NOTES ON SHIP MODEL TESTING IN TRANSIENT WAVES

This document has been approved for public release and sale; its distribution is unlimited.

HYDROMECHANICS LABORATORY
RESEARCH AND DEVELOPMENT REPORT

April 1969

Report 2960
The Naval Ship Research and Development Center is a U.S. Navy center for laboratory effort directed at achieving improved sea and air vehicles. It was formed in March 1967 by merging the David Taylor Model Basin at Carderock, Maryland and the Marine Engineering Laboratory at Annapolis, Maryland. The Mine Defense Laboratory, Panama City, Florida, became part of the Center in November 1967.

Naval Ship Research and Development Center
Washington, D.C. 20007
NOTES ON SHIP MODEL TESTING IN TRANSIENT WAVES

by

Alvin Gersten
and
Robert J. Johnson

This document has been approved for public release and sale; its distribution is unlimited.

April 1969
Report 2960
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>1</td>
</tr>
<tr>
<td>ADMINISTRATIVE INFORMATION</td>
<td>1</td>
</tr>
<tr>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>DESCRIPTION OF MODEL AND TEST EQUIPMENT</td>
<td>3</td>
</tr>
<tr>
<td>TEST PROGRAM AND PROCEDURE</td>
<td>8</td>
</tr>
<tr>
<td>DETAILS OF TRANSIENT WAVE PROGRAMS AND TRANSFER FUNCTION OF WAVEMAKER</td>
<td>10</td>
</tr>
<tr>
<td>VARIATION OF TRANSIENT WAVE TRANSFORMS WITH TANK LOCATION</td>
<td>12</td>
</tr>
<tr>
<td>COMPARISON OF TRANSFER FUNCTIONS OBTAINED IN SEVERAL TRANSIENT WAVE SYSTEMS AND IN REGULAR WAVES</td>
<td>24</td>
</tr>
<tr>
<td>EFFECT OF SAMPLING RATE ON TRANSFORMS</td>
<td>38</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>42</td>
</tr>
</tbody>
</table>

# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 1</td>
<td>Body Plan of Series 60, $C_d = 0.60$ Parent</td>
<td>5</td>
</tr>
<tr>
<td>Figure 2</td>
<td>Impulse Towing Strut</td>
<td>6</td>
</tr>
<tr>
<td>Figure 3</td>
<td>Time Histories of Wavemaker Control Signals</td>
<td>9</td>
</tr>
<tr>
<td>Figure 4</td>
<td>Spectra of Wavemaker Control Signals</td>
<td>11</td>
</tr>
<tr>
<td>Figure 5</td>
<td>Transient Wave Height Spectra for Programs A, B, and C Obtained by Averaging Results at Several Locations in the Tank</td>
<td>13</td>
</tr>
<tr>
<td>Figure 6</td>
<td>Normalized Transfer Function of Hydraulic Actuator Wavemaker System</td>
<td>14</td>
</tr>
<tr>
<td>Figure 7</td>
<td>Time Histories of Waves from Program A at Several Locations in Tank</td>
<td>16</td>
</tr>
<tr>
<td>Figure 8</td>
<td>Spectra for Transient Waves Measured at Several Distances from Wavemaker</td>
<td>17</td>
</tr>
<tr>
<td>Figure 9</td>
<td>Sample of Strip Chart Records Obtained during Tests with the Model</td>
<td>21</td>
</tr>
<tr>
<td>Figure 10</td>
<td>Comparisons of Transient Wave Spectra Derived from Measurements Made on the Carriage and in the Far Field</td>
<td>22</td>
</tr>
</tbody>
</table>
Table 1 - Particulars of Series 60, 0.60 Block Coefficient
Parent Form ............................................ 4

Figure 11 - Comparison of Motion Transfer Functions Obtained from Tests in Transient and Regular Waves at Zero Speed ............................................ 26

Figure 12 - Comparison of Motion Transfer Functions Obtained from Tests in Transient and Regular Waves at a Froude Number of 0.15 ........................................ 28

Figure 13 - Comparison of Motion Transfer Functions Obtained from Tests in Transient and Regular Waves at a Froude Number of 0.25 ........................................ 30

Figure 14 - Comparison of Motion Transfer Functions Obtained from Tests in Transient and Regular Waves at a Froude Number of 0.30 ........................................ 32

Figure 15 - Comparison of Motion Transfer Functions Obtained from Tests in Transient and Regular Waves at a Froude Number of 0.35 ........................................ 34

Figure 16 - Fourier Transform of a Rectangular Pulse as Computed on the IBM 7090 Digital Computer .......................... 37

Figure 17 - Effect of Sampling Rate on the Accuracy of Transient Wave Transforms ........................................ 40
ABSTRACT

Ship model experiments were conducted to evaluate the accuracy of motion transfer functions obtained by means of the transient wave technique. Since such an evaluation was especially needed for higher ship speeds, Froude numbers up to 0.35 were investigated. Also studied was the feasibility of measuring the incoming wave train far ahead of the model, where distortion of the forcing function by model-generated waves cannot occur. Further, in the interest of reducing computer usage time, the effect of sampling rate on the analysis of analog signals obtained during transient wave tests was examined. Results are also presented which demonstrate a need for smoothing the "raw" spectra obtained from the computation of Fourier transformations, in order to provide consistently useful transfer functions.

ADMINISTRATIVE INFORMATION

The study reported herein is part of an extensive development program requested by the Naval Ship Systems Command in letter Serial 341B-116 of 1 August 1963. It was funded under Task 0100 of Project S-R009 01 01.

INTRODUCTION

In 1964, Davis and Zarnick* introduced to the field of seaworthiness the concept of employing transient water waves in the towing tank to obtain the frequency response characteristics of ship motions. Their principal goal was to reduce the number of tests required to characterize a model from that necessary when testing in regular (periodic) waves. Since a transient wave train contains energy distributed over a wide range of frequencies, a single run in transient waves can permit definition of the entire frequency response; in contrast, only one point on the response function (at a single frequency) is obtained by making a pass through regular waves. They developed the testing and analysis techniques for transient head waves by conducting model experiments on Mariner, Series 60, and aircraft carrier

*References are listed on page 42.
forms at Froude numbers \( F \) ranging from 0 to 0.14. The transfer functions of pitch and heave were found to agree closely with those obtained from regular wave tests.

The data obtained in regular waves were utilized as a standard for accuracy since the inputs and responses are not compressed into a short time period and the data superimposed, as in the case of a transient input. Accurate resolution of transient signals into their frequency components places stringent demands on the recording and analysis systems. Especially good agreement was found in the heave to pitch ratio derived from records obtained in the two types of wave systems. This was because obtaining this ratio does not require measurement of the wave forcing function. Accurate delineation of the waves exciting the model proved to be difficult since measurements of the waves coming from ahead were corrupted by waves generated by the pitching and heaving model and by some reflections from the beach directly opposite the wavemaker. It was also believed that nonlinearities might be associated with the water dynamics and/or the wave measurement.

Davis and Zarnick also provided an original contribution to the general field of linear systems analysis in that they proposed and justified the utilization of transient excitations which are a linear frequency sweep rather than those which approximate an impulse. There are several advantages to their proposal. First, when the transient is lengthened, the severe concentration of information over a short time span is relaxed and so, therefore, is the requirement for extremely precise instrumentation. In addition, the very high frequency content of an impulse is not present and so the model is not vibrated at its structural natural frequency. Such structural vibration would, in turn, excite transducers at their natural frequencies and introduce "noise" into the measured signals. A third advantage is the avoidance of nonlinear model and water behavior that can occur with a high wave. And finally, the difficult requirement of meeting the waves exactly at their point of coalescence is obviated.

The purpose of this paper is to report some recent findings on testing in transient waves. The present writers have conducted experiments on a Series 60 model in both transient and regular head waves at higher speeds
than have been investigated heretofore, so that the accuracy of the transient technique could be checked more extensively. As mentioned above, Reference 1 reported difficulty in measuring the wave forcing function undistorted by model-generated waves, even when the wave probe was mounted on the carriage approximately 15 ft forward of the bow of the model. The present study evaluated the practicability of measuring the waves far ahead of the model. This approach was considered since wave theory indicates that the magnitude of the wave transform is independent of where the measurement is made along the direction of wave travel. Motion transfer functions obtained by utilizing wave records made directly ahead of the model are compared with those in the far field.* Another area investigated is the effect of sampling rate on the calculation of Fourier transforms for the type of time history generated by transient wave tests.

DESCRIPTION OF MODEL AND TEST EQUIPMENT

The model used for this study was a Series 60, Block 0.60 form. Table 1 lists the particulars of this vessel, and Figure 1 shows the body plan.

The model was towed and guided by the "impulse towing strut" (ITS) shown in Figure 2. This apparatus is designed to permit model responses in five degrees of freedom (yaw is completely restrained) and the application of a tow force at the center of gravity. Restoring forces in surge and sway are provided by springs. All restraints on the model due to towing and guidance can be measured; heave, surge, and sway forces are sensed by differential reluctance block gages, and yaw restraint by a strain-gaged flexure. Since towing gear inertias and frictions are included in the force measurement, corrections for their effect on the motions can be made. Unrealistically large surging motion occurs if the frequency of wave encounter is close to the natural frequency in surge of the vibratory system consisting of model, towing gear, and springs. Because of cross-coupling between motions, the pitch and heave become erratic.

---

*Sonic-type wave probes were used since they do not physically contact the water surface.
TABLE 1
Particulars of Series 60, 0.60 Block Coefficient
Parent Form

<table>
<thead>
<tr>
<th>Item</th>
<th>Model</th>
<th>Ship</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length between perpendiculars (LBP) ft</td>
<td>10.0</td>
<td>400.0</td>
</tr>
<tr>
<td>Beam, ft</td>
<td>1.33</td>
<td>53.33</td>
</tr>
<tr>
<td>Draft, ft</td>
<td>0.53</td>
<td>21.33</td>
</tr>
<tr>
<td>Displacement</td>
<td>266.3 lb FW</td>
<td>7,807.0 Ltons SW</td>
</tr>
<tr>
<td>Longitudinal center of buoyancy aft of sections as percent of LBP</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>Distance of vertical center of gravity below waterline, ft</td>
<td>0.04</td>
<td>1.76</td>
</tr>
<tr>
<td>Radius of gyration, ft</td>
<td>0.25 LBP</td>
<td>0.25 LBP</td>
</tr>
<tr>
<td>Scale ratio</td>
<td>1:40</td>
<td></td>
</tr>
</tbody>
</table>

In the present series of tests, the surge spring stiffness was quite small,* and so the natural period in surge was appreciably longer than the longest period of wave encounter. As a result, the surge was kept within proper bounds, and pitch and heave, uncorrected for restraints, were in good agreement with results obtained previously at this Center and at other towing tanks. It should be noted that the present results for both transient and regular wave tests are not corrected for restraints since the corrections do not appear to be significant for head seas. This is especially true here because our principal goal is not the accurate characterization of the motions of a ship but rather the comparison of two test procedures (transient wave and regular wave) both of which were executed with the same towing apparatus. The sway and yaw restraints during these head seas tests were extremely small as was the heave restraint due to heave staff friction and inertia. The motions were measured by means of film-type potentiometers. The pitch and roll potentiometers are driven by gears

*The spring constant for each of the two surge springs was 7 lb/in. this yielded a total restoring force of 14 lb/in. at the point of spring attachment. The effective spring constant at the model, however, was only 3.5 lb/in.
Figure 1 - Body Plan of Series 60, $C_b = 0.60$ Parent
Mounting Plate on Carriage

Upper Gimbal Assembly

Sway Axis

Spring for Surge Restoring Force

Surge Axis

Roll Axis

Pitch Axis

Note: Length 12'-1" of Gimbals with Heave Staff in Mid-Position

Heave Staff

Surge Force Gage

Yaw Flexure

Side Force Gage

Heave Force Gage

Lower Gimbal Assembly

Model Attached Here

Figure 2 - Impulse Towing Strut
mounted on the shafts of the lower gimbal, whereas the surge and sway potentiometers are activated by the shafts of a gimbal supporting the top of the tubular strut. Utilization of the ITS simplifies the conduct of a test (as compared to the effort required to test a more freely running model) because no propulsion and steering systems need be installed and operated.

The experiments were carried out at this Center in the Harold E. Saunders Maneuvering and Seakeeping Basin (MASK); it is a rectangular tank 360 ft long by 240 ft wide by 20 ft deep. The facility is equipped with a bank of eight electrohydraulic servosystems which control the flow of air to domes along the shorter side of the tank. The air travels through the domes, impinges on the water surface, and causes the formation of waves which progress away from the source. Regular waves can be generated by employing the electrical signal from a sine-wave generator to drive the servosystems. If long-crested transient or random waves are desired, signals from magnetic tape programs are employed to control the servosystems. Fixed-bar type, concrete wave absorbers are installed along the wall opposite the wavemaker. These absorb approximately 95 percent of the incident wave energy for the range of wavelengths of primary interest, although there is some variation in absorptivity which is dependent upon wave steepness.

The transducer signals were amplified and recorded on three devices: a Sanborn strip chart recorder, analog magnetic tape, and digital magnetic tape. Analog to digital conversion was performed by a digital data acquisition system (DIDAS) during the course of each run. The signals to be digitized were not affected by either alternative recording device but were transmitted directly to DIDAS where they were passed through a 10-cps low-pass filter and then sampled. This was done because of the desirability of bypassing the analog tape recorder which is less accurate than the DIDAS system. DIDAS can digitize an analog signal at various rates up to 6000 points/sec. For a majority of the analyses performed in this investigation, a sampling rate of 125 samples/channel/sec was used. This selection was based on the finding of Smith and Cummins² that the above rate should be used to achieve a maximum difference of 1 percent between the "exact" spectrum (based on a sampling rate of 6000/channel/sec) and the approximate spectrum. Computation of the transforms and transfer functions was performed on the IBM 7090 digital computer.
TEST PROGRAM AND PROCEDURE

Experiments were first conducted in regular head waves to obtain the motion transfer functions by means of a tried and proven method. These results were needed to provide a basis for evaluating the reliability of the transient technique. Wave length to ship length ratios ranging from 0.75 to 2.0 were utilized in increments of 0.25. The speeds investigated corresponded to Froude numbers \((F)\) ranging from 0 to 0.35 in increments of 0.05. As is often the case when tests are run in regular waves, lack of funds and time precluded taking data over a wider range of frequencies and for a closer frequency spacing.

Three different wave programs were used for the transient wave tests. All of these are comprised of a frequency sweep which is a linear function of total elapsed time and which starts at high frequencies (short waves) and proceeds towards low frequencies (long waves). The highest frequency is nominally 1.0 cps since this is usually the highest frequency of importance for seaworthiness model tests. A linear frequency sweep leads to coalescence of the waves at one point in the tank; that is, the later generated long waves catch up to the short ones, and they all merge to form one large wave. As discussed previously, each run was conducted so as to avoid a meeting of waves and model at the point of wave coalescence. It can be seen in Figure 3 that one of the wavemaker control programs consists of a signal which has constant amplitude for all frequencies; the other two initially decrease in amplitude as the frequency is decreased, and then increase in amplitude with further decrease in frequency. The manipulation of amplitudes was performed to produce a water wave system which is characterized by a flat spectrum for the frequency range of interest. This is desirable because it helps maintain a good signal-to-noise ratio in the measured responses. It was known that the frequency response of the wavemaker was not flat and, in fact, reached a peak in the 0.4- to 0.5-cps frequency range. Since the spectrum of the control signal is multiplied by the frequency response of the linear system (wavemaker) through which

*These programs were developed by Davis and Zarnick.1
Figure 3 - Time Histories of Wavemaker Control Signals
the signal passes, it was attempted to provide a control signal with a 
transform magnitude inversely proportional to the magnitude of the wave-
maker transfer function.

During the model tests in transient waves, one sonic wave probe was 
attached to the carriage, at a point 21 ft directly forward of the model 
center of gravity, and traveled down the tank with the model. A second 
probe was attached to the bridge which spans the basin and supports the 
carriage. This stationary probe was located at a point 94 ft from the 
wavemaker dome and approximately 15 ft off the line of travel of the model. 
The model speeds investigated were the same as specified above for the 
tests in regular waves. After the first waves were emitted from the wave-
maker, the model was accelerated in calm water to the desired speed; it 
then passed through the wave train and proceeded in calm water again. When 
wave measurements were made along the centerline of the tank to determine 
the variation of the wave transform with distance from the wavemaker, two 
sonic probes were mounted 40 ft apart on the carriage; by locating the 
carriage at two points in the basin, it was possible to make records 132, 
172, 222, and 262 ft from the wavemaker. No attempt was made to measure 
the waves closer than 132 ft from the source because the longer waves 
(maximum wave length was 120 ft) may not have formed completely at those 
locations.

DETAILS OF TRANSIENT WAVE PROGRAMS AND 
TRANSFER FUNCTION OF WAVEMAKER

The magnitude of the Fourier transform of the wavemaker control 
signals is given in Figure 4. The spectra reveal quite clearly that the 
signal for Program A is constant in amplitude whereas the signal for 
Program C has a decided minimum amplitude at approximately 0.45 cps. It 
should be noted for future reference that even when the spectrum is flat, 
as in Figure 4a, a jaggedness is superposed on the principal shape.

In Reference 3, Davis presents the transfer function of the MASK 
wavemaker which was obtained by the generation and measurement of regular 
waves. This frequency response function was used in the process of making 
up the control signals. To ascertain the shape of the wave spectra
Figure 4a

Figure 4b

Figure 4c

Figure 4 - Spectra of Wavemaker Control Signals

11
generated by Programs A, B, and C, wave measurements were made at several locations in the basin, and average spectra were computed (see Figure 5). Amplitude adjustment in the Program C control signal to achieve a flat wave spectrum was, to a large degree, successful; however, there is some room for further improvement. Where no attempt is made to account for the wavemaker frequency response, as in Program A, the wave spectrum is relatively narrow band and has a pronounced peak. To provide an additional measure of the wavemaker transfer function, the control signal transforms were divided into those of the water waves for corresponding wave programs and frequencies. The three transfer functions obtained differed somewhat (these are not presented here), but it must be remembered that the wave transforms themselves are averages and vary with distance from the wavemaker. An average wavemaker frequency response function is plotted in Figure 6. To some extent, this function was different in shape from the one obtained in regular waves; however, it did peak at approximately the same frequency (0.4 cps from transient waves as compared to 0.45 cps from regular waves). Because of the distinct maximum and minimum in the transfer function, it requires 5.5 times as large an excitation to generate the same wave height at 0.9 cps as at 0.4 cps.

VARIATION OF TRANSIENT WAVE TRANSFORMS WITH TANK LOCATION

According to linearized wave theory, the magnitude of the wave transform should be invariant if the time history of a transient wave packet is measured at various points along its direction of travel. Supposedly, the energy content of the waves is not altered as they progress away from the wave source. The phase of the transform does, of course, change; it lags by $e^{-j\omega|x/g}$, where $\omega$ is the frequency of a particular wave component, $x$ is the distance between measuring points, and $g$ is the acceleration due to gravity. If indeed, the magnitude of the wave transform were constant,

* This was necessary because the spectra vary with location of the wave train in the tank. See the next section for details.
Figure 5 - Transient Wave Height Spectra for Programs A, B, and C Obtained by Averaging Results at Several Locations in the Tank
Figure 6 - Normalized Transfer Function of Hydraulic Actuator Wavemaker System
it would be possible to measure the wave input at a great distance forward of the model, away from the influence of model-generated waves, and to use the transform of this uncorrupted time function to compute motion transfer functions.

As a check on this hypothesis, measurements of the transient waves were made at several locations in the tank along the direction of wave travel. It can be seen from the samples of the records presented in Figure 7 that the time histories vary considerably with tank location. At 132 ft from the wavemaker, the waves are almost coalesced. Gradually, the longer waves overtake the shorter ones and produce a dispersed wave pattern at 262 ft. Although the wave transforms for a particular program (see Figure 8) do not differ as much as the time functions, they are certainly not constant. Among the characteristics evident in Figure 8 is a tendency for the wave amplitudes at frequencies close to that of maximum amplitude to increase with distance from the wavemaker.

One possible explanation for this is as follows. It is known that wave energy is propagated towards the sides of the tank by the wavemaking units at both ends of the wavemaker bank. When reflected back toward the center of the basin, this energy would not be detected by a transducer located on the longitudinal centerline of the tank and close to the point of wave origin, but it could impinge on a probe farther away from the wave source. The waves reflected by the long beach and the dome of the long bank of wavemakers may be sufficiently high (especially since the dome tends to act as a resonator) to cause the seemingly anomalous increase of wave amplitude (shown in Figure 8) by superposing on the waves traveling directly down-tank. The differences in wave amplitude at the low and high frequency ends of the spectra are not as pronounced as they are near the peaks; however, there is an indication in Figure 8c that the energy at the high frequencies tends to decrease with distance from the wavemaker. Waves propagating through a fluid and not acted upon by outside forces, such as wind, will normally lose energy because of viscous dissipation.

A further examination of the practicability of utilizing far-field wave measurements for computing motion transfer functions was implemented by mounting a fixed wave probe 94 ft from the wavemaker and 15 ft off the
Figure 7 - Time Histories of Waves from Program A at Several Locations in Tank

Note: The wave records have been shifted in time for convenience of presentation.
Figure 8 - Spectra for Transient Waves Measured at Several Distances from Wavemaker

Figure 8a - Program A
Figure 8b - Program B
Figure 8c - Program C
tank centerline and a second probe on the towing carriage 21 ft forward of the model center of gravity. Here again, the wave records obtained at the two locations are different (see Figure 9) not only because of the spatial separation of the transducers but also because one measurement (far field) was made at the wave frequency while the other (carriage) includes the Doppler shift on frequencies due to motion of the transducer (so-called frequency of encounter \( f_e \)). The digitized wave records from the fixed probe were used to compute Fourier transforms in the wave frequency domain. These were then converted to the frequency of encounter domain by calculating \( f_e \) from

\[
f_e = f + \frac{2\pi f^2 v}{g}
\]

where \( f \) is the wave frequency and \( v \) is the model speed, and multiplying the ordinates of the spectrum by the Jacobian

\[
J = \frac{1}{\sqrt{1 - 4\alpha_e}}
\]

where \( \alpha_e = -\frac{2\pi f v}{g} \). The modified ordinates were then plotted at the frequency of encounter derived from the applicable wave frequency. Figure 10 compares the converted transforms with those obtained from the moving probe. It can be seen that although they are in fairly good agreement and have the same general shape, the differences in detail are by no means negligible. The curves in Figures 10b-10d are for speeds at which no model-generated waves should reach the wave probe. Nevertheless, the disagreement between the two curves in each of these figures is the same order of magnitude as the disparity between the curves in Figure 10a; at that speed, according to Reference 1, significant wave energy can reach the probe from the model. This would indicate that there is, in general, no increase in accuracy to be obtained by measuring the wave forcing function far away from the model to avoid model-generated waves. However, for certain types of tests (e.g., those carried out with a radio-controlled model in which signals are telemetered to shore) there may be no platform moving with the model on which a wave probe could be mounted, and it may
Figure 9 - Sample of Strip Chart Records Obtained during Tests with the Model

Note: The wave record from the far-field probe has been shifted in time.
Figure 10 - Comparisons of Transient Wave Spectra Derived from Measurements Made on the Carriage and in the Far-Field

Figure 10a - Wave Probe Speed = 0 Knots, Program A

Figure 10b - Wave Probe Speed = 3.72 Knots, Program A
Figure 10c - Wave Probe Speed = 2.65 Knots, Program B

Figure 10d - Wave Probe Speed = 3.72 Knots, Program B
therefore be necessary to make wave measurements far ahead of the model. The results presented above and in the next section of this report show that reasonably accurate motion transfer functions can be obtained from such measurements.

COMPARISON OF TRANSFER FUNCTIONS OBTAINED IN SEVERAL TRANSIENT WAVE SYSTEMS AND IN REGULAR WAVES

The motion transfer functions obtained from tests in transient waves for Froude numbers of 0, 0.15, 0.25, 0.30, and 0.35 are plotted in Figures 11 through 15; these figures also include the response data for regular waves. Because new procedures for processing transient signals had been developed just prior to the analysis required by this study, the records provided served as "guinea pigs" for the debugging process. As a result, errors appeared in several of the computations of transfer function magnitude and/or phase. Since the system frequency response is independent of the excitation used, costly reanalysis of the records was avoided by presenting, where necessary, the response magnitude obtained by means of one wave program along with the phase obtained by means of a different program (e.g., see Figure 14).

Several types of evaluations can be made from these figures. First, by comparing the response functions derived in the transient and regular wave systems, we can determine the accuracy of the transient wave technique (data obtained in regular waves are used as a standard for accuracy of the analysis). In addition, we can check the consistency of the response functions obtained with different transient wave programs. Finally, we can examine the agreement between the magnitude of transfer functions derived from far-field wave measurements and those made directly ahead of the model. Utilization of a fixed, far-field wave transducer makes it extremely difficult to determine the phase between the forcing function (waves) and the responses since it is necessary to know the position of the model relative to the wave probe at all times. Only with this information available is it possible to compute the phase of a motion relative to a wave crest (or trough) located on some station along the hull. For this reason, phase data for the fixed probe are not presented in Figures 11 through 15.
If we compare the magnitude of the transient wave responses which were computed by utilizing the wave transforms from the moving probe (solid line) with the regular wave data, we find that the agreement is fairly good in those cases where the solid line is relatively smooth (e.g., for pitch and heave in Figure 12a). Where the transient wave response function is jagged, as for pitch and heave in Figure 14a, the blackened circles representing regular wave data fall within, or adjacent to, the band defined by the straight-line segments. Naturally, in the latter case, it is difficult to make a definitive judgment on accuracy; nevertheless, the results do look encouraging. The correspondence between transient-wave and regular-wave phase data follows the same general trend exhibited by the magnitudes, although the plots for the former tend to be reasonably smooth throughout the speed range and permit a comparison to be made more easily. In summary, it can be stated that acceptable motion transfer functions (both magnitude and phase) can be obtained from model tests conducted in head transient waves over a wide range of speeds.

The transfer functions obtained from tests in different transient wave systems differ mainly in detail, that is, all the short straight-line segments do not superpose (see Figures 11 and 12 for magnitudes and Figures 11 and 13 for phases), but for the most part, the large-scale trends are in agreement. There are occasional large differences (for example, the heave plot of Figure 12a at a frequency of approximately 0.95 cps and the pitch-wave phase plot of Figure 13b at approximately 1.0 cps), but these would become less significant if mean, smooth curves were faired through the jagged ones shown. The motion frequency responses derived from far-field transient wave measurements compare satisfactorily with the regular wave data, indicating that this method does produce usable results. However, there is no obvious improvement in accuracy over the response magnitudes computed from wave measurements made with a moving probe mounted directly forward of the model.

Even a cursory examination of the transforms and transfer functions presented in this paper reveals the existence of an all too common jagged shape. The jaggedness is characteristic of the type of transient signal being processed rather than an artifact introduced by the digital analysis.
Figure 11 - Comparison of Motion Transfer Functions Obtained from Tests in Transient and Regular Waves at Zero Speed

Figure 11a - Magnitude of Transfer Functions
Figure 11b - Phase of Transfer Functions
Figure 12 - Comparison of Motion Transfer Functions Obtained from Tests in Transient and Regular Waves at a Froude Number of 0.15

Figure 12a - Magnitude of Transfer Functions
Figure 12b - Phase of Transfer Functions
Figure 13 - Comparison of Motion Transfer Functions Obtained from Tests in Transient and Regular Waves at a Froude Number of 0.25

Figure 13a - Magnitude of Transfer Functions
Figure 15b - Phase of Transfer Functions
Figure 14 - Comparison of Motion Transfer Functions Obtained from Tests in Transient and Regular Waves at a Froude Number of 0.30

Figure 14a - Magnitude of Transfer Functions
Figure 14b - Phase of Transfer Functions
Figure 15 - Comparison of Motion Transfer Functions Obtained from Tests in Transient and Regular Waves at a Froude Number of 0.35

Figure 15a - Magnitude of Transfer Functions
Figure 15b - Phase of Transfer Functions
This was demonstrated by utilizing the existing computer programs to obtain the transform of a rectangular pulse; the result (Figure 16) is obviously quite smooth. The irregularity present in the other transforms is probably caused by the fact that their associated time histories are comprised of single cycles of sine functions joined end to end. It is known that the transform of a single cycle of a sine wave is a curve which has its largest peak close to the frequency of the sine wave, with secondary peaks (which are by no means insignificant) at higher and lower frequencies. If many single cycles of sine waves with different frequencies are joined to form a continuous signal and the transform of this function is computed, the secondary peaks associated with each sine wave will superpose on those of the neighboring sine functions. The spectral ordinates tabulated by the digital computer will, in effect, result from addition of those individual ordinates contributed by each sine wave occurring at the same frequency. The net ordinates associated with the secondary peaks will generally not blend with the ordinates falling near the fundamental frequencies of the sine waves to form a smooth curve.

Methods are presently being investigated for smoothing the "raw" spectra obtained from the computation of Fourier transforms. One approach under consideration is to convolve the spectrum with an appropriate filter which smoothes the Fourier coefficients by an averaging process. A form of this averaging, which is known as Hanning, is given below:

\[ A_k = \frac{1}{4}a_{k-1} + \frac{1}{2}a_k + \frac{1}{4}a_{k+1} \]
\[ B_k = \frac{1}{4}b_{k-1} + \frac{1}{2}b_k + \frac{1}{4}b_{k+1} \]

where \( a_k \) and \( b_k \) are the coefficients of the real and imaginary parts of the raw spectrum and \( A_k \) and \( B_k \) are the smoothed coefficients.

* A Hanning-type smoothing process is almost always used in the numerical calculation of energy density spectra from random signals.
Figure 16 - Fourier Transform of a Rectangular Pulse as Computed on the IBM 7090 Digital Computer
Ideally, when the transfer function of a linear response is being calculated, the jaggedness present in the transforms of input and response should divide out. Actual experience reveals that such cancellation does not occur because these irregularities fluctuate rapidly and are of large amplitude. An appropriate method of smoothing the raw amplitude spectra must be incorporated in the analysis in order to make the transfer functions obtained during transient wave tests useful for the objective prediction of ship response in a random seaway (the ultimate goal).

**EFFECT OF SAMPLING RATE ON TRANSFORMS**

In order to employ digital computers in the analysis of transient signals, a continuous time history must be represented by discrete values of the function at finite time intervals apart. The sampling rate should be high enough to permit the computation of transforms with an accuracy that is no greater than required for the user's purposes; excessively high sampling rates result in a waste of computer time. According to the Shannon sampling theorem, 2f samples/sec suffice to represent perfectly a time function containing only frequency components below f cps. This theorem is often difficult to apply because most empirical data do not have a clearly determinable upper frequency bound.

An alternative method of arriving at an appropriate sampling rate is a "cut and try" approach. First, an extremely high digitizing rate is used to compute a transform which is to be the standard for accuracy. Then, the digitizing rate is decreased in discrete steps until the difference between the standard and the approximation has reached what is considered to be the maximum acceptable limit. This method was used by Smith and Cummins* when computing transforms of records obtained during force pulse tests. They determined that a sampling rate of 125/sec** was needed to produce a

---

*The rigid body responses of a ship model to force and moment impulses were being studied.

**All sampling rates given are on a per channel basis.
transform with a maximum error of 1 percent (the standard was based on a
digitizing rate of 6000 samples/sec). In keeping with their findings, the
sampling rate used for computing the transforms presented thus far in this
report is 125/sec.

As a check on whether a lower sampling rate would yield satisfactory
results in the analysis of records from transient wave experiments, such
analyses were performed after digitizing the signals at rates which, in
some cases, were as low as 1.25 samples/sec. Figure 4 shows how the trans-
form of the wavemaker control signals is affected by decreasing the samp-
ling rate to 62.5 and 25/sec. It should be recalled that the highest
frequency sine wave programmed into the control signal was 1.0 cps. Higher
frequencies could, however, be present in the signal because of distortion
of the sine functions (harmonic content) or electronic noise. The only
change in the spectra which can be resolved in Figure 4 occurs at the low
frequency end where the energy content is quite small, and so the effect is
insignificant.

The wave records were sampled at even lower rates with no deterio-
ration in the transforms. The plots in Figure 17a are the transform of
waves generated in the basin by Program A and measured at zero speed. If
harmonic content is neglected, the highest frequency in the time function
should be close to 1.0 cps. The figure clearly shows that the sampling
rate could be decreased to 5/sec without causing an appreciable change in
the spectrum. Similar plots are presented in Figure 17b, here the measure-
ment was made at a forward speed of 3.72 knots (corresponding to $F = 0.35$
for the model) so that the upper frequency limit in the signal was greater
than that associated with Figure 17a. As a result, the minimum sampling
rate that could be utilized without altering the spectrum was increased to
6.25/sec.

Thus, this brief investigation into the effect of sampling rate on
the analysis of analog signals obtained during transient wave tests indi-
cates that in most cases, these signals can be digitized at a rate of
approximately 10 samples/sec without causing a significant error in the
computed spectra. If spurious signals are filtered out, it should be
necessary to increase the sampling rate above 10/sec only when the fre-
quencies of wave encounter are significantly higher than the values reported
herein.
Figure 17 - Effect of Sampling Rate on the Accuracy of Transient Wave Transforms

Figure 17a - Wave Probe Speed = 0 Knots, Program A
Figure 17b - Wave Probe Speed = 3.72 Knots, Program A
The curves in Figure 17 reveal that when the sampling rate is decreased to the point where significant changes in the spectra begin to appear, these differences occur at the high frequency end of the spectra. Usually, when this "folding" or aliasing of the spectrum about \( f \) occurs, it is characterized by an increase in the spectral ordinates. This is due to the inability of the discrete sampled values to adequately represent the higher frequencies in the record, and the energy associated with these high frequencies is attributed to lower ones. In Figure 17, however, the aliased spectra are lower than the true spectra, at least up to the highest frequency analyzed. Additional studies of sampling rate effects will be conducted in the future to provide more comparisons of spectra, especially at higher frequencies than shown in Figure 17, so that this apparent anomaly can be explained.

REFERENCES


## INITIAL DISTRIBUTION

<table>
<thead>
<tr>
<th>Copies</th>
<th>Copies</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>NAVSHIPSYS COM</td>
<td>Dir, Fluid Mech Lab, Univ of Calif, Berkeley</td>
</tr>
<tr>
<td>2 Ships 2052</td>
<td>1</td>
</tr>
<tr>
<td>1 Ships 0341</td>
<td>James Forrestal Res Ctr, Princeton Attn: Mr. Maurice H. Smith, Asst to Dir</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>NAVSEC</td>
<td>NNS &amp; DD Co</td>
</tr>
<tr>
<td>2 Sec 6110</td>
<td>1 Asst Nav Arch</td>
</tr>
<tr>
<td>2 Sec 6122</td>
<td>1 Dir, Hydra Lab</td>
</tr>
<tr>
<td>2 Sec 6115</td>
<td>20</td>
</tr>
<tr>
<td>2 CHONR</td>
<td>CDR, DDC</td>
</tr>
<tr>
<td>1 Code 438</td>
<td>1</td>
</tr>
<tr>
<td>1 Code 466</td>
<td>MARAD (Res &amp; Dev Sec)</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>ONR, New York</td>
<td>Catholic Univ, Mechanical Engineering Dept, School of Eng &amp; Arch</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>ONR, Pasadena</td>
<td>Prof. J.R. Paulling College of Eng, Univ of Calif, Berkeley</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>ONR, Chicago</td>
<td>NYU, Dept of Oceanography &amp; Meteorology</td>
</tr>
<tr>
<td>1</td>
<td>1 Dr. W.J. Pierson, Jr.</td>
</tr>
<tr>
<td>ONR, Boston</td>
<td>1 Mr. R. Johnson</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>ONR, London</td>
<td>DIR, SPO</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>NMDL</td>
<td>General Dynamics Corp, Electric Boat Div</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>NAVUWRES</td>
<td>Oceanics, Inc., Technical Industrial Park</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>NOL</td>
<td>Westinghouse Electric Corp, Annapolis, Md Attn Mr. M.S. Macovsky</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>CDR, NUWC</td>
<td>SNAME Attn: Panel H-7</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>CDR, NWC</td>
<td>Dir, Natl BuStand</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>NRL</td>
<td>Dir, APL/JHU</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>D.L., SIT</td>
<td>Dir, Fluid Mech Lab, Columbia Univ, New York, N.Y.</td>
</tr>
<tr>
<td>1 Dr. J. Breslin</td>
<td>1 Dir, Hydra Lab, Univ of Colorado</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>DIR, DEF R&amp;E</td>
<td>Boulder, Colorado</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Dir, Exptl Nav Tank, Univ of Mich, Ann Arbor</td>
<td>Dir, Hydra Lab, Univ of Minnesota, Minneapolis</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Dir, Inst for Fluid Dyn &amp; Appl Math, Univ of Md</td>
<td>1 Dr. W.J. Pierson, Jr.</td>
</tr>
<tr>
<td>1</td>
<td>1 Mr. R. Johnson</td>
</tr>
<tr>
<td>Dir, Scripps Inst of Ocean, Univ of Calif</td>
<td>1 Mr. R. Johnson</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Dir, WHOI, Woods Hole</td>
<td>Dir, Hydra Lab, Univ of Colorado</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Dir, ORL, Penn State</td>
<td>Boulder, Colorado</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>CO, USNROTC &amp; NAVADMINU, MIT</td>
<td>Dir, Hydra Lab, Univ of Colorado</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Dir, St. Anthony Falls Hydrau Lab, Univ of Minnesota, Minneapolis</td>
<td>Boulder, Colorado</td>
</tr>
</tbody>
</table>
Copies
1 Dir, Hydra Res Lab, Univ of Conn, Storrs, Conn
1 Dir, Robinson Hydra Lab, Ohio St Univ, Columbus, Ohio
1 Admin, Webb Inst of Nav Arch, Glen Cove
2 Dir Iowa Inst of Hydraul Res, St Univ of Iowa, Iowa City
  1 Dr. L. Landweber
1 Dir, Hydra Lab, Penn State Univ, University Park, Pa.
1 Dir, Hydra Lab, Univ of Wisconsin
1 Dir, Hydra Lab, Univ of Washington
1 SAFHL/Univ of Minn
1 Dir of Res, The Tech Inst, Northwestern Univ
  Evanston, Ill.
1 Dr. M.L. Albertson, Head of Fluid Mech Res, Dept of Civil Engr, Colorado St Univ
  Fort Collins, Colo.
1 MIT, Dept of NA & ME
1 Lockheed Missiles & Space Co
  Sunnyvale, Calif
1 Hydronautics, Laurel, Md.
Ship model experiments were conducted to evaluate the accuracy of motion transfer functions obtained by means of the transient wave technique. Since such an evaluation was especially needed for higher ship speeds, Froude numbers up to 0.35 were investigated. Also studied was the feasibility of measuring the incoming wave train far ahead of the model, where distortion of the forcing function by model-generated waves cannot occur. Further, in the interest of reducing computer usage time, the effect of sampling rate on the analysis of analog signals obtained during transient wave tests was examined. Results are also presented which demonstrate a need for smoothing the "raw" spectra obtained from the computation of Fourier transformations, in order to provide consistently useful transfer functions.
<table>
<thead>
<tr>
<th>KEY WORDS</th>
<th>LINK A</th>
<th>LINK B</th>
<th>LINK C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ship Motions</td>
<td>ROLE</td>
<td>WT</td>
<td>ROLE</td>
</tr>
<tr>
<td>Seaworthiness</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ship Models</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transient Waves</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data Reduction</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Ship model experiments were conducted to evaluate the accuracy of motion transfer functions obtained by means of the transient wave technique. Since such an evaluation was especially needed for higher ship speeds, Froude numbers up to 0.35 were investigated. Also studied was the feasibility of measuring the incoming wave train far ahead of the model, where distortion of the forcing function by model-generated waves cannot occur. Further, in the interest of reducing computer usage time, the effect of sampling rate on the analysis of analog signals obtained during transient wave tests was examined. Results
are also presented which demonstrate a need for smoothing the "raw" spectra obtained from the computation of Fourier transformations, in order to provide consistently useful transfer functions.