Report 4102

SHIP RESEARCH AND DEVELOPMENT CENTER

Bethesda, Maryland 20034

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THE CONVERGING FACTORS FOR THE FRESNEL INTEGRALS

[redacted]

by
John W. Wrench Jr.
and
Vicki Alley

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COMPUTATION AND MATHEMATICS DEPARTMENT
RESEARCH AND DEVELOPMENT REPORT

June 1973

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Naval Ship Research and Development Center

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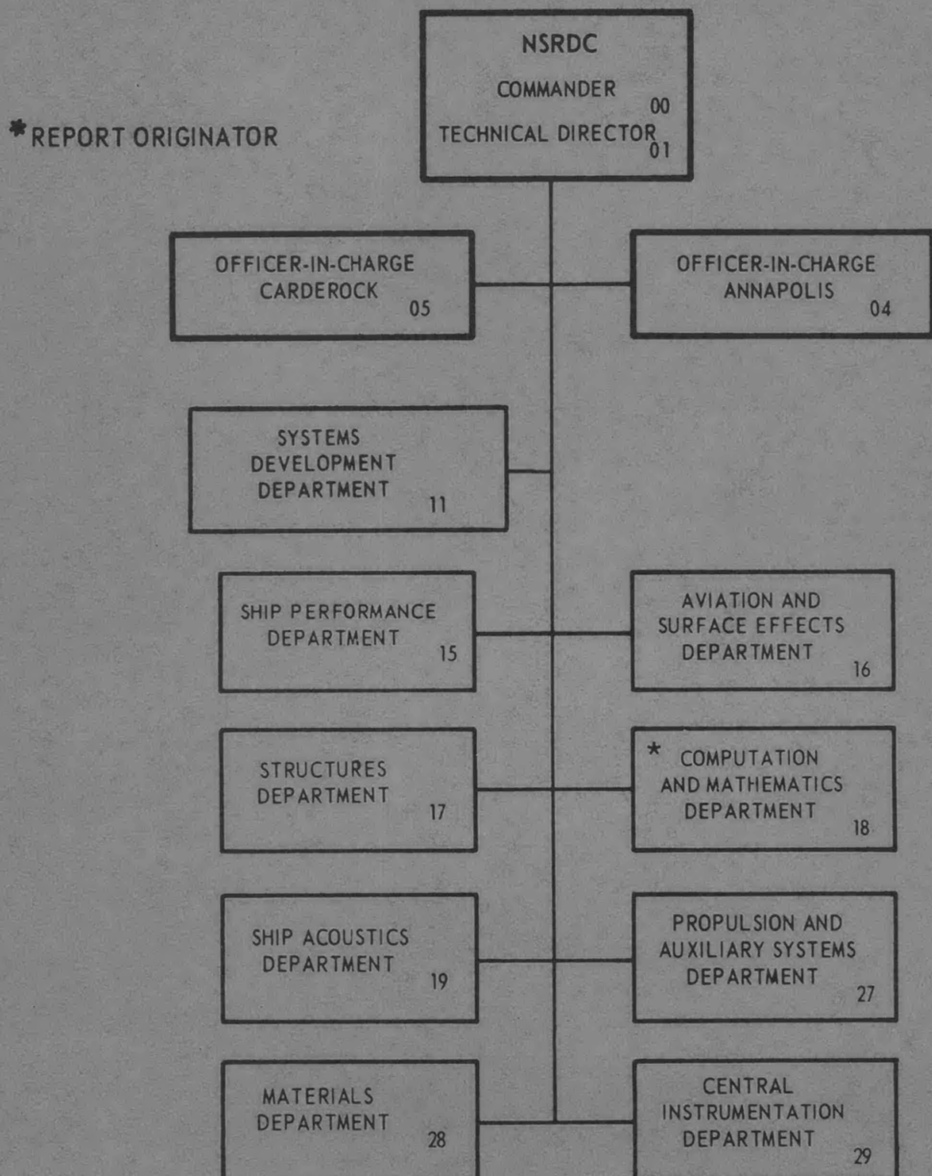

NDW-NSRDC 3960/44 (REV. 8/71)
GPO 917-872

DEPARTMENT OF THE NAVY

NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER

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ABSTRACT

The theory of the converging factors for the Fresnel integrals is developed from that of the converging factors for the sine and cosine integrals, and is then applied to the calculation on a CDC 6700 system of tables of these factors and their reduced derivatives to about 35 decimal places. The factors were used in conjunction with appropriately truncated asymptotic series to produce appended 28-place tables of the Fresnel integrals $S_2(x)$, $C_2(x)$ and of the closely related rocket functions $rr(x)$ and $ri(x)$, for successive integer values of x from 1 through 70. An abridged 28-place table of $S(x)$ and $C(x)$, for x ranging from 1 through 6, is also included.

ADMINISTRATIVE INFORMATION

Work on this research was authorized by the Naval Ship Systems Command under the Mathematical Sciences Program. Necessary funds were allocated under Subproject SR0140301, Task 15324, Program Element 61153N, Work Unit Number 1-1802-001.

INTRODUCTION

The Fresnel integrals are encountered in the mathematical analysis of a variety of physical problems, typified by the diffraction of light passing through an aperture and by the reduction of the level of sound by barriers such as solid walls. These integrals also appear in the parametric equations of transition curves used in the design of highways. Moreover, they are closely related to the error function of a complex argument, and thereby to certain functions involved in the mathematical theory of rocket flight.

It was in connection with his study of the diffraction of light that Fresnel[1] published in 1826 the first table of approximate numerical values of these integrals (correct to about three decimal places). Subsequently, a large number of more elaborate tables have appeared. These are listed by A. Fletcher[2] and his associates. Especially noteworthy tabulations include: the five-place table of Wijngaarden and Scheen[3], published in 1949; a seven-place Russian table[4] published in 1953; the six-place table of Pearcey[5] (1956); and the abridged seven-place table (with auxiliary functions to 15 decimals) in the National Bureau of Standards Handbook of Mathematical Functions[6], first published in 1964.

In this report methods are developed for the expeditious computation of converging factors for the Fresnel integrals, which, in conjunction with appropriately truncated asymptotic series, permit the numerical evaluation of these integrals to high precision. Specifically, these converging factors and their reduced derivatives are herein tabulated to 33 and 35 decimal places (Tables 1 - 8).

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[1] References are listed on page 107.

The corresponding algorithms were programmed for the CDC 6700 system and have been used to calculate 28-place tables of the Fresnel integrals $S_2(x)$ and $C_2(x)$ for $x = 1(1)70$ (Tables 9 and 10). The related rocket functions $rr(x)$ and $ri(x)$ were also calculated to 28 decimals for the same values of the argument (Tables 12 and 13).

This computer program was also used to evaluate the Fresnel integrals $S(x)$ and $C(x)$, which are equivalent to $S_2(\frac{\pi x^2}{2})$ and $C_2(\frac{\pi x^2}{2})$. These results, for $x = 1(1)6$, are also included in this report (Table 11).

THE FRESNEL INTEGRALS

The sine and cosine Fresnel integrals have been defined in a variety of equivalent ways in the mathematical literature[6].

Thus, we find the representations

$$S(z) = \int_0^z \sin(\tfrac{\pi}{2} t^2)\, dt \quad , \tag{1}$$

$$C(z) = \int_0^z \cos(\tfrac{\pi}{2} t^2)\, dt \quad , \tag{2}$$

and

$$S_1(x) = \sqrt{\tfrac{2}{\pi}} \int_0^x \sin t^2\, dt \quad , \tag{3}$$

$$C_1(x) = \sqrt{\tfrac{2}{\pi}} \int_0^x \cos t^2\, dt \quad , \tag{4}$$

and

$$S_2(x) = \frac{1}{\sqrt{2\pi}} \int_0^x \frac{\sin t}{\sqrt{t}}\, dt = \tfrac{1}{2} \int_0^x J_{\frac{1}{2}}(t)\, dt \quad , \tag{5}$$

$$C_2(x) = \frac{1}{\sqrt{2\pi}} \int_0^x \frac{\cos t}{\sqrt{t}}\, dt = \tfrac{1}{2} \int_0^x J_{-\frac{1}{2}}(t)\, dt \quad , \tag{6}$$

where $J_{\frac{1}{2}}(t)$ and $J_{-\frac{1}{2}}(t)$ are the ordinary Bessel functions of the first kind of orders $\frac{1}{2}$ and $-\frac{1}{2}$, respectively.

These three pairs of functions are related by the equations

$$S(x) = S_1(x\sqrt{\tfrac{\pi}{2}}) \div S_2(\tfrac{\pi}{2} x^2) \tag{7}$$

$$C(x) = C_1(x\sqrt{\tfrac{\pi}{2}}) = C_2(\tfrac{\pi}{2} x^2) \tag{8}$$

The Fresnel integrals are related to the probability integral (or error function) of semi-imaginary argument, $x\, i^{\frac{1}{2}}$. To see this, set $t = v\, i^{\frac{1}{2}}$ in the integral

$$\mathrm{Erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt \tag{9}$$

Then we infer

$$\mathrm{Erf}(x\, i^{\frac{1}{2}}) = \frac{2\, i^{\frac{1}{2}}}{\sqrt{\pi}} \int_0^x e^{-i\, v^2}\, dv \tag{10}$$

$$= \frac{2\, i^{\frac{1}{2}}}{\sqrt{\pi}} \left\{ \int_0^x \cos v^2 dv - i \int_0^x \sin v^2\, dv \right\}, \tag{11}$$

or

$$(2\, i\,)^{-\frac{1}{2}}\, \mathrm{Erf}(x\, i^{\frac{1}{2}}) = C_1(x) - i\, S_1(x) \quad . \tag{12}$$

THE ASYMPTOTIC SERIES FOR THE FRESNEL INTEGRALS AND THEIR CONVERGING FACTORS

To derive asymptotic series for the Fresnel integrals we take as starting point the truncated asymptotic series

$$\int_x^\infty e^{-i v^2} dv = \frac{e^{-i x^2}}{2 i x} \left\{ 1 - \frac{1}{2 i x^2} + \frac{1 \cdot 3}{(2 i x^2)^2} - \cdot \cdot \cdot \right.$$

$$\left. + (-1)^n \frac{1 \cdot 3 \cdots (2 n-1)}{(2 i x^2)^n} C_n(x^2) \right\}, \qquad (13)$$

where the converging factor $C_n(x^2)$ is given by

$$C_n(x^2) = 1 - \frac{2 n+1}{2 i x^2} + \frac{(2 n+1)(2 n+3)}{(2 i x^2)^2} - \cdots \qquad (14)$$

$$= 1 - \frac{n + \frac{1}{2}}{i x^2} + \frac{(n + \frac{1}{2})(n + \frac{3}{2})}{(i x^2)^2} - \cdots \qquad (15)$$

$$= \frac{1}{\Gamma(n + \frac{1}{2})} \left\{ \Gamma(n + \tfrac{1}{2}) - \frac{\Gamma(n + \frac{3}{2})}{i x^2} + \frac{\Gamma(n + \frac{5}{2})}{(i x^2)^2} - \cdots \right\} \qquad (16)$$

$$= \frac{1}{\Gamma(n + \frac{1}{2})} \left\{ \int_0^\infty t^{n-\frac{1}{2}} e^{-t} dt - \frac{1}{i x^2} \int_0^\infty t^{n+\frac{1}{2}} e^{-t} dt \right.$$

$$\left. + \frac{1}{(i x^2)^2} \int_0^\infty t^{n+\frac{3}{2}} e^{-t} dt - \cdots \right\} \qquad (17)$$

$$= \frac{1}{\Gamma(n + \frac{1}{2})} \left\{ \int_0^\infty n^{n-\frac{1}{2}} \left(1 - \frac{t}{i x^2} + \frac{t^2}{(i x^2)^2} - \cdots \right) e^{-t} dt \right. \qquad (18)$$

$$= \frac{1}{\Gamma(n + \frac{1}{2})} \int_0^\infty \frac{t^{n-\frac{1}{2}} e^{-t}}{1 + \frac{t}{i x^2}} dt \qquad (19)$$

$$= \frac{1}{\Gamma(n + \frac{1}{2})} \int_0^\infty \frac{t^{n-\frac{1}{2}} e^{-t}}{1 + \frac{t^2}{x^4}} dt + \frac{i x^{-2}}{\Gamma(n + \frac{1}{2})} \int_0^\infty \frac{t^{n+\frac{1}{2}} e^{-t}}{1 + \frac{t^2}{x^4}} dt \qquad (20)$$

Hence, if we define the converging factor $\Pi_s(z)$ by the integral

$$\Pi_s(z) = \frac{1}{\Gamma(s + 1)} \int_0^\infty \frac{t^s e^{-t}}{1 + (\frac{t}{z})^2} dt \quad , \qquad (21)$$

then we have the relation

$$C_n(x^2) = \Pi_{n-\frac{1}{2}}(x^2) + \frac{(2n+1)i}{2x^2} \Pi_{n+\frac{1}{2}}(x^2) \qquad (22)$$

From Equation (13) we then obtain

$$\int_x^\infty \cos v^2 \, dv - i \int_x^\infty \sin v^2 \, dv = (\cos x^2 - i \sin x^2)$$

$$\cdot \left\{ \frac{1}{2ix} - \frac{1}{(2i)^2 x^3} + \frac{1 \cdot 3}{(2i)^3 x^5} - \dots + (-1)^n \frac{1 \cdot 3 \dots (2n-1)}{(2i)^{n+1} x^{2n+1}} \Pi_{n-\frac{1}{2}}(x^2) \right.$$

$$\left. + (-1)^{n+1} \frac{1 \cdot 3 \dots (2n+1)}{(2i)^{n+2} x^{2n+3}} \Pi_{n+\frac{1}{2}}(x^2) \right\} \qquad (23)$$

Multiplying the factors in the right member of Equation (23) and equating the real and imaginary parts of the resulting form of that equation, we obtain the expansions

$$\int_x^\infty \cos v^2 \, dv = P(x) \cos x^2 - Q(x) \sin x^2 \quad , \qquad (24)$$

$$\int_x^\infty \sin v^2 \, dv = P(x) \sin x^2 + Q(x) \cos x^2 \quad , \qquad (25)$$

where

$$P(x) = \frac{1}{2^2 x^3} - \frac{1 \cdot 3 \cdot 5}{2^4 x^7} + \ldots + (-1)^k \frac{1 \cdot 3 \ldots (4k+1)}{2^{2k+2} \quad x^{4k+3}} \Pi_{2k+\frac{1}{2}} (x^2) \tag{26}$$

$$Q(x) = \frac{1}{2x} - \frac{1 \cdot 3}{2^3 x^5} + \ldots + (-1)^k \frac{1 \cdot 3 \ldots (4k-1)}{2^{2k+1} \quad x^{4k+1}} \Pi_{2k-\frac{1}{2}} (x^2) \tag{27}$$

From standard tables of definite integrals it is known that the Fresnel integrals have a common limiting value of ½ as the argument (upper limit) of each tends to infinity.

Thus, we conclude that

$$S_1(x) = \tfrac{1}{2} - (2/\pi)^{\frac{1}{2}} \left\{ P(x) \sin x^2 + Q(x) \cos x^2 \right\}, \tag{28}$$

$$C_1(x) = \tfrac{1}{2} - (2/\pi)^{\frac{1}{2}} \left\{ P(x) \cos x^2 - Q(x) \sin x^2 \right\}, \tag{29}$$

with similar expansions for the other forms of the Fresnel integrals, by virtue of Equation (7) and Equation (8).

By means of the series in Equation (28) and Equation (29) the Fresnel integrals can be numerically evaluated to high precision for large or even moderately large values of x, provided the appropriate converging factors can be calculated. The expeditious calculation of these converging factors is the main purpose of this report.

It may be noted here that although the Maclaurin expansions

$$S_1(x) = (2/\pi)^{\frac{1}{2}} \left\{ \frac{x^3}{3} - \frac{x^7}{7 \cdot 3!} + \frac{x^{11}}{11 \cdot 5!} - \ldots + (-1)^n \frac{x^{4n-1}}{(4n-1)(2n-1)!} + \ldots \right\} \tag{30}$$

$$C_1(x) = (2/\pi)^{\frac{1}{2}} \left\{ x - \frac{x^5}{5 \cdot 2!} + \frac{x^9}{9 \cdot 4!} - \ldots + (-1)^n \frac{x^{4n-3}}{(4n-3)(2n-2)!} + \ldots \right\} \tag{31}$$

converge for all values of x, they are unsatisfactory for calculating the Fresnel integrals when x exceeds 5, say, because of relatively slow convergence and the loss of figures resulting from partial cancellation of nearly equal terms of the alternating series. For example, when $x = 5$ a total of 60 terms of the alternating series in Equations (30) and (31) are required to yield accuracy to 30 decimal places, and nine significant figures before the decimal point are lost through cancellation. On the other hand, a total of 25 terms of the series in Equations (26) and (27) in conjunction with Equations (28) and (29) are required to give $S_1(5)$ and $C_1(5)$ to 40 decimals when 30-place approximations to the converging factors $\Pi_{\frac{49}{2}}(25)$ and $\Pi_{\frac{51}{2}}(25)$ are used, and furthermore no figures are lost through cancellation

CALCULATION OF THE CONVERGING FACTORS AND THEIR DERIVATIVES

If we write Equation (22) in the form

$$C_n(x^2) = \Pi_{n-\frac{1}{2}}(x^2) + i\,\Omega_{n-\frac{1}{2}}(x^2) \tag{32}$$

then we have the relation

$$\Omega_s(z) = \frac{z}{\Gamma(s+1)} \int_0^\infty \frac{t^{s+1}\, e^{-t}}{t^2 + z^2}\, dt \quad (s > -2) \tag{33}$$

$$= \frac{s+1}{z}\, \Pi_{s+1}(z) \quad , \tag{34}$$

where $\Pi_s(z)$ is given by Equation (21).

To derive a similar relation between $\Pi_s(z)$ and $\Omega_{s+1}(z)$, we proceed as follows:

$$\Pi_s(z) = \frac{z^2}{\Gamma(s+1)} \int_0^\infty \frac{t^s\, e^{-t}}{t^2 + z^2}\, dt \qquad (s > -1)$$

$$= 1 - \frac{1}{\Gamma(s+1)} \int_0^\infty t^s e^{-t}\, dt + \frac{z^2}{\Gamma(s+1)} \int_0^\infty \frac{t^s\, e^{-t}}{t^2 + z^2}\, dt \tag{35}$$

$$= 1 - \frac{1}{\Gamma(s+1)} \int_0^\infty \frac{t^{s+2}\, e^{-t}}{t^2 + z^2}\, dt \tag{36}$$

$$= 1 - \frac{s+1}{z}\, \Omega_{s+1}(z) \quad , \tag{37}$$

which is the desired relation.

We next derive relations between the converging factors $\Pi_s(z)$ and $\Omega_s(z)$ and their derivatives.

Differentiating both sides of Equation (21) with respect to z, we obtain

$$\frac{d}{dz}\Pi_s(z) = \frac{2z}{\Gamma(s+1)}\int_0^\infty \frac{t^{s+2}\,e^{-t}}{(t^2+z^2)^2}\,dt \tag{38}$$

Integration by parts then yields

$$2\int_0^\infty \frac{t^{s+2}\,e^{-t}}{(t^2+z^2)^2}\,dt = -\frac{t^{s+1}\,e^{-t}}{t^2+z^2}\Bigg|_0^\infty + \int_0^\infty \frac{(s+1)t^s\,e^{-t} - t^{s+1}\,e^{-t}}{t^2+z^2}\,dt$$

$$= \int_0^\infty \frac{(s+1-t)\,t^s\,e^{-t}}{t^2+z^2}\,dt$$

Hence,

$$\frac{d}{dz}\Pi_s(z) = \frac{z}{\Gamma(s+1)}\int_0^\infty \frac{(s+1-t)\,t^s\,e^{-t}}{t^2+z^2}\,dt \tag{39}$$

$$= \frac{(s+1)\,z}{\Gamma(s+1)}\int_0^\infty \frac{t^s\,e^{-t}}{t^2+z^2}\,dt - \frac{z}{\Gamma(s+1)}\int_0^\infty \frac{t^{s+1}\,e^{-t}}{t^2+z^2}\,dt$$

By Equations (21) and (33), this implies

$$\frac{d}{dz}\Pi_s(z) = \frac{s+1}{z}\Pi_s(z) - \Omega_s(z) \quad , \tag{40}$$

which is the first of the desired relations

Similarly, if we differentiate both sides of Equation (33) with respect to z, we find

$$\frac{d}{dz}\Omega_s(z) = \frac{1}{\Gamma(s+1)}\int_0^\infty \frac{(t^2-z^2)\,t^{s+1}\,e^{-t}}{(t^2+z^2)^2}\,dt \tag{41}$$

Integration by parts then yields

$$\int_0^\infty \frac{(t^2-z^2)\, t^{s+1}\, e^{-t}}{(t^2+z^2)^2}\, dt = \tfrac{1}{2}\int_0^\infty \frac{[(s+2)t^2 - t^3 + z^2 t - sz^2]\, t^{s-1}\, e^{-t}}{t^2+z^2}\, dt$$

$$= \tfrac{1}{2}\int_0^\infty \frac{(s+1-t)\, t^{s+1}\, e^{-t}}{t^2+z^2}\, dt$$

$$+ \tfrac{1}{2}\int_0^\infty \frac{(t^2 + z^2 t - sz^2)\, t^{s-1}\, e^{-t}}{t^2+z^2}\, dt$$

$$= \int_0^\infty \frac{(s+1-t)\, t^{s+1}\, e^{-t}}{t^2+z^2}\, dt \quad ,$$

since

$$\int_0^\infty \frac{(t^2 + z^2 t - sz^2)\, t^{s-1}\, e^{-t}}{t^2+z^2}\, dt = \int_0^\infty \frac{(s+1-t)\, t^{s+1}\, e^{-t}}{t^2+z^2}\, dt$$

$$- \int_0^\infty (s-t)\, t^{s-1}\, e^{-t}\, dt.$$

and

$$\int_0^\infty (s-t)\, t^{s-1}\, e^{-t}\, dt = s\int_0^\infty t^{s-1}\, e^{-t}\, dt - \int_0^\infty t^s\, e^{-t}\, dt$$

$$= s\,\Gamma(s) - \Gamma(s+1) = 0.$$

Therefore, we have

$$\frac{d}{dz}\,\Omega_s(z) = \frac{1}{\Gamma(s+1)}\int_0^\infty \frac{(s+1-t)\, t^{s+1}\, e^{-t}}{t^2+z^2}\, dt \qquad (42)$$

$$= \frac{1}{\Gamma(s+1)}\int_0^\infty \frac{[(s+1)t + z^2]\, t^s\, e^{-t}}{t^2+z^2}\, dt - \frac{1}{\Gamma(s+1)}\int_0^\infty t^s\, e^{-t}\, dt$$

$$= \frac{(s+1)}{\Gamma(s+1)}\int_0^\infty \frac{t^{s+1}\, e^{-t}}{t^2+z^2}\, dt + \frac{z^2}{\Gamma(s+1)}\int_0^\infty \frac{t^s\, e^{-t}}{t^2+z^2}\, dt - 1,$$

whence

$$\frac{d}{dz}\,\Omega_s(z) = \frac{s+1}{z}\,\Omega_s(z) + \Pi_s(z) - 1 \;, \tag{43}$$

which is the second desired relation between the converging factors and their derivatives.

To obtain similar relations for the second derivatives, we differentiate both sides of Equations (40) and (43) after multiplying by z, and find

$$z\,\frac{d^2}{dz^2}\,\Pi_s(z) = s\,\frac{d}{dz}\,\Pi_s(z) - z\,\frac{d}{dz}\,\Omega_s(z) - \Omega_s(z) \quad , \tag{44}$$

$$z\,\frac{d^2}{dz^2}\,\Omega_s(z) = s\,\frac{d}{dz}\,\Omega_s(z) + z\,\frac{d}{dz}\,\Pi_s(z) + \Pi_s(z) - 1 \quad . \tag{45}$$

If we let d_j and δ_j represent, respectively, the values of $\frac{d^j}{dz^j}\,\Pi_s(z)$ and $\frac{d^j}{dz^j}\,\Omega_s(z)$ when $z = s$, then Equations (40), (43), (44), and (45) reduce to

$$d_1 = (1 + \tfrac{1}{s})\,d_0 - \delta_0 \tag{46}$$

$$\delta_1 = (1 + \tfrac{1}{s})\,\delta_0 + d_0 - 1 \tag{47}$$

$$d_2 = d_1 - \delta_1 - \delta_0/s \tag{48}$$

$$\delta_2 = \delta_1 + d_1 + \frac{d_0 - 1}{s} \tag{49}$$

If we proceed in the same manner to find higher derivatives of the converging factors, we find that

$$d_k = (1 - \frac{k-2}{s})\,d_{k-1} - \delta_{k-1} - \frac{k-1}{s}\,\delta_{k-2} \quad , \tag{50}$$

$$\delta_k = \left(1 - \frac{k-2}{s}\right) \delta_{k-1} + d_{k-1} + \frac{k-1}{s} d_{k-2} \quad , \tag{51}$$

when $k \geq 3$.

Thus we can systematically evaluate all the d_k and δ_k once we know the values of $d_0 = \Pi_s(s)$ and $\delta_0 = \Omega_s(s)$. Then we can write at once the Taylor series

$$\Pi_s(s+h) = d_0 + d_1 h + \frac{d_2}{2!} h^2 + \frac{d_3}{3!} h^3 + \ldots \tag{52}$$

and

$$\Omega_s(s+h) = \delta_0 + \delta_1 h + \frac{\delta_2}{2!} h^2 + \frac{\delta_3}{3!} h^3 + \ldots , \tag{53}$$

which permit the evaluation of the converging factors in the neighborhood if a given argument s.

The calculation of the extensive tables in this report was performed in the following manner. For large values of s the following asymptotic series were available:

$$2\,\Pi_s(s) \doteq 1 - \frac{1}{2s} + \frac{1}{(2s)^2} + \frac{3}{(2s)^3} - \frac{55}{(2s)^4} + \frac{599}{(2s)^5}$$
$$- \frac{5823}{(2s)^6} + \frac{49595}{(2s)^7} - \frac{266743}{(2s)^8} - \frac{2679473}{(2s)^9} + \ldots , \tag{54}$$

$$2\,\Omega_s(s) = 1 - \frac{1}{2s} + \frac{3}{(2s)^2} - \frac{13}{(2s)^3} + \frac{59}{(2s)^4} - \frac{185}{(2s)^5}$$
$$- \frac{1309}{(2s)^6} + \frac{45387}{(2s)^7} - \frac{832613}{(2s)^8} + \frac{12609823}{(2s)^9} - \ldots \quad . \tag{55}$$

Indeed, the first 60 coefficients of each of these series have been tabulated in an earlier report[7] by the present authors.

For the evaluation of the Fresnel integrals by means of Equations (24) - (29) it is clearly necessary to specialize s to numbers of the form $n + \frac{1}{2}$, where n is an integer.

To attain final accuracy to about 35 decimal places from 60 terms of the series in Equations (54) and (55), it was found necessary to take $s \geqq 70.5$. From the values of $\Pi_s(s)$, $\Omega_s(s)$, and their derivatives thus calculated on a CDC 6700 system for $s = 70.5$, it was possible to calculate $\Pi_s(s-1)$ and $\Omega_s(s-1)$ by the appropriate Taylor series, and then deduce $\Pi_{s-1}(s-1)$ and $\Omega_{s-1}(s-1)$ by means of the difference equations (34) and (37).

By such a recurrent procedure the appended table of $\Pi_{s+\frac{1}{2}}(s+\frac{1}{2})$ and its reduced derivatives, $D_j = d_j/j!$ was calculated to 35 decimal places for $s = 1(1)70$, that is, for all integral values of s from 1 to 70, inclusive. The final two decimals in this table should be considered as guard figures.

In order to check the stability of this backward recurrence, the final value, namely $\Pi_{\frac{3}{2}}(\frac{3}{2})$, was calculated independently from the following power series given by Dingle[8]:

$$\Pi_s(z) = \frac{z^2}{s(s-1)}\left\{1 - \frac{z^2}{(s-2)(s-3)} + \frac{z^4}{(s-2)(s-3)(s-4)(s-5)} - \cdots\right\}$$

$$+ \frac{\pi z^{s+1} \sin\left(z + \frac{\pi s}{2}\right)}{\Gamma(s+1)\sin \pi s} \quad . \tag{56}$$

Setting $z = s = \frac{3}{2}$ and evaluating 21 terms of the series to more than 40 decimal places, we obtain

$$\Pi_{\frac{3}{2}}\left(\tfrac{3}{2}\right) = 0.38103\ 27723\ 47441\ 35241\ 84636\ 04433\ 15865\ 71377 \ ,$$

which is less than the tabulated values by about $4.3 \cdot 10^{-34}$. This serves to confirm that the tabular entries should be considered consistently accurate to 33 decimals.

Only a portion of the companion table of $\Omega_{s-\frac{1}{2}}(s-\frac{1}{2})$ is reproduced herein; namely, a tabulation of that converging factor to 33 decimal places for $s = 1(1)70$. This limitation does not detract from the practical utility of this report, inasmuch as the converging factor $\Pi_{s-\frac{1}{2}}(s-\frac{1}{2})$ and its reduced derivatives are all that are required to calculate the Fresnel integrals from Equations (26) - (29).

Because of the relation of $\Pi_s(z)$ and $\Omega_s(z)$ to certain definite integrals, as shown in Equations (21) and (33), it is considered useful also to reproduce tables of $\Pi_{s-\frac{1}{2}}(s)$, $\Omega_{s-\frac{1}{2}}(s)$, $\Pi_{s+\frac{1}{2}}(s)$, $\Omega_{s+\frac{1}{2}}(s)$, $\Pi_{s+\frac{1}{2}}(s-\frac{1}{2})$, and $\Omega_{s+\frac{1}{2}}(s-\frac{1}{2})$, all to 33 decimal places, and for $s = 1(1)70$, except for the first two, wherein s ranges up to 71.

As a further partial check on the electronic computer calculations, the value of the converging factor $\Pi_{\frac{3}{2}}(1)$ was found to about 40 places from Equation (56) by means of a desk calculator. The result was

$$\Pi_{\frac{3}{2}}(1) = 0.25396\ 60243\ 36788\ 20750\ 56056\ 53722\ 93693\ 02532,$$

which agrees with the earlier approximation to within $2 \cdot 10^{-34}$.

For completeness, the following value of $\Pi_{\frac{1}{2}}(\frac{1}{2})$, also calculated in two ways, is recorded:

$$\Pi_{\frac{1}{2}}(\tfrac{1}{2}) = 0.26823\ 29533\ 84628\ 45377\ 84421\ 62033\ 05691\ \ldots\ .$$

The corresponding reduced derivatives were not calculated because of their excessive number with respect to convenient tabulation.

APPLICATIONS

The method of converging factors set forth in this report has been programmed and used on the CDC 6700 system in the Computation and Mathematics Department to calculate in double-precision arithmetic a table of the Fresnel integrals $S_2(x)$ and $C_2(x)$ to 28 decimal places for $x = 2(1)70$ (Tables 9 and 10) and a table of $S(x)$ and $C(x)$ for $x = 1(1)6$ (Table 11).

As a partial check, a desk calculator was used to evaluate

$$\int_1^\infty \sin v^2\, dv = P(1) \sin 1 + Q(1) \cos x ,$$

$$\int_1^\infty \cos v^2\, dv = P(1) \cos 1 - Q(1) \sin x ,$$

where

$$P(1) = \frac{1}{4} \Pi_{\frac{1}{2}}(1) \quad ,$$

$$Q(1) = \tfrac{1}{2} - \frac{3}{8} \Pi_{\frac{3}{2}}(1) \quad .$$

Then

$$S_2(1) = S_1(1) = \tfrac{1}{2} - \sqrt{\frac{2}{\pi}} \int_1^\infty \sin v^2\, dv \ ,$$

$$C_2(1) = C_1(1) = \tfrac{1}{2} - \sqrt{\frac{2}{\pi}} \int_1^\infty \cos v^2\, dv \ .$$

The numerical values thus calculated are:

$S_2(1)$ = 0.24755 82876 51610 84260 99050 14405 217,

$C_2(1)$ = 0.72170 59242 92605 08777 15858 15611 907.

As a further check, the same procedure was used to evaluate

$$\int_{\sqrt{2}}^{\infty} \sin v^2 \, dv = P(\sqrt{2}) \sin 2 + Q(\sqrt{2}) \cos 2 \quad ,$$

$$\int_{\sqrt{2}}^{\infty} \cos v^2 \, dv = P(\sqrt{2}) \cos 2 - Q(\sqrt{2}) \sin 2 \quad ,$$

where

$$P(\sqrt{2}) = \frac{\sqrt{2}}{16} [1 - \frac{15}{16} \Pi_{\frac{5}{2}}(2)] \quad ,$$

$$Q(\sqrt{2}) = \frac{\sqrt{2}}{4} [1 - \frac{3}{16} \Pi_{\frac{3}{2}}(2)] \quad .$$

Then, since

$$S_2(2) = S_1(\sqrt{2}) = \tfrac{1}{2} - \sqrt{\frac{2}{\pi}} \int_{\sqrt{2}}^{\infty} \sin v^2 \, dv$$

and

$$C_2(2) = C_1(\sqrt{2}) = \tfrac{1}{2} - \sqrt{\frac{2}{\pi}} \int_{\sqrt{2}}^{\infty} \cos v^2 \, dv \quad ,$$

we deduce the values

$S_2(2) = 0.56284\ 89062\ 30056\ 47929\ 80811\ 09137\ 254,$

$C_2(2) = 0.75330\ 23754\ 67891\ 16558\ 21899\ 71106\ 416.$

The Fresnel integrals $S_2(x)$ and $C_2(x)$ are closely related to the rocket functions introduced by Rosser et al[9] in a study of the exterior ballistics of fin-stabilized rocket projectiles. These functions are defined as the real and imaginary parts of the complex integrals

$$rc(w) = i\, e^{i\, w} \int_{w}^{\infty} \frac{e^{-ix}}{\sqrt{x}} \, dx$$

$$= rr(w) + i\, ri(w)$$

Thus, the rocket functions rr(w) and ri(w) are given by the equations

$$rr(w) = \cos w \int_w^\infty \frac{\sin x}{\sqrt{x}}\, dx - \sin w \int_w^\infty \frac{\cos x}{\sqrt{x}}\, dx \qquad (57)$$

$$ri(w) = \cos w \int_w^\infty \frac{\cos x}{\sqrt{x}}\, dx + \sin w \int_w^\infty \frac{\sin x}{\sqrt{x}}\, dx \qquad (58)$$

If we set $x = y^2$ in these integrals and use Equations (24) and (25), we deduce the relations

$$\begin{aligned} rr(w) &= 2\,Q(\sqrt{w}) \\ ri(w) &= 2\,P(\sqrt{w}) \end{aligned} \qquad (59)$$

Consequently, the rocket functions are computable as a by-product of the evaluation of the Fresnel integrals by means of the series in Equations (26) and (27).

For convenient reference, tables of the rocket functions thus calculated to 28 decimals for integer arguments from 1 to 70, inclusive, are included in this report as Tables 12 and 13.

As a final illustration of the use of the present tables of converging factors and their reduced derivatives, we evaluate $S_2(x)$ when $x = 5.24$ in order to check and extend the calculation of that value as given in the NBS Handbook.[6] We can write

$$\begin{aligned} S_2(5.24) &= S_1(\sqrt{5.24}) \\ &= \tfrac{1}{2} - \left(\tfrac{2}{\pi}\right)^{\frac{1}{2}} \left\{ P(\sqrt{5.24})\, \sin(5.24) + Q(\sqrt{5.24})\, \cos(5.24) \right\}, \end{aligned}$$

where

$$P(\sqrt{5.24}) = \frac{1}{2\sqrt{5.24}} \left\{ \frac{1}{10.48} - \frac{3 \cdot 5}{10.48^3} + \frac{3 \cdot 5 \cdot 7 \cdot 9}{10.48^5}\, \Pi_{\frac{9}{2}}(5.24) \right\},$$

$$Q(\sqrt{5.24}) = \frac{1}{2\sqrt{5.24}} \left\{ 1 - \frac{3}{10.48^2} + \frac{3 \cdot 5 \cdot 7}{10.48^4} - \frac{3 \cdot 5 \cdot 7 \cdot 9 \cdot 11}{10.48^6}\, \Pi_{\frac{11}{2}}(5.24) \right\}.$$

The converging factors $\Pi_{\frac{9}{2}}(5.24)$ and $\Pi_{\frac{11}{2}}(5.24)$ are then calculated by the Taylor series in Equation (52) from the tabulated values of $\Pi_{\frac{9}{2}}(4.5)$ and $\Pi_{\frac{11}{2}}(5.5)$ and the corresponding reduced derivatives, taking $h = 0.74$ and $h = -0.26$, respectively. Thus we calculate

$$\Pi_{\frac{9}{2}}(5.24) = 0.51578\ 34390\ 28829\ 00112\ 57204\ 80642\ 32326 \quad ,$$

$$\Pi_{11}(5.24) = 0.43799\ 18752\ 09521\ 60878\ 79477\ 87444\ 92751 \quad ,$$

whence

$$S_2(5.24) = 0.42717\ 67188\ 77837\ 56118\ 94216\ 34146\ 91721 \quad .$$

APPENDIX A

VALUES OF $\Pi_{s+\frac{1}{2}}(s+\frac{1}{2})$ AND OF ITS REDUCED DERIVATIVES

In this appendix are tabulated to 35 decimal places the values of the converging factor $\Pi_{s+\frac{1}{2}}(s+\frac{1}{2})$ and its reduced derivatives D_i, which represents $\frac{1}{i!}\frac{d^i}{dx^i}\,\Pi_{s+\frac{1}{2}}(x)$ evaluated at $x = s+\frac{1}{2}$. This table (Table 1) has been photographically reproduced from computer output that was left-justified. Accordingly, the position of the decimal point for each tabular entry is determined by the right-hand indentation.

Table 1 – Table of $\Pi_{s+1/2}(s + 1/2)$ and its Reduced Derivatives D_i to 35D for s = 1 (1) 70

S = 1.5

I	D SUB I
0	38103 27723 47441 35241 84636 04433 15909
1	22967 93497 19473 83636 71015 20252 85200
2	- 48614 52731 68446 89744 40849 93040 7999
3	30588 76947 35042 34698 01746 03308 820
4	33162 11359 54622 64195 39451 47664 525
5	- 24411 28662 68706 60575 71471 62814 799
6	12505 87515 65353 66869 88785 45789 940
7	- 57817 11878 75934 99785 70012 30683 69
8	25989 12243 27041 98840 30786 81159 97
9	- 11710 99057 39674 37524 38355 56534 08
10	53618 19256 04236 01695 84530 36451 5
11	- 25078 27723 74956 24399 09623 38396 1
12	11999 27369 89580 62254 60146 59186 7
13	- 58699 60473 35627 42718 27343 28233
14	29316 17283 31390 40832 27234 21911
15	- 14921 20206 38829 69044 26517 33280
16	77258 12753 32304 08586 04693 9759
17	- 40624 85399 39994 34877 10203 7147
18	21660 92447 38871 60069 24612 5860
19	- 11695 05540 50716 30622 31189 3800
20	63861 61437 72768 21229 69528 391
21	- 35231 04178 06305 46523 95768 092
22	19617 83183 91254 44812 69736 522
23	- 11016 85317 88912 81604 70728 667
24	62348 90906 24822 68522 27607 40
25	- 35537 38113 66283 95679 33150 89
26	20388 23086 84506 85181 24307 84
27	- 11767 68353 09452 90397 24358 64
28	68300 40120 13783 03106 57736 9
29	- 39847 44826 39627 83741 60496 6
30	23359 57119 67208 64134 37112 8
31	- 13755 42735 93821 62766 95401 2
32	81339 16160 57680 98751 58973
33	- 48286 44771 12372 54598 42174
34	28770 34043 01129 53475 44205
35	- 17201 35068 30135 86286 80225
36	10317 86586 64538 29316 32855
37	- 62079 23911 93196 62915 8558
38	37459 16585 37594 96369 6270
39	- 22665 06608 84172 76527 7564
40	13749 29189 58323 86526 5209
41	- 83612 30617 72958 99401 969
42	50965 11309 20196 77389 371
43	- 31134 29235 21615 31514 977
44	19059 96322 31305 33005 284
45	- 11691 74188 00879 50413 472
46	71857 22714 46667 09738 58
47	- 44244 40982 10071 63940 76
48	27290 29015 29828 00135 32
49	- 16861 14273 05054 83782 13
50	10434 32399 36885 52777 24
51	- 64671 26102 55424 52648 7
52	40142 18491 42049 95681 5
53	- 24952 14665 83582 81448 8
54	15531 30995 71261 73049 2
55	- 96800 79478 30211 35899
56	60408 64329 51423 54447
57	- 37744 04079 81663 28997
58	23610 64061 62744 24166
59	- 14786 28729 79003 08108
60	92701 20226 56124 9389
61	- 58179 52767 98632 6215
62	36550 90823 88658 8206
63	- 22985 53599 70982 2571
64	14468 57022 04775 3608
65	- 91158 63698 34999 834
66	57485 53656 70830 532
67	- 36282 38594 15805 006
68	22919 12569 17590 127
69	- 14489 52697 13257 504
70	91675 60962 77087 63
71	- 58047 93677 19126 89
72	36782 69367 52179 45
73	- 23324 63315 32916 02
74	14801 02654 45143 60
75	- 93986 63853 85485 9
76	59721 34042 15897 0
77	- 37972 95011 56502 8
78	24159 78224 74743 1
79	- 15380 78117 27371 0
80	97976 87110 95350
81	- 62448 48294 76393
82	39826 02684 42217
83	- 25412 80961 05420
84	16224 56528 55703
85	- 10363 88134 78975
86	66236 17847 4218
87	- 42353 20373 2015
88	27095 07201 0647
89	- 17342 13052 9730
90	11104 98348 2298
91	- 71142 98715 638
92	45597 44754 733
93	- 29237 41122 419
94	18755 25962 443
95	- 12036 18885 118
96	77273 88600 56
97	- 49630 72063 41
98	31888 84716 58
99	- 20497 17087 49
100	13179 91025 91
101	- 84779 54530 0
102	54553 99768 0
103	- 35116 87110 0
104	22612 87281 9
105	- 14566 10410 3
106	93859 00716
107	- 60499 33435
108	39008 98289
109	- 25160 27710
110	16233 06104
111	- 10476 52248
112	67633 7063
113	- 43675 3292
114	28211 9992
115	- 18228 6283
116	11781 3322
117	- 76164 569
118	49252 434
119	- 31857 855
120	20611 876
121	- 13339 170
122	86347 25
123	- 55908 11
124	36208 12
125	- 23455 26
126	15197 63
127	- 98494 4
128	63847 7
129	- 41397 6
130	26847 3
131	- 17414 8
132	11298 7
133	- 73322
134	47591
135	- 30896
136	20061
137	- 13029
138	8463
139	- 5498
140	3573
141	- 2322
142	1509
143	- 981
144	638
145	- 415
146	270
147	- 175
148	114
149	- 74
150	48
151	- 31
152	20
153	- 13
154	8

S = 2.5

I	D SUB I
0	41936 11674 52711 23170 45662 85386 16533
1	15398 90023 97888 40062 78223 35595 30714
2	- 22491 05135 97280 06921 08539 24462 2097
3	82887 58073 31137 57420 52994 12907 78
4	72338 71976 01032 39715 72693 05199 11
5	- 32081 60341 96448 04056 20782 09620 52
6	96088 26434 62492 00928 75337 15773 0
7	- 24853 60552 96852 46925 14666 74001 4
8	59842 88039 33192 82923 31204 07040
9	- 13880 47844 65775 38247 35238 23804
10	31574 24266 31235 13026 59186 1169
11	- 71080 02735 44344 53516 86485 205
12	15885 91399 35215 54536 30980 610
13	- 35178 70852 39920 65892 37128 92
14	76594 93420 17573 59149 91058 9
15	- 16125 79740 16098 12133 17785 7
16	31668 97636 81926 32173 16416
17	- 52750 42009 63613 03284 9032
18	46783 06970 31325 83697 052
19	16731 28172 95156 43323 161
20	- 13906 06201 87123 27847 381
21	68506 89105 49864 55886 30
22	- 28938 07828 47601 19087 79
23	11391 06976 14490 70266 14
24	- 43154 80845 97097 58531 0
25	15983 69620 66375 70261 7
26	- 58381 64184 63789 88687
27	21139 19096 61082 42783
28	- 76128 94133 45245 7973
29	27328 12961 44949 6765
30	- 97929 70984 57778 196
31	35068 07230 32692 161
32	- 12557 92934 48769 895
33	44994 27761 76360 90
34	- 16135 76544 58625 56
35	57933 34630 32929 2
36	- 20828 33293 97721 0
37	74998 33019 74058
38	- 27043 94415 96390
39	97683 22935 9135
40	- 35341 55154 8150
41	12807 72922 5051
42	- 46492 08874 885
43	16904 46665 219
44	- 61564 62154 20
45	22457 30803 52
46	- 82048 10799 5
47	30022 73366 7
48	- 11002 41989 6
49	40380 10899
50	- 14841 32568
51	54624 6098
52	- 20132 5576
53	74300 053
54	- 27456 405
55	10158 923
56	- 37634 52
57	13958 72
58	- 51833 6
59	19269 5
60	- 71714
61	26718
62	- 9964
63	3720
64	- 1390
65	520
66	- 194
67	72
68	- 27
69	10
70	- 3
71	1

S = 3.5

I	D SUB I
0	43894 31929 07890 69179 18511 13098 73086
1	11652 21877 61406 80671 53825 76766 59592
2	- 13079 56012 68424 97337 30367 41430 7225
3	32487 00151 03295 23671 36057 93655 20
4	25257 09606 72633 36878 44374 87799 41
5	- 79817 17047 71895 82256 03337 13306 0
6	16667 33890 89098 75364 98003 33273 6
7	- 28864 42781 67650 38169 03495 53383
8	43969 00456 34239 58863 77005 8308
9	- 59215 10631 19603 47661 73357 715
10	66768 25892 33004 68493 10636 66
11	- 47240 42427 54915 27017 78990 2
12	- 44628 11543 60063 52201 43405
13	31145 82927 57432 86754 09799
14	- 98249 42647 29492 96629 0636
15	25663 34402 73941 54913 4207
16	- 61890 87040 91270 45885 737
17	14346 81729 17119 07127 017
18	- 32589 23173 66465 13747 28
19	73309 53941 05434 10322 1
20	- 16432 75051 16189 33136 6
21	36844 82766 48930 42118
22	- 82830 75484 93600 3583
23	18698 19816 84906 6876
24	- 42422 32921 53885 065
25	96784 11350 78431 69
26	- 22209 87913 40814 00
27	51270 12909 15221 2
28	- 11905 63352 87789 1
29	27808 09313 04185
30	- 65322 02862 8633
31	15429 20168 9572
32	- 36638 70842 582
33	87450 67894 43
34	- 20976 08839 01
35	50551 77947 6
36	- 12238 07119 4
37	29755 72207
38	- 72648 7889
39	17807 7595
40	- 43816 662
41	10820 516
42	- 26814 41
43	66670 9
44	- 16630 0
45	41608
46	- 10441
47	2627
48	- 662
49	167
50	- 42
51	10
52	- 2

S = 4.5

I	D SUB I
0	45086 58915 06093 77010 31417 69925 68262
1	93955 92374 27802 84361 60596 25584 5354
2	- 85844 86396 87968 02438 62814 05419 200
3	15511 94228 21310 06622 68956 14394 19
4	11206 21010 41835 22913 54218 45923 45
5	- 27428 19836 76937 86333 90775 98890 0
6	43580 53953 35722 39852 37622 10695
7	- 55263 81066 85023 59533 47601 0952
8	57353 35587 37589 16594 39197 036
9	- 43779 48838 47853 01943 72571 72
10	68994 84533 86429 29421 26604
11	63607 20262 86860 60091 73429
12	- 18246 40461 30650 69238 71007
13	37141 61270 11729 09950 1326
14	- 66321 54813 70064 68446 254
15	11077 52948 02002 96560 090
16	- 17815 96649 55151 27417 40
17	28029 62137 45019 71580 3
18	- 43553 69750 98828 12075
19	67252 34667 33480 8617
20	- 10361 86092 78246 0940
21	15973 54399 97534 021
22	- 24681 97229 09904 21
23	38270 85378 35954 7
24	- 59586 85475 63877
25	93186 50522 6315
26	- 14637 93684 7620
27	23090 69599 675
28	- 36563 50339 18
29	58084 56210 6
30	- 92501 90062
31	14754 10968
32	- 23542 3358
33	37526 946
34	- 59650 83
35	94333 0
36	- 14795 7
37	22916
38	- 3482
39	514
40	- 72
41	9
42	- 1

S = 5.5

I	C SUB I
0	45889 48235 63063 59858 51194 78075 16545
1	78811 18671 53206 69345 53973 92499 8066
2	- 60775 40683 24952 77688 98304 32337 938
3	83945 97753 13684 77803 68114 35016 5
4	57611 12344 50173 13656 18947 04432 7
5	- 11483 70438 92333 01757 97445 59431 7
6	14636 82382 95230 88581 21140 17122
7	- 14350 08502 79691 53191 39243 9146
8	10513 98360 64104 52834 17243 834
9	- 35555 16958 37194 51269 23514 6
10	- 58147 69323 87888 55754 82213
11	16807 33400 70046 88054 73687
12	- 28682 84715 21538 52269 7230
13	40809 49518 92264 27215 489
14	- 52664 42669 41571 83800 50
15	63796 33554 71490 72749 3
16	- 73761 24753 05895 59217
17	82036 51563 65925 2208
18	- 87910 57924 39464 170
19	90338 46663 48593 73
20	- 87744 62772 35212 0
21	77744 12894 66954
22	- 56738 57178 9818
23	19321 55675 821
24	42603 88017 65
25	- 14114 26194 79
26	29438 13542 7
27	- 52935 54177
28	88652 0533
29	- 14265 1110
30	22403 816
31	- 34651 67
32	53075 9
33	- 80802
34	12257
35	- 1855
36	280
37	- 42
38	6

S = 6.5

I		C SUB I						
0		46467	10459	60702	60960	77533	27010	60107
1		67918	77247	32650	66518	59864	51761	1473
2	-	45331	42670	90097	79036	50707	20062	161
3		49573	94896	04724	11004	97246	91546	0
4		32748	82104	36866	55415	75156	20947	8
5	-	54984	00245	77671	94570	25729	55173	
6		58253	54303	48565	57790	85767	4721	
7	-	45813	35648	11586	08852	77999	636	
8		24007	73117	37046	94984	71649	28	
9		66581	79499	81970	37489	4722		
10	-	23190	99816	53656	36930	32464		
11		40536	30743	21604	24556	9814		
12	-	51456	27624	10084	54838	176		
13		55991	63966	34534	32638	61		
14	-	54936	21593	17437	37186	1		
15		49403	89434	12930	37800			
16	-	40534	55914	42897	6133			
17		29326	32219	13755	560			
18	-	16561	63534	47724	38			
19		27932	95008	35381				
20		11637	15470	62676				
21	-	26567	68649	1140				
22		41982	36339	622				
23	-	57982	40414	57				
24		74764	39671	6				
25	-	92606	30744					
26		11186	09680					
27	-	13295	6408					
28		15640	246					
29	-	18280	33					
30		21287	5					
31	-	24747						
32		2876						
33	-	334						
34		38						
35	-	4						

S = 7.5

I	D SUB I
0	46902 61333 89080 60727 89816 84008 33441
1	59696 99041 70206 23871 74138 03188 8353
2	- 35130 13298 96812 13650 97464 16430 368
3	31233 05734 36702 78694 45826 56018 7
4	20031 22373 37575 32590 01539 42945 8
5	- 29021 58556 42354 06937 46220 59776
6	26222 80317 54525 66562 96259 5002
7	- 16992 24012 05768 26633 49380 856
8	63360 93704 44442 08220 95565 6
9	25264 73432 59913 28915 84692
10	- 82478 60640 83003 45898 9508
11	10815 13372 07546 40121 7053
12	- 10910 79984 37128 26765 460
13	94200 53818 64832 70252 9
14	- 71423 75358 35513 68887
15	46714 68367 43569 0097
16	- 23814 06628 66197 732
17	46454 42232 39364 7
18	10150 81446 82724 3
19	- 20722 14899 44040
20	27623 16047 6652
21	- 31558 97016 084
22	33232 04786 42
23	- 33263 71500 7
24	32163 90973
25	- 30328 9329
26	28053 295
27	- 25547 02
28	22953 6
29	- 20366
30	1784
31	- 154
32	13
33	- 1

S = 8.5

I		D SUB I
0		47242 69817 76795 71806 92806 70488 03952
1		53265 34520 87660 09084 18659 36688 6269
2	-	28033 74494 07026 32582 70868 14132 506
3		20688 03706 33617 56173 93051 33203 9
4		12957 61827 50562 88766 91317 84389 3
5	-	16498 02087 65635 05575 44500 00560
6		12962 98105 22579 16070 83609 6463
7	-	70643 14452 72329 09423 29992 90
8		18272 53879 27694 98060 48232 1
9		15461 63232 18969 02198 52612
10	-	30565 39744 85393 22045 6069
11		32330 64259 37894 70577 765
12	-	26814 66148 63848 54172 34
13		18723 68082 86641 79872 2
14	-	10922 38175 52661 68405
15		47364 93303 00445 813
16	-	48386 73834 53079 0
17	-	20487 30278 33310 1
18		32754 18839 87454
19	-	36229 14810 8473
20		34452 20122 954
21	-	30003 23969 68
22		24579 37394 9
23	-	19188 21306
24		14353 4050
25	-	10287 579
26		70206 3
27	-	44868
28		2579
29	-	118
30		1

S = 9.5

I		D SUB I
0		47515 60994 08877 68463 22570 58637 82620
1		48093 60203 12686 86965 12084 29958 9565
2	-	22895 47756 42708 25920 47818 25429 443
3		14262 20723 93289 98864 06893 23376 5
4		87610 04389 19971 02680 19615 55133
5	-	99435 01784 85844 14875 77456 3991
6		68984 29309 91922 89757 29419 966
7	-	32133 00718 03550 07347 69084 28
8		54470 67983 37886 91438 35015
9		81928 11231 84507 61926 5636
10	-	12092 02826 52497 21362 5332
11		10713 78232 02262 54107 262
12	-	74624 97565 72116 50430 3
13		42548 21440 74009 41221
14	-	18566 84242 28675 2990
15		36941 08542 24488 45
16		39148 97973 63778 5
17	-	67388 58272 22938
18		68862 76408 5001
19	-	58154 39803 419
20		43919 84780 15
21	-	30537 92493 4
22		19733 94176
23	-	11799 3552
24		63754 57
25	-	28968 0
26		8093
27		343
28	-	90
29		11
30	-	1

S = 10.5

I		O SUB I
0		47739 44910 59762 68163 38870 05951 50018
1		43842 88144 31704 49508 62408 06277 9301
2	-	19054 18869 17939 94497 71356 19078 925
3		10159 26389 19042 96065 06752 81334 0
4		61400 45805 46975 38412 20722 99201
5	-	62835 00628 80443 25134 78014 8680
6		38966 19282 82155 71759 98144 475
7	-	15720 23298 23347 29500 00533 70
8		15429 30441 82568 96671 58824
9		42766 07821 82379 77477 9338
10	-	51084 10025 57477 10561 098
11		38829 11138 40750 94410 55
12	-	23052 22791 89323 43243 5
13		10759 71740 49208 58570
14	-	32734 21726 72303 572
15	-	39699 38850 20826 1
16		17011 16712 59134 6
17	-	18052 62875 02838
18		14417 19394 7608
19	-	99072 57434 16
20		60973 20024 2
21	-	33857 02903
22		16628 9074
23	-	66841 43
24		14960 2
25		8625
26	-	1681
27		174
28	-	15
29		1

S = 11.5

I	C SUB I
0	47926 34991 27184 42689 91060 27565 77584
1	40286 24801 45986 14379 94389 46925 2037
2	- 16106 50030 56866 11054 70867 95160 272
3	74369 27378 34768 45141 23861 94542
4	44330 86425 03566 44466 26795 36307
5	- 41290 99916 12438 95464 60675 3484
6	23121 30448 21446 14076 36564 567
7	- 81681 63937 54659 48453 91079 6
8	32526 21485 01285 60411 0175
9	22722 95275 85691 98283 7878
10	- 22924 14156 24606 63812 512
11	15203 09738 40477 96343 21
12	- 77770 60280 28197 56722
13	29599 42531 49560 7352
14	- 52222 97262 57565 31
15	- 41008 33180 46983 8
16	59104 47734 25946
17	- 48506 11698 2963
18	32021 31134 454
19	- 18306 33015 69
20	91902 09419
21	- 39368 0630
22	12618 842
23	- 81050
24	- 33095
25	3958
26	- 332
27	24
28	- 1

S = 12.5

I	C SUB I
0	48084 75169 39036 32322 26323 07039 14934
1	37265 89481 89835 98332 22875 33823 5739
2	- 13794 76675 62822 82791 65976 99595 183
3	55715 60625 89843 94465 19309 10423
4	32819 12701 16245 66635 01259 35459
5	- 28041 34033 37977 46658 84335 3059
6	14299 11296 41237 64388 88683 861
7	- 44648 11481 29736 66388 74403 0
8	- 38731 16274 88861 22260 468
9	12409 51954 28834 25221 1571
10	- 10858 96807 09024 89357 102
11	63644 06940 69033 84833 2
12	- 28282 76590 23174 27131
13	86786 53290 50228 775
14	- 36473 80695 93255 3
15	- 20194 95460 38163 1
16	20090 07856 91966
17	- 13650 83519 4992
18	76328 68787 33
19	- 36543 87291 2
20	14685 85758
21	- 43512 481
22	23311 2
23	98500
24	- 10591
25	796
26	- 51
27	3

S = 13.5

I		D SUE I
0		48220 70575 85944 92382 66929 31624 18354
1		34668 62688 43058 83814 42100 82248 7344
2	-	11948 07575 51119 47438 08500 55145 335
3		42579 13456 06036 26810 72551 56414
4		24822 93057 37845 70074 57475 22773
5	-	19585 40891 39989 12547 57094 4106
6		91606 66567 16193 74116 33933 69
7	-	25484 44733 08452 12904 43133 2
8	-	12635 31661 01984 16796 7217
9		69813 37425 42592 25616 715
10	-	53972 73146 11871 37986 11
11		28241 50944 59857 05270 8
12	-	10970 80366 50805 26101
13		26569 31037 14010 548
14		30339 55719 62188 5
15	-	88580 73002 92983
16		70016 00342 4830
17	-	40651 26204 918
18		19489 05233 89
19	-	77860 93326
20		23822 4600
21	-	30590 79
22	-	29024 8
23		34066
24	-	2457
25		146
26	-	7

S = 14.5

I		C SUB I
0		48338 66416 01180 76585 70931 22682 56731
1		32411 07551 91487 91417 87604 61886 7918
2	-	10449 38264 66790 76030 20481 35140 326
3		33107 11177 42626 17740 74144 74828
4		19126 17549 17237 98273 74970 08415
5	-	14014 98058 79684 51675 68326 3628
6		60501 35047 34833 46792 82334 19
7	-	15100 36380 43632 21362 18617 3
8	-	12761 64371 21261 38928 2381
9		40436 99324 38136 51902 220
10	-	27998 75088 50887 58970 99
11		13189 03767 76254 88023 6
12	-	44998 89349 69955 8888
13		82878 10783 54322 70
14		26495 62094 27389 0
15	-	38173 57052 31625
16		25373 27361 5413
17	-	12814 32735 656
18		52975 51261 7
19	-	17409 89893
20		35668 434
21		61166 1
22	-	12584 6
23		9441
24	-	546
25		27
26	-	1

S = 15.5

I		D SUB I
0		48441 97518 51353 88966 23874 13962 56125
1		30430 50700 03855 29141 25727 50422 1015
2	-	92163 24634 11049 89729 41640 00217 28
3		26135 43585 70101 63737 01104 41592
4		14977 41986 87156 77351 22878 64545
5	-	10243 19567 08766 09271 09622 7020
6		41032 56815 90189 12822 31425 43
7	-	92444 12797 57485 86175 88936
8	-	10599 81048 99247 56630 2636
9		24076 09883 30194 79705 748
10	-	15089 38596 84145 43263 98
11		64438 09541 41417 09717
12	-	19377 12166 98927 5838
13		25388 19715 31373 34
14		16213 16138 00720 3
15	-	16660 45784 65772
16		95957 77218 379
17	-	42630 56564 12
18		15214 43409 7
19	-	39914 5442
20		33480 96
21		46439 8
22	-	42638
23		2528
24	-	123
25		5

S = 16.5

I		D SUB I
0		48533 20537 73463 79624 65494 25330 62195
1		28678 75689 04454 44616 51173 37614 8165
2	-	81895 67396 73422 03288 87626 25886 17
3		20910 44035 16602 39883 21900 40295
4		11897 25883 49116 27131 64978 01552
5	-	76271 44640 91728 22843 11913 421
6		28485 98244 58483 36247 17763 06
7	-	58245 63107 97774 65370 07942
8	-	82084 09221 55265 90451 607
9		14707 00089 79260 28112 211
10	-	84147 15460 68505 12331 0
11		32771 98922 78179 89061
12	-	87071 14468 17490 068
13		70763 65428 10028 0
14		90559 36265 33083
15	-	74466 97040 4028
16		37858 34053 009
17	-	14906 32997 10
18		45792 37790
19	-	90337 064
20	-	81172 6
21		20476 6
22	-	13744
23		686
24	-	28
25		1

S = 17.5

I		C SUB I
0		48614 35504 44326 48943 38349 68960 08404
1		27118 24311 06668 16928 12804 80939 8446
2	-	73254 81342 18782 84311 02255 60034 47
3		16931 21257 39821 62095 67061 84741
4		95711 92165 85882 80689 59158 1540
5	-	57737 77300 83019 16421 66611 128
6		20189 37381 67895 42741 40880 54
7	-	37647 75981 93754 74445 74003
8	-	61741 58623 70635 73437 895
9		91989 90890 81194 22340 95
10	-	48389 14868 20666 55988 1
11		17276 49736 21197 07792
12	-	40617 81552 06424 300
13		13698 78284 18333 7
14		49264 38980 97862
15	-	34224 83961 4566
16		15552 53080 274
17	-	54546 46247 7
18		14318 57064
19	-	18584 651
20	-	78957 1
21		80814
22	-	4439
23		191
24	-	6

S = 18.5

I	D SUB I
0	48687 00620 64300 26073 03943 26999 22367
1	25719 20262 06596 24176 94445 30843 5504
2	- 65914 00451 49561 16623 39923 05422 64
3	13857 02209 60570 73333 93677 43050
4	77877 71081 22831 24655 88027 0267
5	- 44357 24563 20099 99935 94624 433
6	14576 04176 95721 78855 55039 55
7	- 24896 10707 68033 05769 67439
8	- 45911 01604 65784 72195 917
9	58805 86831 89685 88079 88
10	- 28609 23769 22762 72754 6
11	94066 72790 77110 6277
12	- 19582 89790 87316 459
13	- 24816 99785 79688
14	26740 90492 93316
15	- 16189 77820 3696
16	66363 95141 84
17	- 20800 71948 9
18	46059 1092
19	- 25369 36
20	- 43099 1
21	31093
22	- 1463
23	55
24	- 1

S = 19.5

I		C SUB I						
0		48752	42646	66812	68106	46799	16048	30796
1		24457	75520	28879	27905	52701	27555	3921
2	-	59624	72146	53928	72272	63173	93303	39
3		11451	26562	82975	45034	77786	36026	
4		64016	98278	24746	39113	42194	8285	
5	-	34532	45255	16498	67436	23626	905	
6		10699	53785	13079	22810	25701	99	
7	-	16805	21251	53354	28758	38940		
8	-	34045	51543	38601	76019	719		
9		38354	16138	16927	76211	39		
10	-	17345	82050	95013	91620	3		
11		52735	22137	40364	2411			
12	-	97196	27705	46684	18			
13	-	57269	02544	30098				
14		14638	02271	06335				
15	-	78790	99956	847				
16		29337	93771	96				
17	-	82333	12044					
18		15058	8381					
19		52052	1					
20	-	20689	1					
21		12012						
22	-	496						
23		16						

S = 20.5

I		C SUB I						
0		48811	64340	01821	47949	46525	04359	16097
1		23314	51622	98374	65026	41433	78876	8438
2	-	54195	15538	90412	89991	86497	23602	29
3		95465	79200	78472	42246	76639	3126	
4		53111	67531	80497	41442	41736	1467	
5	-	27207	86755	05287	25322	30838	826	
6		79726	27375	27951	78838	85774	4	
7	-	11556	45763	73498	68347	80312		
8	-	25294	02556	92443	88992	140		
9		25481	74472	53653	22963	96		
10	-	10760	64922	74848	02324	5		
11		30359	15306	71987	7599			
12	-	49488	78595	77903	64			
13	-	52468	06458	48200				
14		81210	33329	3652				
15	-	39407	93323	030				
16		13402	71854	15				
17	-	33700	09378					
18		49199	359					
19		75646	9					
20	-	95464						
21		4717						
22	-	173						
23		5						

S = 21.5

I		D SUB I
0		48865 49878 05683 56857 82456 79828 24780
1		22273 58480 34417 96307 02136 54754 9473
2	-	49475 35079 52924 32017 12251 07898 77
3		80225 85522 66153 48715 65902 5148
4		44435 84044 12780 64381 42203 1334
5	-	21671 47053 74806 45347 19001 640
6		60221 01540 24300 23406 81535 2
7	-	80823 74211 86321 41677 0958
8	-	18875 48673 63865 51936 037
9		17220 64917 28078 87309 81
10	-	68168 49035 99421 99187
11		17905 80633 08362 6505
12	-	25766 25033 11682 66
13	-	39950 38886 86945
14		45767 04967 6621
15	-	20229 09951 847
16		63122 46042 2
17	-	14214 09958
18		15614 669
19		52487 9
20	-	43701
21		1893
22	-	62
23		1

S = 22.5

I		D SUB I
0		48914 68874 45833 82518 25565 84814 31540
1		21321 79375 07072 69874 29197 57864 0970
2	-	45346 69876 73344 15354 31707 40702 22
3		67913 84088 66382 21163 39076 5424
4		37463 55227 46406 06605 94164 7060
5	-	17433 90614 01489 91161 22036 808
6		46055 72000 79628 60575 86310 4
7	-	57404 86516 06043 32399 8719
8	-	14168 09321 19095 05536 483
9		11822 61124 95562 05734 01
10	-	44023 03311 51047 38054
11		10797 69656 08716 5309
12	-	13676 43004 09638 90
13	-	28344 48295 18837
14		26223 88307 9320
15	-	10642 15492 682
16		30580 18416 8
17	-	61572 0317
18		45342 61
19		31018 6
20	-	20125
21		778
22	-	23

S = 23.5

I		G SUB I
0		48959 79397 18303 71791 36705 71551 70628
1		20448 14560 35133 12570 11460 26110 7035
2	-	41714 38533 25392 99221 66571 38715 05
3		57878 99444 44110 20419 97200 4252
4		31808 26105 43134 97811 57513 9789
5	-	14153 09803 97092 37296 40198 327
6		35624 83090 92351 21019 54189 9
7	-	41351 98192 21888 65722 7957
8	-	10704 88480 58518 17937 836
9		82359 96894 70915 53298 2
10	-	28937 72448 91649 37008
11		66454 64109 77534 607
12	-	73791 30846 35812 0
13	-	19491 42337 93340
14		15279 54126 5734
15	-	57295 19129 16
16		15208 17887 0
17	-	27302 9642
18		99498 2
19		17296 0
20	-	9386
21		327
22	-	8

S = 24.5

I	C SUB I
0	49001 30266 00556 38854 40515 53481 74715
1	19643 38283 85008 82223 12087 46526 4150
2	- 38501 88134 96959 13737 63010 11440 06
3	49633 43615 04192 46922 21856 0071
4	27182 09131 11281 54414 59972 4420
5	- 11586 19279 19505 35589 34817 062
6	27845 56123 29588 57771 62515 9
7	- 30178 07343 21216 26289 3249
8	- 81444 72691 04667 02261 06
9	58157 13014 78270 72692 1
10	- 19335 20228 67061 55141
11	41676 42070 39508 741
12	- 40353 30686 16044 8
13	- 13223 21860 27913
14	90507 04003 431
15	- 31523 90117 82
16	77497 31194
17	- 12353 4283
18	- 34804
19	94347
20	- 4448
21	141
22	- 3

S = 25.5

I	D SUB I
0	49039 62822 53716 24199 99901 58603 79571
1	18899 65647 87680 59664 48129 75956 7127
2	- 35646 86634 52260 73159 55055 85609 04
3	42807 14019 46528 26307 60659 1751
4	23368 04922 39101 51094 11660 5924
5	- 95583 16162 62617 14871 06455 70
6	21975 56129 70367 80067 75932 5
7	- 22289 52234 14353 79813 2323
8	- 62402 60947 48590 74924 62
9	41587 54880 73283 20910 4
10	- 13116 22671 65995 90209
11	26596 04055 37812 616
12	- 22297 89981 75245 5
13	- 89313 19030 0240
14	54476 08174 820
15	- 17702 10159 26
16	40395 24074
17	- 56840 389
18	- 25860 2
19	51173
20	- 2145
21	62
22	- 1

S = 26.5

I		D SUB I
0		49075 12308 81152 89377 51874 30444 08899
1		18210 26773 41049 40172 21870 49739 5447
2	-	33098 17493 30900 56882 85931 93576 07
3		37116 45720 25049 39612 69937 0576
4		20200 74221 59941 10829 70835 0114
5	-	79418 90775 77190 86129 39455 82
6		17498 12121 53727 47326 96974 7
7	-	16647 11338 27624 00481 1960
8	-	48148 77302 83060 53322 98
9		30090 27651 43899 94296 6
10	-	90235 23829 01566 6725
11		17248 89528 61669 773
12	-	12407 65716 84986 2
13	-	60365 94759 1594
14		33298 36567 586
15	-	10133 02467 95
16		21504 73642
17	-	26499 512
18	-	24761 1
19		27829
20	-	1053
21		28

S = 27.5

I	C SUB I
0	49108 08951 82032 33897 20984 91470 05151
1	17569 46454 88624 02805 91292 97152 2258
2	- 30813 48266 86829 05135 38998 67147 67
3	32341 79641 79207 73181 79855 8296
4	17552 80781 10218 59731 19359 6347
5	- 66427 16535 08126 42969 80603 09
6	14048 52950 58985 12463 78085 3
7	- 12562 10069 43902 08210 9704
8	- 37407 14422 81101 29878 15
9	22011 79220 26715 20920 8
10	- 62896 94708 73128 1041
11	11356 28139 38176 113
12	- 69251 69446 77650
13	- 40950 39466 6258
14	20656 27959 273
15	- 59059 03429 9
16	11675 69837
17	- 12465 788
18	- 18614 8
19	15243
20	- 526
21	13

S = 28.5

I		D SUB I
0		49138 78824 64778 19977 18700 09700 82608
1		16972 27989 79891 29705 52760 73690 7818
2	-	28757 53347 07021 48944 09263 26075 26
3		28311 59676 82033 56370 26433 8732
4		15325 23217 67976 50498 47611 7849
5	-	55904 58465 05413 88699 69418 41
6		11366 01154 65910 85072 52804 3
7	-	95711 16881 39347 59298 396
8	-	29257 07405 38504 79461 98
9		16268 43772 43561 40614 2
10	-	44380 25895 28294 0052
11		75823 42704 96532 73
12	-	38575 58185 03064
13	-	27930 76582 9445
14		12995 84706 562
15	-	35011 15587 4
16		64567 3853
17	-	58868 91
18	-	12793 2
19		8431
20	-	268
21		6

S = 29.5

I	D SUB I
0	49167 44536 10052 68293 43119 18804 73177
1	16414 40215 94685 49288 83832 83186 1482
2	- 26900 76840 23446 14045 42094 35286 69
3	24890 67627 96792 24564 83045 7524
4	13440 35727 61435 77751 14360 4350
5	- 47320 54037 88584 77617 37861 63
6	92618 58591 31274 47775 06457
7	- 73580 49304 98379 17705 063
8	- 23031 40837 08323 48988 60
9	12140 03269 53800 34160 4
10	- 31674 81426 31213 7994
11	51294 11976 21753 80
12	- 21299 77185 23151
13	- 19174 42969 7313
14	82869 31673 40
15	- 21689 86512 8
16	36325 2581
17	- 27720 53
18	- 84372
19	4715
20	- 139
21	2

S = 30.5

I		D SUB I
0		49194 25787 39522 09054 56681 43791 38795
1		15892 07037 18679 71696 85337 24157 9215
2	-	25218 25517 23653 48601 54785 65669 13
3		21971 66932 01357 43096 30471 3921
4		11836 77146 88259 73713 66052 1372
5	-	40270 83710 96774 24400 31375 28
6		75979 78453 64706 10367 39921
7	-	57044 27245 74506 04242 353
8	-	18244 09512 28309 89896 40
9		91416 41982 11723 07902
10	-	22850 26939 25017 2524
11		35129 37818 69060 36
12	-	11539 77042 27992
13	-	13256 78905 4536
14		53523 04498 51
15	-	12897 18678 9
16		20768 0054
17	-	12888 61
18	-	54546
19		2668
20	-	73
21		1

S = 31.5

I		C SUB I
0		49219 39824 86694 45157 96516 24822 05925
1		15401 98897 23054 23654 54078 56977 4889
2	-	23688 84525 14889 79822 76505 61130 97
3		19468 66782 42689 80636 24073 9391
4		10465 53604 07627 40502 70025 6410
5	-	34444 80213 26446 71638 06403 33
6		62722 60162 38365 01192 61642
7	-	44574 21513 33362 37378 130
8	-	14538 92700 83491 99886 39
9		69426 84901 81015 52586
10	-	16650 93794 66917 3036
11		24338 09657 09325 03
12	-	60311 00751 5827
13	-	92335 18801 495
14		34992 74351 65
15	-	80003 02191
16		12054 0812
17	-	58235 1
18	-	34951
19		1528
20	-	39

S = 32.5

I		D SUB I
0		49243 01810 66850 29673 62044 97049 63380
1		14941 25791 21367 26760 99420 12267 1473
2	-	22294 50490 86453 89747 66742 26091 66
3		17312 45335 20172 46156 96181 8279
4		92873 70345 04323 38922 77658 164
5	-	29601 62710 79208 67379 25591 14
6		52084 59073 38193 59111 20932
7	-	35089 02658 61567 16109 682
8	-	11653 23162 75093 96880 17
9		53151 74994 03632 99468
10	-	12248 93072 04999 1369
11		17045 88137 98470 30
12	-	29418 45366 4759
13	-	64798 42852 301
14		23144 62890 14
15	-	50300 43934
16		70961 467
17	-	24815 5
18	-	22335
19		886
20	-	21

S = 33.5

I		C SUB I
0		49265 25128 28170 01910 73307 22684 34258
1		14507 31500 69172 66511 40430 05833 3242
2	-	21019 78031 74770 90835 11540 84107 43
3		15446 88948 58394 48154 95905 1897
4		82705 32289 70123 22247 38384 637
5	-	25553 17740 12707 43983 06249 53
6		43491 21898 07211 16249 70261
7	-	27815 49083 77174 28091 627
8	-	93920 81444 76002 90376 2
9		41001 51393 51636 85289
10	-	90914 62547 26511 312
11		12061 40829 24552 60
12	-	12339 68110 6208
13	-	45817 53025 327
14		15478 03432 78
15	-	32031 70148
16		42333 646
17	-	92914
18	-	14287
19		520
20	-	11

S = 34.5

I		D SUB I
0		49286 21635 83132 72231 26151 92297 86181
1		14097 88809 11531 72361 30489 47831 3732
2	-	19851 36687 20122 65076 45571 43956 97
3		13826 16890 60696 99477 34044 3768
4		73892 08342 06768 01880 15826 240
5	-	22151 37599 26767 47227 06194 07
6		36505 82150 84971 98446 04860
7	-	22195 13882 60446 90861 354
8	-	76099 01575 22982 18119 3
9		31856 30138 02190 76965
10	-	68050 14266 99134 753
11		86173 55095 73140 6
12	-	31466 28803 569
13	-	32638 85246 260
14		10460 35785 82
15	-	20646 33038
16		25572 616
17	-	23430
18	-	9171
19		308
20	-	6

S = 35.5

I		D SUB I
0		49306 01877 31628 24058 77937 08930 59900
1		13710 95508 48273 89049 92139 41270 8092
2	-	18777 76013 17383 76609 33814 31547 13
3		12412 69582 30105 37470 44211 7034
4		66222 80446 20594 42958 11706 748
5	-	19278 85765 72278 12844 14531 03
6		30793 82608 32391 43339 75759
7	-	17820 79125 36388 40841 739
8	-	61972 88927 91896 18171 2
9		24919 47322 53200 92205
10	-	51343 46264 35371 264
11		62132 57024 46179 3
12		15655 85116 568
13	-	23421 60669 811
14		71404 80781 5
15	-	13461 48901
16		15630 259
17		5415
18	-	5915
19		185
20	-	3

S = 36.5

I		D SUB I
0		49324 75259 69300 39096 74643 90849 97391
1		13344 71048 53992 11599 60647 51923 5493
2	-	17788 97117 64887 81077 64845 78891 33
3		11175 44483 28490 04924 79705 3722
4		59523 74261 66369 46677 00684 319
5	-	16841 98454 43234 62567 52655 68
6		26096 92387 92468 75509 89254
7	-	14392 92593 30306 06632 382
8	-	50715 14695 75611 11968 6
9		19619 04790 44518 48348
10	-	39031 82816 08438 097
11		45188 09715 70854 0
12		37591 45107 758
13	-	16927 95476 034
14		49210 33310 6
15	-	88732 4634
16		96594 98
17		15464
18	-	3838
19		112
20	-	2

S = 37.5

I	D SUB I
0	49342 50202 08147 51153 20398 99066 09837
1	12997 53710 99067 66061 58483 18113 6403
2	- 16876 29313 39370 40051 71849 63966 67
3	10088 67994 89760 79064 55093 7393
4	53651 19149 08181 20145 91027 096
5	- 14765 58303 93268 26512 42007 33
6	22214 24611 95089 85231 13168
7	- 11689 35682 36849 04545 941
8	- 41696 41550 26128 62550 4
9	15540 73134 33221 02867
10	- 29885 54168 65504 654
11	33135 57170 38863 3
12	45653 27636 485
13	- 12320 15601 401
14	34224 96305 2
15	- 59099 0477
16	60319 03
17	17218
18	- 2506
19	69
20	- 1

S = 38.5

I		D SUB I
0		49359 34262 08204 74729 68395 80876 24099
1		12667 98215 13860 49544 49680 56020 4084
2	-	16032 10862 69644 62110 72875 76983 47
3		91309 48031 76159 52587 50836 743
4		48485 68804 75036 44578 57588 425
5	-	12988 94652 68682 16464 93453 84
6		18988 52473 43775 40120 68017
7	-	95439 51699 75949 70589 28
8	-	34435 06235 37337 32908 4
9		12381 93765 89576 59436
10	-	23038 56512 84564 870
11		24487 89407 27642 8
12		46342 40954 034
13	-	90274 31523 70
14		24010 92338 2
15	-	39753 2074
16		38036 08
17		15651
18	-	1647
19		42

S = 39.5

I		D SUB I
0		49375 34243 19681 37283 05524 81121 28175
1		12354 73679 98035 03844 38143 33723 8119
2	-	15249 73014 31186 92316 04535 29607 59
3		82842 82797 22064 51597 93370 861
4		43927 44694 55285 73854 45459 092
5	-	11462 77701 12903 36526 36131 76
6		16295 83211 88180 94912 94856
7	-	78315 51996 53839 21545 97
8	-	28560 40182 46717 30260 3
9		99199 14209 96626 0859
10	-	17875 44176 53346 177
11		18231 77731 84787 6
12		43364 10526 821
13	-	66582 18764 44
14		16985 80197 2
15	-	26993 3902
16		24205 99
17		13045
18	-	1090
19		26

S = 40.5

I		D SUB I
0		49390 56286 57236 53039 02456 26194 15762
1		12056 61882 30882 81906 67722 58803 1671
2	-	14523 26704 53250 97366 27394 50762 96
3		75335 71547 80458 37911 25268 360
4		39892 74024 09090 48994 87155 696
5	-	10146 82860 41629 19210 72162 77
6		14037 91401 70364 30228 33164
7	-	64571 91123 37286 00740 82
8	-	23785 55277 68489 86262 9
9		79894 40069 30677 9124
10	-	13955 02339 89860 587
11		13670 14527 71478 1
12		38797 61241 755
13	-	49420 42955 50
14		12112 02579 3
15	-	18494 7908
16		15537 88
17		10389
18	-	727
19		17

S = 41.5

I	C SUB I
0	49405 05949 66983 42874 66623 01213 96614
1	11772 55761 82611 41966 74518 53357 2293
2	- 13847 51425 98822 98349 17800 44508 76
3	68660 48470 92451 42346 75495 135
4	36311 00978 30736 66909 11393 483
5	- 90080 79682 89116 90977 62069 5
6	12136 41705 34353 21329 53106
7	- 53483 20135 08178 47952 88
8	- 19887 29824 89331 14891 4
9	64670 85287 63113 9920
10	- 10958 51115 01422 084
11	10319 06985 29570 6
12	33778 57839 378
13	- 36907 85487 91
14	87026 73436
15	- 12781 2560
16	10054 68
17	8057
18	- 488
19	10

S = 42.5

I		C SUB I
0		49418 88273 98467 58461 73567 90697 01123
1		11501 58133 32565 72957 92502 97125 5235
2	-	13217 85869 47964 87923 08134 05135 97
3		62708 86998 24584 88217 97824 847
4		33122 55112 10011 42541 46689 208
5	-	80193 06123 79383 77649 77529 2
6		10528 51166 79513 00356 96410
7	-	44491 56409 45394 27591 36
8	-	16691 05422 19138 58371 0
9		52600 07708 49345 0427
10	-	86537 24373 31250 10
11		78396 70898 43010
12		28898 48406 497
13	-	27727 17295 64
14		62987 47113
15	-	89056 654
16		65558 3
17		6150
18	-	330
19		7

S = 43.5

I	C SUB I
0	49432 07843 55380 33278 55378 54552 86183
1	11242 80573 22071 28102 50845 73064 5774
2	- 12630 20023 21275 48476 42446 35119 20
3	57388 70157 13680 42917 55331 514
4	30276 64590 96979 26424 82494 008
5	- 71579 60886 04175 96772 16854 6
6	91635 50592 00605 63382 4269
7	- 37165 43863 41818 69089 72
8	- 14059 58784 77655 76606 9
9	42979 15547 78381 6028
10	- 68703 07816 79678 77
11	59926 71323 06394
12	24438 74911 676
13	- 20949 78684 29
14	45908 15477
15	- 62542 445
16	43047 8
17	4648
18	- 224
19	4

S = 44.5

I	D SUB I
0	49444 68835 67959 83453 02713 30489 98876
1	10995 42453 47779 35787 63028 59334 4301
2	- 12080 88475 64759 81921 03337 45617 08
3	52621 22673 73117 35176 77262 288
4	27730 04832 48680 53129 26797 536
5	- 64052 89114 05753 51739 98353 5
6	80005 01375 20230 24204 0521
7	- 31168 98122 05606 79985 34
8	- 11884 50297 99989 39451 4
9	35272 57148 47904 2901
10	- 54823 80745 77410 38
11	46077 83623 92681
12	20507 75700 537
13	- 15916 81144 46
14	33684 94864
15	- 44254 289
16	28452 7
17	3493
18	- 154
19	3

S = 45.5

I		D SUB I
0		49456 75065 05234 33961 55498 83450 00080
1		10758 70100 67092 47464 55829 87500 6699
2	-	11566 64716 83989 51060 48810 82392 20
3		48338 92232 40406 57320 11391 375
4		25445 75244 59523 79426 25152 240
5	-	57456 25423 76497 93269 48432 5
6		70059 61495 93699 70419 7509
7	-	26239 36490 04628 51655 36
8	-	10079 77955 86570 23297 7
9		29069 90119 31981 5947
10	-	43962 94650 82385 16
11		35629 01254 38094
12		17120 67926 096
13	-	12157 74264 58
14		24875 71793
15	-	31540 824
16		18920 5
17		2616
18	-	106
19		1

S = 46.5

I		D SUB I
0		49468 30022 25200 96861 07549 98580 02255
1		10531 96061 64268 26231 27145 89426 9226
2	-	11084 56271 66613 69845 99731 24994 25
3		44483 70066 75595 89311 32967 007
4		23391 98381 32319 38153 81442 832
5	-	51658 32947 75876 30343 97410 1
6		61526 14835 27399 96432 8171
7	-	22169 79814 42069 60109 28
8	-	85768 45706 69869 93735
9		24054 79040 70937 1258
10	-	35419 27773 72196 50
11		27698 32574 93656
12		14245 54308 362
13	-	93345 00849 7
14		18484 12634
15	-	22636 070
16		12652 3
17		1957
18	-	73
19		1

S = 47.5

I	C SUB I
0	49479 36907 44723 74572 37847 11129 09437
1	10314 58460 32520 13322 60089 81903 6300
2	- 10632 00528 96361 76216 77033 38746 79
3	41005 43175 61220 03090 23545 639
4	21541 37074 56422 36523 68286 894
5	- 46548 53305 98494 06504 40691 5
6	54180 24626 55717 84506 0899
7	- 18796 74035 36431 86835 72
8	- 73207 99766 31250 11627
9	19982 05499 69719 5844
10	- 28664 48288 39384 43
11	21644 39639 74887
12	11829 39058 663
13	- 72026 84188 4
14	13816 62058
15	- 16353 848
16	85040
17	1464
18	- 51

S = 48.5

I		C SUB I
0		49489 98659 97613 30527 99827 38709 74768
1		10106 00432 78127 87280 76878 15022 6746
2	-	10206 61155 12096 73501 22104 02074 29
3		37860 72087 10286 33419 64727 106
4		19870 26049 91150 31272 90048 650
5	-	42033 43842 51932 98726 14119 8
6		47836 84459 29736 27632 2718
7	-	15990 21811 06598 38786 30
8	-	62674 99610 56491 24374
9		16660 66926 06824 4795
10	-	23298 14049 19383 06
11		16997 67968 96284
12		98127 38015 46
13	-	55845 68126 8
14		10386 90310
15	-	11890 971
16		57421
17		1096
18	-	36

S = 49.5

I	C SUE I
0	49500 17984 38421 47467 66502 87931 78713
1	99056 96295 83886 30354 84247 60028 725
2	- 98062 50003 47868 27029 30485 98756 6
3	35011 89354 53041 96876 43721 319
4	18358 15266 65754 18612 85293 113
5	- 38033 83869 20920 05766 52453 7
6	42342 68330 09636 83154 2338
7	- 13646 44713 45014 33657 58
8	- 53813 09381 17856 61348
9	13941 05076 16370 5751
10	- 19015 03070 92335 36
11	13412 23926 10059
12	81372 09137 37
13	- 43501 73169 7
14	78516 1051
15	- 86993 53
16	38930
17	821
18	- 25

S = 50.5

I	C SUB I
0	49509 97373 40478 91297 10522 65562 39823
1	97131 77763 49335 48928 10268 59076 890
2	- 94289 94217 53795 26508 37980 39321 3
3	32426 14948 28733 87971 45948 014
4	16987 22789 84978 01912 18349 966
5	- 34482 35746 80186 81782 72872 7
6	37570 35070 54867 23378 9674
7	- 11682 17938 62532 36165 83
8	- 46333 47050 41002 05760
9	11705 50050 88019 5392
10	- 15581 22508 87681 13
11	10631 64886 93955
12	67492 36828 05
13	- 34039 08788 3
14	59666 6805
15	- 64021 91
16	26488
17	617
18	- 18

S = 51.5

I	D SUB I
0	49519 39128 29253 99879 22399 87063 02850
1	95280 02846 44190 55839 72422 62650 458
2	- 90730 99602 62952 13326 74031 45207 0
3	30074 85474 75978 54626 43341 812
4	15741 95444 95985 29125 75909 942
5	- 31321 49749 83699 79004 28716 6
6	33413 53138 10167 62115 3254
7	- 10030 34860 83737 52183 55
8	- 40001 15347 36957 96146
9	98609 74767 72951 045
10	- 12816 49033 56409 96
11	84647 15449 2662
12	56015 46814 45
13	- 26750 94975 3
14	45574 5243
15	- 47385 73
16	18075
17	465
18	- 13

S = 52.5

I		D SUB I
0		49528 45376 85923 48868 81170 85001 15648
1		93497 59066 53584 28360 32996 20710 600
2	-	87369 83188 98740 51262 53967 04111 7
3		27932 94755 59721 87729 84526 382
4		14608 75852 61403 54870 21401 330
5	-	28502 04048 43057 34853 28375 5
6		29783 19724 79895 39901 9135
7	-	86366 98704 94040 42602 4
8	-	34624 27511 38790 59520
9		83335 90071 63076 878
10	-	10581 26001 59555 68
11		67680 79145 4618
12		46534 32866 85
13	-	21111 96496 1
14		34982 4768
15	-	35265 83
16		12364
17		351
18	-	9

S = 53.5

I		C SUB I
0		49537 18089 50886 87129 04378 42412 36298
1		91780 64279 72605 89191 99858 65673 618
2	-	84192 05985 64937 89762 18054 41215 4
3		25978 43776 39350 04463 99939 487
4		13575 74714 00070 09614 60089 184
5	-	25981 72978 46993 51154 03652 6
6		26604 54133 55813 97143 9855
7	-	74571 60149 24740 66188 7
8	-	30045 60813 59751 40188
9		70644 23963 26303 458
10	-	87669 32056 33904 5
11		54336 62137 6620
12		38704 11811 06
13	-	16729 66071 0
14		26980 0131
15	-	26385 37
16		8470
17		265
18	-	6

S = 54.5

I		C SUB I
0		49545 59093 52640 12833 09685 16596 01897
1		90125 63936 97907 85300 52906 78799 901
2	-	81184 57545 08354 55153 38822 00642 6
3		24191 98391 12410 05518 61820 813
4		12632 47433 69808 01199 48301 356
5	-	23724 18195 03158 82113 49085 9
6		23814 49906 86765 23725 6313
7	-	64557 98701 66924 22960 0
8	-	26135 86617 27986 76972
9		60062 89751 19096 050
10	-	72886 02445 05492 6
11		43795 59092 0790
12		32235 95739 94
13	-	13309 38622 8
14		20903 8259
15	-	19842 41
16		5808
17		201
18	-	4

S = 55.5

I	D SUB I
0	49553 70085 83800 67574 42291 65278 91473
1	88529 28636 81788 43853 03482 10412 577
2	- 78335 42424 65895 40263 87452 87276 2
3	22556 53469 28511 08022 60781 547
4	11769 74338 43711 04401 33693 110
5	- 21697 98411 23763 42452 08735 5
6	21359 73642 59369 28486 8566
7	- 56032 04214 83242 41245 4
8	- 22788 39367 85252 97641
9	51212 47325 23453 679
10	- 60795 93989 92543 8
11	35433 86169 7935
12	26889 69801 01
13	- 10628 87138 3
14	16267 9862
15	- 14995 87
16	3982
17	153
18	- 3

S = 56.5

I		D SUB I
0		49561 52644 43022 39673 75086 62641 76136
1		86988 51933 88509 43455 65467 23303 934
2	-	75633 68283 12996 32045 60559 67913 4
3		21057 02413 19595 52249 39609 188
4		10979 43887 42727 64611 74248 286
5	-	19875 94290 51502 50295 03637 8
6		19195 01069 52634 75079 6075
7	-	48752 19500 53957 78115 9
8	-	19914 90359 69128 33989
9		43786 73051 87198 962
10	-	50873 15212 46175 6
11		28773 97211 7522
12		22466 76187 78
13	-	85196 67125
14		12714 5760
15	-	11387 32
16		2727
17		117
18	-	2

S = 57.5

I	D SUB I
0	49569 08238 58959 63127 63620 79331 74502
1	85500 48372 80965 06116 56117 23109 791
2	- 73069 35390 27739 86049 58846 91632 8
3	19680 11166 15933 44234 47716 404
4	10254 38379 77979 88676 17896 855
5	- 18234 45740 16542 73888 99221 3
6	17281 82982 61653 00443 2625
7	- 42519 33201 14439 84924 6
8	- 17442 10181 42155 36689
9	37537 56416 48518 146
10	- 42701 10252 66061 5
11	23448 77733 0055
12	18803 53960 29
13	- 68535 37306
14	99785 821
15	- 86871 2
16	1863
17	90
18	- 1

S = 58.5

I	D SUB I
0	49576 38238 10251 77273 30731 75931 33578
1	84062 51720 80064 89603 42577 51248 041
2	- 70633 27361 15465 54783 40098 42512 1
3	18413 95988 02945 70124 41980 016
4	95882 17530 16448 31261 21626 94
5	- 16752 99390 53807 65817 45749 4
6	15587 35203 48610 41318 3554
7	- 37168 78546 66948 88789 3
8	- 15308 94952 36314 69863
9	32263 23951 58544 031
10	- 35948 51007 90008 3
11	19174 68196 7986
12	15765 55289 02
13	- 55324 31521
14	78627 843
15	- 66568 5
16	1267
17	69
18	- 1

S = 59.5

I	D SUB I
0	49583 43921 53640 39581 18245 54372 05685
1	82672 13375 91622 24760 31786 92853 662
2	- 68317 02954 22738 89339 34599 30097 2
3	17248 04400 93153 77512 92295 567
4	89752 91381 17961 52251 40821 53
5	- 15413 64469 04271 48205 25324 6
6	14083 47946 54431 28209 0355
7	- 32563 93374 57128 03980 6
8	- 13464 46641 69180 96619
9	27799 15324 03438 883
10	- 30350 88004 78839 4
11	15731 66859 0051
12	13242 42737 21
13	- 44810 61031
14	62196 923
15	- 51231 6
16	857
17	53
18	- 1

S = 60.5

I		D SUB I
0		49590 26483 70745 01432 10722 99491 49583
1		81327 00930 96235 31143 42011 81673 033
2	-	66112 88796 04175 62318 20974 66000 8
3		16172 98810 42460 80229 20302 669
4		84105 78945 22961 17655 32239 89
5	-	14200 75616 28551 39985 09862 9
6		12746 10917 81309 78946 0229
7	-	28591 06993 03013 87782 3
8	-	11865 95212 78615 60020
9		24010 54561 38540 829
10	-	25696 23932 82758 4
11		12948 32241 0092
12		11143 63890 86
13	-	36413 76271
14		49384 819
15	-	39593 5
16		574
17		41

S = 61.5

I		D SUB I
0		49596 87042 42667 54590 76649 98535 99925
1		80024 96875 54915 64865 21319 03909 329
2	-	64013 72914 69373 83856 00681 33386 8
3		15180 42391 03182 31568 47154 232
4		78895 98956 60203 76425 45730 76
5	-	13100 61461 59817 67100 34265 5
6		11554 51214 68968 39777 6564
7	-	25155 27221 16808 15700 9
8	-	10477 54284 21221 48114
9		20786 72938 97511 956
10	-	21814 08422 37583 3
11		10690 54840 5784
12		93949 57755 4
13	-	29684 40161
14		39354 962
15	-	30723 4
16		379
17		32

S = 62.5

I		C SUB I
0		49603 26644 60432 93157 19573 69992 01163
1		78763 97421 03917 72168 97303 79949 298
2	-	62012 98980 93887 07799 10729 82529 9
3		14262 86893 50148 72661 75124 430
4		74083 48741 25188 82074 20898 21
5	-	12101 17991 68992 73632 75019 1
6		10490 81677 46780 54340 9624
7	-	22177 06857 15256 02568 1
8	-	92690 36145 93365 0415
9		18036 50225 74618 033
10	-	18566 78103 34168 3
11		88530 30870 417
12		79355 01286 4
13	-	24273 30771
14		31473 041
15	-	23934 3
16		246
17		25

S = 63.5

I	D SUB I
0	49609 46271 78273 95559 36570 58796 19400
1	77542 11435 02041 17527 63437 36545 439
2	- 60104 61169 79786 05501 22602 77827 7
3	13413 62087 35397 94368 73641 109
4	69632 46671 86931 97367 34657 45
5	- 11191 85920 34062 71115 17842 7
6	95395 78032 74528 21476 042
7	- 19589 73362 46131 45478 7
8	- 82149 31277 17166 6380
9	15684 48670 22761 889
10	- 15842 84774 03923 1
11	73527 39739 533
12	67153 05736 6
13	- 19908 06575
14	25256 022
15	- 18716 5
16	155
17	19

S = 64.5

I		D SUB I
0		49615 46845 15907 06711 11065 53256 44996
1		76357 59473 57342 92760 32585 24409 201
2	-	58282 99567 40451 71605 80235 70665 6
3		12626 66598 55442 35797 94220 145
4		65510 82292 71577 99172 87439 19
5	-	10363 31407 72878 18464 07478 4
6		86874 16974 51814 51965 855
7	-	17337 09044 79042 56110 8
8	-	72936 42918 77941 4709
9		13668 19935 74186 063
10	-	13551 68453 97891 2
11		61239 71662 956
12		56933 33965 1
13	-	16375 39158
14		20334 528
15	-	14690 4
16		93
17		15

S = 65.5

I	D SUB I
0	49621 29230 15202 01964 72149 39915 51095
1	75208 72901 01652 97879 75594 50598 891
2	- 56542 96057 95735 95355 81446 06198 0
3	11896 59940 39356 47756 37836 146
4	61689 72994 82169 55090 97544 29
5	- 96072 95919 93012 30454 83847
6	79227 18272 38103 20569 762
7	- 15371 71724 36327 14725 9
8	- 64868 55256 42314 2486
9	11935 69633 47353 397
10	- 11619 42490 98469 8
11	51145 47115 168
12	48358 45664 0
13	- 13507 70004
14	16424 946
15	- 11571 8
16	52
17	12

S = 66.5

I	C SUE I
0	49626 94240 56003 94623 29183 91598 40251
1	74093 93088 03835 50995 01280 26619 458
2	- 54879 70634 26270 24388 83224 70899 6
3	11218 55567 35230 91634 79550 674
4	58143 26299 29211 85842 49366 35
5	- 89165 04883 26674 43639 95017
6	72353 75739 85509 82314 693
7	- 13653 47966 77537 73676 1
8	- 57789 89756 04175 8202
9	10443 67498 59402 414
10	- 99856 56807 68462
11	42828 82119 949
12	41150 66603 6
13	- 11172 85550
14	13308 652
15	- 91470
16	24
17	9

S = 67.5

I	D SUB I
0	49632 42642 35307 42414 70922 42226 37726
1	73011 70680 19143 30807 61617 05017 063
2	- 53288 78082 73243 74662 86607 97480 3
3	10588 14808 06705 26572 23504 214
4	54848 06953 46571 07017 80643 05
5	- 82844 68865 29058 60789 75649
6	66165 77655 27637 78065 106
7	- 12148 32601 40264 94840 3
8	- 51567 65419 11642 5774
9	91559 38639 83801 45
10	- 86008 22170 33806
11	35957 24499 420
12	35081 03289 9
13	- 92663 2084
14	10816 475
15	- 72548
16	7
17	7

S = 68.5

I		D SUB I
0		49637 75157 13495 83528 90360 32894 92365
1		71960 64929 64607 42962 39912 06612 545
2	-	51766 05000 02083 94777 05842 51152 5
3		10001 41555 46870 36267 78735 586
4		51783 08167 00049 67913 65072 54
5	-	77054 39398 23516 19015 79802
6		60586 25193 50095 53987 293
7	-	10827 29524 63659 42443 5
8	-	46088 34462 60054 6495
9		80421 50987 28691 08
0	-	74241 47188 92783
1		30263 97113 340
2		29960 58271 4
3	-	77051 2082
4		88170 21
5	-	57730
6	-	3
7		5

S = 69.5

I		C SUB I
0		49642 92465 30935 50926 92570 72436 36605
1		70939 43083 91149 58181 93078 72003 609
2	-	50307 67103 91500 15677 60334 37581 4
3		94547 76105 18598 58060 06612 59
4		48929 26417 04117 20814 02469 65
5	-	71743 01885 53052 18168 47748
6		55547 78474 36551 61587 356
7	-	96656 97928 41570 87890
8	-	41254 79691 14633 3642
9		70768 22369 90712 79
10	-	64219 89436 70044
11		25534 29453 810
12		25633 09071 4
13	-	64231 8380
14		72078 69
15	-	46086
16	-	10
17		4

S = 70.5

I	C SUB I
0	49647 95208 97843 96657 19022 86389 53791
1	69946 79825 93663 21449 35543 58835 828
2	- 48910 06805 76912 22969 48174 77323 9
3	89449 85913 42174 45730 13743 52
4	46269 39337 03872 80787 46245 32
5	- 66864 98041 87161 82126 97001
6	50991 25709 54133 62995 252
7	- 86424 38027 09133 86902
8	- 36983 62470 06764 2728
9	62384 82341 35920 07
10	- 55665 13548 47927
11	21594 87880 851
12	21969 20331 6
13	- 53677 3398
14	59089 00
15	- 36904
16	- 14
17	3

B L A N K   P A G E

APPENDIX B

VALUES OF $\Pi_{s+\frac{1}{2}}(s)$, $\Pi_{s-\frac{1}{2}}(s)$, AND $\Pi_{s+\frac{1}{2}}(s-\frac{1}{2})$

Table 2

$\Pi_{s+1/2}(s)$ to 33 Decimal Places for s = 1(1)70

s	$\Pi_{s+1/2}(s)$						
1	0.25396	60243	36788	20750	56056	53722	937
2	0.33669	72531	21326	33052	39065	52592	747
3	0.37739	01650	32642	25053	28934	28843	737
4	0.40173	03517	64841	97396	78340	54278	770
5	0.41796	33353	81213	50477	67451	18172	270
6	0.42957	44054	08420	88661	52478	09003	143
7	0.43829	68276	00080	50735	72035	28556	068
8	0.44509	17430	33055	82684	67839	67611	405
9	0.45053	57084	17628	08803	40365	66524	242
10	0.45499	58297	22878	02037	88149	41388	062
11	0.45871	70733	35319	26019	01596	70103	652
12	0.46186	92180	24929	63478	86583	59948	837
13	0.46457	36744	19608	58169	91971	88658	530
14	0.46691	95794	50527	74521	58812	11605	372
15	0.46897	38604	17063	95826	81836	78368	279
16	0.47078	77515	49129	63069	02295	76232	624
17	0.47240	11418	71375	07706	20431	16784	742
18	0.47384	55526	12607	69632	11509	96658	509
19	0.47514	62232	27486	17457	97423	93853	670
20	0.47632	36027	23779	54489	89949	22269	366
21	0.47739	44352	03761	04174	34321	45689	445
22	0.47837	25628	98030	96127	40379	11671	212
23	0.47926	95289	32518	38208	77263	23631	182
24	0.48009	50357	91386	22969	70138	23806	071
25	0.48085	72982	46722	89182	45677	79632	100
26	0.48156	33180	57591	06010	50027	93295	953
27	0.48221	90999	53978	42790	89387	99691	904
28	0.48282	98230	47366	15605	66563	57543	852
29	0.48339	99780	45520	00494	16069	92377	752
30	0.48393	34799	76090	64278	32422	96684	640
31	0.48443	37482	01161	73395	36992	47357	434
32	0.48490	38001	05871	69671	38710	92890	412
33	0.48534	62918	14923	88215	17010	07630	086
34	0.48576	35785	25715	34375	27942	48124	734
35	0.48615	77544	72669	65149	76445	53042	903
36	0.48653	06881	02627	50607	98800	62019	092
37	0.48688	40517	09199	25908	75404	84983	569
38	0.48721	93465	18502	66015	76696	81878	149
39	0.48753	79240	20610	16277	25298	04543	616
40	0.48784	10041	86339	58380	85142	17067	797
41	0.48812	96910	87420	96527	10222	00577	180
42	0.48840	49863	41878	18477	23324	32927	643
43	0.48866	78007	29911	21582	34335	10307	875
44	0.48891	89642	64296	12993	32097	95414	684
45	0.48915	92349	50017	70447	06168	21886	427
46	0.48938	93064	27971	83932	49874	31926	913
47	0.48960	98146	65161	56787	58628	00582	357
48	0.48982	13438	27340	37679	20732	35703	289
49	0.49002	44314	48343	03446	74056	99209	492
50	0.49021	95729	92455	97875	58026	00478	129
51	0.49040	72258	91382	03523	82899	45912	914
52	0.49058	78131	25065	55214	68968	71609	940
53	0.49076	17264	05400	11136	70388	20238	972
54	0.49092	93290	13270	60377	57892	02336	390
55	0.49109	09583	32186	24858	13994	32065	710
56	0.49124	69281	15700	19946	70135	96737	505
57	0.49139	75305	20689	44546	38708	92420	648
58	0.49154	30379	34226	86647	94221	18373	214
59	0.49168	37046	18085	72057	23591	06860	171
60	0.49181	97681	91769	07838	44404	20173	562
61	0.49195	14509	72265	19454	82087	07682	047
62	0.49207	89611	86422	20282	34064	97036	355
63	0.49220	24940	69851	93943	75036	02926	046
64	0.49232	22328	64563	35927	70818	59899	974
65	0.49243	83497	26049	41901	44856	98128	325
66	0.49255	10065	49272	76993	20455	32162	329
67	0.49266	03557	21886	04523	14733	55300	786
68	0.49276	65408	12057	48683	49004	79145	911
69	0.49286	96971	97431	51784	24471	26155	638
70	0.49296	99526	41019	20170	91141	41040	111

Table 3

$\Pi_{s-1/2}(s)$ to 33 Decimal Places for s = 1(1)71

s	$\Pi_{s-1/2}(s)$						
1	0.46439	87801	10529	21148	64127	73365	386
2	0.48424	80053	44947	67937	29726	32292	120
3	0.49087	30296	38979	07642	45235	04173	670
4	0.49398	85363	92239	01453	00994	48869	239
5	0.49572	33324	48920	85537	12366	64053	029
6	0.49679	47886	15819	54316	36416	69197	452
7	0.49750	52270	03106	99601	10046	17630	344
8	0.49800	14446	31123	26927	98453	34966	981
9	0.49836	21570	28137	99771	19016	58746	508
10	0.49863	28138	03644	73606	38642	16782	570
11	0.49884	12116	85596	39534	34785	40715	600
12	0.49900	51547	62134	12056	88009	00788	931
13	0.49913	64882	01260	58485	35601	15015	965
14	0.49924	33475	37418	69430	30633	95638	212
15	0.49933	14738	85750	17473	27293	78808	624
16	0.49940	50143	99905	11996	79833	34095	384
17	0.49946	70264	32708	95306	60942	03225	239
18	0.49951	98046	53157	54378	24768	28927	182
19	0.49956	50988	86103	09805	35132	94687	984
20	0.49960	42625	53474	09387	26671	79097	329
21	0.49963	83559	17424	83755	25976	13246	912
22	0.49966	82192	17247	77618	48705	97081	329
23	0.49969	45253	43631	66507	53765	30561	924
24	0.49971	78183	48884	59791	01633	29832	215
25	0.49973	85419	89293	30795	33044	43559	185
26	0.49975	70611	45911	86682	60539	39809	260
27	0.49977	36780	86444	83369	48642	53883	095
28	0.49978	86449	42142	47239	13359	21652	735
29	0.49980	21733	74928	60369	01629	14914	231
30	0.49981	44421	35914	38385	13033	54434	936
31	0.49982	56030	25430	27328	76949	57928	664
32	0.49983	57856	29849	74294	27717	86078	305
33	0.49984	51011	14126	15350	94778	58322	224
34	0.49985	36452	79353	15993	50795	66463	580
35	0.49986	15010	43841	21458	81475	15160	331
36	0.49986	87404	68740	95419	10800	05413	695
37	0.49987	54264	21374	14827	17133	55760	191
38	0.49988	16139	48519	17773	86187	85750	367
39	0.49988	73514	16076	38255	44685	45587	909
40	0.49989	26814	59477	08005	54745	26657	031
41	0.49989	76417	79938	04782	66204	99936	764
42	0.49990	22658	14501	95110	85926	54575	350
43	0.49990	65833	02230	89509	59557	60289	741
44	0.49991	06207	64555	90068	81062	17720	239
45	0.49991	44019	14347	20620	83706	91205	705
46	0.49991	79480	05546	96417	02895	40774	335
47	0.49992	12781	33037	03021	35346	76217	715
48	0.49992	44094	90678	39344	55475	82458	186
49	0.49992	73575	94062	23303	21789	80446	404
50	0.49993	01364	73383	92147	51037	24950	528
51	0.49993	27588	40935	04846	11209	25139	108
52	0.49993	52362	36931	53509	95680	41683	715
53	0.49993	75791	57024	37375	74221	98412	235
54	0.49993	97971	63496	91366	40213	05130	275
55	0.49994	18989	83417	94554	87538	45875	510
56	0.49994	38925	94576	47857	91840	67404	661
57	0.49994	57853	01418	78632	85479	96538	162
58	0.49994	75838	02130	40917	60362	19186	319
59	0.49994	92942	48046	80625	89199	71819	097
60	0.49995	09222	96379	36657	54310	24568	052
61	0.49995	24731	57102	94029	43505	06524	427
62	0.49995	39516	34732	40518	23064	04210	548
63	0.49995	53621	65615	34170	13358	72388	211
64	0.49995	67088	51282	62697	02547	48987	892
65	0.49995	79954	88326	18655	70038	83413	573
66	0.49995	92255	95211	21582	27711	42973	568
67	0.49996	04024	36377	20653	07956	27398	308
68	0.49996	15290	43936	68646	08771	75837	380
69	0.49996	26082	37241	53326	44951	54305	790
70	0.49996	36426	40553	08617	87914	75764	764
71	0.49996	46346	99023	23743	19435	77420	785

Table 4

$\Pi_{s+1/2}(s-1/2)$ to 33 Decimal Places for $s = 1(1)70$

s	$\Pi_{s+1/2}(s-1/2)$						
1	0.10774	57644	36528	87397	35082	44177	514
2	0.24322	51625	15757	05059	82026	92281	448
3	0.30936	90228	82790	94554	18360	45675	232
4	0.34831	48246	49018	73101	32202	20058	045
5	0.37399	28629	89273	53744	36459	82377	857
6	0.39220	84348	03946	10913	88936	44863	643
7	0.40580	81098	17206	21862	49678	34524	636
8	0.41636	23183	15238	88033	33275	24903	437
9	0.42476	85150	11057	70994	13242	44573	794
10	0.43164	28404	28206	01547	33382	00689	159
11	0.43736	40340	89911	40164	77519	75813	080
12	0.44220	01509	53753	70166	95576	44448	657
13	0.44634	20587	73310	76921	13093	87240	670
14	0.44992	93760	72970	48027	53354	41207	922
15	0.45306	66032	12299	63840	16606	92377	029
16	0.45583	35180	02506	87743	87820	94983	515
17	0.45829	20829	91043	80560	81240	76355	511
18	0.46049	11583	12614	63558	29030	31573	883
19	0.46246	97929	11366	77112	66265	12534	579
20	0.46425	95706	93333	92037	40566	60012	551
21	0.46588	63138	75697	38493	45830	19396	485
22	0.46737	13401	04616	96321	82030	41707	220
23	0.46873	24040	67497	07743	05749	05869	881
24	0.46998	44123	02688	90789	74381	97618	639
25	0.47113	99725	00820	24680	02687	52344	896
26	0.47220	98203	59052	63741	71874	00540	037
27	0.47320	31547	01015	40597	28675	87469	187
28	0.47412	79030	73480	89766	81474	18937	819
29	0.47499	09340	88754	80327	66992	70378	157
30	0.47579	82285	70566	70903	19928	79939	186
31	0.47655	50185	38189	61001	37942	43515	780
32	0.47726	59008	67383	04096	06351	70991	933
33	0.47793	49308	53583	32301	28519	13769	985
34	0.47856	56997	05511	22408	90106	04726	614
35	0.47916	13991	00181	04776	43048	01593	123
36	0.47972	48752	51864	61562	14318	05053	575
37	0.48025	86744	30178	49167	72007	54996	082
38	0.48076	50814	64731	83030	32235	04444	640
39	0.48124	61524	65714	54710	78996	56428	682
40	0.48170	37427	49502	64685	88718	23881	552
41	0.48213	95307	69644	35166	20425	50335	053
42	0.48255	50387	04446	07835	56052	37886	143
43	0.48295	16502	33791	62253	20277	38679	000
44	0.48333	06259	42998	08396	62318	34489	246
45	0.48369	31167	15264	93029	84131	38752	885
46	0.48404	01754	12647	87166	63584	29834	344
47	0.48437	27670	95436	75977	38260	96667	703
48	0.48469	17779	88961	46463	46395	01561	213
49	0.48499	80233	73392	69552	00375	68955	001
50	0.48529	22545	44480	41090	91954	31373	090
51	0.48557	51649	70451	44846	45101	57526	790
52	0.48584	73957	51315	74855	27241	82010	362
53	0.48610	95404	71098	59618	54678	83115	679
54	0.48636	21495	20330	09785	59939	44464	925
55	0.48660	57339	55066	07824	28985	45065	461
56	0.48684	07689	49404	34683	47717	62504	066
57	0.48706	76968	90596	17359	79530	29621	471
58	0.48728	69301	59189	46187	17428	18805	452
59	0.48749	88536	30977	18509	97951	79553	519
60	0.48770	38269	32697	99519	49007	30863	827
61	0.48790	21864	79310	50434	68008	89492	924
62	0.48809	42473	17127	17592	49280	20601	531
63	0.48828	03047	94055	78864	19403	38693	298
64	0.48846	06360	75579	44169	75256	12707	723
65	0.48863	55015	22846	20927	44027	09771	810
66	0.48880	51459	47283	48278	83818	35552	469
67	0.48896	97997	54455	04824	60023	15484	300
68	0.48912	96799	88403	45724	26921	08636	793
69	0.48928	49912	86434	59659	41808	67765	001
70	0.48943	59267	53178	78644	13929	73556	330

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APPENDIX C

VALUES OF $\Omega_{s+\frac{1}{2}}(s)$, $\Omega_{s-\frac{1}{2}}(s)$, $\Omega_{s+\frac{1}{2}}(s-\frac{1}{2})$, AND $\Omega_{s-\frac{1}{2}}(s-\frac{1}{2})$

Table 5

$\Omega_{s+1/2}(s)$ to 33 Decimal Places for s = 1(1)70

s	$\Omega_{s+1/2}(s)$						
1	0.35706	74799	26313	85900	90581	51089	743
2	0.41260	15957	24041	85650	16218	94166	305
3	0.43639	45460	23732	22020	75512	82136	854
4	0.44978	79676	51343	09819	54671	56560	676
5	0.45843	33341	37344	67693	52393	96315	428
6	0.46449	71182	00781	96015	66384	59202	352
7	0.46899	51214	63766	80372	30623	56878	346
8	0.47246	92285	82472	21714	83808	61207	547
9	0.47523	58512	36500	84427	29352	70661	203
10	0.47749	25582	82243	10851	06055	07826	124
11	0.47936	92757	78994	75228	01509	61054	644
12	0.48095	50514	28351	24425	39511	35242	626
13	0.48231	30113	61749	06643	73124	81836	478
14	0.48348	91816	88009	53653	49732	73176	898
15	0.48451	79284	97661	12122	63909	23733	590
16	0.48542	54405	81910	18669	77131	30574	173
17	0.48623	20314	65368	44559	29370	59724	053
18	0.48695	37035	80711	57902	24549	77260	039
19	0.48760	32369	82771	34035	81152	51329	656
20	0.48819	09633	62464	29866	08125	08197	728
21	0.48872	53267	78329	22843	69976	80084	411
22	0.48921	32967	65357	72995	25709	71742	700
23	0.48966	06773	23254	11503	26102	04130	883
24	0.49007	23412	09255	90408	80032	68735	789
25	0.49045	24098	14418	32553	59760	35726	290
26	0.49080	43928	37973	26273	67017	94904	122
27	0.49113	12978	78763	25419	04969	14369	325
28	0.49143	57172	49824	93940	50033	05042	927
29	0.49171	98973	60239	67772	83144	22626	688
30	0.49198	57946	20412	08145	77344	05473	834
31	0.49223	51208	32116	23898	67128	98546	395
32	0.49246	93803	02917	17617	94247	02938	284
33	0.49268	99003	95039	90848	32009	15682	586
34	0.49289	78568	26144	71194	80375	28702	559
35	0.49309	42947	45508	66167	36573	79419	392
36	0.49328	01463	86995	22326	35923	23427	589
37	0.49345	62459	30910	84037	19094	88983	278
38	0.49362	33420	76786	26612	81165	23155	482
39	0.49378	21087	28430	91342	72335	87900	546
40	0.49393	31541	14096	71105	63214	55153	550
41	0.49407	70286	02952	77202	67122	77170	907
42	0.49421	42314	30374	54243	38613	76654	949
43	0.49434	52165	05840	72438	79058	00173	359
44	0.49447	03974	46281	80605	11983	46523	809
45	0.49459	01519	52843	42243	12817	33972	380
46	0.49470	48256	28921	28275	62727	12352	271
47	0.49481	47353	20994	93852	55551	62479	314
48	0.49492	01720	50462	62091	98704	33855	816
49	0.49502	14035	93756	57740	24894	94103	560
50	0.49511	86767	59025	82032	16794	80247	002
51	0.49521	22194	00239	07822	29676	27532	145
52	0.49530	22422	03390	47952	04278	44427	559
53	0.49538	89402	74349	68581	04041	77273	861
54	0.49547	24945	53599	38829	61990	73815	874
55	0.49555	30730	79495	72963	63700	62645	891
56	0.49563	08321	18649	86193	92157	91598	920
57	0.49570	59171	79463	11616	12654	64301	300
58	0.49577	84639	22674	12423	57247	74140	060
59	0.49584	85989	80928	37698	69533	05255	013
60	0.49591	64406	97805	58686	73411	32659	783
61	0.49598	20997	95393	83157	79612	86211	544
62	0.49604	56799	78345	45405	91520	47023	136
63	0.49610	72784	81358	00744	59187	40780	200
64	0.49616	69865	66169	17633	95921	87050	774
65	0.49622	48899	73416	76143	19808	79055	233
66	0.49628	10693	34076	08655	18361	58853	301
67	0.49633	56005	44632	99499	90621	17989	828
68	0.49638	85551	09668	69081	25598	83840	265
69	0.49644	00004	55112	72237	05012	13710	798
70	0.49649	00002	15053	67329	76538	53850	589

Table 6

$\Omega_{s-1/2}(s)$ to 33 Decimal Places for s = 1(1)71

s	$\Omega_{s-1/2}(s)$						
1	0.38094	90365	05182	31125	84084	80584	405
2	0.42087	15664	01657	91315	48831	90740	935
3	0.44028	85258	71415	95895	50423	33651	027
4	0.45194	66457	35447	22071	38133	11063	616
5	0.45975	96689	19334	85525	44196	29989	497
6	0.46537	22725	25789	29383	31851	26420	072
7	0.46960	37438	57229	11502	55752	09167	216
8	0.47290	99769	72621	81602	47079	65587	118
9	0.47556	54699	96385	20403	59274	86886	700
10	0.47774	56212	09021	92139	77556	88457	465
11	0.47956	78493	96015	59019	88032	91471	999
12	0.48111	37687	75968	36957	15191	24946	706
13	0.48244	18926	66516	60407	22432	34376	166
14	0.48359	52787	16618	02183	07341	12019	849
15	0.48460	63224	30966	09021	04564	67647	222
16	0.48549	98687	85039	93164	92992	50489	894
17	0.48629	52931	02886	10874	03385	02572	528
18	0.48700	79290	74069	02121	89607	46565	690
19	0.48765	00712	07156	86338	44724	56849	820
20	0.48823	16927	91874	03352	14697	95326	101
21	0.48876	09693	75279	16178	49424	34872	527
22	0.48924	46666	00258	93766	66296	82391	012
23	0.48968	84317	35399	21648	09377	65449	251
24	0.49009	70157	03706	77614	90349	45135	364
25	0.49047	44442	11657	34966	10591	35224	742
26	0.49082	41510	97160	11895	31759	23936	260
27	0.49114	90832	86459	50990	72524	81167	680
28	0.49145	17841	73211	98027	19537	92499	992
29	0.49173	44604	25615	17744	06002	16384	265
30	0.49199	90359	42358	82016	29630	01629	384
31	0.49224	71957	52793	37482	39202	02959	974
32	0.49248	04219	82525	94197	50253	28716	825
33	0.49270	00235	09089	39551	76358	71382	057
34	0.49290	71605	62858	21645	50412	22361	863
35	0.49310	28652	50850	64651	90394	75229	230
36	0.49328	80587	70719	55477	54339	51769	357
37	0.49346	35659	21485	73556	16964	37483	347
38	0.49363	01273	93746	11621	23758	61902	861
39	0.49378	84102	26002	60075	67930	07165	970
40	0.49393	90167	38668	82860	61206	44781	144
41	0.49408	24921	98243	17216	45712	51803	731
42	0.49421	93314	17376	73459	10506	76176	781
43	0.49434	99844	59328	78810	04501	79032	385
44	0.49447	48615	85481	31322	79053	61271	669
45	0.49459	43375	60573	45674	25125	64351	832
46	0.49470	87554	10884	57670	89546	86621	771
47	0.49481	84297	14790	94625	75209	15482	169
48	0.49492	36494	92208	50571	69906	65241	865
49	0.49502	46807	48836	33073	74812	67568	773
50	0.49512	17687	22380	53854	33606	26482	911
51	0.49521	51398	70709	31009	35672	98323	825
52	0.49530	50036	35883	49014	83093	41529	267
53	0.49539	15540	12998	22562	52184	31750	661
54	0.49547	49709	48578	66492	18613	24580	245
55	0.49555	54215	89751	57811	39576	08720	853
56	0.49563	30614	02447	52267	65405	03851	233
57	0.49570	80351	74379	70375	74136	19547	145
58	0.49578	04779	16418	47739	73481	71117	811
59	0.49585	05156	74171	19278	05994	38274	241
60	0.49591	82662	60033	82070	43107	57008	342
61	0.49598	38399	14660	81089	69645	16761	408
62	0.49604	73399	05667	54316	87565	49431	810
63	0.49610	88630	70406	31990	92298	21996	887
64	0.49616	85003	08817	76052	14340	61930	442
65	0.49622	63370	31634	41454	53663	57344	697
66	0.49628	24535	68585	44243	15307	25587	801
67	0.49633	69255	40706	09034	51410	66907	508
68	0.49638	98242	00381	43894	39806	29727	866
69	0.49644	12167	42340	44188	47836	99533	578
70	0.49649	11665	88455	05314	98935	27761	826
71	0.49653	97336	48883	03848	22493	30222	813

Table 7

$\Omega_{s+\frac{1}{2}}(s-\frac{1}{2})$ to 33 Decimal Places for s = 1(1)70

s	$\Omega_{s+\frac{1}{2}}(s-\frac{1}{2})$						
1	0.24392	23488	71790	51540	71859	45988	980
2	0.37138	03365	91535	18854	89218	37340	105
3	0.41474	20232	48063	40592	53097	96152	739
4	0.43637	75166	27196	12860	63380	23145	432
5	0.44929	15433	13196	00627	92476	42788	078
6	0.45785	82262	15869	26273	56681	33936	398
7	0.46395	17601	67391	07167	32804	49924	146
8	0.46850	63528	91987	69945	97220	43522	058
9	0.47203	90163	04972	25225	38015	05352	807
10	0.47485	87672	01491	61866	60531	37422	919
11	0.47716	15516	41086	24720	38422	98913	848
12	0.47907	75808	02990	32725	28224	54639	486
13	0.48069	67435	74966	36738	64515	67556	343
14	0.48208	30843	16534	03643	72169	25729	208
15	0.48328	34643	08572	83194	01386	91684	050
16	0.48433	29603	82061	49758	98784	89913	958
17	0.48525	83492	99305	56353	89676	84688	271
18	0.48608	04252	55366	83431	93452	99632	353
19	0.48681	55821	44125	39366	60361	51308	429
20	0.48747	69189	75470	86435	31093	48149	170
21	0.48807	50280	44774	86838	88197	05145	916
22	0.48861	85672	07902	36780	30096	83719	674
23	0.48911	46822	32712	29503	79777	37943	741
24	0.48956	93231	27341	33179	70914	92593	261
25	0.48998	74842	46524	25414	39504	68223	419
26	0.49037	33887	74725	88033	96321	11532	197
27	0.49073	06320	59979	93872	57284	76117	514
28	0.49106	22941	22600	37467	60453	15248	196
29	0.49137	10288	05214	28157	63289	73678	863
30	0.49165	91350	32899	86404	71409	30992	145
31	0.49192	86142	36335	75359	86387	81408	339
32	0.49218	12169	74434	60846	89530	40557	081
33	0.49241	84810	54548	21958	42792	19280	206
34	0.49264	17629	05979	83651	89687	18552	885
35	0.49285	22635	60054	11493	56274	89175	317
36	0.49305	10502	88416	36874	33787	21451	061
37	0.49323	90747	23214	28612	49818	59572	692
38	0.49341	71881	08947	22902	72338	64546	008
39	0.49358	61542	02129	55010	30804	08513	031
40	0.49374	66602	31421	87094	30414	07301	466
41	0.49389	93262	50166	75949	86759	55159	918
42	0.49404	47131	49886	76957	67885	76461	657
43	0.49418	33295	53221	32537	38468	13686	828
44	0.49431	56377	64965	29267	03169	28695	517
45	0.49444	20589	28039	28271	21522	15235	066
46	0.49456	29775	05630	91499	98382	86086	558
47	0.49467	87451	90066	42020	21030	01390	083
48	0.49478	96843	22177	77480	66026	02502	433
49	0.49489	60908	91227	36755	39563	06516	712
50	0.49499	82371	74220	53472	28873	41532	209
51	0.49509	63740	64190	58048	46963	22118	425
52	0.49519	07331	29398	45832	76122	03166	743
53	0.49528	15284	39047	04381	07262	24998	865
54	0.49536	89581	85826	64928	36986	31760	341
55	0.49545	32061	31191	22533	26525	37757	062
56	0.49553	44428	95558	62825	12439	17292	394
57	0.49561	28271	12508	42755	35784	44534	617
58	0.49568	85064	63415	74703	60543	66468	798
59	0.49576	16186	06727	24865	73986	42151	544
60	0.49583	22920	14188	37106	10981	65534	919
61	0.49590	06467	24714	25420	44735	91557	146
62	0.49596	67950	25215	13482	68576	41440	577
63	0.49603	08420	66503	02010	63411	71267	705
64	0.49609	28864	21389	20650	85701	82425	452
65	0.49615	30205	91206	01788	29561	42213	114
66	0.49621	13314	66229	58967	07882	92263	670
67	0.49626	79007	44825	74185	94211	40128	981
68	0.49632	28053	15572	97620	54200	53280	577
69	0.49637	61176	06122	80982	30364	28010	039
70	0.49642	79059	02127	40575	58387	72562	731

Table 8

$\Omega_{s-\frac{1}{2}}(s-\frac{1}{2})$ to 33 Decimal Places for s = 1(1)70

s	$\Omega_{s-\frac{1}{2}}(s-\frac{1}{2})$						
1	0.32323	72933	09586	62192	05247	32532	542
2	0.40537	52708	59595	08433	03378	20469	080
3	0.43311	66320	35907	32375	85704	63945	324
4	0.44783	33459	77309	79701	69974	25788	915
5	0.45710	23880	98000	99020	89006	45128	492
6	0.46351	90593	13754	49261	86923	71202	488
7	0.46824	01267	12161	02149	03475	01374	579
8	0.47186	59607	57270	73104	44378	61557	229
9	0.47474	12814	82946	85228	73623	90994	240
10	0.47707	89288	94332	96447	05316	95498	544
11	0.47901	77516	22283	91609	03950	21128	612
12	0.48065	23379	93210	54529	29974	39618	106
13	0.48204	94234	75175	63074	82141	38219	924
14	0.48325	74780	04301	62696	32973	25741	842
15	0.48431	25758	47630	64794	66028	09092	686
16	0.48524	21320	67184	74049	93486	81756	645
17	0.48606	73607	48076	76352	37679	59770	996
18	0.48680	49387	87621	18618	76403	47663	819
19	0.48746	81600	95764	97497	13090	26725	637
20	0.48806	77538	05812	58295	73416	16936	272
21	0.48861	24755	28170	42810	21236	54488	997
22	0.48910	95419	69947	98476	32357	41321	509
23	0.48956	49553	59385	83642	74893	46130	764
24	0.48998	37489	96420	35078	66908	86879	006
25	0.49037	01754	60037	39973	08919	66726	320
26	0.49072	78525	29995	87810	02143	57423	960
27	0.49105	98775	19921	64770	76927	79449	157
28	0.49136	89177	30698	38485	60800	52353	740
29	0.49165	72826	53272	51567	23729	28987	917
30	0.49192	69820	81433	37713	47722	99598	142
31	0.49217	97732	44359	76116	17874	97401	543
32	0.49241	71993	07617	42321	33537	47848	820
33	0.49264	06210	33693	57910	55550	49578	292
34	0.49285	12429	80302	60391	25631	60091	588
35	0.49305	01353	05983	39697	48643	61059	590
36	0.49323	82520	19522	77380	79510	10829	732
37	0.49341	64463	32375	16268	20555	70201	454
38	0.49358	54836	37124	67911	13094	64563	164
39	0.49374	60525	29759	08079	90139	33219	038
40	0.49389	87742	11515	37209	58052	87777	287
41	0.49404	42105	41734	33565	36979	21948	265
42	0.49418	28709	62384	53807	50174	12293	038
43	0.49431	52184	74586	71953	27813	32530	271
44	0.49444	16748	15250	91348	26969	34132	677
45	0.49456	26249	56057	40064	21977	03663	680
46	0.49467	84210	26112	66005	46300	43676	857
47	0.49478	93857	42649	43202	70266	57886	363
48	0.49489	58154	20308	02178	48424	38436	186
49	0.49499	79826	18204	91604	62239	10582	939
50	0.49509	61384	74671	93436	19064	50188	708
51	0.49519	05147	71846	52665	19262	00250	093
52	0.49528	13257	65904	40386	44275	64185	321
53	0.49536	87698	13405	23611	28101	28508	358
54	0.49545	30308	19775	52024	58256	06978	288
55	0.49553	42795	32223	25399	04746	65158	406
56	0.49561	26746	96240	46119	21550	37504	139
57	0.49568	83640	92199	64569	70318	44305	036
58	0.49576	14854	66305	80033	90774	76523	808
59	0.49583	21673	68258	84638	35523	62109	989
60	0.49590	05299	06356	78503	01091	46508	597
61	0.49596	66854	29381	75235	25331	35600	245
62	0.49603	07391	43421	92675	29756	30692	613
63	0.49609	27896	70760	68126	02113	84112	391
64	0.49615	29295	57084	62975	57543	62514	144
65	0.49621	12457	32502	73965	07500	38605	481
66	0.49626	78199	31211	47489	20212	52889	148
67	0.49632	27290	74070	91363	31602	45040	455
68	0.49637	60456	17861	28623	88801	39875	857
69	0.49642	78378	74557	72939	11762	08900	257
70	0.49647	81703	03584	23660	60173	32888	076

B L A N K P A G E

APPENDIX D

VALUES OF THE FRESNEL INTEGRALS $S_2(x)$, $C_2(x)$, $S(x)$, AND $C(x)$

Table 9

$S_2(x)$ to 28 Decimal Places for $x = 1(1)70$

x	$S_2(x)$					
1	0.24755	82876	51610	84260	99050	144
2	0.56284	89062	30056	47929	80811	091
3	0.71168	50216	07530	03251	62245	900
4	0.64211	87357	44514	69533	18002	859
5	0.46594	14967	66258	53239	02386	006
6	0.34985	23653	53978	11417	86162	438
7	0.38119	44739	44967	60991	63048	088
8	0.51200	96184	67464	11649	82642	631
9	0.61721	36970	24189	61244	00929	011
10	0.60843	62590	65110	89720	19547	955
11	0.50478	63386	47342	03809	68183	456
12	0.40581	10077	59143	22235	53644	027
13	0.39826	77211	08448	47737	84957	116
14	0.48176	94215	59744	45667	96895	665
15	0.57580	32698	07805	48219	77229	457
16	0.59612	65594	98017	19736	48629	640
17	0.52925	92129	08924	91400	19054	543
18	0.43998	93396	82881	69933	05126	288
19	0.40933	64957	30567	52374	19924	504
20	0.46164	57788	15957	76010	44402	616
21	0.54588	38021	13002	40369	03426	682
22	0.58493	89064	87810	39959	25854	993
23	0.54578	17221	88624	21912	35877	295
24	0.46702	84356	61254	99740	12891	612
25	0.42121	70480	22836	05724	64907	451
26	0.44830	00011	91069	73629	28809	124
27	0.52105	36692	33784	59264	68564	828
28	0.57214	20631	62520	77264	59409	563
29	0.55621	23973	19162	09274	77896	506
30	0.48996	86291	00923	19993	94160	139
31	0.43497	25874	60339	12318	21081	032
32	0.44060	47712	22625	03026	26065	462
33	0.49987	28381	15131	66742	07038	097
34	0.55748	94930	84749	63207	44223	545
35	0.56131	33650	04911	49641	16543	190
36	0.50941	67298	57160	52096	68003	820
37	0.45039	59263	07966	73341	50882	260
38	0.43797	07054	62434	94397	35566	333
39	0.48218	72763	36492	42823	11962	764
40	0.54146	35717	53990	76653	46122	669
41	0.56160	84504	15086	92834	37102	012
42	0.52528	21742	89865	59053	11247	283
43	0.46682	88117	93015	92018	95391	349
44	0.43987	77286	99008	64373	95784	992
45	0.46820	89680	25314	90768	70354	210
46	0.52483	65418	06007	76809	09045	621
47	0.55764	98008	25884	38939	19197	444
48	0.53730	91333	90219	36635	11992	529
49	0.48342	78283	38064	54076	26341	343
50	0.44572	17064	23986	86844	98133	357
51	0.45818	63426	86199	52226	53223	318
52	0.50849	05833	97445	31036	71419	089
53	0.55010	31159	32906	02708	32485	867
54	0.54529	15267	88878	36961	76398	576
55	0.49929	83294	66241	99614	21360	227
56	0.45477	28701	94827	08670	61660	020
57	0.45225	89692	98187	61020	75946	090
58	0.49331	04258	70790	46170	16743	778
59	0.53975	96898	54026	58435	74679	725
60	0.54917	28343	71156	41336	91662	885
61	0.51358	50973	71291	51284	15073	259
62	0.46618	46490	42827	48925	95156	654
63	0.45038	95102	37434	21969	85256	357
64	0.48010	44103	94744	09082	00212	337
65	0.52751	23621	55070	17331	76681	660
66	0.54909	05218	84463	34413	71882	741
67	0.52554	20559	11094	37262	06700	536
68	0.47902	89894	85267	50275	31239	347
69	0.45233	78517	17457	93249	73496	181
70	0.46954	28510	17320	62465	64952	910

Table 10

$C_2(x)$ to 28 Decimal Places for $x = 1(1)70$

x	$C_2(x)$					
1	0.72170	59242	92605	08777	15858	156
2	0.75330	23754	67891	16558	21899	711
3	0.56102	03289	78138	66929	91502	047
4	0.36819	29762	80974	79631	06624	017
5	0.32845	66248	67552	60617	66040	539
6	0.44327	38563	37623	33740	30799	535
7	0.59011	60610	93977	28750	27047	081
8	0.63930	12479	30604	90750	78986	021
9	0.56080	39810	63954	86486	90870	450
10	0.43696	39527	29382	03550	07688	183
11	0.38039	18718	58184	33069	19940	790
12	0.43455	73415	13101	06382	98818	020
13	0.54251	04114	00767	86698	45105	819
14	0.60472	09589	34283	43617	62030	143
15	0.56933	60588	83420	11025	14977	264
16	0.47431	07173	20327	99317	30365	277
17	0.40798	54159	55980	92358	30735	035
18	0.42783	71578	92569	44281	65610	037
19	0.51133	18949	15923	94675	18493	394
20	0.58038	89720	04910	94064	51525	069
21	0.57384	06247	62014	25706	02197	786
22	0.50116	67664	65156	16986	40365	593
23	0.43066	21163	53179	33670	43223	754
24	0.42563	49063	11197	51548	13703	732
25	0.48787	98923	51789	83957	93421	219
26	0.55862	83863	27546	46744	83741	890
27	0.57376	57770	37074	23404	53464	846
28	0.52169	49544	74128	78657	84831	103
29	0.45183	15477	50914	35713	16176	407
30	0.42790	80908	40306	14524	17381	495
31	0.47001	91383	09000	47664	25911	705
32	0.53794	44618	53456	89560	93373	956
33	0.56940	72903	39672	13076	90869	300
34	0.53702	65412	69461	25664	37707	963
35	0.47201	16032	70986	94618	12969	742
36	0.43421	19897	83205	15912	46474	578
37	0.45713	95302	72218	30841	58459	369
38	0.51835	89847	44665	75610	03439	870
39	0.56132	10368	23016	28421	80192	047
40	0.54750	32143	63865	06819	29762	110
41	0.49087	00405	95054	71618	94969	793
42	0.44389	70230	92958	08432	69774	941
43	0.44902	49039	25601	05219	36799	969
44	0.50038	22120	28141	13522	09821	414
45	0.55023	87665	70790	24448	70103	605
46	0.55330	10449	49385	26102	17100	530
47	0.50780	17801	24741	49054	92799	134
48	0.45615	97793	41237	37822	41076	172
49	0.44548	63431	58052	45336	22627	428
50	0.48465	78973	19108	24740	37663	118
51	0.53702	44360	46413	29862	68174	358
52	0.55465	50271	56142	18161	73739	417
53	0.52216	45976	31517	94804	81338	592
54	0.47008	79288	91381	21654	77886	721
55	0.44622	22922	92059	24194	99293	190
56	0.47179	66342	84999	33138	70715	460
57	0.52262	55347	21282	33912	91202	494
58	0.55194	50050	09827	38847	56842	734
59	0.53340	27306	60555	39477	20239	399
60	0.48471	28776	40926	32055	56984	179
61	0.45076	92343	06427	85096	69328	634
62	0.46228	12441	60707	11789	76822	269
63	0.50801	71379	34074	63602	11897	379
64	0.54571	87525	01344	78406	81929	904
65	0.54112	03685	03845	46865	95484	807
66	0.49906	79604	75024	56442	92846	118
67	0.45849	83107	20878	14222	99509	401
68	0.45640	98730	80451	48931	43025	168
69	0.49414	26457	28350	82197	34184	535
70	0.53668	00384	69336	38967	16854	139

Table 11

S(x) and C(x) to 28 Decimal Places for x = 1(1)6

x	S(x)					
1	0.43825	91473	90354	76607	67566	966
2	0.34341	56783	63698	24219	53008	160
3	0.49631	29989	67375	03609	76122	653
4	0.42051	57542	46928	42444	53431	407
5	0.49919	13819	17116	88675	19283	805
6	0.44696	07612	36930	27762	39202	878

x	C(x)					
1	0.77989	34003	76822	82947	42064	137
2	0.48825	34060	75340	75450	02235	034
3	0.60572	07892	97685	62955	61610	743
4	0.49842	60330	38177	61553	07095	868
5	0.56363	11887	04012	23110	21074	044
6	0.49953	14678	55501	12018	82799	033

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APPENDIX E

VALUES OF THE ROCKET FUNCTIONS rr(x) AND ri(x)

Table 12
rr(x) to 28 Decimal Places for x = 1(1)70

x	rr(x)					
1	0.80952	54817	47408	84437	07957	597
2	0.64290	39596	19896	52163	18463	093
3	0.54689	05730	71946	22244	14808	288
4	0.48289	41401	75925	91510	32300	344
5	0.43654	99085	67707	70674	44352	460
6	0.40110	42924	03541	95840	07503	263
7	0.37291	81029	38842	08094	75218	618
8	0.34984	06791	01947	67584	61958	551
9	0.33051	22229	54062	42492	68511	661
10	0.31402	71771	57729	61278	61320	242
11	0.29975	70820	42102	17696	53920	582
12	0.28725	08576	48330	44498	57282	972
13	0.27617	55473	28321	18721	75088	888
14	0.26627	94996	08486	15680	27268	433
15	0.25736	86190	92354	95019	46576	842
16	0.24929	05505	03437	93323	20925	053
17	0.24192	38515	20391	99022	67697	142
18	0.23517	04003	37679	35184	51200	161
19	0.22894	99575	48894	48288	10150	742
20	0.22319	61971	37070	16879	81356	583
21	0.21785	37606	00010	25964	92287	313
22	0.21287	60371	75852	92268	15848	830
23	0.20822	34681	93205	78971	87397	061
24	0.20386	22356	16878	45084	78548	812
25	0.19976	32361	88183	50051	62472	303
26	0.19590	12705	98034	55402	41557	422
27	0.19225	43964	77024	92595	62592	194
28	0.18880	34075	41892	22591	69988	096
29	0.18553	14108	71212	15994	75441	831
30	0.18242	34812	19687	58435	72522	904
31	0.17946	63763	35949	14568	47297	488
32	0.17664	83009	75158	59792	77446	574
33	0.17395	87100	73327	39592	17352	162
34	0.17138	81436	38055	43971	34917	309
35	0.16892	80874	95268	21051	73220	011
36	0.16657	08552	45015	87165	56659	428
37	0.16430	94877	16058	08607	09497	297
38	0.16213	76669	37462	07898	74546	343
39	0.16004	96422	16125	84048	74788	081
40	0.15804	01663	59231	43665	31097	748
41	0.15610	44404	37900	09982	07432	153
42	0.15423	80657	73633	21337	62754	006
43	0.15243	70020	58290	33190	87645	769
44	0.15069	75307	03436	49178	60596	825
45	0.14901	62226	65148	18466	26453	352
46	0.14738	99101	12959	77852	53307	175
47	0.14581	56614	12127	94302	46490	700
48	0.14429	07589	71146	68689	98650	358
49	0.14281	26795	74886	68418	75334	633
50	0.14137	90768	80569	84625	30258	220
51	0.13998	77658	01178	04148	15122	773
52	0.13863	67085	40554	53701	05843	250
53	0.13732	40020	77769	44800	47469	281
54	0.13604	78669	26399	85154	92903	086
55	0.13480	66370	18122	94555	79616	398
56	0.13359	87505	80169	83365	24961	205
57	0.13242	27418	93336	64764	12989	654
58	0.13127	72338	31889	17658	44409	687
59	0.13016	09310	99280	29348	56438	095
60	0.12907	26140	83959	17500	54573	581
61	0.12801	11332	70213	33328	58621	229
62	0.12697	54041	44177	95726	67986	712
63	0.12596	44025	45563	01319	99743	327
64	0.12497	71604	18846	71724	11699	542
65	0.12401	27619	24298	34381	73472	286
66	0.12307	03398	73336	56796	05047	184
67	0.12214	90724	56746	71886	19448	497
68	0.12124	81802	37751	84822	31142	085
69	0.12036	69233	84976	57388	67198	257
70	0.11950	45991	23017	21488	40045	180

Table 13
ri(x) to 28 Decimal Places for x = 1(1)70

x	ri(x)					
1	0.23219	93900	55264	60574	32063	867
2	0.12097	64818	07033	76319	32619	889
3	0.07654	40951	56604	86935	00467	648
4	0.05364	43356	69216	54762	87781	247
5	0.04010	81659	38640	71691	64769	012
6	0.03136	58945	10539	82203	24798	035
7	0.02535	43867	30896	71710	79174	939
8	0.02102	18912	00551	89748	68998	336
9	0.01778	31668	47962	82735	91670	314
10	0.01529	00854	25035	00877	38645	249
11	0.01332	43365	91499	17457	61557	560
12	0.01174	30120	94976	02196	06721	501
13	0.01044	91661	38869	68480	06791	008
14	0.00937	50215	82057	73966	09740	590
15	0.00847	19555	30599	12141	48528	570
16	0.00770	42979	62835	39589	78126	643
17	0.00704	53645	75548	69409	60608	032
18	0.00647	48459	02052	99440	11483	316
19	0.00597	70456	00652	50357	70984	290
20	0.00553	96651	12159	64306	97010	495
21	0.00515	29484	86711	51256	62336	537
22	0.00480	90698	45615	00389	37531	232
23	0.00450	16875	25886	43017	85235	385
24	0.00422	56147	75816	70614	14433	044
25	0.00397	65732	71078	51524	20173	669
26	0.00375	10063	55584	34383	42606	690
27	0.00354	59359	37727	93299	97118	753
28	0.00335	88516	95676	39523	62244	224
29	0.00318	76244	70800	50126	07121	050
30	0.00303	04379	63450	60117	35835	536
31	0.00288	57344	15269	02436	80011	621
32	0.00275	21710	78964	41265	99089	467
33	0.00262	85850	80166	39797	67459	812
34	0.00251	39648	60739	26650	48462	783
35	0.00240	74268	22774	33442	11591	576
36	0.00230	81961	11490	53443	06685	630
37	0.00221	55907	14171	90649	95644	755
38	0.00212	90082	32701	53703	50956	577
39	0.00204	79148	24618	58491	69121	772
40	0.00197	18359	13000	18740	18204	905
41	0.00190	03483	46884	33321	61681	161
42	0.00183	30737	57279	46267	50897	461
43	0.00176	96729	03386	68334	15827	008
44	0.00170	98408	42714	24438	78159	178
45	0.00165	33027	89705	58024	95137	415
46	0.00159	98105	52153	01146	71717	223
47	0.00154	91394	54412	13999	62443	381
48	0.00150	10856	72324	17712	56830	415
49	0.00145	54639	17607	77796	39517	133
50	0.00141	21054	19927	18873	23382	883
51	0.00137	08561	63368	71105	00295	887
52	0.00133	15753	41045	10417	93328	650
53	0.00129	41339	97298	19851	31088	522
54	0.00125	84138	31721	13788	05594	289
55	0.00122	43061	43161	75861	01017	747
56	0.00119	17108	95147	89385	23530	276
57	0.00116	05358	86914	16756	03474	849
58	0.00113	06960	16504	85105	31597	835
59	0.00110	21126	24356	69779	18134	425
60	0.00107	47129	07392	65083	35755	750
61	0.00104	84293	95033	43229	51151	260
62	0.00102	31994	79701	32542	07556	470
63	0.00099	89649	95383	40262	07899	511
64	0.00097	56718	38668	36445	91560	357
65	0.00095	32696	27395	40067	82002	398
66	0.00093	17113	92674	39900	89940	650
67	0.00091	09533	00570	45784	89916	892
68	0.00089	09544	00205	36340	17718	375
69	0.00087	16763	95425	62288	75451	465
70	0.00085	30834	37530	13937	73609	078

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Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1 ORIGINATING ACTIVITY (Corporate author)

Naval Ship Research and Development Center
Bethesda, Maryland 20034

2a. REPORT SECURITY CLASSIFICATION

UNCLASSIFIED

2b. GROUP

3 REPORT TITLE

The Converging Factors for the Fresnel Integrals

4 DESCRIPTIVE NOTES (Type of report and inclusive dates)

Final

5 AUTHOR(S) (First name, middle initial, last name)

John W. Wrench, Jr.
Vicki Alley

6 REPORT DATE	7a. TOTAL NO OF PAGES	7b. NO OF REFS
June 1973		9

8a CONTRACT OR GRANT NO	9a. ORIGINATOR'S REPORT NUMBER(S)
b PROJECT NO SR 0140301, Task 15324	Report 4102
c.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
d	

10 DISTRIBUTION STATEMENT

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11 SUPPLEMENTARY NOTES	12 SPONSORING MILITARY ACTIVITY
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13 ABSTRACT

The theory of the converging factors for the Fresnel integrals is developed from that of the converging factors for the sine and cosine integrals, and is then applied to the calculation on a CDC 6700 system of tables of these factors and their reduced derivatives to about 35 decimal places. The factors were used in conjunction with appropriately truncated asymptotic series to produce appendixed 28-place tables of the Fresnel integrals $S_2(x)$, $C_2(x)$ and of the closely related rocket functions rr(x) and ri(x), for successive integer values of x from 1 through 70. An abridged 28-place table of S(x) and C(x), for x ranging from 1 through 6, is also included.

DD FORM 1 NOV 65 1473 (PAGE 1)

S/N 0101-807-6801

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Fresnel Integrals						
Asymptotic Expansions						
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Rocket Functions						

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