Report 3232

NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER

NAVAL SHIP RESEARCH AND DEVELOPMENT LABORATORY

ANNAPOLIS, MARYLAND 21402

THE CORRELATION BETWEEN HEAT AND MOMENTUM TRANSFER FOR SOLUTIONS OF DRAG-REDUCING AGENTS

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DEPARTMENT OF MACHINERY TECHNOLOGY

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The Correlation Between Heat and Momentum Transfer for Solutions of Drag-Reducing Agents

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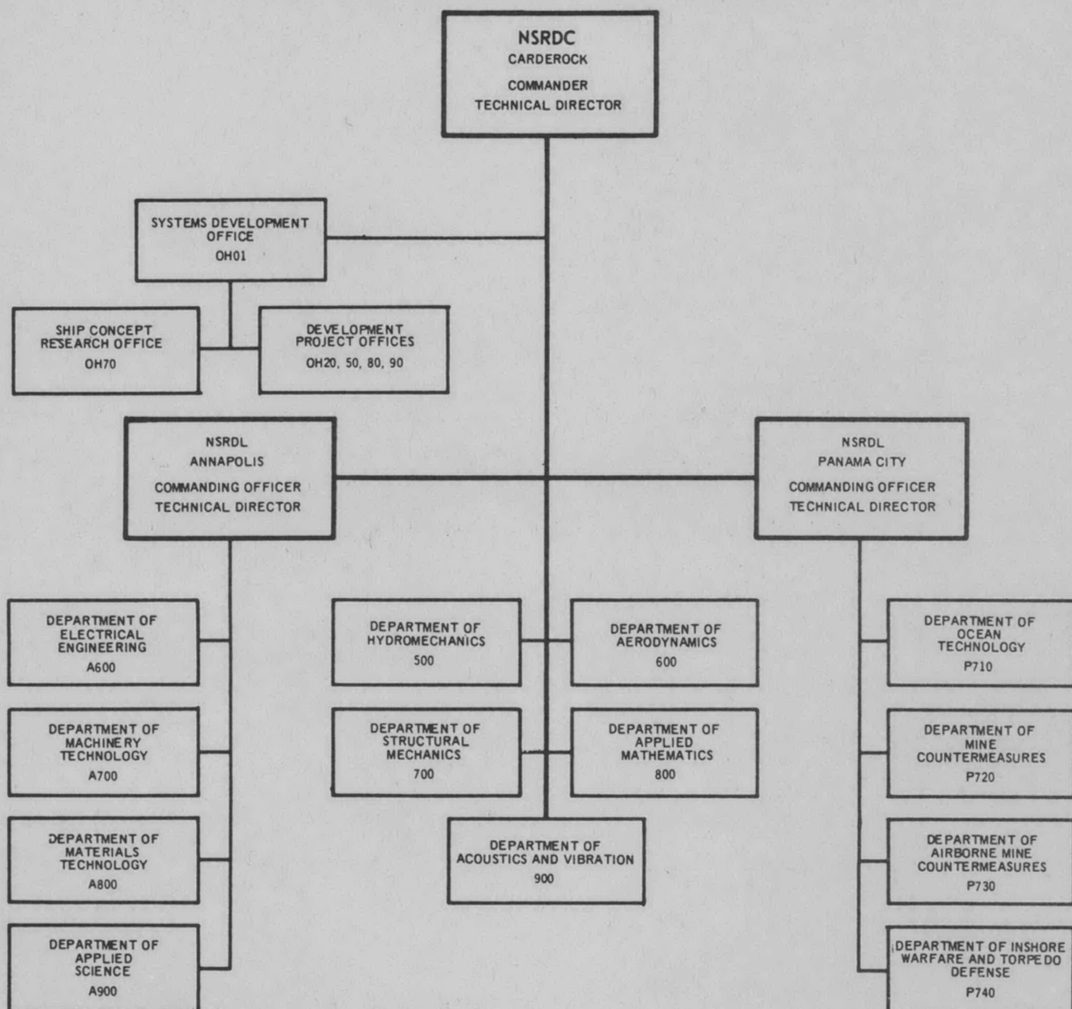

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THE CORRELATION BETWEEN HEAT AND MOMENTUM TRANSFER
FOR SOLUTIONS OF DRAG-REDUCING AGENTS

By
R. G. Howard and D. M. McCrory

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ABSTRACT

Experimental friction factor and heat transfer data from turbulent tube flow of Polyox WSR 301 are correlated through the use of a new method. The Nusselt number is derived for tube flow of drag-reducing solutions in general. The Nusselt number is shown to be a function of Reynolds number, friction factor, and solution properties. This analytical expression for the Nusselt number is compared for accuracy with experimental results.

Studies of relaxation time and refractive index of Polyox are reported.

ADMINISTRATIVE INFORMATION

This investigation was carried out as part of the Independent Exploratory Development Program under Task Area ZFXX 412 00, Work Unit 1-722-162. This work was carried out under the guidance of L. F. Marcous, Head, Piping Systems Branch.

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NOMENCLATURE

A = 2.5 = 1/Prandtl's mixing constant

B = 5.5 = u^+ intercept of $u^+ = A \ln y^+ + B$

$B_1 = \sqrt{\frac{2}{f}} + \frac{3A}{2} - A \ln y_c^+$

C = $q_w D/k \, Re \sqrt{f/2}$, ° F

C_a = Friction factor constant in table 1

C_o = Polyox concentration, ppm

C_p = Fluid specific heat, Btu/lb ° F

D = Inside tube diameter, feet

$f = \frac{2\tau_w}{\rho V^2}$, Friction factor

h = Convection heat transfer coefficient, Btu/hr ° F ft^2

i = Friction factor constant in table 1

I_1 = See equation (21)

I_2 = See equation (22)

I_3 = See equation (23)

I_4 = See equation (24)

I_5 = See equation (B-3)

I_6 = See equation (C-9)

I_7 = See equation (C-10)

I_8 = See equation (C-11)

I_9 = See equation (C-12)

I_{10} = See equation (C-13)

I_{11} = See equation (C-14)

K = Boltzmann constant, 1/° F

k = Fluid thermal conductivity, Btu/hr ft ° F

ℓ^* = Length parameter dependent on Polymer species

M = Polymer molecular weight, per mole

m = Number of rectangles being summed by numerical integration

N = Avogadro number, molecules/slug-mole

n = Index of summation

Nu = hD/k, Nusselt number

P = Polymer species

Pr = $\rho\nu$ Cp/k, Prandtl number

q_w = Heat transfer at the tube wall, Btu/hr ft^2

q_y = Heat transfer at a distance y from the tube wall, Btu/hr ft^2

r = Tube radius, feet

Re = DV/ν, Reynolds number

St = Nu/RePr, Stanton number

T = Temperature at a point, ° F

T_b = Bulk temperature defined in equation (20), ° F

T_L = Temperature at y_L^+, ° F

T_w = Inside tube wall temperature, ° F

t = Polymer relaxation time, sec

u = Fluid velocity at a point, fps

$u^+ = u/u_*$

$u_* = \sqrt{\tau w/\rho}$, shear velocity, fps

$u_L^+ = u^+$ at y_L^+, fps

V = Average fluid velocity, fps

$X_L = [(T_w - T)\ kRe \sqrt{f/2}]\ /q_w\ D \quad 0 \leq y^+ \leq y_L^+$

$X_T' = [(T_L - T)\ kRe\ \sqrt{f/2}]\ /q_w\ D$ when $y_L^+ \leq y^+ \leq y_c^+$

$X_T = [(T_w - T)\ kRe\ \sqrt{f/2}]\ /q_w\ D$ when $y_L^+ \leq y^+ \leq y_c^+$

y = Distance from tube wall, feet

y_L = Distance, y to the edge of the viscous sublayer, feet

$y^+ = y\ u_*/\nu$

$y_L^+ = y_L\ u_*/\nu$

$y_c^+ = r\ u_*/\nu$

α = $k/\rho C_p$, thermal diffusivity, ft^2/sec

β = See equation (13)

ε_h = Eddy diffusivity for heat, ft^2/sec

ε_m = Eddy diffusivity for momentum, ft^2/sec

θ_1 = See equation (B-4)

θ_2 = See equation (B-5)

θ_3 = See equation (B-6)

θ_4 = See equation (C-19)

$\lambda = \varepsilon_h/\varepsilon_m = 1.0$

ν = Solution kinematic viscosity, ft^2/sec

ν_o = Solvent kinematic viscosity, ft^2/sec

ρ = Fluid density, $slug/ft^3$

τ_w = Shear stress at tube wall, lb/ft^2

τ_y = Shear stress at a distance y from wall, lb/ft^2

NAVAL SHIP RESEARCH AND DEVELOPMENT LABORATORY

THE CORRELATION BETWEEN HEAT AND MOMENTUM TRANSFER FOR SOLUTIONS OF DRAG-REDUCING AGENTS

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INTRODUCTION

The phenomenon of turbulent drag reduction[1] through the use of polymer additives promises some useful applications. Polyethylene oxide WSR 301 (Polyox), for example, reduces the coefficient of friction for the turbulent flow of water in the boundary layer of a moving flat plate.[2] The friction factor in turbulent pipe flow is also reduced by Polyox.[3] Van Driest[4] explains the use of drag reduction in obtaining high speeds of ships. Application of the drag-reducing additives to practical situations, however, involves reducing the coefficient of convective heat transfer also. It is necessary to know as well as possible what the heat transfer reduction will be in a given case to avoid any problems that may arise. Heat transfer experiments are expensive and difficult to perform with sufficient accuracy to suit most engineers. With this in mind, many investigators[5,6,7,8] have correlated heat transfer reduction with friction reduction so that pipe friction experiments can be substituted for heat transfer experiments. Howard and Marcous[9] explained the mechanism of the analogy between the two phenomena. This report is an extension of that work. It results in an explicit correlation between reductions in friction factor and in heat transfer coefficient for turbulent tube flow of Polyox solutions.

The correlation is formulated in terms of the Nusselt number, Nu, and the definitions of conduction and convection. Convection in the turbulent core is defined by

[1]Superscripts refer to similarly numbered entries in the Technical References at the end of the text.

$$q_w = h(T_w - T_b) \text{ for } y \geq y_L . \qquad .....(1)$$

Heat conduction across the viscous sublayer is described by

$$q_w = k\left(\frac{\partial T}{\partial y}\right) \text{ for } 0 \leq y \leq y_L . \qquad .....(2)$$

In tube flow, it is obvious that heat conducted across the viscous sublayer equals heat convected by the turbulent core. Eliminating q_w from equations (1) and (2) and introducing D:

$$Nu = \frac{hD}{k} = \frac{-D}{T_w - T_b}\frac{\partial T}{\partial y} . \qquad .....(3)$$

The problem of determining Nu is now reduced to finding T_b and $(\partial T)/(\partial y)$ analytically since D and T_w can be measured. $(\partial T)/(\partial y)$ is determined from the temperature variation across the viscous sublayer. T_b is found by integrating the temperature across the tube.

TEMPERATURE PROFILE

To obtain the bulk temperature, T_b, first the temperature profile across the inside radius of the tube must be derived. A basic description of heat transfer is,

$$\frac{q_y}{\rho C_p} = -(\alpha + \varepsilon_h)\frac{dT}{dy} . \qquad .....(4)$$

A basic description of momentum transport is given as,

$$\frac{\tau_y}{\rho} = (\nu + \varepsilon_m)\frac{du}{dy} . \qquad .....(5)$$

In terms of the wall shear stress,

$$\tau_y = \tau_w\left(1 - \frac{y}{r}\right) . \qquad .....(6)$$

Martinelli[10] obtained,

$$q_y = q_w\left(1 - \frac{y}{r}\right) \qquad .....(7)$$

as a first approximation in the turbulent core. In the viscous sublayer, however, Martinelli used the more accurate,

$$q_y = q_w . \qquad .....(8)$$

The temperature profile can be obtained by using equations (4) through (8) and the following conditions:

$\alpha >> \varepsilon_h$ and $\nu >> \varepsilon_m$ when $0 \leq y^+ \leq y_L^+$ (condition 1)

$\alpha >> \varepsilon_h$ and $\nu >> \varepsilon_m$ when $y_L^+ \leq y^+ \leq y_C^+$ (condition 2).

VISCOUS SUBLAYER TEMPERATURE PROFILE

Equations (4) and (8) may be written as,

$$\int_{T_w}^{T} dT = -\int_0^{y^+} \frac{q_w \nu}{u_* k}\, dy^+$$

by using condition 1.

Integrating this:

$$T_w - T = \frac{q_w \nu}{u_* k} y^+ .$$

Let the dimensionless temperature difference be:

$$X_L = \frac{[(T_w - T) \, kRe\sqrt{f/2}]}{q_w \, D} . \qquad .....(9)$$

Since $_w = (f\rho v^2)/(2)$, we obtain a relationship

$$Re\sqrt{\frac{f}{2}} = \frac{Du_*}{\nu} \text{ or } y_c^+ = \frac{Re}{2}\sqrt{\frac{f}{2}}$$

so

$$X_L = y^+ \text{ for } 0 \leq y^+ \leq y_L^+ . \qquad .....(10)$$

THE QUANTITY $(\partial T)/(\partial y)$

From equations (9) and (10) we can derive

$$\frac{\partial T}{\partial y} = \frac{T_L - T_w}{y_L} = \frac{-q_w}{k} \qquad .....(11)$$

for use in equation (3).

VISCOUS SUBLAYER THICKNESS

Elata[11] used

$$y_L^+ = \frac{y_L u_*}{\nu} = 11.6 + \frac{\beta}{\ln 10}\left[\ln\left(\frac{u_*^2 t}{\nu}\right)\right] \qquad .....(12)$$

Elata used an equation equivalent to

$$\beta = \left[\sqrt{\frac{2}{f}} - A\ln\left(\frac{Re\ \sqrt{f}}{2\ \sqrt{2}}\right) + \frac{3A}{2} - B\right] \frac{\ln 10}{\ln\left(\frac{u_*^2 t}{\nu}\right)} \qquad \ldots\ldots(13)$$

substituting this into equation (12)

$$y_L^+ = 11.6 + \left[\sqrt{\frac{2}{f}} - A\ln\left(\frac{Re\ \sqrt{f}}{2\ \sqrt{2}}\right) + \frac{3A}{2} - B\right] \cdot \qquad \ldots\ldots(14)$$

Note that t, the relaxation time, is eliminated from equation (14) Though t is not explicitly used, it was investigated experimentall as reported in appendix A.

TURBULENT CORE TEMPERATURE PROFILE

Elata suggests

$$u^+ = A\ln\ (y^+) + B + \frac{\beta}{\ln 10} \ln\left(\frac{u_*^2 t}{\nu}\right)$$

as a description of the velocity profile in the turbulent core of a viscoelastic fluid. Substituting equation (13) for β, we get

$$u^+ = A\ln y^+ + \sqrt{\frac{2}{f}} + \frac{3A}{2} - A\ln y_c^+ \cdot \qquad \ldots\ldots(15)$$

Granville[12] has suggested formulating the experimental determination of the thickening of the viscous sublayer in terms of a function $B_1 = B_1\ (\ell^*, C_o, P)$. B_1 is the u^+ intercept of equation (15) when $y^+ = 1$. ℓ^* is an experimental dummy parameter with a value dependent on polymer species, P, and flow conditions. We suggest using

$$B_1 = \sqrt{2/f} + 3A/2 - A \ln y_C^+ \,.$$

B_1 could be determined from pipe flow friction data for various concentrations of a given polymer such as Polyox WSR 301.

We shall use equations (4) and (7) to get

$$\int_{T_L}^{T} dT = \frac{-q_w \nu}{r\rho C_p u_*^2} \int_{y_L^+}^{y^+} \frac{(ru_* - y^+\nu)\, dy^+}{\varepsilon_h + \alpha} \,. \qquad \ldots\ldots (16)$$

Now condition 2 allows us both to neglect α and to convert equations (5) and (6) to:

$$\varepsilon_h = \lambda\varepsilon_m = \lambda\nu \left[\frac{ru_* - y^+\nu}{ru_* \dfrac{du^+}{dy^+}} \right] .$$

Where we have assumed analogous mechanisms for heat and momentum transfer as indicated by $\varepsilon_h = \lambda\varepsilon_m$. Substituting for ε_h into equation (16) then, we have:

$$\int_{T_L}^{T} dT = \frac{-q_w}{\rho C_p u_* \lambda} \int_{y_L^+}^{y^+} \frac{du^+}{dy^+}\, dy^+ \,. \qquad \ldots\ldots (17)$$

From equation (15)

$$\frac{du^+}{dy^+} = \frac{A}{y^+} \,.$$

Using this in equation (17), and integrating

$$X_T' = \frac{A}{Pr\lambda} \ln\left(\frac{y^+}{y_L^+}\right) \quad \text{and finally,}$$

$$X_T = \frac{A}{Pr\lambda} \ln\left(\frac{y^+}{y_L^+}\right) + y_L^+ \qquad \text{.....(18)}$$

for

$$y_L^+ \leq y^+ \leq y_c^+$$

from equation (18):

$$T = T_w - \frac{q_w D}{kRe\sqrt{\frac{f}{2}}}\left[\frac{A}{Pr\lambda} \ln\left(\frac{y^+}{y_L^+}\right) + y_L^+\right] \qquad \text{.....(19)}$$

for $y_L^+ \leq y^+ \leq y_c^+$.

DERIVATION OF THE BULK TEMPERATURE

T_b is defined as

$$T_b \equiv \frac{\int_o^{y_c^+} \left[\left(T\rho C_p \ u^+ y^+ \nu^2\right)/u_*\right] dy^+}{\int_o^{y_c^+} \left[\left(\rho C_p \ u^+ y^+ \nu^2\right)/u_*\right] dy^+} \ . \qquad \text{.....(20)}$$

Integrate T_b in two intervals between the limits $0 \leq y^+ \leq y_L^+$ and $y_L^+ \leq y^+ \leq y_c^+$. If we let

$$I_1 = \int_{o}^{y_L^+} T\, u^+ y^+ dy^+ \qquad .....(21)$$

$$I_2 = \int_{y_L^+}^{y_c^+} T\, u^+ y^+ dy^+ \qquad .....(22)$$

$$I_3 = \int_{o}^{y_L^+} u^+ y^+ dy^+ \qquad .....(23)$$

$$I_4 = \int_{y_L^+}^{y_c^+} u^+ y^+ dy^+ \, . \qquad .....(24)$$

Substitution into equation (20) yields:

$$T_b = \frac{I_1 + I_2}{I_3 + I_4} \, . \qquad .....(25)$$

The integration of equations (21) to (24) is given in appendix B.

CORRELATION BETWEEN HEAT TRANSFER COEFFICIENT AND FRICTION FACTOR

The Nusselt number in equation (3) can now be evaluated using equation (25) for T_b and equation (11) for $(\partial T)/(\partial y)$:

$$\mathbf{Nu} = \left[\frac{D}{T_w - \left(\frac{I_1 + I_2}{I_3 + I_4} \right)} \right] \frac{q_w}{k}$$

Substituting for I, I_2, I_3 and I_4 yields

$$Nu = \frac{hD}{k} = Re \sqrt{\frac{f}{2}} \left[\frac{I_3 + \theta_1}{\frac{3y_L^+ I_3}{4} + y_L^+ \theta_1 + \frac{B_1 A \theta_2}{Pr\lambda} + \frac{A^2 \theta_3}{Pr\lambda}} \right] . \quad \ldots\ldots (26)$$

The algebra necessary to derive equation (26) was considerable. Therefore, the Nusselt number, Nu, was derived by numerical integration of the elementary integrals as a check against algebraic errors. This work is reported in appendix C.

DISCUSSION: AGREEMENT WITH EXPERIMENT

It was important to know how well equation (26) agreed with the experiment. The experimental data on flow of Polyox solutions used to test this agreement was reported earlier.[13] To test the agreement with the experiment, the Nusselt number was converted to the Stanton number, St = (Nu)/(Re Pr). The similarity laws of friction-reduced flow data were considered next.

It must be remembered that the Reynolds number is not a sufficient similarity parameter for modeling drag reduction in tube flow. It has often been observed that for tubes of different diameters $D_1 < D_2$, for equal Polyox concentration, C_b, flowing in these tubes, and for equal Re that the friction factors are not equal

$$f_1 < f_2 .$$

Thus, the constants in

$$f = C_a \, Re^i$$

determined elsewhere[13] and recorded in table 1 are useful only for tube diameters of about 1/2 inch. They are used here to evaluate St in a test of the assumptions of the theory behind equation (26).

Table 1
Polyox Solution Friction
Factor Constants
in $f = C_a \, Re^i$ for D = 0.5 Inch

Concentration of Polyox ppm*	Ca	i
o	0.0775	-0.245
5	0.600	-0.480
12.5	4.80	-0.739
50	5.55	-0.762
100	6.65	-0.787
200	11.3	-0.857

The St versus Re correlations showing the agreement between experiment and theory are given in figures 1 through 6. The following values were used in the computer evaluation (26) for a range of Re.

$$A = 2.5$$

$$Pr = 7$$

$$\lambda = 1$$

$$y_c^+ = \frac{Re}{2} \sqrt{\frac{f}{2}} \; .$$

*Abbreviations used in this text are from the GPO Style Manual, 1967, unless otherwise noted.

From equation (15) and the fact that

$$u^+ = y^+ \quad \text{for} \quad y^+ \leq y_L^+$$

we know that

$$u_L^+ = y_L^+ = A\ln y_L^+ + B_1 \text{ when } y^+ = y_L^+ \cdot \qquad .....(27)$$

Equation (27) was solved for y_L^+ by reiteration. Some of the y_L^+ computed were listed in table 2. Note that y_L^+ grows with Polyox concentration. θ_1, θ_2, θ_3 and I_3 were computed by substitution of the previously determined values into the appropriate equations. Finally, St was computed.

Certain research was carried out incidentally to the task at hand. We were always receptive to new ideas for on-line, continuous, Polyox concentration determination. This receptivity led to the study of refractive index versus Polyox concentration detailed in appendix D.

Table 2
The Thickening of the Viscous Sublayer
Due to Polyox Additive

Concentration of Polyox ppm	Re	Y_L^+
0	10,000-100,000	About 11.6
5	10,000	12.5
	100,000	21.8
12.5	10,000	16.3
	100,000	41.0
50	10,000	17.2
	100,000	44.1
100	10,000	17.8
	100,000	47.1
200	10,000	19.4
	100,000	55.4

CONCLUSIONS

The limitation on internal heat transfer is conduction through the viscous sublayer. Thickening of the sublayer due to Polyox causes a reduction in heat transfer. Using friction factor and associated Reynolds number data for drag-reducing solutions in turbulent tube flow, equation (26) may be used to estimate heat transfer under the same conditions. The specific data provided herein on Polyox can be used to estimate heat transfer in Polyox solutions up to 200 ppm in concentration but only in tubes of about 1/2 inch diameter.

There is insufficient variation of the index of refraction of Polyox solutions to correlate this property to solution concentration.

RECOMMENDATIONS

Turbulent tube flow friction experiments should be substituted for tube flow heat transfer experiments whenever practical. The results from the friction experiments should be used with equation (26) to estimate heat transfer in solutions with reduced drag.

The relationship between the observed phenomena of turbulence suppression and thickening of the viscous sublayer by drag-reducing additives should be investigated.

TECHNICAL REFERENCES

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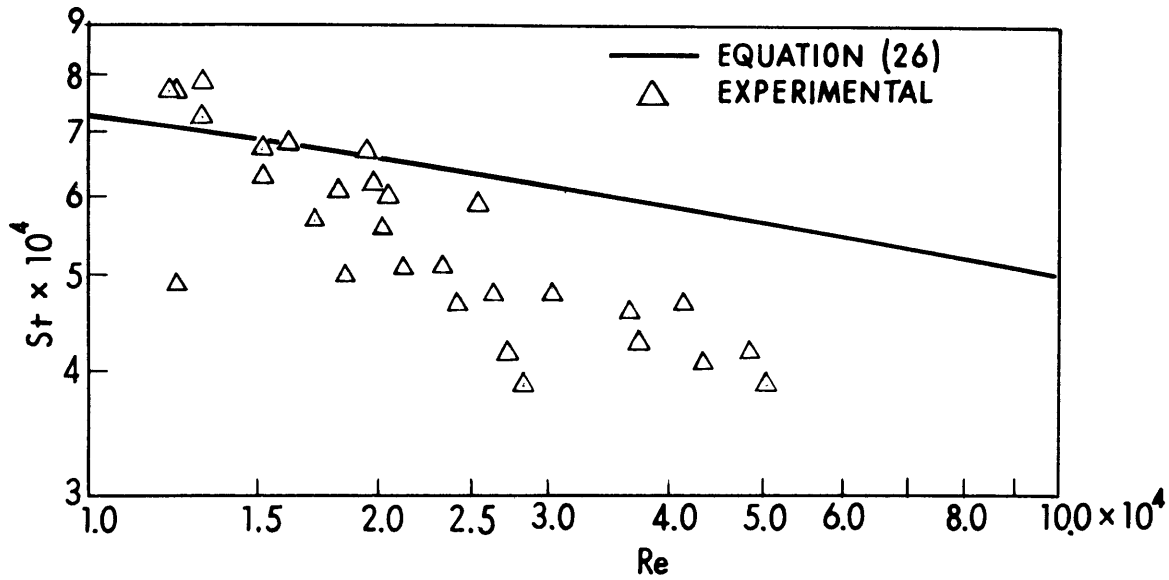

Figure 1
Heat Transfer in Tap Water

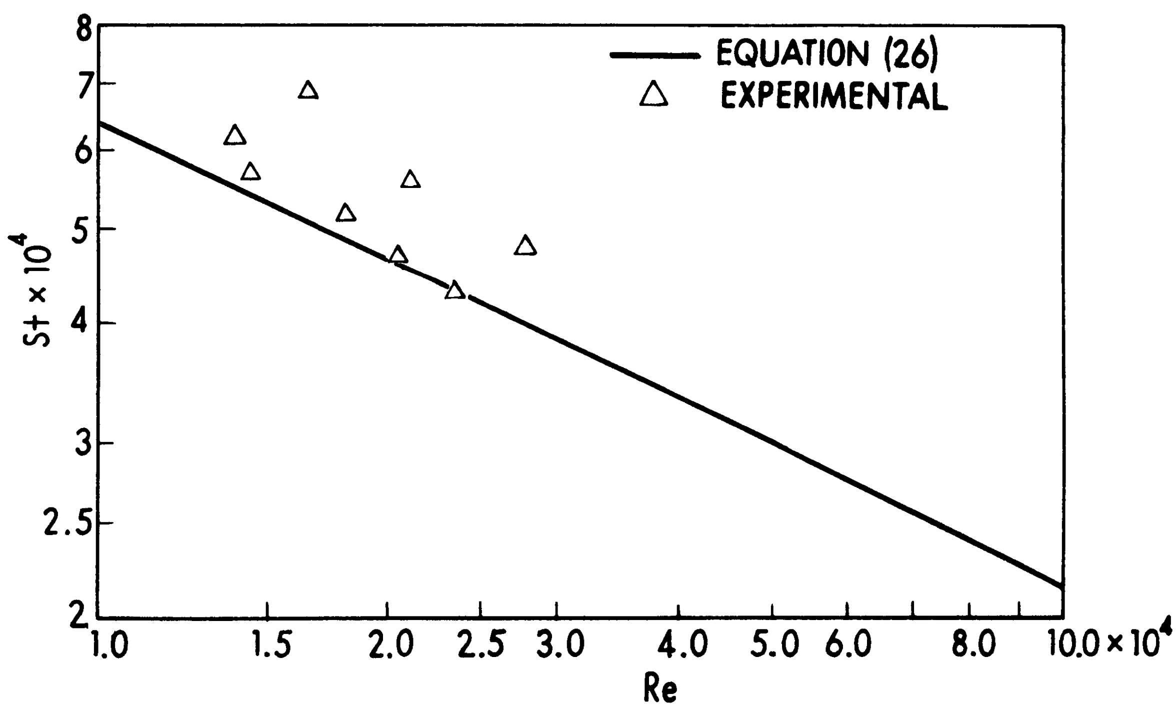

Figure 2
Heat Transfer in 5 PPM Polyox Solution

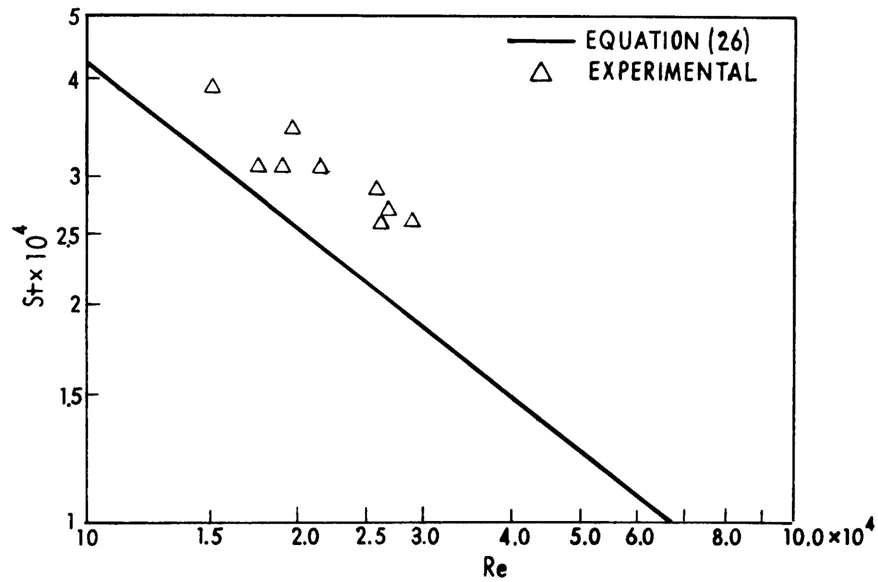

Figure 3
Heat Transfer in 12.5 PPM Polyox Solution

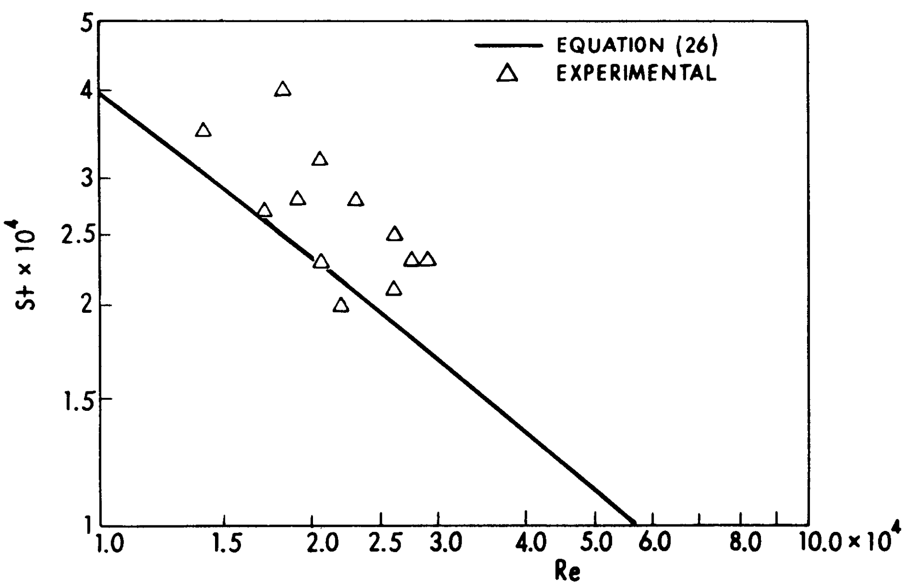

Figure 4
Heat Transfer in 50 PPM Polyox Solution

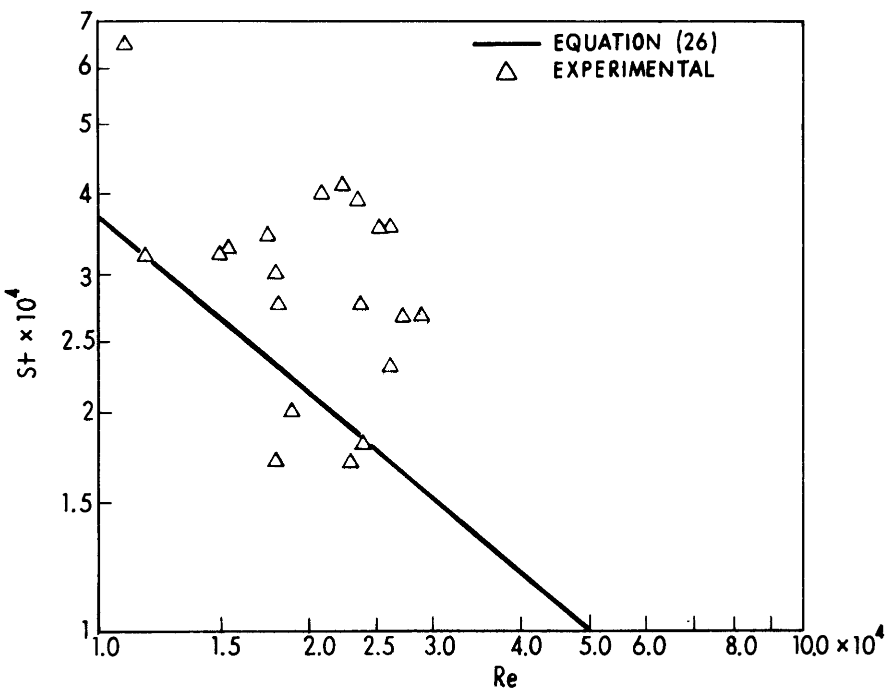

Figure 5
Heat Transfer in 100 PPM Polyox Solution

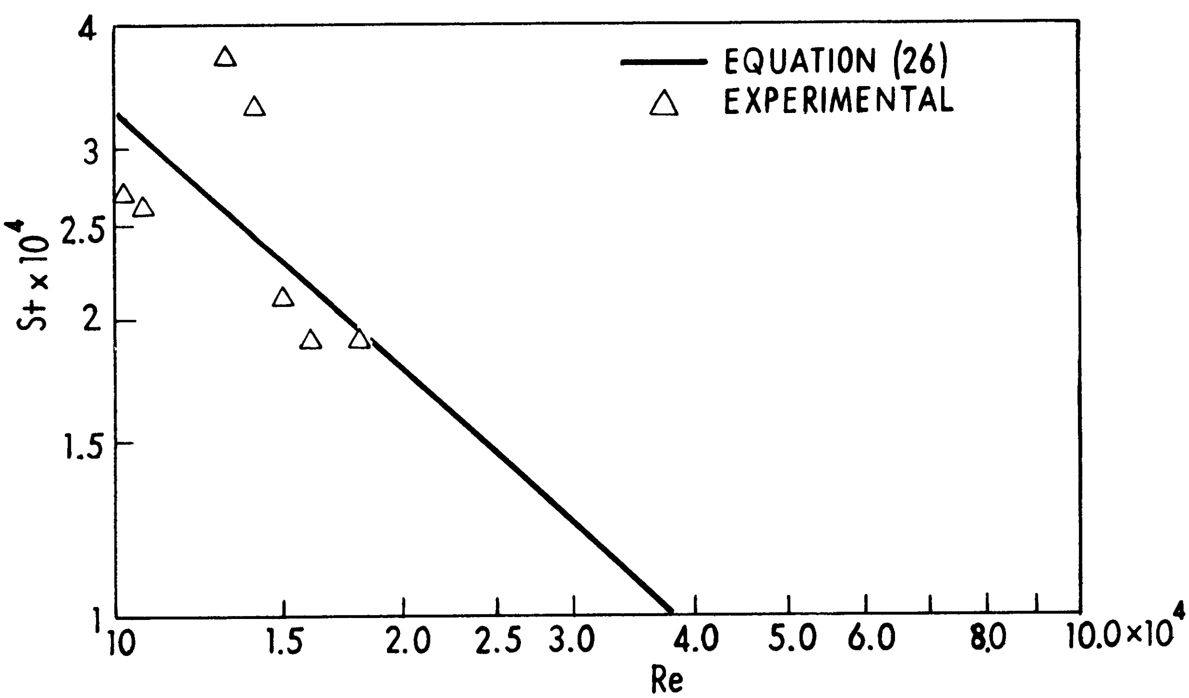

Figure 6
Heat Transfer in 200 PPM Polyox Solution

Appendix A

Relaxation Time for Polyox

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The relaxation time for Polyox was studied because it was a parameter of interest in an earlier investigation (reference (a)) and in the β term of this investigation. Elata (reference (b)) suggested using this formula for relaxation time:

$$t = \frac{6\rho M}{\pi kTN} \left(\frac{\nu - \nu_o}{C_o} \right) \cdot \qquad \text{.....(A-1)}$$

These values were used for the constants

$\rho = 1.927$ slug/ft^3

$\nu_o = 0.739 \times 10^{-5}$ ft^2/sec

$M = 3.8 \times 10^6$ per mole

$\pi = 3.142$

$T = 310.7$ ° K

$k = 13.1 \times 10^{-27}$ Btu/° K

$N = 8.79 \times 10^{27}$ molecules/slug-mole.

The units of concentration C_o were, for example, 1.939×10^{-4} slug/ft^3 for 100 ppm Polyox. The following tabulation gives the Polyox concentrations, viscosities, and relaxation times. Figure 1-A provides a graphic demonstration of Polyox concentratio versus relaxation time, t. Notice that t appears to increase with decrease in Polyox concentration below 200 ppm. This is due to the small change on viscosity in this region and to the subtraction of two nearly equal values in equation (A-1). As C_o decreases and $\nu - \nu_o$ changes little, then t increases.

Polyox Properties

Concentration of Polyox ppm	$\nu \times 10^6$ ft^2/sec	t, sec
12.5	0.760	1.079
50	0.794	0.706
100	0.828	0.568
200	9.14	0.504
500	12.4	0.574
1000	21.2	0.793
1500	32.6	0.966
2000	47.5	1.152

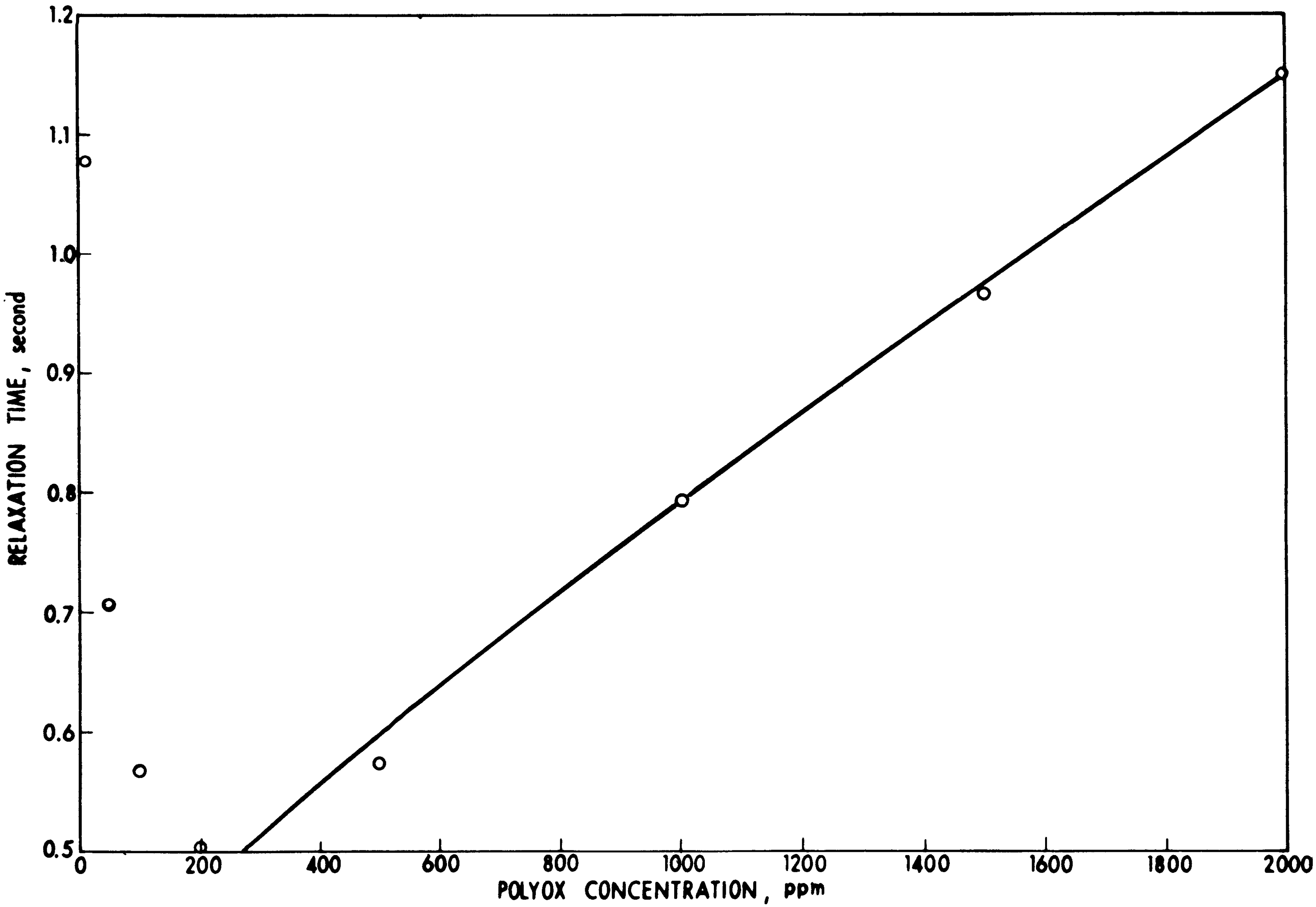

Figure 1-A - Polyox WSR 301 Relaxation Time

Appendix B

Integration of I_1, I_2, I_3 and I_4

We need functions $T = T(y^+)$ and $u^+ = u(y^+)$ for equations 21 through 24. We use $u^+ = y^+$ when $0 \leq y^+ \leq y_L^+$ and from equations (9) and (10)

$$T = T_w - \frac{q_w y^+ \nu}{k u_*} \text{ when } 0 \leq y^+ \leq y_L^+ . \qquad \text{.....(B-1)}$$

Between $y_L^+ \leq y^+ \leq y_C^+$, u^+ is given in equation (15) and T is given in equation (19). Perform the integration and cancel $(\nu^2)/(u_*)$, then using equation (B-1)

$$I_1 = \frac{{y_L^+}^3}{3}\left(T_w - \frac{3 q_w y_L^+ \nu}{4 k u_*}\right)$$

$$I_3 = \frac{{y_L^+}^3}{3}$$

$$I_4 = \frac{{y_c^+}^2}{2}\left(A \ln y_c^+ - \frac{A}{2} + B_1\right) - \frac{{y_L^+}^2}{2}\left(A \ln y_L^+ - \frac{A}{2} + B_1\right).$$

Define: $C \equiv \dfrac{q_w D}{k Re \sqrt{f/2}}$

then

$$I_2 = (T_w - C y_L^+)\left[\frac{B_1}{2}\left({y_c^+}^2 - {y_L^+}^2\right) + A\left({y_c^+}^2 \ln y_c^+ - \frac{{y_c^+}^2}{4} - \frac{{y_L^+}^2}{2} \ln y_L^+ + \frac{{y_L^+}^2}{4}\right)\right] - \frac{C B_1 A}{Pr \lambda}\left(\frac{{y_c^+}^2}{2} \ln \frac{y_c^+}{y_L^+} - \frac{{y_c^+}^2}{4} + \frac{{y_L^+}^2}{4}\right) - I_5$$

.....(B-2)

where

$$I_5 = \frac{q_w DA^2}{\Pr\lambda k \mathrm{Re}\sqrt{f/2}} \left[\frac{(y_c^+ \ln y_c^+)^2}{2} - \left(\frac{y_c^{+2}}{2} \ln y_c^+ - \frac{y_c^{+2}}{4} \right) \left(1 + \ln y_L^+ \right) - \frac{(y_L^+ \ln y_L^+)^2}{2} + \left(\frac{y_L^{+2}}{2} \ln y_L^+ - \frac{y_L^{+2}}{4} \right) (1 + \ln y_L^+) \right].$$

.....(B-3)

To simplify equation B-2, define these constants

$$\theta_1 \equiv \frac{B_1 \left(y_c^{+2} - y_L^{+2} \right)}{2} + A \left(\frac{y_c^{+2} \ln y_c^+}{2} - \frac{y_c^{+2}}{4} - \frac{y_L^{+2} \ln y_L^+}{2} + \frac{y_L^{+2}}{4} \right)$$

.....(B-4)

$$\theta_2 \equiv \frac{y_c^{+2}}{2} \ln\left(\frac{y_c^+}{y_L^+} \right) - \frac{y_c^{+2}}{4} + \frac{y_L^{+2}}{4} \qquad \text{.....(B-5)}$$

$$\theta_3 \equiv \frac{(y_c^+ \ln y_c^+)^2}{2} - \left(\frac{y_c^{+2} \ln y_c^+}{2} - \frac{y_c^{+2}}{4} \right) (1 + \ln y_L^+) - \frac{(y_L^+ \ln y_L^+)^2}{2} + \left(\frac{y_L^{+2} \ln y_L^+}{2} - \frac{y_L^{+2}}{4} \right) \left(1 + \ln y_L^+ \right). \qquad \text{.....(B-6)}$$

Substitute equations (B-3) through (B-6) back into I_2 of equation (B-2). Then

$$I_2 = T_w - \left(\frac{q_w D\, y_L^+}{kRe\,\sqrt{f/2}}\right)\theta_1 - \frac{q_w D B_1 A \theta_2}{k\,Re\,\sqrt{f/2}\,Pr\lambda} - \frac{q_w D A^2 \theta_3}{kRe\,\sqrt{f/2}\,Pr\lambda}\,. \quad \text{.....(B-7)}$$

Since $y_L^+ << y_c^+$, then appropriate simplifications may be made in θ_1, θ_2, and θ_3 when evaluating Nu in equation (26).

Appendix C

Evaluating the Nusselt Number by Numerical Integration

The Nusselt number was computed by numerical integration as a check on our algebra. This method involved summing several integrals over a given area by the rectangular method.

$$Nu = \frac{hD}{k} = \frac{-D}{T_w - T_b} \frac{\partial T}{\partial y}$$

where

$$\frac{\partial T}{\partial y} = - \frac{q_w}{K} .$$

The bulk temperature consisted of four numeric integrations: I_1, I_2, I_3, and I_4, as explained earlier in the text. Note that I_1, I_2, I_3, and I_4 were strict integrations earlier where now they are numeric integrals. Note that I_1 is equal to:

$$I_1 = \sum_{n=1}^{m} T \, u_n^+ \, y_n^+ \, \Delta_n \, y^+ \qquad \text{.....(C-1)}$$

over the interval $0 \leq y^+ \leq y_L^+$.

In subsequent notation, the subscript n and the limits will be understood but not written. With

$$u^+ = y^+$$

and

$$T = T_w - \frac{q_w \, y^+ \nu}{k u_*}$$

substituted into equation (C-1), we have

$$I_1 = T_w \sum {y^+}^2 \Delta y^+ - \frac{q_w \nu}{k u_*} \sum {y^+}^3 \Delta y^+. \qquad \text{.....(C-2)}$$

Now

$$I_2 = \sum_{n=1}^{m} T_n u_n^+ y_n^+ \Delta_n y^+ \qquad \text{.....(C-3)}$$

over the interval

$$y_L^+ \leq y^+ \leq y_c^+$$

with

$$u^+ = A \ln y^+ + B_1$$

and

$$T = T_w - \frac{q_w D}{k Re \sqrt{\frac{f}{2}}} \left[\frac{A}{Pr\lambda} \ln \left(\frac{y^+}{y_L^+} \right) + y_L^+ \right].$$

In subsequent notation the subscript n and the limits will be left out. Substitute for u^+ and T in equation (C-3).

$$I_2 = \left[T_w A - \frac{Aq_w \nu \, y_L^+}{ku_*}\right] \sum y^+ \ln y^+ \, \Delta \, y^+$$

$$+ \left[T_w B - \frac{Bq_w \nu \, y_L^+}{ku_*}\right] \sum y^+ \, \Delta \, y^+ - \frac{A^2 q_w \nu}{ku_* Pr\lambda} \sum y^+ \ln y^+ \ln \frac{y^+}{y_L^+} \Delta \, y^+$$

$$- \frac{Bq_w \nu A}{ku_* Pr\lambda} \sum y^+ \ln \frac{y^+}{y_L^+} \Delta \, y^+ \, . \qquad \text{.....(C-4)}$$

Now

$$I_3 = \sum_{n=1}^{m} u_n^+ \, y_n^+ \, \Delta_n \, y^+ \qquad \text{.....(C-5)}$$

over the interval

$$0 \leq y^+ \leq y_L^+ \, .$$

The subscript n and the limits will not be written hereafter. We use

$$u^+ = y^+$$

and substitute u^+ in equation (C-5)

$$I_3 = \sum {y^+}^2 \, \Delta \, y^+ \qquad \text{.....(C-6)}$$

$$I_4 = \sum u^+ y^+ \Delta y^+ \qquad \text{.....(C-7)}$$

over the interval

$$y_L^+ \leq y^+ \leq y_c^+ .$$

The subscript n and the limits are understood. In this interval, $u^+ = A\ln y^+ + B_1$. Substitute u^+ in equation (C-7). Then

$$I_4 = A \sum y^+ \ln y^+ \Delta y^+ + B_1 \sum y^+ \Delta y^+ .$$

.....(C-8)

I_1, I_2, I_3, I_4, can be simplified by using

$$I_6 = {y^+}^2 \Delta y^+ \qquad \text{.....(C-9)}$$

$$I_7 = {y^+}^3 \Delta y^+ \qquad \text{.....(C-10)}$$

$$I_8 = y^+ \ln y^+ \Delta y^+ \qquad \text{.....(C-11)}$$

$$I_9 = y^+ \Delta y^+ \qquad \text{.....(C-12)}$$

$$I_{10} = y^+ \ln y^+ \ln \frac{y^+}{y_L^+} \Delta y^+ \qquad .....(C\text{-}13)$$

$$I_{11} = y^+ \ln \frac{y^+}{y_L^+} \Delta y^+ \ . \qquad .....(C\text{-}14)$$

Substitute equations (C-9) to (C-14) into equations (C-2), (C-4), (C-6) and (C-8). Then,

$$I_1 = T_w \sum I_6 - \frac{q_w \nu}{k u_*} \sum I_7 \qquad .....(C\text{-}15)$$

$$I_2 = \left[T_w A - \frac{A q_w \nu \ y_L^+}{k u_*} \right] \sum I_8 + \left[T_w B - \frac{B q_w \nu \ y_L^+}{k u_*} \right] \sum I_9$$

$$- \frac{A^2 q_w \nu}{k u_* Pr \lambda} \sum I_{10} - \frac{B q_w \nu A}{k u_* Pr \lambda} \sum I_{11} \qquad .....(C\text{-}16)$$

$$I_3 = \sum I_6 \qquad .....(C\text{-}17)$$

$$I_4 = A \sum I_8 + B_1 \sum I_9 \ . \qquad .....(C\text{-}18)$$

Let

$$\theta_4 = \sum I_6 + A \sum I_8 + B_1 \sum I_9 . \qquad .....(C\text{-}19)$$

Remember that T_b is defined as

$$T_b \equiv \frac{I_1 + I_2}{I_3 + I_4}$$

and that Nu is defined as

$$Nu = \frac{D}{T_w - T_b} \frac{q_w}{k} \;.$$

Substituting for T_b we get

$$Nu = \frac{D}{T_w - \left(\dfrac{I_1 + I_2}{I_3 + I_4}\right)} \frac{q_w}{k} \;.$$

Substitute for I_1, I_2, I_3, and I_4 from equations (C-15) to (C-18) and use equation (C-19). Then

$$Nu = Re \, \frac{f}{2} \left[\frac{\theta_4}{\sum I_7 + \frac{A^2}{Pr} \sum I_{10} + \frac{B_1 A}{Pr} \sum I_{11} + Ay_L^+ \sum I_8 + By_L^+ \sum I_9} \right] .$$

.....(C-20)

Equation (C-20) agreed with equation (26) to four significant figures when n = 20 and to six significant figures when n = 200.

Appendix D

Refractive Index Study

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PURPOSE

Tests were performed to determine whether the refractive index of Polyox could be used as an indication of concentration. An Abbe-type refractometer (references (a), (b), and (c)) was used to compare the refractive indices of Polyox solutions with that of water.

METHOD

Our technique was developed by taking the refractive index of several organic solvents and water and checking our results with standard references (references (d) through (h)).

The Polyox solutions were prepared by adding a CCl_3F slurry of Polyox to distilled water revolving at 100 rpm. The dispersions were then allowed to solvate motionlessly overnight. Next, the solutions were brought to volume in a volumetric flask. The refractometer was calibrated with distilled water and with isopropanol. Finally, the refractive index of Polyox solutions of 500 and 2000 ppm were measured.

APPARATUS

Refractometer - Bausch & Lomb precision Abbe-type, having a range in refractive index of 1.30 to 1.63.

Mixer - Corning Glass Works, Model PC 351.

Light Source - The light was a sodium arc lamp furnished with the instrument.

Transformer - George W. Gates & Co.

DATA AND DISCUSSION

The data collected were recorded in the following tabulation. Though the refractive index was measured to six significant figures, notice that the fourth significant figure was not reproducible. This was because no temperature control was used while this cursory experiment was performed. Had the index of 2000 ppm Polyox solution varied by at least 0.01, we would have used temperature control to gain exact data.

Refractive Indices

Date	Substance	Index of Refraction
4/23/70	500 ppm Polyox	1.33497
	Distilled Water	1.33484
	500 ppm Polyox	1.33458
	Isopropanol (Calibration)	1.37760
4/24/70	2000 ppm Polyox	1.33278
	Isopropanol (Calibration)	1.37542
	Isopropanol (Calibration)	1.37635
	2000 ppm Polyox	1.33303
	Distilled Water (Calibration)	1.33293

CONCLUSIONS

Refractive index is not a useful method of measuring Polyox concentration because the change in refractive index lacks sufficient magnitude.

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13. ABSTRACT

Experimental friction factor and heat transfer data from turbulent tube flow of Polyox WSR 301 are correlated through the use of a new method. The Nusselt number is derived for tube flow of drag-reducing solutions in general. The Nusselt number is shown to be a function of Reynolds number, friction factor, and solution properties. This analytical expression for the Nusselt number is compared for accuracy with experimental results. Studies of relaxation time and refractive index of Polyox are reported.

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