Report 2223

MODEL BASIN
WASHINGTON, D.C.

DEPARTMENT OF THE NAVY

HYDROMECHANICS

AERODYNAMICS

STRUCTURAL MECHANICS

APPLIED MATHEMATICS

ACOUSTICS AND VIBRATION

CALCULATIONS ON THE COLLAPSE OF A SPHERICAL GAS-FILLED CAVITY IN A COMPRESSIBLE LIQUID

by

Russel R. Lilliston

STRUCTURAL MECHANICS LABORATORY
RESEARCH AND DEVELOPMENT REPORT

August 1966

Report 2223

DAVID TAYLOR MODEL BASIN
WASHINGTON, D. C. 20007

CALCULATIONS ON THE COLLAPSE OF A SPHERICAL GAS-FILLED CAVITY IN A COMPRESSIBLE LIQUID

by

Russel R. Lilliston

August 1966

Report 2223
SP 6-0002

TABLE OF CONTENTS

LIST OF FIGURES

NOTATION

B	A constant which characterizes the adiabatic nature of the liquid medium (for water $B = 3000$ atmospheres)
C	Isentropic sound speed in the medium as a function of ambient and transient pressures
c_∞	Sound speed in the undisturbed liquid medium
H	Specific enthalpy of the liquid medium
n	A constant which characterizes the adiabatic nature of the liquid medium (for water, $n = 7$)
P	Pressure in the liquid at the bubble wall
p	Pressure of the gas inside the sphere
p_0	Initial pressure of the gas inside the sphere
$\bar{p}$	Pressure in the liquid outside the bubble wall
p_∞	Pressure in the undisturbed liquid medium; ambient pressure
R	Instantaneous radius of the imploding sphere
R_0	Initial radius of the imploding sphere
r	Standoff (measured from the bubble center); component in the direction of the radial spherical coordinate
t	Time
t_R	Time measured at the bubble wall; time measured when the Eulerian position vector $r = R$
U	Instantaneous velocity of the bubble wall
u	Eulerian velocity in the fluid outside the bubble wall
v	Instantaneous specific volume of the gas inside the bubble
v_0	Initial specific volume of the gas inside the bubble
γ	Polytropic gas constant for an adiabatic process
ρ_∞	Density of the undisturbed liquid medium

ABSTRACT

This paper presents a method for calculating the instantaneous pressure, velocity, acceleration, and radius associated with the collapse of a spherical gas-filled cavity in an infinite compressible liquid. The method is an independent approach which makes use of Hamming's technique to numerically integrate Gilmore's differential equations which describe the collapse.

Included is a computer program which will perform the necessary calculations on a IBM 7090/1401 digital computer. Results obtained are in good agreement with those of Hickling and Plesset, whose work was unknown to the present author when he undertook the study.

It may be inferred that the peak shock wave pressure is significantly reduced by a decrease in ambient pressure, an increase in internal pressure, and/or a variation of the specific heat ratio by proper selection of the gas. Control of the last two parameters can be investigated as a possible means of protecting glass spheres against sympathetic implosion in multiple sphere buoyancy systems.

ADMINISTRATIVE INFORMATION

This work was funded under Special Projects Office Project Order Number 6-0002.

INTRODUCTION

PURPOSE

Because of the excessive weight-displacement ratios obtained with tough metals such as steel or aluminum, designers are turning toward nonductile materials for use in buoyancy systems for all depth vehicles. Spherical glass shells are among the components for such systems.[1,2] In a system which contains a number of buoyancy spheres, it is essential to know the effect that the collapse of one sphere will have on neighboring spheres in order to prevent catastrophic failure. A two-part investigation has been initiated:

1. The definition of the free-field pressure-time history due to the implosion of a single sphere.

2. Determination of the loading and response of a sphere to the pressure field generated by the implosion of a neighboring sphere.

---

[1]References are listed on page 36.

This report deals with the analytical determination of Part 1, based on the assumptions that the spherical shell has negligible weight and thickness and that it contains air at arbitrary pressure.

BACKGROUND

The need for more complete understanding of the hydromechanical problem of cavitation and the gas bubble phenomena of underwater explosions has encouraged more and more detailed investigations into the pulsations of underwater gas bubbles. One of the earliest of these investigations was made by Rayleigh in 1917.[3] A more refined treatment was successfully completed by Herring in 1941.[4] Sometime later (1952), Gilmore[5] took a different approach and postulated equations to describe the growth or collapse of a spherical bubble in a viscous compressible liquid. Gilmore's description is presented in the next section.

THEORY

GILMORE'S BUBBLE WALL EQUATION

On the basis of the Kirkwood-Bethe hypothesis,[6] Gilmore has derived an equation (which he calls a "second order" approximation) which accurately describes the (nonmigratory) oscillations of a spherical gas-filled cavity in an infinite compressible liquid. If R is the radius of the sphere, H the specific enthalpy of the surrounding liquid, and C the isentropic sound speed in the liquid, then Gilmore's equation is:

$$\ddot{R}R\left(1-\frac{\dot{R}}{C}\right)+\frac{3}{2}\dot{R}^2\left(1-\frac{\dot{R}}{3C}\right)=H\left(1+\frac{\dot{R}}{C}\right)+\frac{R\dot{H}}{C}\left(1-\frac{\dot{R}}{C}\right) \quad [1.1]$$

$$R(0)=R_0, \qquad \dot{R}(0)=0$$

where

$$C=c_\infty\left(\frac{P+B}{p_\infty+B}\right)^{\frac{n-1}{2n}} \quad [1.2]$$

and

$$H = \int_{p_\infty}^{p} \left(\frac{P + B}{p_\infty + B}\right)^{-1/n} \frac{dp}{\rho_\infty} = \frac{n(p_\infty + B)}{(n-1)\,\rho_\infty} \left[\left(\frac{P + B}{p_\infty + B}\right)^{\frac{n-1}{n}} - 1\right] \qquad [1.3]$$

Here c_∞, p_∞, and ρ_∞ are respectively sound speed, pressure, and density in the undisturbed liquid. B and n are constants which characterize the adiabatic compression of the liquid (for water, $B = 3000$ atm, $n = 7$). P is the pressure in the liquid at the bubble wall. If viscosity and surface tension are neglected and the pressure p inside the bubble is uniform, then pressure is continuous across the boundary of the sphere, i.e., $P = p$ except at time $t = 0$ when the pressure in the fluid is artificially and discontinuously reduced from $P = p_\infty$ to $P = p(0) = p_0$. The gas can be assumed to undergo an adiabatic expansion (or compression). From thermodynamics, for an ideal gas,

$$p_0 v_0^\gamma = p v^\gamma \qquad [1.4a]$$

where γ is a constant (the specific heat ratio), v is specific volume, and the subscript 0 refers to some initial state. Since the volume of a sphere is proportional to its radius

$$\frac{v_0}{v} = \left(\frac{R_0}{R}\right)^3 \qquad [1.4b]$$

Elimination of volume between Equation [1.4a] and Equation [1.4b] yields

$$P = \begin{cases} p_\infty & t = 0 \\ p = p_0 \left(\dfrac{R_0}{R}\right)^{3\gamma} & t > 0 \end{cases} \qquad [1.4c]$$

For air ($\gamma = 4/3$), the exponent 3γ becomes 4. Usually the value of γ is taken to be 1.4 for air. This value represents the behavior of air fairly accurately, but since γ decreases with increasing pressure, 4/3 represents a rough average. It will be seen later that the use of a constant value of γ leads to a deficiency in the model.

Combining Equations [1.1], [1.2], [1.3], and [1.4c] and taking $n = 7$ yields the following ordinary differential equation for R:

$$R\ddot{R}\left[1 - \dot{R}\left(\frac{p_\infty + B}{p_0\left(\frac{R_0}{R}\right)^4 + B}\right)^{3/7}\right] + \frac{7}{6}\left(\frac{p_\infty + B}{\rho_\infty}\right)\left[1 - \left(\frac{p_0\left(\frac{R_0}{R}\right)^4 + B}{p_\infty + B}\right)^{6/7}\right.$$

$$+ \frac{\dot{R}}{c_\infty}\left(\frac{p_\infty + B}{p_0\left(\frac{R_0}{R}\right)^4 + B}\right)^{3/7} - \left.\frac{\dot{R}}{c_\infty}\left(\frac{p_0\left(\frac{R_0}{R}\right)^4 + B}{p_\infty + B}\right)^{3/7}\right] + \frac{3}{2}\dot{R}^2$$

$$- \frac{\dot{R}^3}{2c_\infty}\left(\frac{p_\infty + B}{p_0\left(\frac{R_0}{R}\right)^4 + B}\right)^{3/7} + \frac{4p_0}{\rho_\infty c_\infty}\dot{R}\left(\frac{R_0}{R}\right)^4\left[\left(\frac{p_0\left(\frac{R_0}{R}\right)^4 + B}{p_\infty + B}\right)^{4/7}\right.$$

$$\left.- \frac{\dot{R}}{c_\infty}\left(\frac{p_\infty + B}{p_0\left(\frac{R_0}{R}\right)^4 + B}\right)\right] = 0 \qquad [1.5]$$

$$R(0) = R_0, \qquad \dot{R}(0) = 0$$

Once B, R_0, p_0, p_∞, and c_∞ are specified, it is possible to find a numerical solution for $R(t)$, $\dot{R}(t)$, $\ddot{R}(t)$, and $p(t)$.

The initial velocity $\dot{R}(0)$ is taken throughout this paper to be zero. With the help of Equation [1.5], Gilmore has pointed out that near $t = 0$, there is a small finite jump in velocity during an infinitesimally small interval of time, i.e.,

$$\dot{R}(0_+) = \frac{p_0 - p_\infty}{\rho_\infty c_\infty}$$

Hickling and Plesset[7] give a good physical explanation of this jump in terms of the initial pressure discontinuity between p_0 and p_∞. This velocity jump may lead one to choose $\dot{R}(0_+) = (p_0 - p_\infty)/\rho_\infty c_\infty$ as the initial condition on the velocity. Since the difference between $(p_0 - p_\infty)/\rho_\infty c_\infty$ and zero is small compared to the magnitude of the velocities of interest, the question as to whether to start the solution at $\dot{R}(0) = 0$ or at $\dot{R}(0_+) = (p_0 - p_\infty)/\rho_\infty c_\infty$ is somewhat academic. The plots of bubble wall velocity do not show this initial jump because the writer's solution of Gilmore's equation was actually carried out from $t = 0_+$ to avoid an infinite initial acceleration. (One of the terms appearing in the expression for the initial acceleration is the derivative with respect to time of P as given in Equation [1.4c]; this derivative is infinite at $t = 0$). The approximation $(p_0 - p_\infty)/\rho_\infty c_\infty \approx 0$ was made to simplify the calculations of the initial values of R and $\dot{R}$ which are tedious even with such an approximation.

It should be emphasized that one of the inherent assumptions upon which Equation [1.5] is based is that the cavity remain spherical throughout the collapse and subsequent oscillations. In some bubble collapse experiments,[8] however, the single bubble has occasionally been observed to dissociate into many smaller bubbles at the end of the first collapse.

THE INTEGRATION: HAMMING'S METHOD

An ordinary differential equation of the form

$$\frac{dy}{dx} = f(x, y), \qquad y(x_0) = y_0 \tag{2.1}$$

can be integrated numerically by one of various finite difference methods. One of these, the Hamming method, which is particularly well suited for the solution of Equation [1.5] can be found in Ralston and Wilf.[9] It is outlined briefly here:

1. The x-axis is equally divided into a large number of small intervals. The value of y at the end of the nth interval (i.e., the interval between x_n and x_{n-1}) is denoted by y_n.

2. Knowing the values of y_i and y_i' at previous intervals x_i up to and including the nth ($i = n$), it is possible to calculate p_{n+1}, a first approximation to the $(n+1)^{st}$ value of y_{n+1} at x_{n+1}, by means of

$$p_{n+1} = y_{n-3} + \frac{4h}{3}\left(2y_n' - y_{n-1}' + 2y_{n-2}'\right)$$

where h is the width of the interval; p_{n+1} is called the predictor.

3. Since the prediction p_{n+1} is based on the value of a series expansion of y, error due to truncation is incurred. Most of the difference between the true value of y and the estimated value of y is taken into account by the modifier (denoted by m_{n+1})

$$m_{n+1} = p_{n+1} - \frac{112}{121} (p_n - c_n)$$

where c_n is as defined in the next paragraph and the derivative of m_{n+1} is $m'_{n+1} = f(x_{n+1}, m_{n+1})$.

4. The predicted value is compared with a quantity called the corrector

$$c_{n+1} = \frac{1}{8} [9y_n - y_{n-1} + 3h (m'_{n+1} + 2y'_n - y'_{n-1})]$$

5. If the predictor p_{n+1} lies close to c_{n+1} within some specified tolerance, then the final value of y_{n+1} at x_{n+1} is taken as

$$y_{n+1} = c_{n+1} + \frac{9}{121} (p_{n+1} - c_{n+1})$$

6. If the predictor p_{n+1} does not lie close enough to c_{n+1}, then either (1) a new value of p_{n+1}

$$p_{n+1} = c_{n+1} + \frac{9}{121} (p_{n+1} - c_{n+1})$$

may be calculated and an interative process carried out or (2) the interval may be halved. This is discussed in more detail in Appendix A.

In order to solve Equation [1.5] by the procedure just outlined, it is necessary to reduce the equation to a system of two simultaneous differential equations of the form of Equation [2.1]. This method is mentioned in Hildebrand.[10] Write U for $\dot{R}$; then Equation [1.5] can be written

$$\dot{R} = U \qquad [1.6a]$$

and

$$\dot{U} = \left\{ \frac{U^3}{2c_\infty} \left(\frac{p_\infty + B}{p_0\left(\frac{R_0}{R}\right)^4 + B} \right)^{3/7} - \frac{3}{2} U^2 - \frac{4 U p_0}{\rho_\infty c_\infty} \left(\frac{R_0}{R}\right)^4 \left[\left(\frac{p_0\left(\frac{R_0}{R}\right)^4 + B}{p_\infty + B} \right)^{4/7} \right. \right.$$

$$\left. - \frac{U}{c_\infty} \left(\frac{p_\infty + B}{p_0\left(\frac{R_0}{R}\right)^4 + B} \right) \right] - \frac{7}{6} \left(\frac{p_\infty + B}{\rho_\infty} \right) \left[1 - \left(\frac{p_0\left(\frac{R_0}{R}\right)^4 + B}{p_\infty + B} \right)^{6/7} \right. \qquad [1.6b]$$

$$\left. \left. + \frac{U}{c_\infty} \left(\frac{p_\infty + B}{p_0\left(\frac{R_0}{R}\right)^4 + B} \right)^{3/7} - \frac{U}{c_\infty} \left(\frac{p_0\left(\frac{R_0}{R}\right)^4 + B}{p_\infty + B} \right)^{3/7} \right] \right\} \Bigg/ \left[1 - U \left(\frac{p_\infty + B}{p_0\left(\frac{R_0}{R}\right)^4 + B} \right)^{3/7} \right]$$

Hamming's method can be applied simultaneously to Equations [1.6a] and [1.6b] to yield a numerical solution. The justification for use of this particular method is discussed in Appendix B.

The function $R(t)$ (from which $U(t)$ and $p(t)$ at the bubble wall can be obtained), determined by Equation [1.6], constitutes one of the boundary conditions necessary to find the Eulerian velocity and pressure fields in the fluid outside the bubble wall.

THE EULERIAN VELOCITY AND PRESSURE FIELDS IN THE LIQUID

In his "second order" approximation, Gilmore uses the Kirkwood-Bethe hypothesis in conjunction with the method of characteristics to determine the (Eulerian) velocity and pressure fields in the liquid. If the standoff r is greater than or equal to the initial radius R_0, then the velocity u associated with the standoff will not be of the same order of magnitude as the sound speed except for the most severe implosions. Provided the approximation $u^2 << c^2$ is valid, the following set of equations (the expressions derived by Gilmore) are sufficient to determine the velocity and pressure fields u and $\overline{p}$ in the liquid when U and R, the bubble wall velocity and radius, are known functions of time.

$$u(r, t) = \frac{y}{c_\infty r} + \frac{K_3 y^2}{c_\infty^3 r^2} \left(1 - \frac{y}{c_\infty^2 r} + \frac{K_3^2 y^4}{2c_\infty^8 r^4} \right) \qquad [3.1]$$

where y and K_3 are given by

$$y = \frac{RU^2}{2} + \frac{R\left(p_0\left(\frac{R_0}{R}\right)^4 - p_\infty\right)}{\rho_\infty}\left(1 - \left(\frac{p_0\left(\frac{R_0}{R}\right)^4 - p_\infty}{2\rho_\infty c_\infty^2}\right)\right) \quad [3.2]$$

$$K_3 = \frac{c_\infty^3 R^2 U}{y^2}\left(1 - \frac{U^2}{2c_\infty^2}\right) - \frac{c_\infty^2 R}{y}\left(1 - \frac{U}{c_\infty}\right) \quad [3.3]$$

$$\bar{p}(r, t) = \rho_\infty\left(\frac{y}{r} - \frac{u^2}{2}\right) + \frac{\rho_\infty}{2c_\infty^2}\left(\frac{y}{r} - \frac{u^2}{2}\right)^2 \quad [3.4]$$

$$t = t_R + \left(\frac{r - R}{c_\infty}\right)\left(1 - \frac{UR}{c_\infty r}\right) \quad [3.5]$$

Here t_R is the time at which the bubble radius is R and the bubble wall velocity is U. An event which occurs at the bubble wall at time t_R requires finite time $t - t_R$ to propagate from the bubble wall to the point r in the fluid. This time lag is indicated by Equation [3.5].

If the approximation $u^2 << c^2$ is not made, then the numerical integration for u and $\bar{p}$ is more complicated and requires a great deal more time on the computer. Hickling and Plesset[9] avoid making this approximation. Instead of using values of U and R in Equations [3.1] through [3.5] to find $\bar{p}$ and u at discrete points, they use U and R as coefficients in a differential equation which determines $\bar{p}$ and u. Then each time U and R are determined at a single point, another differential equation must be solved to find $\bar{p}$ and u. Each solution of this second differential equation gives $\bar{p}$ and u as a function of distance from the bubble at one specific instant in time (i.e., that instant in time at which the bubble radius is that value of $R(t)$ used in this second differential equation).

RESULTS

The integration of Equation [1.6] (the equation which determines bubble radius and bubble wall velocity and acceleration) has been coded in FORTRAN for a 7090/1401 computer according to the procedure outlined. The program and some sample input and output are given in Appendix C. Plots from computer output of R, u, $\dot{U}$, and p (the bubble radius, bubble wall velocity and acceleration, and pressure at the bubble wall, respectively) as functions of time can be found among Figures 1 through 5 for various ambient and internal pressures.

Equations [3.1] through [3.5] (those equations which determine the Eulerian velocity and pressure fields in the fluid outside the bubble wall) have been incorporated into the program. From computer output, plots of u and p (Eulerian velocity and pressure) were obtained and are found with the corresponding plots of R, U, $\dot{U}$, and p (Figures 1 through 4).

DISCUSSION

Dynamically, the air inside the bubble behaves in a peculiar fashion which is quite evident in the more violent implosions (those at great depths). During the greater part of the collapse, the air offers insignificant resistance to the inrushing water. Just before the instant of minimum radius, however, the air violently arrests further decrease in volume, behaving very much like a rigid sphere. This is borne out especially by the curves for bubble wall acceleration. At the instant of minimum radius, the water in the immediate vicinity of the bubble "sees" a rigid sphere, but that water a little further from the wall continues to rush in since the water is compressible. The result is a spherical shock wave propagating from the wall out into the fluid.

Comparisons among Figures 1 through 5 lead to the following observations:

1. As the depth at which the implosion occurs becomes greater, the peak pressure increases, the rise time decreases, and the collapse time decreases.

2. Increasing the initial internal pressure of the gas inside the sphere has roughly the same effect as decreasing the depth of implosion. Specifically, the peak pressure can be effectively attenuated by increasing the initial internal pressure. Comparison between Figures 2e and 3e, for example, shows that the peak pressure pulse from an implosion at 1000 ft of water is reduced by almost 40 percent when the initial internal pressure is increased from 1 to 2 atm.

3. As the pressure peak propagates away from the cavity wall, it suffers an attenuation proportional to $1/r$.

4. Comparison between Figures 1, 2, and 5 indicates that for constant initial internal pressure and radius, the collapse time varies approximately inversely with the square root of the depth at which the implosion occurs.

It should be noted that the present theory does not account for the effect of migration. The model presented here is excellent for a ratio of initial internal pressure to ambient pressure which is less than perhaps one-tenth.

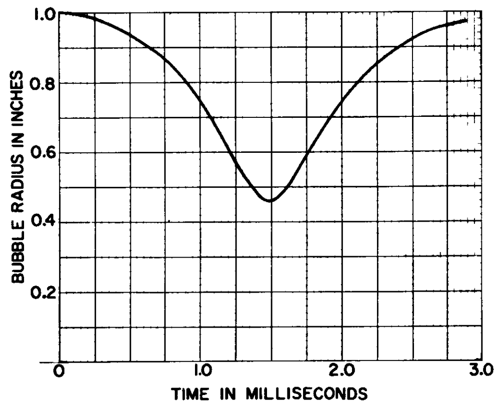

Figure 1a – Bubble Radius

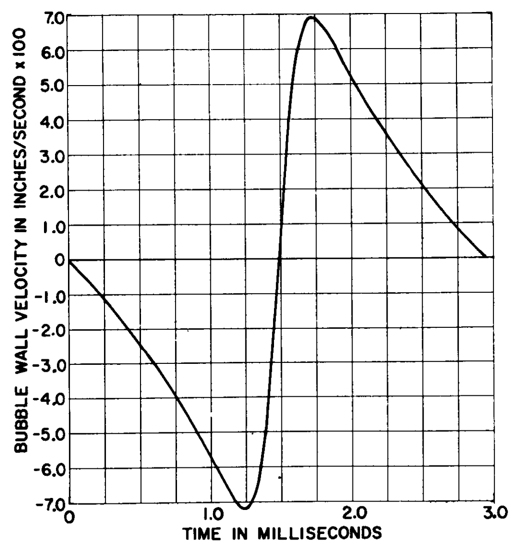

Figure 1b – Bubble Wall Velocity

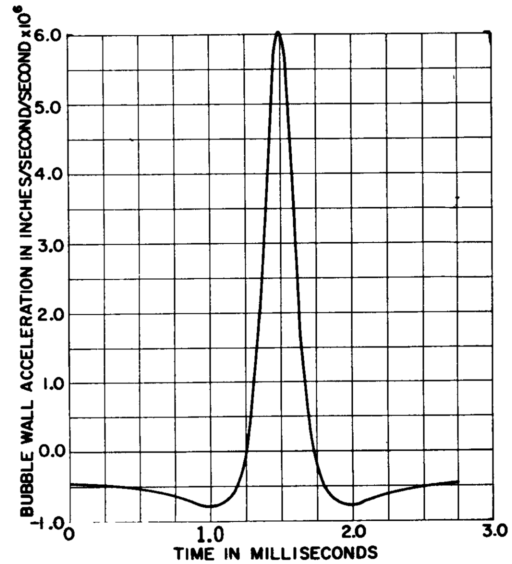

Figure 1c – Bubble Wall Acceleration

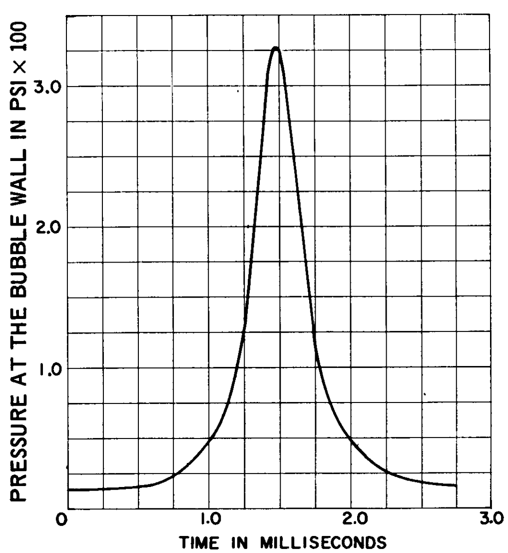

Figure 1d – Pressure at the Bubble Wall

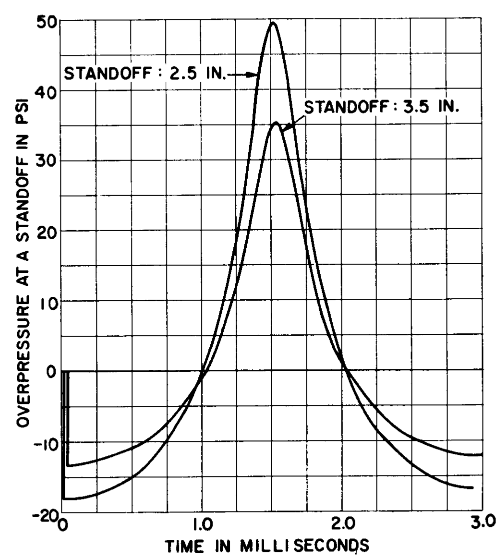

Figure 1e – Overpressures

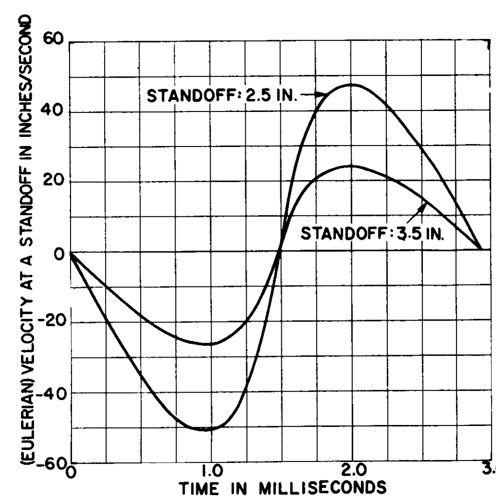

Figure 1f – (Eulerian) Velocity

Figure 1 – Spherical Collapse as a Function of Time for a Water Depth of 100 Feet, an Initial Radius of 1 Inch, and an Initial Internal Pressure of 1 Atmosphere

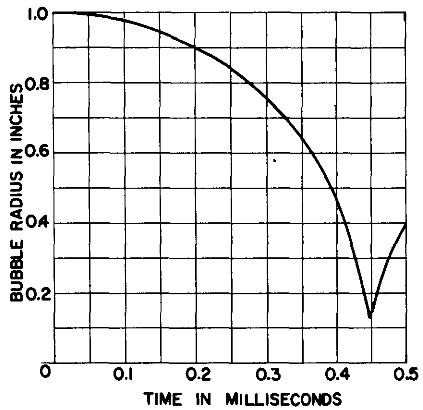

Figure 2a – Bubble Radius

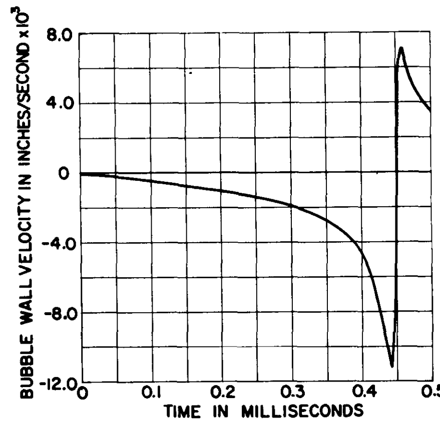

Figure 2b – Bubble Wall Velocity

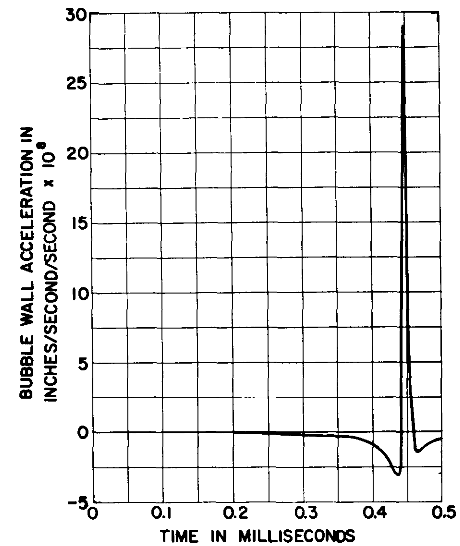

Figure 2c – Bubble Wall Acceleration

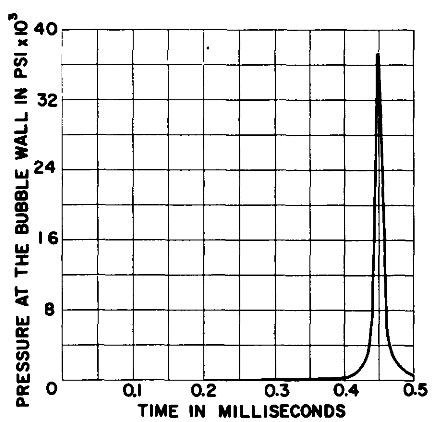

Figure 2d – Pressure at the Bubble Wall

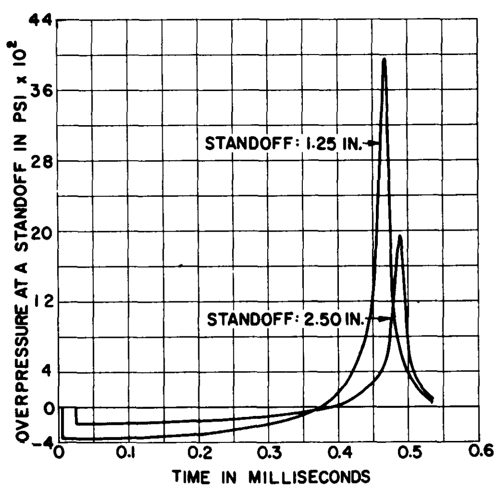

Figure 2e – Overpressures

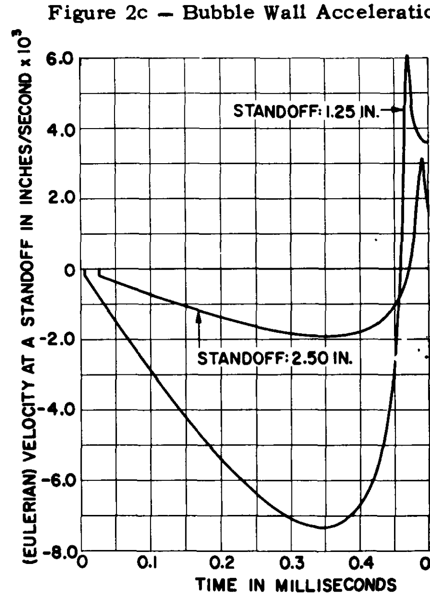

Figure 2f – (Eulerian) Velocity

Figure 2 – Spherical Collapse as a Function of Time for a Water Depth of 1000 Feet, an Initial Radius of 1 Inch, and an Initial Internal Pressure of 1 Atmosphere

Figure 3 – Spherical Collapse as a Function of Time for a Water Depth of 1000 Feet, an Initial Radius of 1 Inch, and an Initial Internal Pressure of 2 Atmospheres

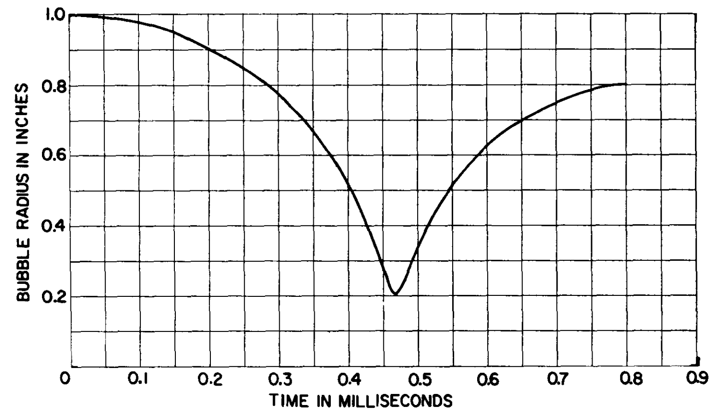

Figure 3a – Bubble Radius

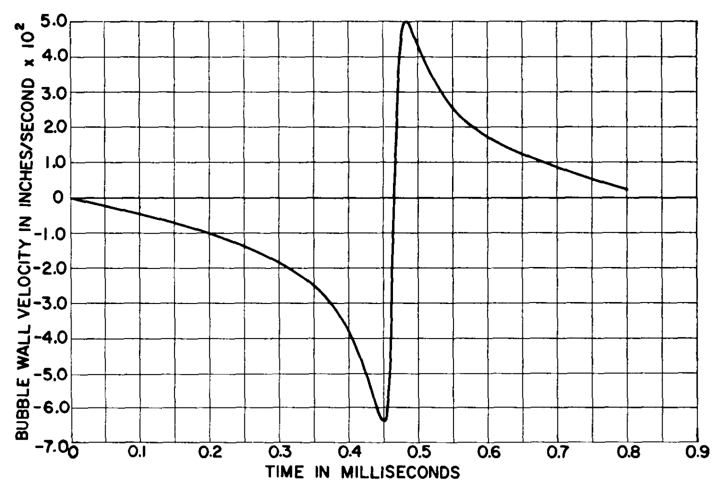

Figure 3b – Bubble Wall Velocity

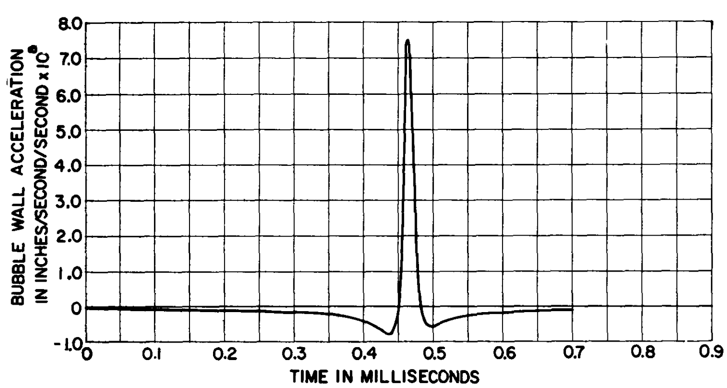

Figure 3c – Bubble Wall Acceleration

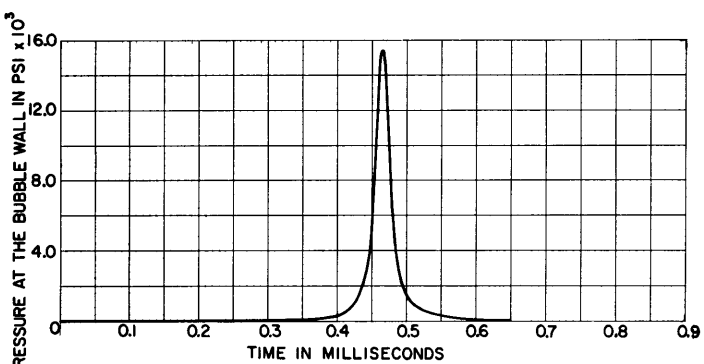

Figure 3d — Pressure at the Bubble Wall

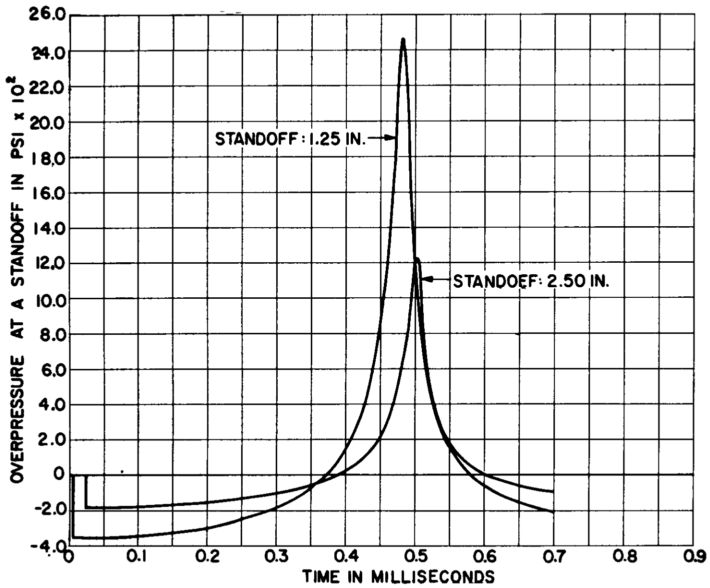

Figure 3e — Overpressures

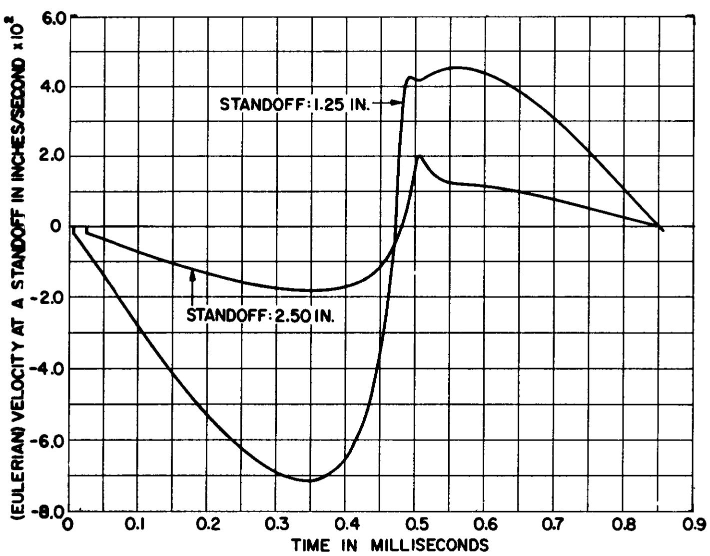

Figure 3f — (Eulerian) Velocity

Figure 4 – Spherical Collapse as a Function of Time for a Water Depth of 1000 Feet, an Initial Radius of 1 Inch, and an Initial Internal Pressure of 10 Atmospheres

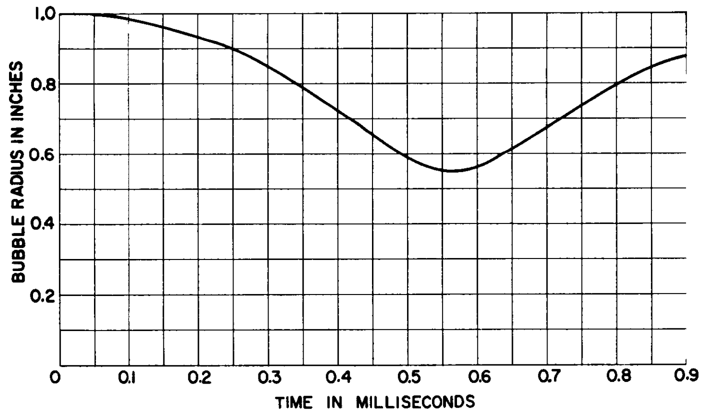

Figure 4a – Bubble Radius

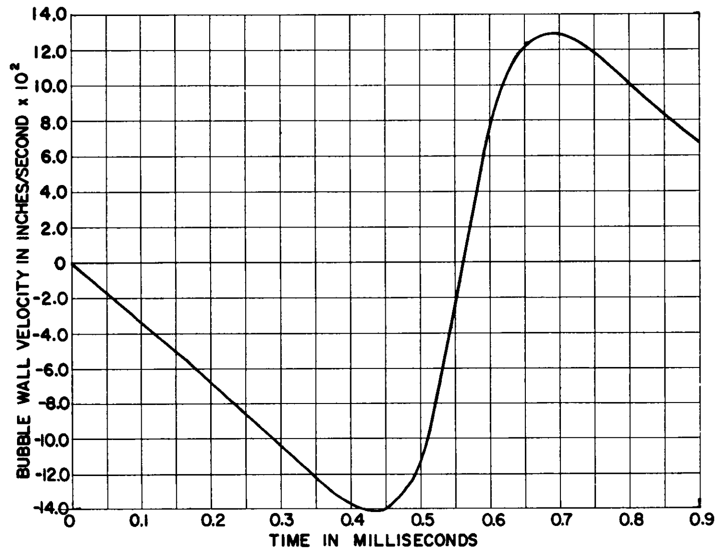

Figure 4b – Bubble Wall Velocity

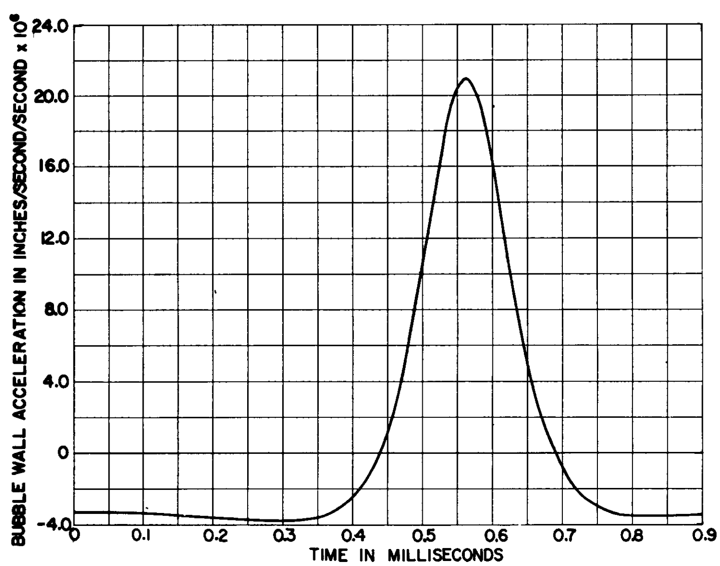

Figure 4c – Bubble Wall Acceleration

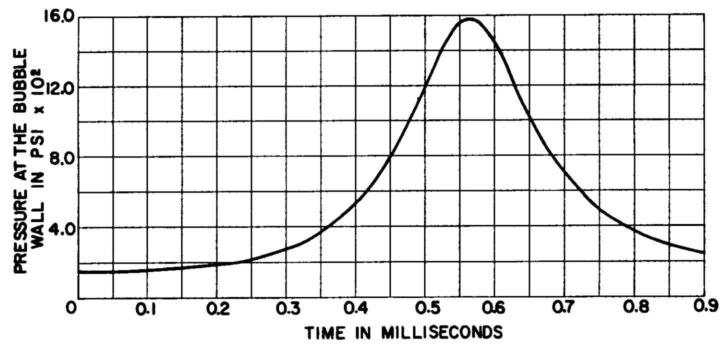

Figure 4d — Pressure at the Bubble Wall

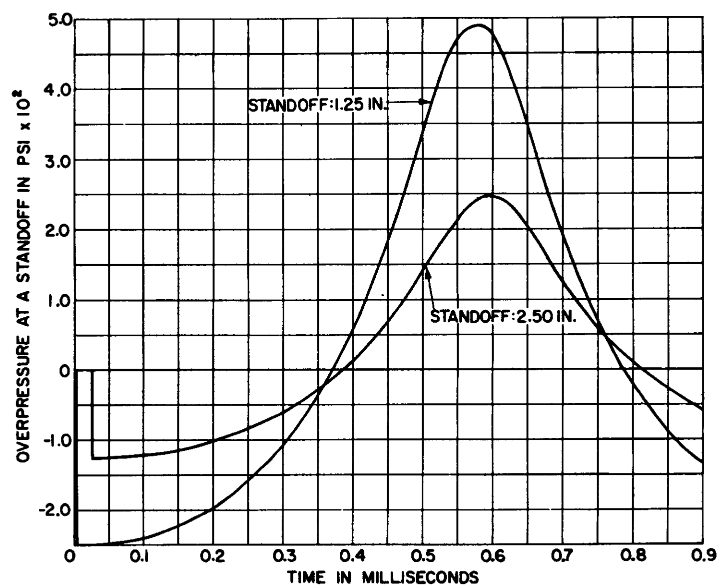

Figure 4e — Overpressures

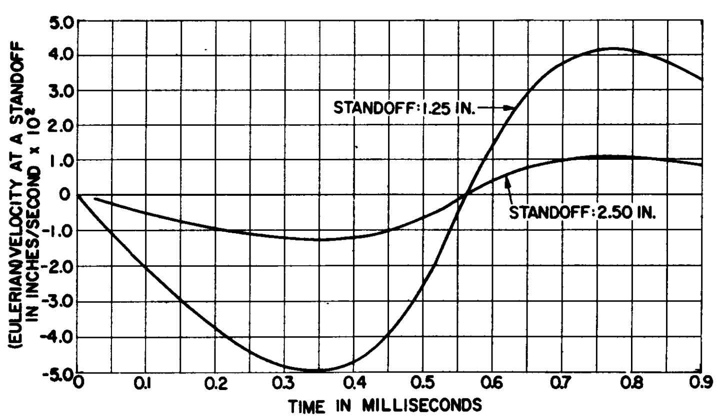

Figure 4f — (Eulerian) Velocity

Figure 5 – Spherical Collapse as a Function of Time for a Water Depth of 10,000 Feet, an Initial Radius of 1 Inch, and an Initial Internal Pressure of 1 Atmosphere

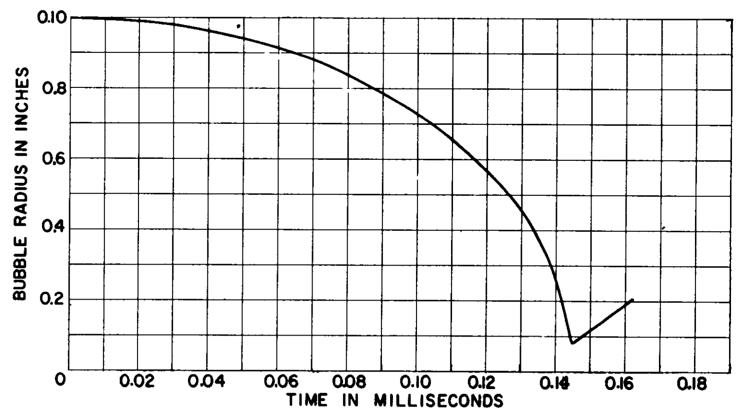

Figure 5a – Bubble Radius

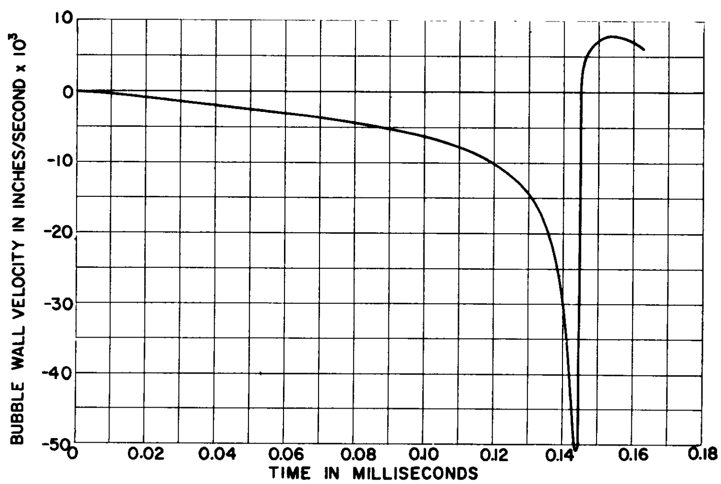

Figure 5b – Bubble Wall Velocity

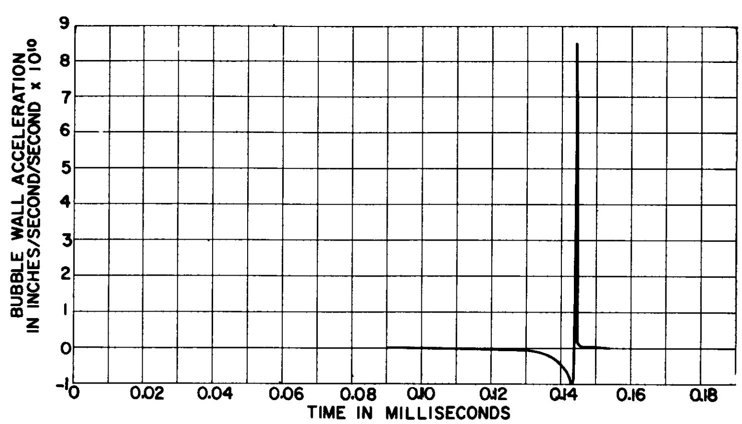

Figure 5c – Bubble Wall Acceleration

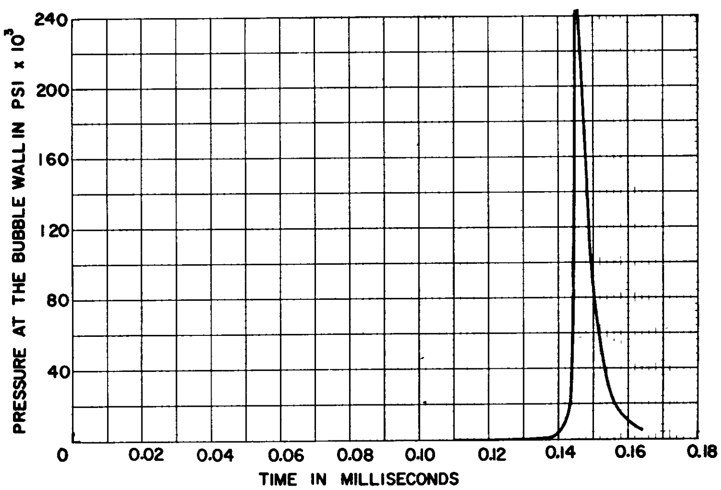

Figure 5d – Pressure at the Bubble Wall

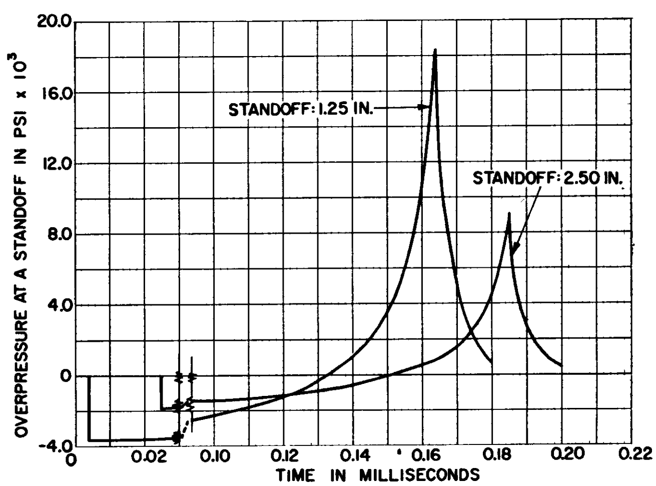

Figure 5e – Overpressures

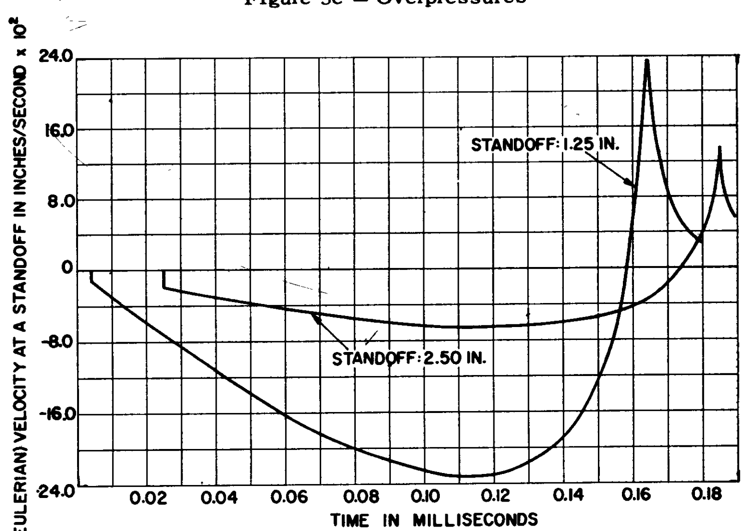

Figure 5f – (Eulerian) Velocity

As stated earlier, the "well behaved" spherical collapse of the model occasionally may not conform to the behavior of a real bubble in its final stage of implosion. At very high ratios of ambient to initial internal pressure, the possible dissociation of the cavity into numerous smaller bubbles represents a departure from the behavior of the model. At the standoff of interest ($r > R_0$), however, the field variables (pressure and Eulerian velocity) are thought not to deviate significantly from values obtained using the model.

All the results mentioned so far may be extended to cases for spheres of any radius. Suppose that at depth h a solution exists for a sphere with initial radius R_0 and initial internal pressure p_0. The radius, velocity, acceleration, and pressure are known functions of time at the bubble wall or at some standoff in the fluid. If the initial radius is multiplied by λ = constant, then pressure and velocity will remain the same if radius, time, and standoff are multiplied by λ and acceleration divided by λ.

After the writer's program was completed, it was discovered that Hickling and Plesset had solved the free-field implosion problem numerically in a similar but more elaborate manner without making the approximation $u^2 << c^2$. Their program requires 20 min of computer time for each case compared to only 2 min for the program presented here. They report two cases; the first ($p_0 = 10^{-3}$ atm, $p_\infty = 1$ atm) was solved by the program presented here, but the second ($p_0 = 10^{-4}$ atm, $p_\infty = 1$ atm) is too violent an implosion for the writer's program to be applicable because the approximation $u^2 << c^2$ may not be valid. (Note that this second case reported by Hickling and Plesset represents such a violent implosion that it has little in common with the type of implosion expected in a buoyancy sphere system even as deep as 30,000 feet of water).

In an attempt to verify the soundness of his approach, the writer used the initial conditions of Hickling and Plesset's first case ($p_0 = 10^{-3}$ atm, $p_\infty = 1$ atm) as input in his program. Very little discrepancy (less than 2 percent) can be found in the bubble radius and bubble wall velocity even though the Hickling and Plesset results are based on a value of $\gamma = 1.4$ and the writer's are based on a value of $\gamma = 4/3$. It can be seen from the differential (Equation [1.5]) that a small change in γ has little effect on the radius-time curve.

However, a marked difference appeared between the results of the two programs when the peak pressures inside and outside the bubble were compared. The Hickling and Plesset results showed peak pressures which were about twice those obtained in this study. This discrepancy can be readily resolved by noting the different values of γ used. The results of both programs indicate that a minimum radius $R_{MIN} = 0.0170$ will be obtained when a sphere of initial radius $R_0 = 1$ and initial internal pressure $p_0 = 10^{-3}$ atmospheres is imploded at the ambient pressure $p_\infty = 1$ atm. The pressure at the boundary, by Equation [1.4c], is

$$p = p_0 \left(\frac{R_0}{R} \right)^{3\gamma}$$

When $\gamma = 1.4$

$$p_{MAX}^* = p_0 \left(\frac{1}{.0170}\right)^{4.2} \qquad [4.1]$$

and when $\gamma = 4/3$

$$p_{MAX} = p_0 \left(\frac{1}{.0170}\right)^{4.0} \qquad [4.2]$$

By dividing Equation [4.1] by Equation [4.2], p_{MAX}^* is $(1/.0170)^{.2}$ or 2.25 times p_{MAX}. Had a value of $\gamma = 1.4$ been used in the writer's program, the peak pressure at the bubble wall would have compared well with that obtained by Hickling and Plesset. A similar statement is true for those peak pressures in the fluid outside the bubble wall, because the peak pressure varies inversely as the distance from the center of the bubble (i.e. as $1/r$). This verifies the validity of the writer's program and the assumption $u^2 << c^2$ for the range of interest ($p_\infty \leq 1000$ atm, $p_0 \leq 1$ atm).

The verification against the work of Hickling and Plesset shows that small variations in γ, the specific heat ratio, can lead to large variations in the peak pressure associated with a collapse. Such behavior suggests that the present equation of state, the ideal gas law, is a deficient description of the gas inside the cavity and that the use of a more elaborate equation of state (e.g., the Beattie-Bridgeman[11] equation of state) would give more accurate results. Use of the Beattie-Bridgeman equation could be made to investigate the differences in behavior of the collapse for different gases.(representing different values of γ). Variations of the kind of gas inside the cavity along with variations in its initial internal pressure may be used to control the characteristics of the pressure pulse emitted when a glass buoyancy sphere collapses. Such control might ultimately be used to reduce the distances between glass spheres in buoyancy sphere systems without increased risk of sympathetic implosions.

SUMMARY AND CONCLUSIONS

1. A program to integrate Gilmore's equations describing the collapse of a spherical gas filled cavity has been written (see Appendix C). The program is general enough to be used in the study of such phenomena as cavitation and underwater explosion gas bubble pulses, provided behavior is adiabatic.

2. Parameters from the program for various ambient and initial internal pressures have been plotted (see Figures 1 through 5).

3. The pressure shock wave associated with the implosion may be controllable in two different ways:

a. Proper variation of the initial internal pressure of the gas inside the sphere.

b. Proper variation of the specific heat ratio γ by changing the kind of gas inside the sphere.

These two effects should not be overlooked as possible means of protecting glass buoyancy spheres from sympathetic implosions.

FUTURE WORK PLANNED

1. An experimental verification will be carried out to determine how good Gilmore's model is.

2. The velocity and pressure curves can be used to synthesize analytical functions to describe the free-field implosion. This information will then form a foundation for the analysis of the effects of a single implosion in a system of buoyancy spheres. Such an analysis has, in fact, been started.

3. The present computer program is now being altered by replacing the ideal gas law by the Beattie-Bridgeman equation of state.

ACKNOWLEDGMENTS

Appreciation is expressed to Dr. W. W. Murray, who proposed that this work be undertaken, and to Mr. S. Zilliacus for valuable discussions during preparation of the paper. Thanks are also due Dr. W. W. Murray, Dr. W. J. Sette, and Dr. H. M. Schauer for their helpful comments.

APPENDIX A

A PROCEDURE FOR HALVING THE INTERVAL OF INTEGRATION

The description of Hamming's method pointed out that if the predictor does not lie close enough to the corrector, then one alternative is to halve the interval. According to Ralston and Wilf,[9] a suitable set of interpolation formulas for Hamming's method is:

$$y_{n-1/2} = \frac{1}{256}\,(80y_n + 135y_{n-1} + 40y_{n-2} + y_{n-3}) + \frac{h}{256}\,(-15y_n' + 90y_{n-1}' + 15y_{n-2}')$$

$$y_{n-3/2} = \frac{1}{256}\,(12y_n + 135y_{n-1} + 108y_{n-2} + y_{n-3}) + \frac{h}{256}\,(-3y_n' - 54y_{n-1}' + 27y_{n-2}')$$

where h is the original step size.

In the particular program which was written for the solution of Equation [1.5], the following procedure was adopted:

1. If p_{n+1} was not close enough to c_{n+1} then an iteration was carried out.

2. If, after the first iteration, the value of p_{n+1} was still not close enough to c_{n+1}, then the interval was halved and the entire integration process at that step was carried out from the beginning using the new half interval.

As the radius of the bubble approaches its minimum, the radius and wall velocity become more and more difficult to predict, that is, it becomes more and more likely that p_{n+1} will not fall within the desired limits of c_{n+1}. With the above procedure, more coordinates will be calculated near the minimum where the functions are changing most rapidly than are calculated in regions of small slopes.

APPENDIX B

JUSTIFICATION FOR THE USE OF HAMMING'S METHOD

The choice between an elaborate procedure like Hamming's method and some other iterative routine to integrate the equations is easy to make if it is based on economy of computer time. When an iterative technique is used to converge on the correct value of the dependent variable at each step, the finite difference form of the differential equation may have to be evaluated many times because the initial prediction of the dependent variable is not likely to be very accurate. In the case of Equations [1.5], it is desirable to avoid the evaluation of the finite difference equation as often as possible because it involves so many calculations. Hamming's method, on the other hand, makes a much more accurate initial prediction of the dependent variable at each step simply because more information about past values is utilized in making such predictions. Consequently, Hamming's method yields a more rapid convergence because the finite difference form of the equation has to be evaluated only once or twice at each step to obtain an accurate value of the dependent variable there.

One disadvantage in using this technique is that it is not self-starting. Values of R, U, and N in at least four equally spaced intervals near $t = 0$ are required. Such values may be computed by expressing $R(t)$ in a Taylor Series in t about zero and using Equations [1.5] to evaluate the coefficients. This approach for solving differential equations can be found in any elementary text on the subject, for instance, Coddington.[12] Because of the difficulty of differentiating Equation [1.5], Herring's[4] equation

$$R \frac{d^2R}{dt^2} + \frac{3}{2}\left(\frac{dR}{dt}\right)^2 - \frac{2}{c_\infty}\left(\frac{dR}{dt}\right)^3 - \frac{2R}{c_\infty}\left(\frac{dR}{dt}\right)\left(\frac{d^2R}{dt^2}\right)$$

$$= \frac{p(t) - p_\infty}{\rho_\infty} + \frac{R}{\rho_\infty c_\infty} \frac{dp(t)}{dt}\left(1 - \frac{\frac{dR}{dt}}{c_\infty}\right) \qquad [5.1]$$

rather than Equation [1.5] was used to find the first four values of R, U, and $\dot{U}$. This equation has also been derived by Gilmore and called the "first order approximation."

As mentioned in the discussion under Equation [1.5], to find the initial acceleration of the bubble wall, Equation [5.1] should be evaluated at $t = 0_+$ rather than at $t = 0$ in order to avoid an infinite initial acceleration.   It can be seen from Equation [1.4c] that Equation [5.1] evaluated at $t = 0$ contains the infinite term $\left.\frac{dP}{dt}\right|_{t=0}$. The result is

$$\left.\frac{d^2R}{dt^2}\right|_{t=0_+} = \frac{p_0 - p_\infty}{\rho_\infty c_\infty}$$

in agreement with acoustic theory.

APPENDIX C

THE COMPUTER PROGRAM FOR IBM 7090

The following is a list of FORTRAN IV symbols used in the program.

FORTRAN Symbol	Corresponding Symbol Used in Discussion	Explanation
A	R	Instantaneous radius of the imploding sphere.
B	B	A constant which characterizes the adiabatic nature of the liquid medium (for water, B = 3000 atm)
BDYP	P	Pressure in the liquid at the bubble wall
BØ	R_0	Initial radius of the imploding sphere
BØD	–	A dimensioned variable name under which the BØ, the unitial radius is read in
B2, B3, B4, B5, B6	–	Coefficients of the Taylor Series expansion of R about $t = 0$
C	c_∞	Sound speed in the undisturbed liquid medium
D	ρ_∞	Density of the undisturbed liquid medium (for water, ρ_∞ = 2 slugs/cu ft)
DU	$\dot{U}$	Instantaneous acceleration of the bubble wall
H	–	Depth at which the implosion takes place
HD	–	A dimensioned variable name under which H, the depth of implosion, is read in
PA	–	Atmospheric pressure (14.7 psi)
PL	p_∞	Pressure in the undisturbed liquid medium
PØ	p_0	Initial pressure of the gas inside the sphere
PØD	–	A dimensioned variable name under which PØ, the initial internal pressure, is read in
R and T	t_R	Time measured at the bubble wall
S	h	The step size, or size of the interval over which the integration is to be performed
STF1, STF2, STF(I) (I = 1, 2, 3)		The five standoffs for which Eulerian velocities and pressures are calculated; note STF(1) = STF3, etc.
STFD1, STFD2, STFD(I) (I = 1, 2, 3)		Dimensioned variable names under which the above five standoffs are read in

FORTRAN Symbol	Corresponding Symbol Used in Discussion	Explanation
STFPC1, STFCP2, STFPC(I) I = 1, 2, 3	$\overline{p}$	Five instantaneous pressures evaluated at the standoffs STF1, STF2, STF3, STF4 and STF5, respectively; note: STFPC(1) = STFPC3 etc.
STFUC1, STFUC2, STFUC(I) I = 1, 2, 3	u	Five instantaneous Eulerian velocities evaluated at the standoffs STF1, STF2, STF3, STF4, and STF5, respectively
T and R	t_R	Time measured at the bubble wall
TSTF1, TSTF2, TSTF(I) I = 1, 2, 3	t	The time for each of the five pressures and velocities evaluated at each of the five standoffs, respectively
U	U	Instantaneous velocity of the bubble wall
Y	y	A constant used in the expression for the Eulerian velocity (see Equations [3.1] and [3.2])
YK	K_3	A constant used in the expression for the Eulerian velocity (see Equations [3.1] and [3.3])
AP4, AP5	p_n, p_{n+1}	Predicted values of the radius, AP 5 is the predicted value being tested at the $(n + 1)$th interval, and AP4 is the previous predicted value of R which was cloest to the actual value of R at the nth interval
A4MH, A3MH	$y_{n-1/2}$, $y_{n-3/2}$	When R at the $(n + 1)$th interval is being predicted and the half interval routine is required, then these are the interpolated values of R between the nth and $(n - 1)$th interval and between the $(n - 1)$th and $(n - 2)$th interval, respectively
CØD	m_{n+1}, m'_{n+1}	Modifier for U, also derivative of the modifier for R
C4, C5	c_n, c_{n+1}	Correctors for U; C5 is the corrector being tested at the $(n + 1)$th interval and C4 is the corrector at the previous interval
DØD	m_{n+1}	Modifier for R
DCØD	m'_{n+1}	Derivative of the modifier for U
D4, D5	c_n, c_{n+1}	Correctors for R (see C4, C5)
UP4, UP5	p_n, p_{n+1}	Predicted values of the velocity (see AP4, AP5)
U4MH, U3MH	$y_{n-1/2}$, $y_{n-3/2}$	Half interval values of velocity (see A4MH, A3MH)

THE COMPUTER PROGRAM

The simplified flow chart is shown in Figure 6.

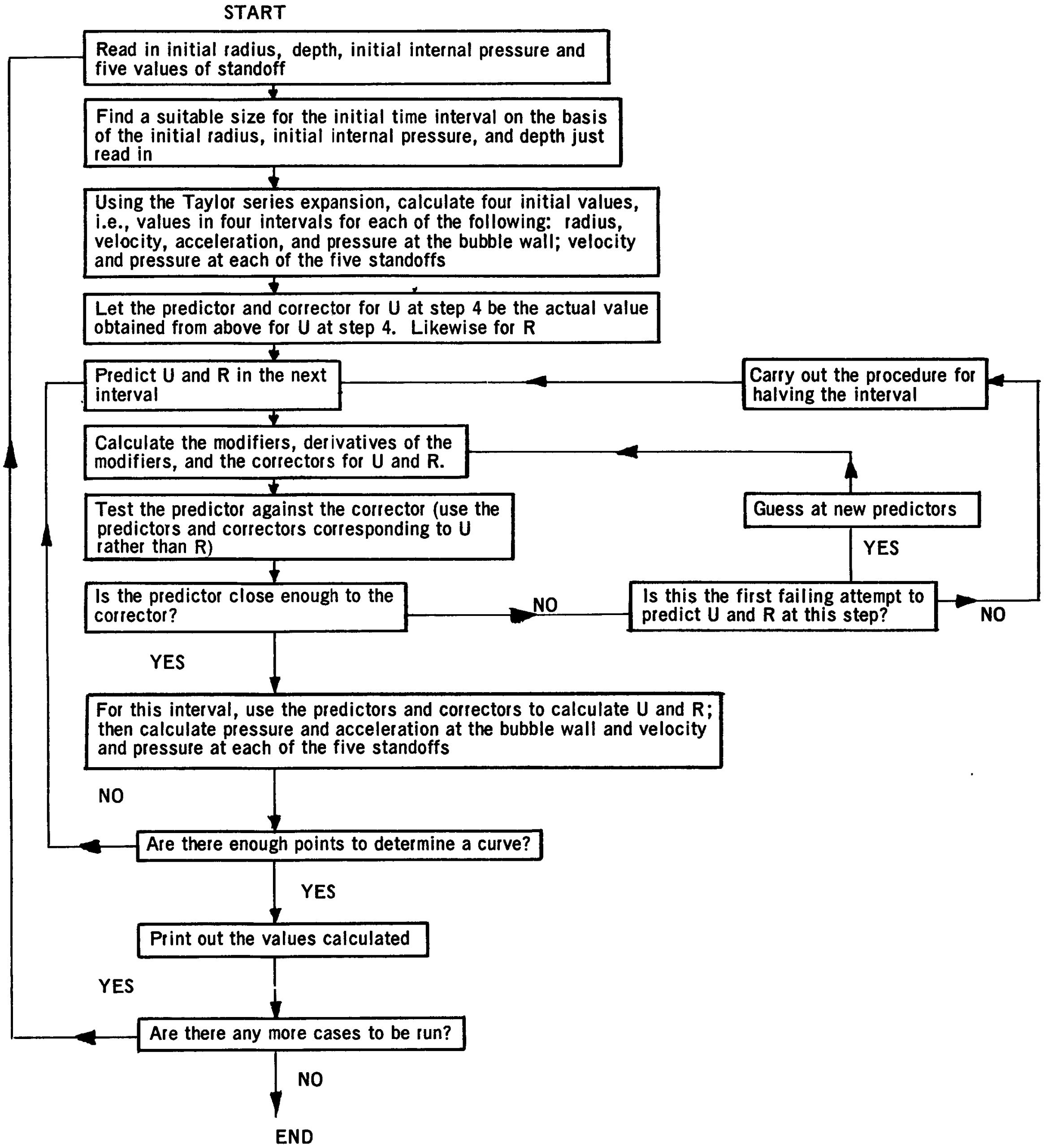

Figure 6 – Simplified Flow Chart

DATA INPUT

The first data card is read according to the format (I2) and must have a number no greater than 50 in the first two columns (see Figure 7). This number should equal the number of data cards to follow. Each of the following data cards should contain the initial information for a single collapse. Eight pieces of information are placed on each card; the card is divided equally into eight parts, 10 spaces each. Information is read in according to the format (8F10.4). The first two pieces of information are the depth of collapse in feet of water and the initial bubble radius in inches, respectively. The next five divisions are set off for five values of standoff in inches from the bubble center at which a pressure and (Eulerian) velocity time history are desired. The last division is reserved for the initial internal pressure inside the bubble in pounds per square inch. The first eight pieces of information, corresponding to one collapse, will require no more than 2 min of running time. Each additional card, i.e., each additional collapse, requires no more than 1 min.

DATA OUTPUT

For each data card, the computer will print 2000 lines of output. The first 1000 lines of numbers are printed under headings TIME, RADIUS, VELOCITY, ACCELERATION, BDYP, TSTF1, STFPC1, STFUC1, TSTF2, STFPC2, STFUC2 (see Figure 8). The first five quantities refer to the bubble wall; BDYP is the internal pressure inside the bubble (absolute pressure, not overpressure). Each line refers to the state of the motion at one instant in time. STFPC1 and STFUC1 are the overpressure (pressure above ambient pressure) and (Eulerian) velocity as functions of the time TSTF1 for the first standoff given in the data. Similarly for STFPC2, STFUC2, and TSTF2. The next 1000 lines give similar information for the third, fourth, and fifth standoffs specified. The headings are TSTF3, STFPC3, STFUC3, TSTF4, STFPC4, STFUC4, TSTF5, STFPC5, STFUC5. All time is given in milliseconds, the radius in inches, all velocities in inches per second, the acceleration in inches per second per second, and all pressures in pounds per square inch.

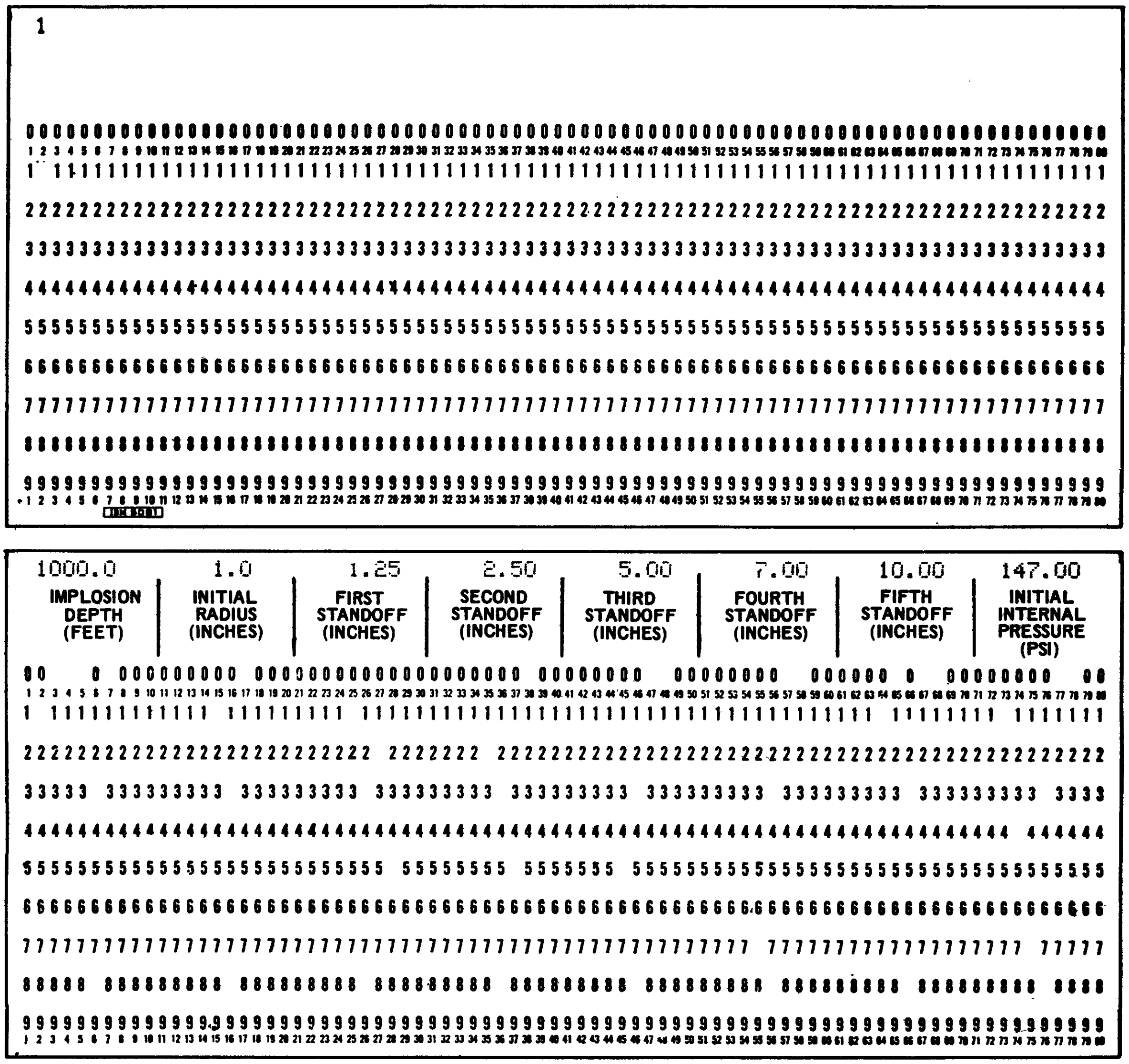

Figure 7 – Data Input for Computer Program

(MILLISECONDS)	(INCHES)	(IN/SEC)	(IN/SEC/SEC)	(PSI)	(MILLISECONDS)	(PSI)	(IN/SEC)	(MILLISECONDS)	(PSI)	(IN/SEC)
TIME	RADIUS	VELOCITY	ACCELERATION	BDYP	TSTF1	STFPC1	STFUC1	TSTF2	STFPC2	STFUC2
0.	1.00000	-0.	-0.32668E 07	0.14700E 03	0.004167	-0.25196E 03	-0.86856E 01	0.025000	-0.12601E 03	-0.13063E 02
0.006469	0.99993	-0.21109E 02	-0.32628E 07	0.14704E 03	0.010638	-0.25192E 03	-0.22191E 02	0.031474	-0.12598E 03	-0.16435E 02
0.012938	0.99973	-0.42195E 02	-0.32598E 07	0.14716E 03	0.017112	-0.25176E 03	-0.35671E 02	0.037950	-0.12589E 03	-0.19796E 02
0.019407	0.99939	-0.63265E 02	-0.32577E 07	0.14736E 03	0.023588	-0.25148E 03	-0.49119E 02	0.044428	-0.12573E 03	-0.23142E 02
0.025876	0.99891	-0.84335E 02	-0.32565E 07	0.14764E 03	0.030066	-0.25109E 03	-0.62534E 02	0.050909	-0.12551E 03	-0.26474E 02
0.032345	0.99829	-0.10540E 03	-0.32563E 07	0.14801E 03	0.036546	-0.25058E 03	-0.75902E 02	0.057391	-0.12521E 03	-0.29789E 02
0.038814	0.99754	-0.12647E 03	-0.32570E 07	0.14845E 03	0.043029	-0.24995E 03	-0.89216E 02	0.063876	-0.12486E 03	-0.33085E 02
0.045284	0.99666	-0.14754E 03	-0.32586E 07	0.14898E 03	0.049514	-0.24921E 03	-0.10247E 03	0.070364	-0.12444E 03	-0.36360E 02
0.051753	0.99564	-0.16863E 03	-0.32611E 07	0.14959E 03	0.056002	-0.24835E 03	-0.11565E 03	0.076853	-0.12395E 03	-0.39611E 02
0.058222	0.99448	-0.18974E 03	-0.32644E 07	0.15029E 03	0.062491	-0.24737E 03	-0.12876E 03	0.083345	-0.12339E 03	-0.42838E 02
0.064691	0.99318	-0.21087E 03	-0.32687E 07	0.15108E 03	0.068983	-0.24627E 03	-0.14178E 03	0.089840	-0.12277E 03	-0.46038E 02
0.071160	0.99175	-0.23203E 03	-0.32738E 07	0.15195E 03	0.075477	-0.24505E 03	-0.15472E 03	0.096336	-0.12208E 03	-0.49209E 02
0.077629	0.99018	-0.25323E 03	-0.32798E 07	0.15292E 03	0.081974	-0.24370E 03	-0.16755E 03	0.102835	-0.12133E 03	-0.52350E 02
0.084098	0.98847	-0.27447E 03	-0.32866E 07	0.15398E 03	0.088473	-0.24223E 03	-0.18028E 03	0.109336	-0.12050E 03	-0.55458E 02
0.090567	0.98663	-0.29575E 03	-0.32943E 07	0.15513E 03	0.094974	-0.24064E 03	-0.19289E 03	0.115839	-0.11961E 03	-0.58532E 02
0.097036	0.98465	-0.31709E 03	-0.33028E 07	0.15639E 03	0.101477	-0.23891E 03	-0.20539E 03	0.122345	-0.11864E 03	-0.61569E 02
0.103505	0.98252	-0.33849E 03	-0.33121E 07	0.15774E 03	0.107983	-0.23706E 03	-0.21775E 03	0.128853	-0.11761E 03	-0.64567E 02
0.109974	0.98027	-0.35994E 03	-0.33223E 07	0.15920E 03	0.114491	-0.23507E 03	-0.22997E 03	0.135363	-0.11650E 03	-0.67525E 02
0.116443	0.97787	-0.38147E 03	-0.33332E 07	0.16077E 03	0.121002	-0.23295E 03	-0.24205E 03	0.141875	-0.11532E 03	-0.70440E 02
0.122913	0.97533	-0.40307E 03	-0.33449E 07	0.16245E 03	0.127514	-0.23069E 03	-0.25397E 03	0.148390	-0.11407E 03	-0.73309E 02
0.129382	0.97265	-0.42475E 03	-0.33574E 07	0.16424E 03	0.134030	-0.22829E 03	-0.26573E 03	0.154908	-0.11274E 03	-0.76132E 02
0.135851	0.96983	-0.44651E 03	-0.33706E 07	0.16616E 03	0.140547	-0.22575E 03	-0.27731E 03	0.161427	-0.11134E 03	-0.78904E 02
0.142320	0.96687	-0.46836E 03	-0.33846E 07	0.16820E 03	0.147067	-0.22305E 03	-0.28872E 03	0.167949	-0.10986E 03	-0.81625E 02
0.148789	0.96377	-0.49030E 03	-0.33992E 07	0.17038E 03	0.153589	-0.22021E 03	-0.29994E 03	0.174473	-0.10830E 03	-0.84292E 02
0.155258	0.96053	-0.51234E 03	-0.34145E 07	0.17269E 03	0.160114	-0.21721E 03	-0.31095E 03	0.181000	-0.10666E 03	-0.86902E 02
0.161727	0.95715	-0.53448E 03	-0.34305E 07	0.17515E 03	0.166641	-0.21406E 03	-0.32176E 03	0.187529	-0.10493E 03	-0.89452E 02
0.168196	0.95362	-0.55673E 03	-0.34470E 07	0.17776E 03	0.173171	-0.21074E 03	-0.33235E 03	0.194060	-0.10313E 03	-0.91941E 02
0.174665	0.94994	-0.57908E 03	-0.34641E 07	0.18052E 03	0.179703	-0.20725E 03	-0.34271E 03	0.200594	-0.10124E 03	-0.94366E 02
0.181134	0.94612	-0.60155E 03	-0.34817E 07	0.18345E 03	0.186237	-0.20359E 03	-0.35284E 03	0.207130	-0.99261E 02	-0.96724E 02
0.187603	0.94216	-0.62413E 03	-0.34998E 07	0.18656E 03	0.192774	-0.19976E 03	-0.36271E 03	0.213669	-0.97194E 02	-0.99012E 02
0.194072	0.93805	-0.64683E 03	-0.35182E 07	0.18985E 03	0.199314	-0.19574E 03	-0.37233E 03	0.220210	-0.95036E 02	-0.10123E 03
0.200541	0.93379	-0.66965E 03	-0.35369E 07	0.19334E 03	0.205856	-0.19153E 03	-0.38167E 03	0.226754	-0.92784E 02	-0.10337E 03
0.207011	0.92938	-0.69259E 03	-0.35558E 07	0.19703E 03	0.212400	-0.18714E 03	-0.39074E 03	0.233300	-0.90436E 02	-0.10543E 03
0.213480	0.92483	-0.71566E 03	-0.35748E 07	0.20094E 03	0.218947	-0.18254E 03	-0.39951E 03	0.239848	-0.87989E 02	-0.10741E 03
0.219949	0.92012	-0.73884E 03	-0.35939E 07	0.20508E 03	0.225496	-0.17773E 03	-0.40797E 03	0.246399	-0.85440E 02	-0.10930E 03
0.226418	0.91527	-0.76215E 03	-0.36128E 07	0.20947E 03	0.232049	-0.17271E 03	-0.41612E 03	0.252953	-0.82788E 02	-0.11111E 03
0.232887	0.91026	-0.78558E 03	-0.36314E 07	0.21412E 03	0.238603	-0.16747E 03	-0.42394E 03	0.259509	-0.80028E 02	-0.11282E 03
0.239356	0.90510	-0.80914E 03	-0.36495E 07	0.21904E 03	0.245160	-0.16201E 03	-0.43142E 03	0.266067	-0.77157E 02	-0.11444E 03
0.245825	0.89979	-0.83280E 03	-0.36670E 07	0.22426E 03	0.251720	-0.15630E 03	-0.43853E 03	0.272628	-0.74172E 02	-0.11596E 03
0.252294	0.89433	-0.85658E 03	-0.36836E 07	0.22979E 03	0.258282	-0.15035E 03	-0.44528E 03	0.279192	-0.71069E 02	-0.11738E 03
0.258763	0.88871	-0.88046E 03	-0.36990E 07	0.23565E 03	0.264847	-0.14415E 03	-0.45165E 03	0.285758	-0.67844E 02	-0.11869E 03
0.265232	0.88294	-0.90443E 03	-0.37130E 07	0.24188E 03	0.271415	-0.13768E 03	-0.45761E 03	0.292327	-0.64494E 02	-0.11989E 03
0.271701	0.87701	-0.92849E 03	-0.37252E 07	0.24849E 03	0.277985	-0.13095E 03	-0.46316E 03	0.298898	-0.61013E 02	-0.12098E 03
0.278170	0.87092	-0.95262E 03	-0.37352E 07	0.25550E 03	0.284558	-0.12392E 03	-0.46827E 03	0.305472	-0.57398E 02	-0.12194E 03
0.284639	0.86468	-0.97681E 03	-0.37425E 07	0.26296E 03	0.291134	-0.11661E 03	-0.47294E 03	0.312048	-0.53643E 02	-0.12279E 03
0.291109	0.85829	-0.10010E 04	-0.37467E 07	0.27089E 03	0.297712	-0.10899E 03	-0.47714E 03	0.318627	-0.49744E 02	-0.12350E 03
0.297578	0.85173	-0.10253E 04	-0.37472E 07	0.27932E 03	0.304293	-0.10105E 03	-0.48086E 03	0.325209	-0.45695E 02	-0.12408E 03
0.304047	0.84502	-0.10495E 04	-0.37433E 07	0.28830E 03	0.310876	-0.92778E 02	-0.48407E 03	0.331793	-0.41490E 02	-0.12452E 03
0.310516	0.83815	-0.10737E 04	-0.37341E 07	0.29787E 03	0.317462	-0.84166E 02	-0.48676E 03	0.338379	-0.37124E 02	-0.12482E 03
0.316985	0.83113	-0.10978E 04	-0.37190E 07	0.30807E 03	0.324051	-0.75197E 02	-0.48891E 03	0.344969	-0.32591E 02	-0.12496E 03
0.323454	0.82395	-0.11218E 04	-0.36967E 07	0.31894E 03	0.330642	-0.65858E 02	-0.49050E 03	0.351560	-0.27883E 02	-0.12496E 03
0.329923	0.81662	-0.11456E 04	-0.36662E 07	0.33056E 03	0.337236	-0.56132E 02	-0.49150E 03	0.358154	-0.22994E 02	-0.12479E 03
0.336392	0.80913	-0.11692E 04	-0.36262E 07	0.34296E 03	0.343833	-0.46006E 02	-0.49189E 03	0.364751	-0.17917E 02	-0.12445E 03
0.342861	0.80149	-0.11925E 04	-0.35751E 07	0.35623E 03	0.350432	-0.35463E 02	-0.49165E 03	0.371350	-0.12644E 02	-0.12393E 03
0.349330	0.79370	-0.12154E 04	-0.35113E 07	0.37042E 03	0.357033	-0.24486E 02	-0.49075E 03	0.377951	-0.71686E 01	-0.12324E 03
0.355799	0.78576	-0.12379E 04	-0.34327E 07	0.38561E 03	0.363637	-0.13060E 02	-0.48917E 03	0.384555	-0.14821E 01	-0.12235E 03
0.362268	0.77768	-0.12598E 04	-0.33372E 07	0.40189E 03	0.370243	-0.11671E 01	-0.48688E 03	0.391161	0.44232E 01	-0.12128E 03
0.368737	0.76947	-0.12810E 04	-0.32221E 07	0.41933E 03	0.376852	0.11209E 02	-0.48386E 03	0.397769	0.10556E 02	-0.11999E 03

Figure 8 – Data Output for Computer Program

(MILLISECONDS) TSTF3	(PSI) STFPC3	(IN/SEC) STFUC3	(MILLISECONDS) TSTF4	(PSI) STFPC4	(IN/SEC) STFUC4	(MILLISECONDS) TSTF5	(PSI) STFPC5	(IN/SEC) STFUC5
0.066667	-0.630109E 02	-0.871052E 01	0.100000	-0.450085E 02	-0.666637E 01	0.150000	-0.315062E 02	-0.489984E 01
0.073142	-0.629950E 02	-0.955200E 01	0.106475	-0.449969E 02	-0.709509E 01	0.156475	-0.314980E 02	-0.510947E 01
0.079619	-0.629460E 02	-0.103876E 02	0.112953	-0.449616E 02	-0.751963E 01	0.162953	-0.314732E 02	-0.531620E 01
0.086098	-0.628637E 02	-0.112168E 02	0.119433	-0.449025E 02	-0.793976E 01	0.169433	-0.314317E 02	-0.551990E 01
0.092580	-0.627483E 02	-0.120397E 02	0.125915	-0.448197E 02	-0.835548E 01	0.175916	-0.313736E 02	-0.572057E 01
0.099064	-0.625996E 02	-0.128554E 02	0.132399	-0.447132E 02	-0.876633E 01	0.182400	-0.312989E 02	-0.591801E 01
0.105550	-0.624177E 02	-0.136634E 02	0.138885	-0.445828E 02	-0.917209E 01	0.188887	-0.312076E 02	-0.611208E 01
0.112039	-0.622024E 02	-0.144632E 02	0.145374	-0.444286E 02	-0.957249E 01	0.195376	-0.310994E 02	-0.630265E 01
0.118529	-0.619535E 02	-0.152543E 02	0.151865	-0.442504E 02	-0.996730E 01	0.201867	-0.309746E 02	-0.648962E 01
0.125022	-0.616710E 02	-0.160363E 02	0.158359	-0.440481E 02	-0.103563E 02	0.208361	-0.308328E 02	-0.667285E 01
0.131518	-0.613546E 02	-0.168087E 02	0.164854	-0.438216E 02	-0.107391E 02	0.214857	-0.306741E 02	-0.685221E 01
0.138015	-0.610041E 02	-0.175710E 02	0.171352	-0.435708E 02	-0.111156E 02	0.221355	-0.304983E 02	-0.702757E 01
0.144515	-0.606193E 02	-0.183227E 02	0.177852	-0.432953E 02	-0.114855E 02	0.227855	-0.303054E 02	-0.719879E 01
0.151017	-0.601998E 02	-0.190633E 02	0.184355	-0.429952E 02	-0.118485E 02	0.234358	-0.300951E 02	-0.736573E 01
0.157522	-0.597454E 02	-0.197922E 02	0.190860	-0.426700E 02	-0.122043E 02	0.240863	-0.298673E 02	-0.752824E 01
0.164028	-0.592557E 02	-0.205088E 02	0.197367	-0.423197E 02	-0.125526E 02	0.247370	-0.296218E 02	-0.768618E 01
0.170537	-0.587303E 02	-0.212128E 02	0.203876	-0.419438E 02	-0.128931E 02	0.253880	-0.293585E 02	-0.783939E 01
0.177049	-0.581689E 02	-0.219034E 02	0.210388	-0.415421E 02	-0.132256E 02	0.260392	-0.290772E 02	-0.798772E 01
0.183562	-0.575708E 02	-0.225802E 02	0.216901	-0.411143E 02	-0.135498E 02	0.266906	-0.287775E 02	-0.813099E 01
0.190078	-0.569357E 02	-0.232425E 02	0.223418	-0.406600E 02	-0.138652E 02	0.273422	-0.284593E 02	-0.826904E 01
0.196596	-0.562629E 02	-0.238897E 02	0.229936	-0.401788E 02	-0.141716E 02	0.279941	-0.281222E 02	-0.840171E 01
0.203117	-0.555520E 02	-0.245212E 02	0.236457	-0.396703E 02	-0.144687E 02	0.286462	-0.277661E 02	-0.852880E 01
0.209640	-0.548022E 02	-0.251365E 02	0.242980	-0.391341E 02	-0.147561E 02	0.292985	-0.273906E 02	-0.865013E 01
0.216165	-0.540130E 02	-0.257347E 02	0.249506	-0.385697E 02	-0.150334E 02	0.299511	-0.269953E 02	-0.876552E 01
0.222693	-0.531836E 02	-0.263153E 02	0.256034	-0.379765E 02	-0.153003E 02	0.306039	-0.265799E 02	-0.887476E 01
0.229223	-0.523131E 02	-0.268776E 02	0.262564	-0.373541E 02	-0.155564E 02	0.312570	-0.261440E 02	-0.897766E 01
0.235755	-0.514009E 02	-0.274208E 02	0.269097	-0.367019E 02	-0.158014E 02	0.319103	-0.256873E 02	-0.907399E 01
0.242290	-0.504460E 02	-0.279443E 02	0.275632	-0.360191E 02	-0.160347E 02	0.325638	-0.252092E 02	-0.916353E 01
0.248827	-0.494476E 02	-0.284473E 02	0.282169	-0.353053E 02	-0.162560E 02	0.332175	-0.247093E 02	-0.924607E 01
0.255367	-0.484045E 02	-0.289290E 02	0.288709	-0.345596E 02	-0.164650E 02	0.338715	-0.241871E 02	-0.932136E 01
0.261909	-0.473158E 02	-0.293886E 02	0.295251	-0.337813E 02	-0.166610E 02	0.345258	-0.236421E 02	-0.938916E 01
0.268453	-0.461803E 02	-0.298253E 02	0.301795	-0.329696E 02	-0.168437E 02	0.351802	-0.230738E 02	-0.944922E 01
0.275000	-0.449969E 02	-0.302382E 02	0.308343	-0.321237E 02	-0.170126E 02	0.358350	-0.224815E 02	-0.950126E 01
0.281549	-0.437643E 02	-0.306264E 02	0.314892	-0.312427E 02	-0.171672E 02	0.364899	-0.218646E 02	-0.954502E 01
0.288101	-0.424811E 02	-0.309891E 02	0.321444	-0.303256E 02	-0.173069E 02	0.371451	-0.212225E 02	-0.958020E 01
0.294655	-0.411460E 02	-0.313252E 02	0.327998	-0.293714E 02	-0.174313E 02	0.378006	-0.205544E 02	-0.960651E 01
0.301212	-0.397575E 02	-0.316338E 02	0.334555	-0.283790E 02	-0.175397E 02	0.384563	-0.198596E 02	-0.962364E 01
0.307771	-0.383138E 02	-0.319139E 02	0.341115	-0.273474E 02	-0.176317E 02	0.391123	-0.191374E 02	-0.963126E 01
0.314332	-0.368134E 02	-0.321644E 02	0.347677	-0.262752E 02	-0.177066E 02	0.397685	-0.183867E 02	-0.962903E 01
0.320897	-0.352545E 02	-0.323842E 02	0.354241	-0.251613E 02	-0.177637E 02	0.404249	-0.176069E 02	-0.961661E 01
0.327463	-0.336352E 02	-0.325721E 02	0.360808	-0.240042E 02	-0.178025E 02	0.410816	-0.167968E 02	-0.959361E 01
0.334033	-0.319534E 02	-0.327270E 02	0.367377	-0.228026E 02	-0.178222E 02	0.417386	-0.159556E 02	-0.955966E 01
0.340604	-0.302070E 02	-0.328476E 02	0.373949	-0.215549E 02	-0.178221E 02	0.423958	-0.150822E 02	-0.951436E 01
0.347179	-0.283939E 02	-0.329326E 02	0.380523	-0.202595E 02	-0.178015E 02	0.430532	-0.141754E 02	-0.945727E 01
0.353755	-0.265116E 02	-0.329806E 02	0.387100	-0.189148E 02	-0.177597E 02	0.437109	-0.132341E 02	-0.938797E 01
0.360335	-0.245577E 02	-0.329903E 02	0.393680	-0.175190E 02	-0.176957E 02	0.443689	-0.122570E 02	-0.930599E 01
0.366917	-0.225294E 02	-0.329602E 02	0.400262	-0.160701E 02	-0.176088E 02	0.450271	-0.112428E 02	-0.921085E 01
0.373501	-0.204241E 02	-0.328887E 02	0.406846	-0.145663E 02	-0.174981E 02	0.456855	-0.101901E 02	-0.910205E 01
0.380088	-0.182388E 02	-0.327743E 02	0.413433	-0.130054E 02	-0.173627E 02	0.463442	-0.909751E 01	-0.897907E 01
0.386677	-0.159704E 02	-0.326152E 02	0.420023	-0.113851E 02	-0.172016E 02	0.470032	-0.796342E 01	-0.884135E 01
0.393269	-0.136157E 02	-0.324098E 02	0.426615	-0.970337E 01	-0.170138E 02	0.476624	-0.678625E 01	-0.868832E 01
0.399864	-0.111714E 02	-0.321562E 02	0.433209	-0.795762E 01	-0.167983E 02	0.483218	-0.556433E 01	-0.851938E 01
0.406460	-0.863393E 01	-0.318525E 02	0.439806	-0.614538E 01	-0.165539E 02	0.489815	-0.429588E 01	-0.833391E 01
0.413059	-0.599962E 01	-0.314969E 02	0.446405	-0.426806E 01	-0.162797E 02	0.496414	-0.297909E 01	-0.813127E 01
0.419661	-0.326471E 01	-0.310872E 02	0.453006	-0.231096E 01	-0.159743E 02	0.503015	-0.161208E 01	-0.791079E 01
0.426264	-0.425312E 00	-0.306214E 02	0.459610	-0.283286E 00	-0.156365E 02	0.509619	-0.192885E 00	-0.767176E 01
0.432870	0.252256E 01	-0.300972E 02	0.466216	0.182179E 01	-0.152653E 02	0.516225	0.128047E 01	-0.741347E 01
0.439478	0.558295E 01	-0.295125E 02	0.472823	0.400716E 01	-0.148591E 02	0.522832	0.281001E 01	-0.713520E 01

Figure 8 (Continued)

COMPUTER PROGRAM

```
          RU 1        -  EFN   SOURCE STATEMENT  -  IFN(S)  -

C  PROGRAMMER  R. LILLISTON, CODE 745, EXT 3357
C D=DENSITY IN SLUGS/CU.FT,H=DEPTH IN FT,PO=INITIAL INTERNAL PRESSURE
C IN LBS/SQ.IN.BO=INITIAL RADIUS IN INCHES,C=SPEED OF SOUNDIN MEDIUM IN
C IN/SEC,PA=ATMOSPHERIC PRESSURE IN LBS/SQ.IN,STF=STANDOFF IN INCHES
C AND TIME IS IN MILLISECONDS
C  INITIAL VALUES
      READ(5,101)MM
  101 FORMAT(I2)
      DIMENSION TSTF(3,1000),STFPC(3,1000),STFUC(3,1000),STF(3)
      DIMENSION A(1000),U(1000),DU(1000)
      DIMENSION HD(50),BOD(50),STFD1(50),STFD2(50),STFD(3,50),POD(50)
      READ(5,99)(HD(J),BOD(J),STFD1(J),STFD2(J),(STFD(I,J),I=1,3),
     1POD(J),J=1,MM)
   99 FORMAT(8F10.4)
      DO1 J=1,MM,1
      H=HD(J)
      BO=BOD(J)
      STF1=STFD1(J)
      STF2=STFD2(J)
      DO 9 I=1,3
    9 STF(I)=STFD(I,J)
      PO=POD(J)
C VALUES OF CONSTANTS FOR THE LIQUID - WATER
      D=2.0
      B=4.41E4
      C=6.0E4
      PA=14.7
      PL=D*H*32.2/144.0+PA
C HEADING
      WRITE(6,88)BO,H,PO,STF1,STF2,(STF(I),I=1,3)
   88 FORMAT(1H1/1H3/////42X,44H GILMORES SECOND ORDER APPROXIMATION FOR
     1 THE///45X,14HIMPLOSION OF A,F4.1,19H INCH RADIUS SPHERE///45X,14H
     2 AT A DEPTH OF,F8.1,14H FEET OF WATER///33X,51HWHEN THE INITIAL IN
     3TERNAL PRESSURE IN THE SPHERE IS,F5.1,5H PSIA///47X,33H AND THE ST
     4ANDOFFS ARE, IN INCHES///48X,10HSTANDOFF 1,F21.2///48X,10HSTANDOFF
     5 2,F21.2///48X,10HSTANDOFF 3,F21.2///48X,10HSTANDOFF 4,F21.2///
     648X,10HSTANDOFF 5,F21.2)
C CALCULATE INITIAL VALUES OF THE RADIUS AND VELOCITY
      B2=(2.0736E4)*(PO-PL)/(BO*D*2.0)
      B3=4.0*B2**2/(3.0*C)-(2.7648E4)*PO*B2/(BO*D*C)
      B4=3.0*B2*B3/C-4.0*(B2**2)/(3.0*BO)+(2.0736E4)*(-PO*B3/(BO*D*C)
     1-PL*B2/(3.0*D*BO**2)+4.0*PO*(B2**2)/(3.0*BO*D*C**2))
      B5=2.8*(B2**3)/(C*BO)-2.9*B2*B3/BO+1.8*(B3**2)/C+3.2*B2*B4/C
     1+(2.0736)*(-2.0*PL*B3/(D*BO**2)-8.0*PO*B4/(BO*C*D)
     2+24.0*PO*B3*B2/(BO*D*C**2))
      B6=146.0*(B2**2)*B3/(15.0*C*BO)+4.0*B4*B3/C+1.0*B2*B5/(3.0*C)
     1-11.2*B2**3/(9.0*BO**2)-2.9*B3**2/(2.0*BO)-9.4*B2*B4/(3.0*BO)
     2+(2.0736E4)*(1.2*PO*B3**2/(BO*D*C**2)+6.4*PO*B2*B4/(3.0*BO*D*C**2)
     3-2.0*PO*B5/(3.0*BO*D*C)-PL*B3/(10.0*D*BO**3)
     4-0.2*PL*B2**2/(D*BO**3)-4.0*PL*B4/(30.0*D*BO**2))
      R=0.0
      S=(1.44E-4)*SQRT(D)*PO**(1.0/3.0)*BO/PL**(5.0/6.0)
      WRITE(6,100)
C  HEADINGS
  100 FORMAT(123H1    TIME     RADIUS    VELOCITY    ACCELERATION     BDYP
```

```
    1    TSTF1      STFPC1      STFUC1      TSTF2      STFPC2      STFU
    2C2)
     DO66 I=1,4,1
     A(I)=B0+B2*R**2+B3*R**3+R**2*B4*R**2+R**2*B5*R**3+R**3*B6*R**3
     U(I)=2.0*B2*R+3.0*B3*R**2+4.0*R*B4*R**2+5.0*R**2*B5*R**2+
    16.0*R**3*B6*R**2
     BDYP=P0*B0**4/A(I)**4
     DU(I)=((U(I)**3)*(((PL+B)/(BDYP+B))**(3.0/7.0))/(2.0*C)
    1-3.0*U(I)**2/2.0-7.0*(PL+B)/(6.0*D)*(2.0736E4)*(1.0
    2-((BDYP+B)/(PL+B))**(6.0/7.0))+U(I)/C*((PL+B)/(BDYP+B))**(3.0/7.0)
    3-U(I)/C*((BDYP+B)/(PL+B))**(3.0/7.0))
    4-4.0*U(I)*BDYP/(D*C)*(2.0736E4)*(((BDYP+B)/(PL+B))**(4.0/7.0'
    5-U(I)/C*(PL+B)/(BDYP+B)))/(A(I)-A(I)*U(I)/C*((PL+B)/(BDYP
    6+B))**(3.0/7.0))
     Y=A(I)*U(I)**2/2.0+(2.0736E4)*A(I)*(BDYP-PL)/D*(1.0-
    1(2.0736E4)*(BDYP-PL)/(2.0*D*C**2))
     YK=U(I)*(A(I)**2)*(C**3)*(1.0-U(I)**2/(2.0*C**2))/Y**2
    1-A(I)*(C**2)*(1.0-U(I)/C)/Y
     C4STFU=Y*C/STF1+YK*Y**2/(C**3*STF1**2)*(C**2-Y/STF1
    1+(YK**2/C**3)*(Y**4/C**3)/(2.0*STF1**4))
     STFUC1=C4STFU/C**2
     STFPC1=D*(Y/STF1-STFUC1**2/2.0)/(2.0736E4)
    1+D*((Y/STF1-STFUC1**2/2.0)**2)/(2.0*(C**2)*(2.0736E4))
     TSTF1=((STF1-A(I))/C)*(1.0-U(I)*A(I)/(C*STF1))+R
     C4STFU=Y*C/STF2+YK*Y**2/(C**3*STF2**2)*(C**2-Y/STF2
    1+(YK**2/C**3)*(Y**4/C**3)/(2.0*STF2**4))
     STFUC2=C4STFU/C**2
     STFPC2=D*(Y/STF2-STFUC2**2/2.0)/(2.0736E4)
    1+D*((Y/STF2-STFUC2**2/2.0)**2)/(2.0*(C**2)*(2.0736E4))
     TSTF2=((STF2-A(I))/C)*(1.0-U(I)*A(I)/(C*STF2))+R
     DO 709 KK=1,3
     C4STFU=Y*C/STF(KK)+YK*Y**2/(C**3*STF(KK)**2)*(C**2-Y/STF(KK)
    1+(YK**2/C**3)*(Y**4/C**3)/(2.0*STF(KK)**4))
     STFUC(KK,I)=C4STFU/C**2
     STFPC(KK,I)=D*(Y/STF(KK)-STFUC(KK,I)**2/2.0)/(2.0736E4)
    1+D*((Y/STF(KK)-STFUC(KK,I)**2/2.0)**2)/(2.0*(C**2)*(2.0736E4))
 709 TSTF(KK,I)=((STF(KK)-A(I))/C)*(1.0-U(I)*A(I)/(C*STF(KK)))+R
     R=R*1.0E3
     TSTF1=TSTF1*1.0E3
     TSTF2=TSTF2*1.0E3
     WRITE(6,110)R,A(I),U(I),DU(I),BDYP,TSTF1,STFPC1,STFUC1,TSTF2,
    1STFPC2,STFUC2
 110 FORMAT(F11.6,F8.5,2E13.5,E12.5,2(F9.6,2E13.5))
     R=R*1.0E-3
     TSTF1=TSTF1*1.0E-3
     TSTF2=TSTF2*1.0E-3
  66 R=R+S
     T=R
C INITIAL VALUES FOR PREDICTORS AND CORRECTORS
     UP4=U(4)
     AP4=A(4)
     C4=U(4)
     D4=A(4)
     N=4
C USING THE FOUR CALCULATED INITIAL VALUES ABOVE, BEGIN INTEGRATION
     DO669 I=5,1000,1
```

```
      N=N+1
  250 L=0
  700 UP5=U(I-4)+4.0*S*(2.0*DU(I-1)-DU(I-2)+2.0*DU(I-3))/3.0
      AP5=A(I-4)+4.0*S*(2.0*U(I-1)-U(I-2)+2.0*U(I-3))/3.0
  230 COD=UP5-112.0*(UP4-C4)/121.0
      DOD=AP5-112.0*(AP4-D4)/121.0
      DCOD=((COD**3)*(((PL+B)/(PO*(BO/DOD)**4+B))**(3.0/7.0))/(2.0*C)
     1-3.0*COD**2/2.0-7.0*(PL+B)/(6.0*D)*(2.0736E4)*(1.0
     2-((PO*(BO/DOD)**4+B)/(PL+B))**(6.0/7.0)+COD/C*((PL
     3+B)/(PO*(BO/DOD)**4+B))**(3.0/7.0)-COD/C*((PO*(BO/DOD)**4+B)/(PL
     4+B))**(3.0/7.0))
     5-4.0*COD*PO*(BO/DOD)**4/(D*C)*(2.0736E4)*(((PO*(BO/DOD)**4
     6+B)/(PL+B))**(4.0/7.0)-COD/C*(PL+B)/(PO*(BO/DOD)**4+B)))/(DOD
     7-DOD*COD/C*((PL+B)/(PO*(BO/DOD)**4+B))**(3.0/7.0))
      C5=(9.0*U(I-1)-U(I-3)+3.0*S*(DCOD+2.0*DU(I-1)-DU(I-2)))/8.0
      D5=(9.0*A(I-1)-A(I-3)+3.0*S*(COD+2.0*U(I-1)-U(I-2)))/8.0
      Q=ABS(UP5-C5)
      IF(Q-1.0)210,210,260
C HALF INTERVAL PROCEDURE
  400 S=S/2.0
      A4MH=(80.0*A(I-1)+135.0*A(I-2)+40.0*A(I-3)+A(I-4))/256.0
     4+S*(-15.0*U(I-1)+90.0*U(I-2)+15.0*U(I-3))/128.0
      A3MH=(12.0*A(I-1)+135.0*A(I-2)+108.0*A(I-3)+A(I-4))/256.0
     4+S*(-3.0*U(I-1)-54.0*U(I-2)+27.0*U(I-3))/128.0
      U4MH=(80.0*U(I-1)+135.0*U(I-2)+40.0*U(I-3)+U(I-4))/256.0
     4+S*(-15.0*DU(I-1)+90.0*DU(I-2)+15.0*DU(I-3))/128.0
      U3MH=(12.0*U(I-1)+135.0*U(I-2)+108.0*U(I-3)+U(I-4))/256.0
     4+S*(-3.0*DU(I-1)-54.0*DU(I-2)+27.0*DU(I-3))/128.0
      A(I-4)=A3MH
      A(I-3)=A(I-2)
      A(I-2)=A4MH
      U(I-4)=U3MH
      U(I-3)=U(I-2)
      U(I-2)=U4MH
      BDYP=PO*(BO/A(I-4))**4
      DU(I-4)=((U(I-4)**3)*(((PL+B)/(BDYP+B))**(3.0/7.0))/(2.0*C)
     1-3.0*U(I-4)**2/2.0-7.0*(PL+B)/(6.0*D)*(2.0736E4)*(1.0
     2-((BDYP+B)/(PL+B))**(6.0/7.0)+U(I-4)/C*((PL+B)/(BDYP
     3+B))**(3.0/7.0)-U(I-4)/C*((BDYP+B)/(PL+B))**(3.0/7.0))
     4-4.0*U(I-4)*BDYP/(D*C)*(2.0736E4)*(((BDYP+B)/(PL
     5+B))**(4.0/7.0)-U(I-4)/C*(PL+B)/(BDYP+B)))/(A(I-4)
     6-A(I-4)*U(I-4)/C*((PL+B)/(BDYP+B))**(3.0/7.0))
      DU(I-3)=DU(I-2)
      BDYP=PO*(BO/A(I-2))**4
      DU(I-2)=((U(I-2)**3)*(((PL+B)/(BDYP+B))**(3.0/7.0))/(2.0*C)
     1-3.0*U(I-2)**2/2.0-7.0*(PL+B)/(6.0*D)*(2.0736E4)*(1.0
     2-((BDYP+B)/(PL+B))**(6.0/7.0)+U(I-2)/C*((PL+B)/(BDYP
     3+B))**(3.0/7.0)-U(I-2)/C*((BDYP+B)/(PL+B))**(3.0/7.0))
     4-4.0*U(I-2)*BDYP/(D*C)*(2.0736E4)*(((BDYP+B)/(PL
     5+B))**(4.0/7.0)-U(I-2)/C*(PL+B)/(BDYP+B)))/(A(I-2)
     6-A(I-2)*U(I-2)/C*((PL+B)/(BDYP+B))**(3.0/7.0))
      L=L+1
      IF(L-10)700,700,71
C SET UP FOR ITERATION
  260 UP5=C5+9.0*(UP5-C5)/121.0
      AP5=D5+9.0*(AP5-D5)/121.0
```

```
      L=L+1
      IF(L-2)230,400,400
C CALCULATION OF FINAL VALUES
  210 U(I)=C5+9.0*(UP5-C5)/121.0
  500 A(I)=D5+9.0*(AP5-D5)/121.0
      BDYP=PO*BO**4/A(I)**4
      DU(I)=((U(I)**3)*(((PL+B)/(BDYP+B))**(3.0/7.0))/(2.0*C)
     1-3.0*U(I)**2/2.0-7.0*(PL+B)/(6.0*D)*(2.0736E4)*(1.0
     2-((BDYP+B)/(PL+B))**(6.0/7.0)+U(I)/C*((PL+B)/(BDYP+B))**(3.0/7.0)
     3-U(I)/C*((BDYP+B)/(PL+B))**(3.0/7.0))
     4-4.0*U(I)*BDYP/(D*C)*(2.0736E4)*(((BDYP+B)/(PL+B))**(4.0/7.0)
     5-U(I)/C*(PL+B)/(BDYP+B)))/(A(I)-A(I)*U(I)/C*((PL+B)/(BDYP
     6+B))**(3.0/7.0))
      Y=A(I)*U(I)**2/2.0+(2.0736E4)*A(I)*(BDYP-PL)/D*(1.0-
     1(2.0736E4)*(BDYP-PL)/(2.0*D*C**2))
      YK=U(I)*(A(I)**2)*(C**3)*(1.0-U(I)**2/(2.0*C**2))/Y**2
     1-A(I)*(C**2)*(1.0-U(I)/C)/Y
      C4STFU=Y*C/STF1+YK*Y**2/(C**3*STF1**2)*(C**2-Y/STF1
     1+(YK**2/C**3)*(Y**4/C**3)/(2.0*STF1**4))
      STFUC1=C4STFU/C**2
      STFPC1=D*(Y/STF1-STFUC1**2/2.0)/(2.0736E4)
     1+D*((Y/STF1-STFUC1**2/2.0)**2)/(2.0*(C**2)*(2.0736E4))
      TSTF1=((STF1-A(I))/C)*(1.0-U(I)*A(I)/(C*STF1))+T
      C4STFU=Y*C/STF2+YK*Y**2/(C**3*STF2**2)*(C**2-Y/STF2
     1+(YK**2/C**3)*(Y**4/C**3)/(2.0*STF2**4))
      STFUC2=C4STFU/C**2
      STFPC2=D*(Y/STF2-STFUC2**2/2.0)/(2.0736E4)
     1+D*((Y/STF2-STFUC2**2/2.0)**2)/(2.0*(C**2)*(2.0736E4))
      TSTF2=((STF2-A(I))/C)*(1.0-U(I)*A(I)/(C*STF2))+T
      DO 708 KK=1,3
      C4STFU=Y*C/STF(KK)+YK*Y**2/(C**3*STF(KK)**2)*(C**2-Y/STF(KK)
     1+(YK**2/C**3)*(Y**4/C**3)/(2.0*STF(KK)**4))
      STFUC(KK,I)=C4STFU/C**2
      STFPC(KK,I)=D*(Y/STF(KK)-STFUC(KK,I)**2/2.0)/(2.0736E4)
     1+D*((Y/STF(KK)-STFUC(KK,I)**2/2.0)**2)/(2.0*(C**2)*(2.0736E4))
  708 TSTF(KK,I)=((STF(KK)-A(I))/C)*(1.0-U(I)*A(I)/(C*STF(KK)))+T
      T=T*1.0E3
      TSTF1=TSTF1*1.0E3
      TSTF2=TSTF2*1.0E3
      WRITE(6,169)T,A(I),U(I),DU(I),BDYP,TSTF1,STFPC1,STFUC1,TSTF2,
     1STFPC2,STFUC2
  169 FORMAT(F11.6,F8.5,2E13.5,E12.5,2(F9.6,2E13.5))
      T=T*1.0E-3
      TSTF1=TSTF1*1.0E-3
      TSTF2=TSTF2*1.0E-3
C  RELOCATE CERTAIN QUANTITIES FOR THE NEXT STEP OF THE INTEGRATION
      C4=C5
      D4=D5
      UP4=UP5
      AP4=AP5
      IF(A(I)-A(I-1))669,669,240
  240 IF(U(I))70,669,669
  669 T=T+S
      GO TO 7`
   71 WRITE(6,75)
   75 FORMAT(57HTHE PROCESS IS NOT CONVERGING QUICKLY ENOUGH AT SOME STE
```

```
      1P)
 70   IF(M-1000)7,223,223
223   WRITE(6,224)
224   FORMAT(90H 1000 VALUES HAVE BEEN CALCULATED BUT THESE VALUES DO NO
     1T COMPLETE THE ENTIRE FIRST PERIOD)
  7   WRITE(6,107)
107   FORMAT(120H1    TSTF3          STFPC3          STFUC3          TSTF4
     1 STFPC4          STFUC4          TSTF5          STFPC5          STFUC5     )
      DO 715 I=1,3
      DO 715 N=1,1000
715   TSTF(I,N)=TSTF(I,N)*1.0E3
      WRITE(6,701)((TSTF(I,N),STFPC(I,N),STFUC(I,N),I=1,3),N=1,1000)
701   FORMAT(3(F10.6,2E15.6))
  1   CONTINUE
222   STOP
      END
```

REFERENCES

1. Krenzke, M. et. al, "Potential Hull Structures for Rescue and Search Vehicles of the Deep-Submergence Systems Project," David Taylor Model Basin Report 1985 (Mar 1965).

2. Krenzke, M. and Charles, R., "The Elastic Buckling Strength of Spherical Glass Shells," David Taylor Model Basin Report 1759 (Sep 1963).

3. Rayleigh, J.W.S., "On the Pressure Developed in a Liquid during the Collapse of a Spherical Cavity," The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science, London Sixth Series, No. 200 (Aug 1917).

4. Herring, "Theory of the Pulsations of the Gas Bubble Produced by an Underwater Explosion," Office of Scientific Research and Development Report 236 (Oct 1941).

5. Gilmore, F.R., "The Growth or Collapse of a Spherical Bubble in a Viscous Compressible Liquid," California Institute of Technology Report 26-4 (Apr 1952).

6. Kirkwood, J.G. and Bethe, H.A., "The Pressure Wave Produced by an Underwater Explosion," Office of Scientific Research and Development Report 588 (1942).

7. Hickling, R. and Plesset, M.S., "The Collapse of a Spherical Cavity in a Compressible Liquid," California Institute of Technology Report 85-24 (Mar 1963).

8. Schmid, J., "Cinematographic Investigation of Single Bubble Cavitation," (German), Acustica, Vol. 9 (1959).

9. Ralston, A. and Wilf, H.S., "Mathematical Methods for Digital Computers," John Wiley and Sons, Inc., New York (1960).

10. Hildebrand, F.B., "Introduction to Numerical Analysis," McGraw-Hill Book Company, Inc., New York (1956).

11. Otto, J., "Handbook of Experimental Physics," (German), Academic Publishing Company, Ltd., Vol. 8, Leipzig (1929).

12. Coddington, E.A., "An Introduction to Ordinary Differential Equations," Prentice-Hall, Inc., New York (1961).

INITIAL DISTRIBUTION

Copies	
5	DSSP
1	CHBUWEPS, SP-001
3	CHNAVMAT 1 MAT 0331
5	NAVSHIPSYSCOM 3 Tech Info Br (Code 2021) 1 Advanced Ship Devel Sec (Code 03411) 1 Sub Br (Code 525)
10	NAVSEC 2 Sci & Res Sec (Code 6442) 1 Prelim Des Br (Code 6420) 2 Prelim Des Sec (Code 6421) 1 Ship Protec (Code 6423) 1 Hull Arrgt, Struc, & Preserv (Code 6633) 1 Boiler & Heat Exchange Br (Code 6651) 1 Mat & Chemistry Br (Code 6634) 1 Polymer, Fiber & Packing Sec (Code 6634C)
2	CHONR 1 Struc Mech Br (Code 439) 1 Undersea Programs (Code 466)
4	CNO 1 Tech Anal & Adv Gr (Op 07T) 1 Plans, Programs & Req Br (Op 311) 1 Sub Program Br (Op 713) 1 Tech Support Br (Op 725)
20	DDC
1	CO & DIR, USNMEL
1	CDR, USNOL (Dept W)
1	USNASL, Brooklyn (Code 9360)
1	DIR, USNRL (Code 2027)
1	CO & DIR, USNUSL
1	CO & DIR, USNEL
1	CDR, USNOTS, China Lake
1	CDR, USNOTS, Pasadena
1	CO, USNUOS

Copies	
2	NAVSHIPYD PTSMH
2	NAVSHIPYD MARE
1	NAVSHIPYD CHASN
1	SUPSHIP, Groton
1	EB Div, Gen Dyn Corp
1	SUPSHIP, Newport News
1	NNSB & DD Co
1	SUPSHIP, Pascagoula
1	Ingalls Shipbldg Corp
1	SUPSHIP, Camden
1	New York Shipbldg Corp
1	DIR, EEF R&E, Attn: Tech Lib
1	CO, USNROTC & NAVADMINU, MIT
1	O in C, PGSCOL, Webb
1	DIR, APL, Univ of Washington, Seattle
1	NAS, Attn: Comm on Undersea Warfare
1	WHOI 1 Mr. J. Mavor
1	Dr. R. Dehart, SWRI

UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)* David Taylor Model Basin Washington, D.C.	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b GROUP

3. REPORT TITLE

CALCULATIONS ON THE COLLAPSE OF A SPHERICAL GAS-FILLED CAVITY IN A COMPRESSIBLE LIQUID

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

Final

5. AUTHOR(S) *(Last name, first name, initial)*

Lilliston, Russel R.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
August 1966	41	12

8a. CONTRACT OR GRANT NO. b. PROJECT NO. SP 6-0002	9a. ORIGINATOR'S REPORT NUMBER(S) 2223
c. d.	9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*

10. AVAILABILITY/LIMITATION NOTICES

Distribution of this report is unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Deep Submergence Systems Project Office Washington, D.C.

13. ABSTRACT

This paper presents a method for calculating the instantaneous pressure, velocity, acceleration, and radius associated with the collapse of a spherical gas-filled cavity in an infinite compressible liquid. The method is an independent approach which makes use of Hamming's technique to numerically integrate Gilmore's differential equations which describe the collapse.

Included is a computer program which will perform the necessary calculations on a IBM 7090/1401 digital computer. Results obtained are in good agreement with those of Hickling and Plesset, whose work was unknown to the present author when he undertook the study.

It may be inferred that the peak shock wave pressure is significantly reduced by a decrease in ambient pressure, an increase in internal pressure, and/or a variation of the specific heat ratio by proper selection of the gas. Control of the last two parameters can be investigated as a possible means of protecting glass spheres against sympathetic implosion in multiple sphere buoyancy systems.

DD FORM 1 JAN 64 1473

UNCLASSIFIED

Security Classification

UNCLASSIFIED
Security Classification

14. KEY WORDS	LINK A ROLE	LINK A WT	LINK B ROLE	LINK B WT	LINK C ROLE	LINK C WT
Collapse of Spherical Gas-Filled Cavity						
Bubble Collapse						
Buoyancy Sphere						
Bubble Pressure						
Bubble Radius						
Bubble Wall Velocity						
Bubble Wall Acceleration						
Numerical Solution of Differential Equations						
Shock Pressure from Spherical Collapse						

INSTRUCTIONS

1. ORIGINATING ACTIVITY: Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization *(corporate author)* issuing the report.

2*a*. REPORT SECURITY CLASSIFICATION: Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

2*b*. GROUP: Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

3. REPORT TITLE: Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.

4. DESCRIPTIVE NOTES: If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

5. AUTHOR(S): Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. REPORT DATE: Enter the date of the report as day, month, year; or month, year. If more than one date appears on the report, use date of publication.

7*a*. TOTAL NUMBER OF PAGES: The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7*b*. NUMBER OF REFERENCES: Enter the total number of references cited in the report.

8*a*. CONTRACT OR GRANT NUMBER: If appropriate, enter the applicable number of the contract or grant under which the report was written.

8*b*, 8*c*, & 8*d*. PROJECT NUMBER: Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.

9*a*. ORIGINATOR'S REPORT NUMBER(S): Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9*b*. OTHER REPORT NUMBER(S): If the report has been assigned any other report numbers *(either by the originator or by the sponsor)*, also enter this number(s).

10. AVAILABILITY/LIMITATION NOTICES: Enter any limitations on further dissemination of the report, other than those imposed by security classification, using standard statements such as:

(1) "Qualified requesters may obtain copies of this report from DDC."

(2) "Foreign announcement and dissemination of this report by DDC is not authorized."

(3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through
____________________________________."

(4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through
____________________________________."

(5) "All distribution of this report is controlled. Qualified DDC users shall request through
____________________________________."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. SUPPLEMENTARY NOTES: Use for additional explanatory notes.

12. SPONSORING MILITARY ACTIVITY: Enter the name of the departmental project office or laboratory sponsoring *(paying for)* the research and development. Include address.

13. ABSTRACT: Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as *(TS)*, *(S)*, *(C)*, or *(U)*.

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. KEY WORDS: Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, roles, and weights is optional.

UNCLASSIFIED
Security Classification