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A DIGITAL FILTER FOR SEPARATING HIGH- AND LOW-FREQUENCY COMPONENTS OF A TRANSIENT SIGNAL

by

LTJG Thomas L. Geers, USNR
and
Stewart A. Denenberg

STRUCTURAL MECHANICS LABORATORY
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A DIGITAL FILTER FOR SEPARATING HIGH- AND LOW-FREQUENCY COMPONENTS OF A TRANSIENT SIGNAL

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Report 1795
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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

NOTATION

A	Smallest value of truncated portion of signal
B	Largest value of truncated portion of signal
C	Amplitude of a sinusoidal signal
$C_1(x)$	A function for determining E_T, defined in Equation [30]
$C_2(x)$	A function for determining E_T, defined in Equation [31]
E_T	Conservative estimate for the error at time t due to truncation at time T_T, defined in Equation [6]
$f(t)$	Portion of signal from T_S to T_T
$f'(t)$	Portion of signal from T_T to T_F
$f''(t)$	Portion of signal from $T_T{}'$ to T_F
$f_1(t)$	Low-frequency portion of $f(t)$ after filtering
$F(nh)$	$F(t)$ sampled at times nh
$F(t)$	Signal, recorded as a function of time
$F_1(kh)$	Digital approximation to $F_1(t)$ using trapezoidal integration
$F_1(t)$	Low-frequency portion of $F(t)$ after filtering
$F_2(t)$	High-frequency portion of $F(t)$ after filtering
$\mid F(\omega) \mid$	Spectrum of $F(t)$
$G(x)$	A periodic function, defined in Equation [17]
h	A time increment in seconds
k	An integer
$L(t)$	An undamped sinusoidal oscillation with frequency 0.16 cps
m	A positive integer
n	An integer
$S(kh)$	$S(t)$ sampled at times kh
$S(t)$	A sinusoidal signal
$S_1(kh)$	Digital approximation to $S_1(t)$ using trapezoidal integration
$S_1(t)$	Low-frequency portion of $S(t)$ after filtering
t	Time in seconds
t'	Variable of integration representing time in seconds

T_F	Time at which a transient signal ends, in seconds
T_S	Time at which a transient signal begins, in seconds
T_T	Time at which a signal to be filtered is truncated, in seconds
$T_T{}'$	Time at which the function $f'(t)$ is truncated, in seconds
u	A variable of integration
$U(t)$	A damped sinusoidal oscillation with frequency 0.96 cps and decay time 8.727 seconds
x	x_T or x_F
x_F	$x_F = \frac{\omega_c}{2\pi} (T_F - t)$, in cycles
x_T	$x_T = \frac{\omega_c}{2\pi} (T_T - t)$, in cycles
ϕ	Phase of a sinusoidal signal in radians
ω	Circular frequency in radians per second
ω_c	Filter cutoff frequency in radians per second
ω_H	Highest frequency of interest in a signal, in radians per second

ABSTRACT

A filter is described which separates a transient signal into high- and low-frequency portions and introduces no phase shifts or amplitude changes in either portion. A program written in FORTRAN for an IBM 7090 computer which will filter a sampled time-history record is presented. Simple formulas are given for estimating errors in the filter that are due to sampling and truncation of the continuous record. Application of the filter to signals produced by two or more physical sources is discussed.

INTRODUCTION

In many instances, the recorded time-history of a transient phenomenon is significantly obscured by an unwanted signal. Ideally, the unwanted signal should be silenced at the source. If this cannot be done, methods may exist whereby the corrupted record can be treated with a filter designed to preserve the desired signal while eliminating the unwanted signal. To design this optimum filter, we would require detailed knowledge of the characteristics of the desired and unwanted signals.

Often, however, it is very difficult to obtain the signal information necessary to design an optimum filter with acceptable accuracy. In these situations, we might abandon optimum filtering and utilize a filter requiring less specialized information.

Such a filter is presented here. It separates the high-frequency components of a record from the low-frequency components. The filter is a symmetrical digital filter that suppresses the portion of the signal to be rejected and that produces no change in amplitude or phase for the portion of the signal passing through the filter. It is most useful when the major frequency components of an unwanted signal lie well outside the region in which the major frequency components of the desired signal lie.

FILTER EQUATION

We assume that the record to be filtered $F(t)$ can be expressed by Fourier's integral formula:[1]

$$F(t) = \frac{1}{\pi} \int_0^\infty dw \int_{-\infty}^{\infty} F(t') \cos w(t - t')\, dt' \qquad [1]$$

The integral over frequency can be considered in two parts, one covering the range from 0 to ω_c (to represent the portion of the record made up of low-frequency components) and the

[1] References are listed on page 32.

other covering the range from ω_c to ∞ (to represent the portion of the record made up of high-frequency components).

If $F(t)$ is a transient record, Figure 1, the integrand will be nonzero only over a finite range of time and the integral over time will always be well-behaved. The integral over the

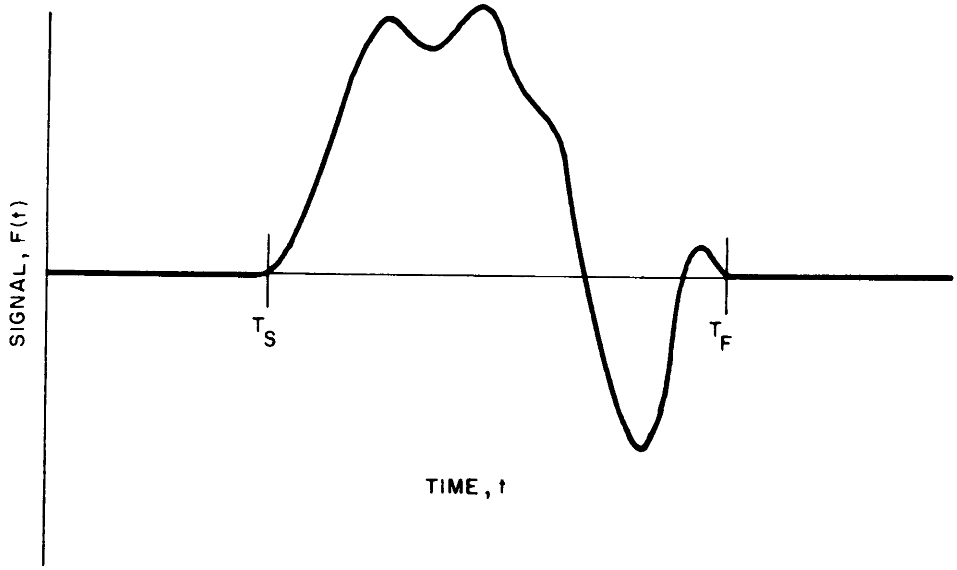


Figure 1 – Transient Signal

frequency can be evaluated for the low-frequency portion of the record to obtain

$$F_1(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} F(t') \frac{\sin \omega_c (t - t') \, dt'}{t - t'} \qquad [2]$$

for the low-frequency portion. The high-frequency portion can be obtained from

$$F_2(t) = F(t) - F_1(t) \qquad [3]$$

If the record $F(t)$ is sampled at a uniform time interval h to produce a sequence of samples $F(nh)$, where n is an integer, the integral of Equation [2] may be approximated by trapezoidal integration to produce the formula

$$F_1(t) = \frac{h}{\pi} \sum_{n=-\infty}^{\infty} F(nh) \frac{\sin \omega_c (t - nh)}{t - nh} \qquad [4]$$

Since $F(t)$ is a transient, the sum involves only a finite number of nonzero terms. Terms with $t = nh$ are to be defined in the limit as t approaches nh.

If $F_1(t)$ is evaluated only at times when $F(t)$ is sampled, Equation [4] may be written

$$F_1(kh) = \frac{1}{\pi} \sum_{n=-\infty}^{\infty} F(nh) \frac{\sin \omega_c h (k-n)}{k-n} \qquad [5]$$

where k takes on only integral values and the value of the term in the summation when $n = k$ is defined to be $F(kh) \cdot \omega_c h$.

FILTER PROGRAM

The filter program, Table 1, is a coding of Equations [3] and [5] in FORTRAN[2] for the IBM 7090 digital computer and peripheral equipment available at the David Taylor Model Basin. Minor changes in the program may be necessary to adapt it to computers at other installations.

The program accepts up to 2000 samples taken at equal time intervals from a transient signal. It evaluates the filtered values $F_1(kh)$ and $F_2(kh)$ for three different cutoff frequencies. Output consists of a table of $F(kh)$ and tables of $F_1(kh)$ and $F_2(kh)$ for each cutoff frequency selected, a listing of the largest absolute value of $F(kh)$ and the largest absolute values of $F_1(kh)$ and $F_2(kh)$ for each cutoff frequency selected, and plots of F, F_1, and F_2 as a function of time on an SC 4020 Microfilm Recorder.[3]

About 2 min of computer time are required to evaluate F_1 and F_2 for a single cutoff frequency when the record consists of 1000 samples. Computing time is approximately proportional to the number of cutoff frequencies supplied and to the square of the number of samples in the record to be filtered.

Details of input and output, including a number of options available in the program, are described in the following pages.

INPUT

The filter program is shown in Table 1. The arrows indicate where input is accepted by the program.

Tape

The record to be filtered must be on binary tape in IBM format and must consist of samples taken from a transient record at equal increments of time. If any other type of input record is to be used (e.g., punch cards), Statements 999 and 1000 of the program must be modified.

TABLE 1

FORTRAN Filter Program

```
      DIMENSION F(2000,7),TAU(2000),SINE(4000),T(2000),FREQ(7),BIGG(7),
     1BIGT(7),TITLE(10),GTITLE(20)
      COMMON F
      IX=XTAPNF(6)
      IPLOT = -1
    1 READ 500,NF,NR,NPSKP,ID,KTH,NSPL,NTF,NIP,NFP,LOCK,KEY
      IF (KEY)250,2,253
  253 CALL LGCHAR(8,4HYD08)
      IPLOT =  1
    2 READ 501,TINC,ZEROL,CAL,TSI,TSF,TMAX
      NL = NTF + 1
      NN= NTF + 2
      READ502,(FREQ(L),  L = 2,NL)
      READ9999, TITLE
      IF(IPLOT)  93,1191,1191
B1191 GTITLE(20) = 740101053060
B     GTITLE(19) = 606060606060
      DO 9 I = 1,10
      K = 19 - I
    9 GTITLE(K) = TITLE(I)
   93 ERASE F
      IX=XTAPSF(NF,NR)
C
C     THE INPUT TAPE IS READ AT 999 OR 1000
C
      IF(NPSKP) 999,999,1000
  999 READ TAPE 6,(F(K,1),K=1,NIP)
      GO TO 1001
 1000 READ TAPE 6,(A,I=1,NPSKP),(F(K,1),K=1,NIP)
 1001 T(1) = TSF
      TAU(1)= TSI
      KK = XINTF((TSF-TSI)/TINC)
      IF(NFP-NIP) 20,10,10
   10 N=NFP
      GO TO 30
   20 N=NIP
   30 DO 40  I=1,NIP
   40 TAU(I+1) = TAU(I) + TINC
      DO 41 I = 1,NFP
   41 T(I+1) = T(I) + TINC
      NNN = N +XABSF(KK)
      BIGG(1) = 0.
      DO 16 K = 1, NIP
      F(K,1) = (F(K,1) -ZEROL)/CAL
      IF(ABSF(BIGG(1)) - ABSF(F(K,1))) 15,16,16
   15 BIGG(1) = F(K,1)
      BIGT(1) =T(K)
   16 CONTINUE
      DO 24 L = 2, NL
      OMEGA = FREQ(L) *  6.28318
      BIGG(L) = 0.
      BIGT(L) =  0.
      DO 23 IK=1, NNN
       TFX=FLOATF(IK)
   23 SINE(IK)= SINF( (     OMEGA)*(TINC*TFX ))/((TINC*TFX )*OMEGA)
      DO 24 I=1,NFP
       II = I+KK
   25 SUM = 0.
      DO 8 K =1, NIP
```

```
      IF(II-K) 3,4,5
    3 IK = K-II
      GO TO 6
    4 SUM = SUM + F(K,1)
      GO TO 8
    5 IK =II-K
    6 SUM = SUM + (F(K,1) * SINE(IK))
    8 CONTINUE
      F(I,L)=(SUM*TINC  *     OMEGA/3.14159)
      IF(ABSF(BIGG(L))-ABSF(F(I,L)))7,24,24
    7 BIGG(L)=F(I,L)
      BIGT(L)=T(I)
   24 CONTINUE
      DO129 L = 2, NL
      JJ = L + NTF
      BIGG(JJ) = 0.
      FREQ(JJ) = FREQ(L)
      IF(TSI-TSF) 730,777,740
  730 IM = NIP-KK
      IMM =IM+1
      DO 731 I = 1,IM
      IMMM = I+KK
  731 F(I,JJ) = F(IMMM,1) - F(I,L)
      DO 732 I = IMM,NFP
  732 F(I,JJ) = -F(I,L)
      GO TO 778
  740 M = XABSF(KK)
      MM = 1+M
      DO 741 I = 1,M
  741 F(I,JJ) =-F(I,L)
      DO 742 I = MM,NFP
      IMMM = I-MM+1
  742 F(I,JJ) = F(IMMM  ,1) - F(I,L)
      GO TO 778
  777 DO 29I = 1,NFP
   29 F(I,JJ) = F(I,1) - F(I,L)
  778 DO 129 I = 1,NFP
      IF(ABSF(BIGG(JJ)) -ABSF(F(I,JJ))) 28,129,129
   28 BIGG(JJ) = F(I,JJ)
      BIGT(JJ) = T(I)
  129 CONTINUE
   32 IF(KEY)  250,249,245
C
C     STATEMENTS 245 THRU 2451 PICK THE LARGEST PEAK VALUE OF THE INPUT
C     AND FILTERED RECORDS
C
  245 YY = 0.
      DO 2451 L = 1,JJ
      IF(ABSF(YY)-ABSF(BIGG(L))) 2450,2451,2451
 2450 YY = BIGG(L)
 2451 CONTINUE
C
C     STATEMENTS 2461 THRU 2462 PICK THE UPPER AND LOWER SCALES FOR THE
C     RESPONSE
C
 2461 BBIG    = ABSF(YY)
      IY = XINTF(BBIG   ) + 1
      YU = FLOATF(IY)
```

```
 2462 YL = -YU
C
C     STATEMENTS 2463 THRU 2401 PICK THE UPPER AND LOWER SCALES FOR THE
C     TIME
C
 2463 XL = TSF
      IF(TMAX-T(NFP)) 2400,2401,2401
 2400 XU = T(NFP)
      GO TO 2452
 2401 XU = TMAX
 2452 DO 248 L = 1,JJ
      IF(L-1) 2482,2481,2482
B2481 GTITLE(3) = 314547646360
B     GTITLE(2) = 602674633460
B     GTITLE(1) = 606060606060
      DO 2483 IZ = 4,8
B2483 GTITLE(IZ) = 606060606060
      GO TO 2402
B2482 GTITLE(1) = 606023476260
      REWIND 4
      WRITE OUTPUT TAPE 4 , 1500,FREQ(L)
 1500 FORMAT (F8.2,4X)
      REWIND 4
      READ INPUT TAPE 4 ,246,GTITLE(3),GTITLE(2)
  246 FORMAT(2A6)
B     GTITLE(6) = 236463462626
B     GTITLE(5) = 602651255064
B     GTITLE(4) = 254523706013
      IF(L-NN) 2485,2484,2484
B2484 GTITLE( 8) = 602602746334
B     GTITLE( 7) = 606060606060
      GO TO 2402
B2485 GTITLE( 8) = 602601746334
B     GTITLE( 7) = 606060606060
 2402 DO 248 J=1,NSPL
      MTH =    J*KTH
      CALL PNTFN(MTH)
      CALL FNPLOT(GTITLE(20),22H(17H  TIME IN SECONDS),15H(10H  RESPONSE
     1),XL,XU,YL,YU,20,2,20,1 ,6H(F9.3),6H(F9.3))
  247 CALL CURVE(T( NFP),F( NFP,L), NFP,6H       )
  248 CONTINUE
  249 PRINT 998, TITLE
      PRINT 100,TINC,TSF,TSI,NTF,NFP,NIP,ZEROL,ID,CAL,NF,NR,TMAX
  100 FORMAT        ( /11H PARAMETERS    /8H TINC   =F11.8 /8H TSF    = F8.5
     1  /8H TSI    =  F8.5
     1                      /8H NTF    = I2 /8H NFP    = I4 /8H NIP    = I4 /8H
     2 ZEROL =F8.3 /8H ID     = I4 /8H CAL    = F8.3 / 8H NF     = I4 /
     3 8H NR     = I4 /8H TMAX   = F7.3/////)
      PRINT31,BIGG(1),BIGT(1)
   31 FORMAT(29H THE PEAK RESPONSE OF F(T)  = F6.3,8H AT TIME F9.5//)
   36 PRINT37,(BIGG(L),BIGT(L),FREQ(L),L=2,NL)
   37 FORMAT(29H0THE PEAK RESPONSE OF F1(T) = F6.3,8H AT TIME F9.5,25H
     1 FOR CUTOFF FREQUENCY = F8.2,4H CPS    )
      PRINT34,(BIGG(L),BIGT(L),FREQ(L),L=NN,JJ)
   34 FORMAT(29H0THE PEAK RESPONSE OF F2(T) = F6.3,8H AT TIME F9.5,25H
     1 FOR CUTOFF FREQUENCY = F8.2,4H CPS    )
      IF (LOCK)79,91,79
   79 PRINT 888,(FREQ(L),L=2,JJ)
```

TABLE 1 (Continued)

```
      IF(NTF-2)84,82,81
   82 PRINT 83
   83 FORMAT                     (34X,56H   CPS        CPS        CPS
     1 CPS                       //
     2       91H     TAU      F(T)       TIME       F1(T)      F1(T)      F
     32(T)     F2(T)                          //)
      GO TO 89
   84 PRINT 85
   85 FORMAT                     (34X,56H   CPS        CPS
     1                           //
     2       91H     TAU      F(T)       TIME       F1(T)      F2(T)
     3                                        //)
      GO TO 89
   81 PRINT 80
   80 FORMAT                     (34X,56H   CPS        CPS        CPS
     1 CPS       CPS       CPS   //
     2       91H     TAU      F(T)       TIME       F1(T)      F1(T)      F
     31(T)     F2(T)     F2(T)     F2(T)    //)
   89 DO 90 I=1,N
 102  FORMAT(F10.5,F10.3,F10.5,6F10.3)
 90   PRINT 102,TAU(I),F(I,1),T(I),(F(I,L),L=2,JJ)
   91 GO TO 1
  250 REWIND 6
      IF(IPLOT)  251,251,252
  252 CALL LGCHAR(8,4HYD08)
      END FILE 8
  251 CALL ENDJOB
  500 FORMAT (11I4)
  501 FORMAT(6F10.3)
  502 FORMAT(3F6.2)
 9999 FORMAT (10A6)
  998 FORMAT (1H1, 12A6  //)
  888 FORMAT(////1H ,F39.2,5F10.2/)
      END
```

Card 1

Card 1 is the first of four IBM cards required to implement the program. It must contain the following 11 integers in order, in FORMAT (11I4):

NF – Number of files to skip forward on the input tape.

NR – Number of records to skip forward within the desired file.

NPSKP – Number of points to skip forward within the desired record. (For example, if NF is 3, NR is 6, and NPSKP is 60, the input to be filtered will begin with point 61 of record 7 in file 4 on the tape.)

ID – An identification number which can be assigned to the input record and which will appear on the printed output from the computer.

KTH – A number directing the plotter to plot only every KTH point.

NSPL – Number of times each record is to be plotted. The first plot is obtained with every KTH point plotted, the second plot is obtained with every 2KTH point plotted, etc., with decreasing density of plot. Finally, every NSPL multiplied by KTH point will be plotted.

NTF – A number 1, 2, or 3 to indicate that one, two, or three different cutoff frequencies for filtering will be supplied on Card 3.

NIP – Number of points to be read from the input tape; must be 2000 or less.

NFP – Number of points to be filtered; must be 2000 or less.

LOCK – A number 0 directes the computer to print out peak values of the input and filtered records and their times of occurrence only. A number 1 or -1 directs the computer to print out complete time histories.

KEY – A number 1, 0, or -1 respectively directs the computer to plot all input and filtered records, not to plot any records, or to end the program immediately.

Card 2

The second card must contain the following six fixed-point numbers in order, in FORMAT (6F10.3):

TINC – Time increment for input and filtered records, in seconds.

ZEROL – Value of the input record which represents the zero value of the signal.

CAL – A calibration constant to convert values of the input record to units of some physical parameter.

TSI – An integral multiple of TINC, giving the value of time in seconds at the first point read from the input record.

TSF – An integral multiple of TINC, giving the value of time in seconds at the first point in the filtered record. It is required that $\frac{|TSF - TSI|}{TINC}$ not exceed 2000.

TMAX – The value of time in seconds to be used as the largest value on the abscissa of the plots. If this number is not punched, TMAX will be chosen by the computer as NFP multiplied by TINC.

Card 3

This card contains one, two, or three cutoff frequencies from the number NTF on Card 1. FORMAT (3F6.2) is used, and each frequency is to be expressed in cycles per second.

Card 4

The final card contains 60 alphanumeric characters. The 60 characters entered are printed as a title on each plot and are used as a heading for each block of printing produced by the program.

OUTPUT

Table 2 and Figure 2 show typical outputs from the computer for a transient record filtered at three different cutoff frequencies.

Printout

The first page of a complete printout (obtained with LOCK = 1 on Card 1) is shown in Table 2. The title from Card 4 is printed at the top of the page, directly followed by a list of the input parameters used in the calculations. The next section of printout consists of the peak values and times of occurrence of peaks for the input signal $F(t)$ and for the filtered values $F_1(t)$ and $F_2(t)$ at each of the three cutoff frequencies. Following the peak values is the tabular array of every value of each variable, $F(t)$, $F_1(t)$, and $F_2(t)$ as a function of time. TAU is the time associated with $F(t)$, and TIME is the time associated with $F_1(t)$ and $F_2(t)$. TAU begins with the value TSI, and TIME begins with the value TSF, as specified on Card 2.

Plots

Figure 2 shows the plots obtained by setting KEY = 1 on Card 1. Seven plots were obtained with every point plotted by setting KTH = 1 and NSPL = 1 on Card 1. The plotter draws straight lines to connect the plotted points. (If this feature is not desired, Statement 247 of the program must be modified.)

The smallest value of the abscissa of the plots is the time chosen as TSF on Card 2, and the largest value of the abscissa is the time TMAX. The computer automatically chooses upper and lower ordinates for the plots by the following procedure:

1. The seven peak values of the records are compared and the absolute value of the largest peak is found.
2. This largest value is rounded up to the next largest whole number.
3. The upper limit for the ordinate is set equal to this whole number, and the lower limit is set equal to the negative of this whole number.

TABLE 2

Typical Filter Program Printout

FILE 15 RECORD 1 ID 1000 OCTOBER 21, 1965

```
PARAMETERS
TINC   = 0.17450000
TSF    = 0.
TSI    = 0.
NTF    = 3
NFP    = 450
NIP    = 380
ZEROL  =    0.
ID     =1000
CAL    =    1.000
NF     =   14
NR     =    0
TMAX   = 80.000
```

THE PEAK RESPONSE OF F(T) = 1.354 AT TIME 4.71150

THE PEAK RESPONSE OF F1(T) = 1.049 AT TIME 36.29600 FOR CUTOFF FREQUENCY = 0.18 CPS

THE PEAK RESPONSE OF F1(T) =-1.015 AT TIME 64.56499 FOR CUTOFF FREQUENCY = 0.56 CPS

THE PEAK RESPONSE OF F1(T) = 1.111 AT TIME 4.53700 FOR CUTOFF FREQUENCY = 0.94 CPS

THE PEAK RESPONSE OF F2(T) = 0.543 AT TIME 5.58400 FOR CUTOFF FREQUENCY = 0.18 CPS

THE PEAK RESPONSE OF F2(T) = 0.390 AT TIME 4.53700 FOR CUTOFF FREQUENCY = 0.56 CPS

THE PEAK RESPONSE OF F2(T) =-0.384 AT TIME 3.31550 FOR CUTOFF FREQUENCY = 0.94 CPS

TAU	F(T)	TIME	0.18 CPS F1(T)	0.56 CPS F1(T)	0.94 CPS F1(T)	0.18 CPS F2(T)	0.56 CPS F2(T)	0.94 CPS F2(T)
0.	0.	0.	-0.042	-0.000	0.072	0.042	0.000	-0.072
0.17450	0.	0.17450	-0.079	-0.003	0.062	0.079	0.003	-0.062
0.34900	0.	0.34900	-0.115	-0.005	-0.010	0.115	0.005	0.010
0.52350	0.	0.52350	-0.148	-0.005	-0.077	0.148	0.005	0.077
0.69800	0.	0.69800	-0.177	-0.004	-0.072	0.177	0.004	0.072
0.87250	0.	0.87250	-0.198	-0.001	0.005	0.198	0.001	-0.005
1.04700	0.	1.04700	-0.212	0.004	0.083	0.212	-0.004	-0.083
1.22150	0.	1.22150	-0.217	0.007	0.085	0.217	-0.007	-0.085
1.39600	0.	1.39600	-0.211	0.009	0.002	0.211	-0.009	-0.002
1.57050	0.	1.57050	-0.195	0.008	-0.091	0.195	-0.008	0.091
1.74500	0.	1.74500	-0.167	0.003	-0.103	0.167	-0.003	0.103
1.91950	0.	1.91950	-0.127	-0.005	-0.012	0.127	0.005	0.012
2.09400	0.	2.09400	-0.076	-0.014	0.102	0.076	0.014	-0.102
2.26850	0.	2.26850	-0.015	-0.020	0.129	0.015	0.020	-0.129
2.44300	0.	2.44300	0.056	-0.021	0.029	-0.056	0.021	-0.029
2.61750	0.	2.61750	0.135	-0.013	-0.120	-0.135	0.013	0.120
2.79200	0.	2.79200	0.219	0.010	-0.178	-0.219	-0.010	0.178

Figure 2 – Typical Filter Program Plots

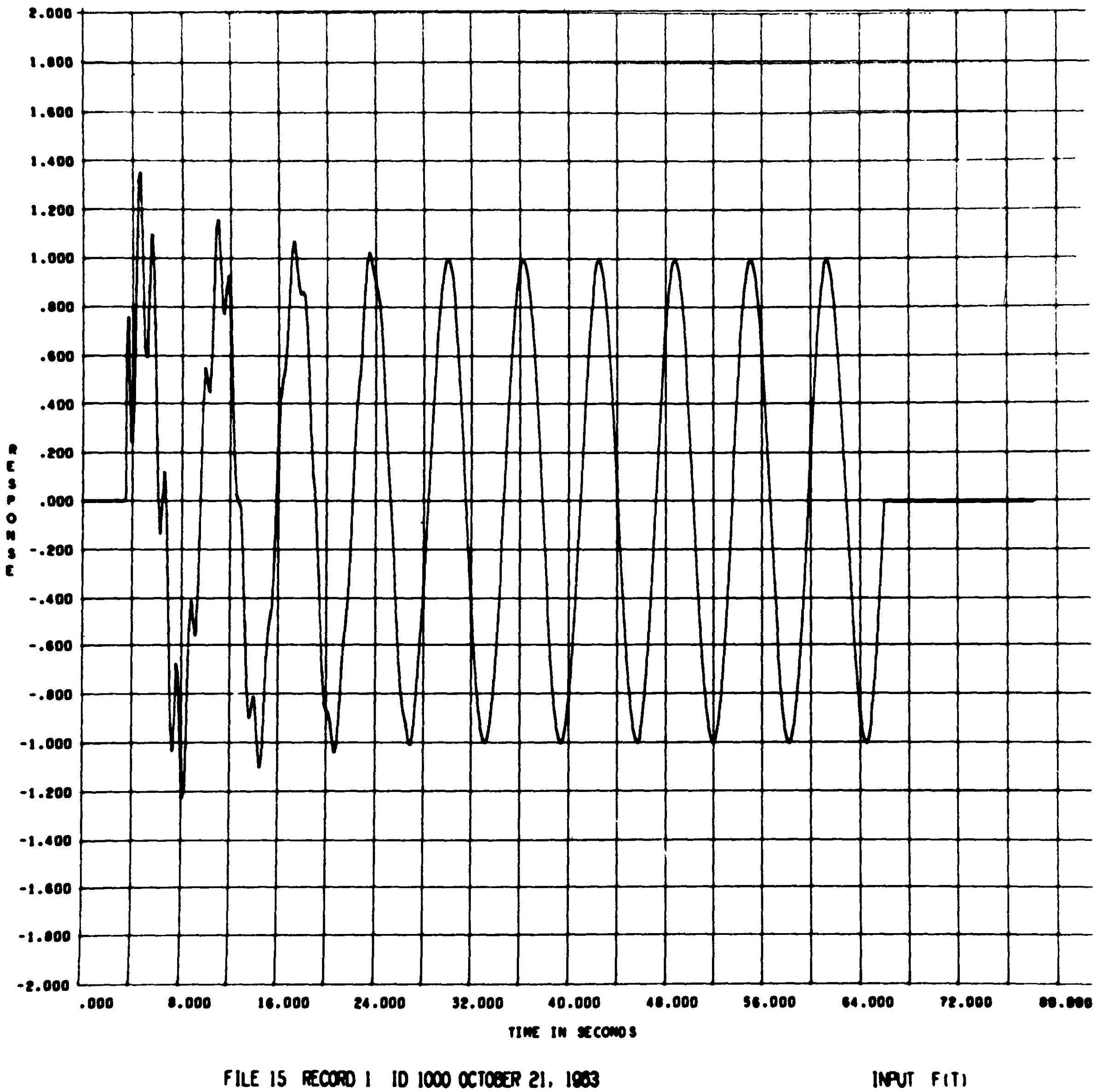
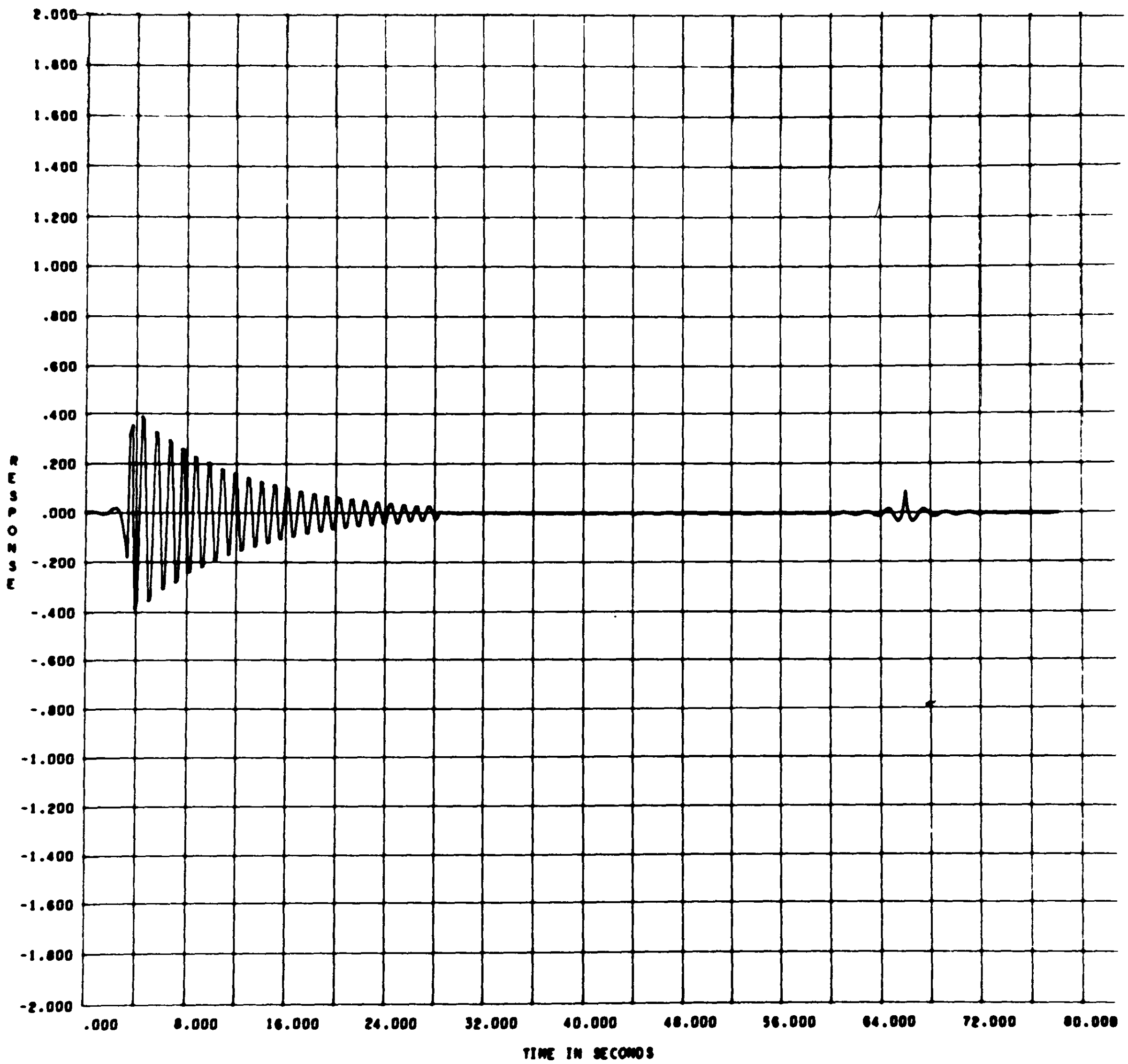


Figure 2a – Input Record $F(t) = L(t) + U(t)$

$L(t)$ is an undamped sinusoidal oscillation with frequency $\frac{1}{2\pi} = 0.16$ cps, suddenly beginning at time 3.3155 sec and ending at time 66.1355 sec. $U(t)$ is a damped sinusoidal oscillation with frequency $\frac{6}{2\pi} = 0.96$ cps and decay time 8.727 sec, suddenly starting at time 3.3155 sec and ending at time 25.4435 sec. The sudden start of the oscillations and the damping generate frequency components over the entire frequency spectrum. The signal has been sampled at a time increment of 0.1745 sec.

Figure 2b – Input Record $F(t)$ Filtered to Obtain a Record Consisting Entirely of Frequency Components below (Lower Curve) and above (Upper Curve) 0.56 CPS

The upper curve contains most of the frequency components of $U(t)$ and thus very nearly reproduces the time history of $U(t)$. The lower curve contains most of the frequency components of $L(t)$ and thus very nearly reproduces the time history of $L(t)$.



FILE 15 RECORD 1 ID 1000 OCTOBER 21, 1963 F2(T) CUTOFF FREQUENCY = 0.56 CPS

Figure 2b – Upper

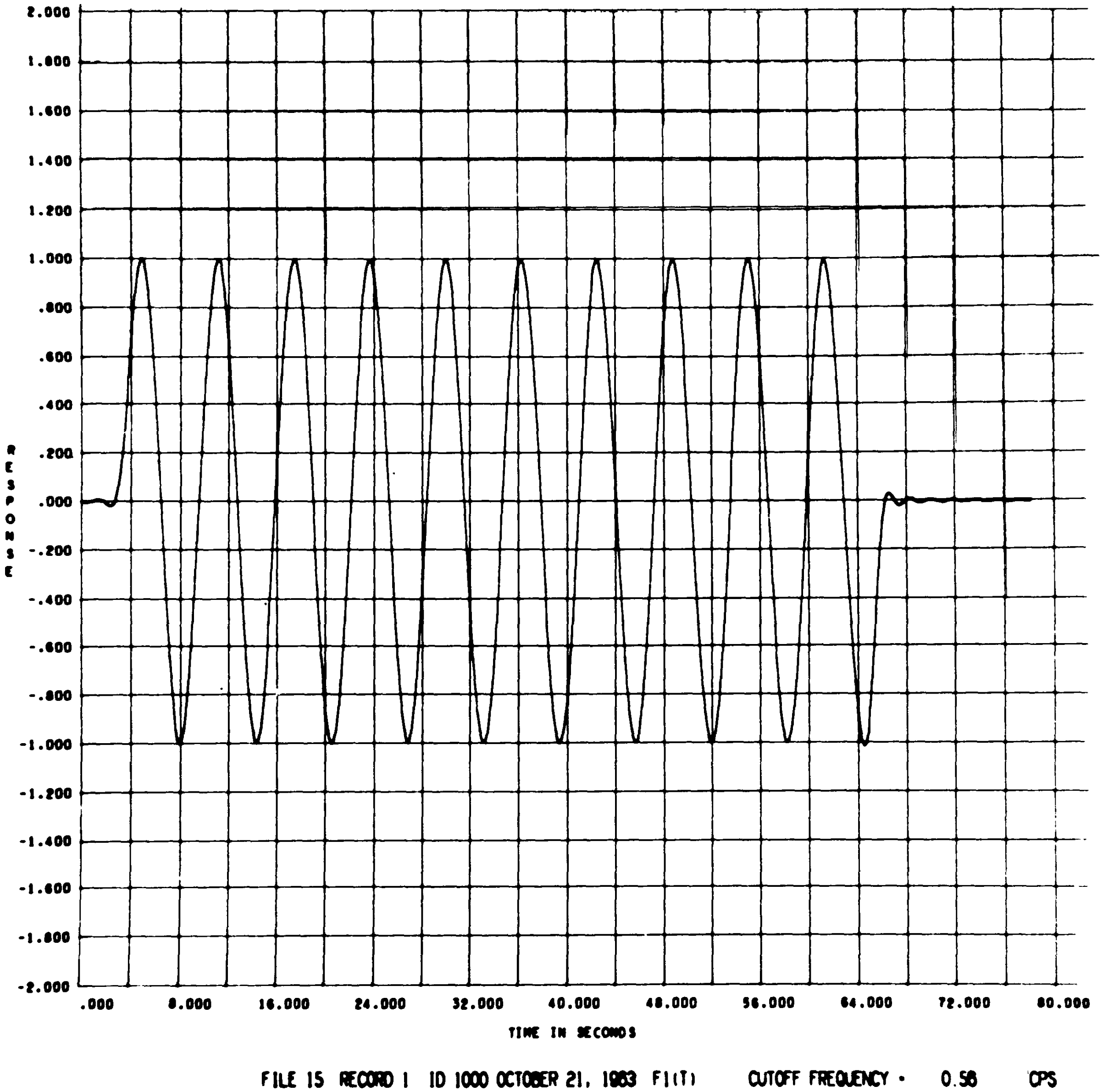


Figure 2b – Lower

Figure 2c — Input Record $F(t)$ Filtered to Obtain a Record Consisting Entirely of Frequency Components below (Lower Curve) and above (Upper Curve) 0.18 CPS

The upper curve contains frequency components of $U(t)$ along with an appreciable proportion of the frequency components of $L(t)$. The lower curve does not contain a large portion of the frequency components of $L(t)$. Thus, neither $U(t)$ nor $L(t)$ is well determined with this choice of filter cutoff frequency.

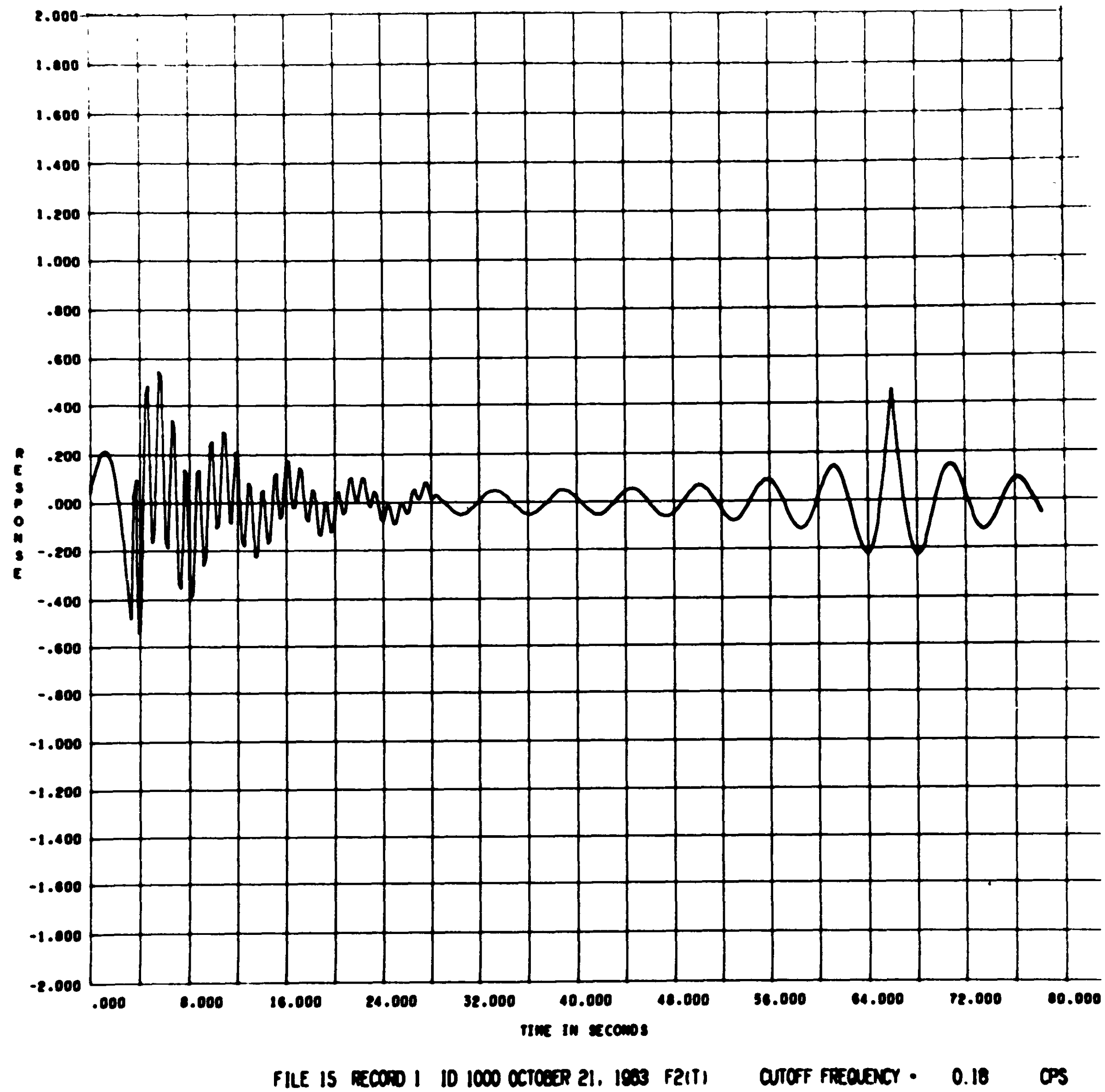


Figure 2c — Upper

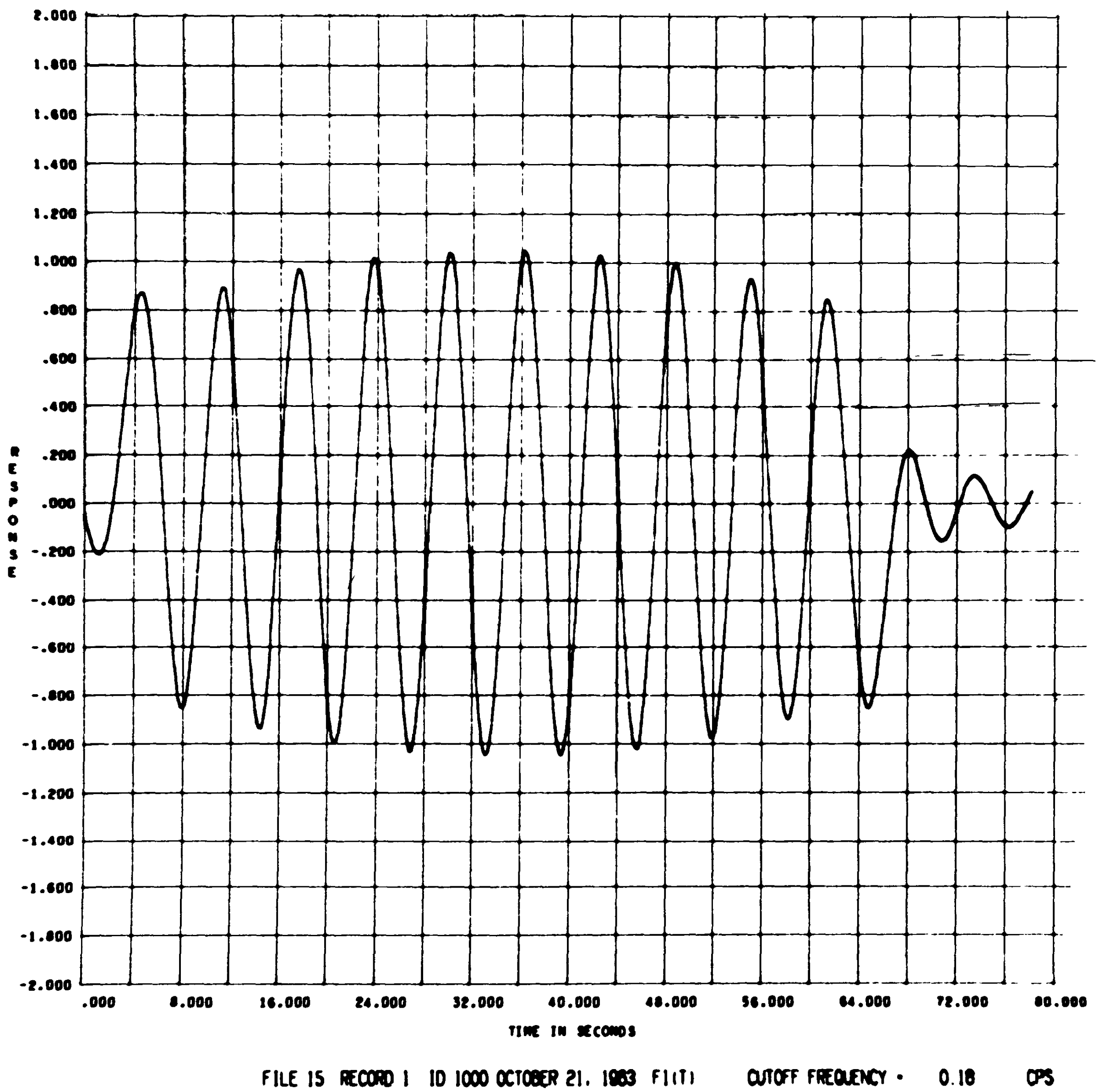


Figure 2c – Lower

Figure 2d – Input Record $F(t)$ Filtered to Obtain a Record Consisting Entirely of Frequency Components below (Lower Curve) and above (Upper Curve) 0.94 CPS

The lower curve contains frequency components of $L(t)$ along with an appreciable proportion of the frequency components of $U(t)$. The upper curve does not contain a large portion of the frequency components of $U(t)$. Thus, neither $L(t)$ nor $U(t)$ is well determined with this choice of filter frequency.

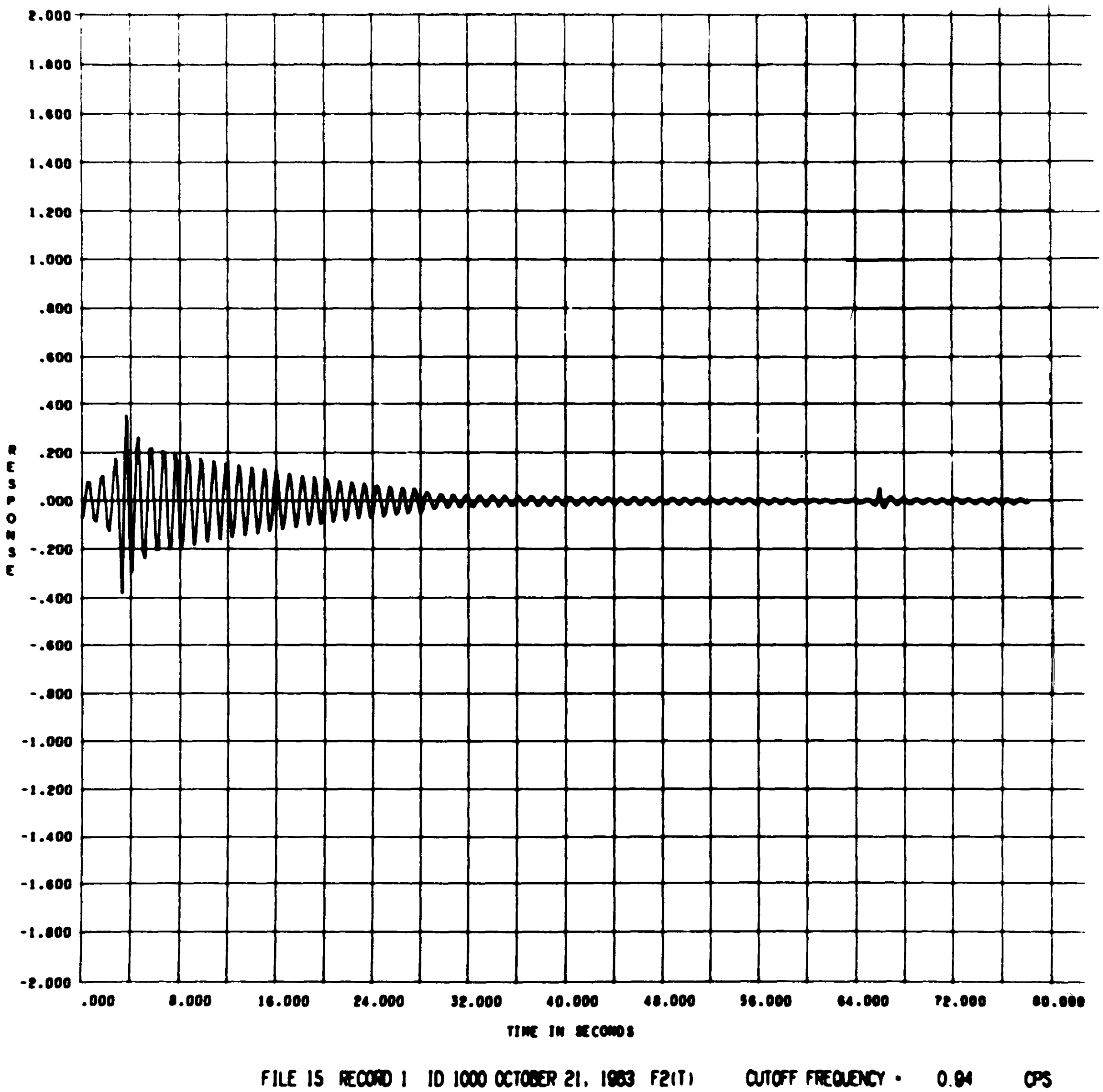


Figure 2d – Upper

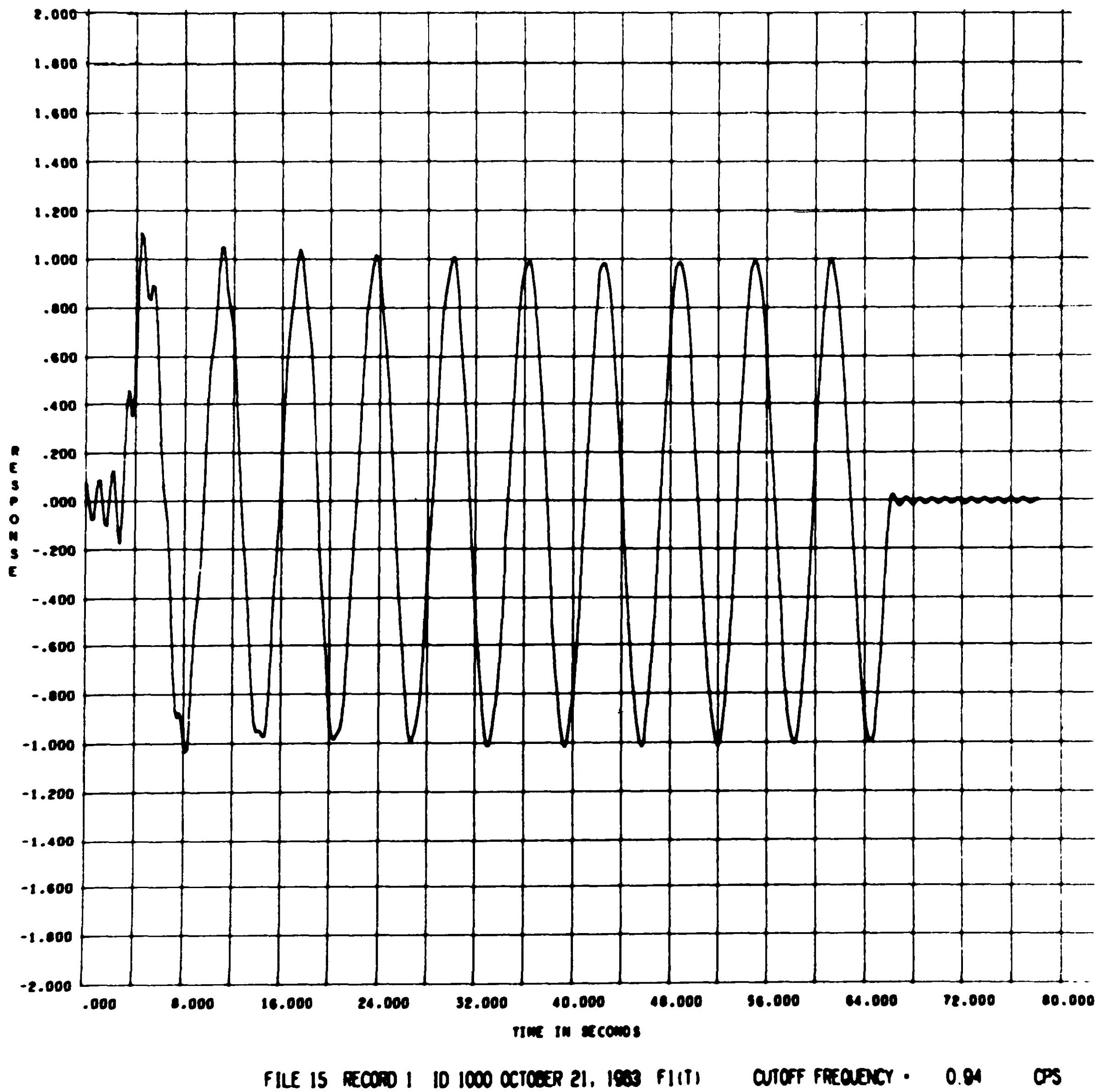


Figure 2d – Lower

This procedure produces a uniform scale for all of the plots in which at least one of the functions covers more than one-half of the available height on the plot, unless the largest value of all of the records is less than 0.5. The computer also automatically draws and labels grid lines, dividing the abscissa and the ordinate into 20 equal parts.

ERROR ANALYSIS

The digital-computer program for the filter requires that the record be presented in the form of no more than 2000 samples taken at equal time increments. If the time increment chosen for the samples is too large, the computer program may not properly represent the actual integral required to represent the filter. On the other hand, if too small a time increment is chosen, more than 2000 samples may be required to represent the entire time over which the transient signal has a value different from zero. Methods of estimating the errors due to these effects are described in this section.

SAMPLING ERROR

It is assumed that any record to be analyzed will have an upper frequency limit, or a highest frequency of interest. Frequency components above this limit are either irrelevant to the analysis being conducted or are not correctly reproduced by the recording system used to obtain the original record. The upper frequency limit of the recording system may be used to define the highest frequency of interest in default of any other criterion.

Appendix A shows that the integration formula used in the computer program gives an exact representation of the filter integral whenever the sampling rate for the record is high enough to produce at least two samples in each period of the highest frequency of interest. If this requirement is met, error from this source vanishes completely.

If any frequency of interest has a period shorter than two time increments, the filter cannot be used since the samples do not adequately represent the original record.

TWO METHODS OF EVALUATING TRUNCATION ERROR

The computer program cannot accept a record consisting of more than 2000 samples. After the time increment has been chosen to fit the foregoing requirement, the record may consist of more than 2000 samples and must be truncated so that only 2000 samples will be presented to the computer for analysis. An error will result from this truncation since the filter uses the total time-history record; see Equation [2]. Two methods of estimating this error will be presented.

Conservative Estimation of Truncation Error

Figure 3 represents a record that consists of more than 2000 samples. $F(t)$, the record to be filtered, has nonzero values from the time at which the record starts T_S to the time at which the record ends T_F. The truncated record $f(t)$ extends from T_S to the time at which $F(t)$ was truncated T_T. The unanalyzed portion of the record $f'(t)$ starts at T_T and ends at T_F. It is assumed that $f'(t)$ is bounded below by A and above by B and may be considered an oscillation taking place about a median value $\frac{1}{2}(B+A)$ with maximum amplitude $\frac{1}{2}(B-A)$; see Figure 3.

A conservative estimate for the truncation error at the time of interest t may be found as follows:

1. Calculate the time $T_F - t$.

2. Calculate the time $T_T - t$.

3. Convert the time $(T_F - t)$ to the number of periods of the cutoff frequency ω_c by multiplying $(T_F - t)$ by $\frac{\omega_c}{2\pi}$. Call this number x_F.

4. Convert the time $(T_T - t)$ to the number of periods of the cutoff frequency ω_c by multiplying $(T_T - t)$ by $\frac{\omega_c}{2\pi}$. Call this number x_T.

5. Using Figure 4, read the value of C_1 at x_F: $C_1(x_F)$. Read the value of C_1 at x_T: $C_1(x_T)$.

6. Using Figure 5, read the value of C_2 at x_F: $C_2(x_F)$. Read the value of C_2 at x_T: $C_2(x_T)$.

7. Calculate a conservative estimate for the truncation error E_T:

$$E_T = \frac{|B+A|}{2}\left[C_1(x_F) - C_1(x_T)\right] + \frac{|B-A|}{2}\left[C_2(x_F) - C_2(x_T)\right] \quad [6]$$

E_T is an upper limit for the error at time t due to the truncation of the record at time T_T. The formula for E_T is derived in Appendix B.

Exact Estimation of Truncation Error

If the preceding error estimate indicates that unacceptable errors in the filtered function may occur at times of interest within the analyzed portion of the record, the errors may be eliminated entirely or reduced in magnitude by the following procedure.

The unanalyzed portion of the record, designated as $f'(t)$ in Figure 3, is sampled and filtered as if it were a transient beginning at time T_T. Filtered values of $f'(t)$ are obtained

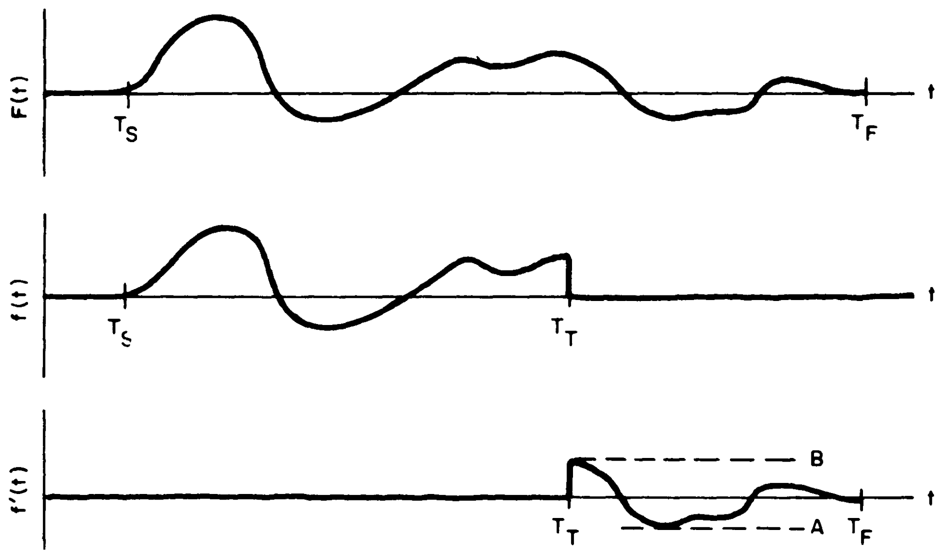


Figure 3 – Truncating a Record To Be Filtered

from the filter program for the time interval from T_S to T_T and are added point by point to the filtered values of $f(t)$ previously obtained over the same interval of time.

If the entire transient signal may be fitted into the computer by this procedure, truncation errors will be eliminated completely. If, on the other hand, the time interval from T_T to T_F contains more than 2000 samples, it will be necessary to truncate $f'(t)$ in its turn at a new truncation time $T_T{}'$ and to leave an unanalyzed portion of the record $f''(t)$ beginning at time $T_T{}'$. The error estimate of the preceding section may be applied to the truncation at $T_T{}'$, by using upper and lower bounds B and A appropriate to the unanalyzed portion of the record $f''(t)$. An appreciable reduction in truncation error over the interval from T_S to T_T will usually be found after this procedure.

APPLICATION

If the filter presented here is used as a frequency filter, the preceding section describes the corrections to be made concerning sampling and truncating the continuous record for computer processing.

However, the record to be filtered is often assumed to have been produced by a combination of two or more physical processes which contribute record components concentrated about different frequencies. If the filter is used to separate the contributions of each physical process (that is, to separate the high- and low-frequency modes of a signal), a new source of error is introduced. This error is an approximation error and is a measure of how well the filter presented here approximates a mode filter.

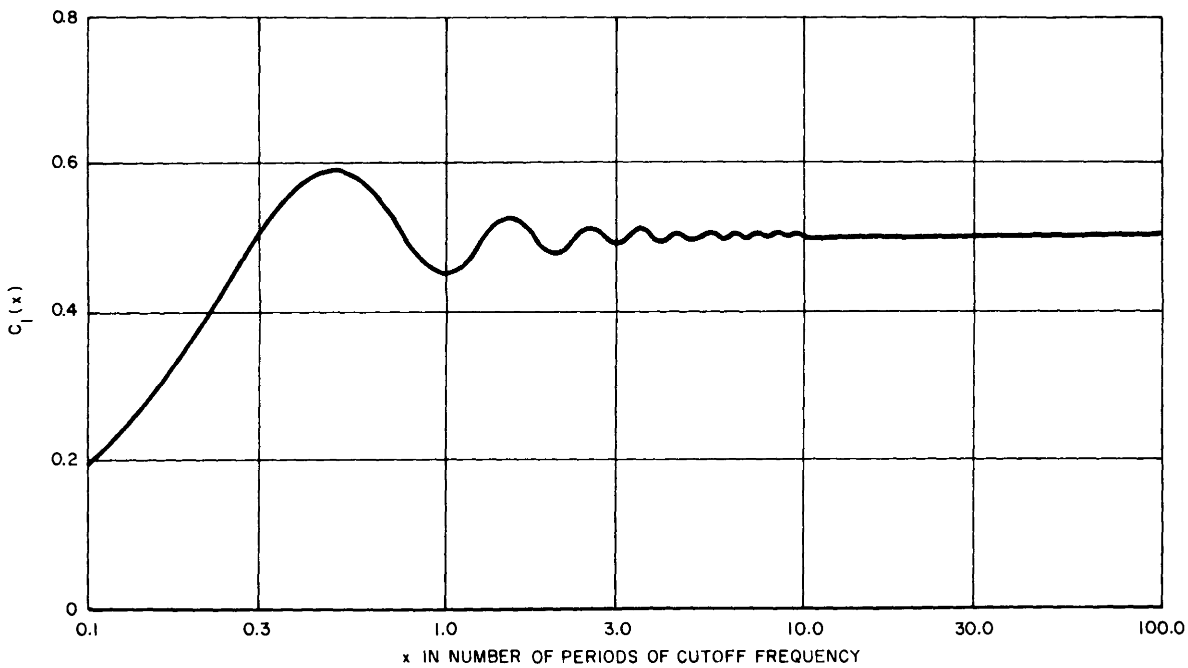


Figure 4 – Factors for Estimating Truncation Error Associated with Median Value of Truncated Portion of Record

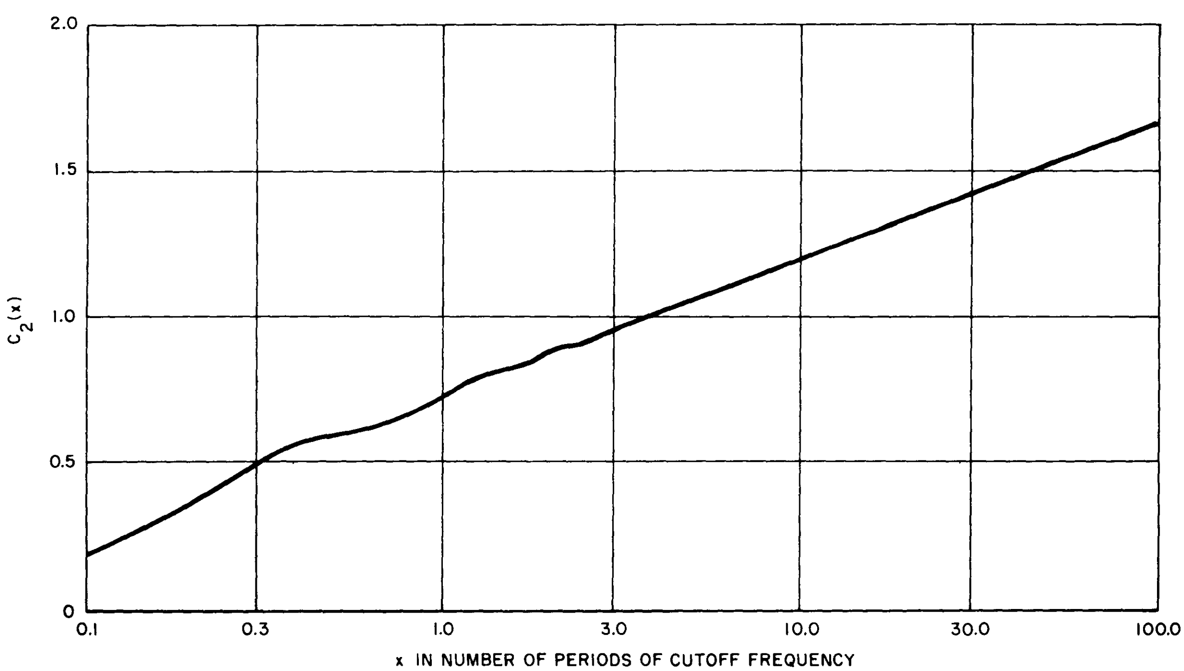


Figure 5 – Factors for Estimating Truncation Error Associated with Oscillations in Truncated Portion of Record

EXAMPLE OF APPROXIMATION ERROR

As an example, consider the spectrum of a record as represented by curve a of Figure 6. A mode filter would operate on this spectrum and might result in a low-mode spectrum represented by curve b of Figure 6. The filter presented here would operate on this spectrum strictly on the basis of frequency separation with no regard to individual modes. Used as a low-pass filter, it would not affect any of the frequencies in the spectrum below the cutoff frequency ω_c and it would completely suppress the frequencies above ω_c, as shown in curve c of Figure 6. Although the spectrum of the frequency filter closely approximates the spectrum of the mode filter for this particular case, there will be differences in the respective time-history responses after filtering. Since the spectrum of the mode filter has nonzero values for all frequencies from 0 to ∞, its time-history record will be able to rise sharply after the signal begins, as is characteristic of transient record. The spectrum of the frequency filter contains no frequencies higher than ω_c. Thus, if the original record contained any sharp changes (e.g., at the beginning of a transient), the filtered record could approximate these only by combining components with frequencies below ω_c. It does this by looking ahead in time to anticipate the beginning of the transient and produces precursor oscillations to smooth away discontinuities in the derivatives of the signal at the beginning of the transient. The lower curve of Figure 2a shows these precursor oscillations; they are characteristic of a filter designed to produce no phase shift in the filtered record.

If the filter described here is used as an approximation to a mode filter, evaluation of the accuracy of the approximation will be inexact. However, two methods of estimating approximation error which should yield reasonable error estimates are now presented.

TWO METHODS OF EVALUATING APPROXIMATION ERROR

Vary the Cutoff Frequency

The record being investigated may be filtered at two or more different cutoff frequencies, and the results compared. If a change in the cutoff frequency produces little change in the appearance or in the peak values of the filtered records, the spectrum of the original record consists of relatively independent high- and low-frequency processes, and the cutoff frequencies used fall between the frequencies of the two processes; see Figure 7. The filtered records may then be associated with the processes, with an uncertainty which can be estimated by the changes observed as a result of a change in the cutoff frequency.

If a change in the cutoff frequency produces a large change in the appearance or peak values of the filtered records, a significant portion of the spectrum of the record falls between the cutoff frequencies chosen; see Figure 8. Here, the separation of the record into high- and low-frequency portions is less likely to have physical significance in terms of assumed

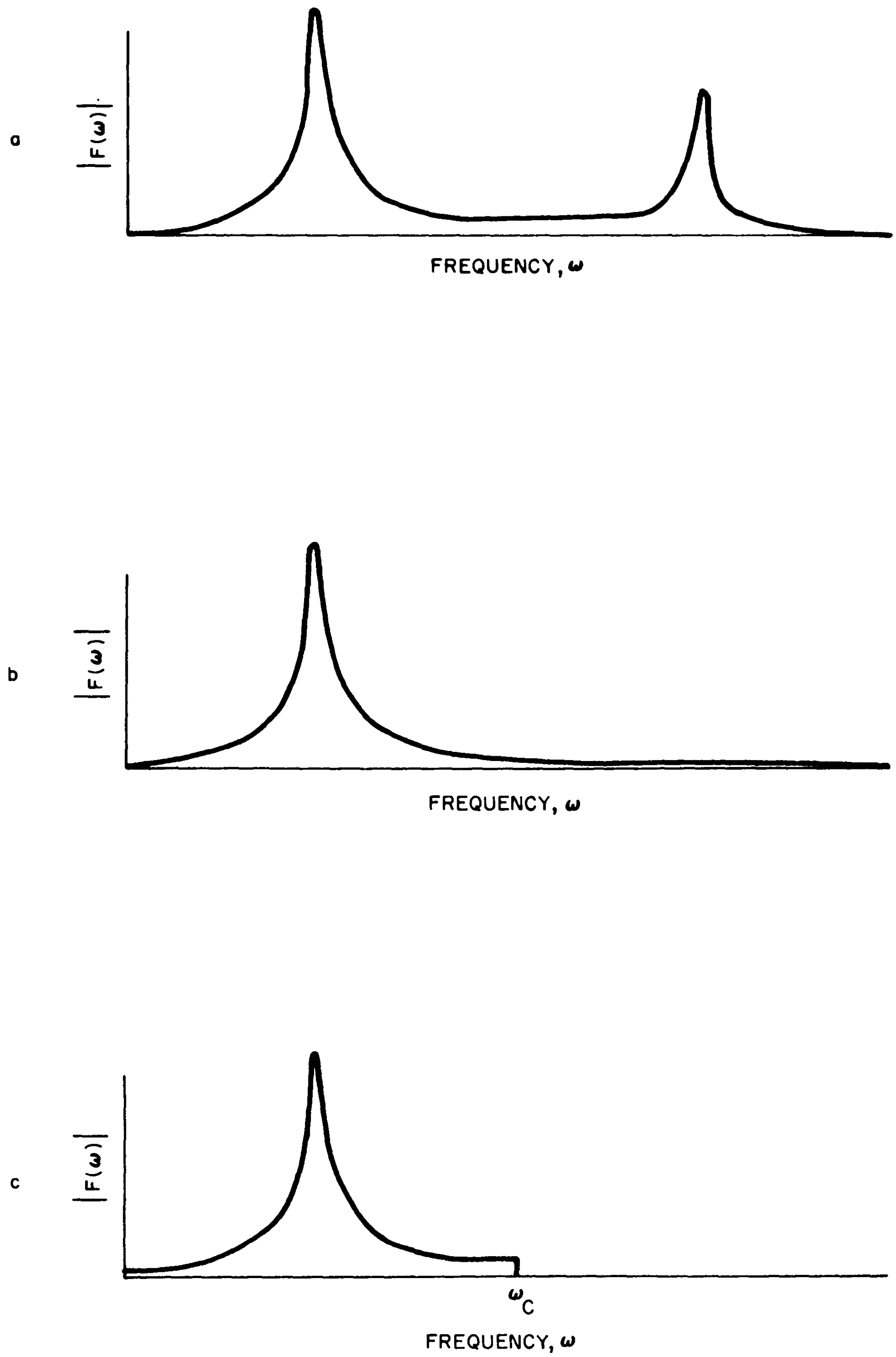


Figure 6 – Mode Filter Compared to Frequency Filter

Curve a is the spectrum of a record produced by two combined modes of response.

Curve b is the spectrum of the low mode only.

Curve c is the spectrum of the low frequencies only.

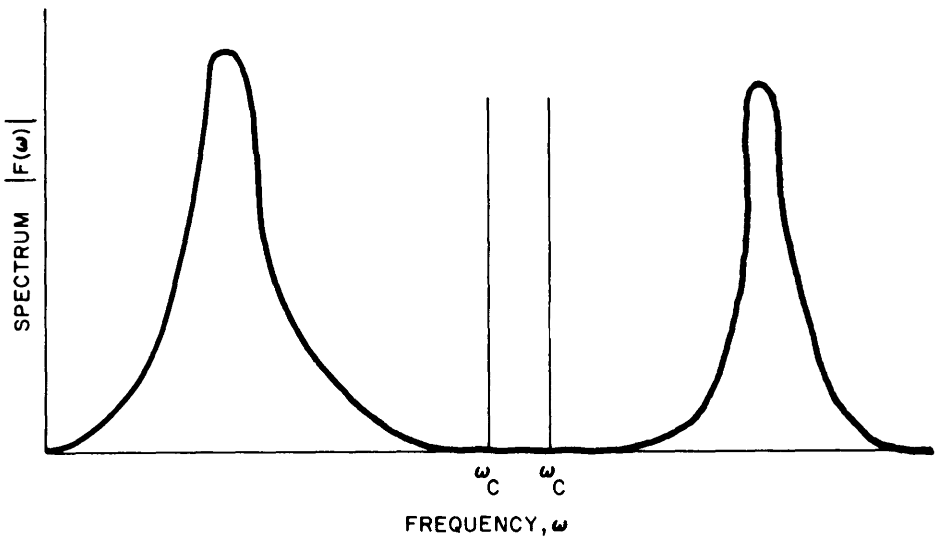


Figure 7 – Example of Record Spectrum Giving Small Approximation Error

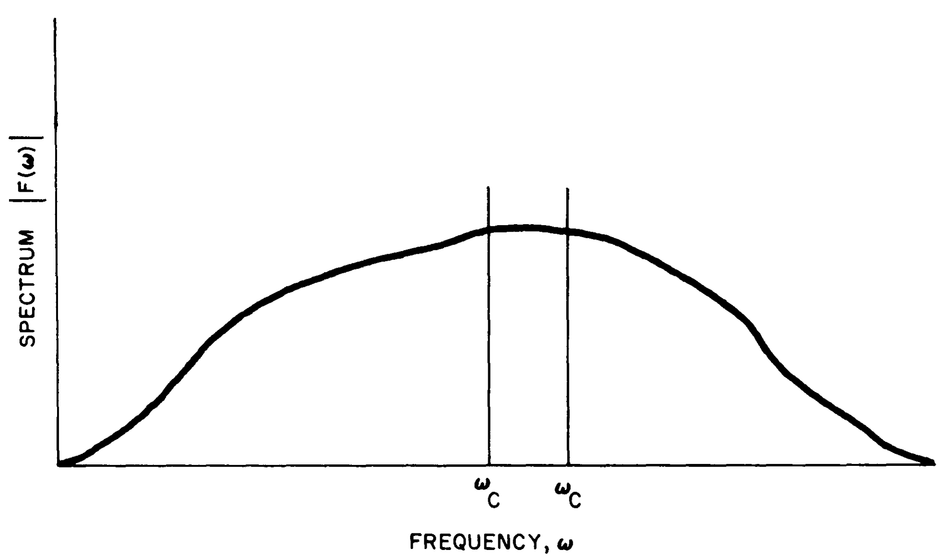


Figure 8 – Example of Record Spectrum Giving Large Approximation Error

high-frequency and low-frequency physical processes; although the filter may be useful for other purposes, it cannot be used to separate the record into physically meaningful portions.

Construct a Model of the Process

A record may be manufactured by combining simple signals similar to those expected from two or more physical processes. The ability of the filter to separate the simple signals may then be used as a measure of its ability to separate the similar signals expected in an experimental record and to associate the filtered records with the individual processes.

Figure 2a shows a record constructed to simulate some of the features expected from the transient response of a cantilever beam to an impulsive load. The record combines responses in the first and second modes of the beam. The second mode is at a frequency six times the frequency of the first mode, with appreciable damping in the second mode. The accuracy with which the low-frequency portion of the record represents the first-mode response when the cutoff frequency is properly chosen is shown as the lower curve in Figure 2b, and the accuracy with which the high-frequency portion of the record represents the second-mode response is shown as the upper curve in Figure 2b. Interpretation of the filtered portions in terms of modal contributions is less valid for other choices of the cutoff frequency; see Figures 2c and 2d.

CONCLUSIONS

1. A program written in FORTRAN for an IBM 7090 digital computer will separate a transient record into low- and high-frequency components without phase shift or amplitude change and with a sharp cutoff frequency between the components.

2. Digital calculations used in the program are mathematically exact if the transient is represented by 2000 (or fewer) samples taken at a time increment equal to (or less than) one-half period of the highest frequency of interest in the record. A method is given for estimating error if more than 2000 samples are required to represent a particular transient record.

3. Application of the filter to separation of modes in a transient record is described. Two methods are given for estimating the error which results from interpreting the separation of a transient into frequency components as a separation into different modal components.

APPENDIX A

SAMPLING ERROR

It was stated that the trapezoidal integration formula of Equation [5] was an exact representation of the filter for each sampled time provided there were at least two samples taken from $F(t)$ per period of the highest frequency of interest in the calculation. The statement will be proved in this appendix. Let

$$S(t) = C \cos(\omega t + \phi) \qquad [7]$$

be a single sinusoidal component of the record $F(t)$. The low-frequency portion of $S(t)$ may be expressed in terms of Equation [2] as

$$S_1(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} C \cos(\omega t' + \phi) \frac{\sin \omega_c (t - t')}{t - t'} dt' \qquad [8]$$

which may be expanded to

$$\begin{aligned} S_1(t) &= \frac{C}{\pi} \cos(\omega t + \phi) \int_{-\infty}^{\infty} \frac{\cos \omega (t - t') \sin \omega_c (t - t')}{t - t'} dt' \\ &+ \frac{C}{\pi} \sin(\omega t + \phi) \int_{-\infty}^{\infty} \frac{\sin \omega (t - t') \sin \omega_c (t - t')}{t - t'} dt' \end{aligned} \qquad [9]$$

The integrals may be evaluated to give

$$\begin{aligned} S_1(t) &= 0 && \text{if } \omega > \omega_c \\ S_1(t) &= \tfrac{1}{2} C \cos(\omega t + \phi) && \text{if } \omega = \omega_c \\ S_1(t) &= C \cos(\omega t + \phi) && \text{if } \omega < \omega_c \end{aligned} \qquad [10]$$

Equation [10] describes the filter characteristic. The single component of the record is completely suppressed if its frequency is above the cutoff frequency, and it is unchanged in amplitude or phase if its frequency is below the cutoff frequency.

Equation [10] gives the exact results of using Equation [2] to filter $S(t)$. Now let $S(t)$ be sampled at an interval h

$$S(kh) = C \cos(\omega kh + \phi) \qquad [11]$$

and apply the trapezoidal approximation of Equation [5]

$$S_1\,(kh) = \frac{1}{\pi} \sum_{n=-\infty}^{\infty} C \cos\,(\omega nh + \phi)\, \frac{\sin\,\omega_c h\,(k-n)}{k-n} \qquad [12]$$

which may be expanded to

$$S_1\,(kh) = \frac{C}{\pi} \cos\,(\omega kh + \phi) \sum_{n=-\infty}^{\infty} \frac{\cos\,\omega h\,(k-n)\,\sin\,\omega_c h(k-n)}{k-n}$$

$$+ \frac{C}{\pi} \sin\,(\omega kh + \phi) \sum_{n=-\infty}^{\infty} \left[\frac{\sin\,\omega h(k-n)\,\sin\,\omega_c h(k-n)}{k-n}\right] \qquad [13]$$

In Equation [13], the term with $n = k$ in the second sum has the value zero. The remaining terms cancel in pairs symmetrical about $n = k$, and the sum vanishes.

The term with $n = k$ in the first sum has the value $\omega_c h$. The remaining terms add in pairs symmetrical about $n = k$. Let $m = n - k$

$$S_1\,(kh) = \frac{C}{\pi} \cos\,(\omega kh + \phi) \left[\omega_c h + 2 \sum_{m=1}^{\infty} \frac{\cos\,m\omega h\,\sin\,m\omega_c h}{m}\right] \qquad [14]$$

$$= \frac{C}{\pi} \cos\,(\omega kh + \phi) \left[\omega_c h + \sum_{m=1}^{\infty} \frac{\sin\,m\,(\omega_c h + \omega h)}{m} + \sum_{m=1}^{\infty} \frac{\sin\,m\,(\omega_c h - \omega h)}{m}\right] \qquad [15]$$

Equation [15] can now be evaluated by recognizing that

$$G\,(x) = \sum_{m=1}^{\infty} \frac{\sin\,mx}{m} \qquad [16]$$

is the Fourier series expansion of the odd periodic function:

$$\begin{aligned} G\,(x) &= \tfrac{1}{2}\,(\pi - x) \qquad \text{for} \quad 0 < x < 2\pi \\ G\,(x \pm 2\pi) &= G\,(x) \\ G\,(0) &= 0 \end{aligned} \qquad [17]$$

If $\omega_c h + \omega h < 2\pi$, the arguments of $G(\omega_c h + \omega h)$ and $G(\omega_c h - \omega h)$ will be within the range from -2π to 2π. The argument of $G(\omega_c h - \omega h)$ will be positive, zero, or negative as $\omega < \omega_c$, $\omega = \omega_c$, and $\omega > \omega_c$, respectively.

Thus by Equation [17]

$$\begin{aligned} G(\omega_c h - \omega h) &= \tfrac{1}{2}(\pi - \omega_c h + \omega h) && \text{if} \quad \omega < \omega_c \\ G(\omega_c h - \omega h) &= 0 && \text{if} \quad \omega = \omega_c \\ G(\omega_c h - \omega h) &= -\tfrac{1}{2}(\pi + \omega_c h - \omega h) && \text{if} \quad \omega > \omega_c \end{aligned} \qquad [18]$$

Since the argument of $G(\omega_c h + \omega h)$ will always be positive, $G(\omega_c h + \omega h) = \tfrac{1}{2}(\pi - \omega_c h - \omega h)$ for all cases.

Direct substitution gives

$$\begin{aligned} S_1(kh) &= 0 && \text{if} \quad \omega > \omega_c \\ S_1(kh) &= \tfrac{1}{2} C \cos(\omega kh + \phi) && \text{if} \quad \omega = \omega_c \\ S_1(kh) &= C \cos(\omega kh + \phi) && \text{if} \quad \omega < \omega_c \end{aligned} \qquad [19]$$

This result is identical with the result of Equation [10].

Let ω_H be a "highest frequency of interest" assigned to a particular record by some criterion outside of the present development, such as the characteristics of the sensing and recording equipment or the ultimate use of the data from the record. Components with frequency ω greater than ω_H are then not of interest, and cutoff frequencies ω_c greater than ω_H would be redundant. The requirement

$$\omega_H h < \pi \qquad [20]$$

ensures that $\omega_c h + \omega h < 2\pi$ for all cutoff frequencies and component frequencies of interest and causes the trapezoidal integration formula to represent the filter exactly.

By Equations [5] or [12], the special case

$$\omega_c h = \omega h = \omega_H h = \pi \qquad [21]$$

causes the trapezoidal integration formula to reproduce the sampled record exactly. For example, Equation [12] becomes

$$S_1(kh) = \frac{1}{\pi} \sum_{n=-\infty}^{\infty} \frac{C \cos(\omega nh + \phi) \sin \pi (k-n)}{k-n} \qquad [22]$$

Every term in the sum vanishes except when $n = k$ and the value of the sum is $\pi C \cos(\omega kh + \phi)$.

Thus Equation [22] becomes

$$S_1(kh) = C \cos(\omega kh + \phi) \qquad [23]$$

and $S_1(kh) = S(kh)$ by Equation [11].

This illustrates the theorem[4] that a sampled record contains no information about any frequencies higher than the frequency corresponding to one-half the sampling rate. The relation of Equation [20] may be combined with the special case, Equation [21], to give the requirement

$$\frac{2\pi}{\omega_H} \geq 2h \qquad [24]$$

for zero sampling error in the program. Equation [24] indicates that at least two samples must be taken from the record in each period of the highest frequency of interest.

APPENDIX B

TRUNCATION ERROR

It was stated that a conservative formula for the truncation error E_T is given by:

$$E_T = \frac{|B + A|}{2} \left[C_1(x_F) - C_1(x_T)\right] + \frac{|B - A|}{2} \left[C_2(x_F) - C_2(x_T)\right] \quad [25]$$

If the values of $f'(t)$ in the truncated portion of the record are always greater than A and always less than B, $f'(t)$ may be considered an oscillation with maximum amplitude $\frac{1}{2}(B - A)$ taking place about a median value $\frac{1}{2}(B + A)$; see Figure 3. The difference between the filtered value of the complete record $F(t)$ and the filtered value of the truncated record $f(t)$ will then have an upper limit given by

$$F_1(t) - f_1(t) \leqq \frac{B + A}{2\pi} \int_{T_T}^{T_F} \frac{\sin \omega_c (t - t')}{t - t'} \, dt' + \frac{B - A}{2\pi} \int_{T_T}^{T_F} \frac{|\sin \omega_c (t - t')|}{t - t'} \, dt' \quad [26]$$

and a lower limit given by

$$F_1(t) - f_1(t) \geqq \frac{B + A}{2\pi} \int_{T_T}^{T_F} \frac{\sin \omega_c (t - t')}{t - t'} \, dt' - \frac{B - A}{2\pi} \int_{T_T}^{T_F} \frac{|\sin \omega_c (t - t')|}{t - t'} \, dt' \quad [27]$$

for any value of t less than the truncation time T_T.

The first term on the right-hand side of Equations [26] and [27] represents the error produced by neglecting the median value of $f'(t)$; the second term shows the effects of the oscillations in $f'(t)$. An upper limit for the effects of the oscillations was obtained in

Equation [26] by assuming that the oscillations caused $f'(t)$ to have its largest possible value whenever the sine function in the filter integral was positive and its smallest value whenever the sine function was negative. The opposite assumption was made to obtain the lower limit shown in Equation [27].

A useful measure of the error may be obtained conservatively by combining the terms in Equations [26] and [27] in the most unfavorable way. An upper limit E_T for the error is then given in the form:

$$E_T = \frac{|B+A|}{2}\left[C_1(x_F) - C_1(x_T)\right] + \frac{|B-A|}{2}\left[C_2(x_F) - C_2(x_T)\right] \qquad [28]$$

where $$x_F = \frac{\omega_c}{2\pi}(T_F - t) \quad \text{and} \quad x_T = \frac{\omega_c}{2\pi}(T_T - t) \qquad [29]$$

and $$C_1(x) = \frac{1}{\pi}\int_0^{2\pi x} \frac{\sin u}{u}\, du \qquad [30]$$

$$C_2(x) = \frac{1}{\pi}\int_0^{2\pi x} \frac{|\sin u|}{u}\, du \qquad [31]$$

Figure 4 shows the function C_1 in terms of the upper limit of integration, and Figure 5 shows the function C_2.

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David Taylor Model Basin. Report 1795.
A DIGITAL FILTER FOR SEPARATING HIGH- AND LOW-FREQUENCY COMPONENTS OF A TRANSIENT SIGNAL, by Thomas L. Geers and Stewart A. Denenberg. Jan 1964. v, 33 p. illus., refs. UNCLASSIFIED

A filter is described which separates a transient signal into high- and low-frequency portions and introduces no phase shifts or amplitude changes in either portion. A program written in FORTRAN for an IBM 7090 computer which will filter a sampled time-history record is presented. Simple formulas are given for estimating errors in the filter that are due to sampling and truncation of the continuous record. Application of the filter to signals produced by two or more physical sources is discussed.

1. Frequency filters--Data analysis
2. Transient signals--Filtering--Data analysis
3. FORTRAN (Computer program language)--Applications
I. Geers, Thomas L.
II. Denenberg, Stewart A.
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