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A DIGITAL COMPUTER TECHNIQUE FOR PREDICTION OF STANDARD MANEUVERS OF SURFACE SHIPS

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in the expansion. If the expansion is limited to the first order terms, the well-known linearized expansion will be obtained.

If straight ahead motion at constant speed with rudder amidships is chosen as the initial equilibrium condition, the linearized expansion of the forces and moment (Equation (4)) becomes

$$X = X_* + X_u \Delta u + X_v v + X_r r + X_{\dot{u}} \dot{u} + X_{\dot{v}} \dot{v} + X_{\dot{r}} \dot{r} + X_\delta \delta \qquad (5)$$

where $\Delta u = (u - u_1)$, with similar expressions for Y and N.

Similarly, the Taylor expansion, including terms up to third order, becomes

$$\begin{aligned} X = X_* &+ \left[X_u \Delta u + X_v v + X_r r + X_{\dot{u}} \dot{u} + X_{\dot{v}} \dot{v} + X_{\dot{r}} \dot{r} + X_\delta \delta \right] \\ &+ \frac{1}{2!} \left[X_{uu} \Delta u^2 + X_{vv} v^2 + \dots\dots X_{\delta\delta} \delta^2 + \right. \\ &\quad \left. 2 \cdot X_{uv} \Delta u \cdot v + 2 \cdot X_{ur} \Delta u \cdot r + \dots\dots + 2 \cdot X_{\dot{r}\delta} \dot{r} \delta \right] \\ &+ \frac{1}{3!} \left[X_{uuu} \Delta u^3 + X_{vvv} v^3 + \dots\dots + X_{\delta\delta\delta} \delta^3 + \right. \\ &\quad 3 \cdot X_{uuv} \Delta u^2 v + 3 \cdot X_{uur} \Delta u^2 r + \dots\dots + 3 \cdot X_{\dot{r}\delta\delta} \dot{r} \delta^2 + \\ &\quad \left. 6 \cdot X_{uvr} \Delta u \cdot vr + 6 \cdot X_{uv\dot{u}} \Delta u \cdot u\dot{v} + \dots\dots + 6 \cdot X_{\dot{v}\dot{r}\delta} \dot{v} \dot{r} \delta \right] \end{aligned} \qquad (6)$$

with similar expressions for Y and N.

LINEAR MATHEMATICAL MODEL FOR STEERING AND MANEUVERING

Equating the linearized expansion, Equation (5), with the dynamic response terms given on the right-hand side of the equations of motion, Equations (3), and neglecting dynamic response of second-order smallness in the same way as second-order terms have been neglected in the force and moment expansions, the linearized equations of motion for steering and maneuvering are obtained

NONLINEAR MATHEMATICAL MODEL

To obtain realistic predictions of maneuvers such as tight turns for large rudder angles and to predict the performance of a dynamically unstable ship, it becomes necessary to develop and solve a nonlinear mathematical model, which includes higher order terms in the Taylor expansion of forces and moments.

The nonlinear mathematical model used as a basis for the computer program has been based on a Taylor expansion of forces and moments including terms of up to third order; see Equations (6). The inclusion of terms higher than third order was not considered to increase the accuracy of prediction significantly. Furthermore, practical limitations of measurement techniques and the state of refinement of present theory did not justify the inclusion of higher terms.

Symmetry considerations demonstrate that the X-equation should be an even function of the parameters v, r, δ, $\dot{v}$, and $\dot{r}$; similarly, the Y- and N-equations are odd functions of the same parameters. Consequently, odd terms in v, r, δ, $\dot{v}$, and $\dot{r}$ have been eliminated from the X-equation, and even terms in the same parameters from the Y- and N-equations. An alternative solution would have been to introduce absolute values of the parameters v, r, δ, $\dot{v}$, and $\dot{r}$ into the equations, but this was considered less attractive.

As a further consequence of the body symmetry, Y_u, Y_{uu}, Y_{uuu}, $Y_{\dot{u}}$ and corresponding derivatives in the moment equation N_u, N_{uu}, N_{uuu}, $N_{\dot{u}}$ are all zero.

An unsymmetrical force (for instance, the side force from a single propeller) has been taken into account by constant terms Y_* and N_* in the Taylor expansion. An unsymmetrical side force has been considered a function of speed, and terms Y_{*u}, Y_{*uu}, N_{*u}, N_{*uu} have consequently been introduced into the mathematical model to facilitate that changes of side force with speed are taken into account.*

The nonlinear equations can be reduced further by considering the nature of the acceleration forces. Abkowitz states,[4] that no second or higher order acceleration terms can be expected. This is based on the assumption that there is no significant interaction between viscous and inertia properties of the fluid and that acceleration forces calculated from potential theory give only linear terms when applied to submerged bodies.

Abkowitz further reasons that terms representing cross-coupling between acceleration and velocity parameters are zero or negligibly small for reasons similar to those just given.

The validity of these basic considerations of Abkowitz' has been verified by the experimental measurements reported in Reference 5.

Equating the nonlinear Taylor expansion, Equations (6), with dynamic response terms, Equations (3), and taking the above considerations into account, the nonlinear equations of motion finally become

$$\text{X-Equation:} \quad (m-X_{\dot{u}})\dot{u} = f_1(u,v,r,\delta)$$

$$\text{Y-Equation:} \quad (m-Y_{\dot{v}})\dot{v} + (mx_G-Y_{\dot{r}})\dot{r} = f_2(u,v,r,\delta) \qquad (10)$$

*If an unsymmetrical force should turn out to be a function of other parameters than speed, this unsymmetry could easily be introduced into the present mathematical model. It would have been more difficult to do this if absolute values of the parameters had been applied.

N-Equation: $(mx_G - N_{\dot v})\dot v + (I_z - N_{\dot r})\dot r = f_3(u,v,r,\delta)$ (10) cont'd

where

$$f_1(u,v,r,\delta) = X_* + X_u\Delta u + \frac{1}{2}X_{uu}\Delta u^2 + \frac{1}{6}X_{uuu}\Delta u^3 +$$

$$\frac{1}{2}X_{vv}v^2 + (\frac{1}{2}X_{rr} + mx_G)r^2 + \frac{1}{2}X_{\delta\delta}\delta^2 + \frac{1}{2}X_{vvu}v^2\Delta u + \frac{1}{2}X_{rru}r^2\Delta u + \frac{1}{2}X_{\delta\delta u}\delta^2\Delta u +$$

$$(X_{vr} + m)vr + X_{v\delta}v\delta + X_{r\delta}r\delta + X_{vru}vr\Delta u + X_{v\delta u}v\delta\Delta u + X_{r\delta u}r\delta\Delta u$$

$$f_2(u,v,r,\delta) = Y_* + Y_{*u}\Delta u + Y_{*uu}\Delta u^2 +$$

$$Y_v v + \frac{1}{6}Y_{vvv}v^3 + \frac{1}{2}Y_{vrr}vr^2 + \frac{1}{2}Y_{v\delta\delta}v\delta^2 + Y_{vu}v\Delta u + \frac{1}{2}Y_{vuu}v\Delta u^2 +$$

$$(Y_r - mu)r + \frac{1}{6}Y_{rrr}r^3 + \frac{1}{2}Y_{rvv}rv^2 + \frac{1}{2}Y_{r\delta\delta}r\delta^2 + Y_{ru}r\Delta u + \frac{1}{2}Y_{ruu}r\Delta u^2 +$$

$$Y_\delta\delta + \frac{1}{6}Y_{\delta\delta\delta}\delta^3 + \frac{1}{2}Y_{\delta vv}\delta v^2 + \frac{1}{2}Y_{\delta rr}\delta r^2 + Y_{\delta u}\delta\Delta u + \frac{1}{2}Y_{\delta uu}\delta\Delta u^2 + Y_{vr\delta}vr\delta$$

$$f_3(u,v,r,\delta) = N_* + N_{*u}\Delta u + N_{*uu}\Delta u^2 +$$

$$N_v v + \frac{1}{6}N_{vvv}v^3 + \frac{1}{2}N_{vrr}vr^2 + \frac{1}{2}N_{v\delta\delta}v\delta^2 + N_{vu}v\Delta u + \frac{1}{2}N_{vuu}v\Delta u^2 +$$

$$(N_r - mx_G u)r + \frac{1}{6}N_{rrr}r^3 + \frac{1}{2}N_{rvv}rv^2 + \frac{1}{2}N_{r\delta\delta}r\delta^2 + N_{ru}r\Delta u + \frac{1}{2}N_{ruu}r\Delta u^2 +$$

$$N_\delta\delta + \frac{1}{6}N_{\delta\delta\delta}\delta^3 + \frac{1}{2}N_{\delta vv}\delta v^2 + \frac{1}{2}N_{\delta rr}\delta r^2 + N_{\delta u}\delta\Delta u + \frac{1}{2}N_{\delta uu}\delta\Delta u^2 + N_{vr\delta}vr\delta$$

PRINCIPLES FOR SOLUTION OF MATHEMATICAL MODEL USING DIGITAL COMPUTER

METHOD OF NUMERICAL SOLUTION

The mathematical model, Equations (10), can be solved with respect to the accelerations $\dot u$, $\dot v$, and $\dot r$, which become

$$\dot u = \frac{f_1(u,v,r,\delta)}{(m - X_{\dot u})}$$

$$\dot v = \frac{(I_z - N_{\dot r})\, f_2(u,v,r,\delta) - (mx_G - Y_{\dot r})\, f_3(u,v,r,\delta)}{(m - Y_{\dot v})(I_z - N_{\dot r}) - (mx_G - N_{\dot v})(mx_G - Y_{\dot r})} \quad (11)$$

$$\dot{r} = \frac{(m-Y_{\dot{v}})\, f_3(u,v,r,\delta) - (mx_G-N_{\dot{v}})\, f_2(u,v,r,\delta)}{(m-Y_{\dot{v}})(I_z-N_{\dot{r}}) - (mx_G-N_{\dot{v}})\,(mx_G-Y_{\dot{r}})} \qquad \text{(11) cont'd}$$

These solutions can be rewritten in the form

$$\begin{aligned}
\frac{du}{dt} &= g_1\left[\, t, u(t), v(t), r(t), \delta(t)\right] \\
\frac{dv}{dt} &= g_2\left[\, t, u(t), v(t), r(t), \delta(t)\right] \\
\frac{dr}{dt} &= g_3\left[\, t, u(t), v(t), r(t), \delta(t)\right]
\end{aligned} \qquad (12)$$

It is seen that the mathematical model has been reduced to a set of three first-order differential equations. An approximate numerical solution for this type of equations is readily obtained on a digital computer. The process in the numerical solution is that the values of u, v, and r at time $t+\Delta t$ are obtained from knowledge of the values of u, v, r, and δ at time t.

A simple first-order method has been applied in the computer program; the values at time $t+\Delta t$ are obtained simply by the first-order Taylor series expansion

$$\begin{aligned}
u(t+\Delta t) &= u(t) + \Delta t \cdot \dot{u}(t) \\
v(t+\Delta t) &= v(t) + \Delta t \cdot \dot{v}(t) \\
r(t+\Delta t) &= r(t) + \Delta t \cdot \dot{r}(t)
\end{aligned} \qquad (13)$$

This method is found to give adequate accuracy for the present type of differential equations, because of the fact that the accelerations $\dot{u}$, $\dot{v}$, and $\dot{r}$ vary only slowly with time. This is due to the large mass and inertia of a ship compared to the relatively small forces and moments produced by its control surfaces.

Furthermore, digital computers enable long repetitive calculations to be made fast and accurately, and any desired accuracy of the solutions can be obtained using small time intervals Δt.

CALCULATION PROCEDURE FOR PREDICTION OF TRAJECTORY

So far, the mathematical model has been developed in dimensional form. The development has on the other hand been completely general, and the equations are equally valid in the nondimensional form.*

In the computer program, the mathematical model has been adopted in its nondimensional form. To describe the calculation of a trajectory in nondimensional form on the basis of the nondimensional equations, the nondimensionalized form of a given quantity will be indicated by the prime of that quantity in the following discussion.

Assuming that a full set of nondimensional hydrodynamic coefficients (X_u', X_{uu}', Y_v', N_r', etc.) is available and that the rudder deflection δ is defined as a function of time, the first step in the calculation of the trajectory of a ship maneuver would be to define the initial condition, i.e., set the nondimensional values

$$
\begin{aligned}
u(t)' &= u(t)/u(t) \\
v(t)' &= v(t)/u(t) \\
r(t)' &= r(t)/(u(t)/LBP) \\
\delta(t)' &= \delta(t)
\end{aligned}
\qquad (14)
$$

VELOCITY SHOULD BE VELOCITY OF ORIGIN VS VELOCITY OF ORIGIN IN X DIRECTION STEC P 364

at time t=0. Having done this, the nondimensional accelerations $\dot{u}'$, $\dot{v}'$, and $\dot{r}'$ can be calculated from equations (11), and the corresponding accelerations in dimensional form from

*The velocity used for nondimensionalization should be the velocity at any time, t rather than the initial velocity.

$$\dot{u}(t) = \dot{u}(t)' \cdot (u(t)^2/LBP)$$

$$\dot{v}(t) = \dot{v}(t)' \cdot (u(t)^2/LBP) \tag{15}$$

$$\dot{r}(t) = \dot{r}(t)' \cdot (u(t)^2/LBP^2)$$

The new velocities in dimensional form at time t=Δt can be obtained from Equations (13) and the corresponding nondimensional values from Equations (14). The process is then repeated using the new values for u', v', r', and δ' in Equations (11), and so on. The values of the velocities at a time t are thus obtained from

$$u(t) = u(0) + \sum_{\tau=0}^{t-\Delta t} \dot{u}(\tau) \cdot \Delta t$$

$$v(t) = v(0) + \sum_{\tau=0}^{t-\Delta t} \dot{v}(\tau) \cdot \Delta t \tag{16}$$

$$r(t) = r(0) + \sum_{\tau=0}^{t-\Delta t} \dot{r}(\tau) \cdot \Delta t$$

where u(0), v(0), and r(0) are the values of u, v, and r at t=0, and τ represents intermediate values of time (between time, 0 and time, t-Δt) at which the accelerations $\dot{u}(t)$, $\dot{v}(t)$, and $\dot{r}(t)$ are determined.

The instantaneous coordinates of the path of the origin of the ship $x_{o0}(t)$ and $y_{o0}(t)$ relative to the fixed earth axes, the instantaneous radius of curvature R(t), angle of yaw $\psi(t)$, etc., can be obtained similarly from the velocities by using the formulas

$$\psi(t) = \psi(0) + \sum_{\tau=0}^{t-\Delta t} r(\tau) \cdot \Delta t$$

$$x_{o0}(t) = x_{o0}(0) + \sum_{\tau=0}^{t-\Delta t} \left[-v(\tau) \cdot \cos\psi(\tau) + (u(\tau) - u(0)) \cdot \sin\psi(\tau) \right] \cdot \Delta t \tag{17}$$

$$y_{o0}(t) = y_{o0}(0) + \sum_{\tau=0}^{t-\Delta t} \left[(u(\tau)-u(0)) \cdot \cos\psi(\tau) + v(\tau) \cdot \sin\psi(\tau) \right] \cdot \Delta t$$

$$R(t) = \frac{\sqrt{(u(t)+u(0))^2 + v(t)^2}}{r(t)} \qquad (17) \text{ cont'd}$$

The accuracy of the predicted trajectory can be controlled by running the calculation with different values of the time interval Δt. It is found that a high accuracy is easily obtainable, and a time interval of $\Delta t = 1$ sec has been chosen as standard in the computer program.

DEFINITION OF RUDDER DEFLECTION

It is necessary in the calculation of a ship trajectory, as mentioned above, to define the rudder deflection as a function of time. This has been accomplished in the computer program by assuming the rudder to move with a certain constant rate of deflection and assuming a certain timelag between the instant the rudder deflection is ordered, and the instant the rudder begins to move. A rudder deflection up to a certain given angle δ_{const} would be executed in the program as indicated in the following example:

$$\delta(t) = \delta(t_1) \qquad \text{until} \quad t > t_{lag} + t_1$$

$$\text{then } \delta(t) = \delta(t_1) + \text{rate} \cdot (t - t_1 - t_{lag}) \qquad \text{until } \delta(t) = \delta_{const}$$

$$\text{then } \delta(t) = \delta_{const}.$$

A rudder function of this type gives a close approximation to the actual time history of a ship's rudder when a certain maneuver is ordered on the bridge, and almost any practical rudder sequence encountered when considering ship maneuvers can be built up. The zig-zag maneuver can, for example, be built up as follows, using these principles:

$\delta(t) = \delta(t_1)$	until $t > t_{lag} + t_1$
then $\delta(t) = \delta(t_1) + rate \cdot (t - t_{lag} - t_1)$	until $\delta(t) = \delta_{const}$
┌→ then $\delta(t) = \delta_{const}$	until $t = t_2$ when $\psi = \delta_{const}$
then $\delta(t) = \delta_{const}$	until $t > t_{lag} + t_2$
then $\delta(t) = \delta_{const} - rate \cdot (t - t_{lag} - t_2)$	until $\delta(t) = -\delta_{const}$
then $\delta(t) = -\delta_{const}$	until $t = t_3$ when $\psi = -\delta_{const}$
then $\delta(t) = -\delta_{const}$	until $t > t_{lag} + t_3$
then $\delta(t) = -\delta_{const} + rate \cdot (t - t_{lag} - t_3)$	until $\delta(t) = \delta_{const}$
└─ then repeat.	

COEFFICIENTS IN MATHEMATICAL MODEL

EXPERIMENTAL TECHNIQUES FOR MEASUREMENT OF COEFFICIENTS

To perform the computations of ship maneuvers, it is necessary to know the various hydrodynamic derivatives (X_u, Y_v, N_{vvr}, etc.) which appear in the mathematical model, Equations (10). These coefficients depend largely upon the ship geometry and design, and in general they differ significantly from one hull form to another. For most of the coefficients, it is necessary to rely on model testing techniques of special nature in order to determine the values for the particular ship form.

The coefficients are by definition partial derivatives of a force or moment with respect to one or more of the motion parameters. To obtain the different coefficients, it is necessary to let the model execute various forced motions and to measure the forces and moments as functions of the different motion parameters. An example might illustrate this principle. For a model which has been towed at different specific drift angles, corresponding forces Y and moments N have been measured. Figure 1 shows the nondimensional values Y' and N' plotted as a function of the nondimensional side velocity $v'=v/u$. From these measurements, it is now possible to obtain the derivatives with respect to the side velocity v, namely, Y_v, Y_{vvv} as well as N_v and N_{vvv}. The derivatives are related in a simple manner to the coefficients in the third-order polynominals, which give the best curve fitting to the experimental values. Thus, if the third-order polynomials fitted, e.g., by a least squares procedure, are of the form:

$$Y' = a_0 + a_1 \cdot v' + a_3 \cdot v'^3 \qquad (18)$$
$$N' = b_0 + b_1 \cdot v' + b_3 \cdot v'^3$$

then the derivatives would be directly related to the polynom-coefficients as follows:

$$Y_v' = a_1 \qquad \frac{1}{6} Y_{vvv}' = a_3 \qquad (19)$$
$$N_v' = b_1 \qquad \frac{1}{6} N_{vvv}' = b_3.$$

Different testing facilities such as rotating arm, oscillators, and planar motion mechanism are capable of executing model tests with various types of forced motions. The most versatile instrumentation is probably the planar motion mechanism because any type of motion with respect to which derivatives are desired can be produced by this instrumentation. A detailed discussion of a planar motion mechanism and the technique for measuring the different derivatives for a surface ship is presented in Reference 6. Here it is sufficient to mention that measuring techniques are available, which different assumptions that depend upon the type of engine and the engine setting to be maintained during the maneuver.

The propeller thrust can thus be calculated, either assuming constant propeller revolutions or assuming the propeller torque to vary proportionally to the revolutions in a certain power. If torque is assumed to vary inversely proportional to propeller revolutions, the thrust values corresponding to a turbine power plant capable of maintaining a constant power output would be obtained. If torque is assumed to be constant during the maneuver, the corresponding condition for a Diesel power plant would be obtained.

SCALE EFFECTS

Most of the coefficients to be used in the mathematical model would be obtained from model tests, and in this connection it is reasonable to give some considerations to scale effects in the measurement of the coefficients.

The model tests would be conducted according to Froude's law, hence the Reynolds number would not be satisfied, and the possibility of Reynolds number effects should be recognized.

Tests with airfoils covering a wide range of Reynolds numbers indicate that change of Reynolds number apparently has no systematic effect on the lift-curve slope. However, the variation of maximum lift might be appreciable because separation or flow breakdown occur earlier for the relatively thicker boundary layer around a model body at the lower Reynolds number. These results from airfoil testing can be applied in the present discussion of scale effects, as most of the Y-forces and N-moments would be due to similar lift and circulation effects. Thus, according to the nature of the

Reynolds number effect, scale effects should not be expected for any of the first-order derivatives, e.g., Y_v, Y_r, Y_δ, N_v, N_r, N_δ, etc., which in general only represent lift slope characteristics. In the case of the higher order derivatives, however, the possibility of scale effects should be considered, as it is likely that these coefficients would be influenced if separation or flow breakdown occurred. Normally, higher order derivatives of the motion parameters v and r, for instance Y_{vvv}, Y_{rrr} etc., are determined for relatively small values of v and r corresponding to angles of attack before any separation effect takes place. For this reason, scale effects would probably be negligible also for these coefficients. This is not true for the rudder, as the rudder deflection for which rudder characteristics are measured also will cover the range of rudder breakdown. For the derivatives $Y_{\delta\delta\delta}$ and $N_{\delta\delta\delta}$, in particular, a rational correction for scale effects should be considered.

The maximum lift is sensitive to surface roughness, especially near the leading edge. Thus, model rudders should be finished as smooth as possible in order to operate in a well-defined condition and to obtain repeatable measurements. Similarly, the surface roughness of the full-scale rudder should be taken into consideration and corrected for as part of the above-mentioned correction of rudder derivatives $Y_{\delta\delta\delta}$ and $N_{\delta\delta\delta}$ for Reynolds number effect.

Model tests should be carried out for propeller revolutions corresponding to the ship propulsion point and not to the model propulsion point, which, e.g., normally would have to be applied using free-running, self-propelled models. The propeller slipstream can thus be correctly modeled. This has been found to be very important, as it has a great effect not only upon the rudder derivatives Y_δ and N_δ, but also upon the hull derivatives Y_v, Y_r, N_v and N_r.

As outlined previously, the coefficients X_u, X_{uu} and X_{uuu} in this computer program are calculated on the basis of the proper ship resistance values and a power assumption corresponding to the engine setting which would be attempted during an actual maneuver. As these coefficients are of prime importance in obtaining the correct speed reduction during a maneuver, it is found that a principal scale effect problem has thus been taken properly into account. This procedure would be contrary to the free-running model technique, where the difference between model and ship resistance would be a serious problem and result in the measurement of a too small speed reduction in model scale.

The foregoing discussion of factors influencing scale effect should indicate that it is possible to take scale effect problems into account in the determination of the different coefficients for the mathematical model. Present experience might be insufficient to introduce a correction for Reynolds number effect as suggested for the rudder derivatives $Y_{\delta\delta\delta}$ and $N_{\delta\delta\delta}$; nevertheless, a correction is thought to be feasible. It is emphasized that this is in contrast to the free-running model technique, where the scale effect problems caused by incorrect propulsion point, Reynolds number effects, etc., would be completely mixed up in the model results, leaving only very little room for introduction of scale effect corrections based on a proper physical understanding of the problem.

VARIATIONS OF COEFFICIENTS WITH SPEED

The computer program has been based on a solution of the mathematical model in nondimensional form; consequently, the coefficients used as input data to the program should be applied in their corresponding nondimensional form.

The calculation of a full-scale trajectory of a ship maneuver is based on dimensionalizing by the instantaneous forward velocity u(t); see Equations (14) and (15). When a certain speed loss takes place during a maneuver, forces and moments are thus basically considered as being proportional with the instantaneous speed squared, and coefficients such as Y_{vu}, Y_{vuu}, Y_{ru}, Y_{ruu}, $Y_{\delta u}$, $Y_{\delta uu}$, etc., which represent the change of forces and moments with speed, should only reflect the extent to which this proportionality does not hold true.

Measurements of the nondimensional coefficients Y_v', Y_r', N_v', and N_r' carried out for various ship models at different speed values have indicated that these coefficients are largely independent of speed. Thus coefficients Y_{vu}', Y_{vuu}', Y_{ru}', Y_{ruu}', N_{vu}', N_{vuu}', N_{ru}', and N_{ruu}', which should represent the change with speed, are negligible. Consequently, at present it has been found reasonable to eliminate these coefficients in the computer program.

For the rudder derivatives Y_δ' and N_δ', a noteworthy effect has been measured for a change in forward speed especially on ships where the rudder is situated in the propeller slipstream. Apparently, this is due to the fact that the propeller slipstream is nearly constant even for a considerable change of forward speed, because propeller revolutions are kept more or less constant during a maneuver. Thus, the velocity of the inflow to the rudder is not dependent on forward speed alone; consequently, the nondimensional coefficients Y_δ' and N_δ' must vary as a function of forward speed. The coefficients $Y_{\delta u}'$ and $N_{\delta u}'$, which represent the first order change of the rudder derivatives with speed, are for this reason thought to be of considerable importance, and they should be included in an experimental determination of the various coefficients.

The coefficients $Y_{\delta uu}'$, $N_{\delta uu}'$, representing only the second-order change of Y_{δ}' and N_{δ}' with speed, have nevertheless, been considered negligible and eliminated in the program.

The coefficients X_{vvu}', X_{rru}', $X_{\delta\delta u}'$, X_{vru}', $X_{v\delta u}'$, and $X_{\delta ru}'$ in the X-equation, which represent the change of X_{vv}', X_{rr}', $X_{\delta\delta}'$, X_{vr}', $X_{v\delta}'$, and $X_{\delta r}'$ with forward speed, have similarly been omitted from the computer program as they are thought be be of minor importance at least in comparison with the dominating coefficients X_{u}, X_{uu}, and X_{uuu}.

RESUMÉ OF COEFFICIENTS

The mathematical model developed in Equations (10) include 17 coefficients in the X-equation and 24 coefficients in each of the Y- and N-equations. As mentioned in the previous section, several of the coefficients representing change of nondimensional forces and moments with forward speed have been found negligible and are eliminated in the computer program.

Obviously, coefficients are of varying importance with respect to the accuracy of a prediction, and a classification of the coefficients has been attempted in the summary of the coefficients given in Tables 1-3, pages 25-27.

The tables also show the identifiers that have been used for the coefficients in the computer program as well as nondimensional factors and examples of the numerical values taken from Reference 5. The planar motion mechanism test technique, which could be used to measure the coefficients, is mentioned briefly.

Table 1 - Summary of Coefficients in X-Equation

Variable	X - Equation					Planar Motion Mechanism Test Technique or Calculation Method
	Taylor Expansion And Dynamic Response Terms	Identifier in FORTRAN Program (1)	Nondim. Factor	Nondim. Coeff. $\cdot 10^5$ from Example (2)	Relative Importance of Coeff. (3)	
$\dot{u}$	$(m-X_{\dot{u}})$	X UDOT	$\frac{1}{2}\rho LBP^3$	-840.0	I	Estimated from theory $X_{\dot{u}} \sim -0.05\ m$
Δu	X_u	X U	$\frac{1}{2}\rho LBP^2 u$	-120.0	I	Calculated on the basis of ship EHP-data and results from open-water propeller test.
Δu^2	$\frac{1}{2} X_{uu}$	X UU	$\frac{1}{2}\rho LBP^2$	45.0	I	
Δu^3	$\frac{1}{6} X_{uuu}$	X UUU	$\frac{1}{2}\rho LBP^2/u$	-10.3	I	
v^2	$\frac{1}{2} X_{vv}$	X VV	$\frac{1}{2}\rho LBP^2$	-898.8	MI	Static drift angle test
r^2	$(\frac{1}{2} X_{rr}+mx_G)$	X RR	$\frac{1}{2}\rho LBP^4$	18.0	MI	Pure yaw (angular motion) test
δ^2	$\frac{1}{2} X_{\delta\delta}$	X DD	$\frac{1}{2}\rho LBP^2 u^2$	-94.8	MI	Static drift angle test
$v^2\Delta u$	$\frac{1}{2} X_{vvu}$		$\frac{1}{2}\rho LBP^2/u$			
$r^2\Delta u$	$\frac{1}{2} X_{rru}$		$\frac{1}{2}\rho LBP^4/u$			
$\delta^2\Delta u$	$\frac{1}{2} X_{\delta\delta u}$		$\frac{1}{2}\rho LBP^2 u$	-190.0		
vr	$(X_{vr}+m)$	X VR	$\frac{1}{2}\rho LBP^3$	798.0	N	Yaw and drift angle test - m is known
$v\delta$	$X_{v\delta}$	X VD	$\frac{1}{2}\rho LBP^2 u$	93.2	N	Static drift angle test
$r\delta$	$X_{r\delta}$	X RD	$\frac{1}{2}\rho LBP^3 u$	0.0	N	Yaw and rudder angle test
$vr\Delta u$	X_{vru}		$\frac{1}{2}\rho LBP^3/u$			
$v\delta\Delta u$	$X_{v\delta u}$		$\frac{1}{2}\rho LBP^2$	93.0		
$r\delta\Delta u$	$X_{r\delta u}$		$\frac{1}{2}\rho LBP^3$			
-	X_*	X O	$\frac{1}{2}\rho LBP^2 u^2$	0.0	N	Static drift angle test

(1) The Fortran program does not include all terms in the mathematical model, Equations (10). Certain coefficients have been left out, as they have been considered unimportant for the accuracy of the predictions.

(2) The nondimensional coefficients have been taken from Reference 5.

(3) The coefficients have been divided into three grades according to their importance for the accuracy of a prediction. The most important coefficients are indicated by I; coefficients of minor importance by MI; coefficients, which apparently are negligible, by N.

Table 2 - Summary of Coefficients in Y-Equation

Variable	Y - Equation: Taylor Expansion And Dynamic Response Terms	Identifier in FORTRAN Program (1)	Nondim. Factor	Nondim. Coeff. $\times 10^5$ from Example (2)	Relative Importance of Coeff. (3)	Planar Motion Mechanism Test Technique or Calculation Method
$\dot{v}$	$(m-Y_{\dot{v}})$	Y VDOT	$\frac{1}{2}\rho LBP^3$	1546.0	I	Pure sway (transverse motion) test
$\dot{r}$	$(mx_G-Y_{\dot{r}})$	Y RDOT	$\frac{1}{2}\rho LBP^4$	-8.6	I	Pure yaw (angular motion) test
v	Y_v	Y V	$\frac{1}{2}\rho LBP^2u$	-1160.4	I	Static drift angle test
v^3	$\frac{1}{6}Y_{vvv}$	Y VVV	$\frac{1}{2}\rho LBP^2/u$	-8078.2	MI	Static drift angle test
vr^2	$\frac{1}{2}Y_{vrr}$	Y VRR	$\frac{1}{2}\rho LBP^4/u$	0.0	N	Yaw and drift angle test
$v\delta^2$	$\frac{1}{2}Y_{v\delta\delta}$	Y VDD	$\frac{1}{2}\rho LBP^2u$	-3.8	N	Static drift angle test
$v\Delta u$	Y_{vu}		$\frac{1}{2}\rho LBP^2$	-1160.0		
$v\Delta u^2$	$\frac{1}{2}Y_{vuu}$		$\frac{1}{2}\rho LBP^2/u$			
r	(Y_r-mu)	Y R	$\frac{1}{2}\rho LBP^3u$	-499.0	I	Pure yaw (angular motion) test
r^3	$\frac{1}{6}Y_{rrr}$	Y RRR	$\frac{1}{2}\rho LBP^5/u$	0.0	N	Pure yaw (angular motion) test
rv^2	$\frac{1}{2}Y_{rvv}$	Y RVV	$\frac{1}{2}\rho LBP^3/u$	15356.0	I	Yaw and drift angle test
$r\delta^2$	$\frac{1}{2}Y_{r\delta\delta}$	Y RDD	$\frac{1}{2}\rho LBP^3u$	0.0	N	Yaw and rudder angle test
$r\Delta u$	Y_{ru}		$\frac{1}{2}\rho LBP^3$	-499.0		
$r\Delta u^2$	$\frac{1}{2}Y_{ruu}$		$\frac{1}{2}\rho LBP^3/u$			
δ	Y_δ	Y D	$\frac{1}{2}\rho LBP^2u^2$	277.9	I	Static drift angle test
δ^3	$\frac{1}{6}Y_{\delta\delta\delta}$	Y DDD	$\frac{1}{2}\rho LBP^2u^2$	-90.0	MI	Static drift angle test
δv^2	$\frac{1}{2}Y_{\delta vv}$	Y DVV	$\frac{1}{2}\rho LBP^2$	1189.6	MI	Static drift angle test
δr^2	$\frac{1}{2}Y_{\delta rr}$	Y DRR	$\frac{1}{2}\rho LBP^4$	0.0	N	Yaw and rudder angle test
$\delta\Delta u$	$Y_{\delta u}$	Y DU	$\frac{1}{2}\rho LBP^2u$	556.0 (0.0)	MI	Static drift angle test executed at various speed values
$\delta\Delta u^2$	$\frac{1}{2}Y_{\delta uu}$		$\frac{1}{2}\rho LBP^2$	278.0		
$vr\delta$	$Y_{vr\delta}$	Y VRD	$\frac{1}{2}\rho LBP^3$	0.0	N	Yaw and drift angle test executed at various speed values
-	Y_*	Y O	$\frac{1}{2}\rho LBP^2u^2$	-3.6	MI	Static drift angle test
Δu	Y_{*u}	Y OU	$\frac{1}{2}\rho LBP^2u$	(0.0)	N	Static drift angle test executed at various speed values
Δu^2	Y_{*uu}		$\frac{1}{2}\rho LBP^2$			

(1) The FORTRAN program does not incorporate all terms in the mathematical model, Equations (10). Certain coefficients have been left out, as they have been considered without importance for the accuracy of the predictions.

(2) The nondimensional coefficients have been taken from Reference 5 except values enclosed in parenthesis, for which no data were available.

(3) The coefficients have been divided into three grades according to their importance for the accuracy of a prediction. The most important coefficients, which should be available in order to obtain a prediction, are marked by I; coefficients of minor importance by MI; coefficients which apparently are negligible, by N.

Table 3 - Summary of Coefficients in N-Equation

Variable	N-Equation: Taylor Expansion And Dynamic Response Terms	Identifier in FORTRAN Program (1)	Nondim. Factor	Nondim. Coeff. $\times 10^5$ from Example (2)	Relative Importance of Coeff. (3)	Planar Motion Mechanism Test Technique or Calculation Method
$\dot v$	$(mx_G - N_{\dot v})$	N VDOT	$\frac{1}{2}\rho LBP^4$	-22.7	I	Pure sway (transverse motion) test
$\dot r$	$(I_z - N_{\dot r})$	N RDOT	$\frac{1}{2}\rho LBP^5$	82.9	I	Pure yaw (angular motion) test
v	N_v	N V	$\frac{1}{2}\rho LBP^3 u$	-263.5	I	Static drift angle test
v^3	$\frac{1}{6} N_{vvv}$	N VVV	$\frac{1}{2}\rho LBP^3/u$	1636.1	MI	Static drift angle test
vr^2	$\frac{1}{2} N_{vrr}$	N VRR	$\frac{1}{2}\rho LBP^5/u$	0.0	N	Yaw and drift angle test
$v\delta^2$	$\frac{1}{2} N_{v\delta\delta}$	N VDD	$\frac{1}{2}\rho LBP^3 u$	12.5	N	Static drift angle test
$v\Delta u$	N_{vu}		$\frac{1}{2}\rho LBP^3$	- 264.0		
$v\Delta u^2$	$\frac{1}{2} N_{vuu}$		$\frac{1}{2}\rho LBP^3/u$			
r	$(N_r - mx_G u)$	N R	$\frac{1}{2}\rho LBP^4 u$	-166.0	I	Pure yaw (angular motion) test
r^3	$\frac{1}{6} N_{rrr}$	N RRR	$\frac{1}{2}\rho LBP^6/u$	0.0	N	Pure yaw (angular motion) test
rv^2	$\frac{1}{2} N_{rvv}$	N RVV	$\frac{1}{2}\rho LBP^4/u$	-5483.0	I	Yaw and drift angle test
$r\delta^2$	$\frac{1}{2} N_{r\delta\delta}$	N RDD	$\frac{1}{2}\rho LBP^4 u$	0.0	N	Yaw and rudder angle test
$r\Delta u$	N_{ru}		$\frac{1}{2}\rho LBP^4$	- 166.0		
$r\Delta u^2$	$\frac{1}{2} N_{ruu}$		$\frac{1}{2}\rho LBP^4/u$			
δ	N_δ	N D	$\frac{1}{2}\rho LBP^3 u^2$	-138.8	I	Static drift angle test
δ^3	$\frac{1}{6} N_{\delta\delta\delta}$	N DDD	$\frac{1}{2}\rho LBP^3 u^2$	45.0	MI	Static drift angle test
δv^2	$\frac{1}{2} N_{\delta vv}$	N DVV	$\frac{1}{2}\rho LBP^3$	-489.0	MI	Static drift angle test
δr^2	$\frac{1}{2} N_{\delta rr}$	N DRR	$\frac{1}{2}\rho LBP^4$	0.0	N	Yaw and rudder angle test
$\delta\Delta u$	$N_{\delta u}$	N DU	$\frac{1}{2}\rho LBP^3 u$	-278.0 (0.0)	MI	Static drift angle test executed at various speed values
$\delta\Delta u^2$	$\frac{1}{2} N_{\delta uu}$		$\frac{1}{2}\rho LBP^3$	-139.0		
$vr\delta$	$N_{vr\delta}$	N VRD	$\frac{1}{2}\rho LBP^4$	0.0	N	Yaw and drift angle test executed for various speed values
-	N_*	N O	$\frac{1}{2}\rho LBP^3 u^2$	2.8	MI	Static drift angle test
Δu	N_{*u}	N OU	$\frac{1}{2}\rho LBP^3 u$	(0.0)	N	Static drift angle test executed at various speed values
Δu^2	N_{*uu}		$\frac{1}{2}\rho LBP^3$			

(1) The FORTRAN program does not incorporate all terms in the mathematical model, Equations (10). Certain coefficients have been left out, as they have been considered without importance for the accuracy of the predictions.

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(3) The coefficients have been divided into three grades according to their importance for the accuracy of a prediction. The most important coefficients, which should be available in order to obtain a prediction, are marked by I; coefficients of minor importance by MI; coefficients, which apparently are negligible, by N.