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Naval Ship Research and Development Center
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DEPARTMENT OF THE NAVY
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FLUSH-MOUNTED PRESSURE TRANSDUCER SYSTEMS AS SPATIAL AND SPECTRAL FILTERS

by

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Flush-Mounted Pressure Transducer Systems as Spatial and Spectral Filters

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Analysis of the responses of a flush-mounted transducer system to the pressure field in a turbulent boundary layer and to the boundary pressure field induced by an incident plane acoustic wave is formulated and discussed. The transducer system is considered as a spatial and a spectral filter. The properties of the filters are defined and illustrated for the case where the system consists of nominally identical transducers. The analysis is limited to geometries where the centers of the transducers are regularly placed at the centers of a plane rectangular grid. Time delays between transducers are allowed for in this analysis.

INTRODUCTION

IN recent papers,[1,2] the properties of flush-mounted pressure transducer systems were examined. The transducer systems of particular concern were those possessing a number of transducer units set flush in a flat infinite rigid boundary. The transducers are placed so as to bear a definite geometrical relationship with respect to each other. The utilization of these transducer systems for the study of turbulent boundary-layer pressure fields and as devices for increasing the "signal-to-noise ratio" of planar array systems was the major theme of these papers. A point of view was taken that a transducer system is basically a spatial filter rejecting some components of the pressure field to which it is subjected and readily accepting others. By proper adjustments of the spatial characteristics of the transducer system, one can predetermine which of the pressure-field components are rejected and which are accepted.[2] In one of these papers,[1] the analysis of the spatial filtering action of the transducer system was conducted in **k** space; the Fourier conjugate of the spatial space. The transducer system in **k** space was analyzed in terms of its properties as a wave-vector filter. Insertion of a frequency filter in the circuit of the transducer system provided one with a spectral filter; a filter operating in $\{\mathbf{k},\omega\}$ space. It was shown that such a filter can be employed to analyze the spectral nature of a convecting boundary pressure field.[1,2]

In this paper, time delays between the transducers of the transducer system are introduced. The possibility of introducing time delays to increase the versatility of the filtering action of the transducer system was implied in Ref. 1; here the effects that time delays have on the spectral filter (a filter whose properties are defined in the $\{\mathbf{k},\omega\}$ space) are analyzed and illustrated.

The analysis is limited to nominally identical transducers and to special geometrical relationships between them. The elements that underlie the response of the transducer system to turbulent boundary layer and incident plane acoustic pressure fields are discussed and illustrated.

I. RESPONSE OF FLUSH-MOUNTED PRESSURE TRANSDUCER SYSTEM TO A STATIONARY BOUNDARY PRESSURE FIELD

The pressure field is considered stationary, both spatially and temporally. The transducer system is considered to consist of several individual transducers feeding the same output channel. The cross-frequency spectral density of the pressure field $\tilde{\phi}_m(\mathbf{y},\omega)$ as measured by the transducer system is given by[1,3]

$$\tilde{\phi}_m(\mathbf{y},\omega)=2\pi\tilde{p}\sum_i\sum_j\tilde{\phi}^{ij}(\mathbf{y},\omega)F_i(\omega)F_j^*(\omega)\times\exp[-i\omega(\tau_i-\tau_j)],\quad(1)$$

[1] G. Maidanik and D. W. Jorgensen, "Boundary Wave-vector Filters for the Study of the Pressure Field in a Turbulent Boundary Layer," J. Acoust. Soc. Am. 42, 494–501 (1967).

[2] D. W. Jorgensen and G. Maidanik, "The Response of Point Transducer System to Turbulent Boundary Layer Pressure Field" (to be published).

[3] G. M. Corcos, "Resolution of Pressure in Turbulence," J. Acoust. Soc. Am. 35, 192–199 (1963).

where

$$\tilde{\phi}^{ij}(\mathbf{y},\omega)=\tilde{\phi}^{ij}\{\mathbf{y}-(\mathbf{y}_i-\mathbf{y}_j),\,\omega\}, \tag{2}$$

$$\tilde{\phi}^{ij}(\mathbf{y},\omega)=\int\!\!\int_{-\infty}^{\infty} d\mathbf{z}\tilde{\phi}\{\mathbf{z}-[\mathbf{y}-(\mathbf{y}_i-\mathbf{y}_j)],\,\omega\}\times s_is_jm_{ij}(\mathbf{z}), \tag{3}$$

$$s_is_jm_{ij}(\mathbf{z})=\int\!\!\int_{-\infty}^{\infty} d\mathbf{z}'h_i(\mathbf{z}')h_j(\mathbf{z}'+\mathbf{z}), \tag{4}$$

$\tilde{\phi}$ is the true cross-frequency spectral density of the pressure field, h_i is the spatial sensitivity function of the ith transducer, $\mathbf{y}$ is a spatial vector defining a spatial translation of the transducer system, $\mathbf{y}_i$ is the spatial vector defining the location of the center of the ith transducer, $F_i(\omega)$ is the frequency sensitivity function of the ith transducer, ω is the frequency variable, τ_i is the time delay associated with the ith transducer, and s_i is the sensitivity of the ith transducer. [Unlike Ref. 1 the mean square measured pressure and the mean square true pressure, $\langle p_{\mathrm{m}}^2\rangle$ and $\langle p^2\rangle$, are considered embodied in $\tilde{\phi}_{\mathrm{m}}$ and $\tilde{\phi}^{ij}$, respectively.[1]]

The spectral density of the pressure field $\Phi_{\mathrm{m}}(\mathbf{k},\omega)$ as measured by the transducer system is given by[1]

$$\Phi_{\mathrm{m}}(\mathbf{k},\omega)=8\pi^3\Phi(\mathbf{k},\omega)\sum_i\sum_j F_i(\omega)F_j^*(\omega)s_is_jM_{ij}(\mathbf{k})\times\exp\{-i\mathbf{k}\cdot(\mathbf{y}_i-\mathbf{y}_j)-i\omega(\tau_i-\tau_j)\}, \tag{5a}$$

where $\Phi(\mathbf{k},\omega)$ is the true spectral density of the pressure field, $\mathbf{k}$ is the wave-vector variable and $M_{ij}(\mathbf{k})$ is the Fourier transform of $m_{ij}(\mathbf{z})$. Note that $s_is_jM_{ij}(\mathbf{k})=H_i(\mathbf{k})H_j^*(\mathbf{k})$, where $H_i(\mathbf{k})$ is the Fourier transform of $h_i(\mathbf{z})$.

It is noted that the function

$$W(\mathbf{k},\omega)=8\pi^3\sum_i\sum_j F_i(\omega)F_j^*(\omega)s_is_jM_{ij}(\mathbf{k})\times\exp\{-i\mathbf{k}\cdot(\mathbf{y}_i-\mathbf{y}_j)-i\omega(\tau_i-\tau_j)\}, \tag{5b}$$

in Eq. 5a describes the spectral filtering action of the transducer system; it is a functional of the parameters of the transducer system only and is independent of the parameters of the pressure field.[4]

The relationship between $\tilde{\phi}_{\mathrm{m}}(\mathbf{y},\omega)$ and $\Phi_{\mathrm{m}}(\mathbf{k},\omega)$ is given by the Fourier integral transform

$$\tilde{\phi}_{\mathrm{m}}(\mathbf{y},\omega)=\frac{1}{2\pi}\int\!\!\int_{-\infty}^{\infty} d\mathbf{k}\Phi_{\mathrm{m}}(\mathbf{k},\omega)\exp(i\mathbf{k}\cdot\mathbf{y}). \tag{6}$$

[4] In this analysis, it is considered that no feedback is established between the pressure field and the transducer system.

II. FORM OF THE TRUE CROSS-FREQUENCY SPECTRAL DENSITY AND THE SPECTRAL DENSITY OF A CONVECTING BOUNDARY PRESSURE FIELD

In this Section some relevant features of a typical boundary pressure field are considered and stated. The predominant feature of the boundary pressure field considered in this paper is its convection. The cross-frequency spectral density $\tilde{\phi}$ of a convecting boundary pressure field has the approximate form[3]

$$\tilde{\phi}(\boldsymbol{\varepsilon},\omega)=\tilde{\phi}(\omega)Q'\left(\frac{\omega\epsilon_1}{c_t},\frac{\omega\epsilon_3}{c_t}\right)\exp\left(-\frac{i\omega\epsilon_1}{c_t}\right), \tag{7}$$

where c_t and Q' are the convection velocity and the normalized spatial correlation function of the frequency spectral components of the boundary pressure field, respectively;

$$\tilde{\phi}(\omega)=\tilde{\phi}(0,\omega);\quad \boldsymbol{\varepsilon}=\{\epsilon_1,\epsilon_3\}$$

and

$$\mathbf{c}_t\cdot\{\epsilon_1,\epsilon_3\}=\{c_t\epsilon_1,0\}.$$

The convective nature of the pressure field is described by the exponential phase factor in Eq. 7. The parameter ω/c_t is used here as a normalizing factor for the spatial separation vector variable $\boldsymbol{\varepsilon}$. (That such a normalization is useful in describing the pressure field in a turbulent boundary layer has been discussed by Corcos.[3])

A cross-frequency spectral density must obey the following conditions[5]:

$$\tilde{\phi}(\boldsymbol{\varepsilon},\omega)=\tilde{\phi}(-\boldsymbol{\varepsilon},\,-\omega)=\tilde{\phi}^*(-\boldsymbol{\varepsilon},\,\omega). \tag{8}$$

For the expression given in Eq. 7 to satisfy the conditions stated in Eq. 8, Q' must be real and of the form

$$Q'\left(\frac{\omega\epsilon_1}{c_t},\frac{\omega\epsilon_3}{c_t}\right)=Q\left(\left|\frac{\omega\epsilon_1}{c_t}\right|,\left|\frac{\omega\epsilon_3}{c_t}\right|\right). \tag{9}$$

The spectral density $\Phi(\mathbf{k},\omega)$ of a convective pressure field can be derived by the appropriate Fourier transformation of its cross-frequency spectral density $\tilde{\phi}(\boldsymbol{\varepsilon},\omega)$. It can be shown that the form of $\Phi(\mathbf{k},\omega)$ that describes a convecting pressure field is[6]

$$\Phi(\mathbf{k},\omega)=\Phi^\circ(\mathbf{k},\,\omega-\mathbf{c}_t\cdot\mathbf{k}), \tag{10}$$

[5] J. S. Bendat and A. G. Piersol, *Measurement and Analysis of Random Data* (John Wiley & Sons, Inc., New York, 1966), Chap. 3.

[6] The convection velocity $\mathbf{c}_t$ is a parameter that is employed in this paper to describe the functional form of the pressure field. In Eq. 7, it approximates the velocity of propagation of those frequency spectral components of the pressure field whose spatial correlation is given by the function Q of Eq. 9. Most pressure fields that are encountered in practice possess spectral densities that have regions of high concentration. In Eq. 10, the convection velocity approximates the phase velocities of those spectral components of the pressure field that lie in these regions of high density. In this sense, the convection velocity in Eq. 10 is a parameter that is employed to describe, in an approximate fashion, the spectral density distribution of the pressure field.

where $\mathbf{c}_t\cdot\{k_1,k_3\}=\{c_tk_1,0\}$. The wavenumbers k_1 and k_3 are the Fourier conjugate variables of ϵ_1 and ϵ_3, respectively.

That the Fourier conjugate of the temporal variable appears in Eq. 10 in the form $(\omega-\mathbf{c}_t\cdot\mathbf{k})$ is directly related to the convective nature of the pressure field (a Doppler effect). It is further observed that if the convection velocity were constant, independent of $\boldsymbol{\varepsilon}$ and t, and in a moving coordinate system (moving with this convection velocity), the cross correlation of the pressure field were independent of the temporal variable (a frozen pressure field), Φ° would have a delta function factor in the variable $(\omega-\mathbf{c}_t\cdot\mathbf{k})$.

In the subsequent discussions, the functional forms of the pressure field [$\tilde{\phi}(\boldsymbol{\varepsilon},\omega)$ and $\Phi(\mathbf{k},\omega)$] are specialized to describe the pressure field in a turbulent boundary layer and the boundary pressure field due to an incident plane acoustic field. For the moment, the analysis is further developed in its more general form.

III. MEASURED FREQUENCY SPECTRAL DENSITY OF A CONVECTIVE PRESSURE FIELD

The flush-mounted transducer system is considered to be subjected to a convective pressure field of the form described in the preceding section. In practice, one invariably measures the response of the transducer system in a chosen frequency band. This response is obtained by inserting a frequency filter in the output circuit of the transducer system. One may account for this filtering operation by appropriately modifying the frequency sensitivity function $F_i(\omega)$. However, it is preferable to reserve this function to describe the inherent frequency characteristics of a transducer in the transducer system. The frequency filter filtering action is designated by a frequency function $D(\omega,\omega_0,\Delta)$. If the function $D(\omega,\omega_0,\Delta)$ describes an ideal band frequency filter whose center frequency is ω_0 and its bandwidth is 2Δ, then the expression for this function is

$$D^0(\omega,\omega_0,\Delta)=(2\pi)^{\frac{1}{2}}[U(\omega_0+\Delta-|\omega|)-U(\omega_0-\Delta-|\omega|),\quad (11)$$

where

$$U(a)=\begin{cases}1 & a>0\\ 0 & a<0\end{cases}.\quad (12)$$

From Eqs. 1 and 3, the measured frequency spectral density is

$$\tilde{\phi}_m(0,\omega)=\sum_i\sum_j\tilde{\phi}_m{}^{ij}(0,\omega),\quad (13)$$

where

$$\tilde{\phi}_m{}^{ij}(0,\omega)=[\tilde{\phi}_m{}^{ji}(0,\omega)]^*=2\pi s_is_j\tilde{\phi}(\omega)F_i(\omega)F_j{}^*(\omega)D(\omega,\omega_0,\Delta)\times\int\int_{-\infty}^{\infty}d\mathbf{z}Q(|z_1'+\xi_{ij}'|,\,|z_3'+\eta_{ij}'|)m_{ij}(\mathbf{z})\exp[-i(\xi_{ij}'+\tau_{ij}'+z_1')],\quad (14)$$

$$\{\xi_{ij},\eta_{ij}\}=\boldsymbol{\xi}_{ij}=\mathbf{y}_i-\mathbf{y}_j;\quad \mathbf{c}_t\cdot\boldsymbol{\xi}_{ij}=\{c_t\xi_{ij},0\};\quad \xi_{ij}'=(\omega/c_t)\xi_{ij};$$

$$\eta_{ij}'=\frac{\omega}{c_t}\eta_{ij};\quad z_1'=\frac{\omega}{c_t}z_1;\quad z_3'=\frac{\omega}{c_t}z_3;\quad \tau_{ij}'=\omega(\tau_i-\tau_j).$$

The mean-square response (or simply the response) of the transducer system is

$$\phi_m(\omega_0,\Delta)=\frac{1}{(2\pi)^{\frac{1}{2}}}\int_{-\infty}^{\infty}d\omega\tilde{\phi}_m(0,\omega).\quad (15)$$

Corcos[3] examined extensively the functional form of $\tilde{\phi}_m{}^{ij}(0,\omega)/\tilde{\phi}(\omega)$ or its equivalent forms. His arguments, by and large, are valid for a single transducer, i.e., for $\tilde{\phi}_m{}^{ii}(0,\omega)/\tilde{\phi}(\omega)$.[7] His arguments concerning finite size transducers with $i\neq j$ are not valid even if one is to accept that the spatial correlation Q is exponential in form. This is so because of the condition imposed on the frequency spectral density as stated in Eq. 8.

The expression for $m_{ii}(\mathbf{z})$ was evaluated by Corcos[3] for cases of a uniform rectangular and a uniform circular transducer. Thus, if one knows the functional form of Q, one may compute the integral term in Eq. 14 for such transducers. This is essentially the procedure followed by Corcos.[3] To extend these computations to other types of transducers, one has to determine the appropriate functional form of $m_{ii}(\mathbf{z})$. Similarly, in order to compute this integral term for cases where $i\neq j$, one has to determine the appropriate functional forms of $m_{ij}(\mathbf{z})$. In the special case of a transducer system consisting of nominally identical transducers,

[7] K. L. Chandiramani, "Interpretation of Wall Pressure Measurements under a Turbulent Boundary Layer," Bolt Beranek and Newman Rept. No. 1310, Contract No. Nonr 2321(00) (Aug. 1965).

one has $m_{ij}(\mathbf{z})=m_{ii}(\mathbf{z})=m_{ji}(\mathbf{z})$ and of course $F_i(\omega)=F_j(\omega)=F(\omega)$. This special case is of particular interest in this paper. Since $m_{ij}(\mathbf{z})=m_{ji}(-\mathbf{z})$ (see Eq. 4), one readily concludes that

$$m_{ij}(\mathbf{z})=m_{ii}(\mathbf{z})=m(|\mathbf{z}|), \tag{16}$$

and Eq. 14 reduces to

$$\tilde{\phi}_m{}^{ij}(0,\omega)=2\pi s_i s_j \tilde{\phi}(\omega)|F(\omega)|^2 D(\omega,\omega_0,\Delta) \times\iint_{-\infty}^{\infty} d\mathbf{z} Q(|z_1'+\xi_{ij}'|, |z_3'+\eta_{ij}'|)m(|\mathbf{z}|) \times\exp[-i(\xi_{ij}'+\tau_{ij}'+z_i')]. \tag{17}$$

In Eq. 17, s_i is not set equal to s_j for it is assumed that the values of these parameters can be adjusted at will by electrical means external to the transducer units. (The sensitivities s_i are considered real in this paper.[1])

Equation 17 can also be written in the form

$$\tilde{\phi}_m{}^{ij}(0,\omega)=2\pi s_i s_j \tilde{\phi}(\omega)|F(\omega)|^2 D(\omega,\omega_0,\Delta) \times\iint_{-\infty}^{\infty} d\mathbf{z} Q(|z_1'|, |z_3'|)m(|\mathbf{z}+\xi_{ij}|) \times\exp[-i(z_1'+\tau_{ij}')]. \tag{18}$$

With $\tau_{ij}=0$ and $|F(\omega)|^2=1$, Eqs. 13 and 18 become those equations on which Jorgensen and Maidanik[2] based the evaluation of the "signal-to-noise ratio" of a doublet and a triplet transducer system.

Of particular interest, is the manner in which the normalized time delay parameter τ_{ij}' influences the response of the transducer system to the pressure field. Each pair of transducers in the transducer system may be thought to be subjected to a pressure field of the form

$$\tilde{\phi}(\mathbf{z}',\omega,\tau_{ij}')=\tilde{\phi}(\omega)Q(|z_1'|,|z_3'|) \times\exp[-i(z_i'+\tau_{ij}')]. \tag{19}$$

(cf. Eqs. 7 and 9). Thus, the time delay between a pair of transducers can be accounted for by imposing a phase factor on the expression describing the cross-frequency spectral density of the pressure field; this phase factor is simply $\exp(-i\tau_{ij}')$. Once the time delay is accounted for as just described, one may proceed in the usual manner to account for the spatial characteristics of the pair of transducers; one defines a spatial filter operator[2]

$$0^{ij}=2\pi D(\omega,\omega_0,\Delta)|F_i(\omega)|^2\iint_{-\infty}^{\infty} d\mathbf{z}\cdots\theta^{ij}, \tag{20}$$

where

$$\theta^{ij}=s_i s_j m(|\mathbf{z}+\xi_{ij}|). \tag{21}$$

The graphical form of θ^{ij} for uniform rectangular transducers was presented by Jorgensen and Maidanik in a recent paper.[2] In the special case of point transducers,

$$\theta^{ij}=s_i s_j \delta(\mathbf{z}+\xi_{ij}), \tag{22}$$

where δ is the Dirac delta function and $\delta(\mathbf{z})=\delta(z_1)\delta(z_3)$. The expression for θ^{ij} as given in Eq. 22 can be considered to represent a good approximation for transducers whose sizes are such that $b\omega/c_t\ll 1$, where b is a typical linear spatial dimension of the transducers.[2]

It is observed that for uniform transducers the size of the transducers, as measured by $b\omega/c_t$, tends to reduce the value of the ratio $\tilde{\phi}_m{}^{ij}(0,\omega)/\tilde{\phi}(\omega)$;[3] the larger the value of b, (ω and c_t are assumed fixed), the smaller is the value of this ratio. It is clear from Eq. 17 that the value of this ratio is also influenced by the frequency factor $|F(\omega)|$.[2] In most situations, particularly those associated with the measurements of the response of the transducer system to turbulent boundary layer pressure fields, the spatial transducer size effects on this ratio are dominant. However, there are situations where the reverse is true. In the case of turbulent boundary layer pressure fields, these reverse situations may arise when the transducers are in the form of "pin holes." When the back cavities are held constant and the sizes of the pin holes are reduced, a stage is reached where the influence on the ratio in question due to the frequency response function overtakes that due to the transducer sizes.[8]

IV. MEASURED SPECTRAL DENSITY OF A CONVECTIVE FIELD

Just as it is convenient in certain problems to analyze the situation in terms of the Fourier conjugate ω of the temporal variable, it is convenient in certain problems to analyze the situation further in terms of the Fourier conjugate $\mathbf{k}$ of the spatial vector variable. In this Section, the analysis of the response of the transducer system to a convecting pressure field is made in Fourier conjugate space, that is, in terms of the vector variable $\{\mathbf{k},\omega\}$ in this space. Specifically, in this Section, Eq. 5 is examined in greater detail. As was done in the preceding section, the analysis is primarily limited to nominally identical transducers and, in addition, to particular geometrical relationship between the transducers.

For nominally identical transducers, and taking account of the frequency filtering action, Eq. 5b becomes

$$W(\mathbf{k},\omega,\omega_0,\Delta)=8\pi^3 D(\omega,\omega_0,\Delta)|F(\omega)|^2 M(\mathbf{k}) \times\sum_i\sum_j s_i s_j \exp\{-i[\mathbf{k}\cdot\xi_{ij}+\tau_{ij}']\}, \tag{23}$$

where $F_i(\omega)=F(\omega)$; $M_{ij}(\mathbf{k})=M(\mathbf{k})$; and the function $D(\omega,\omega_0,\Delta)$ is as defined previously.

[8] F. E. Geib, Jr., "Measurements on the Effect of Transducer Size on the Resolution of Boundary-Layer Pressure Fluctuations," Naval Ship Res. and Develop. Ctr. Rept. 2503 (Aug. 1967).

The term involving the double summation can be readily evaluated in closed form under special conditions. These conditions are:

1. The transducer system possesses two orthogonal principal axes.
2. The separations between the centers of successive transducers lying in the direction of a principal axis are nominally equal.
3. The time delays between successive transducers lying in the direction of a principal axis are nominally equal.
4. The sensitivities s_i of the transducers lying in the direction of a principal axis are either equal to $(-1)^{2i}$ or to $(-1)^i$.

Under these special conditions, Eq. 23 becomes[1,9,10]

$$W(\mathbf{k},\omega,\omega_0,\Delta)=8\pi^3 D(\omega,\omega_0,\Delta)\,|F(\omega)|^2 \times M(\mathbf{k}) S_N(d,k_N,\tau_N') S_P(e,k_P,\tau_P'), \quad (24)$$

where

$$S_\alpha(\gamma,k_\alpha,\tau_\alpha')=\begin{cases} \dfrac{\sin^2(\alpha\nu_\alpha/2)}{\sin^2(\nu_\alpha/2)}; & s_i=(-1)^{2i}, \quad (25a)\\[2ex] \dfrac{\sin^2(\alpha\nu_\alpha/2)}{\cos^2(\nu_\alpha/2)}; & s_i=(-1)^i;\ \alpha \text{ even}, \quad (25b)\\[2ex] \dfrac{\cos^2(\alpha\nu_\alpha/2)}{\cos^2(\nu_\alpha/2)}; & s_i=(-1)^i;\ \alpha \text{ odd}, \quad (25c)\end{cases}$$

$$\nu_\alpha=(k_\alpha\gamma+\tau_\alpha'), \quad (26)$$

the integer N is the number of transducers in one principal direction (designated by N), d is the separation between the centers of successive transducers lying in this direction, k_N is the wavenumber associated with this direction, and τ_N' is the normalized time delay between successive transducers lying in this direction. The integer P is the number of transducers in the other principal direction (designated by P), e, k_P and τ_P' are the corresponding parameters in this direction (see Fig. 1).

The function $M(\mathbf{k})$ describes the wave-vector filtering action of a single transducer. For uniform circular transducer,[3,9]

$$M(\mathbf{k})=(2J_1(kR)/kR)^2; \quad k=|\mathbf{k}|, \quad (27)$$

where J_1 is the Bessel function of the first order and R is the radius of the transducer.

For uniform rectangular transducer[3,9]

$$M(\mathbf{k})=\frac{\sin^2(k_b b/2)}{(k_b b/2)^2}\,\frac{\sin^2(k_l l/2)}{(k_l l/2)^2}, \quad (28)$$

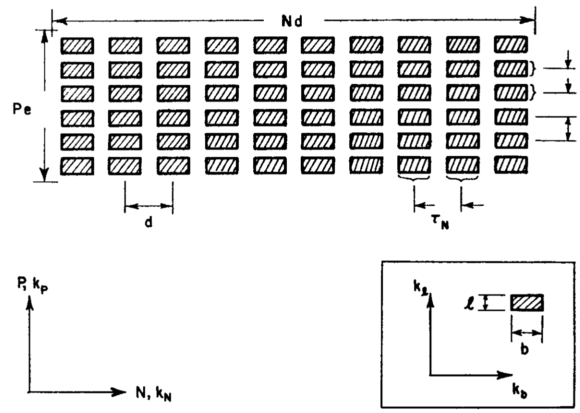

FIG. 1. Typical flush-mounted pressure transducer system consisting of $N\times P$ transducers (N and P also designate the directions of the two principal axes of the transducer system, respectively).

where b is the width of the transducer and l is its length. The wavenumbers k_b and k_l are the Fourier conjugates of the spatial variable along the width and the length of the transducer, respectively (see Fig. 1).

For the purposes of this paper, only uniform rectangular transducers are considered. It is further assumed that the widths of the transducers are aligned such that they lie in the direction of one of the principal axes, the one along which the separation between the centers of successive transducers is designated by d; the principal axis N (see Fig. 1). With these geometrical impositions $k_b=k_N$ and $k_l=k_P$. A graphical presentation of the factor K_N in Eq. 24

$$K_N(d,b,k_N,\tau_N')=\frac{\sin^2(k_N b/2)}{(k_N b/2)^2} S_N(d,k_N,\tau_N'), \quad (29)$$

as a function of k_N for two typical representative cases is illustrated in Figs. 2 and 3.

It is apparent from Eqs. 25, 26, and 29 that

$$K_N(d,\ b,\ -k_N,\ \tau_N')=K_N(d,\ b,\ k_N,\ -\tau_N'). \quad (30)$$

It is noted that $\tau_N'=\omega\tau_N$ and, therefore, a change of sign in either τ_N or ω induces a change of sign in τ_N'. It is also noted that K_P and K_N are similar functions, and hence, by replacing N by P, d by e, k_N by k_P and τ_N' by τ_P', Figs. 2 and 3 become graphical presentations of the factor $K_P(e,l,k_P,\tau_P')$ in Eq. 24 as a function of k_P.

The nature of the filter operator $W(\mathbf{k},\omega,\omega_0,\Delta)$ in the $\{k_N,\omega\}$ plane is illustrated in Fig. 4 for a typical transducer system.[11] It is observed that the time delay

[9] M. Born and E. Wolf, *Principles of Optics* (Pergamon Press Ltd., New York, 1959), Chap. 9.

[10] S. A. Schelkunoff, *Electromagnetic Waves* (D. Van Nostrand Co., Inc., New York, 1943), Chap. 9.

[11] Strictly the filter operator is the integral operator

$$(2\pi)^{-3}\int_{-\infty}^{\infty} d\omega \iint_{-\infty}^{\infty} d\mathbf{k}\cdots W(\mathbf{k},\omega,\omega_0,\Delta).$$

The function $W(\mathbf{k},\omega,\omega_0,\Delta)$ is essentially the "joint acceptance" of the transducer system as defined by A. Powell. Indeed the nature of $W(\mathbf{k},\omega,\omega_0,\Delta)$ with $\tau_N=\tau_P=0$ is closely related to the modal joint acceptance function of a responding flat plate [see for example: A. Powell, "Aero-Acoustic Excitation of Structures," David Taylor Model Basin Rept. 2232 (Sept. 1966)].

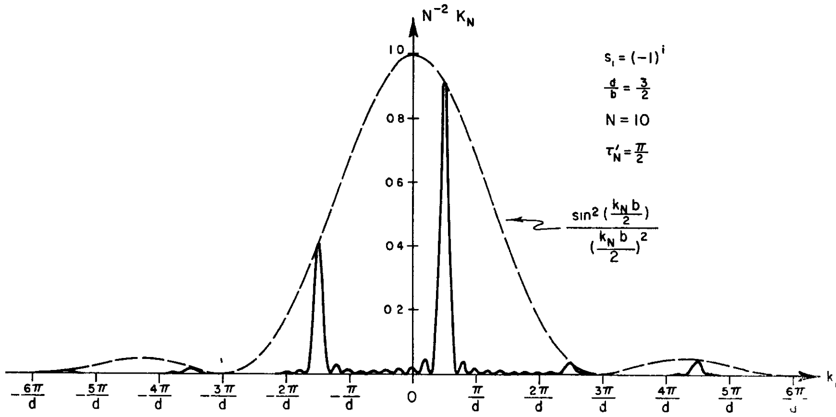

FIG. 2. The effect of time delay τ_N between successive transducers on a typical single-band wave-vector filter.

between transducers can be used to induce variation in the location of the wavenumber bands of the transducer system. In this sense, time delays may be used to advantage in making the transducer system a more versatile instrument than was previously described.[1] In Fig. 4, only the first and fourth quadrants are shown. This limited representation is made possible because the first quadrant is identical to the third, and the fourth to the second. Further remarks concerning Fig. 4 are made in the next Section.

V. RESPONSE TO A TURBULENT BOUNDARY-LAYER PRESSURE FIELD

As in Ref. 1, it is assumed that the spectral density $\Phi_t(\mathbf{k},\omega)$ of the pressure field in a turbulent boundary layer is separable.

$$\Phi_t(\mathbf{k},\omega)=\Phi_{t1}(k_1,\omega)\Phi_{t3}(k_3,\omega)\Phi_{t0}(\omega-k_1U), \qquad (31)$$

where U is the convection velocity[6] of the spectral components of the pressure field $[U=c_t]$ and Φ_{t1} and Φ_{t3} are assumed symmetric and reasonably smooth functions of ω, k_1 and k_3.

As was discussed in Ref. 1, the function $\Phi_{t0}(\omega-k_1U)$ peaks substantially in the range where $\omega\simeq k_1U$. Although the convection velocity may be a function of $\mathbf{k}$ and ω, it is found that the spectral density is substantially confined to a narrow region in $\{\mathbf{k},\omega\}$ space; the region is roughly defined so that $\omega\simeq k_1\sigma U_\infty$, where U_∞ is the free-flow velocity and $0.6\lesssim\sigma\lesssim 0.8$.[3,7] Elsewhere in this space the spectral density is substantially lower. This situation is illustrated graphically in Fig. 4; the situation illustrated is that where the direction of the flow coincides with that of a principal direction of the transducer system, i.e., $k_1=k_N$ and $k_3=k_P$. A few remarks about Fig. 4 may be in order. Figure 4 consists of two quadrants, the first and the fourth. The situation in the first is identical with the third, and the fourth with the second, and hence, only the two quadrants need be considered. Examination of the foregoing analysis shows that the first quadrant illustrates those spectral components that propagate in the positive direction of the principal axis N, while those in the fourth quadrant propagate in the negative direction. (The propagation of concern in this connection is associated with the phase velocities; these velocities are equal to $\mathbf{k}\omega/k^2$.) It is clear from Fig. 4 that the filtering action in the first quadrant is different from that in the fourth quadrant for a nonzero τ_N'; the filtering action of the transducer system is different for a positively and a negatively propagating spectral component. Further, a change of sign in the time delay τ_N can be accounted

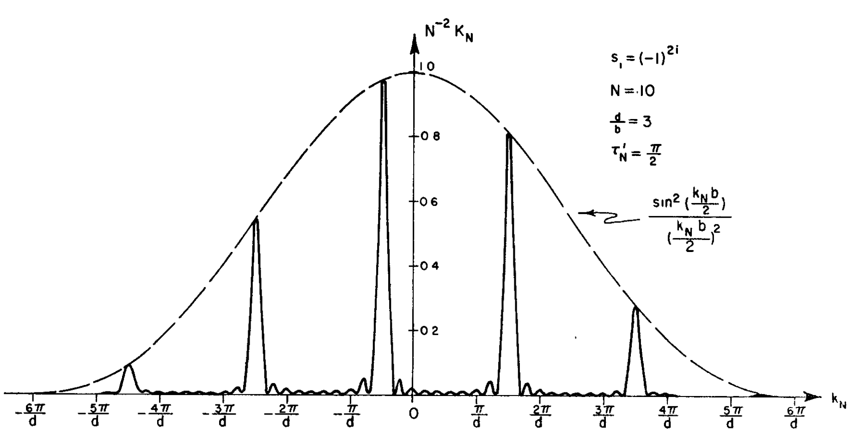

FIG. 3. The effect of time delay τ_N between successive transducers on a typical multiband wave-vector filter.

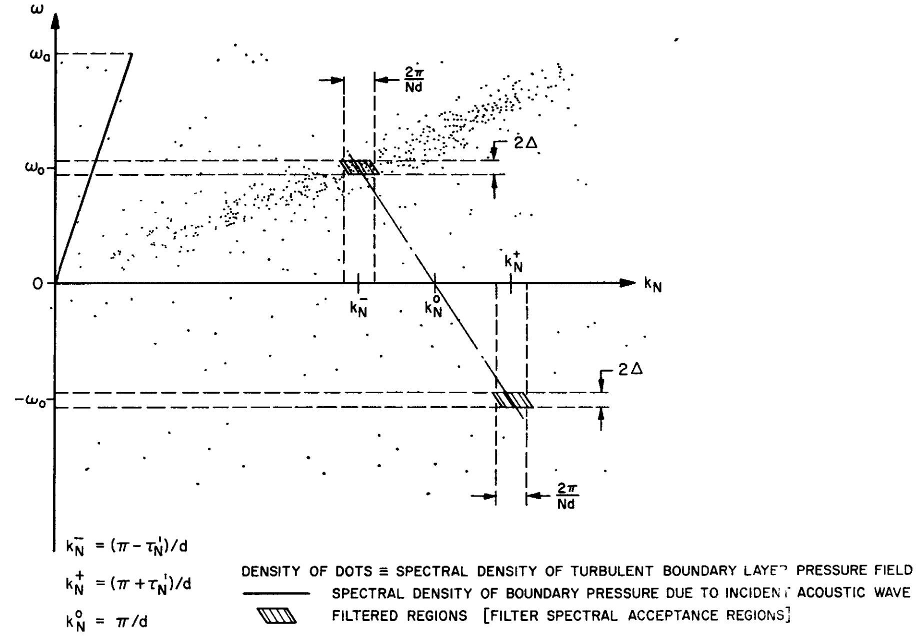

FIG. 4. Typical filtering action of a single-band wave-vector filter with time delay τ_N ($\omega\tau_N=\tau'_N$) between successive transducers. Also illustrated are the spectral density of a turbulent boundary-layer pressure field and the spectral density of a boundary pressure field due to an incident plane acoustic wave.

for by reflecting the filtering action on the k_N axis and using the images as the true filtering action. The spectral density remains unchanged. On the other hand, a change of sign in the convection velocity can be accounted for by reflecting the spectral density on the k_N axis and using the images as the true spectral density. The filtering action remains unchanged. If the time delay τ_N is zero, the filtering action in both quadrants is identical. In this case, one can illustrate the entire situation in one quadrant. The spectral density in the fourth quadrant can be reflected on the k_N axis and the image of this spectral density is simply added to the true density in the first quadrant. The first quadrant then illustrates the total situation. Indeed, this is precisely what was done in Ref. 1 where time delays were considered equal to zero.

It is apparent that the appropriate use of time delays between transducers can be employed, over a limited range of wavenumbers, to shift the center wavenumber of a given single-band wave-vector transducer system.[1] This is so, provided that the examination of the turbulent boundary-layer pressure field is limited to those spectral components that lie in the region defined so that $\omega\simeq k_1U$.[12] It should be noted, however, that τ_N' is a function of ω and consequently as the frequency is varied, to keep the center wavenumber at a fixed value the time delays must be made to vary inversely to the frequency.

Finally, the transducer system described in this paper can be employed to study some aspects of the dependence of the spectral density on the wavenumber k_3. This can be brought about by appropriately choosing the parameters on which K_P depends. (It is apparent that in the $\{k_P,\omega\}$ plane normal to the convection velocity, the description of the spectral density in the first and fourth quadrants is identical; a description commensurate with that of a nonconvecting pressure field.)

VI. RESPONSE TO AN INCIDENT PLANE ACOUSTIC PRESSURE FIELD

It is assumed that a plane acoustic wave is incident on the boundary. This wave is incident so that it gives rise to a boundary pressure field that is uniform in one principal direction of the transducer system; the principal axis P. The angle of incidence is designated by θ; it is the angle between the plane of incidence and a plane normal to the boundary containing the principal axis P. The spectral density Φ_a of this pressure field is expressed in the form

$$\Phi_a(\mathbf{k},\omega)=\phi_a(\omega)\delta(\omega-k_Nc_a)\delta(k_P), \qquad (32)$$

where $c_a=c_0{}'\sin\theta$, c_0 being the sound velocity in the fluid medium occupying the semi-infinite space above the boundary. The angle θ is considered positive when the convection velocity c_a of the pressure field is in the positive direction of the principal axis N.

The nature of the spectral density $\Phi_a(\mathbf{k},\omega)$ is illustrated in Fig. 4 for the case where $\phi_a(\omega)=AU(\omega_a-|\omega|)$, A and ω_a are constants. It is apparent that by proper choice of the parameters of the transducer system the degree of rejection of the acoustic pressure field by the transducer system can be made substantial. It is also

[12] The extent of the region that can be explored is contingent both on the nature of the minor maxima in the wave-vector filtering action of the transducer system and on the nature of the spectral distribution of the pressure field.

apparent that one can choose the parameters so as to reject substantially the pressure field in a subsonic turbulent boundary layer and make the transducer system highly receptive to a narrow frequency band of the acoustic pressure field. This latter use of the transducer system is more commensurate with the employment of such a system as a planar array where the acoustic pressure field is the sought after signal and the turbulent boundary layer pressure field constitutes the unwanted noise.[13]

The analysis is not restricted to the specific pressure fields just described. Rather, the analysis can be generalized to any boundary pressure field that is stationary, both spatially and temporally, provided one can assume that the filter operator of the transducer system is independent of the parameters of the pressure field to which the system is subjected.[4]

Finally, as stated in Ref. 1, should a method be devised to obtain the "simultaneous" signals of the transducers in the transducer system, a computer program can be instituted to generate the required wave-vector filtering. Time delays between transducers can be simulated in this program. Such a program would substitute mathematical procedures for the many actual experiments that would otherwise be required to utilize the full range of a given transducer system.

[13] It is emphasized that the degree of rejection (or acceptance) of the turbulent boundary-layer pressure field (or any other boundary pressure field) by the transducer system is contingent both on the spectral nature of this pressure field and on the nature of the operational filtering action mode of the transducer system. It is not dependent solely on the filtering action of a single transducer; although, as the analysis shows, the filtering action of a single transducer plays a definite role in determining the filtering action of the entire transducer system (see Figs. 2 and 3).

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