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A SPLINE-FUNCTION METHOD FOR GENERATING THE THERMODYNAMIC PROPERTIES OF WATER SUBSTANCE

NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER
Washington, D.C. 20007

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Report 2916

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Naval Ship Research and Development Center
Washington, D.C. 20007

DEPARTMENT OF THE NAVY
NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER
WASHINGTON, D. C. 20007

A SPLINE-FUNCTION METHOD FOR GENERATING THE THERMODYNAMIC PROPERTIES OF WATER SUBSTANCE

by

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A paper presented to the Seventh International Conference on the Properties of Steam, Tokyo, Japan, September 1968.

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

ABSTRACT

A computer formulation of the thermodynamic properties of water in the supercritical region is described, based on an application of cubic spline functions to fit data given in the 1963 International Skeleton Table: Specific Volume of Compressed Water and Superheated Steam. A thermal equation of state is obtained in the form of a doubly cubic spline function, representing specific volume as a function of pressure and temperature. This expression is then used directly in the equations defining enthalpy, entropy, and the Gibbs free-enthalpy function to generate these thermodynamic properties over the indicated range of the International Tables. The cubic spline function has also been used to fit the saturation line as well as the specific volume data for saturated water and saturated steam. Double precision arithmetic is used to retain a sufficient number of significant figures. The accuracy of the computed output is well within the range of tolerances specified in the skeleton tables. In this report, the mathematical formulation is described, the computer program is outlined, and the computational results are presented.

ADMINISTRATIVE INFORMATION

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1. INTRODUCTION

The problem of finding a satisfactory computer method for generating the thermodynamic properties of steam was first brought to the attention of the author in 1965. At that time an ambitious task was being defined at the Naval Ship Research and Development Center which would make it possible to design ships in their entirety with the aid of advanced computer techniques. In one of the problem areas of this task, computer programs to be used in the design of steam propulsion plants and piping systems required an efficient way of computing thermodynamic properties of water substance for repeatedly varying situations. The methods which were used to compute, by hand, the well-known Keenan and Keyes steam tables of 1936[1] are not altogether satisfactory for specialized high-speed applications in the active design process; hence, an investigation was made of the work of the Sixth International Conference on the Properties of Steam.[2]

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[1]References are listed on page 50.

This paper describes an application of cubic spline functions in computing the thermodynamic properties of water substance. The method consists of two parts. First, cubic splines are used to fit data on specific volume in such a way that a thermal equation of state is obtained in the form of a doubly cubic spline function of pressure and temperature. Secondly, this equation of state is used directly in the thermodynamic relations defining enthalpy, entropy, and the Gibbs free-enthalpy function to compute desired thermodynamic properties. The method has been applied to the data on specific volume given in the 1963 International Skeleton Tables,[3*] and a successful formulation has been obtained for the supercritical region.

The justification for using spline functions to fit experimental data stems from their mathematical properties. Historically, the derivation of cubic spline functions arose from the technique of draftsmen using splines (long thin pieces of wood, plastic, or other flexible material) held in place by special weights to draw smooth curves through, or near, a set of plotted points. In general, mathematical spline functions are composed of polynomials of degree n, where n is a positive integer greater than or equal to 3, which are joined at prescribed points, so that $n-1$ continuous derivatives exist at these points. Thus, by using a spline function of degree 3, a given set of data may be satisfied exactly by a function which is not only continuous but also possesses continuous first and second derivatives over the range of the specified data. Since the curvature of a mathematical spline is most readily controlled when $n=3$, the use of simple cubics to construct spline functions is especially attractive for fitting data representing a physical relationship.

Mathematical splines were first introduced by Schoenberg in 1946,[4] and interest in both the theory and application of such functions has recently accelerated, as demonstrated by Reference 5 and its bibliography. An early application of cubic splines to the geometric problem of fairing lines defining ship hulls was developed by Theilheimer and Starkweather[6] and their method is now being used at NSRDC in computer-aided ship design.

We shall begin with the definition of cubic spline functions and then state the problem in terms suitable for defining the doubly cubic interpolating-spline function. After deriving the resulting thermodynamic equations, the computer formulation for the supercritical region is presented in the form of the Gibbs function. The major features of the computer program THERMOSPLINE, which was developed to generate this formulation, are then described.

The versatility of the spline-fitting procedure makes it possible to use the THERMOSPLINE program to fit all or portions of the skeleton data. Thus, by using cubic and doubly cubic splines, values of specific volume have been obtained for the entire range of the skeleton table, and all of these values fall within the specified tolerances. A spline function representation of the saturation line (vapor pressure curve) has also been obtained.

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*This application of spline functions was suggested to the author by Prof. J. Kestin. These Skeleton tables are reproduced in Appendix B.

Furthermore, the method may be applied to any set of experimental data on steam or, indeed, to data on any pure substance. Representative computer output for various cases using the skeleton data only are presented in the appendix.

2. MATHEMATICAL FORMULATION

CUBIC SPLINE FUNCTION

Where appropriate, the symbols and physical constants adopted by the International Formulation Committee on the Properties of Steam[7] will be used.

The first definition will be of the cubic spline function of one independent variable. Let a mesh Δ be defined over an interval $\alpha \leq x \leq \beta$ by a set of distinct points x_j, $j = 1, \ldots, J$ as follows:

$$\Delta : \alpha = x_1 < x_2 < x_3 < \cdots < x_{J-1} < x_J = \beta$$

Let a corresponding set of ordinates by given,

$$y : y_1, y_2, y_3, \ldots, y_J \qquad [2.1]$$

Assume that the pairs (x_j, y_j) represent points defining a function $f(x)$; i.e., $y_j = f(x_j)$ and $J \geq 3$. Then a cubic spline approximation $s_\Delta(x)$ of $f(x)$ which satisfies

$$s_\Delta(x_j) = y_j, \quad j = 1, \ldots, J \qquad [2.2]$$

may be constructed by piecing together a number of cubics, each cubic representing an approximation to $f(x)$ over a subinterval $x_{j-1} \leq x \leq x_j$ of the mesh Δ. The cubics must be joined at the given data points (x_j, y_j), which are also called knots, juncture points, or joints, in such a way that the resulting function and its first two derivatives are continuous at these points. An explicit formula for this "piecewise cubic," called the cubic spline function, is given by

$$s_\Delta(x) = a + bx + cx^2 + A_1 x^3 + A_2(x - x_2)_+^3 + \cdots + A_{J-1}(x - x_{J-1})_+^3 \qquad [2.3]$$

where

$$(x - x_j)_+^3 = \begin{cases} 0 & \text{for } x \leq x_j \\ (x - x_j)^3 & \text{for } x > x_j \end{cases} \qquad [2.4]$$

A derivation of the cubic spline is given in Reference 5. The previous notation has been used in Reference 6 and suggested in Reference 8. Equation [2.3] defines a value $s_\Delta(x)$ approximating $f(x)$ for every x in the interval $\alpha \leq x \leq \beta$. Note that Equation [2.3] reduces to a simple cubic for each value of x in the interval; for example, if x is a point in the fourth mesh subinterval $x_4 < x \leq x_5$, then $s_\Delta(x)$ is given by

$$s_\Delta(x) = a + bx + cx^2 + A_1 x^3 + A_2(x - x_2)^3 + A_3(x - x_3)^3 + A_4(x - x_4)^3$$

which reduces to a simple cubic. Writing Equation [2.3] for each data point and using Equation [2.2] leads to the following system of J equations in the $J + 2$ unknowns a, b, c, $A_1, \ldots, A_{J-1}$

$$\begin{cases} y_1 = a + bx_1 + cx_1^2 + A_1 x_1^3 \\ y_2 = a + bx_2 + cx_2^2 + A_1 x_2^3 \\ y_3 = a + bx_3 + cx_3^2 + A_1 x_3^3 + A_2(x_3 - x_2)^3 \\ \cdot \\ \cdot \\ \cdot \\ y_J = a + bx_J + cx_J^2 + A_1 x_J^3 + A_2(x_J - x_2)^3 + A_3(x_J - x_3)^3 \\ \qquad + \cdots + A_{J-1}(x_J - x_{J-1})^3 \end{cases} \qquad [2.5]$$

The two additional conditions required to solve this system of equations may be obtained by prescribing appropriate conditions at the two ends of the interval $\alpha \leq x \leq \beta$. These conditions usually take the form of specifying values for the first or second derivatives of the spline function at the ends of the interval. For the purposes of fitting skeleton data on specific volume, spline curves are desired which will fit the data within the specified tolerances and still be free of unwanted changes in curvature. The additional conditions actually imposed in this application will take the form of minimizing the following expression

$$\sum_{j=1}^{J} [s_\Delta(x_j) - y_j]^2 + w \sum_{j=2}^{J-1} [s_\Delta''(x_j) - r_j]^2 \qquad [2.5a]$$

where w is a positive number, and r_j is the second difference

$$r_j = \frac{2}{x_{j+1} - x_{j-1}} \left[\frac{y_{j+1} - y_j}{x_{j+1} - x_j} - \frac{y_j - y_{j-1}}{x_j - x_{j-1}} \right], \quad \text{for } j = 2, \ldots, J - 1 \qquad [2.6]$$

If $w = 0$ were used, only the first sum would be minimized, which corresponds to the method of least squares. However, in this case, w must be positive and has been set equal to unity to obtain the formulation for the supercritical region. In experimenting with the method in other regions, $w = 1$ was also used with success. However, $w = 0.5$ was preferred when fitting specific volume data for the liquid phase along the saturation line. With this one exception, therefore, equal weight is given to each sum in Equation [2.5a], and the data are fitted as closely as possible while minimizing the sum of the squares of the differences between the second derivatives and second differences. This procedure tends to make the second derivatives and differences agree in sign, which in effect minimizes the presence of fluctuations in the resulting spline curve. These conditions were also used by Theilheimer in Reference 6.

APPLICATION TO 1963 SKELETON TABLES

Consider the data given in the 1963 International Skeleton Tables (IST).* Table 1 (IST) provides values in cubic centimeters per gram of v_f, the specific volume of saturated fluid, and v_g, the specific volume of saturated vapor, for 44 values of the saturation temperature t_s over the interval $0 \leq t_s \leq 374.15$ (°C). The corresponding values of the saturation pressure p_s are also given. Table 2 (IST) provides values of the specific volume v of pressurized water and superheated steam for 580 combinations of temperature t and pressure p given over the intervals $1 \leq p \leq 1000$ (bars) and $0 \leq t \leq 800$ (°C). Let D_Δ denote the mesh or grid of points (p_j, t_k), $j = 1, \ldots, 29$ and $k = 1, \ldots, 20$, for which a value $v(p_j, t_k)$ is given in Table 2 (IST). Let $\delta_{j,k}$ denote the corresponding tolerance specified for $v(p_j, t_k)$. Also, let D denote the domain of all points in the (p, t)-plane defined by

$$D: \ \{(p, t) \mid 0 \leq p \leq 1000 \text{ (bar)}, \quad 0 \leq t \leq 800 \text{ (°C)} \}$$

Figure 1 illustrates distribution of the points of the mesh D_Δ in the rectangular domain D. Furthermore, the points (p_s, t_s) of the saturation line given in Table 1 (IST) define a set of mesh points D_s contained in D.

Although the specific volume v is given in Table 2 (IST) as a function of the independent variables p and t, it is preferable to fit the product of $p \cdot v$ as a function of p and the thermodynamic temperature T, where T is in degrees Kelvin and hence $T/\text{deg } K = t/\text{deg } C + 273.15$. Accordingly, Table 2 (IST) is converted so that the data points are defined to be $(p_j, p_j v_j)_k$ for each temperature T_k, and an additional row of data is inserted for zero pressure by using the relation $pv = RT$, where R is the specific gas constant for water. The mesh D_Δ now contains 20 additional points. This is done so that the equation of state obtained by fitting the skeleton data will satisfy the ideal gas law in the limit as p tends to zero. Hence, the problem we have studied may now be stated as follows. We seek a spline function $F_\Delta(p, T)$ or set of spline functions, expressible in the form

*See Appendix B.

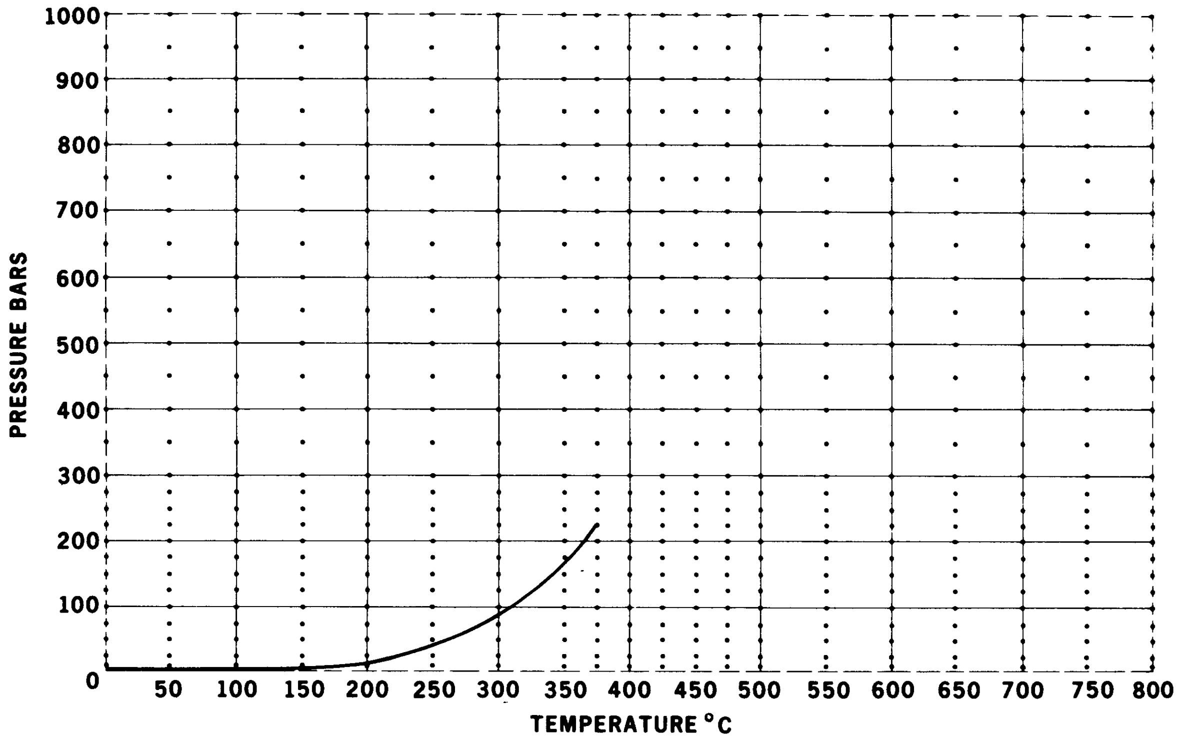

Figure 1 – Distribution of Points in p, t-Plane for which Data Are Given in 1963 International Skeleton Tables

Solid curve is the saturation line.

$$(p\nu)_\Delta = F_\Delta(p, T) \qquad [2.7]$$

which defines a value $p\nu$, and hence an approximation ν of the specific volume v, for any point $(p, T) \in D$ and which satisfies the relation

$$\left| \frac{1}{p} F_\Delta(p, T) - v(p, T) \right| \leq \delta(p, T), \quad p > 0 \qquad [2.8]$$

for $(p, T) = (p_j, T_k) \in D_\Delta + D_s$. Note that T_k now replaces t_k in referring to the temperature coordinate of the grid points defined previously when degrees Kelvin are indicated.

The stated problem has been solved by dividing the domain D into sets of rectangular subdomains, each determined by a sensible choice of mesh points of D_Δ. This partitioning may have merit for certain applications, but it results in a separate doubly cubic spline function for each subdomain and hence has not led to one formulation for the thermodynamic properties of steam that is valid for the entire domain D. However, a single spline function, and thus a single formulation, has been obtained for the supercritical region, i.e., for $0 \leq p \leq 1000$ (bar) and $400°C \leq t \leq 800°C$. This function was obtained by first applying cubic spline functions to fit the skeleton data as a function of p only for each isotherm and then by fitting sets of corresponding coefficients as functions of T, as described in the following paragraphs.

To fit $p \cdot v$ data as a function of p in the supercritical region, a cubic spline function of the same form as Equation [2.3] is applied to the data along the appropriate isotherms $T = T_k$, and the following set of K equations is obtained:

$$(p\nu)_k \equiv F_\Delta(p)_k = a_k + b_k p + c_k p^2 + A_{k,1} p^3 + \sum_{j=2}^{J-1} A_{k,j}(p - p_j)_+^3 \qquad [2.9]$$

$$k = 1, \ldots, K$$

where the subscript k designates that the equation represents $p\nu$ for the kth isotherm. For notational generality and programming convenience, the index k is counted from 1 to K in Equation [2.9], even though the isotherms in the supercritical region of Table 2 (IST) are those corresponding to $k = 10, \ldots, 20$ and, hence, $K = 11$. Furthermore, if every data point $(p_j, p_j v_j)_k$ is used in the fitting procedure, then $J = 30$ (including the values inserted for zero pressure).

Now to obtain one doubly cubic spline function of the two independent variables p and T, the coefficients $a_k, b_k, c_k, \ldots, A_{k, J-1}$ of the equations given by Equation [2.9] are regarded as data representing functional relationships between the variable T and each type of coefficient. For example, the pairs (T_k, a_k), $k = 1, \ldots, K$ may be used as data points to obtain a spline fit

$$a_\Delta = a_\Delta(T) \qquad [2.10]$$

Thus, a cubic spline fit of each set of data points (T_k, a_k), (T_k, b_k), (T_k, c_k), $\ldots$, $(T_k, A_{k, J-1})$ for $k = 1, \ldots, K$ leads to a second set of spline functions with T as the independent variable, namely,

$$
\begin{cases}
a_\Delta(T) = \alpha_1 + \beta_1 T + \gamma_1 T^2 + B_{1,1} T^3 + \sum_{k=2}^{K-1} B_{1,k}(T - T_k)_+^3 \\
b_\Delta(T) = \alpha_2 + \beta_2 T + \gamma_2 T^2 + B_{2,1} T^3 + \sum_{k=2}^{K-1} B_{2,k}(T - T_k)_+^3 \\
\cdot \\
\cdot \\
\cdot \\
A_{J-1,\Delta}(T) = \alpha_{J+2} + \beta_{J+2} T + \gamma_{J+2} T^2 + B_{J+2,1} T^3 + \sum_{k=2}^{K-1} B_{J+2,k}(T - T_k)_+^3
\end{cases}
\qquad [2.11]
$$

Then the spline function of Equation [2.7] may be written in the following form

$$
\begin{aligned}
(p\nu)_\Delta &= a_\Delta(T) + b_\Delta(T)p + c_\Delta(T)p^2 + A_{1,\Delta}(T)p^3 + \sum_{j=2}^{J-1} A_{j,\Delta}(T)(p - p_j)_+^3 \qquad [2.12] \\
&\equiv F_\Delta(p, T)
\end{aligned}
$$

where the coefficients are given by Equation [2.11] and, hence, $F_\Delta(p, T)$ denotes a doubly cubic spline expansion. Interpolated values of the specific volume ν are obtained by evaluating

$$
\begin{cases}
\nu(p, T) = p^{-1} F_\Delta(p, T), & p > 0 \\
(p\nu) = a_\Delta(T), & p = 0
\end{cases}
\qquad [2.13]
$$

Thus, with $673.15 \leq T \leq 1073.15$ (°K) the spline function of Equation [2.12] defines a thermal equation of state for steam in the supercritical region. This basic equation may then be used directly in the usual relations to obtain the remaining thermodynamic properties, described as follows. The calculations involved in determining the $(J+2) \times (K+2)$ coefficients in the system of Equations [2.11] are described in Section 3.

The method used to obtain Equation [2.12] may also be applied to fit the skeleton data over any rectangular region defined by at least 3×3 of the given grid points D_Δ, provided that the region does not contain any points of the saturation line in its interior. The reason for the restriction to rectangular regions is that a valid doubly cubic spline fit of the two independent variables p and T can be made only when exactly the same set of abscissa values p_j, $j = 1, \ldots, J$ is used for all of the selected isotherms T_k, $k = 1, \ldots, K$. This is because all of the coefficients of Equation [2.12] must be obtained in such a way that they are each functions of T defined over the same range of T. Hence, the same number of data points $(p_j, v_j)_k$ must be used for each isotherm $T = T_k$, which implies fitting over a rectangular subdomain of the mesh points D_Δ. It follows that many such rectangles must be used to fit data on each side of the saturation line. Isotherms which cross the saturation phase boundary have been separated into two portions $0 \le p \le p_s$ and $p_s \le p \le 1000$ (bars), and two separate cubic spline fits of the independent variable p have been made, each one satisfying the appropriate saturation data; i.e., either (p_s, v_g) or (p_s, v_f), respectively, at the phase boundary. Thus, by defining a set of rectangular subdomains of D_Δ on each side of the saturation line, it has been possible to fit the entire table of skeleton data on specific volume by a set of spline functions. The saturation line $p(T_s)$ as well as the specific volume $\nu_f(T_s)$ of saturated liquid and the specific volume $\nu_g(T_s)$ of saturated vapor have each been represented by a cubic spline function of the saturation temperature T_s.

PROPERTIES DERIVED FROM DOUBLY CUBIC SPLINE FUNCTION

Specific Enthalpy

Assuming that the thermal equation of state has been explicitly determined in Equation [2.12], the remaining thermodynamic properties may be derived from it, providing an expression for the specific heat $c_p^0(T)$ extrapolated to zero pressure is available. An accurate equation for $c_p^0(T)$ is

$$c_p^0(T) = 46.174T^{-1} + 7.58050 \times 10^{-4}\, T + 1.48286 \qquad [2.14]$$

which is an empirical formulation developed by Keyes and reported by Kestin et al,[9] based on data given by Friedman and Haar.[10] The equation for the specific enthalpy h in terms of p and T is derived from the general thermodynamic relation

$$\left(\frac{\partial h}{\partial p}\right)_T = -T\left(\frac{\partial v}{\partial T}\right)_p + v$$

By convention, the liquid phase at the triple point of water substance is the state for which the specific internal energy and the specific entropy are each made exactly zero. Integration of the previous equation with respect to this standard reference state yields

$$h(p, T) = h^f(p_t, T_t) + 2501 - \int_{p_t}^{p} T^2\left[\frac{\partial(v/T)}{\partial T}\right]_p dp + \int_{T_t}^{T} c_p^0(p, T)\, dT \qquad [2.15]$$

where $p_t = 0.006112$ bar is the triple point pressure,

$T_t = 273.16°K$ is the triple point temperature, and

the superscript f refers to the liquid phase.

The constant 2501 corresponds to the difference $h^g(p_t, T_t) - h^f(p_t, T_t)$. By Equation [2.13], Equation [2.15] may be written

$$h(p, T) = h^f(p_t, T_t) + 2501 - \int_{p_t}^{p} T^2 p^{-1} \left[\frac{\partial F_\Delta(p, T)/T}{\partial T}\right]_p dp + \int_{T_t}^{T} c_p^0(T)\, dT \qquad [2.16]$$

After substituting the complete expression for $F_\Delta(p, T)$ given by Equations [2.11] and [2.12], and using Equation [2.14], integration of Equation [2.16] yields

$$h(p, T) = h^f(p_t, T_t) + 2501 - T\,\{[\beta_1 + 2\gamma_1 T + 3B_{1,1}T^2 + 3\sum_{k=2}^{K-1} B_{1,k}(T - T_k)_+^2] \ln \frac{p}{p_t}$$

$$+ [\beta_2 + 2\gamma_2 T + 3B_{2,1}T^2 + 3B_{2,2}(T - T_2)_+^2 + \dots + 3B_{2,K-1}(T - T_{K-1})_+^2](p - p_t)$$

$$+ \frac{1}{2}[\beta_3 + 2\gamma_3 T + 3B_{3,1}T^2 + 3\sum_{k=2}^{K-1} B_{3,k}(T - T_k)_+^2](p^2 - p_t^2)$$

$$+ \frac{1}{3}[\beta_4 + 2\gamma_4 T + 3B_{4,1}T^2 + 3B_{4,2}(T - T_2)_+^2 + \dots + 3B_{4,K-1}(T - T_{K-1})_+^2](p^3 - p_t^3)$$

$$+ [\beta_5 + 2\gamma_5 T + 3B_{5,1}T^2 + 3\sum_{k=2}^{K-1} B_{5,k}(T - T_k)_+^2]\, G(p, p_2)_+ + \dots \qquad [2.17]$$

$$+ [\beta_{J+2} + 2\gamma_{J+2,1} T + 3B_{J+2,1}T^2 + 3\sum_{k=2}^{K-1} B_{J+2,k}(T - T_k)_+^2]\, G(p, p_{J-1})_+\}$$

$$+ \{[\alpha_1 + \beta_1 T + \gamma_1 T^2 + B_{1,1}T^3 + \sum_{k=2}^{K-1} B_{1,k}(T - T_k)_+^3] \ln \frac{p}{p_t} + [\alpha_2 + \beta_2 T^2$$

$$+ \gamma_2 T^2 + B_{2,1}T^3 + \sum_{k=2}^{K-1} B_{2,k}(T - T_k)_+^3](p - p_t) + \frac{1}{2}[\alpha_3 + \beta_3 T + \gamma_3 T^2$$

$$+ B_{3,1}T^3 + \sum_{k=2}^{K-1} B_{3,k}(T - T_k)_+^3](p^2 - p_t^2) + \frac{1}{3}[\alpha_4 + \beta_4 T + \gamma_4 T^2 + B_{4,1}T^3$$

$$+ \sum_{k=2}^{K-1} B_{4,k}(T-T_k)_+^3](p^3-p_t^3) + [\boldsymbol{\alpha}_5 + \beta_5 T + \gamma_5 T^2 + B_{5,1}T^3$$

$$+ \sum_{k=2}^{K-1} B_{5,k}(T-T_k)_+^3]\, G(p,\, p_2)_+ + \dots + [\boldsymbol{\alpha}_{J+2} + \beta_{J+2} T + \gamma_{J+2} T^2 + B_{J+2,1}T^3$$

[2.17 (continued)]

$$+ \sum_{k=2}^{K-1} B_{J+2,k}(T-T_k)_+^3]\, G(p,p_{J-1})_+ \} + 46.174 \ln \frac{T}{T_t}$$

$$+ 1.48286\,(T - T_t) + 3.79025 \times 10^{-4}(T^2 - T_t^2)$$

where for $j = 2, \dots, J-1$

$$G(p,p_j)_+ = \begin{cases} 0 & \text{if } p \le p_j \\ \left[\frac{1}{3}(p^3 - p_j^3) - \frac{3}{2} p_j(p^2 - p_j^2) + 3p_j^2(p-p_j) - p_j^3 \ln \frac{p}{p_j}\right] & \text{if } p > p_j \end{cases} \qquad [2.17\text{a}]$$

This equation may be written in a more compact form as follows:

$$h^f(p,\, T) = h(p_t,\, T_t) + [a_\Delta(T) - T\, a_\Delta'(T)] \ln \frac{p}{p_t} + [b_\Delta(T) - T b_\Delta'(T)](p - p_t)$$

$$+ \frac{1}{2}[c_\Delta(T) - T\, c_\Delta'(T)]\,(p^2 - p_t^2) + \frac{1}{3}[A_{1,\Delta}(T) - TA_{1,\Delta}'(T)]\,(p^3 - p_t^3)$$

$$+ [A_{2,\Delta}(T) - TA_{2,\Delta}'(T)]\, G(p,\, p_2)_+ + \dots + [A_{J-1,\Delta}(T) \qquad [2.18]$$

$$- TA_{J-1,\Delta}'(T)]\, G(p,\, p_{J-1})_+ + 46.174 \ln \frac{T}{T_t} + 1.48286\,(T - T_t)$$

$$+ 3.79025 \times 10^{-4}(T^2 - T_t^2) + 2501$$

where the spline functions $a_\Delta(T)$, $b_\Delta(T)$, etc., are given by Equation [2.11], and the primes denote differentiation with respect to T.

Specific Entropy

From the thermodynamic relation

$$\left(\frac{\partial s}{\partial p}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_p \qquad [2.19]$$

an equation is derived for evaluating the specific entropy s. As in the case of enthalpy, the liquid phase at the triple point is the chosen reference state. Since $s \to \infty$ as $p \to 0$, a convenient path of integration is chosen and

$$s(p, T) = s^f(p, T) + 9.15580 - \int_{p_t}^{p} \left[\frac{1}{p}\left(\frac{\partial F_\Delta(p, T)}{\partial T}\right)_p - \frac{R}{p}\right]_T dp$$

$$+ \int_{T_t}^{T} T^{-1}\, c_p^0(T)\, dt - R \ln \frac{p}{p_t} \qquad [2.20]$$

is obtained, where $s^f(p, T)$ is zero by convention, and the constant is calculated from the values given for enthalpy in the skeleton tables as follows:

$$s^g(p_t, T_t) - s^f(p_t, T_t) = s^g(p_t, T_t) = \frac{h^g(p_t, T_t) - h^f(p_t, T_t)}{T_t} = 9.15580 \qquad [2.21]$$

By substituting the expressions for $c_p^0(T)$ and $F_\Delta(p, T)$ given by Equations [2.14] and [2.12], integration of Equation [2.20] yields, in compact form,

$$s(p, T) = 9.15580 - \left\{a_\Delta'(T) \ln \frac{p}{p_t} + b_\Delta'(T)(p - p_t) + \frac{1}{2} c'(T)(p^2 - p_t^2)\right.$$

$$\left. + \frac{1}{3} A_{1,\Delta}'(T)(p^3 - p_t^3) + A_{2,\Delta}'(T)\, G(p, p_2)_+ + \ldots + A_{J-1,\Delta}'(T)\, G(p, p_{J-1})_+\right\} \qquad [2.22]$$

$$+ 1.48286 \ln \frac{T}{T_t} - 46.174\,(T^{-1} - T_t^{-1}) + 7.58050 \times 10^{-4}\,(T - T_t)$$

where again the spline functions $a_\Delta(T)$, $b_\Delta(T)$, etc., are given by Equation [2.11]; $G(p, p_j)_+$ is defined by Equation [2.17a]; and primes denote differentiation with respect to T.

Gibbs Free-Enthalpy Function

Equations [2.17] and [2.22] for enthalpy and entropy, respectively, lead directly to a formulation for the supercritical region of the Gibbs free-enthalpy function g, which is defined by the thermodynamic relation

$$g = h - Ts \qquad [2.23]$$

The result of performing the indicated substitutions is given in expanded form as follows:

$$\begin{aligned}
g(p, T) = {} & h^f(p_t, T_t) + 2501 - 9.1580\,T + 46.174\left(1 - \frac{T}{T_t}\right) + [46.174 - 1.48286\,T]\ln\frac{T}{T_t} \\
& + [1.4828 - 7.58050\times10^{-4}\,T](T - T_t) + 3.79025\times10^{-4}(T^2 - T_t^2) \\
& + \{[\alpha_1 + \beta_1 T + \gamma_1 T^2 + B_{1,1}T^3 + \sum_{k=2}^{K-1} B_{1,k}(T - T_k)_+^3]\ln\frac{p}{p_t} \\
& + [\alpha_2 + \beta_2 T + \gamma_2 T^2 + B_{2,1}T^3 + \sum_{k=2}^{K-1} B_{2,k}(T - T_k)_+^3](p - p_t) \\
& + [\alpha_3 + \beta_3 T + \gamma_3 T^2 + B_{3,1}T^3 + \sum_{k=2}^{K-1} B_{3,k}(T - T_k)_+^3](p^2 - p_t^2) \\
& + [\alpha_4 + \beta_4 T + \gamma_4 T^2 + B_{4,1}T^3 + \sum_{k=2}^{K-1} B_{4,k}(T - T_k)_+^3](p^3 - p_t^3) \\
& + [\alpha_5 + \beta_5 T + \gamma_5 T^2 + B_{5,1}T^3 + \sum_{k=2}^{K-1} B_{5,k}(T - T_k)_+^3]\,G(p, p_2)_+ \\
& + \ldots + [\alpha_{J+2} + \beta_{J+2}T + \gamma_{J+2}T^2 + B_{J+2,1}T^3 + \sum_{k=2}^{K-1} B_{J+2,k}(T - T_k)_+^3]\,G(p - p_{J-1})_+\}
\end{aligned} \qquad [2.24]$$

where $G(p, p_j)$ is defined as before by

$$G(p, p_j)_+ = \begin{cases} 0 & \text{if } p \leq p_j \\ \left[\frac{1}{3}(p^3 - p_j^3) - \frac{3}{2}p_j(p^2 - p_j^2) + 3p_j^2(p - p_j) - p_j^3\ln\frac{p}{p_j}\right] & \text{if } p > p_j \end{cases} \qquad [2.17a]$$

for $j = 2, \ldots, J-1$. The numerical values of the coefficients appearing in Equation [2.24] as computed by the THERMOSPLINE program are given on the next few pages. Since Greek letters are not available in FORTRAN printed output, the following notation is used for the α, β, and γ-coefficients:

$$\left.\begin{aligned} \alpha_j &= AAj \\ \beta_j &= BBj \\ \gamma_j &= CCj \end{aligned}\right\} \quad \text{for } j = 1, \ldots, 32$$

Note that for the supercritical region, $J = 30$, and $K = 11$.

Since the equations presented in this paper for h, s, and g have all been derived from the spline formulation of the thermal equation of state, thermodynamic consistency is guaranteed. The Gibbs function (see Table 1) constitutes a fundamental thermodynamic equation, and hence all thermodynamic properties, including, for example, the Helmholtz free-energy function f, can now be obtained directly from the formulation in Equation [2.24]. This fact also provides a means of checking the calculation of h, s, and v.

Table 1 – Coefficients of the Gibbs Function for the Supercritical Region $400 \leq t \leq 800°C$; $0 \leq p \leq 1000$ (bar)

A(T)

AA 1	BB 1	CC 1	B 1,1	B 1,2	B 1,3	B 1,4	B 1,5
-0.74544542D 03	0.79353304D 01	-0.49207899D-02	0.24269082D-05	-0.39763461D-05	-0.21653716D-05	0.94060076D-05	-0.76338258D-05
B 1,6	**B 1,7**	**B 1,8**	**B 1,9**	**B 1,10**			
0.27638922D-05	-0.10533739D-05	0.35499271D-06	-0.17591342D-06	-0.11829785D-06			

B(T)

AA 2	BB 2	CC 2	B 2,1	B 2,2	B 2,3	B 2,4	B 2,5
0.55940610D 03	-0.18800470D 01	0.19035997D-02	-0.52509127D-06	-0.92609599D-05	-0.85678017D-05	0.50296651D-04	-0.43011051D-04
B 2,6	**B 2,7**	**B 2,8**	**B 2,9**	**B 2,10**			
0.15915090D-04	-0.61363677D-05	0.15113894D-05	-0.11197094D-06	-0.40037906D-06			

C(T)

AA 3	BB 3	CC 3	B 3,1	B 3,2	B 3,3	B 3,4	B 3,5
-0.70530239D 02	0.15492354D 00	-0.12396497D-04	-0.92651850D-07	0.17565904D-05	0.46385532D-05	-0.15952928D-04	0.12962994D-04
B 3,6	**B 3,7**	**B 3,8**	**B 3,9**	**B 3,10**			
-0.47010088D-05	0.18323758D-05	-0.81218218D-06	0.61940141D-06	0.30266204D-06			

A 1(T)

AA 4	BB 4	CC 4	B 4,1	B 4,2	B 4,3	B 4,4	B 4,5
0.48040342D 00	0.58931797D-02	-0.19654254D-04	0.14638347D-07	-0.69752678D-07	-0.35751031D-06	0.99333245D-06	-0.77317177D-06
B 4,6	**B 4,7**	**B 4,8**	**B 4,9**	**B 4,10**			
0.27056678D-06	-0.10375788D-06	0.52530098D-07	-0.46916338D-07	-0.20865513D-07			

A 2(T)

AA 5	BB 5	CC 5	B 5,1	B 5,2	B 5,3	B 5,4	B 5,5
-0.10256017D 01	0.78420385D-02	-0.16030065D-04	0.98822195D-08	-0.19813992D-07	-0.19211204D-06	0.47039862D-06	-0.35371231D-06
B 5,6	**B 5,7**	**B 5,8**	**B 5,9**	**B 5,10**			
0.11683966D-06	-0.49642563D-07	0.55259626D-07	-0.69659570D-07	-0.24708206D-07			

A 3(T)

AA 6	BB 6	CC 6	B 6,1	B 6,2	B 6,3	B 6,4	B 6,5
0.76105220D 01	-0.44453519D-01	0.80058579D-04	-0.45819930D-07	0.38923494D-07	0.79507687D-06	-0.17921001D-05	0.13163737D-05
B 6,6	**B 6,7**	**B 6,8**	**B 6,9**	**B 6,10**			
-0.42524139D-06	0.16848965D-06	-0.16942644D-06	0.21485613D-06	0.77100684D-07			

A 4(T)

AA 7	BB 7	CC 7	B 7,1	B 7,2	B 7,3	B 7,4	B 7,5
-0.77579569D 01	0.33440931D-01	-0.47900656D-04	0.22800945D-07	0.57588704D-07	-0.25647993D-06	0.32852008D-06	-0.18440747D-06
B 7,6	**B 7,7**	**B 7,8**	**B 7,9**	**B 7,10**			
0.32033160D-07	-0.13553149D-07	0.73140305D-07	-0.11821686D-06	-0.37627543D-07			

Table 1 (Continued)

A 5(T)

AA 8	BB 8	CC 8	B 8,1	B 8,2	B 8,3	B 8,4	B 8,5
0.68040282D 00	-0.25944152D-02	0.32338803D-05	-0.13101564D-08	-0.71568303D-08	0.70601319D-08	0.69948785D-08	-0.37931825D-08
B 8,6	B 8,7	B 8,8	B 8,9	B 8,10			
0.63534014D-08	0.55561687D-09	-0.18335418D-07	0.28454755D-07	0.89056592D-08			

A 6(T)

AA 9	BB 9	CC 9	B 9,1	B 9,2	B 9,3	B 9,4	B 9,5
0.26441488D-01	-0.23931190D-03	0.52347759D-06	-0.33590759D-09	0.39344058D-09	0.68057109D-08	-0.12209703D-07	0.57795387D-08
B 9,6	B 9,7	B 9,8	B 9,9	B 9,10			
0.56131531D-09	-0.56240759D-08	0.13227376D-07	-0.14864979D-07	-0.50364199D-08			

A 7(T)

AA10	BB10	CC10	B10,1	B10,2	B10,3	B10,4	B10,5
0.79720248D-03	0.96839128D-04	-0.28939005D-06	0.20908919D-09	-0.69433577D-09	-0.53316919D-08	0.10789286D-07	-0.46640730D-08
B10,6	B10,7	B10,8	B10,9	B10,10			
-0.24310941D-08	0.63415001D-08	-0.99667101D-08	0.93676522D-08	0.33554705D-08			

A 8(T)

AA11	BB11	CC11	B11,1	B11,2	B11,3	B11,4	B11,5
-0.95176828D-01	0.25723880D-03	-0.15122482D-06	-0.31273218D-10	0.17901259D-08	0.45630163D-08	-0.12964402D-07	0.66876169D-08
B11,6	B11,7	B11,8	B11,9	B11,10			
0.19055967D-08	-0.44833420D-08	0.47384713D-08	-0.33922388D-08	-0.13379987D-08			

A 9(T)

AA12	BB12	CC12	B12,1	B12,2	B12,3	B12,4	B12,5
0.30580578D 00	-0.10816983D-02	0.12179592D-05	-0.42412939D-09	-0.39310511D-08	-0.11203241D-08	0.13830610D-07	-0.96565462D-08
B12,6	B12,7	B12,8	B12,9	B12,10			
0.23854434D-09	0.23820978D-08	-0.13875799D-08	-0.34839248D-09	0.39165049D-10			

A10(T)

AA13	BB13	CC13	B13,1	B13,2	B13,3	B13,4	B13,5
-0.63551364D 00	0.24416900D-02	-0.30835648D-05	0.12749340D-08	0.69239645D-08	-0.73082305D-08	-0.85247056D-08	0.10211441D-07
B13,6	B13,7	B13,8	B13,9	B13,10			
-0.26721634D-08	-0.47230080D-09	-0.14883936D-11	0.14332413D-08	0.40174133D-09			

A11(T)

AA14	BB14	CC14	B14,1	B14,2	B14,3	B14,4	B14,5
0.69694340D 00	-0.28328690D-02	0.38198270D-05	-0.17073540D-08	-0.69996359D-08	0.14692793D-07	-0.34465281D-08	-0.46992919D-08
B14,6	B14,7	B14,8	B14,9	B14,10			
0.26092404D-08	-0.32590095D-09	0.37368975D-09	-0.97523306D-09	-0.31323433D-09			

Table 1 (Continued)

A12(T)

AA15	BB15	CC15	B15.1	B15.2	B15.3	B15.4	B15.5
-0.82480658D 00	0.34740301D-02	-0.48642299D-05	0.22636155D-08	0.74364888D-08	-0.22886590D-07	0.17884108D-07	-0.46413244D-08
B15.6	B15.7	B15.8	B15.9	B15.10			
0.67707047D-09	-0.14541056D-08	0.80903323D-09	-0.46894256D-10	-0.11088791D-09			

A13(T)

AA16	BB16	CC16	B16.1	B16.2	B16.3	B16.4	B16.5
0.14793123D 00	-0.99467166D-03	0.19075731D-05	-0.11265927D-08	-0.14572572D-08	0.17017222D-07	-0.20005896D-07	0.60789721D-08
B16.6	B16.7	B16.8	B16.9	B16.10			
-0.25571387D-08	0.44973520D-08	-0.38858367D-08	0.16017171D-08	0.81902721D-09			

A14(T)

AA17	BB17	CC17	B17.1	B17.2	B17.3	B17.4	B17.5
0.20515451D 01	-0.74853190D-02	0.88784550D-05	-0.33880824D-08	-0.17363318D-07	0.17247781D-07	0.17225034D-09	0.52500674D-08
B17.6	B17.7	B17.8	B17.9	B17.10			
-0.80852196D-10	-0.58210294D-08	0.71083023D-08	-0.34553989D-08	-0.15401129D-08			

A15(T)

AA18	BB18	CC18	B18.1	B18.2	B18.3	B18.4	B18.5
0.23093710D 01	-0.90046786D-02	0.11591161D-04	-0.49154664D-08	-0.22151368D-07	0.36031921D-07	-0.30046054D-08	-0.91225831D-08
B18.6	B18.7	B18.8	B18.9	B18.10			
0.22631383D-08	0.47660665D-08	-0.74739528D-08	0.40095135D-08	0.15826558D-08			

A16(T)

AA19	BB19	CC19	B19.1	B19.2	B19.3	B19.4	B19.5
-0.81456586D 01	0.31491564D-01	-0.40138319D-04	0.16822172D-07	0.74600546D-07	-0.12161664D-06	0.32589060D-07	-0.12075505D-08
B19.6	B19.7	B19.8	B19.9	B19.10			
-0.78925741D-09	-0.24066154D-08	0.40741040D-08	-0.23176571D-08	-0.97051293D-09			

A17(T)

AA20	BB20	CC20	B20.1	B20.2	B20.3	B20.4	B20.5
0.59849444D 01	-0.23513752D-01	0.30525790D-04	-0.13071871D-07	-0.55010574D-07	0.10153422D-06	-0.41464589D-07	0.84177174D-08
B20.6	B20.7	B20.8,	B20.9	B20.10			
-0.64985508D-09	0.76973050D-09	-0.11304879D-08	0.74566122D-09	0.32212322D-09			

A18(T)

AA21	BB21	CC21	B21.1	B21.2	B21.3	B21.4	B21.5
-0.15210220D 01	0.61418264D-02	-0.82248706D-05	0.36500973D-08	0.13691315D-07	-0.32302137D-07	0.21146155D-07	-0.66963858D-08
B21.6	B21.7	B21.8	B21.9	B21.10			
0.69615717D-09	-0.44308031D-09	0.62589704D-09	-0.59158920D-09	-0.22805030D-09			

Table 1 (Continued)

A19(T)

AA22	BB22	CC22	B22,1	B22,2	B22,3	B22,4	B22,5
-0.48971210D 00	0.19686963D-02	-0.26180289D-05	0.11503200D-08	0.48503033D-08	-0.91787230D-08	0.25755173D-08	0.98546965D-09
B22,6	B22,7	B22,8	B22,9	B22,10			
-0.57286751D-09	0.44763509D-09	-0.67670890D-09	0.77224238D-09	0.26722568D-09			

A20(T)

AA23	BB23	CC23	B23,1	B23,2	B23,3	B23,4	B23,5
0.11150481D 00	-0.48525865D-03	0.70172076D-06	-0.33716596D-09	-0.97711045D-09	0.37726109D-08	-0.33820793D-08	0.71435231D-09
B23,6	B23,7	B23,8	B23,9	B23,10			
0.44581872D-09	-0.56006916D-09	0.86174411D-09	-0.98707759D-09	-0.33838100D-09			

A21(T)

AA24	BB24	CC24	B24,1	B24,2	B24,3	B24,4	B24,5
0.15996985D 00	-0.66577394D-03	0.92113783D-06	-0.42354150D-09	-0.13101541D-08	0.42695734D-08	-0.44202312D-08	0.23591277D-08
B24,6	B24,7	B24,8	B24,9	B24,10			
-0.72421709D-09	0.50838808D-09	-0.70507620D-09	0.83303123D-09	0.29114360D-09			

A22(T)

AA25	BB25	CC25	B25,1	B25,2	B25,3	B25,4	B25,5
-0.13400306D 00	0.56366154D-03	-0.78891499D-06	0.36735180D-09	0.95020083D-09	-0.40088900D-08	0.53706182D-08	-0.34028307D-08
B25,6	B25,7	B25,8	B25,9	B25,10			
0.94456990D-09	-0.22439064D-09	-0.81601737D-11	-0.43981301D-10	-0.25507591D-10			

A23(T)

AA26	BB26	CC26	B26,1	B26,2	B26,3	B26,4	B26,5
0.10500466D 00	-0.44019589D-03	0.61343584D-06	-0.28407329D-09	-0.85579850D-09	0.29108471D-08	-0.31388979D-08	0.17135719D-08
B26,6	B26,7	B26,8	B26,9	B26,10			
-0.38074191D-09	-0.26599958D-09	0.88988544D-09	-0.98814745D-09	-0.32913903D-09			

A24(T)

AA27	BB27	CC27	B27,1	B27,2	B27,3	B27,4	B27,5
-0.75933440D-01	0.31392093D-03	-0.43104195D-06	0.19648063D-09	0.68744867D-09	-0.18760292D-08	0.13972461D-08	-0.39260609D-09
B27,6	B27,7	B27,8	B27,9	B27,10			
-0.17497389D-09	0.63851098D-09	-0.13696058D-08	0.15735592D-08	0.53252186D-09			

A25(T)

AA28	BB28	CC28	B28,1	B28,2	B28,3	B28,4	B28,5
0.57630252D-01	-0.23270859D-03	0.31129107D-06	-0.13779919D-09	-0.57570121D-09	0.11417999D-08	-0.30390672D-09	-0.26479155D-09
B28,6	B28,7	B28,8	B28,9	B28,10			
0.36039944D-09	-0.73129338D-09	0.14368504D-08	-0.16414556D-08	-0.55926430D-09			

Table 1 (Concluded)

A26(T)

AA29	BB29	CC29	B29,1	B29,2	B29,3	B29,4	B29,5
-0.18438311D-01	0.73738138D-04	-0.97446691D-07	0.42474567D-10	0.23428636D-09	-0.28640326D-09	-0.40744724D-09	0.64622533D-09
B29,6	B29,7	B29,8	B29,9	B29,10			
-0.47847899D-09	0.79022963D-09	-0.15033022D-08	0.16496889D-08	0.56862944D-09			

A27(T)

AA30	BB30	CC30	B30,1	B30,2	B30,3	B30,4	B30,5
0.44064464D-03	-0.23635900D-05	0.37548570D-08	-0.182072295D-11	-0.698588889D-10	-0.48630315D-10	0.57184124D-09	-0.66704684D-09
B30,6	B30,7	B30,8	B30,9	B30,10			
0.44696560D-09	-0.81102099D-09	0.16147915D-08	-0.17118046D-08	-0.59356346D-09			

A28(T)

AA31	BB31	CC31	B31,1	B31,2	B31,3	B31,4	B31,5
-0.16570316D-01	0.68973227D-04	-0.95263384D-07	0.43641285D-10	0.131112870D-09	-0.39040993D-09	0.361828870D-10	0.232137030D-09
B31,6	B31,7	B31,8	B31,9	B31,10			
-0.24450596D-09	0.57341531D-09	-0.11932913D-08	0.12466708D-08	0.43236561D-09			

A29(T)

AA32	BB32	CC32	B32,1	B32,2	B32,3	B32,4	B32,5
-0.54120535D-02	0.22636890D-04	-0.31407362D-07	0.14448515D-10	0.622502950D-10	-0.12767624D-09	-0.10846720D-10	0.10605964D-09
B32,6	B32,7	B32,8	B32,9	B32,10			
-0.10383499D-09	0.22998088D-09	-0.47167593D-09	0.49603481D-09	0.17190130D-09			

3. COMPUTER PROGRAM

GENERAL DESCRIPTION OF PROGRAM

The spline function method has been programmed in FORTRAN IV language for the IBM 7090 computer. The program, named THERMOSPLINE, is described in detail in Reference 11. Only the major features will be mentioned here.

THERMOSPLINE is designed to generate the thermodynamic properties of steam from a spline fit of any given set of data. The 1963 International Skeleton Tables 1 and 2 have been used as the input data in this study, but the program is by no means restricted to this particular collection of data. Indeed, the method could be applied directly to compute the thermodynamic properties of other pure substances if experimental data of sufficient quantity and quality were provided as input. A considerable degree of versatility in applying the cubic spline fitting procedure is built into the program. By varying certain control parameters in the input, the program is capable of fitting only specified portions of the available data. For example, a single isotherm, an isobar, or a section of the superheated region may be selected from the skeleton data and used in the fitting process. No iteration procedures are necessary. The system of equations is set up as indicated by Equations [2.5] but is augmented by the equations resulting from the additional conditions of Equation [2.5a]. Thus, for each cubic spline fit performed over a set of isotherms $k = 1, \ldots, K$, the following expression is minimized

$$\sum_{j=1}^{J} [F_\Delta(p_j)_k - p_j v_{j,k}]^2 + w \sum_{j=2}^{J-1} [F_\Delta''(p_k)_k - r_{j,k}]^2 \qquad [3.0]$$

where

$$r_{j,k} = \frac{2}{p_{j+1} - p_{j-1}} \left[\frac{p_{j+1} v_{j+1,k} - p_j v_{j,k}}{p_{j+1} - p_j} - \frac{p_j v_{j,k} - p_{j-1} v_{j-1,k}}{p_j - p_{j-1}} \right] \text{ for } j = 2, \ldots, J-1 \qquad [3.0a]$$

and $F_\Delta(p_j)_k$ is given by Equation [2.9]. The program does this by setting up the matrix

$$M = \begin{pmatrix} 1 & p_1 & p_1^2 & p_1^3 & 0 & 0 & \cdots & 0 & 0 \\ 1 & p_2 & p_2^2 & p_2^3 & 0 & 0 & \cdots & 0 & 0 \\ 1 & p_3 & p_3^2 & p_3^3 & (p_3-p_2)^3 & 0 & \cdots & 0 & 0 \\ 1 & p_4 & p_4^2 & p_4^3 & (p_4-p_2)^3 & (p_4-p_3)^3 & \cdots & 0 & 0 \\ \cdot & & & & & & & \cdot & \cdot \\ \cdot & & & & & & & \cdot & \cdot \\ \cdot & & & & & & & \cdot & \cdot \\ 1 & p_J & p_J^2 & p_J^3 & (p_J-p_2)^3 & (p_J-p_3)^3 & \cdots & (p_J-p_{J-2})^3 & (p_J-p_{J-1})^3 \\ 0 & 0 & 2w & 6wp_2 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 2w & 6wp_3 & 6w(p_3-p_2) & 0 & \cdots & 0 & 0 \\ 0 & 0 & 2w & 6wp_4 & 6w(p_4-p_2) & 6w(p_4-p_3) & \cdots & 0 & 0 \\ \cdot & & & & & & & \cdot & \cdot \\ \cdot & & & & & & & \cdot & \cdot \\ \cdot & & & & & & & \cdot & \cdot \\ 0 & 0 & 2w & 6wp_{J-1} & 6w(p_{J-1}-p_2) & 6w(p_{J-1}-p_3) & \cdots & 6w(p_{J-1}-p_{J-2}) & 0 \end{pmatrix} \qquad [3.1]$$

where p_j are the given values of pressure for which values of $p \cdot v$ are known, and w is the parameter in Equation [3.0]. Unless otherwise specified, w is preset to unity. Let M' denote the transpose of the matrix M and M denote the product $M'M$. Then the matrix equation

$$\mathsf{M}\,\vec{c} = M'\vec{d} \qquad [3.2]$$

is formed and solved for the vector $\vec{c}$ of the $J+2$ unknown coefficients, where

$$\vec{c} = \begin{pmatrix} a \\ b \\ c \\ A_1 \\ \cdot \\ \cdot \\ \cdot \\ A_{J-1} \end{pmatrix} \qquad [3.3]$$

$$\vec{d} = \begin{pmatrix} p_1 v_1 \\ p_2 v_2 \\ \cdot \\ \cdot \\ \cdot \\ p_J v_J \\ r_2 \\ r_3 \\ \cdot \\ \cdot \\ \cdot \\ r_{J-1} \end{pmatrix} \qquad [3.4]$$

and the r_j are given by Equation [2.6]. Note that M is a square matrix of order $J+2$.

The matrix M is inverted using a Gauss-Jordan elimination procedure and the solution of Equation [3.2], namely

$$\vec{c} = \mathsf{M}^{-1} M' \vec{d} \qquad [3.5]$$

yields the desired coefficients. The program determines the vector of coefficients in this manner for every isotherm specified. Note that M^{-1} need only be computed once for a specified set of input abscissa values p_j, $j=1, \ldots, J$ to compute the cubic spline coefficients for a set of isotherms T_k, $k=1, \ldots, K$. Thereafter, the spline fitting of each set of K isothermal coefficients (e.g., a_k for $k=1, \ldots, K$) is carried out by the same procedure for the $J+2$ sets a_k, b_k, c_k, $A_{k,1}$, $\ldots$, $A_{k,J-1}$. The result is an array of $(J+2)$ times $(K+2)$ coefficients for the thermal equation of state exactly as given by Equations [2.12] and [2.11]. The remaining calculations consist of evaluating the equations derived for h, s, and g. All calculations are performed in double precision arithmetic to retain the required number of significant figures. All properties are computed in the units adopted by the International Formulating Committee on the Properties of Steam.[7] Options are available which permit the user of the program to select which properties are to be calculated, in which system of units (MKS or English), and how the output is to be edited. Nondimensional ratios have not been used in the calculations presented in this paper. However, the program contains subroutines to perform the conversions to nondimensional units. If desired, computed results may be plotted automatically by the program on a microfilm recorder.

COMPUTATIONAL RESULTS

The appendix to this paper consists of results obtained using the THERMOSPLINE program to fit data from International Skeleton Tables 1 and 2. The coefficients generated for each spline fit of the specific volume data are presented, followed by selected interpolated results. Saturation values are given first, followed by selected output for the supercritical region in the ranges from $400°C \leq t \leq 800°C$ and $0 \leq p \leq 1000$ bar. This output is a partial listing of the results obtained with the program and serves to illustrate the effectiveness of the method. Most of the printouts are self-explanatory; the heading "CAL VALUES" refers to the calculated values; "TAB VALUES" are those taken from the skeleton tables; "VOL DIFF" indicates the difference in absolute value between the calculated and tabular values; and "TOL" stands for tolerances specified in the skeleton tables. The specific volume v is given in the units cubic centimeters per gram; enthalpy h is in Joules per gram; and entropy is in Joules per kilogram in degrees Kelvin. The effectiveness of the spline fit as applied to the sparse data of the 1963 Skeleton Tables indicates that an improved formulation may be obtained by applying the method directly to experimental data.

ACKNOWLEDGMENTS

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APPENDIX A

REPRESENTATIVE COMPUTER OUTPUT FROM THE THERMOSPLINE PROGRAM

THERMOSPLINE: THE CALCULATION OF THERMODYNAMIC PROPERTIES FOR WATER SUBSTANCE

INPUT DATA DATE 32568 RUN C

SETTING OF OPTIONS

I1 = 0 I2 = 0 K4 = 0 K5 = 0 K6 = 1

K7 = -1 KK7= 0 K8 = 1 K9 = 0 K10= 0

VALUES OF PARAMETERS

JJ1 = 2 JJ2 = 44 N = 43 NN = 1 K1 = 1 K2 = 1

K3 = 1 K12 = 0 NSV = 79 PR = -0. TR = -0.

FT =273.16 FP = -0. TINC= 5.00 PINC= -0.

OUTPUT

SIGNIFICANT PART OF COEFFICIENTS $p_\Delta(T_s)$		SIGNIFICANT PART OF COEFFICIENTS of $v^f_\Delta(T_s)$		SIGNIFICANT PART OF COEFFI	
A =	-0.18384568D 01	A =	-0.14158208D 01	A =	-0.41913864D 03
B =	0.24384165D-01	B =	0.19898014D-01	B =	0.85681358D 01
C =	-0.10590862D-03	C =	-0.90042806D-04	C =	-0.12139044D-01
A(0) =	0.15142169D-06	A(0) =	0.13272659D-06	A(0) =	0.11590974D-04
A(1) =	0.39078788D-06	A(1) =	0.42727920D-06	A(1) =	0.11887334D-04
A(2) =	0.15628296D-06	A(2) =	0.15990161D-06	A(2) =	-0.89535711D-04
A(3) =	0.29459222D-06	A(3) =	0.31363948D-06	A(3) =	0.10628840D-03
A(4) =	0.32179488D-06	A(4) =	0.36071735D-06	A(4) =	-0.10489258D-03
A(5) =	0.40498386D-06	A(5) =	0.45705638D-06	A(5) =	0.10080588D-03
A(6) =	0.42516719D-06	A(6) =	0.47038848D-06	A(6) =	-0.12234070D-03
A(7) =	0.52562937D-06	A(7) =	0.65548877D-06	A(7) =	0.15124379D-03
A(8) =	0.49115142D-06	A(8) =	0.65830851D-06	A(8) =	-0.15071993D-03
A(9) =	0.63062999D-06	A(9) =	0.62069295D-06	A(9) =	0.61225826D-04
A(10) =	0.51577467D-06	A(10) =	0.10521949D-05	A(10) =	-0.79302962D-04
A(11) =	0.62662255D-06	A(11) =	0.58916084D-06	A(11) =	0.10983308D-03
A(12) =	0.76746993D-06	A(12) =	0.13376211D-05	A(12) =	0.30617308D-04
A(13) =	0.35365465D-06	A(13) =	0.66632699D-06	A(13) =	-0.20360085D-03
A(14) =	0.81794312D-06	A(14) =	0.12709235D-05	A(14) =	0.22720086D-03
A(15) =	0.57451447D-06	A(15) =	0.13980595D-05	A(15) =	-0.14664687D-03
A(16) =	0.38360010D-06	A(16) =	0.80987692D-06	A(16) =	0.54007192D-04
A(17) =	0.12923785D-05	A(17) =	0.24866652D-05	A(17) =	0.34657160D-04
A(18) =	-0.12553272D-05	A(18) =	-0.40127247D-06	A(18) =	-0.18188974D-03
A(19) =	0.33317232D-05	A(19) =	0.46884011D-05	A(19) =	0.31763797D-03
A(20) =	-0.18744935D-05	A(20) =	0.55253663D-06	A(20) =	-0.30170854D-03
A(21) =	0.11689582D-05	A(21) =	0.54228076D-06	A(21) =	0.11318788D-03
A(22) =	0.11966828D-05	A(22) =	0.81782183D-05	A(22) =	0.20203301D-04
A(23) =	0.44990806D-07	A(23) =	-0.28420009D-05	A(23) =	-0.45987586D-05
A(24) =	0.62365159D-06	A(24) =	0.10852048D-04	A(24) =	-0.92600314D-04
A(25) =	0.46021996D-06	A(25) =	0.85975382D-06	A(25) =	0.45358558D-04
A(26) =	0.53488078D-06	A(26) =	0.10498687D-04	A(26) =	-0.20446448D-04
A(27) =	0.24015370D-05	A(27) =	0.78760762D-05	A(27) =	-0.72221251D-05
A(28) =	-0.11426147D-05	A(28) =	0.22947726D-04	A(28) =	-0.77157230D-04
A(29) =	0.21708610D-05	A(29) =	0.20465920D-05	A(29) =	-0.62346623D-04
A(30) =	0.45546540D-06	A(30) =	0.76583804D-04	A(30) =	-0.50720580D-04
A(31) =	0.70167824D-05	A(31) =	-0.78325050D-04	A(31) =	0.27503895D-03
A(32) =	-0.35505181D-05	A(32) =	0.64984971D-03	A(32) =	-0.76017290D-03
A(33) =	0.82716296D-05	A(33) =	-0.22510884D-02	A(33) =	-0.56213667D-03
A(34) =	-0.57971233D-05	A(34) =	0.10166726D-01	A(34) =	-0.25231100D-03
A(35) =	0.85699167D-04	A(35) =	-0.34626203D-01	A(35) =	-0.65474662D-02
A(36) =	-0.22516305D-03	A(36) =	0.15511116D 00	A(36) =	-0.22415398D-01
A(37) =	0.85833949D-02	A(37) =	-0.29106583D 01	A(37) =	0.33635160D 00
A(38) =	-0.10581228D-01	A(38) =	-0.66869048D 01	A(38) =	-0.42678658D 00
A(39) =	0.59314028D-01	A(39) =	0.10789438D 02	A(39) =	-0.55990273D 01
A(40) =	-0.52423039D 00	A(40) =	0.28205386D 03	A(40) =	-0.79986892D 02
A(41) =	0.31061104D 02	A(41) =	-0.26402735D 05	A(41) =	0.42132846D 04

1. THE VAPOR-PRESSURE CURVE

OUTPUT FOR SATURATION VALUES

TEMPERATURE CENTIGRADE	Kelvin	CAL PRES	TAB PRES	PRES DIFF	TOLERANCE
0.01	273.16	0.006112	0.006112	0.000000	0.000006
10.00	283.15	0.012271	0.012271	0.000000	0.000010
15.00	288.15	0.017048			
20.00	293.15	0.023368	0.023368	0.000000	0.000020
25.00	298.15	0.031657			
30.00	303.15	0.042418	0.042418	0.000000	0.000030
35.00	308.15	0.056213			
40.00	313.15	0.073750	0.073750	0.000000	0.000038
45.00	318.15	0.095814			
50.00	323.15	0.123350	0.123350	0.000000	0.000060
55.00	328.15	0.157396			
60.00	333.15	0.199190	0.199190	0.000000	0.000100
65.00	338.15	0.250076			
70.00	343.15	0.311610	0.311610	0.000000	0.000160
75.00	348.15	0.385465			
80.00	353.15	0.473580	0.473580	0.000000	0.000240
85.00	358.15	0.578018			
90.00	363.15	0.701090	0.701090	0.000000	0.000360
95.00	368.15	0.845245			
100.00	373.15	1.013250	1.013250	0.000000	0.
105.00	378.15	1.208013			
110.00	383.15	1.432700	1.432700	0.000000	0.001000
115.00	388.15	1.690622			
120.00	393.15	1.985400	1.985400	0.000000	0.001300
125.00	398.15	2.320833			
130.00	403.15	2.701100	2.701100	0.000000	0.001600
135.00	408.15	3.130523			

1. THE VAPOR-PRESSURE CURVE (Continued)

140.00	413.15	3.613600	3.613600	0.000000	0.002100
145.00	418.15	4.154975			
150.00	423.15	4.759700	4.759700	0.000000	0.003200
155.00	428.15	5.433004			
160.00	433.15	6.180400	6.180400	0.000000	0.004200
165.00	438.15	7.007523			
170.00	443.15	7.920200	7.920200	0.000000	0.005300
175.00	448.15	8.924465			
180.00	453.15	10.027000	10.027000	0.000000	0.007000
185.00	458.15	11.234492			
190.00	463.15	12.553000	12.553000	0.000000	0.008060
195.00	468.15	13.988841			
200.00	473.15	15.550000	15.550000	0.000000	0.008000
205.00	478.15	17.244643			
210.00	483.15	19.080000	19.080000	0.000000	0.008000
215.00	488.15	21.063210			
220.00	493.15	23.202000	23.202000	0.	0.009000
225.00	498.15	25.504090			
230.00	503.15	27.979000	27.979000	0.	0.010000
235.00	508.15	30.634604			
240.00	513.15	33.480000	33.480000	0.	0.012000
245.00	518.15	36.524067			
250.00	523.15	39.776000	39.776000	0.	0.013000
255.00	528.15	43.245125			
260.00	533.15	46.941000	46.941000	0.	0.015000
265.00	538.15	50.873308			
270.00	543.15	55.052000	55.052000	0.000000	0.017000
275.00	548.15	59.487391			
280.00	553.15	64.191000	64.191000	0.	0.020000
285.00	558.15	69.174501			

1. THE VAPOR-PRESSURE CURVE (Continued)

290.00	563.15	74.448999	74.449000	0.000001	0.022000
295.00	568.15	80.025726			
300.00	573.15	85.917000	85.917000	0.	0.024000
305.00	578.15	92.135467			
310.00	583.15	98.693999	98.694000	0.000001	0.030000
315.00	588.15	105.606406			
320.00	593.15	112.890003	112.890000	0.000003	0.030000
325.00	598.15	120.562540			
330.00	603.15	128.639990	128.639999	0.000010	0.040000
335.00	608.15	137.138920			
340.00	613.15	146.080029	146.080000	0.000029	0.040000
345.00	618.15	155.484324			
350.00	623.15	165.369919	165.370001	0.000082	0.040000
355.00	628.15	175.764912			
360.00	633.15	186.740250	186.740000	0.000250	0.050000
365.00	638.15	198.349449			
370.00	643.15	210.533445	210.530001	0.003445	0.050000
371.00	644.15	213.040127	213.059999	0.019873	0.100000
372.00	645.15	215.606535	215.629999	0.023464	0.110000
373.00	646.15	218.289879	218.200001	0.089878	0.100000
374.00	647.15	220.849812	220.900000	0.050188	0.100000
374.15	647.30	221.199999	221.200001	0.000002	0.100000

2. SPECIFIC VOLUME OF SATURATED WATER

TEMPERATURE CENTIGRADE	Kelvin	CAL VOL(F)	TAB VOL(F)	VOL DIFF	TOLERANCE
0.01	273.16	1.000210	1.000210	0.000000	0.000050
10.00	283.15	1.000400	1.000400	0.000000	0.000100
15.00	288.15	1.001008			
20.00	293.15	1.001800	1.001800	0.000000	0.000100
25.00	298.15	1.002949			
30.00	303.15	1.004400	1.004400	0.000000	0.000100
35.00	308.15	1.006056			
40.00	313.15	1.007900	1.007900	0.000000	0.000100
45.00	318.15	1.009899			
50.00	323.15	1.012100	1.012100	0.000000	0.000200
55.00	328.15	1.014493			
60.00	333.15	1.017099	1.017100	0.000001	0.000200
65.00	338.15	1.019881			
70.00	343.15	1.022800	1.022800	0.000000	0.000200
75.00	348.15	1.025824			
80.00	353.15	1.029000	1.029000	0.000000	0.000300
85.00	358.15	1.032341			
90.00	363.15	1.035900	1.035900	0.000000	0.000300
95.00	368.15	1.039650			
100.00	373.15	1.043500	1.043500	0.000000	0.000300
105.00	378.15	1.047410			
110.00	383.15	1.051500	1.051500	0.000000	0.000400
115.00	388.15	1.055812			
120.00	393.15	1.060300	1.060300	0.000000	0.000400
125.00	398.15	1.064918			
130.00	403.15	1.069700	1.069700	0.000000	0.000400
135.00	408.15	1.074658			
140.00	413.15	1.079800	1.079800	0.000000	0.000400
145.00	418.15	1.085114			

2. SPECIFIC VOLUME OF SATURATED WATER (Continued)

150.00	423.15	1.090600	1.090600	0.	0.000400
155.00	428.15	1.096253			
160.00	433.15	1.102100	1.102100	0.000000	0.000400
165.00	438.15	1.108149			
170.00	443.15	1.114400	1.114400	0.	0.000400
175.00	448.15	1.120843			
180.00	453.15	1.127500	1.127500	0.	0.000400
185.00	458.15	1.134379			
190.00	463.15	1.141500	1.141500	0.000000	0.000400
195.00	468.15	1.148871			
200.00	473.15	1.156500	1.156500	0.000000	0.000400
205.00	478.15	1.164392			
210.00	483.15	1.172600	1.172600	0.000000	0.000400
215.00	488.15	1.181145			
220.00	493.15	1.190000	1.190000	0.000000	0.000400
225.00	498.15	1.199154			
230.00	503.15	1.208700	1.208700	0.000000	0.000400
235.00	508.15	1.218692			
240.00	513.15	1.229100	1.229100	0.000000	0.000400
245.00	518.15	1.239905			
250.00	523.15	1.251200	1.251200	0.000000	0.000400
255.00	528.15	1.263055			
260.00	533.15	1.275500	1.275500	0.000000	0.000400
265.00	538.15	1.288555			
270.00	543.15	1.302300	1.302300	0.000000	0.000400
275.00	548.15	1.316797			
280.00	553.15	1.332100	1.332100	0.000000	0.000400
285.00	558.15	1.348274			
290.00	563.15	1.365500	1.365500	0.000000	0.000500
295.00	568.15	1.383921			

2. SPECIFIC VOLUME OF SATURATED WATER (Continued)

300.00	573.15	1.403599	1.403600	0.000001	0.000700
305.00	578.15	1.424649			
310.00	583.15	1.447504	1.447500	0.000004	0.000700
315.00	588.15	1.472431			
320.00	593.15	1.499190	1.499200	0.000010	0.000700
325.00	598.15	1.528134			
330.00	603.15	1.562030	1.562000	0.000030	0.001000
335.00	608.15	1.601435			
340.00	613.15	1.638921	1.639000	0.000079	0.001000
345.00	618.15	1.675096			
350.00	623.15	1.741209	1.741000	0.000209	0.001000
355.00	628.15	1.843629			
360.00	633.15	1.893525	1.894000	0.000475	0.004000
365.00	638.15	1.898173			
370.00	643.15	2.233446	2.220000	0.013446	0.020000
371.00	644.15	2.361371	2.290000	0.071371	0.020000
372.00	645.15	2.408316	2.380000	0.028316	0.030000
373.00	646.15	2.196283	2.510000	0.313717	0.040000
374.00	647.15	3.001092	2.800000	0.201092	0.150000
374.15	647.30	3.170000	3.170000	0.	0.150000

3. SPECIFIC VOLUME OF SATURATED STEAM

TEMPERATURE CENTIGRADE	Kelvin	CAL VOL(G)	TAB VOL(G)	VOL DIFF	TOLERANCE
0.01	273.16	206144.490234	206146.000000	1.509766	210.000000
10.00	283.15	106422.695313	106422.000000	0.695313	110.000000
15.00	288.15	77931.199219			
20.00	293.15	57835.866699	57836.000000	0.133301	58.000000
25.00	298.15	43410.542480			
30.00	303.15	32929.020508	32929.000000	0.020508	33.000000
35.00	308.15	25246.946045			
40.00	313.15	19545.982666	19546.000000	0.017334	19.000000
45.00	318.15	15276.621704			
50.00	323.15	12045.012451	12045.000000	0.012451	12.000000
55.00	328.15	9578.474365			
60.00	333.15	7677.595276	7677.599976	0.004700	7.700000
65.00	338.15	6201.450989			
70.00	343.15	5045.300842	5045.299988	0.000854	5.000000
75.00	348.15	4133.301270			
80.00	353.15	3408.299957	3408.299988	0.000031	3.400000
85.00	358.15	2828.233856			
90.00	363.15	2360.899933	2360.899994	0.000061	2.400000
95.00	368.15	1982.080032			
100.00	373.15	1673.000015	1673.000000	0.000015	1.700000
105.00	378.15	1419.393417			
110.00	383.15	1210.099960	1210.100006	0.000046	1.200000
115.00	388.15	1036.481857			
120.00	393.15	891.710014	891.709999	0.000015	0.890000
125.00	398.15	770.424629			
130.00	403.15	668.320007	668.320000	0.000008	0.670000
135.00	408.15	581.986229			
140.00	413.15	508.659966	508.660000	0.000034	0.510000
145.00	418.15	446.120510			

3. SPECIFIC VOLUME OF SATURATED STEAM (Continued)

150.00	423.15	392.570030	392.570000	0.000031	0.390000
155.00	428.15	346.550400			
160.00	433.15	306.849964	306.849998	0.000034	0.310000
165.00	438.15	272.477974			
170.00	443.15	242.620026	242.620001	0.000025	0.240000
175.00	448.15	216.600733			
180.00	453.15	193.849981	193.850000	0.000019	0.190000
185.00	458.15	173.896889			
190.00	463.15	156.350006	156.350000	0.000006	0.160000
195.00	468.15	140.876131			
200.00	473.15	127.190009	127.190000	0.	0.130000
205.00	478.15	115.053717			
210.00	483.15	104.264997	104.265000	0.000003	0.104000
215.00	488.15	94.650501			
220.00	493.15	86.062003	86.062000	0.000003	0.086000
225.00	498.15	78.372498			
230.00	503.15	71.471997	71.472000	0.000003	0.071000
235.00	508.15	65.266206			
240.00	513.15	59.674002	59.674000	0.000002	0.060000
245.00	518.15	54.624835			
250.00	523.15	50.055999	50.056000	0.000001	0.050000
255.00	528.15	45.913030			
260.00	533.15	42.149000	42.149000	0.000000	0.042000
265.00	538.15	38.723019			
270.00	543.15	35.599000	35.599000	0.	0.036000
275.00	548.15	32.745139			
280.00	553.15	30.133000	30.133000	0.	0.030000
285.00	558.15	27.737647			
290.00	563.15	25.537000	25.537000	0.	0.030000
295.00	568.15	23.511417			

3. SPECIFIC VOLUME OF SATURATED STEAM (Continued)

300.00	573.15	21.643001	21.643000	0.000001	0.035000
305.00	578.15	19.915911			
310.00	583.15	18.315997	18.316000	0.000003	0.035000
315.00	588.15	16.831014			
320.00	593.15	15.451004	15.451000	0.000004	0.035000
325.00	598.15	14.166660			
330.00	603.15	12.966993	12.967000	0.000007	0.035000
335.00	608.15	11.841430			
340.00	613.15	10.779020	10.779000	0.000020	0.035000
345.00	618.15	9.769915			
350.00	623.15	8.804909	8.805000	0.000090	0.035000
355.00	628.15	7.871754			
360.00	633.15	6.943293	6.943000	0.000293	0.040000
365.00	638.15	5.980313			
370.00	643.15	4.899472	4.930000	0.030528	0.100000
371.00	644.15	4.663669	4.680000	0.016331	0.100000
372.00	645.15	4.426591	4.400000	0.026591	0.110000
373.00	646.15	4.161135	4.050000	0.111135	0.120000
374.00	647.15	3.378785	3.470000	0.091215	0.120000
374.15	647.30	3.170000	3.170000	0.	0.150000

4. SUPERHEATED STEAM ($400^\circ C \leq t \leq 800^\circ C; 0 \leq p \leq 1000$ bars):

a. Specific Volume


```
                 NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER,APPLIED MATHEMATICS LABORATORY, NOV    1967

                           THE CALCULATION OF THERMODYNAMIC PROPERTIES FOR WATER SUBSTANCES

INPUT DATA    DATE  111367    RUN 305
```

IST RANGE: $0 \le p \le 1000$ (bar)
$400 \le T \le 800$ (^{o}C)

```
                                          SETTING OF OPTIONS
                                          ------------------

                        I1 =   0    I2 =   1    K4 =   0    K5 =   0    K6 =   0

                        K7 =   0    KK7=   0    K8 =   1    K9 =   0    K10=   0

                                          VALUES OF PARAMETERS
                                          --------------------

          JJ1 =   2     JJ2 =  31      N =  30      NN =   1      K1 =  11      K2 =  21

           K3 =  11     K12 =  71    NSV =   0                    PR =  0.      TR =  0.

           FT =400.00    FP =  0.    TINC=  0.    PINC= 15.00

THE AB ARRAY (PRESSURE VALUES(BARS))
   0.        1.C000    5.0000  10.0000  25.0000  50.0000  75.0000 100.C000 125.0000 150.0000 175.0000 20C.COC0 225.0000 250.0000
 275.CC00 3CC.COC0 350.0000 400.C00C 45C.C000 500.C000 550.0000 600.C00C 65C.0000 70C.CC00 75C.C000 8C0.CCC0 850.0000 9C0.C000
 950.00001000.COC0

THE OR ARRAY(SPECIFIC VOLUME VALUES AT 400.DEGREES CENTIGRADE)
3106.65463103.CCC03086.00003065.00003000.00002888.CCC02768.25002640.C0002501.25002347.5CCC218C.5C001990.C0C01768.500015CC.CCC0
1152.25C0 846.CCC0 738.85C0 764.ECOC 811.8C0C 865.5CC0 922.35C0 980.40001035.35001098.3C001157.25001215.2CC01273.3C001332.CCC0
1389.85001447.C0C0
```

SIGNIFICANT PART OF COEFFICIENTS of $v_{\Delta}(p)$ for T = 400^{o}C

```
 A     =   0.31C67280D 04
 B     =  -0.37336072D 01
 C     =  -0.12183419D 00
 A(0) =    0.65099436D-02
 A( 1) =   0.38733783D-02
 A( 2) =  -0.12574505D-01
 A( 3) =   0.23974505D-02
 A( 4) =  -0.28718056D-03
 A( 5) =   0.92329190D-04
 A( 6) =  -0.23369941D-04
 A( 7) =  -0.80287817D-04
 A( 8) =   0.18542831D-03
 A( 9) =  -0.26173981D-03
 A(10) =   0.93603987D-04
 A(11) =   0.63271199D-04
 A(12) =  -0.89067615D-03
 A(13) =   0.24595215D-02
 A(14) =   0.84449938D-03
 A(15) =  -0.38415351D-02
 A(16) =   0.15932619D-02
 A(17) =  -0.22351266D-03
```

4. SUPERHEATED STEAM ($400°C \leq t \leq 800°C$; $0 \leq p \leq 1000$ bars): (Continued)

a. Specific Volume: Coefficients (Continued)

```
A(18) =   0.82412050D-04
A(19) =  -0.19746123D-04
A(20) =   0.93760984D-05
A(21) =  -0.45590972D-05
A(22) =   0.40602156D-05
A(23) =  -0.44816116D-05
A(24) =   0.58662625D-05
A(25) =  -0.17834919D-05
A(26) =  -0.43323029D-05
A(27) =   0.39125902D-05
A(28) =   0.14821392D-05
THE OR ARRAY(SPECIFIC VOLUME VALUES AT 425.DEGREES CENTIGRADE)
3222.03213218.C0003203.00003184.00003125.C0003026.50C02922.0C002812.C0002695.CC002571.CC002437.75002294.00002139.75001972.5C00
1787.50001585.40C01200.5C001C18.40OC 985.95C01005.CC001042.8CC01089.60001141.40001194.2C001248.75CC1304.8CC01361.7CC01418.4C00
1474.40001530.CC00
SIGNIFICANT PART OF COEFFICIENTS of v_Δ(p) for T = 425°C
 A     =   0.32219934D 04
 B     =  -0.39907051D 01
 C     =   0.59125364D-01
 A(0)  =  -0.37712884D-02
 A( 1) =  -0.11535548D-02
 A( 2) =   0.49657159D-02
 A( 3) =   0.28714025D-03
 A( 4) =  -0.48158235D-03
 A( 5) =   0.21067629D-03
 A( 6) =  -0.1C037662D-03
 A( 7) =   0.63541704D-04
 A( 8) =  -0.58249521D-04
 A( 9) =   0.25689701D-04
 A(10) =   0.19355489D-04
 A(11) =  -0.22965755D-04
 A(12) =  -0.87684233D-04
 A(13) =   0.22992287D-03
 A(14) =  -0.23062694D-03
 A(15) =   0.59920234D-03
 A(16) =  -0.69832791D-03
 A(17) =   0.64950155D-04
 A(18) =   0.11051838D-03
 A(19) =   0.16180210D-04
 A(20) =   0.87595168D-05
 A(21) =  -0.52181555D-05
 A(22) =   0.12113390D-04
 A(23) =  -0.52357191D-05
 A(24) =   0.82958255D-06
 A(25) =  -0.12824690D-05
 A(26) =   0.11001908D-05
 A(27) =   0.12815688D-05
 A(28) =   0.17377226D-06
THE OR ARRAY(SPECIFIC VOLUME VALUES AT 450.DEGREES CENTIGRADE)
3337.40963334.C0003320.50003303.00003250.C0003162.00003069.75002973.00002872.50002767.50002658.25002542.C0002421.00002292.5C00
2158.75C0202C.8CC01734.6C001474.40OC1312.20001246.CCC01234.75CC1251.COCC1284.4CC01324.4CCC1371.CCCC142C.CCCC1471.35CC1523.7CCC
1577.CCCC1630.CCCO
SIGNIFICANT PART OF COEFFICIENTS of v_Δ(p) for T = 450°C
 A     =   0.33373995D 04
 B     =  -0.33877372D 01
 .
 .
 .
```

etc. for all coefficients for each isotherm specified

4. SUPERHEATED STEAM (400°C $\leq t \leq$ 800°C; 0 $\leq p \leq$ 1000 bars): (Continued)

a. Specific Volume: Coefficients (Continued)

```
A(21) =  -0.10736954D-04
A(22) =   0.55360036D-04
A(23) =  -0.78703655D-04
A(24) =   0.67456055D-04
A(25) =  -0.35122084D-04
A(26) =   0.1[illegible]333352D-05
A(27) =   0.14987827D-04
A(28) =   0.30158002D-05
THE OR ARRAY(SPECIFIC VOLUME VALUES AT 800.DEGREES CENTIGRADE)
4952.69454952.COC04947.99994942.99994930.00004905.00004881.75C04857.59994834.59994813.50C04791.50004769.59994747.49994727.5C(
4705.24994686.COC04644.49994607.59994571.59994535.CCCC4504.5CCC4475.59994452.5CO4423.59994402.5CCC4384.CCC4368.99994355.5999
4341.49994339.5999
```

SIGNIFICANT PART OF COEFFICIENTS of $v_{\Delta}(p)$ for T = 800°C

```
A     =   0.49527371D 04
B     =  -0.74398323D 00
C     =  -0.59100328D-01
A(0)  =   0.35689139D-02
A( 1) =   0.64115138D-03
A( 2) =  -0.34401059D-02
A( 3) =  -0.12761955D-02
A( 4) =   0.69277095D-03
A( 5) =  -0.27902174D-03
A( 6) =   0.13214638D-03
A( 7) =  -0.26000216D-04
A( 8) =  -0.59817439D-04
A( 9) =   0.89036432D-04
A(10) =  -0.10416182D-03
A(11) =   0.16747730D-03
A(12) =  -0.24561993D-03
A(13) =   0.28688459D-03
A(14) =  -0.26183951D-03
A(15) =   0.14895248D-03
A(16) =  -0.51206280D-04
A(17) =   0.12603917D-04
A(18) =   0.24791367D-04
A(19) =  -0.39771567D-04
A(20) =   0.38296907D-04
A(21) =  -0.53416541D-04
A(22) =   0.71368920D-04
A(23) =  -0.56059101D-04
A(24) =   0.24867943D-04
A(25) =  -0.74136259D-05
A(26) =  -0.11212352D-04
A(27) =   0.36262768D-04
A(28) =   0.10160443D-04
GENERAL COEF ISO  673.15

  0.31067280D 04
 -0.37336073D 01
 -0.12183423D 00
  0.65099491D-02
  0.38733819D-02
```

4. SUPERHEATED STEAM (400°C ≤ t ≤ 800°C; 0 ≤ p ≤ 1000 bars): (Continued)

a. Specific Volume: Coefficients (Continued)

-0.12574522D-01

0.23974582D-02

-0.28718098D-03

0.92329072D-04

-0.23369865D-04

-0.80287836D-04

0.18542818D-03

-0.26173940D-03

0.93603424D-04

0.63271955D-04

-0.89067654D-03

0.24595204D-02

0.84449775D-03

-0.38415295D-02

0.15932575D-02

-0.22351145D-03

0.82412430D-04

-0.19746236D-04

0.93759559D-05

-0.45589723D-05

0.40601199D-05

-0.44815461D-05

0.58662171D-05

-0.17834784D-05

-0.43323029D-05

0.39126044D-05

0.14821439D-05

a. Specific Volume: Interpolated Values

OUTPUT

NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER,APPLIED MATHEMATICS LABORATORY, NOV 1967

TEMPERATURE = 400. DEGREES CENTIGRADE

PRESS (BARS)	CAL VOL	TAB VOL	VOL DIFF	TOLERANCE
0.	3106.7280	3106.6546	0.0735	2.000
2.	1549.4147			
3.	1031.5458			
4.	772.5914			
5.	617.2151	617.2000	0.0152	0.200
20.	151.0964			
35.	84.4520			
50.	57.7600	57.7600	0.0000	0.080
65.	43.3413			
80.	34.2911			
95.	28.0669			
110.	23.5099			
125.	20.0100	20.0100	0.0000	0.040
140.	17.2185			
155.	14.9377			
170.	13.0317			
185.	11.3929			
200.	9.9500	9.9500	0.0000	0.030
215.	8.6559			
230.	7.4789			
245.	6.3690			
260.	5.2571			
275.	4.1900	4.1900	0.0000	0.020
290.	3.2783			
305.	2.6475			
320.	2.3186			

PRESS (BARS)	CAL VOL	TAB VOL	VOL DIFF	TOLERANCE
335.	2.1755			
350.	2.1110	2.1110	0.0000	0.007
365.	2.0502			
380.	1.9868			
395.	1.9289			
410.	1.8828			
425.	1.8481			
440.	1.8205			
455.	1.7960			
470.	1.7727			
485.	1.7510			
500.	1.7310	1.7310	0.0000	0.005
515.	1.7130			
530.	1.6968			
545.	1.6818			
560.	1.6677			
575.	1.6544			
590.	1.6419			
605.	1.6302			
620.	1.6192			
635.	1.6088			
650.	1.5990	1.5990	0.0000	0.004
665.	1.5895			
680.	1.5804			
695.	1.5718			
710.	1.5635			
725.	1.5556			
740.	1.5480			
755.	1.5405			
770.	1.5331			

PRESS (BARS)	CAL VOL	TAB VOL	VOL DIFF	TOLERANCE
785.	1.5259			
800.	1.5190	1.5190	0.0000	0.003
815.	1.5123			
830.	1.5060			
845.	1.4999			
860.	1.4942			
875.	1.4887			
890.	1.4834			
905.	1.4782			
920.	1.4731			
935.	1.4680			
950.	1.4630	1.4630	0.0000	0.003
965.	1.4580			
980.	1.4532			
995.	1.4485			

GENERAL COEF ISO 698.15

0.32219934D 04
-0.39907063D 01
0.59126081D-01
-0.37713439D-02
-0.11535847D-02
0.49658397D-02
0.28710003D-03
-0.48158108D-03
0.21067721D-03
-0.10037731D-03
0.63542266D-04
-0.58249557D-04
0.25688477D-04
0.19357743D-04

a. Specific Volume: Interpolated Values (Continued)

OUTPUT

NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER, APPLIED MATHEMATICS LABORATORY, NOV 1967

TEMPERATURE = 800. DEGREES CENTIGRADE

PRESS (BARS)	CAL VOL	TAB VOL	VOL DIFF	TOLERANCE
0.	4952.7370	4952.6945	0.0425	2.000
2.	2475.5209			
3.	1650.0249			
4.	1237.2653			
5.	989.6053	989.6000	0.0053	0.200
20.	246.7140			
35.	140.5825			
50.	98.1000	98.1000	0.0000	0.080
65.	75.2442			
80.	60.9636			
95.	51.1871			
110.	44.0781			
125.	38.6800	38.6800	0.0000	0.070
140.	34.4435			
155.	31.0266			
170.	28.2109			
185.	25.8539			
200.	23.8500	23.8500	0.0000	0.050
215.	22.1223			
230.	20.6235			
245.	19.3127			
260.	18.1486			
275.	17.1100	17.1100	0.0000	0.040
290.	16.1846			
305.	15.3511			
320.	14.5928			

PRESS (BARS)	CAL VOL	TAB VOL	VOL DIFF	TOLERANCE
335.	13.9013			
350.	13.2700	13.2700	0.0000	0.030
365.	12.6929			
380.	12.1632			
395.	11.6747			
410.	11.2219			
425.	10.8008			
440.	10.4078			
455.	10.0401			
470.	9.6954			
485.	9.3724			
500.	9.0700	9.0700	0.0000	0.020
515.	8.7870			
530.	8.5212			
545.	8.2705			
560.	8.0331			
575.	7.8082			
590.	7.5953			
605.	7.3943			
620.	7.2042			
635.	7.0234			
650.	6.8500	6.8500	0.0000	0.020
665.	6.6829			
680.	6.5225			
695.	6.3693			
710.	6.2240			
725.	6.0860			
740.	5.9545			
755.	5.8286			
770.	5.7076			

PRESS (BARS)	CAL VOL	TAB VOL	VOL DIFF	TOLERANCE
785.	5.5914			
800.	5.4800	5.4800	0.0000	0.020
815.	5.3731			
830.	5.2705			
845.	5.1720			
860.	5.0772			
875.	4.9859			
890.	4.8975			
905.	4.8116			
920.	4.7282			
935.	4.6475			
950.	4.5700	4.5700	0.0000	0.020
965.	4.4960			
980.	4.4261			
995.	4.3607			

END OF JOB

b. Specific Enthalpy

NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER.APPLIED MATHEMATICS LABORATORY. JAN. 1968

INPUT RANGE OF $v_{\Delta}(p, T)$ SPLINE FIT: $1 \leq p \leq 1000$ (bar)

OUTPUT $400 \leq T \leq 800$ (^{o}C)

TEMPERATURE = 500. DEGREES CENTIGRADE

PRESS (BARS)	CAL ENTH	TAB ENTH	ENTH DIFF	TOLERANCE
1.	3488.00			
2.	3487.73			
3.	3487.03			
4.	3486.17			
5.	3485.24	3484.00	1.24	4.00
10.	3480.36	3478.00	2.36	4.00
25.	3464.98	3462.00	2.98	4.00
50.	3436.83	3434.00	2.83	4.00
75.	3407.17	3404.00	3.17	4.00
100.	3375.71	3374.00	1.71	4.00
125.	3343.13	3343.00	0.13	5.00
150.	3309.50	3310.00	0.50	5.00
175.	3274.57	3277.00	2.43	6.00
200.	3238.72	3241.00	2.28	6.00
225.	3201.49	3205.00	3.51	6.00
250.	3162.93	3167.00	4.07	6.00
275.	3123.50	3125.00	1.50	6.00
300.	3082.84	3084.00	1.16	6.00
325.	3040.25			
350.	2995.81	2998.00	2.19	6.00
375.	2950.21			
400.	2904.09	2906.00	1.91	6.00
425.	2857.65			
450.	2811.14	2813.00	1.86	6.00
475.	2765.24			
500.	2720.79	2723.00	2.22	6.00
525.	2678.58			
550.	2639.27	2641.00	1.73	8.00
575.	2603.25			
600.	2570.61	2571.00	0.39	8.00
625.	2541.12			
650.	2514.45	2514.00	0.45	8.00
675.	2490.35			
700.	2468.62	2468.00	0.62	8.00
725.	2449.02			
750.	2431.33	2430.00	1.33	8.00
775.	2415.33			
800.	2400.83	2399.00	1.83	8.00
825.	2387.67			
850.	2375.72	2373.00	2.72	8.00
875.	2364.82			
900.	2354.86	2351.00	3.86	8.00
925.	2345.73			
950.	2337.37	2333.00	4.37	8.00
975.	2329.72			
1000.	2322.74	2318.00	4.74	8.00

b. Specific Enthalpy (Continued)

NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER, APPLIED MATHEMATICS LABORATORY, JAN. 1968

INPUT RANGE OF $v_\Delta(p,T)$ SPLINE FIT: $1 \le p \le 1000$ (bar)

OUTPUT $400 \le T \le 800$ (^{o}C)

TEMPERATURE = 550. DEGREES CENTIGRADE

PRESS (BARS)	CAL ENTH	TAB ENTH	ENTH DIFF	TOLERANCE
1.	3596.00			
2.	3594.37			
3.	3593.20			
4.	3592.20			
5.	3591.26	3592.00	0.74	4.00
10.	3586.68	3587.00	0.32	5.00
25.	3572.73	3574.00	1.27	5.00
50.	3549.78	3550.00	0.22	5.00
75.	3525.79	3526.00	0.21	5.00
100.	3501.26	3501.00	0.26	6.00
125.	3476.08	3476.00	0.08	8.00
150.	3450.41	3450.00	0.41	8.00
175.	3424.07	3423.00	1.07	8.00
200.	3396.91	3396.00	0.91	8.00
225.	3369.15	3368.00	1.15	8.00
250.	3340.91	3339.00	1.91	8.00
275.	3311.68	3308.00	3.68	8.00
300.	3281.55	3278.00	3.55	8.00
325.	3251.20			
350.	3220.39	3216.00	4.39	8.00
375.	3189.02			
400.	3157.14	3153.00	4.14	8.00
425.	3124.89			
450.	3092.43	3088.00	4.43	8.00
475.	3059.89			
500.	3027.43	3023.00	4.43	8.00
525.	2995.24			
550.	2963.55	2960.00	3.55	8.00
575.	2932.53			
600.	2902.38	2900.00	2.38	8.00
625.	2873.40			
650.	2845.83	2844.00	1.83	8.00
675.	2819.73			
700.	2795.09	2793.00	2.09	8.00
725.	2771.91			
750.	2750.17	2748.00	2.17	8.00
775.	2729.81			
800.	2710.77	2709.00	1.77	8.00
825.	2692.96			
850.	2676.32	2674.00	2.32	8.00
875.	2660.78			
900.	2646.27	2644.00	2.27	8.00
925.	2632.72			
950.	2620.05	2618.00	2.05	8.00
975.	2608.19			
1000.	2597.07	2595.00	2.07	8.00

b. Specific Enthalpy (Continued)

NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER.APPLIED MATHEMATICS LABORATORY. JAN. 1968

INPUT RANGE OF $v_\Delta(p,T)$ SPLINE FIT: $1 \le p \le 1000$ (bar)

OUTPUT $400 \le T \le 800$ (^oC)

TEMPERATURE = 800. DEGREES CENTIGRADE

PRESS (BARS)	CAL ENTH	TAB ENTH	ENTH DIFF	TOLERANCE
1.	4159.00			
2.	4157.74			
3.	4156.13			
4.	4154.64			
5.	4153.42	4157.00	3.58	4.00
10.	4151.99	4155.00	3.01	6.00
25.	4146.85	4147.00	0.15	6.00
50.	4134.85	4136.00	1.15	6.00
75.	4128.82	4124.00	4.82	6.00
100.	4116.06	4112.00	4.06	8.00
125.	4105.31	4100.00	5.31	10.00
150.	4092.54	4089.00	3.54	10.00
175.	4079.32	4077.00	2.32	11.00
200.	4067.16	4065.00	2.16	11.00
225.	4055.42	4053.00	2.42	12.00
250.	4043.83	4041.00	2.83	13.00
275.	4032.42	4030.00	2.42	13.00
300.	4020.85	4018.00	2.85	13.00
325.	4009.15			
350.	3997.75	3994.00	3.75	13.00
375.	3986.70			
400.	3975.68	3971.00	4.68	14.00
425.	3964.16			
450.	3952.11	3948.00	4.11	13.00
475.	3940.39			
500.	3929.47	3925.00	4.47	14.00
525.	3918.53			
550.	3906.91	3902.00	4.91	14.00
575.	3895.27			
600.	3884.44	3879.00	5.44	13.00
625.	3874.39			
650.	3864.57	3857.00	7.57	15.00
675.	3854.33			
700.	3843.56	3836.00	7.56	16.00
725.	3833.15			
750.	3823.69	3814.00	9.69	17.00
775.	3814.51			
800.	3804.93	3793.00	11.93	18.00
825.	3795.41			
850.	3786.38	3773.00	13.38	19.00
875.	3777.40			
900.	3768.01	3753.00	15.01	20.00
925.	3758.61			
950.	3749.78	3734.00	15.78	20.00
975.	3741.61			
1000.	3733.43	3715.00	18.43	20.00

END OF JOB

c. Specific Entropy

TEMPERATURE = 550. DEGREES CENTIGRADE

PRESS (BARS)	CAL ENTROPY
1.	8.9680
1.	8.9680
5.	8.2201
10.	7.8961
25.	7.4606
50.	7.1200
75.	6.9109
100.	6.7556
125.	6.6293
150.	6.5213
175.	6.4255
200.	6.3383
225.	6.2577
250.	6.1822
275.	6.1103
300.	6.0411
325.	5.9750
350.	5.9110
375.	5.8487
400.	5.7879
425.	5.7284
450.	5.6703
475.	5.6134
500.	5.5579
525.	5.5039
550.	5.4514

PRESS (BARS)	CAL ENTROPY
575.	5.4007
600.	5.3517
625.	5.3049
650.	5.2605
675.	5.2183
700.	5.1785
725.	5.1408
750.	5.1053
775.	5.0718
800.	5.0402
825.	5.0104
850.	4.9823
875.	4.9557
900.	4.9306
925.	4.9068
950.	4.8843
975.	4.8629
1000.	4.8425

TEMPERATURE = 650. DEGREES CENTIGRADE

PRESS (BARS)	CAL ENTROPY
1.	9.2200
1.	9.2200
5.	8.4742
10.	8.1513
25.	7.7198
50.	7.3849
75.	7.1839
100.	7.0357
125.	6.9177
150.	6.8182
175.	6.7312
200.	6.6539
225.	6.5836
250.	6.5189
275.	6.4583
300.	6.4016
325.	6.3481
350.	6.2972
375.	6.2485
400.	6.2018
425.	6.1569
450.	6.1135
475.	6.0715
500.	6.0309
525.	5.9915
550.	5.9530

PRESS (BARS)	CAL ENTROPY
575.	5.9157
600.	5.8794
625.	5.8442
650.	5.8100
675.	5.7766
700.	5.7441
725.	5.7124
750.	5.6817
775.	5.6519
800.	5.6229
825.	5.5948
850.	5.5677
875.	5.5414
900.	5.5158
925.	5.4909
950.	5.4671
975.	5.4449
1000.	5.4250

APPENDIX B*

THE 1963 INTERNATIONAL SKELETON TABLES

TABLE 1

THERMODYNAMIC PROPERTIES OF SATURATED WATER AND SATURATED STEAM
(with tolerances)

Temperature °C	Pressure bar		Specific volume cm^3/g				Specific enthalpy J/g				Notes
			Water		Steam		Water		Steam		(1)
		±		±		±		±		±	
0	0.006 108	0.000 006	1.000 21	0.000 05	206 288	210	-0.041 6	0.000 4	2501	3	(2)
0.01	0.006 112	0.000 006	1.000 21	0.000 05	206 146	210	0.000 611	0.000 001	2501	3	
10	0.012 271	0.000 010	1.000 4	0.000 1	106 422	110	41.99	0.04	2519	3	
20	0.023 368	0.000 020	1.001 8	0.000 1	57 836	58	83.86	0.08	2538	2	
30	0.042 418	0.000 030	1.004 4	0.000 1	32 929	33	125.66	0.08	2556	2	
40	0.073 750	0.000 038	1.007 9	0.000 1	19 546	19	167.47	0.08	2574	2	
50	0.123 35	0.000 06	1.012 1	0.000 2	12 045	12	209.3	0.1	2592	2	
60	0.199 19	0.000 10	1.017 1	0.000 2	7 677.6	7.7	251.1	0.1	2609	2	
70	0.311 61	0.000 16	1.022 8	0.000 2	5 045.3	5.0	293.0	0.1	2626	2	
80	0.473 58	0.000 24	1.029 0	0.000 3	3 408.3	3.4	334.9	0.2	2643	2	
90	0.701 09	0.000 36	1.035 9	0.000 3	2 360.9	2.4	376.9	0.2	2660	2	
100	1.013 25		1.043 5	0.000 3	1 673.0	1.7	419.1	0.2	2676	2	(3)
110	1.432 7	0.001 0	1.051 5	0.000 4	1 210.1	1.2	461.3	0.2	2691	2	
120	1.985 4	0.001 3	1.060 3	0.000 4	891.71	0.89	503.7	0.2	2706	2	
130	2.701 1	0.001 6	1.069 7	0.000 4	668.32	0.67	546.3	0.3	2720	2	
140	3.613 6	0.002 1	1.079 8	0.000 4	508.66	0.51	589.1	0.3	2734	2	
150	4.759 7	0.003 2	1.090 6	0.000 4	392.57	0.39	632.2	0.3	2747	3	
160	6.180 4	0.004 2	1.102 1	0.000 4	306.85	0.31	675.5	0.3	2758	3	
170	7.920 2	0.005 3	1.114 4	0.000 4	242.62	0.24	719.1	0.4	2769	3	
180	10.027	0.007	1.127 5	0.000 4	193.85	0.19	763.1	0.4	2778	4	
190	12.553	0.008	1.141 5	0.000 4	156.35	0.16	807.5	0.4	2786	4	
200	15.550	0.008	1.156 5	0.000 4	127.19	0.13	852.4	0.4	2793	4	
210	19.080	0.008	1.172 6	0.000 4	104.265	0.104	897.7	0.4	2798	4	
220	23.202	0.009	1.190 0	0.000 4	86.062	0.086	943.7	0.4	2802	4	
230	27.979	0.010	1.208 7	0.000 4	71.472	0.071	990.3	0.5	2803	4	
240	33.480	0.012	1.229 1	0.000 4	59.674	0.060	1037.6	0.5	2803	4	
250	39.776	0.013	1.251 2	0.000 4	50.056	0.050	1085.8	0.5	2801	4	
260	46.941	0.015	1.275 5	0.000 4	42.149	0.042	1135.0	0.7	2796	4	
270	55.052	0.017	1.302 3	0.000 4	35.599	0.036	1185.2	0.8	2790	4	
280	64.191	0.020	1.332 1	0.000 4	30.133	0.030	1236.8	0.8	2780	4	
290	74.449	0.022	1.365 5	0.000 5	25.537	0.030	1290	1	2766	4	
300	85.917	0.024	1.403 6	0.000 7	21.643	0.035	1345	1	2749	4	
310	98.694	0.030	1.447 5	0.000 7	18.316	0.035	1402	2	2727	5	
320	112.89	0.03	1.499 2	0.000 7	15.451	0.035	1462	2	2700	6	
330	128.64	0.04	1.562	0.001	12.967	0.035	1526	2	2666	6	
340	146.08	0.04	1.639	0.001	10.779	0.035	1596	3	2623	7	
350	165.37	0.04	1.741	0.001	8.805	0.035	1672	3	2565	8	
360	186.74	0.05	1.894	0.004	6.943	0.040	1762	3	2481	8	
370	210.53	0.05	2.22	0.02	4.93	0.10	1892	6	2331	12	
371	213.06	0.10	2.29	0.02	4.68	0.10	1913	6	2305	14	
372	215.63	0.11	2.38	0.03	4.40	0.11	1937	9	2273	16	
373	218.2	0.1	2.51	0.04	4.05	0.12	1969	14	2230	18	
374	220.9	0.1	2.80	0.15	3.47	0.12	2032	20	2146	30	
374.15 ± 0.10	221.2	0.1	3.17	0.15	3.17	0.15	2095	30	2095	30	(4)

Notes: (1) The specific internal energy is made exactly zero for the liquid phase at the triple point (see also Appendix B).
(2) The states here shown are metastable.
(3) At a pressure of exactly 1.013 25 bar the saturation temperature has the exact assigned value of 100°C on the International Practical Scale of Temperature, 1948.
At a temperature of exactly 100°C on the Thermodynamic Celsius Scale the saturation pressure is 1.013 25 bar, with a tolerance of 0.000 04 bar.
(4) Near the critical point, the tolerances on the specific volume and on the specific enthalpy in the vapor phase are correlated with the corresponding tolerances in the liquid phase. The tolerances on the changes in specific volume and in specific enthalpy on evaporation tend to zero as the critical point is approached.

*These tables were reproduced from those given in Reference 12.

TABLE 2

SPECIFIC VOLUME OF COMPRESSED WATER AND SUPERHEATED STEAM (cm^3/g)

Note	Pressure bar	TEMPERATURE °C 0	50	100	150	200	250	300	350	375	400
(1)	1	1.0002	1.0121	1696	1936	2173	2406	2639	2871	2987	3103
		.0001	.0002	1	1	2	2	2	2	2	2
	5	0.9999	1.0119	1.0433	1.0906	425.1	474.4	522.5	570.1	593.7	617.2
		.0002	.0002	.0002	.0003	.4	.4	.4	.4	.4	.4
	10	0.9997	1.0117	1.0431	1.0903	206.0	232.7	257.9	282.4	294.5	306.5
		.0002	.0002	.0002	.0003	.3	.2	.2	.2	.2	.2
	25	0.9989	1.0110	1.0423	1.0894	1.1556	87.0	98.9	109.7	114.9	120.0
		.0002	.0002	.0002	.0003	.0003	.2	.1	.1	.1	.1
	50	0.9976	1.0099	1.0410	1.0878	1.1531	1.2495	45.34	51.93	54.90	57.76
		.0002	.0002	.0002	.0003	.0003	.0004	.07	.08	.09	.09
	75	0.9964	1.0088	1.0398	1.0862	1.1507	1.2452	26.71	32.44	34.75	36.91
		.0002	.0002	.0003	.0004	.0004	.0004	.05	.07	.08	.08
	100	0.9952	1.0077	1.0386	1.0846	1.1483	1.2409	1.397	22.44	24.53	26.40
		.0002	.0002	.0004	.0004	.0004	.0004	.001	.05	.05	.05
	125	0.9940	1.0066	1.0373	1.0830	1.1460	1.2367	1.387	16.14	18.25	20.01
		.0002	.0002	.0004	.0004	.0004	.0004	.001	.05	.04	.04
	150	0.9928	1.0055	1.0361	1.0813	1.1436	1.2327	1.378	11.49	13.91	15.65
		.0002	.0002	.0004	.0004	.0004	.0005	.001	.04	.04	.04
	175	0.9915	1.0044	1.0348	1.0798	1.1414	1.2288	1.369	1.716	10.57	12.46
		.0002	.0002	.0004	.0004	.0004	.0005	.001	.002	.04	.04
	200	0.9904	1.0033	1.0336	1.0782	1.1391	1.2251	1.360	1.665	7.68	9.95
		.0002	.0002	.0004	.0004	.0004	.0005	.001	.002	.03	.03
	225	0.9892	1.0023	1.0324	1.0766	1.1369	1.2215	1.352	1.630	2.49	7.86
		.0002	.0002	.0004	.0004	.0004	.0005	.001	.002	.04	.03
	250	0.9880	1.0012	1.0313	1.0751	1.1347	1.2179	1.345	1.600	1.98	6.00
		.0002	.0002	.0004	.0004	.0004	.0005	.001	.002	.02	.03
	275	0.9868	1.0002	1.0301	1.0736	1.1326	1.2144	1.338	1.576	1.865	4.19
		.0002	.0002	.0004	.0004	.0004	.0005	.001	.002	.010	.03
	300	0.9856	0.9992	1.0289	1.0721	1.1304	1.2111	1.331	1.555	1.797	2.82
		.0002	.0002	.0004	.0004	.0004	.0005	.001	.002	.008	.02
	350	0.9834	0.9972	1.0267	1.0692	1.1264	1.2046	1.319	1.519	1.705	2.111
		.0002	.0002	.0004	.0004	.0004	.0006	.001	.003	.006	.010
	400	0.9811	0.9951	1.0244	1.0664	1.1224	1.1984	1.308	1.489	1.644	1.912
		.0002	.0002	.0004	.0004	.0004	.0006	.001	.003	.005	.007
	450	0.9788	0.9932	1.0222	1.0636	1.1186	1.1925	1.297	1.464	1.599	1.804
		.0002	.0002	.0004	.0004	.0004	.0006	.001	.003	.005	.006
	500	0.9766	0.9912	1.0200	1.0609	1.1148	1.1868	1.288	1.443	1.564	1.731
		.0002	.0003	.0004	.0005	.0005	.0006	.001	.003	.005	.005
	550	0.9745	.09892	1.0178	1.0582	1.1111	1.1813	1.278	1.424	1.533	1.677
		.0003	.0003	.0004	.0005	.0005	.0006	.001	.003	.005	.005
	600	0.9723	0.9873	1.0157	1.0556	1.1075	1.1760	1.270	1.407	1.507	1.634
		.0003	.0003	.0004	.0005	.0006	.0006	.001	.003	.005	.004
	650	0.9703	0.9854	1.0137	1.0530	1.1040	1.1709	1.261	1.393	1.484	1.599
		.0003	.0003	.0004	.0005	.0005	.0007	.001	.003	.005	.004
	700	0.9682	0.9836	1.0116	1.0505	1.1006	1.1660	1.254	1.380	1.464	1.569
		.0003	.0003	.0004	.0005	.0005	.0007	.001	.003	.005	.004
	750	0.9662	0.9818	1.0096	1.0480	1.0973	1.1614	1.246	1.367	1.446	1.543
		.0003	.0003	.0004	.0005	.0005	.0008	.001	.003	.004	.004
	800	0.9642	0.9800	1.0076	1.0456	1.0941	1.1568	1.239	1.355	1.430	1.519
		.0003	.0003	.0004	.0005	.0005	.0008	.001	.003	.004	.003
	850	0.9622	0.9782	1.0057	1.0432	1.0910	1.1524	1.232	1.345	1.415	1.498
		.0003	.0003	.0004	.0005	.0005	.0008	.002	.004	.004	.003
	900	0.9603	0.9765	1.0038	1.0409	1.0879	1.1481	1.226	1.334	1.401	1.480
		.0003	.0003	.0004	.0005	.0005	.0009	.002	.004	.004	.003
	950	0.9584	0.9748	1.0019	1.0386	1.0848	1.1439	1.220	1.324	1.388	1.463
		.0003	.0003	.0004	.0005	.0005	.0010	.003	.004	.004	.003
	1000	0.9566	0.9731	1.0000	1.0363	1.0818	1.1398	1.214	1.314	1.376	1.447
		.0003	.0003	.0004	.0005	.0005	.0012	.003	.004	.004	.003

Note: (1) The entry shown for 0°C and 1 bar relates to a metastable liquid state. The Stable state is here solid.
Of each pair of figures the upper represents the adopted value and the lower the tolerance (±)

TABLE 2 (CONT.)

SPECIFIC VOLUME OF COMPRESSED WATER AND SUPERHEATED STEAM (cm^3/g)

TEMPERATURE °C 425	450	475	500	550	600	650	700	750	800	Pressure bar
3218	3334	3450	3565	3797	4028	4259	4490	4721	4952	1
2	2	2	2	2	2	2	2	2	2	
640.6	664.1	687.5	710.8	757.4	803.9	850.4	896.9	943.2	989.6	5
.4	.4	.4	.4	.4	.4	.4	.4	.4	.4	
318.4	330.3	342.2	354.0	377.5	401.0	424.4	447.7	471.1	494.3	10
.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	
125.0	130.0	135.0	139.9	149.6	159.2	168.8	178.3	187.7	197.2	25
.1	.1	.1	.1	.1	.1	.2	.2	.2	.2	
60.53	63.24	65.89	68.50	73.61	78.62	83.6	88.4	93.3	98.1	50
.09	.09	.09	.10	.10	.10	.1	.1	.1	.1	
38.96	40.93	42.83	44.69	48.28	51.76	55.16	58.52	61.82	65.09	75
.08	.08	.08	.08	.08	.08	.08	.08	.08	.08	
28.12	29.73	31.26	32.76	35.61	38.32	40.96	43.55	46.09	48.58	100
.05	.05	.06	.07	.07	.07	.08	.08	.08	.08	
21.56	22.98	24.31	25.59	27.99	30.26	32.44	34.56	36.64	38.68	125
.04	.04	.04	.05	.05	.05	.06	.06	.07	.07	
17.14	18.45	19.65	20.80	22.91	24.88	26.77	28.59	30.35	32.09	150
.04	.04	.04	.04	.04	.04	.05	.06	.06	.07	
13.93	15.19	16.31	17.36	19.28	21.04	22.71	24.31	25.86	27.38	175
.03	.03	.03	.04	.04	.04	.04	.05	.05	.06	
11.47	12.71	13.79	14.78	16.55	18.16	19.67	21.11	22.50	23.85	200
.03	.03	.03	.03	.03	.04	.04	.05	.05	.06	
9.51	10.76	11.81	12.76	14.42	15.92	17.31	18.62	19.88	21.10	225
.03	.03	.03	.03	.03	.03	.04	.05	.05	.05	
7.89	9.17	10.22	11.14	12.72	14.12	15.42	16.63	17.79	18.91	250
.02	.02	.02	.02	.02	.02	.03	.04	.05	.05	
6.50	7.85	8.90	9.79	11.32	12.65	13.86	15.00	16.08	17.11	275
.02	.02	.02	.02	.02	.02	.03	.03	.04	.04	
5.298	6.736	7.799	8.682	10.16	11.43	12.58	13.64	14.65	15.62	300
.020	.020	.020	.020	.02	.02	.02	.03	.03	.04	
3.430	4.956	6.054	6.928	8.340	9.516	10.56	11.52	12.42	13.27	350
.012	.014	.014	.015	.016	.018	.02	.03	.03	.04	
2.546	3.686	4.758	5.620	6.980	8.086	9.051	9.93	10.75	11.52	400
.009	.012	.012	.013	.014	.016	.018	.02	.03	.03	
2.191	2.916	3.814	4.628	5.934	6.982	7.885	8.70	9.45	10.16	450
.007	.009	.010	.010	.011	.013	.015	.02	.02	.03	
2.010	2.492	3.170	3.884	5.114	6.108	6.960	7.72	8.42	9.07	500
.006	.006	.008	.008	.010	.012	.014	.02	.02	.03	
1.896	2.245	2.750	3.342	4.464	5.404	6.209	6.93	7.58	8.19	550
.005	.005	.006	.007	.008	.010	.012	.02	.02	.02	
1.816	2.085	2.474	2.950	3.950	4.831	5.592	6.27	6.89	7.46	600
.004	.004	.005	.006	.008	.009	.011	.02	.02	.02	
1.756	1.976	2.283	2.672	3.543	4.360	5.080	5.72	6.31	6.85	650
.004	.004	.005	.005	.007	.008	.010	.02	.02	.02	
1.706	1.892	2.144	2.466	3.221	3.971	4.648	5.26	5.81	6.32	700
.004	.004	.004	.005	.006	.007	.009	.01	.02	.02	
1.665	1.828	2.040	2.310	2.965	3.648	4.283	4.86	5.39	5.87	750
.004	.004	.004	.005	.006	.007	.008	.01	.02	.02	
1.631	1.775	1.958	2.189	2.760	3.380	3.972	4.52	5.02	5.48	800
.003	.004	.004	.004	.006	.007	.008	.01	.01	.02	
1.602	1.731	1.892	2.092	2.594	3.155	3.706	4.22	4.70	5.14	850
.003	.003	.004	.004	.005	.006	.008	.01	.01	.02	
1.576	1.693	1.837	2.014	2.458	2.966	3.478	3.97	4.42	4.84	900
.003	.003	.004	.004	.005	.006	.007	.01	.01	.02	
1.552	1.660	1.790	1.948	2.344	2.806	3.282	3.74	4.17	4.57	950
.003	.003	.004	.004	.005	.006	.007	.01	.01	.02	
1.530	1.630	1.750	1.892	2.248	2.670	3.111	3.54	3.95	4.34	1000
.003	.003	.004	.004	.005	.005	.006	.01	.01	.02	

TABLE 3

SPECIFIC ENTHALPY OF COMPRESSED WATER AND SUPERHEATED STEAM (J/g)

Notes	Pressure bar	TEMPERATURE °C 0	50	100	150	200	250	300	350	375	400
(1)	0	2502	2595	2689	2784	2880	2978	3077	3178	3229	3280
		2	2	2	2	2	2	2	2	2	2
(2)	1	0.06	209.3	2676	2777	2876	2975	3074	3175	3227	3278
		.01	.1	2	2	2	2	3	3	3	3
	5	0.47	209.6	419.4	632.2	2857	2961	3064	3168	3220	3272
		.02	.2	.2	.3	3	3	4	4	4	4
	10	0.98	210.1	419.7	632.4	2830	2943	3051	3158	3211	3264
		.02	.2	.4	.4	4	3	4	4	4	4
	25	2.50	211.3	421.0	633.4	852.8	2881	3009	3126	3184	3240
		.05	.2	.4	.4	.4	5	5	4	4	4
	50	5.05	213.5	422.8	634.9	853.8	1085.8	2925	3068	3134	3196
		.10	.2	.4	.4	.4	.5	5	5	4	4
	75	7.58	215.7	424.7	636.5	855.0	1085.9	2814	3003	3079	3149
		.15	.2	.4	.4	.5	.5	6	5	4	4
	100	10.1	217.9	426.6	638.1	856.1	1086.0	1343	2924	3017	3098
		.2	.2	.4	.4	.5	.5	1	5	4	4
	125	12.6	220.0	428.5	639.7	857.2	1086.1	1340	2826	2946	3041
		.3	.2	.4	.4	.5	.6	1	6	6	5
	150	15.1	222.1	430.4	641.3	858.3	1086.3	1338	2692	2861	2978
		.3	.2	.4	.4	.5	.6	1	8	8	6
	175	17.6	224.3	432.3	642.9	859.5	1086.5	1336	1663	2755	2905
		.4	.3	.4	.4	.5	.6	1	3	8	6
	200	20.1	226.5	434.2	644.5	860.6	1086.8	1334	1646	2605	2819
		.4	.3	.4	.4	.6	.6	1	3	8	8
	225	22.6	228.6	436.1	646.1	861.8	1087.3	1332	1633	1980	2715
		.5	.3	.4	.4	.6	.7	1	3	12	8
	250	25.1	230.7	438.0	647.7	863.0	1087.7	1331	1623	1850	2580
		.5	.3	.4	.4	.6	.8	1	3	8	8
	275	27.5	232.8	439.9	649.3	864.2	1088.2	1330	1615	1814	2383
		.5	.3	.4	.4	.6	.8	1	3	8	8
	300	30.0	235.0	441.8	650.9	865.4	1088.7	1329	1609	1791	2157
		.5	.3	.4	.4	.6	.8	1	3	6	8
	350	34.9	239.2	445.6	654.1	867.9	1090	1327	1598	1762	1992
		.6	.3	.4	.4	.6	1	1	3	6	8
	400	39.7	243.5	449.4	657.4	870.4	1091	1325	1590	1743	1934
		.7	.3	.4	.4	.6	1	1	3	6	8
	450	44.6	247.7	453.2	660.7	873.0	1092	1324	1582	1729	1901
		.8	.4	.4	.4	.6	1	1	3	6	8
	500	49.3	252.0	457.0	664.0	875.6	1094	1324	1577	1717	1878
		.8	.4	.4	.4	.6	1	2	3	6	8
	550	54.1	256.2	460.8	667.3	878.4	1096	1323	1572	1709	1860
		.8	.4	.4	.4	.6	1	2	3	6	8
	600	58.8	260.4	464.6	670.6	881.1	1097	1323	1568	1702	1847
		.9	.4	.4	.4	.7	1	2	3	6	8
	650	63.5	264.6	468.4	674.0	883.8	1099	1323	1565	1696	1836
		1.0	.4	.4	.5	.8	1	2	3	6	8
	700	68.1	268.8	472.1	677.3	886.6	1101	1323	1562	1691	1828
		1.0	.5	.5	.5	.8	1	2	3	6	8
	750	72.7	273.0	476.0	680.7	889.3	1103	1324	1560	1687	1820
		1.1	.6	.5	.5	.9	1	2	4	6	8
	800	77.3	277.1	479.8	684.0	892.2	1105	1324	1559	1684	1814
		1.2	.7	.7	.7	.9	1	2	4	6	8
	850	81.9	281.3	483.6	687.4	895.0	1107	1325	1557	1681	1808
		1.2	.8	.8	.8	1.0	2	2	4	6	8
	900	86.5	285.4	487.3	690.8	898.0	1109	1326	1557	1678	1804
		1.2	.9	.9	.9	1.0	2	2	4	6	8
	950	91.1	289.6	491.2	694.2	900.9	1111	1327	1556	1676	1799
		1.2	1.0	1.0	1.0	1.3	2	3	5	6	8
	1000	95.7	293.7	495.0	697.6	903.8	1114	1328	1555	1674	1796
		1.2	1.2	1.2	1.2	1.5	2	3	5	6	8

Notes: (1) The specific internal energy is made exactly zero for the liquid phase at the triple point (see also Appendix B).

(2) The entry shown for 0°C and 1 bar relates to a metastable liquid state. The stable state is here solid.

Of each pair of figures the upper represents the adopted value and the lower the tolerance (±)

TABLE 3 (CONT.)

SPECIFIC ENTHALPY OF COMPRESSED WATER AND SUPERHEATED STEAM (J/g)

TEMPERATURE °C										
425	450	475	500	550	600	650	700	750	800	Pressure bar
3332	3384	3436	3489	3597	3706	3817	3929	4043	4159	0
2	2	2	2	3	3	4	4	4	4	
3330	3383	3435	3488	3596	3705	3816	3928	4043	4159	1
3	3	3	3	3	3	4	4	4	4	
3325	3377	3430	3484	3592	3702	3813	3926	4040	4157	5
4	4	4	4	4	4	4	4	4	4	
3317	3371	3425	3478	3587	3698	3810	3923	4038	4155	10
4	4	4	4	5	5	5	5	6	6	
3295	3350	3406	3462	3574	3686	3799	3914	4030	4147	25
4	4	4	4	5	5	5	6	6	6	
3257	3317	3375	3434	3550	3666	3782	3898	4016	4136	50
4	4	4	4	5	5	5	6	6	6	
3216	3280	3342	3404	3526	3645	3764	3883	4003	4124	75
4	4	4	4	5	6	6	6	6	6	
3172	3242	3309	3374	3501	3625	3747	3868	3990	4112	100
4	4	4	4	6	8	8	8	8	8	
3125	3201	3273	3343	3476	3604	3729	3852	3976	4100	125
4	4	4	5	8	10	10	10	10	10	
3073	3157	3235	3310	3450	3582	3711	3836	3962	4089	150
5	5	5	5	8	10	10	10	10	10	
3017	3111	3196	3277	3423	3560	3692	3821	3949	4077	175
6	6	6	6	8	10	10	11	11	11	
2955	3062	3155	3241	3396	3538	3673	3805	3935	4065	200
6	6	6	6	8	10	10	11	11	11	
2885	3009	3112	3205	3368	3515	3654	3789	3922	4053	225
6	6	6	6	8	10	10	11	11	12	
2807	2952	3066	3167	3339	3492	3635	3773	3908	4041	250
6	6	6	6	8	10	10	12	12	13	
2718	2890	3018	3125	3308	3467	3615	3757	3894	4030	275
6	6	6	6	8	10	10	12	13	13	
2614	2822	2967	3084	3278	3444	3596	3740	3880	4018	300
6	6	6	6	8	10	10	13	13	13	
2375	2672	2858	2998	3216	3396	3557	3708	3853	3994	350
6	6	6	6	8	10	10	13	13	13	
2203	2514	2741	2906	3153	3347	3518	3676	3826	3971	400
6	6	6	6	8	10	10	13	13	14	
2115	2380	2624	2813	3088	3298	3478	3643	3798	3948	450
6	6	6	6	8	10	10	13	13	13	
2064	2288	2522	2723	3023	3249	3439	3611	3771	3925	500
6	6	6	6	8	10	10	13	13	14	
2030	2228	2439	2641	2960	3200	3400	3579	3744	3902	550
8	8	8	8	8	10	10	13	13	14	
2005	2183	2378	2571	2900	3153	3362	3547	3718	3879	600
8	8	8	8	8	10	10	13	13	13	
1986	2151	2330	2514	2844	3107	3324	3516	3692	3857	650
8	8	8	8	8	10	10	13	13	15	
1971	2126	2294	2468	2793	3062	3288	3486	3666	3836	700
8	8	8	8	8	10	10	13	14	16	
1958	2106	2265	2430	2748	3021	3253	3456	3641	3814	750
8	8	8	8	8	10	10	13	15	17	
1948	2090	2241	2399	2709	298[illegible]	3219	3428	3617	3793	800
8	8	8	8	8	10	10	13	15	18	
1938	2077	2222	2373	2674	2948	3187	3400	3593	3773	850
8	8	8	8	8	10	10	13	16	19	
1932	2065	2206	2351	2644	2916	3157	3373	3570	3753	900
8	8	8	8	8	10	10	13	16	20	
1925	2056	2193	2333	2618	2887	3129	3348	3548	3734	950
8	8	8	8	8	10	10	13	16	20	
1920	2047	2181	2318	2595	2861	3103	3324	3527	3715	1000
8	8	8	8	8	10	10	13	16	20	

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13. ABSTRACT

A computer formulation of the thermodynamic properties of water in the supercritical region is described, based on an application of cubic spline functions to fit data given in the 1963 International Skeleton Table: Specific Volume of Compressed Water and Superheated Steam. A thermal equation of state is obtained in the form of a doubly cubic spline function, representing specific volume as a function of pressure and temperature. This expression is then used directly in the equations defining enthalpy, entropy, and the Gibbs free-enthalpy function to generate these thermodynamic properties over the indicated range of the International Tables. The cubic spline function has also been used to fit the saturation line as well as the specific volume data for saturated water and saturated steam. Double precision arithmetic is used to retain a sufficient number of significant figures. The accuracy of the computed output is well within the range of tolerances specified in the skeleton tables. In this report, the mathematical formulation is described, the computer program is outlined, and the computational results are presented.

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A SPLINE-FUNCTION METHOD FOR GENERATING THE THERMODYNAMIC PROPERTIES OF WATER SUBSTANCE, by Joanna W. Schot. (A paper presented to the Seventh International Conference on the Properties of Steam, Tokyo, Japan, September 1968.) Nov 1968. ii, 52p. UNCLASSIFIED

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