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Naval Ship Research and Development Center
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DEPARTMENT OF THE NAVY
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DOMED SONAR SYSTEM

by

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Domed Sonar System

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An analysis of the changes that are induced by a dome in the signal-to-noise (S/N) ratio of a sonar system is performed. Both the transducer system and the dome are idealized in this analysis. A blanket dome and a plate dome are considered. It is shown that a properly designed dome can be made transparent to supersonic spectral components in the external pressure field and can reduce the response of the transducer system to subsonic spectral components in this pressure field. A dome can thus be employed to increase the S/N ratio of a sonar system that is subjected to noise-pressure fields that possess high density of subsonic spectral components. The external pressure field considered in this paper is composed of the blocked pressure field induced by an incident plane acoustic-pressure field (the signal-pressure field) and the blocked pressure field of a subsonic turbulent boundary layer (the noise-pressure field).

INTRODUCTION

A SONAR system provides the most efficient system, if not the only system, for underwater communications. The proper design of a sonar system is, therefore, of the utmost importance to those agencies that have the task of executing underwater communications. Not unlike other systems of communication, the sonar system is beset with noise problems. It is towards the analysis of a method to alleviate some of these noise problems that this paper is directed—in particular, those noise problems that are associated with the noise fields that arise as a consequence of the motion of a planar sonar system through a fluid medium. The method of concern here is the placement of a dome over the transducer system of a sonar system. The dome serves as means for displacing the pressure field in the turbulent boundary layer (that forms over the moving surfaces that are exposed to the fluid) away from the surface of the transducer system. The hope is that this displacement will reduce the response of the transducer system to this noise field without impairing the response to incident acoustic-pressure fields and will thereby increase the signal-to-noise (S/N) ratio of the sonar system. The analysis is conducted primarily in spectral space, the Fourier conjugate space of the real physical space. Although several works are available that bear on this subject, and some of those employ spectral formalism, the reference material is, by and large, limited to Refs. 1–4. Acknowledgments of those other works are contained in these references.

In two recent papers,[1,2] the analysis of a transducer system as a device for measuring the pressure fields in a turbulent boundary layer was presented. It was shown that the transducer system can be considered as a spectral filter. The filtering action of the transducer system was derived as a function of the sensitivities, sizes, and separations between the transducers of the transducer system. For the most part, the analysis dealt with transducer systems of small sizes (of the order of an inch or so). The object was to establish design criteria for a device for measuring the large wavenumbers that are associated with the highly dense spectral region of the pressure fields in the subsonic turbulent boundary layer. Since sonar systems are devices designed to respond to acoustic pressure fields in the low-wavenumber range, their sizes are correspondingly larger, by approximately the ratio of the respective wavenumbers. However, the spectral analysis of both systems is identical. The analysis presented in the two

[1] G. Maidanik and D. W. Jorgensen, "Boundary Wave Vector Filters for the Study of the Pressure Field in a Turbulent Boundary Layer," J. Acoust. Soc. Am. **42**, 494–501 (1967).
[2] G. Maidanik, "Flush-Mounted Pressure Transducer Systems as Spatial and Spectral Filters," J. Acoust. Soc. Am. **42**, 1017–1024 (1967).
[3] G. Maidanik, "Acoustic Radiation from a Driven Infinite Plate Backed by a Parallel Infinite Baffle," J. Acoust. Soc. Am. **42**, 27–31 (1967).
[4] G. Maidanik, "Acoustic Radiation from a Driven Coated Infinite Plate Backed by a Parallel Infinite Baffle," J. Acoust. Soc. Am. **42**, 32–35 (1967).

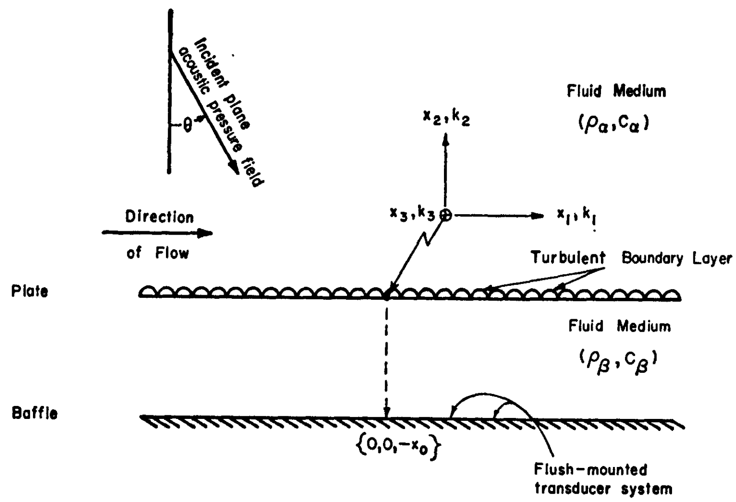

FIG. 1. Domed transducer system and coordinate system.

papers just mentioned can, therefore, be taken whole and applied to the analysis of the transducer system of a planar sonar system. In particular, the concept of the transducer system as a spectral filter is retained and employed extensively in this paper.

In two recent papers[3,4] by the author, the analysis of the motion of an infinite plate placed above an infinite baffle was presented. The baffle constituted a boundary whose spectral reflection coefficient could be specified. The space between the baffle and the plate is filled with a liquid medium; the semi-infinite space above the plate is occupied by the same or a different liquid medium. This construction is closely related to the construction of a dome in a sonar system; the transducer system is embedded flush in the baffle. The analysis in these papers[3,4] was also conducted in spectral space. Thus, the two analyses[1–4] are compatible and can, therefore, be readily combined. In combination, the analyses constitute a format that appears most suitable for the analysis of a domed sonar system.

I. DOMED TRANSDUCER SYSTEM

The transducer system consists of an infinite plane boundary in which are embedded a number of pressure transducers. The transducers are mounted flush with the boundary. The sizes, orientations, and locations of the transducers are specified. Also specified are the sensitivity functions of the transducers.[1] The surface formed by the boundary and the surfaces of the transducers is termed the "baffle." The baffle, therefore, constitutes the surface of the transducer system. It is assumed that the baffle has no surface impedance discontinuities. This assumption allows one to specify the spectral reflection coefficient of the baffle in terms of an algebraic function of the spectral variables.

An infinite plane plate is placed parallel to and a normal distance x_0 above the baffle. The fluid medium that occupies the space between the baffle and the plate possesses a characteristic acoustic impedance $\rho_\beta c_\beta$ (ρ and c denote the density and the speed of sound in a fluid, respectively). The characteristic acoustic impedance of the fluid medium that occupies the semi-infinite space above the plate is denoted by $\rho_\alpha c_\alpha$. The plate and the fluid medium between the plate and the baffle constitute the "dome" over the transducer system. A sketch of the baffle–dome system is shown in Fig. 1. The coordinate system is also sketched in this figure.

The properties of the domed transducer system are assessed in terms of the response of the system (the mean-square of the transducer system output) to external pressure fields. These external pressure fields are assumed to act directly only on the plate member of the system. In order to ascertain the response of the transducer system to such external pressure fields, a procedure is adopted where the motion of the plate is first determined. The knowledge of the motion of the plate is then employed to compute the pressure field on the baffle. Once the expression for the pressure field on the baffle is derived, the response of the transducer system can be calculated from knowledge of the transducer system filtering action.[1,2]

II. FLEXURAL MOTION OF THE PLATE

Recently, the author analyzed and ascertained the motion of a plate, the environment of the plate being identical to that of the plate in the dome system.[3] The plate is acted upon by an arbitrary external pressure field $p_e(\mathbf{x},t)$. The term *external pressure field* is here defined as the blocked pressure field on the plane of the plate; it is assumed that no feedback mechanism exists between the external pressure field system and the baffle–dome system. The equation of motion of the plate can then be expressed[3]

$$z_p(\mathbf{x},t)v_p(\mathbf{x},t)=p_e(\mathbf{x},t)-p_{\alpha\beta}(\mathbf{x},t), \tag{1}$$

where $z_p(\mathbf{x},t)$ is the plate impedance operator, $v_p(\mathbf{x},t)$ is the flexural velocity field on the plate, $p_{\alpha\beta}(\mathbf{x},t)$ is the fluid loading pressure field, $\mathbf{x}$ is the spatial vector variable in the plane of the plate, $\mathbf{x}=\{x_1,x_3\}$, and t is the temporal variable.

It is convenient to cast Eq. 1 in spectral form[3]

$$Z_p(\mathbf{k},\omega)V_p(\mathbf{k},\omega)=P_e(\mathbf{k},\omega)-P_{\alpha\beta}(\mathbf{k},\omega), \tag{2}$$

where Z_p, V_p, P_e, and $P_{\alpha\beta}$ are the Fourier transforms of z_p, v_p, p_e, and $p_{\alpha\beta}$, respectively, $\mathbf{k}$ is the Fourier conjugate vector variable of $\mathbf{x}$; and ω is the Fourier conjugate variable of t.

In Eq. 2, it is assumed that $z_p(x,t)$ is a purely differential operator so that $Z_p(\mathbf{k},\omega)$ is an algebraic function in the vector variable $\{\mathbf{k},\omega\}$. In the case where the plate is thin enough and isotropic[5,6]

$$Z_p(\mathbf{k},\omega)=i\omega m\{1-(k/k_p)^4(1+i\eta_p)\}, \tag{3}$$

where m is the mass per unit area of the plate, k_p is the plate free-wave wavenumber in vacuum, $k=|\mathbf{k}|$, and η_p is the loss factor of the plate.[7,8] This loss factor is assumed to be associated with the stiffness of the plate.[7]

The expression for the spectral fluid loading pressure field was previously derived by the author.[3] This expression is

$$P_{\alpha\beta}(\mathbf{k},\omega)=Z_{\alpha\beta}(\mathbf{k},\omega)V_p(\mathbf{k},\omega), \tag{4}$$

where

$$Z_{\alpha\beta}=(Z_\alpha-Z_\beta)+2Z_\beta[1-R_b\exp(-2ik_{2\beta}x_0)]^{-1}, \tag{5}$$

$$Z_\alpha=\rho_\alpha c_\alpha[1-(c_\alpha k/\omega)^2]^{-\frac{1}{2}}, \tag{6}$$

$$Z_\beta=\rho_\beta c_\beta[1-(c_\beta k/\omega)^2]^{-\frac{1}{2}}, \tag{7}$$

$$k_{2\beta}=[(\omega/c_\beta)^2-k^2]^{\frac{1}{2}}, \tag{8}$$

and R_b is the spectral reflection coefficient of the baffle, $R_b-R_b(\mathbf{k},\omega)$.

The bracketed term in Eq. 5 arises as a consequence of the multiple reflections of the spectral pressure field. These multiple reflections take place between the plate and the baffle. If the spectral reflection coefficient is made zero, Eq. 4 reduces to that equation of the fluid loading that would be obtained if the baffle were removed to infinity $[x_0\to-\infty]$.[3] The reader can readily satisfy himself that setting $R_b=0$ is commensurate with the removal of the baffle to infinity so that the plate is in contact with semi-infinite fluid media on both sides.

From Eq. 2 and 4, one obtains[3]

$$V_p(\mathbf{k},\omega)=P_e(\mathbf{k},\omega)[Z_p(\mathbf{k},\omega)+Z_{\alpha\beta}(\mathbf{k},\omega)]^{-1}. \tag{9}$$

Equation 9 expresses the spectral flexural velocity field on the plate member of the domed sonar system in terms of the external spectral pressure field and the passive elements of the system. Equation 9 is a linear equation in the dynamical quantities V_p and P_e. The spectral pressure field on the baffle is now derived in terms of the external spectral pressure field.

III. PRESSURE FIELD OF THE BAFFLE

Since it is assumed that no external pressure fields are applied directly to the baffle or to the fluid medium that occupies the space between the plate and the baffle, the pressure field on the baffle is directly associated with the flexural motion of the plate. Further, since the plate motion is specified, the boundary that is formed by the plate is an infinite impedance boundary, and its spectral reflection coefficient is, therefore, unity.[3] Taking account of multiple reflections in the space between the plate and the baffle and following a procedure similar to that used in Ref. 3, one can show that the spectral pressure field $P_b(\mathbf{k},\omega)$ on the baffle is given by

$$P_b(\mathbf{k},\omega)=V_p(\mathbf{k},\omega)Z_{\beta b}(\mathbf{k},\omega), \tag{10}$$

where

$$Z_{\beta b}(\mathbf{k},\omega)=\tfrac{1}{2}(1+R_b)2Z_\beta\exp(-ik_{2\beta}x_0)\times[1-R_b\exp(-2ik_{2\beta}x_0)]^{-1}. \tag{11}$$

The term $(1+R_b)$ in Eq. 11 accounts for the incident and reflected spectral components of the pressure field that interact with the baffle. The term $\exp(-ik_{2\beta}x_0)$ accounts for the transfer of the dynamic quantities from the boundary on the plate to that on the baffle. Again, the bracketed term accounts for the multiple reflections. From Eqs. 9 and 10, one obtains

$$P_p(\mathbf{k},\omega)=P_e(\mathbf{k},\omega)\times\{Z_{\beta b}(\mathbf{k},\omega)/[Z_p(\mathbf{k},\omega)+Z_{\alpha\beta}(\mathbf{k},\omega)]\}. \tag{12}$$

Equation 12 is a linear equation that relates the spectral pressure field on the baffle to the external spectral pressure field that drives the plate. The bracketed term is simply the spectral transfer function relating these two pressure fields. If the external pressure field on the plate is stationary, both spatially and temporally, then one can define this pressure field in terms of its spectral density $\Phi_e(\mathbf{k},\omega)$. Denoting by $\Phi_b(\mathbf{k},\omega)$ the spectral density of the pressure field on the baffle, one can readily show that the relationship between these two spectral densities is

$$\Phi_b(\mathbf{k},\omega)=\Phi_e(\mathbf{k},\omega)W_{\alpha\beta}(\mathbf{k},\omega), \tag{13}$$

$$W_{\alpha\beta}(\mathbf{k},\omega)=|Z_{\beta b}/(Z_p+Z_{\alpha\beta})|^2. \tag{14}$$

The quantity $W_{\alpha\beta}(\mathbf{k},\omega)$ may be termed the filtering action of the dome system in analogy with the filtering action of the transducer system.[1,2]

In Refs. 1 and 2, there was derived the expression for the filtering action of a transducer system in which the baffle was considered to be rigid. In this situation, the baffle possesses a spectral reflection coefficient of unity $R_b=1$. In the present analysis, the baffle is considered to possess reflective properites that are less restrictive.

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[5] M. Heckl, "Schallabstrahlung von Platten bei Punktformiger Anregung," Acustica 9, 371–380 (1959).

[6] M. Heckl, "Untersuchungen an Orthotropen Platten," Acustica 10, 109–115 (1960).

[7] G. Maidanik, "The Influence of Fluid Loading on the Radiation from Orthotropic Plates," J. Sound Vibration 3, 288–299 (1966).

[8] G. Maidanik and E. M. Kerwin, Jr., "Influence of Fluid Loading on the Radiation from Infinite Plates below the Critical Frequency," J. Acoust. Soc. Am. 40, 1034–1038 (1966).

However, the reflection of the spectral components of the pressure field on the baffle have been accounted for in Eq. 11 and, therefore, the reflection properties of the baffle are considered in this analysis as part of the dome. In computing the filtering action of the transducer system, it is therefore appropriate to consider the baffle rigid. Thus, the filtering action of the transducer system as computed in Refs. 1 and 2 is applicable whole to the present analysis. {One must, however, remember that in reality the term $[(1+R_b)/2]$ in Eq. 11 describes the transducer system rather than the dome.} If one denotes by $W_b(\mathbf{k},\omega)$ the filtering action of the transducer system under the condition just prescribed, the combined filtering action of the domed transducer system to external boundary pressure fields that act on the plane of the plate is given by

$$W(\mathbf{k},\omega)=W_b(\mathbf{k},\omega)W_{\alpha\beta}(\mathbf{k},\omega). \tag{15}$$

The nature of $W_b(\mathbf{k},\omega)$ has been extensively studied and is, consequently, well known.[1,2] It is assumed that the reader is familiar with the salient features of the filtering action $W_b(\mathbf{k},\omega)$ as described in Refs. 1 and 2. It is further assumed that the reader is familiar with the operational requirement of a transducer system in a sonar system, and that he would therefore have no difficulty in adopting the appropriate functional form of $W_b(\mathbf{k},\omega)$ to satisfy this requirement. In this paper, emphasis is placed on the influence of a dome on a sonar system, and knowledge of $W_b(\mathbf{k},\omega)$ is taken for granted.

The response S of the domed transducer system to an external pressure field having a spectral density $\Phi_e(\mathbf{k},\omega)$ is given by[1,2]

$$S=(2\pi)^{-\frac{3}{2}}\int_{-\infty}^{\infty}d\omega\iint_{-\infty}^{\infty}d\mathbf{k}\Phi_e(\mathbf{k},\omega)W(\mathbf{k},\omega). \tag{16}$$

IV. FILTERING ACTION OF A DOMED TRANSDUCER SYSTEM

In this Section, some aspects of the nature of $W(\mathbf{k},\omega)$, Eq. 15, are examined. Of particular interest in this examination is the rôle played by the filtering action $W_{\alpha\beta}(\mathbf{k},\omega)$ of the dome system. The functional form of $W_{\alpha\beta}(\mathbf{k},\omega)$, Eq. 14, shows no dependence on the choice of the origin of the coordinate system. [This feature is in direct consequence of the spatial homogeneity of the dome system. Should one have introduced some spatial inhomogeneity into the dome system—e.g., a series of ribs on the plate—the functional form of $W_{\alpha\beta}(\mathbf{k},\omega)$ would include information concerning the choice of the origin of the coordinate system.] It is clear then that the dome affects the response of each transducer in the transducer system equally. Indeed, if one designates the spectral sensitivity function of the ith transducer by $H_i(\mathbf{k})$,[1,2] then the spectral sensitivity function of the domed ith transducer is given by $H_i(\mathbf{k})Z_{\beta b}(\mathbf{k},\omega)[Z_p(\mathbf{k},\omega)+Z_{\alpha\beta}(\mathbf{k},\omega)]^{-1}$; see Eq. 12. In fact, once this modification of the spectral sensitivity function of a transducer in the transducer system has been introduced, the dome is fully accounted for. Thus, a dome of the type under consideration in this paper can be accounted for by simply multiplying the auto- and cross-wave-vector filtering action $M_{ii}(\mathbf{k})$ and $M_{ij}(\mathbf{k})$ of the transducers by $W_{\alpha\beta}(\mathbf{k},\omega)$:

$$M_{ij}'(\mathbf{k},\omega)=M_{ij}(\mathbf{k})W_{\alpha\beta}(\mathbf{k},\omega),$$

and by computing the filtering action of the transducer system in the usual manner—i.e., using $M_{ij}'(\mathbf{k},\omega)$ in place of $M_{ij}(\mathbf{k})$ in the formalism.[1,2]

Perusal of Eqs. 5–8, 11, and 14 indicates that it is convenient to distinguish three régimes in spectral space ({$\mathbf{k},\omega$} space) when deciphering the nature of $W_{\alpha\beta}(\mathbf{k},\omega)$. These régimes are defined as

$$k<|\omega/c_\alpha|,\quad |\omega/c_\alpha|<k<|\omega/c_\beta|,\quad\text{and}\quad k>|\omega/c_\beta|, \tag{17}$$

where it is assumed that $c_\alpha>c_\beta$. If this assumption is violated so that $c_\beta>c_\alpha$, c_α and c_β must be interchanged in Eq. 17. Also, if $c_\alpha=c_\beta$, the three régimes degenerate into two régimes. To simplify the discussion as well as the computation, it is assumed that $c_\alpha=c_\beta=c_0$ and $\rho_\alpha=\rho_\beta=\rho_0$. This assumption diminishes somewhat the generality of the consideration; however, many practical situations substantially fall within this assumption. Moreover, many of the features of the influence of the dome on the response of the transducer system are sufficiently general even under this limiting assumption. The removal of this limitation will be considered under separate cover. The function $W_{\alpha\beta}(\mathbf{k},\omega)$, under these conditions, reduces to

$$W_{\alpha\beta}(\mathbf{k},\omega)=|(1+R_b)/2|^2|Z_p'+1|^{-2};\ k<|\omega/c_0|=k_0, \tag{18}$$

$$W_{\alpha\beta}(\mathbf{k},\omega)=|(1+R_b)/2|^2|Z_p''+1|^{-2}\times\exp[-2x_0(k^2-k_0^2)^{\frac{1}{2}}];\ k>k_0, \tag{19}$$

where

$$Z_p'=(Z_p/2\rho_0c_0)[1-(k/k_0)^2]^{\frac{1}{2}}\times\{1-R_b\exp[-2ix_0(k_0^2-k^2)^{\frac{1}{2}}]\}, \tag{20}$$

$$Z_p''=-i(Z_p/2\rho_0c_0)[(k/k_0)^2-1]^{\frac{1}{2}}\times\{1-R_b\exp[-2x_0(k^2-k_0^2)^{\frac{1}{2}}]\}. \tag{21}$$

A feature of particular interest is the exponential decaying term in Eq. 19 and its absence from Eq. 18. The presence of the dome and the consequent displacement of the external pressure field away from the baffle makes it difficult for subsonic ($k>k_0$) spectral components to reach the baffle. On the other hand, the supersonic ($k<k_0$) spectral components do not encounter the same difficulty.[7] Thus, grossly speaking, the dome discriminates primarily against subsonic spectral components; the degree of discrimination is determined, by and large, by the magnitude of the distance parameter x_0 and the disparity between k_0 and k. This feature is well understood, and the absence of the exponential term in Eq. 18 and the appearance of this term in Eq. 19 could have been anticipated on heuristic grounds.

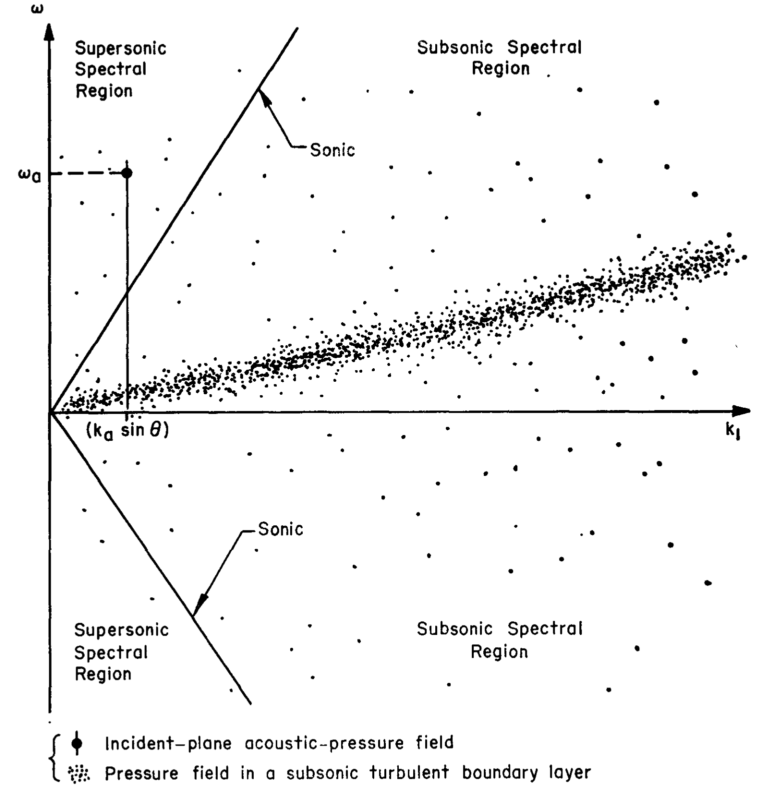

FIG. 2. Spectral density of external pressure field and spectral regions (only first and fourth quadrants in $\{k_1,\omega\}$ plane are shown).

A feature of practical interest is the manner in which the spectral impedance of the plate as well as the multiple reflections influence the filtering action of the domed transducer system. To make the discussion specific, the nature of the external pressure field and the function of the sonar system should be considered. Unfortunately, the detailed nature of the external pressure fields encountered by planar sonar systems are not known at the present time. However, in the analysis, as presented here, the filtering action of the sonar system and the spectral density of the external pressure field are mutually exclusive quantities (see Eqs. 13 and 15). Indeed, this mutual exclusiveness is the feature that makes this method of analysis so attractive. For the purposes of the present discussion, it suffices, therefore, to choose an external pressure field that possesses the variety of spectral components one would expect to find in practical pressure fields.

V. EXTERNAL PRESSURE FIELDS AND THE FUNCTION OF SONAR SYSTEMS

Consideration is limited to two types of external pressure fields, the pressure field in a subsonic turbulent boundary layer and an incident plane acoustic pressure field. The turbulent boundary layer is assumed to be formed on the plane of the plate as shown in Fig. 1. The incident plane acoustic-pressure field is assumed to be substantially pure tone in nature and to possess a single angular direction. The angular direction is chosen so that the boundary pressure field induced by this acoustic pressure propagates in the same direction as does the flow that generates the turbulent boundary layer.[2] The angle of incidence is denoted by θ; it is the angle formed by the plane of incidence and a plane normal to the plane of the plate. It is assumed that the incident plane acoustic pressure field and the pressure field in the turbulent boundary layer are uncorrelated. The spectral density of the external pressure field can then be written in the form

$$\Phi_e(\mathbf{k},\omega)=\Phi_t(\mathbf{k},\omega)+\Phi_a(\mathbf{k},\omega), \tag{22}$$

where $\Phi_t(\mathbf{k},\omega)$ is the spectral density of the (blocked) pressure field in the subsonic turbulent boundary layer, and $\Phi_a(\mathbf{k},\omega)$ is the spectral density of the (blocked) boundary pressure field due to the incident plane acoustic pressure field. It is readily deduced that[2]

$$\Phi_a(\mathbf{k},\omega)=(2\pi)^{\frac{2}{3}}\varphi_a(\omega_a)\delta(\omega c_a^{-1}-k_1)\delta(k_3)\delta(\omega-\omega_a), \tag{23}$$

where $c_a=c_0/\sin\theta$, k_1 is the wave-vector component in the direction of the flow over the plate, k_3 is the wave-vector component in the direction normal to the flow, and ω_a is the pure tone frequency of the incident plane acoustic pressure. The quantity $\varphi_a(\omega_a)$ is the mean-square amplitude of the acoustic-pressure field on the plate taking account of the pressure doubling at the boundary (the blocked pressure field). The spectral density of the external pressure field as defined in Eq. 23 is illustrated in Fig. 2.

For the purposes of the present discussion, it is assumed that the spectral density of the pressure field of the turbulent boundary layer is separable in the form

$$\Phi_t(\mathbf{k},\omega)=\Phi_{t1}(k_1,\omega)\Phi_{t3}(k_3,\omega)\Phi_{t0}(\omega-k_1U), \quad (24)$$

where U is the subsonic convection velocity of the pressure field of the turbulent boundary layer.[2] The nature of the spectral density $\Phi_t(\mathbf{k},\omega)$ is discussed in Refs. 1 and 2; its distribution is sketched in Fig. 2.

The acoustic-pressure field as stated in Eq. 23 is considered to represent the useful signal; the pressure field in the turbulent boundary layer is considered to represent the noise field. Therefore, a properly designed sonar system must have the properties of seeking out the useful signal by responding strongly to it and by simultaneously excluding much of the noise field from inducing a response. In this way, an adequately high S/N ratio in the response can be achieved. It is clear from Fig. 2 that the locus of the spectral density of the external pressure field induced by the incident acoustic-pressure field lies in the supersonic region of $\{\mathbf{k},\omega\}$ space. (Note that the wave vector $\mathbf{k}_a$ of the projected spectral component of a pure tone incident plane acoustic field on the plane of the plate is given by $\mathbf{k}_a=\{k_a \sin\theta,0\}$, $k_a=\omega_a/c_0$, and consequently the spectral components of the external pressure fields induced by incident acoustic fields are invariably supersonic.) The spectral density of the pressure field in a subsonic turbulent boundary layer is chiefly concentrated in the subsonic region of $\{\mathbf{k},\omega\}$ space. The pressure field in a turbulent boundary layer does possess supersonic spectral components; however, their density is relatively low. Thus, a properly designed sonar system must be made to respond efficiently to specific supersonic spectral components in the external pressure field and to respond inefficiently to all other spectral components, particularly to the subsonic spectral components. The degree to which a given design can achieve this requirement is assessed in terms of the filtering action $W(\mathbf{k},\omega)$ of the sonar system. The filtering action $W(\mathbf{k},\omega)$ is composed of two mutually exclusive factors, one pertaining to the filtering action of the transducer system and the other to the filtering action of the dome (see Eq. 15). These factors must be made to complement each other if a high S/N ratio is to be achieved in the response of a domed sonar system.

As was explained in Ref. 2, in order to achieve a high S/N ratio in a transducer system (of a sonar system without a dome), the transducers should possess large geometrical sizes so that a typical linear dimension of a transducer will be large as compared with (U/ω_a) but small as compared with (c_0/ω_a).[9] The referenced typical linear dimension is particularly relevant in the direction of the flow. The effect that the placement configuration of the transducers has on the filtering action of the transducer system was also explained in Ref. 2. The configuration of the transducer system has a marked effect on the filtering action of a transducer system. It is assumed that a suitable configuration is adopted so as to attain a high S/N ratio. Even if these measures are utilized, it is impossible to prevent all subsonic spectral components of the external pressure field from inducing a response because of the difficulty of properly shading a single transducer in the transducer system,[1,2] the periodicity of the major maxima in a multitransducer system,[2] the range of steering capability required, and the presence of minor maxima.[1,2] In this connection, it is noted that the supersonic spectral components of the pressure field in the turbulent boundary layer cannot all be impeded from inducing a response in the transducer system. This is so because the transducer system cannot distinguish between the spectral components of the incident acoustic field and those spectral components of the pressure field in a turbulent boundary layer that occupy the spectral region within the steered maxima in the filtering action of the transducer system. A method that can be employed to reduce the contribution of the supersonic spectral components of the pressure field in a turbulent boundary layer to the response of the transducer system, and thus increase the S/N ratio, is to decrease the major region of the filtering action (using a narrower main beam) of the transducer system.[2] However, with the exception of a remark made in Sec. IX, this topic is outside the scope of the present paper.

A dome is constructed in the hope of modifying the sonar system so that the S/N ratio can be further increased without resorting to further modification of the transducer system itself. An inkling that a dome may be a useful device for this purpose has already been discussed. It was noted that a dome has a tendency to inhibit subsonic spectral components in the external pressure field from inducing a response without affecting as severely the response to the supersonic spectral components.

VI. NO-DOME

In order to establish a criterion for assessing the effectiveness of a domed transducer system as a planar sonar system, it is useful to enquire into the properties of the simplest of domes in the present context, a degenerate dome structure. The degenerate dome is one from which the plate is removed, and the plane of the plate coincides with the surface of the baffle. The properties of the baffle differ from those of the ideal case in that the spectral acoustic reflection coefficient of the baffle need not be set to unity. A degenerate dome is then equivalent to a "no-dome" sonar system. For a degenerate dome, Eqs. 18 and 19 reduce to a single equation

$$W_{\alpha\beta}{}^d(\mathbf{k},\omega)=|(1+R_b)/2|^2. \quad (25)$$

To establish the properties of $W_{\alpha\beta}{}^d(k,\omega)$, Eq. 25, a

[9] G. M. Corcos, "Resolution of Pressure in Turbulence," J. Acoust. Soc. Am. 35, 192–199 (1963).

knowledge of R_b is required. Should one be able to state the expression for the spectral surface impedance $Z_b(\mathbf{k},\omega)$ of the baffle, the spectral reflection coefficient could be derived[10]:

$$R_b = (Z_b - Z_0)(Z_b + Z_0)^{-1}, \tag{26}$$

where $Z_0 = Z_\beta(c_\beta = c_0, \rho_\beta = \rho_0)$; the expression for Z_β was previously stated in Eq. 7.

In the ideal case, the baffle spectral surface impedance Z_b is assumed infinite (rigid baffle), and $R_b = 1$. Should, however, the baffle spectral surface impedance be finite (nonrigid baffle), the situation becomes more complex. In the case of a nonrigid baffle (a situation that includes all baffles of practical construction), it is deduced from Eqs. 7 and 26 that however large Z_b is for near-sonic and sonic spectral components, the spectral reflection coefficient R_b is substantially minus unity. For these spectral components, $W_{\alpha b}{}^d(\mathbf{k},\omega)$ is substantially zero, and the transducer system is rendered ineffective as a sonar system for receiving (or for that matter, transmitting) acoustic pressure fields that are associated with boundary spectral components that lie in the near sonic range. (This phenomenon may be termed the "end-fire catastrophe"!).

Since the spectral reflection coefficient R_b satisfies, in general, the inequality $|R_b| \leq 1$ (see Eq. 26), the sonar system is most effective when the baffle reflective properties can be constructed so that $R_b = 1$ in as wide a range of the supersonic spectral region as the desired range of operation would dictate and as practicality would allow. (This statement does not entertain cases where substantial and controlled changes in R_b in the subsonic region can be made that will result only in slight changes in the values of R_b in the supersonic region, an unlikely practical proposition.) Thus, as a matter of course, one would wish to construct a baffle so that $|Z_b/\rho_\beta c_\beta|$ is large as compared with unity. It is assumed that this condition can be and is achieved. In this situation, the ineffectiveness of the transducer system sets in gradually as the sonar system is steered more and more towards grazing incidence (increase in θ towards $\pi/2$).

The analysis of the influence of the variations in the spectral reflection coefficient on the noise in the response is rather complex and, therefore, only a simplified and brief discussion on this subject is given. Any deterioration in the sonar system with respect to the "main beam" affects the response of the sonar system to supersonic spectral components in the pressure field of a turbulent boundary layer in exactly the same manner as it affects the response to the useful signal. Any change in the S/N ratio from that of an ideal sonar system must then arise from spectral components that do not lie in spectral space within this main beam (the main beam in spectral space of concern here corresponds to the major filtering action region of the sonar system[1,2]). One would expect then to find a decrease in S/N ratio, as compared with that of an ideal system, when the sonar system is steered towards grazing incidence. In support of this expectation, one may argue that the deterioration in the response is confined to spectral components that are near sonic; and since some contribution to the noise is induced by spectral components that are substantially removed from the sonic region, the response to this spectral noise field does not suffer any deterioration. Although such an argument may in general be true, caution must be exercised,nevertheless. To demonstrate the need for caution, it is assumed that Z_b can be expressed in terms of the surface impedance of a thin isotropic plate

$$Z_b(\mathbf{k},\omega) = i\omega m_b[1 - (k/k_b)^4(1+\eta_b),] \tag{27}$$

where the symbols correspond to those defined in Eq. 3. It is assumed that $k_b \gg k_0$ at the frequency of interest. It is further assumed that $\rho_0 c_0/\omega m_b \ll 1$. Thus, the conditions imposed on the sonar system are identical with those previously discussed. From Eqs. 26 and 27, it is deduced that if η_b is small enough so that $k_b m_b \eta_b/\rho_0$ is small as compared with unity, the spectral reflection coefficient becomes minus unity in the limited spectral region defined by $k \simeq k_b$. If a significant part of the noise in the response of an ideal sonar system is induced by spectral components that lie in this confined region of spectral space, the noise in the response is diminished. This reduction in the noise may compensate in part for the deterioration in the response near the sonic region of spectral space, provided that this deterioration is not too severe. The example just cited shows that it is conceivable that a nonideal sonar system may have the better S/N ratio; however, for many reasons the likelihood of a practical situation of this kind is rather remote. Further, such situations, if they exist, would, in general, have ranges in spectral space that span a rather small portion of the sonar operational range.

VII. BLANKET DOME

In this Section, consideration is given to the effect on the performance of a sonar system when provisions are introduced so as to displace the external pressure away from the baffle. To achieve this situation, the plate is considered absent, and, in lieu of the liquid medium in the space between the plane of the "plate" and the baffle, a rubbery material is introduced. The properties of this rubbery blanket are such that the motion in it is substantially identical to that of the fluid medium that it replaced, and yet it should be able to maintain an external pressure field on its top surface. These conditions can be approximately achieved in practice.

For the situation just described, $Z_p(\mathbf{k},\omega) \equiv 0$ and Eqs. 18 and 19 reduce to

$$W_{\alpha\beta}(\mathbf{k},\omega) \simeq |(1+R_b)/2|^2;\ k < k_0, \tag{28}$$

[10] P. M. Morse and H. Feshbach, "Method of Theoretical Physics," (McGraw-Hill Book Co., New York, 1953), Chap. 11.

$$W_{\alpha\beta}(\mathbf{k},\omega) \simeq |(1+R_b)/2|^2\{\exp[-2x_0(k^2-k_0^2)^{\frac{1}{2}}]\};\ k>k_0. \quad (29)$$

It is apparent from Eqs. 28 and 29 that the blanket dome has achieved nothing to alleviate the problem associated with the reflective properties of the baffle. On the other hand, neither has the filtering action of the sonar system with respect to the supersonic spectral components been altered by the blanket; the blanket is transparent to these spectral components. However, with respect to the subsonic spectral components, the situation is changed. One finds that some improvement in the filtering action of the sonar system is achieved; the subsonic spectral components in the external pressure field are inhibited by the blanket from reaching the baffle. The filtering action of the transducer system of the sonar system has to handle, under these circumstances, a reduced density in the subsonic spectral components of the external pressure field. The degree of reduction is determined both by the distance x_0 and the disparity between the wavenumber k of the subsonic spectral components of the external pressure field and the acoustic wavenumber k_0. Thus, the imposition of a rubbery blanket on a transducer system can achieve an improved S/N ratio in a sonar system as compared with that of an equivalent but bare sonar system.

A rubbery blanket of the type just described was examined experimentally employing a single transducer. The external pressure field in this experimental investigation was that of a turbulent boundary layer.[11] It was found that the effect of the blanket was to introduce an apparent increase in the geometrical size of the transducer. Figure 3 demonstrates the analytical form of a typical filtering action of a blanketed transducer, using Eqs. 15, 28, and 29 with $R_b=1$.[12] Figure 3 shows that although the experimental interpretations of the increase in size are in most practical situations grossly valid, in detail, this description is inaccurate. One needs only to consider the locations of the secondary maxima and the minima in the wave-vector filtering action of the transducer to realize that an increase in size is not the proper interpretation. The interpretation, of course, was motivated by the fact that both the increase in size and the imposition of a blanket reduce the normalized response of a transducer to the pressure field in a turbulent boundary layer.[9] A more extensive experimental investigation of this phenomenon will be undertaken in the near future.

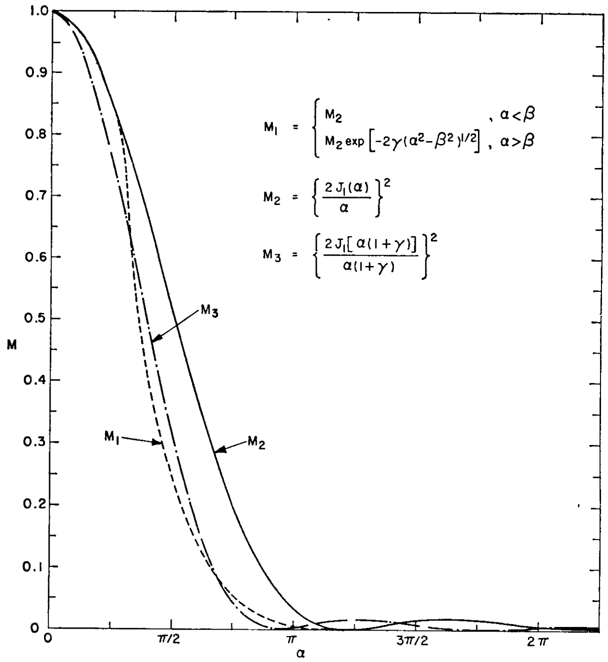

FIG. 3. Wavenumber filtering actions of single uniform circular transducers, one with a blanket dome (M_1). ($\alpha=ka$, $\beta=k_0a=0.3\pi$: $\gamma=0.3$, ratio of thickness of blanket to radius a of transducer, k_0 is acoustic wavenumber in the fluid medium. Dynamic properties of blanket equivalent to those of fluid medium.)

VIII. PLATE DOME

In this Section, the full dome, as defined in Sec. I, is analyzed in some detail. In this situation, Eqs. 18 and 19 apply in full. It is apparent from Eqs. 18, 19, 28, and 29 that the filtering action of the plate in the dome and the filtering action of the "blanket"—the space between the plate and the baffle—are uncoupled. Thus, the difference between the filtering action of the plate dome and an equivalent blanket dome—a blanket dome having the same separation distance x_0 as the plate dome—is completely accounted for by the factor $|Z_p'+1|^{-2}$ and $|Z_p''+1|^{-2}$ in Eqs. 18 and 19, respectively. Consideration, therefore, needs to be focused primarily on these two factors. The behavior of the filtering action of the remaining factors was discussed previously.

In order not to stray far afield, the plate dynamic properties are simplified so that they correspond, throughout the spectral range of significance, with those of a thin isotropic plate. (Modifications in the analysis to account for more complex plate dynamic properties can be readily made; however, these will add much to the complexity of the analysis without substantially contributing to the physical understanding of the phenomena involved.) Thus, $Z_p(\mathbf{k},\omega)$ in Eqs. 20 and 21 is expressed as stated in Eq. 3. It is reasonable to assume that the free-wave wavenumber k_p on the plate is large as compared with the acoustic wavenumber k_0 in the range of frequency of interest. The range of frequency of interest is, therefore, assumed to lie below the critical frequency of the plate.[8] One may then approximate Eq. 20:

$$Z_p' \simeq (i\omega m/2\rho_0c_0)[1-(k/k_0)^2]^{\frac{1}{2}} \times\{1-R_b\exp[-2ix_0(k_0^2-k^2)^{\frac{1}{2}}]\}. \quad (30)$$

[11] F. Schloss, private communcation.

[12] Heaps [H. S. Heaps, "Effects of Turbulence on a Shielded Transducer," J. Acoust. Soc. Am. 40, 1331–1336 (1966)] has conducted a corresponding analysis, utilizing the cross-frequency spectral density as proposed by Corcos, to evaluate the effect of a blanket on the response of a transducer to the pressure field in a turbulent boundary layer. The methodology of his analysis, however, differs from the one presented in this paper.

One of the important requirements of a functional sonar system is that its response to a well-defined acoustic-pressure field be smooth and regular over the desired operational range. Otherwise, the interpretation of the response may become too complex to handle. In addition, under operational conditions, slight but finite variations in the parametric description of the dome and the transducer system occur. These variations may cause fluctuations in the filtering action of the sonar system that would interfere with the reception of the useful signal and its subsequent identification. Because of these circumstances, it is advisable not to introduce delicate dynamic and passive balances into the construction of the sonar system. As a measure in this direction, the absolute value of the normalized spectral impedance $|Z_p'|$ in the supersonic spectral region is chosen small as compared with unity. This measure prevents the sensitive complexity and variability that is inherent in this normalized spectral impedance from influencing significantly the filtering action of the sonar system in the supersonic spectral region. (Some aspects of the variability in Z_p' as a consequence of the multiple-reflections term can be found in Ref. 3.) To ensure that $|Z_p'|\ll 1$, it suffices to ensure that the fluid-loading parameter $2\rho_0c_0/\omega m$ of the plate is large as compared with unity, $2\rho_0c_0/\omega m\gg 1$.[8] It is assumed that the design requirement imposed on the fluid-loading parameter can be accomplished in practice. Under this assumption, the dome is substantially transparent to the supersonic spectral components in the external pressure field; the spectral surface impedance discontinuity in the plane of the plate, and because of the plate, is not sufficiently strong to affect the supersonic spectral components appreciably. For the supersonic spectral components in the external pressure field, the situation is then substantially commensurate with that of an equivalent blanket dome. In particular, the plate dome, as just designed, does not alleviate the problems associated with acoustic pressure fields that are incident at grazing angles ($\theta\rightarrow\pi/2$). It is noted, in this connection, that in the sonic and near-sonic spectral region, $|Z_p'|$ tends towards zero, irrespective of the form of the plate spectral impedance; in this region the acoustic spectral impedance $\rho_0c_0[1-(k/k_0)^2]^{-\frac{1}{2}}$ dominates the dynamic behavior of the plate, see Eq. 30.[7,8]

The influence of a plate in the dome on the filtering action of the domed sonar system in the subsonic spectral region is now considered. In the sonic and near-sonic spectral region, $|Z_p''|$ is substantially zero for the same reasons that were just discussed with respect to $|Z_p'|$; this is particularly true if the fluid-loading parameter is properly chosen. Thus, for this spectral region the influence of the plate in the dome is substantially negligible. For the subsonic spectral region that is removed from the near sonic spectral region, Eq. 21 can be approximated

$$Z_p''\simeq\left(\frac{\omega m}{2\rho_0c_0}\right)\left(\frac{k}{k_0}\right)\left\{1-\left(\frac{k}{k_p}\right)^4(1+i\eta_p)\right\}, \qquad (31)$$

provided that k_0x_0 is not very much smaller than unity, an assumption that is adopted in this paper. Under this assumption, the subsonic spectral components of the external pressure field do not undergo multiple reflections; this is the reason for the absence of the multiple-reflections term in Eq. 31.

It is clear from Eq. 31 that the term $|Z_p''+1|^{-2}$ exhibits a resonance phenomenon. The region in spectral space where this resonance can occur is defined by

$$\frac{\omega m}{2\rho_0c_0}\left(\frac{k}{k_0}\right)\left\{1-\left(\frac{k}{k_p}\right)^4\right\}\simeq -1. \qquad (32)$$

This resonance occurs when the plate impedance, including the fluid loading term, undergoes a resonance. The fluid loading here has the influence of increasing the effective mass of the plate.[7] In the rest of the subsonic spectral region, the influence of inserting the plate in the dome is, if not negligible, to decrease the filtering action of the domed sonar system—a desirable feature. Thus, in the idealized analysis presented in this paper, the major difference between the filtering action of a plate dome and an equivalent blanket dome depends on the behavior of the factor $|Z_p''+1|^{-2}$ in the spectral region where the resonance phenomenon can occur. The value of $|Z_p''+1|^{-2}$ in this spectral region is

$$|Z_p''+1|_r^{-2}\simeq\{\eta_p[1+(\omega m/2\rho_0c_0)(k_r/k_0)]\}^{-2}, \qquad (33)$$

where the subscript r denotes quantities that fall within the spectral region of resonance.

It is readily assessed from Eq. 32 that $k_r>k_p$, and consequently in accordance with the assumption that the frequency of interest is below the critical frequency of the plate, the ratio k_r/k_0 substantially exceeds unity. However, it was also assumed, in order to ensure a proper filtering action in the supersonic spectral region, that the fluid loading parameter $2\rho_0c_0/\omega m$ is large as compared with unity. Thus, unless an appropriate balance can be found so that the quantity $(\omega m/2\rho_0c_0)\times(k_r/k_0)$ can be made large as compared with unity, the loss factor of the plate will have to be made of a value close to unity (which is of considerable practical difficulty to achieve) in order to maintain the filtering action of the plate dome comparable with that of an equivalent blanket dome in this resonance spectral region. However, if other considerations prevent one from either rendering $(\omega m/2\rho_0c_0)(k_r/k_0)$ large or from treating the plate so that η_p approaches the value of unity, it may be necessary to adjust the location of the resonance spectral region so that the density of the spectral components in the external pressure field is low in this spectral region. As compared with the no-dome situation, the resonance phenomenon in the plate dome is not as severe as one may conclude from Eq. 33. The expression for the filtering action of the plate dome has the exponential decaying term $\exp(-2x_0k_r)$ as a factor. With this factor, the filtering action of the domed sonar system can be made superior to that of a no-dome sonar system even in this resonance region of spectral space.

IX. COMPUTATIONAL PROCEDURES

It is not the intention in this Section to embark on a program of computations of the S/N ratio of various sonar systems; this will be done under separate cover. It is, however, the intention in this Section to discuss briefly some of the computational procedures that may prove useful if a computational program is to be undertaken. This discussion should, therefore, be of some value to those who wish to perform computations of S/N ratios of sonar systems that may fall within the scope of the analysis as presented in this paper.

To make the discussion specific, a suitable transducer system is typified. It is assumed that the transducer system of the sonar system consists of nominally identical uniform rectangular transducers. The number of transducers is $N_1 \times N_3$; N_1 is the number of transducers in the x_1 direction and N_3 is the number of transducers in the x_3 direction. The transducers are aligned so that their widths b lie in the x_1 direction (and their lengths l lie in the x_3 direction). The centers of the transducers form a regular rectangular array. The separation between successive transducers that lie in the x_1 direction is designated by d, and the separation between successive transducers that lie in the x_3 direction is designated by e. The time delays between successive transducers that lie in the x_1 direction are assumed equal and are designated by τ_1; the time delays between successive transducers that lie in the x_3 direction are assumed equal and are designated by τ_3. It is further assumed that, in the electrical circuitry of the transducer system, a narrow-band frequency filter is inserted. The width of the band is denoted by 2Δ and the center frequency by ω_0. The frequency filtering action of the frequency band filter is designated by the function $D(\omega,\omega_0,\Delta)$.[1,2] Since the filtering action of the transducer system in a sonar system must provide for a major maximum to lie at the origin of the wave-vector region of spectral space, the appropriate expression for the filtering action of the transducer system is[2]

$$W_b(\mathbf{k},\omega)=8\pi^3D(\omega,\omega_0,\Delta)\,|F(\omega)|^2 \times M(\mathbf{k})S_1(d,k_1,\omega\tau_1)S_3(e,k_3,\omega\tau_3), \quad (34)$$

where

$$S_\alpha(\gamma,k_\alpha,\omega\tau_\alpha)=\frac{\sin^2(N_\alpha\nu_\alpha/2)}{\sin^2(\nu_\alpha/2)}, \quad (35)$$

$$\nu_\alpha=\gamma(k_\alpha+\omega\tau_\alpha/\gamma), \quad (36)$$

$$M(\mathbf{k})=\frac{\sin^2(k_1b/2)}{(k_1b/2)^2}\frac{\sin^2(k_3l/2)}{(k_3l/2)^2}, \quad (37)$$

and $|F(\omega)|^2$ is the frequency filtering action of a transducer in the transducer system. The absolute value of the sensitivity of a transducer is set equal to unity in Eq. 34.[1,2]

In order to steer the sonar system in the direction of the incident-plane acoustic-pressure field as defined in Sec. V, the parameters in the filtering action of the transducer system must be adjusted so that $\tau_3=0$, $\omega_a\tau_1/d=-k_a\sin\theta$; $k_a=\omega_a/c_0$; and $\omega_0=\omega_a$. The resulting filtering action is denoted by $W_b{}^a(\mathbf{k},\omega)$. From Eqs. 16, 23, and 34, one can readily obtain the response S_s of the sonar system to the incident-plane acoustic-pressure field

$$S_s=(8\pi^3)N_1^2N_2^2\varphi_a(\omega_a)D(\omega_a,\omega_a,\Delta)\,|F(\omega_a)|^2 \times[\sin^2(\tfrac{1}{2}bk_a\sin\theta)/(\tfrac{1}{2}bk_a\sin\theta)^2] \times W_{\alpha\beta}(k_a\sin\theta,0,\omega_a). \quad (38)$$

A few features in Eq. 38 are of special interest. The response S_s is proportional to $N_1^2N_2^2$. The frequency filtering action of a transducer should be maintained high over the entire operational frequency range. In order to dispose of the diffraction term, the quantity $\frac{1}{2}bk_a$ must be kept less than unity.

The computation of the response S_n to the pressure field in a turbulent boundary layer is more complex. It is assumed that the frequency-band filter is narrow and has sharp skirts.[1,2] From Eq. 16, one obtains, then

$$S_n=8\Delta(2\pi)^{-\frac{3}{2}}\int_0^\infty dk_1\int_0^\infty dk_3\{\Phi_t(\mathbf{k},\omega_a) \times W_b{}^a(\mathbf{k},\omega_a)W_{\alpha\beta}(\mathbf{k},\omega_a)+\Phi_t(\mathbf{k},-\omega_a) \times W_b{}^a(\mathbf{k},-\omega_a)W_{\alpha\beta}(\mathbf{k},-\omega_a)\}. \quad (39)$$

That the integration can be confined to the first and fourth quadrants in the $\{\omega,k_1\}$ plane was explained in some detail in Ref. 2. Comparison of Eq. 38 with Eq. 39 shows that while S_s is not dependent on the bandwidth Δ, the response S_n is. Thus, in this idealized situation, increasing the frequency bandwidth decreases the S/N ratio of the system. It can be shown that if the quantity N_3lk_a is made large as compared with unity and the transducers are closely packed in the x_3 direction, the integration in Eq. 39 with respect to k_3 can be readily performed[2]:

$$S_n=2\frac{\pi^{-\frac{1}{2}}}{2}\Delta N_3\Phi_t\,(0,\omega_a)\int_0^\infty dk_1\{\Phi_{t_1}(k_1,\omega_a) \times[\Phi_{t_0}(\omega_a-k_1U)W_{b_1}{}^a(k_1,\omega_a) \times W_{\alpha\beta}(k_1,0,\omega_a)+\Phi_{t_0}(\omega_a+k_1U) \times W_{b_1}{}^a(k_1,-\omega_a)W_{\alpha\beta}(k_1,0,-\omega_a)]\}, \quad (40)$$

where

$$W_{b_1}{}^a(k_1,\omega_a)=8\pi^3 D(\omega_a,\omega_a,\Delta)\,|F(\omega_a)|^2 \times S_1(d,\,k_1,\,-dk_a\sin\theta)\frac{\sin^2(k_1b/2)}{(k_1b/2)^2}. \quad (41)$$

From the preceding arguments and discussions, it is clear that it is convenient to split the integral into two parts, one with the limits of integration extending from 0 to k_a and the other from k_a to ∞. The first part covers the supersonic spectral range, and the second covers the subsonic spectral range.

A point of interest is that S_n is proportional to N_3 rather than N_3^2 as is S_s. It can be shown that if $\Phi_{t_1}\Phi_{t_0}$ is substantially constant in the supersonic spectral range, then the supersonic part of the integral is substantially proportional to N_1. Thus, the noise in the response due to the supersonic spectral components in the turbulent boundary layer is proportional to N_1N_3, while the signal in the response due to the incident-plane acoustic-pressure field is proportional $(N_1N_3)^2$. If then the transducer system can be prevented from responding to the subsonic spectral components in the pressure field of a turbulent boundary layer, increasing the number of transducers in the transducer system will greatly enhance the S/N ratio. For a given transducer size and configuration, increasing N_1 and N_3 amounts to an increase in the spatial extent of the array. Such an increase is accompanied by a decrease in the main beam width.[2] Hence, a narrower main beam, under the prescribed conditions, leads to a higher S/N ratio.

In passing, it may be of interest to point out that the closely packed transducer system, both in the x_1 and the x_3 directions, exhibits, in general, a configuration that leads to a higher S/N ratio than any other configuration that employs uniform transducers of similar sizes. This is so provided that one limits the sizes of the transducers such that $k_0b,\ k_0l_2<1$ in the range of frequency of interest. The reason for this is that the closely packed configuration then possesses essentially a single major maximum in the filtering action. These other major maxima that would have appeared if the transducers had degenerated into vanishing sizes would be substantially obliterated by the effect of the finite sizes of the transducers.[1,2] This point is brought out in order to stress the importance of the transducer system configuration in the estimation of the S/N ratio of the sonar system.[2]

At the present time, one does not have an intimate knowledge of the distribution of the spectral components of the pressure field in a turbulent boundary layer. Indeed, the analysis of the wave-vector filter conducted in Refs. 1 and 2 was tailored for the purpose of providing new tools for the study of the spectral content of the pressure field in a turbulent boundary layer. However, for the most part these tools can be employed readily only in the region of high concentration of spectral components.[1,2] It is apparent that in conjunction with blanket domes of the type described previously, the range of these tools can be further extended. Nevertheless, means for obtaining the distribution of the spectral components of the pressure field of a turbulent boundary layer in the supersonic region of the ranges of sonar systems are still lacking. It is hoped that extended utilization of existing devices and some new devices for the study of these spectral components are forthcoming. Because of this experimental situation and a deficiency of a theoretical description of the nature of the pressure field in a turbulent boundary layer, the available data[9] must be used if one wishes to perform computations of which Eq. 40 is typical. Although these data are appropriate primarily to the spectral region of high concentration of spectral components, the extrapolation of these data to other regions is, presently, the only feasible method of obtaining the necessary information.

X. ADDITIONAL COMMENTS

The analysis presented in this paper deals with the idealization of a highly complex system. Further, even with this idealization, it was found that a number of simplifying assumptions had to be made in order to render the analysis amenable to physical interpretation. One may then enquire as to whether these idealizations and assumptions cause the analysis to be so far removed from reality as to make it useless. It will be impossible, within the present text, to argue this point in full. Rather, a few examples are presented with the aim of showing that the analysis in this paper can serve as a format from which an assessment of a more realistic sonar system can be generated.

Following Eq. 17, it was assumed that the liquid media in the spaces of concern be identical in their dynamic properties. If this assumption were relaxed, it would be necessary only to replace Z_p in Eqs. 20 and 21 by $Z_p+(Z_\alpha-Z_\beta)$. The surface spectral impedance $Z_p+(Z_\alpha-Z_\beta)$ simply represents the discontinuity in the surface spectral impedance in the plane of the plate, which is now increased by $(Z_\alpha-Z_\beta)$. Much of the analysis relating to the plate dome (which now is relevant even when $Z_p\equiv 0$) is applicable to this more complex situation. Related to this problem is the more reaslistic description of the dynamical properties of the rubbery material in the case of a blanket dome.

In many practical situations, sonar systems are subjected to flows that are bubble infested in a region close to the surface of the dome. The bubbles contribute spectral components to the external pressure field. The distribution of these spectral components is not unlike those of the pressure field in a turbulent boundary layer since the bubbles are convecting with substantially the flow velocity of the fluid. However, because their sizes are usually fairly uniform, they tend to have a distribution that is more localized in spectral space than do those of the pressure field in a turbulent boundary layer.

Bubbles do not only contribute to the external pressure field, they also impose a compliant layer over the surface of the dome. If the properties of this compliant layer can be stated, the analysis in Ref. 4 can be employed to account for the compliant layer within the format presented in this paper. Related to this problem is a situation where the "plate" may have compliant properties.

The surface of a no-dome sonar system cannot, in practice, be maintained smooth and of uniform surface impedance. These nonuniformities constitute mechanisms whereby spectral components in the external pressure field that lie in a given location of spectral space can be converted into spectral components that lie in other locations of spectral space. The formalism of these mechanisms involves, in general, convolutions of the dynamic quantities, and, therefore, they are not readily handled. Nevertheless, in a few cases the formalisms of such mechanisms have been resolved and are well understood. Reference is made to the problem of acoustic radiation from finite and ribbed plates.[13–15] In the no-dome sonar system, the nonuniformities may convert subsonic spectral components in the external pressure field into supersonic spectral components. Such a conversion will substantially decrease the S/N ratio of the sonar system. A dome may then be used not only as a device to prevent the normal subsonic components of the external pressure field from inducing a response in the transducer system but also to keep these components away from the surface discontinuities. In this sense, a dome may be more effective than is estimated by the present analysis. The argument just presented is valid provided that the surface of the dome does not possess more efficient spectral converters than the baffle. Estimation of the efficiency of the various spectral converting mechanisms that are present in practical sonar systems is an area where further and intensive research is needed. Once the mechanisms are understood, the estimation of their influence can again be accounted for within the format presented in this paper.

Outstanding problems, whose analysis is still wanting and for which the format presented here may require modifications, are those associated with nonstationary external pressure fields (nonstationary, both spatially and temporally). Accounting for the finiteness of practical baffles are other such problems. Related to all these problems are the plate curvature effects. The development of analytical techniques to handle such problems must be forthcoming if the question of "dome–no dome" is to be properly answered.

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1	ASTSECNAV (INSLOG)
2	ONR
1	CO & DIR, USNASL
1	CO & DIR, USNUSL
1	CO & DIR, NMDL
1	CO & DIR, NRDL
1	DIR, NRL
1	CDR, NWL
1	CDR, NOL
1	CO & DIR, NAVCIVENGRLAB
1	CO, NAVMISCEN
1	CO, NAVAIRENGCEN
1	CO, NAVAIRDEVCEN
1	CO, NAVUWRES
1	CDR, NUWC
1	CDR, NWC CHINA LAKE
1	CDR, NWC, Corona Laboratories
1	CDR, NCCCLC
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13. ABSTRACT

An analysis of the changes that are induced by a dome in the signal-to-noise (S/N) ratio of a sonar system is performed. Both the transducer system and the dome are idealized in this analysis. A blanket dome and a plate dome are considered. It is shown that a properly designed dome can be made transparent to supersonic spectral components in the external pressure field and can reduce the response of the transducer system to subsonic spectral components in this pressure field. A dome can thus be employed to increase the S/N ratio of a sonar system that is subjected to noise-pressure fields that possess high density of subsonic spectral components. The external pressure field considered in this paper is composed of the blocked pressure field induced by an incident plane acoustic-pressure field (the signal-pressure field) and the blocked pressure field of a subsonic turbulent boundary layer (the noise-pressure field).

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