresses will be quite small when double-arm struts are installed, whereas dangerously high stresses, of the order of 30,000 psi for the DD692-Class ships, can be expected at the root of the struts if single-arm struts are used.

18. The first three modes of flexural vibration of the DD865 vary approximately as the ratios 1, 2, 3. The same general relationship has been observed on a number of ships of different form such as an APA and a submarine. It is believed that a fair estimate of the frequency corresponding to the first few flexural modes can be obtained by applying the factors 1, 2, 3 to the fundamental mode of vibration which may be obtained by a simple experiment or calculation. In calculating the higher natural frequencies of flexural vibration it is necessary to take shear deflections into consideration.

19. The mass of entrained water associated with torsional hull vibration appears to be relatively small, as compared to that which acts with the ship during flexural vibration; see Section 7.1.

20. The damping associated with the hull vibration is of the order of 0.5 percent of critical damping for the first three modes of flexural vibration.

21. The hull stresses associated with the vibratory motions during steady-state condition are negligibly small.

22. At the present time the torsion theory of Bredt appears to be the most suitable approach to the problem of computing the torsional rigidity of hulls; see Section 6.6. Experimental verification of the shear-flow theory for multiple-deck ships would be of considerable value.

23. The center of shear must be considered in calculating the natural frequencies of torsional vibration of beams. Formula [24] page 91 may be adapted to estimate the fundamental resonance of torsional vibration of ships.

24. It is probable that, on occasion, reported athwartship vibration on various ships may actually have been due to torsional vibration of the vessels.

### 9. RECOMMENDATIONS

The following recommendations are intended to apply to all naval vessels except where indicated otherwise.

1. Set up specifications for the straightness of propulsion shafting as proposed in Section 7.4.

2. Set up specifications for balancing of propulsion shafting as follows: (Also see Section 7.4)

(a) Each section of shafting shall be dynamically balanced.

(b) The shafting shall be supported as shown in Figure 61 during the balancing operation.

(c) The shaft shall be balanced at a speed which should be as near the highest operating rpm of the shaft as practicable. Care shall be taken that the balancing speed is not a natural whipping speed.

(d) The driving torque shall be applied to the shaft in such a way so as to avoid applying bending moments to the shaft.

(e) The allowable unbalance in each correction plane shall be  $U = CWe$  in-lb, the symbols are defined in Section 7.4. It is suggested that an attempt be made to reduce the unbalance to within 10 in-lb in each correction plane.

3. The present BuShips specification for balancing propellers should be retained, although an attempt should be made to balance within 10 in-lb.

4. Set up a specification for the permissible runout at the center of the propeller taper as follows:

(a) The runout at the center of the propeller taper shall not exceed  $2/100(10/f^2)$ , where  $f$  is the maximum rated rps. The runout is two times the eccentricity. (This specification permits an additional unbalanced force equal to that permitted by the specification for propeller balance.)

Adherence to Recommendations 1 through 4 should result in an acceptably low level of first-order hull vibration of the DD692-Class destroyers.

5. It is recommended that the present practice of making propulsion shafting hollow be reconsidered and that, if possible, solid shafting be specified; this is especially suggested for tailshafts.

6. DD692-Class only. It is suggested that the permissible first-order athwartship single-amplitude vibration at Frame 72, 02 Deck be set at 15 mils for both the upper and lower hull criticals, (about 240 and 310 rpm respectively)

7. For DD692-Class destroyers, it is suggested that the tentative permissible athwartship single amplitude of propeller-blade frequency be 5 mils at Frame 72, 02 Deck for speeds over 200 rpm. Ships equipped with three-bladed propellers which exceed this tolerance should be fitted with four-bladed propellers.

8. It is recommended that whenever underway vibration measurements are made on DD692-Class destroyers, the minimum number of measurements include athwartships measurements made at Frame 72, 02 Deck for both the lower and the upper hull critical. This location is convenient to the bridge, one man can make the measurements, and the data can be compared directly with those obtained on almost all previous trials; see Table 1.

9. DD692-Class only. It is recommended that the propeller and tailshafts be pulled, balanced, and checked for runout according to the suggested procedure whenever the amplitudes of first-order athwartship vibration measured at Frame 72, 02 level, are considerably greater at the upper-hull critical than at the lower-hull critical.

10. DD692-Class Only. It is recommended that consideration be given to increasing the torsional stiffness of the hull along the lines indicated in Section 7.3. As a first attempt, the after portion of the vessel could be stiffened--with promise of an appreciable beneficial effect.

11. It is suggested that in future designs of lightly built, heavily loaded, high-power ships, such as the class here considered, attention be given to the probable vibrational behavior of the vessel. Torsional

vibration will be most likely to become a factor if the vessel operates at relatively high shaft rpm since in this case the fundamental torsional mode may synchronize with the propeller rpm.

12. The preceding recommendations apply particularly to the USS TIMMERMAN (DD828) design.

13. The propeller overhang should be kept to a minimum and the strut bearings should be loaded at all times.

14. The present method used by the Bureau of Ships for calculating the fundamental whipping frequency of propulsion shafting should be revised. Model tests, if properly simulated models are used, should prove helpful.

#### ACKNOWLEDGMENTS

The experimental work which forms the basis of this report was the joint effort of the Model Basin and the Material Laboratory of the New York Naval Shipyard. The rather difficult work was made possible by the excellent cooperation between all parties concerned. Special acknowledgments are due to the Commanding Officer and personnel of the USS WARE (DD865) and to Messrs. G. Dashefsky and M. Berg of the Material Laboratory for their patience and for the generous assistance that they provided throughout the long and sometimes trying tests. Most of the personnel of the Vibration Section of the Material Laboratory participated in the tests at one time or another.

Almost all the members of the Vibration Branch of the David Taylor Model Basin have taken some part in this project. Messrs. Q.R. Robinson, V.S. Hardy and R.B. Allnut were especially helpful in working up the test data. Miss E.J. Adams and Dr. A.N. Gleyzal made extensive mathematical calculations in connection with the work described in Section 3. Mr. McGoldrick, head of the Vibrations Branch, offered many helpful suggestions throughout the tests and during the writing of the report. Mr. E. Noonan, Code 371, Bureau of Ships, was instrumental in getting this project underway; the overall test program was planned during conferences between Mr. Noonan and the author. The recommended balancing procedure outlined in this report has been worked out together with Mr. Noonan.



APPENDIX

DD692C1/S29(332-514)

NAVY DEPARTMENT  
BUREAU OF SHIPS  
WASHINGTON 25, D. C.

21 September 1948

To: Commander Destroyers, Pacific Fleet  
Commander Destroyers, Atlantic Fleet  
Commander, Naval Shipyard:  
Puget Sound, Bremerton, Washington  
Boston 29, Massachusetts  
Mare Island, Vallejo, California  
Philadelphia 12, Pennsylvania  
Norfolk, Portsmouth, Virginia  
San Francisco 24, California

Subj: DD692 Class Destroyers - Investigation of Vibratory Characteristics of

Encl: (H.W.)  
(A) "Suggested Vibration Measurements - (DD692 Class)" dtd. 24 Aug. 1948  
(B) "Suggested Trial Schedule - (DD692 Class)" dtd. 24 Aug. 1948

1. Vibration tests recently conducted by the David Taylor Model Basin aboard the DD865 under research problem NS-712-059 have indicated the presence of a torsional mode of vibration of the hull at approximately 320 rpm in addition to a three noded transverse vibration occurring at approximately 250 rpm. It is believed that this characteristic is mainly responsible for the extreme sensitivity of this class to first order vibration. These vibrations are of the first order or shaft frequency. The exciting force is probably due to shaft or propeller unbalance.

2. It is considered necessary to obtain sufficient data on other vessels of the class to determine whether the vibration of the CHARLES R. WARE (DD865) is characteristic of the class as a whole or an isolated case. Type Commanders are requested to schedule ship operations, immediately prior to scheduled regular overhauls, to permit conducting trials witnessed by Naval Shipyard personnel for obtaining the necessary data. Naval Shipyards addressed are requested to obtain vibration data on DD692 Class Destroyers, as outlined in enclosures (A) and (B), and forward these data together with all necessary related information to the David W. Taylor Model Basin for analysis. Displacement, wind and sea conditions, and any other pertinent information should be included.

3. Enclosures (A) and (B) are forwarded for guidance in conducting the vibration tests. The measurements listed are considered necessary to establish the modes of vibration. The instruments used by the Model Basin, while not mandatory, provided phase relationship between readings. This phase relationship is particularly useful in determining torsional modes, but may also be obtained by other electrical pickup and oscillographs.

4. If suitable electrical test equipment is not available, mechanical equipment such as the Geiger or Cox vibrographs may be used. As a last resort, hand instruments may be used. The Askania hand vibrograph is recommended in this case.

5. In carrying out the tests, the following points should be observed:

- (a) A reasonable attempt should be made to synchronize propellers before taking readings,
- (b) When taking measurements at any particular speed, care should be taken that the propeller turns remain constant.
- (c) If mechanical or hand instruments are used, measurements at several frames in addition to those specified in "D" of Enclosure (A) should be made.

/s/ E. E. Paro

E. E. Paro  
By direction of  
Chief of Bureau

CC:

DTMB  
CINCLANT  
CINCPAC

S87-19/A11-(1)

NAVY DEPARTMENT  
DAVID TAYLOR MODEL BASIN  
Washington 7, D. C.  
24 August 1948

SUGGESTED VIBRATION MEASUREMENTS  
(DD692 Class)

- A. Transverse vibration amplitudes at the base of the forward gun director. 02 level, centerline, Frame 72.
- B. Transverse vibration amplitudes at the centerline of the First Platform Deck, Frame 72.

- C. Transverse vibration amplitudes on the Main and First Platform Decks, centerline near Frame 198.
- D. Measure the vertical vibration amplitudes at the outboard edges and centerline of the Main Deck at Frames 68, 102, and 198.

Note 1. Measurements A, B and C are to be made throughout the vessel's speed range from about 150 RPM to Full Power. Measurement D is to be made at the two critical ship speeds which occur near 240 and 310 RPM.

Note 2. The David Taylor Model Basin has found that the following instrumentation will be fairly convenient and reliable for making Measurements A, B and C. The vibration was measured by a General Radio crystal accelerometer, the output signal integrated with the General Radio Meter and then amplified and recorded with the Brush amplifier-oscillograph combination. This arrangement permits phase determination between two signals. The instrumentation should be calibrated both before and after the test.

ENCLOSURE (A)

S87-19/All-(1)

NAVY DEPARTMENT  
DAVID TAYLOR MODEL BASIN  
Washington 7, D. C.  
24 August 1948

SUGGESTED TRIAL SCHEDULE - (DD692 Class)

1. Vessel to be as near full load displacement as practicable.
2. Ship to make speeds from about 150 RPM to Full Power in 10 RPM intervals, approximately 10 minutes at each speed, rudder to be amidships 5 degrees.  
Make measurements A, B and C of Enclosure (A).
3. Determine the critical speeds.
4. Operate ship at upper critical speed.  
Make Measurement D.
5. Operate ship at lower critical speed.  
Make Measurement D.

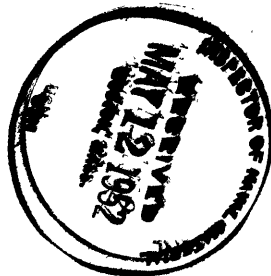
ENCLOSURE (B)

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