Report 3610

Effect of Damping Ratio on Longitudinal Dynamic Stability and Control of a Submarine

NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER

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DEPARTMENT OF THE NAVY · NAVAL SHIP R&D CENTER

EFFECT OF DAMPING RATIO ON LONGITUDINAL DYNAMIC STABILITY AND CONTROL OF A SUBMARINE

by

W. L. Stracke

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

NOTATION

a, b, c, d, A, B	Parameters of stability equation
I_y	Moment of inertia about y axis
Im $\sigma_{1,3}$	Imaginery part of complex root
M	Hydrodynamic pitching moment about y axis
$M_w{}'$, $M_{\dot{w}}{}'$, $M_q{}'$, $M_{\dot{q}}{}'$, $M_\theta{}'$, $M_{\delta s}{}'$	Pitching moment derivatives with respect to subscript
m	Mass of submarine
q	Absolute angular velocity about y axis
$\dot{q}$	Derivative with respect to time of q
Re $\sigma_{1,3}$	Real part of complex root
t	Time
w	Component of absolute linear velocity along Z axis
$\dot{w}$	Derivative with respect to time of w
Z	Component of hydrodynamic force along Z axis
$Z_w{}'$, $Z_{\dot{w}}{}'$, $Z_q{}'$, $Z_{\dot{q}}{}'$, $Z_{\delta s}{}'$	Derivatives of Z with respect to subscript
δ_s	Sternplane deflection
ζ	Damping ratio
θ	Pitch angle referred to X, Y, Z axes
$\theta_{1,3}$	Departure of θ from equilibrium value due to oscillatory part of transient variation
θ_ε	Equilibrium value of θ
θ_{o1}	Initial value of envelope term $\theta_{o1} e^{Re\sigma_{1,3} t}$
θ_1, θ_2, θ_3	Parameters of equation for pitch angle variation

$\sigma_1, \sigma_2, \sigma_3$	Stability roots; roots of characteristic equation corresponding to differential stability equation.
$\sigma_{1,3}$	Complex stability root
ω_n	Natural frequency in radians per second
γ	Constant of integration in Equation (20)

ABSTRACT

The effect of changing damping ratio upon vertical overshoot characteristics of a submarine are investigated in a simulator study based on model data. The change in damping ratio for the model is effected through a change in horizontal stabilizer surface at the stern. It was found that the optimum combination of overshoot characteristics occurs at values of damping ratio of from 0.6 to 1.0. This is also the range of values at which optimum stability is obtained.

INTRODUCTION

This project was sponsored as part of SSN Concept Formulation.[1] The general purpose of this report is to present to the submarine designer a criterion for dynamic longitudinal stability and to show that this criterion can be effectively applied in the selection of desirable handling qualities in the vertical plane. This criterion is "damping ratio," chosen because of its simplicity and because of its familiarity to technical personnel.

The main purpose of this report is to demonstrate the practical significance of damping ratio. This is done by showing the effect of varying damping ratio upon a simple maneuver, the vertical overshoot. For this purpose a simulator study was made for a model for which damping ratio was varied by changing the horizontal stern-stabilizer surface.

DESCRIPTION OF DAMPING RATIO

Damping ratio used in this report applies to the motion of a submerged submarine in the vertical plane, specifically variation in pitch. Damping ratio is described as a stability criterion for submarines in Reference 2. Subsequent to publication of this description, it has been regularly reported with the results of full-scale stability and control trials. Currently, values of damping ratio are also reported with the results of captive model stability and control studies.

A brief outline of the theory on which these applications are based is given in Appendix A. The equation for a damped oscillation with a single degree of freedom is shown to be effectively equivalent to the equation for pitch variation of a submerged submarine as derived from the equations of motion. For sternplanes fixed at neutral, the variations of pitch angle from the netural angle are thus described in Appendix A in Equation (12). The terms damping ratio ζ and undamped natural angular frequency ω_n are applicable to the pitch variation of a submerged submarine and may be used as criteria of pitch stability.

Damping ratio and natural angular frequency are significant whenever metacentric stability makes an effective contribution to longitudinal stability. This includes almost every practical case. The high speeds at which the effects of metacentric stability become unimportant are rarely attainable by an operational submarine.

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[1] References are listed on page 27.

Natural frequency is a function of metacentric height as well as of hydrodynamic characteristics. It is found to be similar for submarines of a given hull type and displacement. The differences in ω_n which have been found to occur between submarines have been insufficient to have a significant effect on pitch variation. The more significant criterion of pitch stability is thus the damping ratio. In practice damping ratio indicates definitely the degree of stability.

Stability applies to the approach to or divergence from a condition of equilibrium. For a stable submarine, Equation (12) in Appendix A describes the approach of the pitch angle to the steady equilibrium condition for longitudinal motion when any departure from equilibrium occurs. Damping ratio as applied in this equation thus indicates the decay of any disturbance to equilibrium which might occur.

Such a disturbance might result either from environmental causes or from control action. Residual disturbances usually remain after the completion of a maneuver and even after corrective control action. An immediate application of damping ratio could be in the selection of the optimum degree of disturbance decay. Optimum disturbance decay should correspond to relative freedom from disturbance and should contribute to minimizing control effort.

The selection of optimum disturbance decay might be based on a number of considerations. But it seems reasonable to base such selection on the minimum time required for a disturbance to become negligible. In general a disturbance may be considered to have become negligible when it has been reduced to one-tenth of its initial value. From Figure 4 in Reference 1 the minimum time for this reduction is found to occur at values of damping ratio from 0.7 to 1.0.

Within the linear range, Equation (13) in Appendix A should apply to the approach to equilibrium after each sternplane change, even if the resulting direction of motion is not horizontal. Damping ratio as applied in this equation should have practical significance for control during a maneuver in addition to mere reduction of uncorrected residual disturbances.

Damping ratio has been found to have a significant relation to control in other applications of damped vibration theory such as for servomechanisms. In this application the design objective appears to be to secure the highest degree of correlation between response and control. But submarine designers have a similar objective which is to secure the optimum degree of response to control. Damping ratio should similarly be of significance in pursuing this objective.

The remainder of this report will be concerned with a practical demonstration of this significance by showing the relationship between damping ratio and vertical overshoot characteristics.

PROCEDURE FOR COMPUTER AND SIMULATOR STUDY

To explore the practical application of damping ratio, it was essential to apply a method by which the value of damping ratio could be varied without unduly affecting the other dynamic characteristics of a submarine. The method selected for this study was to vary the horizontal stern stabilizer surface. The effects of such variation were then applied in simulated vertical overshoot maneuvers. A vertical overshoot maneuver is diagrammed in Appendix B.

Existing model data were utilized for the simulator study. The model for which the most pertinent data had been obtained was a body of revolution similar to the hull of the USS ALBACORE (AGSS-569) without superstructure or propeller. Stability and control studies had been made for this model with a series of movable stern surfaces differing in area and control effectiveness.[2]

For the present study, the horizontal surfaces are regarded as fixed stern stabilizer surfaces, and a selection has been made of the most pertinent ones. The designations 3A, 5B, 5C, 9C, and 11D correspond to those of Reference 2.

A separate computer study was made to obtain the stability roots, damping ratio, and natural angular frequency for the model with each of these surfaces.

So as not to complicate results with varied sternplane effectiveness, the same pair of values of sternplane derivatives was applied in the simulations for all the series. These were the values for stern 5B

$$Z_{\delta s}' = -0.0086 \text{ and } M_{\delta s}' = -0.00437$$

selected as most like those for ALBACORE II. The metacentric height of 0.99 feet, which was used, was that for ALBACORE II.

Vertical overshoot maneuvers were simulated for execute pitch angles of 5, 10, and 20 degrees at 5, 10, 15, and 20 knots. An initiating sternplane angle of 15 degrees and an equal but opposite checking angle were used in each of the simulations.

The pertinent values of the derivatives and coefficients for the model with each of the stern surfaces are given in Table 1. The linear values were used in calculating the values of ζ and ω_n. With the exception of Z_w' and M_w', the linear values were also applied in the simulator study. For the simulation, coefficients taking into account nonlinearities were substituted for the linear values of Z_w' and M_w'. The values of $Z_{\delta s}'$ and $M_{\delta s}'$ obtained for each of the configurations from the model study were included in the tabulation as an indication of their stabilizing effect.

TABLE 1

Derivatives for Model with Various Stern Surfaces

	Stern Surface 3A	Stern Surface 5B	Stern Surface 5C	Stern Surface 9C	Stern Surface 11D
Linear Values					
Z_w'	-0.01117	-0.01662	-0.02199	-0.02500	-0.03572
$Z_{\dot{w}}'$	-0.1596	-0.01622	-0.01638	-0.01681	-0.01750
Z_q'	-0.002131	-0.00532	-0.007843	-0.01023	-0.01478
$Z_{\dot{q}}'$	-0.000070	-0.000200	-0.000280	-0.000496	-0.000840
$Z_{\delta s}'$	-0.00315	-0.00860	-0.01397	-0.01698	-0.02770
M_w'	0.01117	0.00825	0.00585	0.00469	0.00072
$M_{\dot{w}}'$	-0.000070	-0.000200	-0.000280	-0.000496	-0.000840
M_q'	-0.001524	-0.00340	-0.004565	-0.005343	-0.007494
$M_{\dot{q}}'$	-0.000477	-0.000541	-0.000581	-0.000689	-0.000861
$M_{\delta s}'$	-0.00145	-0.00437	-0.006764	-0.007929	-0.01189
I_y'	0.000924	0.000925	0.000926	0.000924	0.000935
m'	0.01747	0.01749	0.01751	0.01747	0.01767
Nonlinearity Coefficients					
Z_w'	-0.00636	-0.01159	-0.01761	-0.01880	-0.02968
$Z_{w\|w\|}'$	-0.06887	-0.07534	-0.07418	-0.08601	-0.09619
M_w'	0.01207	0.009539	0.007002	0.00629	0.00207
$M_{w\|w\|}'$	-0.01582	-0.01911	-0.01886	-0.02212	-0.026744

Note: Prototype length = 200 feet; Prototype metacentric height = 0.99 feet

RESULTS OF COMPUTER AND SIMULATOR STUDY

The variation of damping ratio with speed for each of the stern surfaces is presented in Figure 1. There is a wide range of variation in each case. Selection of a stern surface cannot be based on a single value of damping ratio but can depend only on a satisfactory range of values. Stern surfaces 3A, 5B, and 11D result in either instability or a high degree of overdamping. The range of potentially satisfactory stabilizer surfaces seems quite limited.

The variation of natural frequency with speed for each of the stern surfaces is presented in Figure 2. In dimensional form the values of ω_n remain relatively constant at speeds greater than 8 knots.

At each speed greater than 6 knots the data represent a substantial range of values of damping ratio. The value of natural frequency at each speed changes but little between the various stern surfaces; therefore, the variation in overshoot pitch angle, in overshoot depth, and in time to execute will principally show the effect of damping ratio. Effectively at each speed these data illustrate the variation of overshoot characteristics, resulting from a controlled variation of damping ratio, the control being effected through stern stabilizer surface design.

The results of the simulator study of overshoot tests are presented in Table 2 and in Figures 3 through 17. Initiating and checking sternplane angles of 15 degrees were used for each simulated overshoot test. Results from execute pitch angles of 5, 10, and 20 degrees have been presented whenever they could be obtained.

The variation of time to reach execute with damping ratio, the variation of overshoot pitch angle with damping ratio, and the variation of overshoot depth with damping ratio are presented for 5, 10, 15, and 20 knots.

DISCUSSION

At 5 knots the entire series is stable and oscillatory. At this speed the equilibrium pitch angles that would be approached if the 15-deg sternplane angle were held fixed are in the same ranges of magnitude as are the scheduled execute angles resulting in a saturation effect. A 20-deg execute could not be reached for most of the series since this value exceeded the equilibrium value of pitch angle. For the same reason 10-deg execute could not be reached for the stern surface having the highest value of damping ratio. The time to reach 10-degree execute increases sharply and the overshoot pitch angle approaches zero as damping ratio increases. Even at 5-degree execute, overshoot pitch angle is approaching zero at the highest value of damping ratio. At 5 knots the efficiency of the sternplanes in changing depth is low, resulting in small depth changes during the tests.

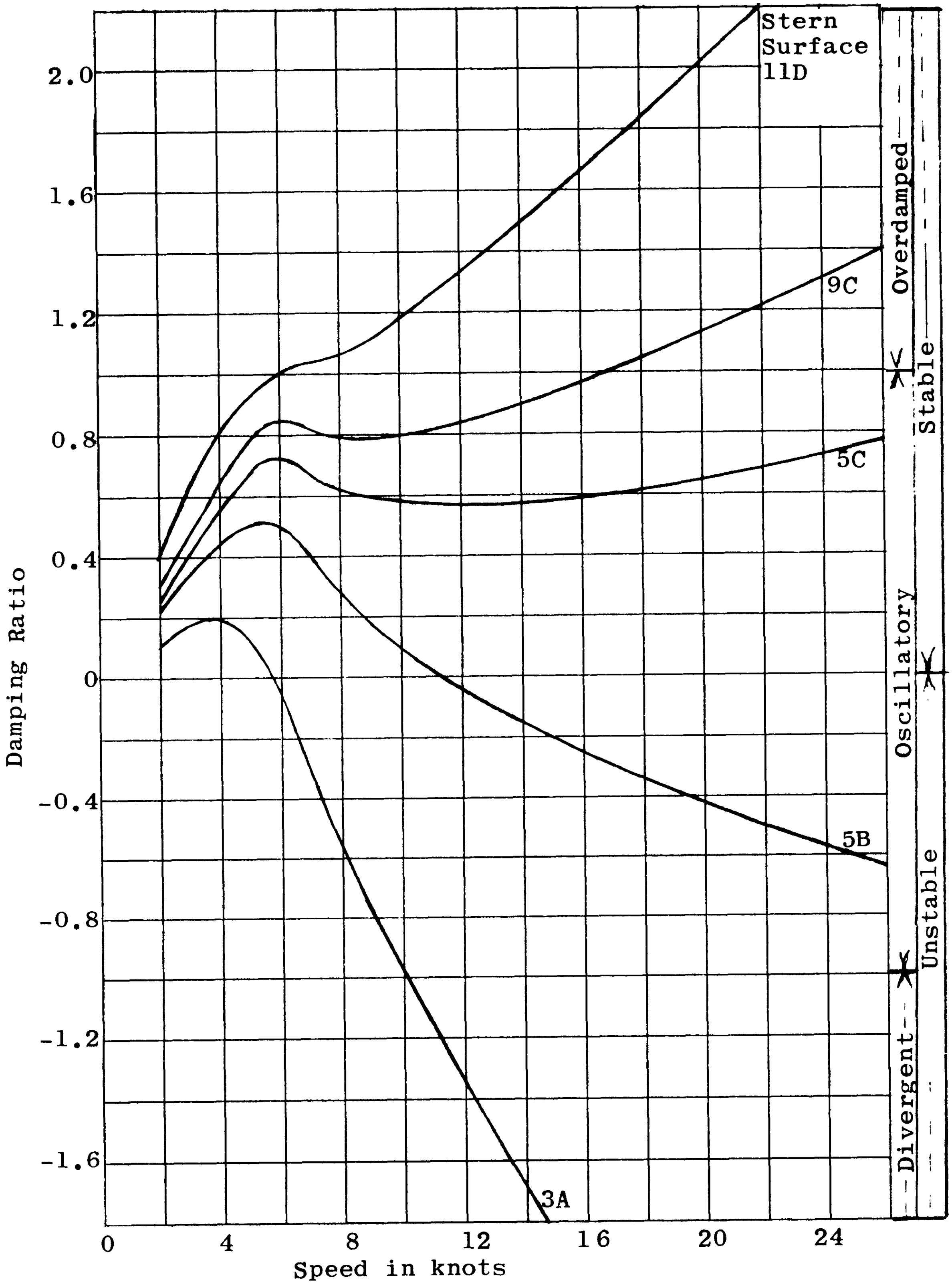

Figure 1 - Variation of Damping Ratio with Speed for Stern Surface Series

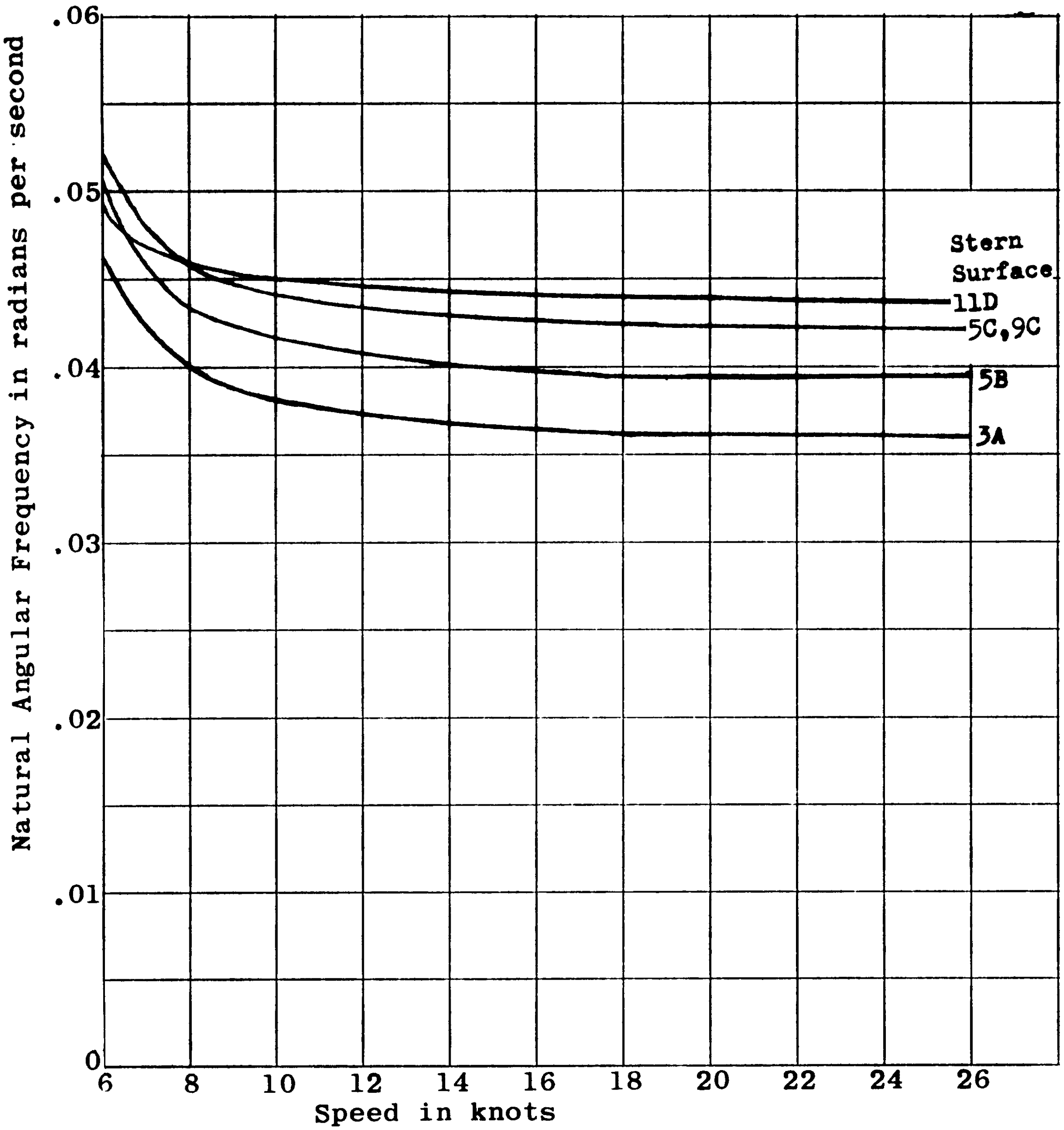

Figure 2 - Variation of Natural Frequency with Speed for Stern Surface Series

TABLE 2

Effect of Damping Ratio on Overshoot Pitch Angle and Overshoot Depth at Various Speeds

			5-Degree Execute Pitch Angle			10-Degree Execute Pitch Angle			20-Degree Execute Pitch Angle		
Stern Surface	Speed knots	Damping Ratio	Time to Execute	Overshoot Pitch Angle	Overshoot Depth	Time to Execute	Overshoot Pitch Angle	Overshoot Depth	Time to Execute	Overshoot Pitch Angle	Overshoot Depth
3A	5	0.12	12.6	4.76	14.7	18.1	5.40	26.0	28.0	3.55	41.2
5B	5	0.53	14.8	2.33	9.9	23.5	1.69	18.3			
5C	5	0.69	16.6	1.41	8.3	30.7	0.55	14.7			
9C	5	0.78	18.3	1.08	8.0	38.7	0.21	14.2			
11D	5	0.91	24.2	0.46	7.3						
3A	10	-0.98	6.6	14.97	80.5	8.9	23.43	169.6	12.2	32.17	298.6
5B	10	0.09	7.5	5.74	30.8	9.8	7.32	63.6	16.1	7.82	123.9
5C	10	0.58	8.3	3.69	22.6	12.5	3.99	46.6	20.5	3.34	98.8
9C	10	0.80	9.0	2.94	20.9	13.9	2.89	43.5	24.4	2.19	95.0
11D	10	1.19	11.0	1.58	17.5	19.2	1.21	39.8	42.2	0.54	98.4
3A	15	-1.87	4.7	30.52	239.8	6.2	51.10	501.5	8.3	73.33	795.9
5B	15	-0.19	5.3	8.61	52.2	7.4	11.38	106.8	10.6	13.61	208.0
5C	15	0.58	5.8	5.30	34.4	8.4	6.08	69.3	13.1	6.45	148.0
9C	15	0.94	6.3	4.25	31.1	9.4	4.67	64.2	15.1	4.43	138.8
11D	15	1.58	7.5	2.25	24.0	12.4	2.06	54.2	23.3	1.66	135.6
3A	20	-2.67	3.8	59.40	615.1	4.9	97.75	1062.2	6.4	-	-
5B	20	-0.42	4.2	11.22	75.3	5.8	15.15	150.2	8.1	18.27	281.4
5C	20	0.66	4.6	6.79	46.3	6.5	7.86	89.8	9.9	8.87	187.6
9C	20	1.14	4.9	5.26	39.7	7.2	6.00	80.3	11.3	6.16	172.0
11D	20	2.03	5.9	2.91	30.5	9.4	2.68	65.6	16.8	2.39	160.4
5C	25	0.75	3.9	8.33	59.5	5.4	9.50	110.1	8.0	10.67	219.4
9C	25	1.36	4.1	6.26	49.1	6.0	7.38	98.0	9.2	7.81	203.0

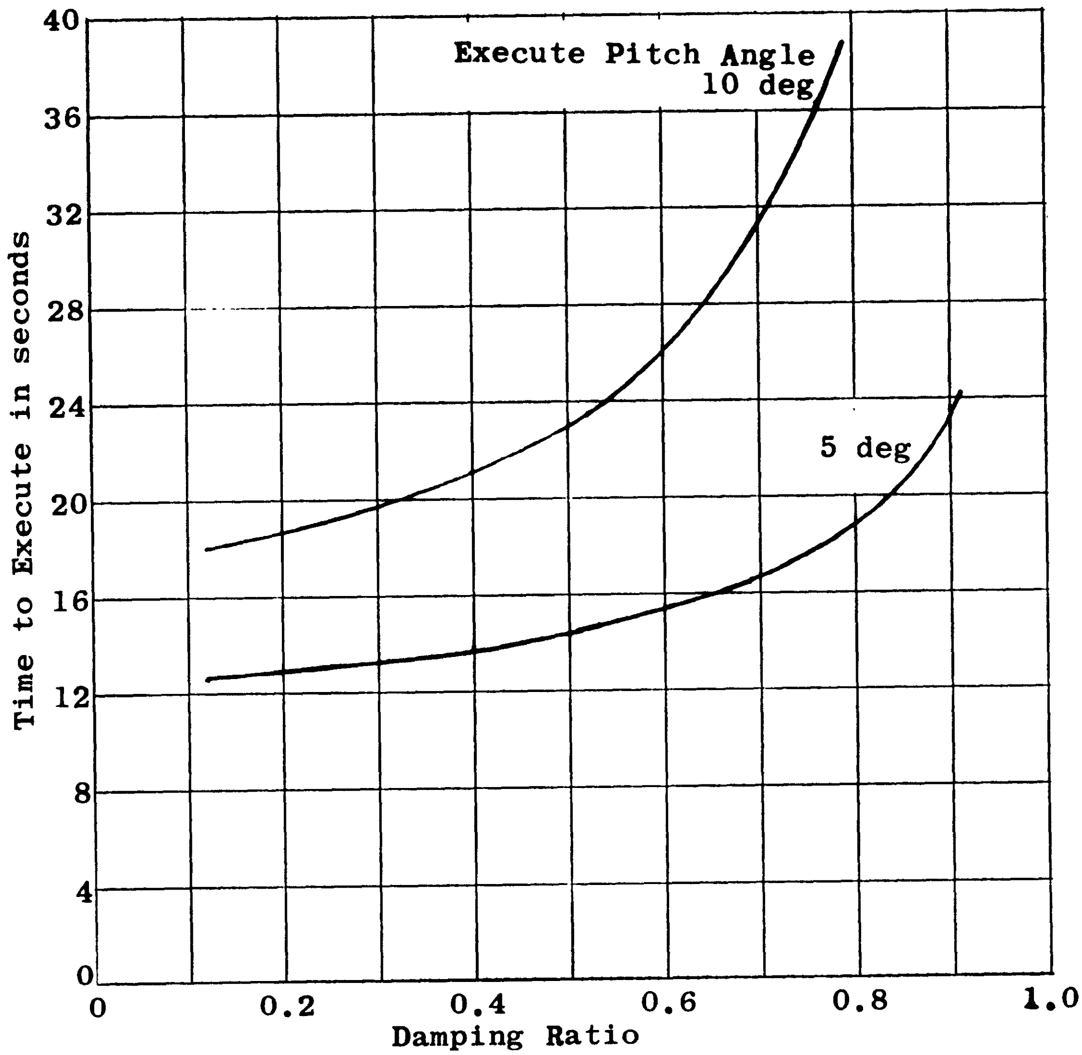

Figure 3 - Effect of Damping Ratio on Time to Execute at 5 Knots

15-Deg Entrance and Checking Sternplane Angles

1-Ft Metacentric Height

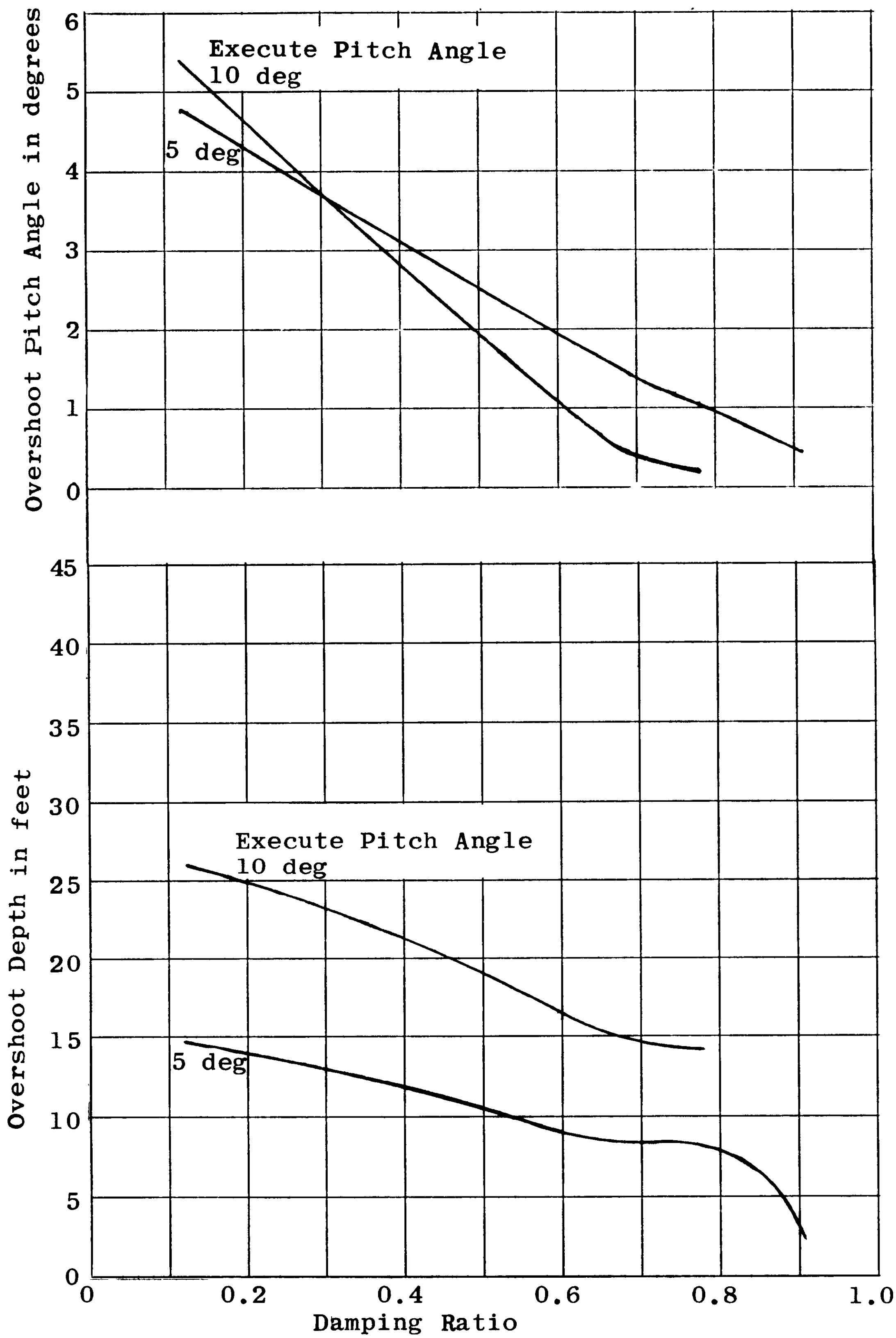

Figure 4 - Effect of Damping Ratio on Overshoot Pitch Angle and Overshoot Depth at 5 Knots
15-Deg Entrance and Checking Sternplane Angles
1-Ft Metacentric Height

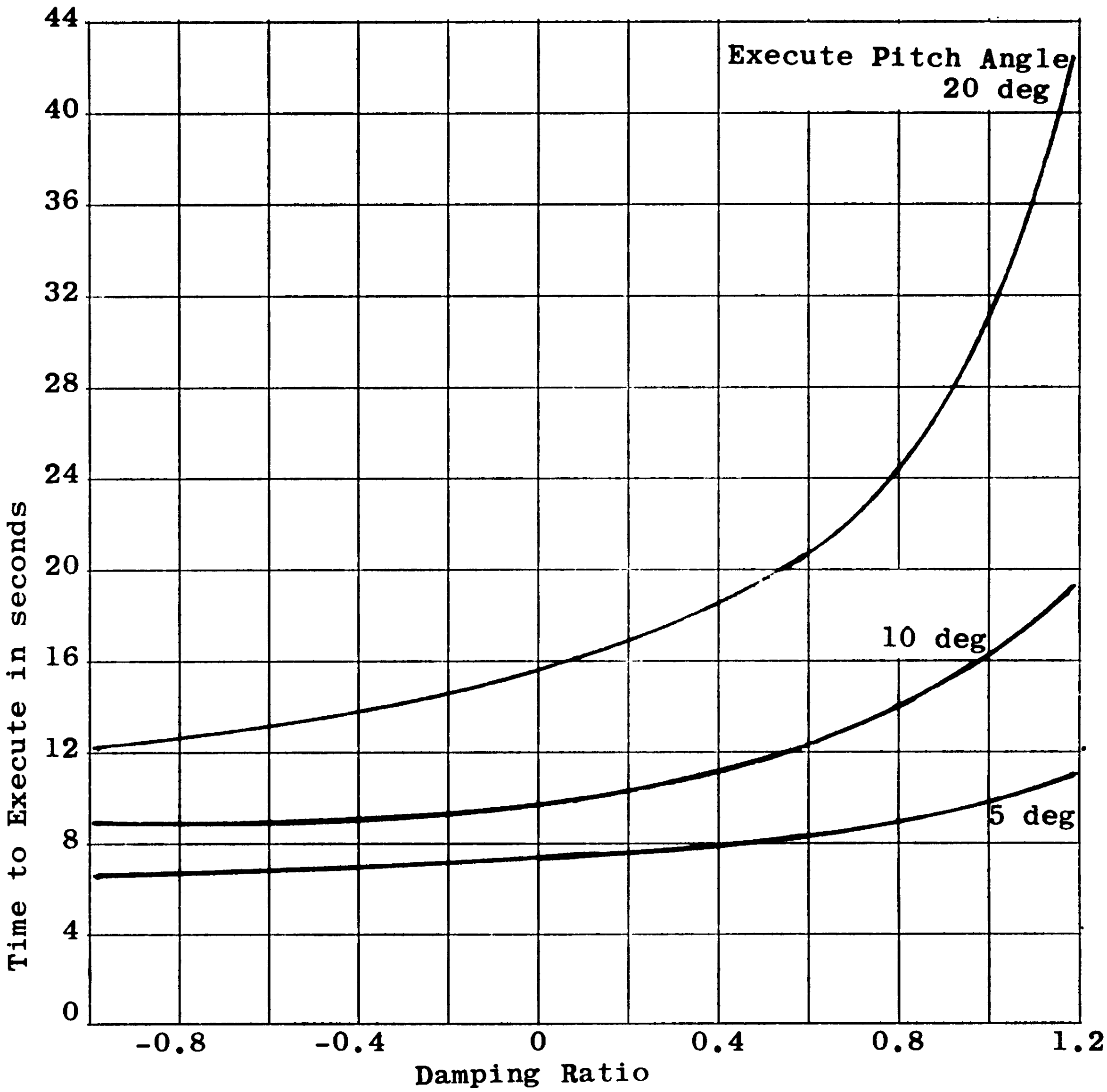

Figure 5 - Effect of Damping Ratio on Time to Execute at 10 Knots

15-Deg Entrance and Checking Sternplane Angles
1-Ft Metacentric Height

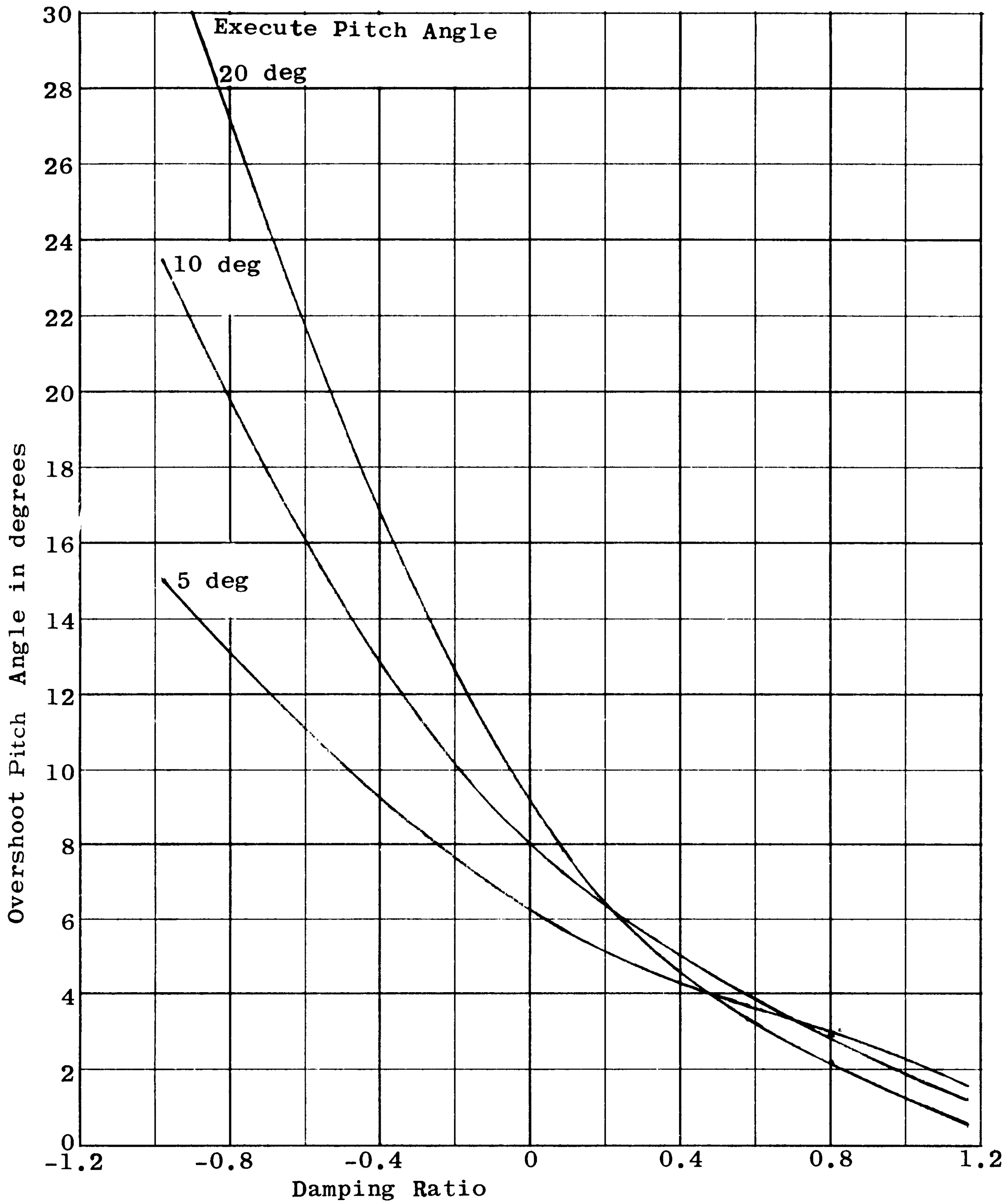

Figure 6 - Effect of Damping Ratio on Overshoot Pitch at 10 Knots

15-Deg Entrance and Checking Sternplane Angles
1-Ft Metacentric Height

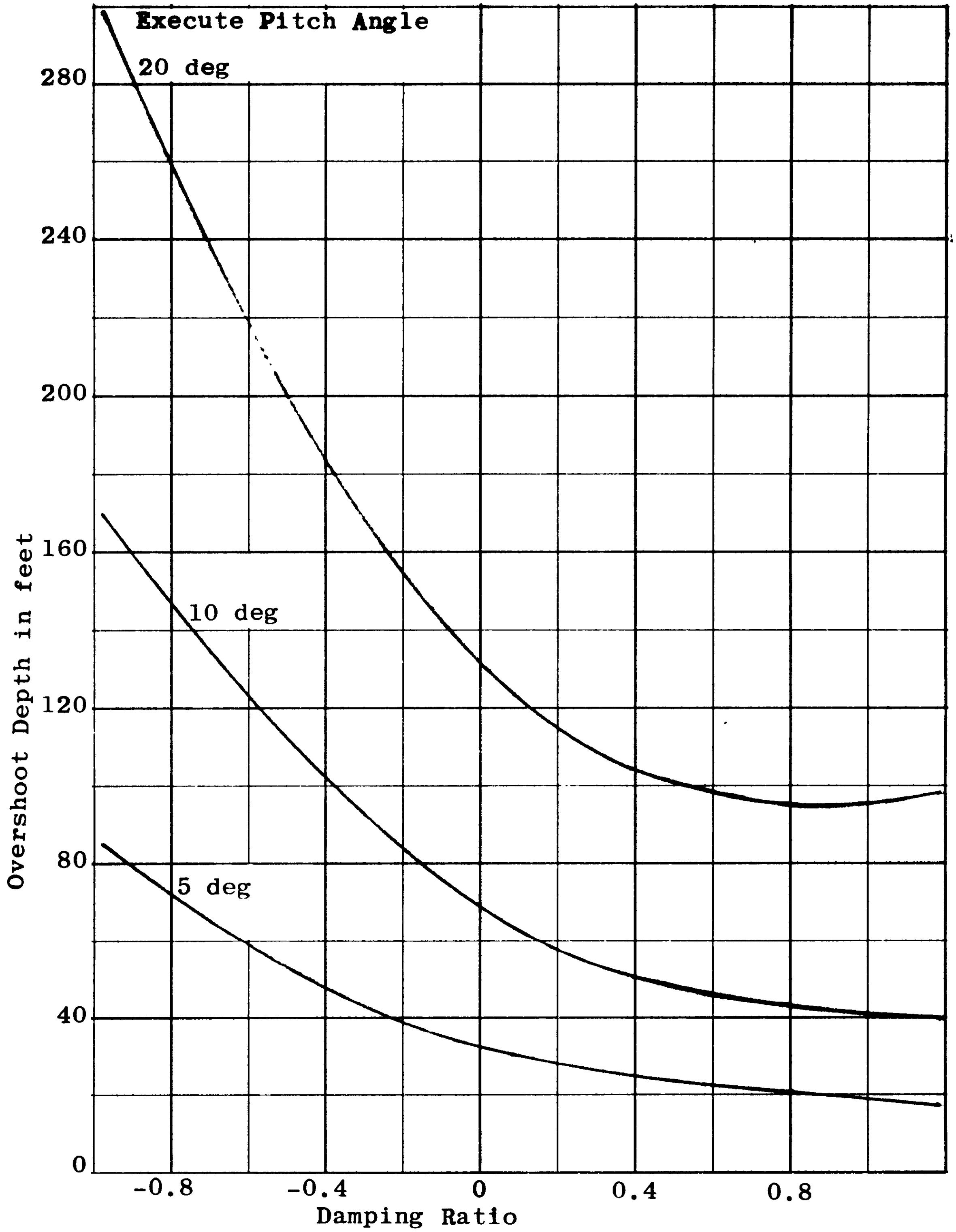

Figure 7 - Effect of Damping Ratio on Overshoot Depth at 10 Knots

15-Deg Entrance and Checking Sternplane Angles
1-Ft Metacentric Height

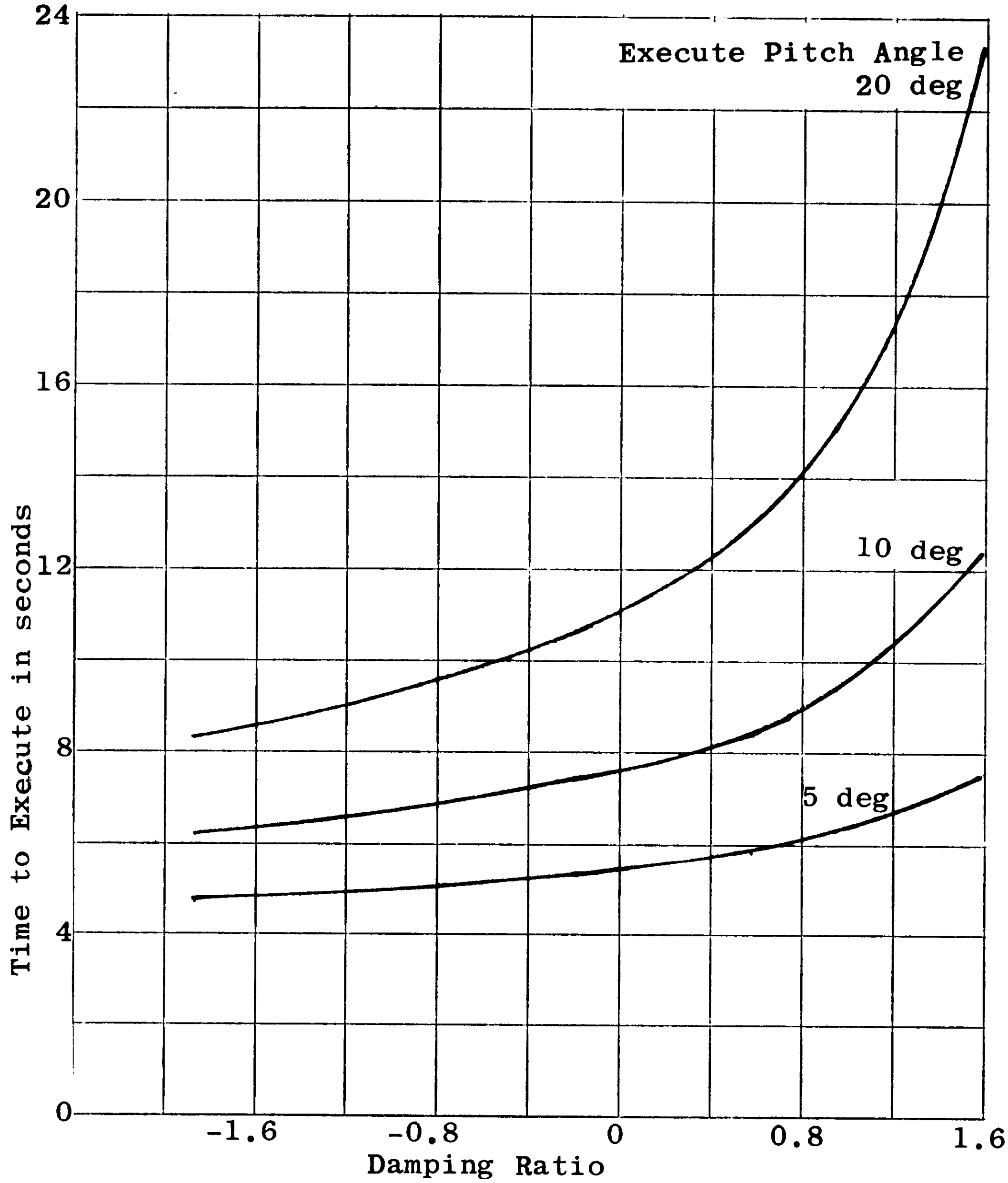

Figure 8 - Effect of Damping Ratio on Time to Execute at 15 Knots

15-Deg Entrance and Checking Sternplane Angles
1-Ft Metacentric Height

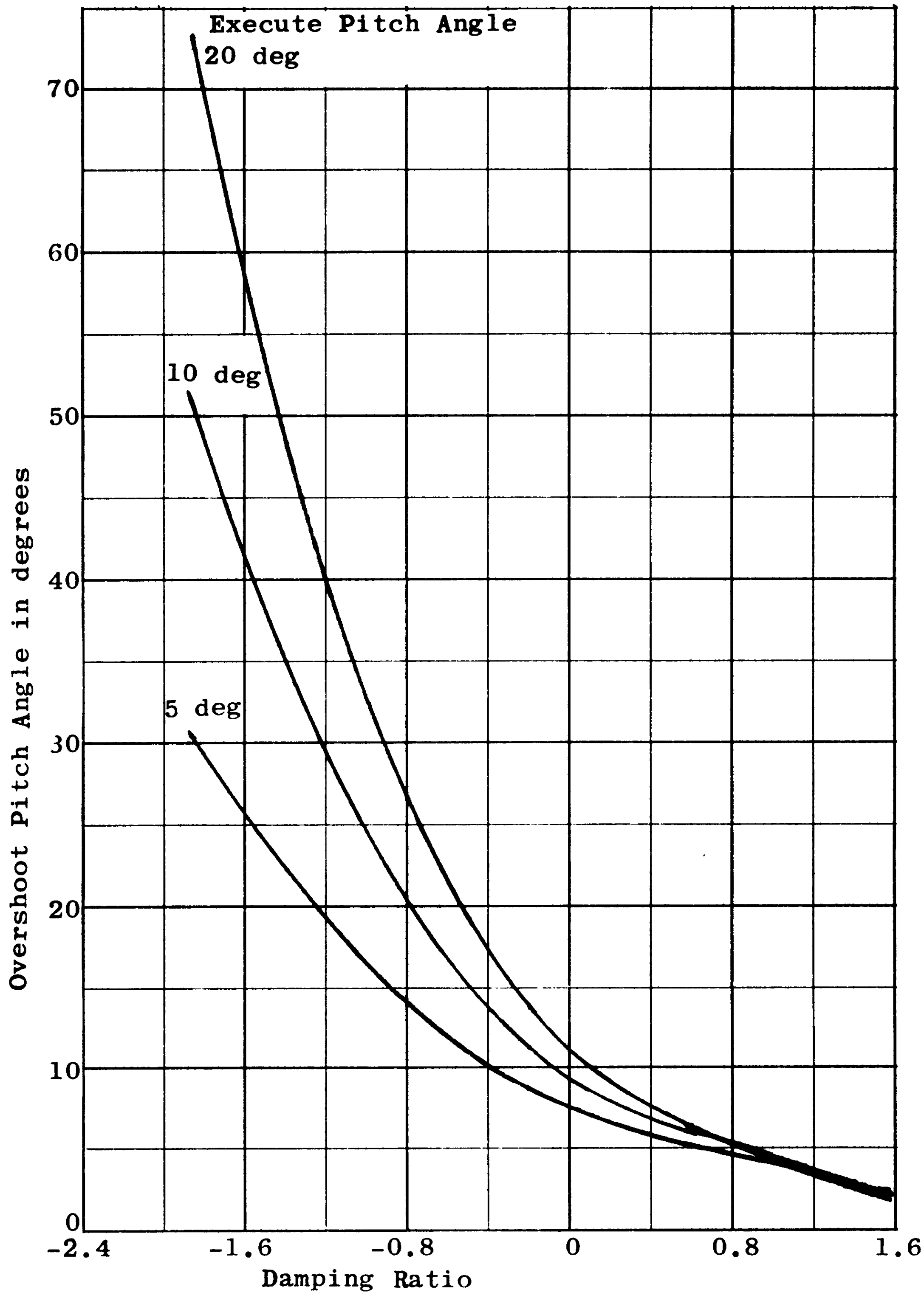

Figure 9 - Effect of Damping Ratio on Overshoot Pitch at 15 Knots

15-Deg Entrance and Checking Sternplane Angles
1-Ft Metacentric Height

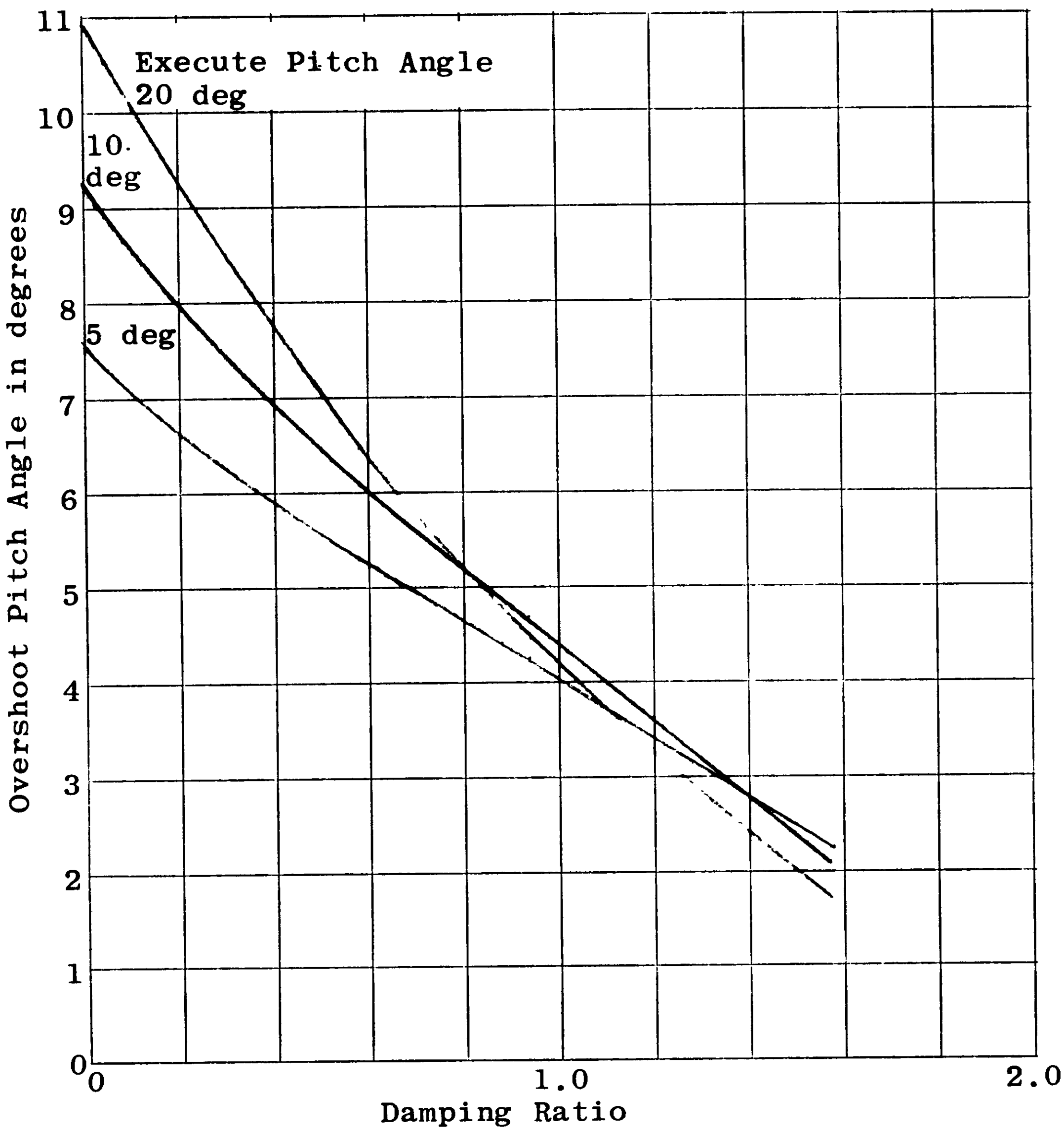

Figure 10 - Effect of Damping Ratio on Overshoot Pitch for Stable Range at 15 Knots

15-Deg Entrance and Checking Sternplane Angles
1-Ft Metacentric Height

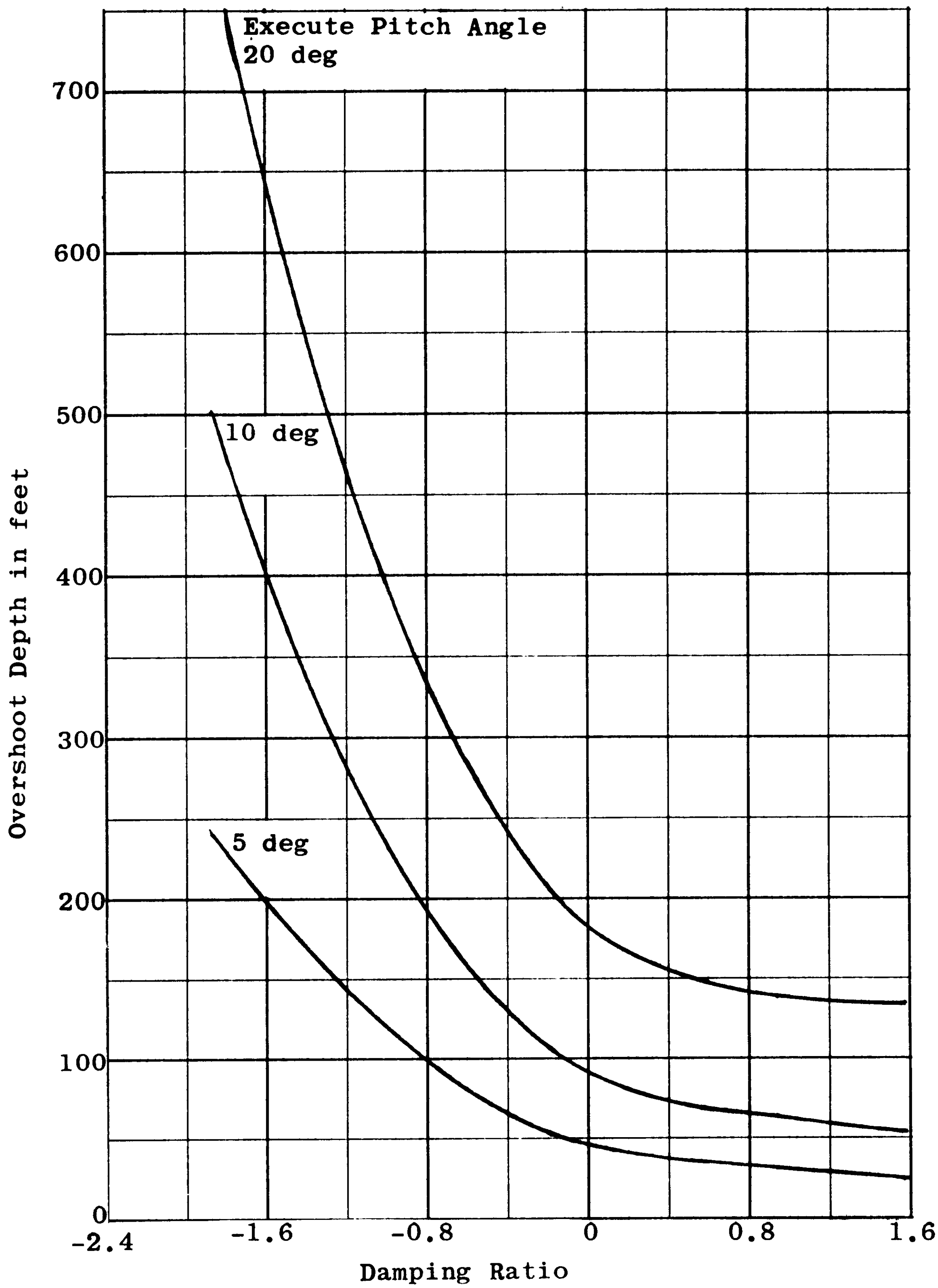

Figure 11 - Effect of Damping Ratio on Overshoot Depth at 15 Knots

15-Deg Entrance and Checking Sternplane Angles
1-Ft Metacentric Height

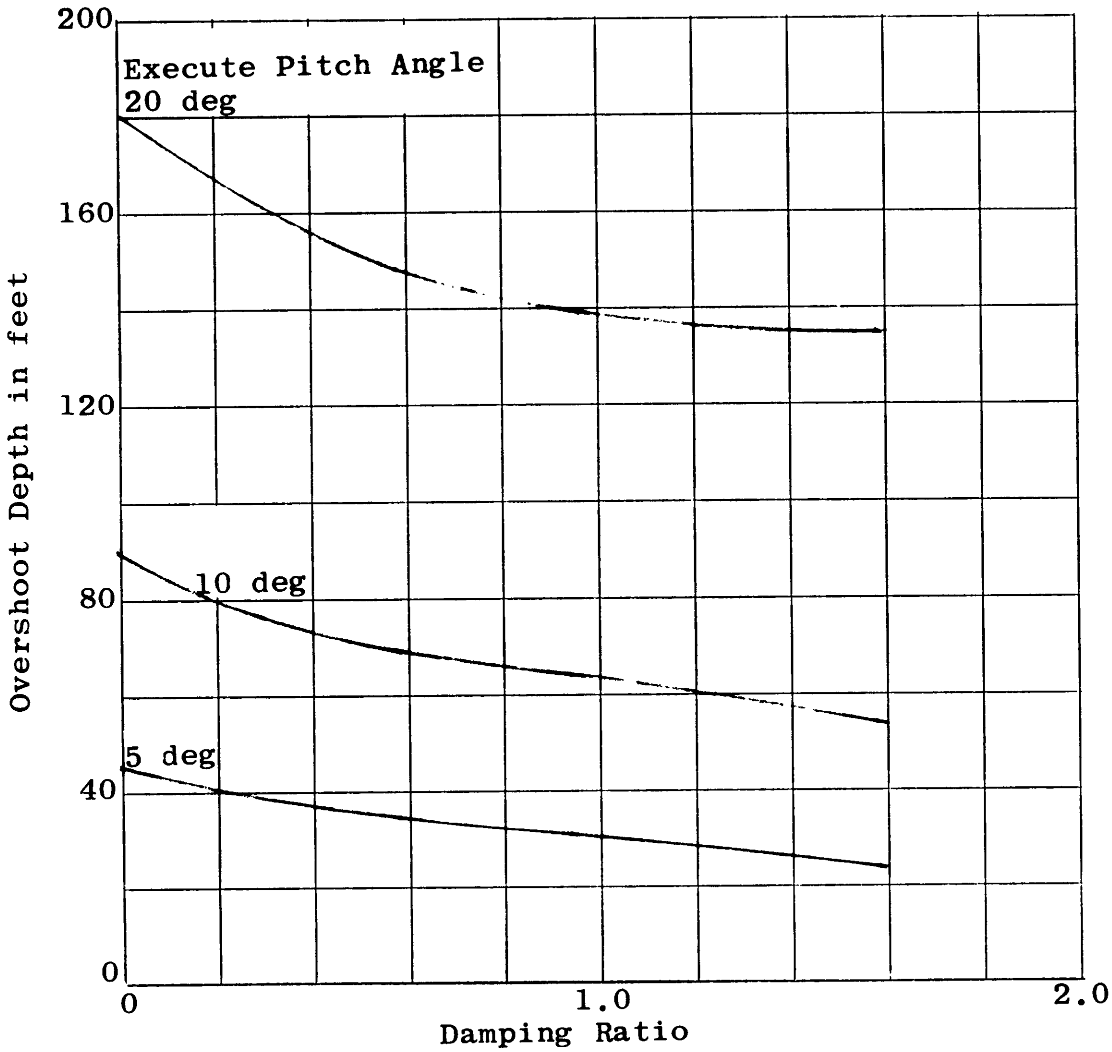

Figure 12 - Effect of Damping Ratio on Overshoot Depth for Stable Range at 15 Knots

15-Deg Entrance and Checking Sternplane Angles
1-Ft Metacentric Height

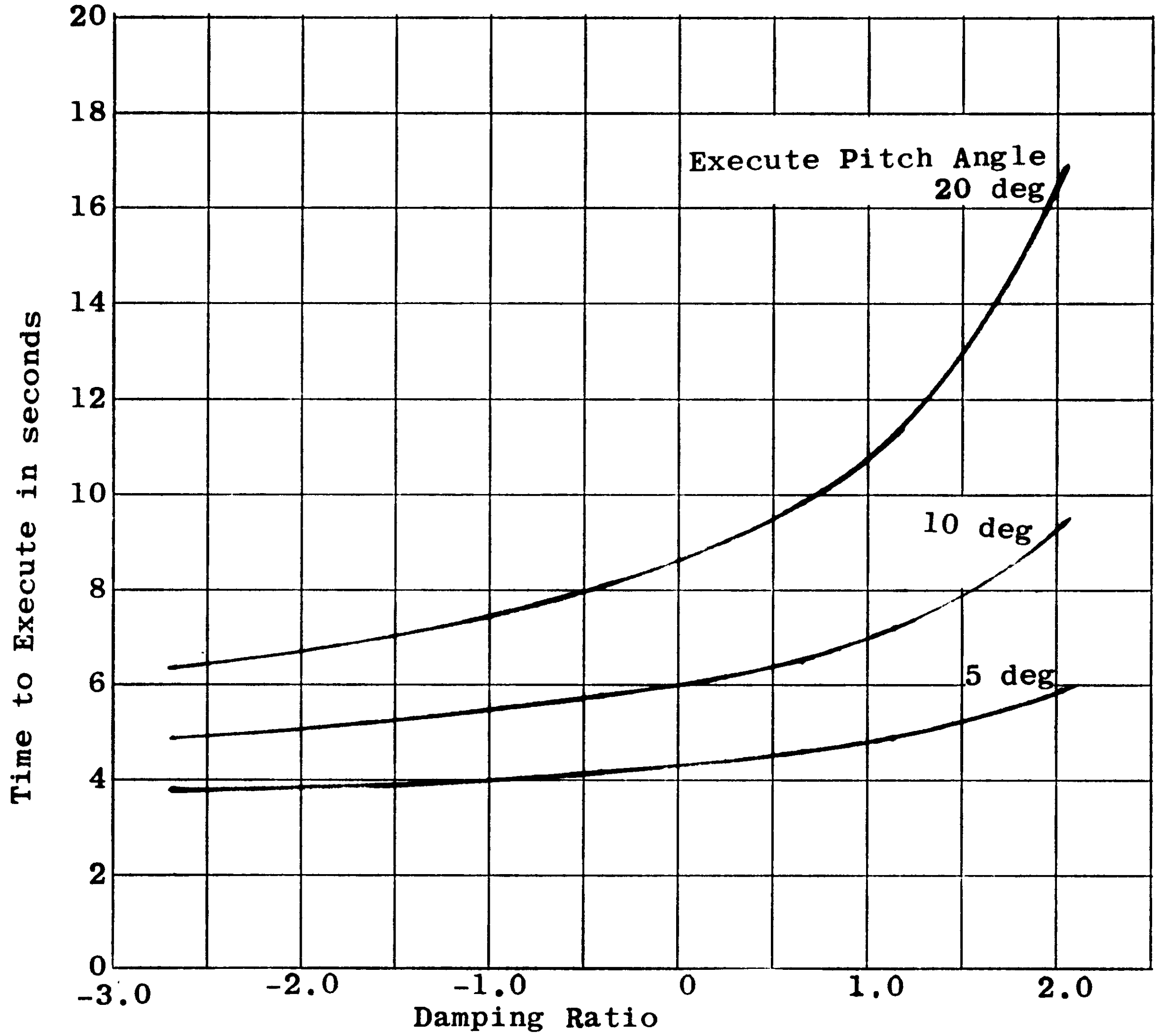

Figure 13 - Effect of Damping Ratio on Time to Execute at 20 Knots

15-Deg Entrance and Checking Sternplane Angles
1-Ft Metacentric Height

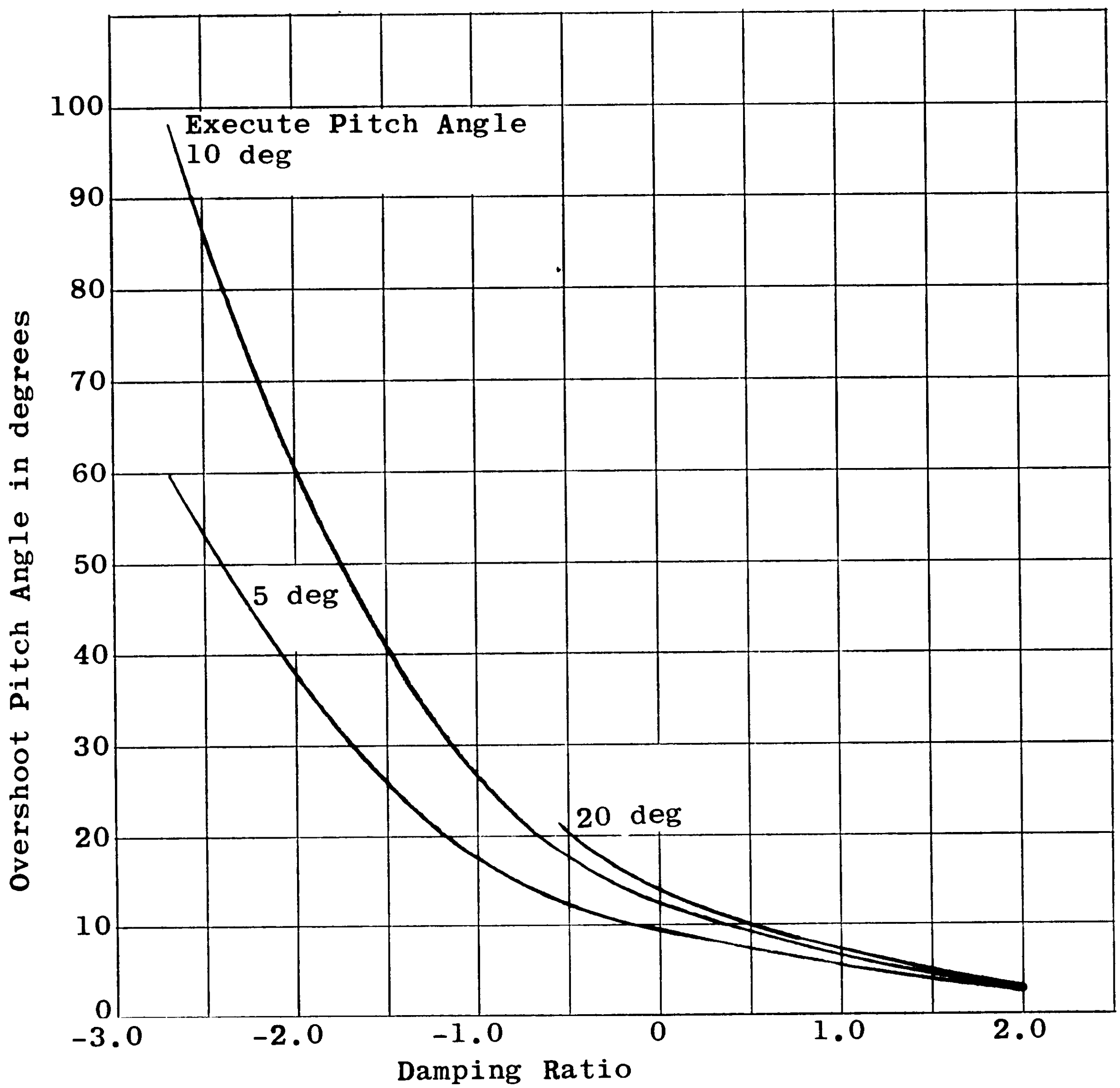

Figure 14 - Effect of Damping Ratio on Overshoot Pitch at 20 Knots

15-Deg Entrance and Checking Sternplane Angles
1-Ft Metacentric Height

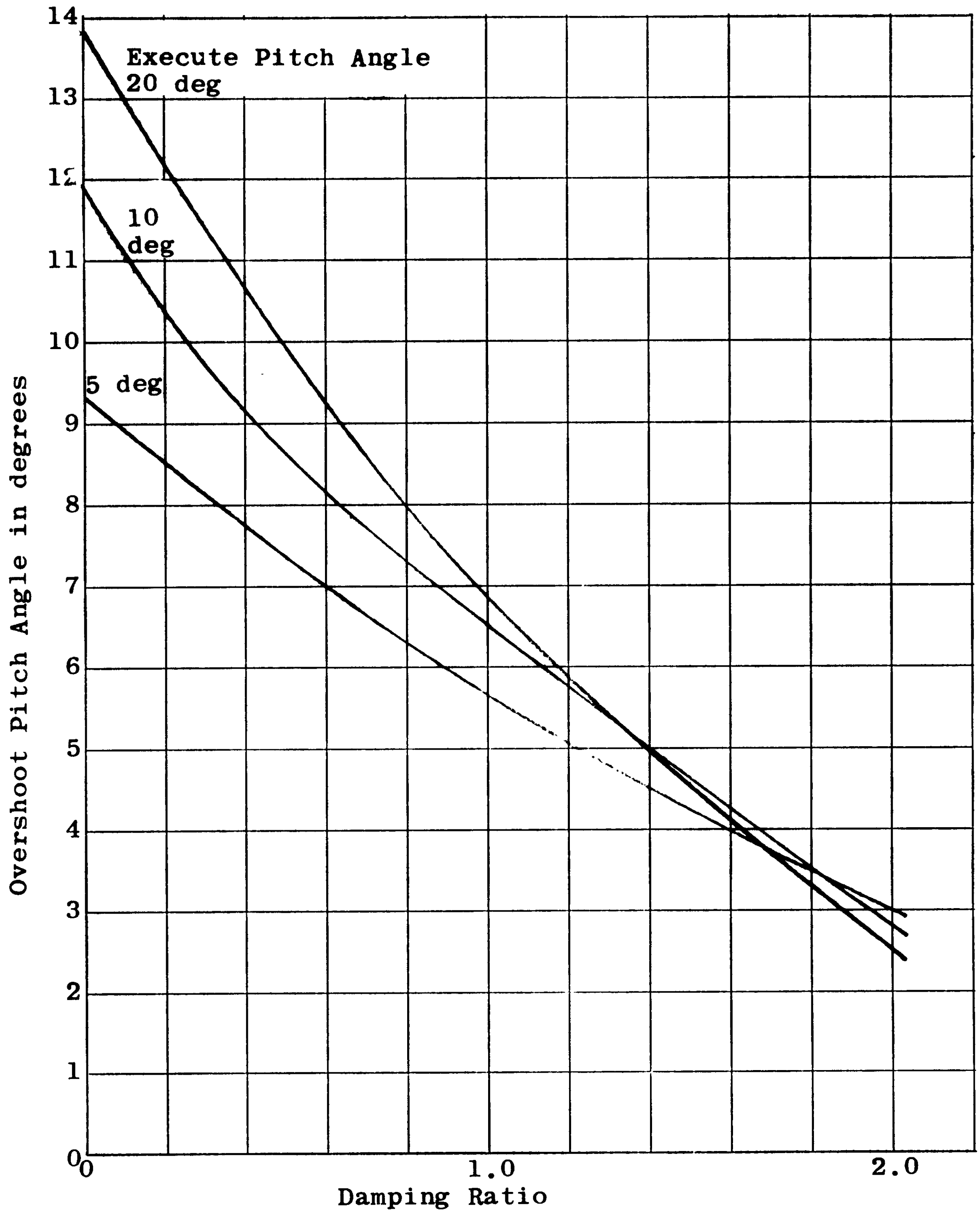

Figure 15 - Effect of Damping Ratio on Overshoot Pitch for Stable Range at 20 Knots

15-Deg Entrance and Checking Sternplane Angles
1-Ft Metacentric Height

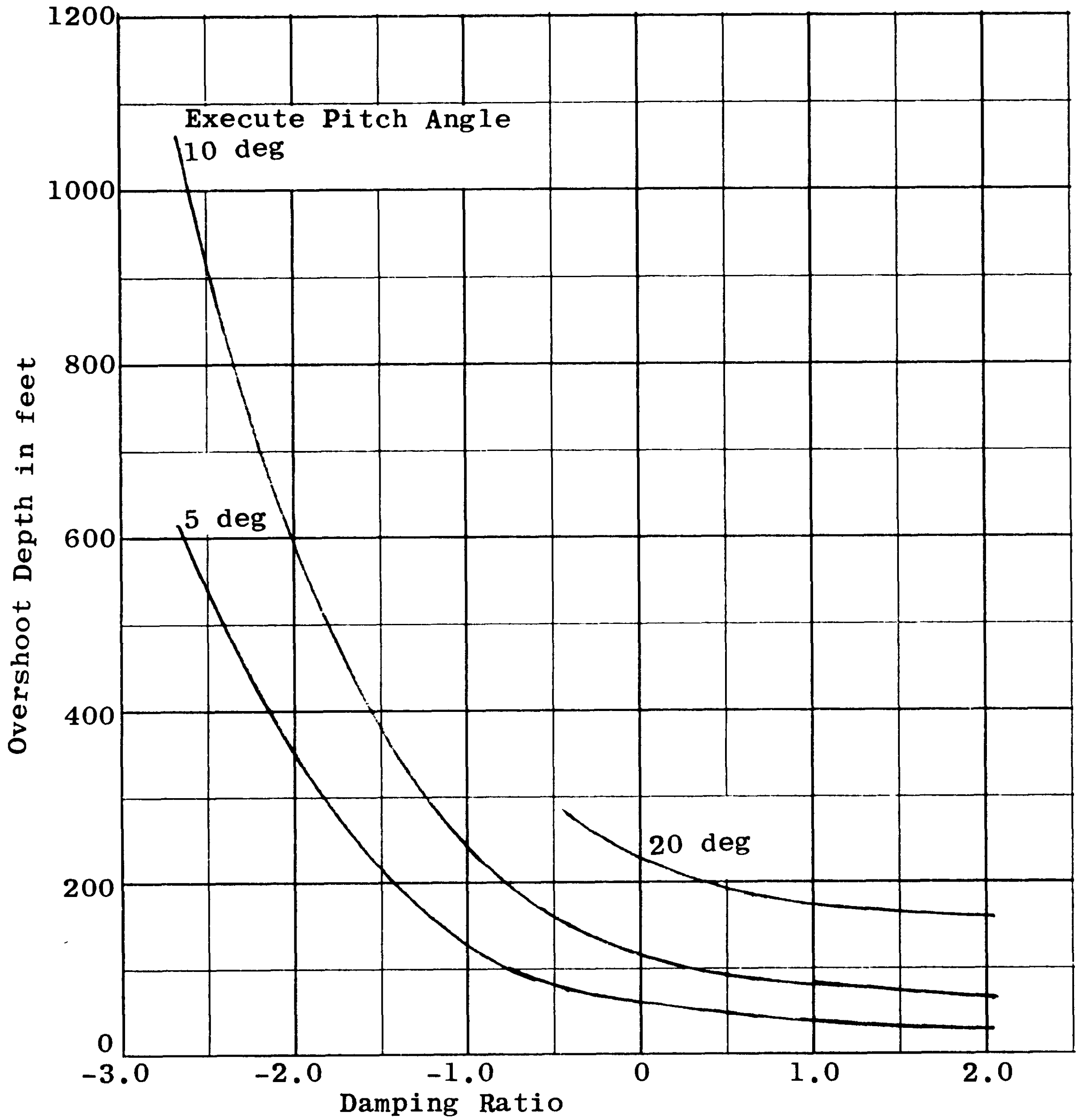

Figure 16 - Effect of Damping Ratio on Overshoot Depth at 20 Knots

15-Deg Entrance and Checking Sternplane Angles
1-Ft Metacentric Height

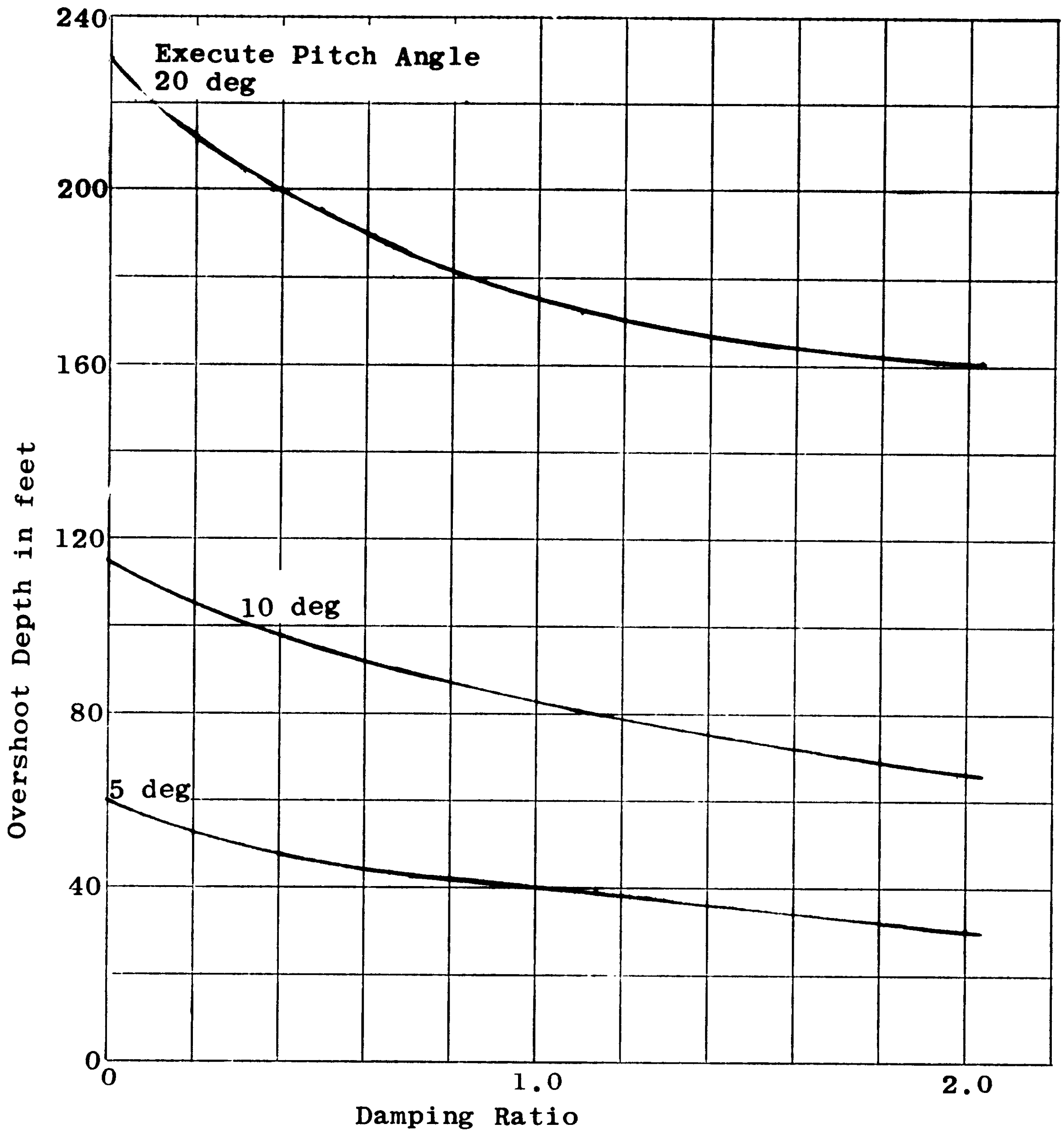

Figure 17 - Effect of Damping Ratio on Overshoot Depth for Stable Range at 20 Knots

15-Deg Entrance and Checking Sternplane Angles
1-Ft Metacentric Height

Even at higher speeds the pitch angle saturation effect, as equilibrium pitch angle is approached, is apparent at high damping ratios. Here the highest pitch overshoot angles occur at the lowest execute pitch angles, reversing the order of occurrence at lower damping ratios.

At 10, 15, and 20 knots, the variations of overshoot characteristics with damping ratio are similar, so the same comments can be applied to each.

For each execute angle and each speed, there is little variation in time to reach execute in the unstable range, where the value of damping ratio is negative. In the oscillatory range, as the value of damping ratio increases from 0 to 1.0, time to reach execute increases moderately. In the overdamped range for values of damping ratio greater than 1.0, the rate of increase is more rapid, indicating a greater amount of time required to initiate a maneuver.

Overshoot pitch angles are found to be excessive in the unstable region. As they decrease with increasing damping ratio, they reach acceptable values in the upper oscillatory range. They continue to decrease in the overdamped range; however, the small remaining values are relatively unimportant.

Overshoot depth is also found to be excessive in the unstable range, at least at 15 knots and more. As overshoot depth decreases with damping ratio, it also reaches acceptable proportions in the oscillatory range. Overshoot depth continues to decrease as damping ratio increases in the overdamped range. At low execute pitch angles this overshoot depth is small and unimportant in this range. At 20-degree execute, overshoot depth is more substantial but its value tends to level off as damping ratio increases.

In selecting the optimum values of overshoot characteristics, as affected by damping ratio, the advantages of a decreased overshoot pitch angle and decreased overshoot depth must be balanced against the disadvantage of increased time to reach execute pitch angle. Appraisal must be based on the best combination of these characteristics obtained over the significant speed range. No exact method has been found for comparing the relative value of these different characteristics. The selection must be based, at least in part, on personal judgment. It can be observed from Figures 6, 7, 9 through 12 and 14 through 17 that in the overdamped range, the reduction in value of overshoot depth has reached a point of diminishing returns. From Figures 5, 8, and 13, it can be noted that in the same range the values of elapsed time to reach execute pitch angle become sufficiently large to indicate slow response time.

As damping decreases, time to reach execute pitch angle reaches a point of diminishing returns in the lower stable oscillatory range. At the same time overshoot pitch angle and overshoot depth start to become excessive.

Analogous to control in the horizontal plane, the control of a submarine in the vertical plane may be satisfactory even for a zero value of metacentric height. Without metacentric stability the ship would not be stable in pitch. However, a hydrodynamically stable submarine would be stable in pitch rate. Control would then be effected through control of pitch rate. This control situation is approached at high speed for some submarines.

Preliminary examination suggests that, in the absence of metacentric stability, the stern surface which would produce the most satisfactory longitudinal control characteristics, differs only slightly from the one which for normal metacentric stability, would produce satisfactory values of damping ratio. However these control characteristics will be improved by the adding of an adequate degree of metacentric stability, sufficient to result in a satisfactory damping ratio.

Metacentric stability is required for considerations other than longitudinal dynamic stability, for roll stability and for pitch stability at zero speed. A satisfactory metacentric height, then appears to be necessary and may be presupposed in selecting the horizontal stabilizer surface to obtain the desired range of values for damping ratio. The effect of change in metacentric height on damping ratio is discussed in Appendix C

CONCLUSIONS

1. Because of the variation in values of damping ratio with speed for every submarine, it is not possible to select a single value that will apply to each design throughout the significant speed range. Selection must be based on an optimum range of values of damping ratio that can be obtained over this speed range.

2. For the model under study, the optimum combination of "overshoot" characteristics appears to occur in the upper part of the stable oscillatory range at values of damping ratio between 0.6 and 1.0. This selection appears to apply to operational submarines as well.

3. This selection of values from 0.6 to 1.0 corresponds approximately to the selection for optimum disturbance decay from 0.7 to 1.0. It also includes the value 0.7 most frequently recommended in servomechanism applications.

4. For the model under study the optimum stern surface would have characteristics between those of 5C and 9C. Exact selection would depend on the speed range under consideration.

5. For a fixed metacentric height and fixed sternplane effectiveness, the vertical overshoot characteristics of a submarine can be controlled in the design of the stern stabilizer surface, using damping ratio as a criterion.

6. Combined with customary metacentric height and with satisfactory stern-plane effectiveness, a range of values of damping ratio from 0.6 to 1.0 will result in optimum vertical overshoot characteristics, nearly optimum stability and disturbance decay, and should contribute to optimum stability and control characteristics.

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REFERENCES

1. NAVSHIPS P. O. #0-0002 Amend 19.

2. Clawson, P. C., "A Full-Scale Evaluation of the Handling Qualities of USS ALBACORE (AGSS 569) with the Second Stern Configuration," David Taylor Model Basin Report C-827 (May 1957) CONFIDENTIAL.

3. Dempsey, E. M., "Static Stability Characteristics of a Systematic Series of Cruciform and X-Stern Control Surfaces on a Body of Revolution," NSRDC Report (in preparation).

4. Hoyt, E. D. and Imlay, F. H., "The Influence of Metacentric Stability on the Dynamic Longitudinal Stability of a Submarine," David Taylor Model Basin Report C-158 (Oct 1948) CONFIDENTIAL.

5. Pode, Leonard and Church, J. W., "A Study of the Effects of Stability and Control Parameters on the Performance and Stability of Submarines in Pitching Motion," David Taylor Model Basin Report C-626 2v. (Nov 1955) CONFIDENTIAL.

APPENDIX A

REVIEW OF BASIC THEORY

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Within the linear range, the significant terms of the equation of motion in the vertical plane for a submerged submarine may be given by[4,5]

$$(Z_{\dot{w}}' - m')\,\dot{w}' + Z_w'\,w' + Z_{\dot{q}}'\,\ddot{\theta} + (m' + Z_q')\,\dot{\theta} = -Z_{\delta s}'\,\delta s \tag{1}$$

$$M_{\dot{w}}'\,\dot{w}' + M_w'\,w' + (M_{\dot{q}}' - I_y')\,\ddot{\theta} + M_q'\,\theta + M_\theta'\,\theta = -M_{\delta s}'\,\delta s \tag{2}$$

From this may be derived an expression from which terms in w and its time derivatives have been eliminated. Differentiating with respect to time

$$(Z_{\dot{w}}' - m')\,\ddot{w}' + Z_{\dot{q}}'\,\dddot{\theta} + (m' + Z_q')\,\ddot{\theta} + Z_w'\,\dot{w}' = -Z_{\delta s}'\,\dot{\delta}_s \tag{3}$$

$$M_{\dot{w}}'\,\ddot{w}' + M_w'\,\dot{w}' + (M_{\dot{q}}' - I_y')\,\dddot{\theta} + M_q'\,\theta + M_\theta'\,\dot{\theta} = -M_{\delta s}'\,\dot{\delta}_s \tag{4}$$

Combining

$$a\dddot{\theta} + b\ddot{\theta} + c\dot{\theta} + d\theta = -(A\delta_s + B\dot{\delta}_s) \tag{5}$$

where

$$a = (Z_{\dot{w}}' - m')(M_{\dot{q}}' - I_y') - Z_{\dot{q}}'\,M_{\dot{w}}'$$

$$b = Z_w'\,(M_{\dot{q}}' - I_y') - M_w'\,Z_{\dot{q}}' - M_{\dot{w}}'\,(m' + Z_q') + M_q'\,(Z_{\dot{w}}' - m')$$

$$c = Z_w'\,M_q' + M_\theta'\,(Z_{\dot{w}}' - m') - M_w'\,(m' + Z_q')$$

$$d = Z_w'\,M_\theta'$$

$$A = Z_w'\,M_{\delta s}' - M_w'\,Z_{\delta s}'$$

$$B = (Z_{\dot{w}}' - m')\,M_{\delta s}' - M_{\dot{w}}'\,Z_{\delta s}'$$

For the condition of fixed controls the solution is

$$\theta = \theta_E + \theta_1 e^{\sigma_1 t} + \theta_2 e^{\sigma_2 t} + \theta_3 e^{\sigma_3 t} \tag{6}$$

where θ_E is the equilibrium value of θ and the remaining right hand terms represent the transient variations of θ. σ_1, σ_2, and σ_3 are conventionally referred to as stability roots. Their values may be determined by solving the corresponding characteristic equation

$$a\sigma^3 + b\sigma^2 + c\sigma + d = 0 \tag{7}$$

One of the roots, which by long usage is designated σ_2 will always have a real value. The other two roots σ_1 and σ_3 may either have real values or may form a complex pair. For a normal hull configuration having metacentric stability, it can be shown that the value of σ_2 will be negative. Values of both σ_1 and σ_3 will be either negative or positive. For a dynamically stable submarine, all real values, including real parts of complex values of the roots, will be negative. For complex values of σ_1 and σ_3 the pitch variation will be oscillatory.

For the speed range for which dynamic stability is of interest θ_2 always has a relatively large negative value (typically -3.0 in nondimensional form). The contribution of the term $\theta_2 e^{\sigma_2 t}$ becomes negligible very soon after each disturbance, after which the transient variation of θ is described to a high degree of approximation by the remaining variable terms $\theta_1 e^{\sigma_1 t}$ and $\theta_3 e^{\sigma_3 t}$. Since the contribution of these terms is by direct addition, they be treated separately.

$$\theta_{1,3} = \theta_1 e^{\sigma_1 t} + \theta_3 e^{\sigma_3 t} \tag{8}$$

This is the equation for that part of the pitch variation which is oscillatory when the values of σ_1 and σ_3 are complex. It has the form of the equation for a damped oscillation with a single degree of freedom. It is thus practicable to apply established vibration theory to the pitching motion of a submarine, and the applicable equations may be restated accordingly

$$\ddot{\theta}_{1,3} + 2\zeta\omega_n\dot{\theta}_{1,3} + \omega_n^2\theta_{1,3} = 0 \tag{9}$$

$$\theta_{1,3} = \theta_1 e^{-\omega_n(\zeta + \sqrt{\zeta^2-1})t} + \theta_3 e^{-\omega_n(\zeta - \sqrt{\zeta^2-1})t} \tag{10}$$

$$\theta = \theta_E + \theta_2 e^{\sigma_2 t} + \theta_{1,3} \tag{11}$$

For steady horizontal motion, θ_ε is the neutral angle and may be assigned the value zero. Since the value of $\theta_3 e^{\sigma_3 t}$ is negligible during most of the variation, $\theta_{1,3}$ is effectively identical to θ. Equation (10) may thus be rewritten

$$\theta = \theta_1 e^{-\omega_n (\zeta + \sqrt{\zeta^2 - 1})t} + \theta_3 e^{-\omega_n (\zeta - \sqrt{\zeta^2 - 1})t} \tag{12}$$

For steady motion in the rise or dive direction but in the linear range, it is more realistic to retain the term θ_ε.

$$\theta = \theta_\varepsilon + \theta_1 e^{-\omega_n (\zeta + \sqrt{\zeta^2 - 1})t} + \theta_3 e^{-\omega_n (\zeta - \sqrt{\zeta^2 - 1})t} \tag{13}$$

The parameters ζ and ω_n were selected for their application in vibration theory. ζ is damping ratio and ω_n is the natural angular frequency. These terms may be similarly and appropriately applied to variations in pitch. As they are defined by Equation (9) they may be applied to the entire stability range. Then positive values of ζ indicate stability in pitch, and negative values indicate instability.

Both ζ and ω_n can readily be expressed in terms of the stability roots and can thus be determined from captive model results.

$$\sigma_1 = -\omega_n (\zeta + \sqrt{\zeta^2 - 1}) \tag{14}$$

$$\sigma_3 = -\omega_n (\zeta - \sqrt{\zeta^2 - 1}) \tag{15}$$

$$\omega_n = \sqrt{\sigma_1 \sigma_3} \tag{16}$$

$$\zeta = -\frac{\sigma_1 + \sigma_3}{2\omega_n} \tag{17}$$

For complex values of σ_1 and σ_3

$$\sigma_1 \cdot \sigma_3 = (R_e \sigma_{1,3})^2 + (I_m \sigma_{1,3})^2 \tag{18}$$

$$R_e \sigma_{1,3} = -\zeta \omega_n \tag{19}$$

This is the oscillatory range where the value of ζ lies between +1 and -1. In this range the solution of Equation (9) has an alternate form.

$$\theta_{1,3} = \theta_{01} e^{-\zeta \omega_n t} \operatorname{Cos} (\sqrt{1-\zeta^2}\, \omega_n t - \gamma) \tag{20}$$

This the form most readily applicable to observable oscillations in pitch such as occur during full-scale trials.

The damping ratio and the natural angular frequency convey the same information as the significant stability roots σ_1 and σ_3 in more tangible form. Damping ratio relates directly to the observable similarity of recorded pitch variations to the decay of a damped oscillation.

APPENDIX B

DIAGRAM OF VERTICAL OVERSHOOT MANEUVER

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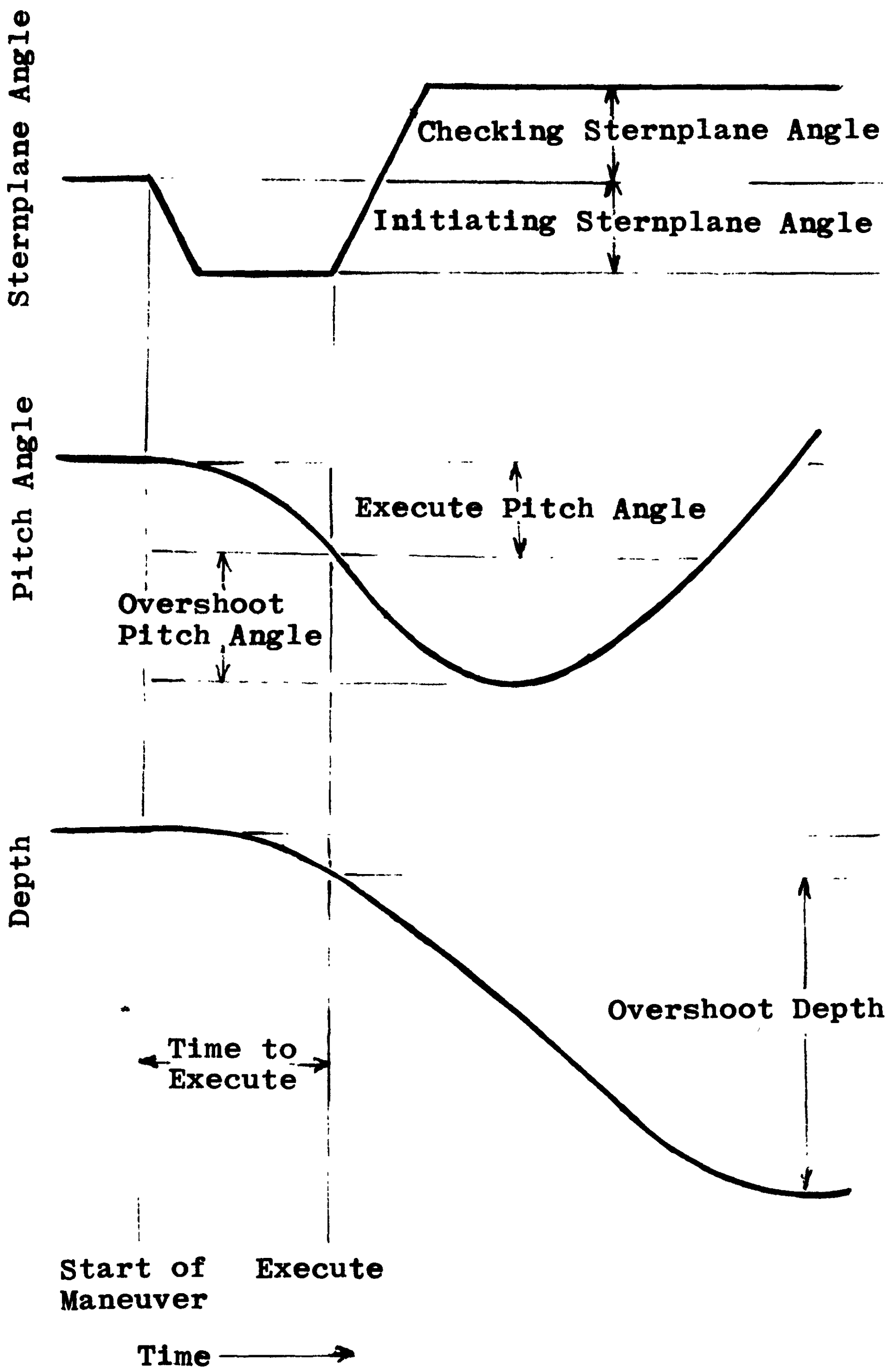

APPENDIX C

EFFECT OF CHANGES IN METACENTRIC HEIGHT ON DAMPING RATIO

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EFFECT OF CHANGES IN METACENTRIC HEIGHT ON DAMPING RATIO

Among the dimensionless values of derivatives, which determine the dimensionless values of damping ratio, natural angular frequency, and the stability roots, there is only one which varies with speed, the metacentric moment derivative. The variations of ζ, ω_n', σ_1', σ_2', and σ_3' which occur with speed are due solely to the variation of M_θ' with speed.

Reported captive model results such as damping ratio, or the stability roots, are based on a design value of metacentric moment or metacentric height. But the actual effective value of metacentric height of an operational submarine frequently differs from the design value.

In applying captive model results to the actual submarine, a simple adjustment may be made to speed to correct for this difference between design and actual values of metacentric height. It is merely necessary to find the reported model prototype speed at which the design value of M_θ' matches the actual full-scale value:

$$V_{KD} = V_K \sqrt{\frac{Z_{BD}}{Z_B}}$$

where V_K is actual full-scale speed

V_{KD} is adjusted prototype speed at which reported results apply to actual full-scale speed

Z_B is actual metacentric height as determined from a submerged inclining experiment

Z_{BD} is design value of metacentric height.

The reported captive model values of ζ, ω_n', σ_1', σ_2', σ_3' can thus be adjusted to any metacentric height. For graphical presentation this can be done by a simple speed-scale change. This adjustment should be considered in applying model results to actual full-scale operation and in making comparisons between model and full-scale results. It also indicates an alternate or supplementary method, with limited application, for the design control of damping ratio.

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13. ABSTRACT

The effect of changing damping ratio upon vertical overshoot characteristics of a submarine are investigated in a simulator study based on model data. The change in damping ratio for the model is effected through a change in horizontal stabilizer surface at the stern. It was found that the optimum combination of overshoot characteristics occurs at values of damping ratio of from 0.6 to 1.0. This is also the range of values at which optimum stability is obtained.

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