Report 1689

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# A COMPARISON OF EXPERIMENTAL AND THEORETICAL FORCES AND MOMENTS ACTING ON A RESTRAINED SURFACE SHIP IN REGULAR WAVES

by

Alvin Gersten



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HYDROMECHANICS LABORATORY

RESEARCH AND DEVELOPMENT REPORT

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# A COMPARISON OF EXPERIMENTAL AND THEORETICAL FORCES AND MOMENTS ACTING ON A RESTRAINED SURFACE SHIP IN REGULAR WAVES

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# A Comparison of Experimental and Theoretical Forces and Moments Acting on a Restrained Surface Ship in Regular Waves

**By Alvin Gersten**[1]

**A mathematically defined, fully restrained, ship form has been towed in regular waves to measure the exciting forces and moments acting on it. The results of these tests are presented and compared with theoretical values computed both on the basis of assumptions made in the Froude-Krylov hypothesis and an extension of this hypothesis which accounts for ship-wave interference. For low values of draft to wavelength ratio, a number of the presently available prediction techniques are in agreement and are shown to be reasonably accurate. The need for a generally applicable correction for speed effect, however, is demonstrated.**

THE attainment of an adequate mathematical representation of the forces acting on a ship in a seaway and the response of the ship to these forces is of primary importance if ship motions are to be reduced and the ability of ships to travel at high speed in rough seas enhanced. If and when this mathematical theory can be evolved, it will be possible to understand clearly the influence of ship form on motions and to modify these form characteristics early in the design stage so that an optimum design can be achieved.

It has been assumed by most investigators in the field of seaworthiness that the oscillatory forces on a ship in sinusoidal waves are directly proportional to the instantaneous displacement from equilibrium position, velocity and acceleration; that the coefficients are dependent on the frequency of oscillation and forward speed, and are independent of the amplitude of oscillation; and that the wave action can be considered to produce exciting forces proportional to the incident wave height. The resulting equations for heave and pitch are

$$a(\omega)\,\ddot{z} + b(\omega)\dot{z} + cz + d(\omega)\,\ddot{\theta} + e(\omega)\dot{\theta} + f\,\theta = F_0\,e^{i(\omega t - \delta)} \quad (1a)$$

$$A(\omega)\,\ddot{\theta} + B(\omega)\,\dot{\theta} + C\theta + D(\omega)\,\ddot{z} + E(\omega)\,\dot{z} + Fz = M_0 e^{i(\omega t - \alpha)} \quad (1b)$$

where $a$ is the virtual mass and $A$ the virtual moment of inertia; $z$ is the heaving displacement and $\theta$ the angular displacement in pitch; $b$ and $B$ are the damping coefficients; $c$ and $C$ are the restoring coefficients; $d, e, f, D, E$, and $F$ are the coupling coefficients. The terms on the right side of equations (1) are the exciting force and moment with $\delta$ and $\alpha$ the phase between the heaving force and wave elevation amidships and pitching moment and wave elevation, respectively.

A different mathematical model has recently been proposed by Cummins [1].[2] He writes the equations of

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[2] Numbers in brackets designate References at end of paper.

## Nomenclature

- $A_{wp}$ = waterplane area (10.05 sq ft)
- $C^*$ = wave velocity
- $C_z = \dfrac{\bar{F}_z}{\rho g\, A_{wp}\, r}$
- $C_\theta = \dfrac{\bar{M}_\theta}{\rho g\, L\, A_{wp}\, r}$
- $C_x = \dfrac{\bar{F}_x}{\rho g\, m\, A_{wp}\, Hr}$
- $\bar{F}_z$ = oscillatory heaving force, single amplitude
- $\bar{F}_x$ = oscillatory surging force, single amplitude
- $\mathfrak{F}$ = Froude number = $\dfrac{V}{(gL)^{1/2}}$ (+ head seas, − following seas)
- $g$ = acceleration due to gravity
- $H$ = draft of model (0.53 ft)
- $h$ = wave height = $2r$, crest to trough
- $L$ = length of model (11.33 ft)
- $\bar{M}_\theta$ = oscillatory pitching moment, single amplitude
- $m$ = wave number = $\dfrac{2\pi}{\lambda}$
- $\hat{n}$ = angular frequency = $\dfrac{2\pi}{\tau}$
- $p$ = total gage pressure
- $p^*$ = oscillatory component of pressure
- $r$ = wave single amplitude, h/2
- $V$ = model velocity
- $X, Y, Z$ = axes of fixed coordinate systems
- $x, y, z$ = coordinates referred to the fixed axes
- $\theta$ = angular displacement in pitch
- $\lambda$ = wavelength
- $\rho$ = mass density of fresh water (1.94 slug/cu ft)
- (0), (1), (2) = superscripts defined in text

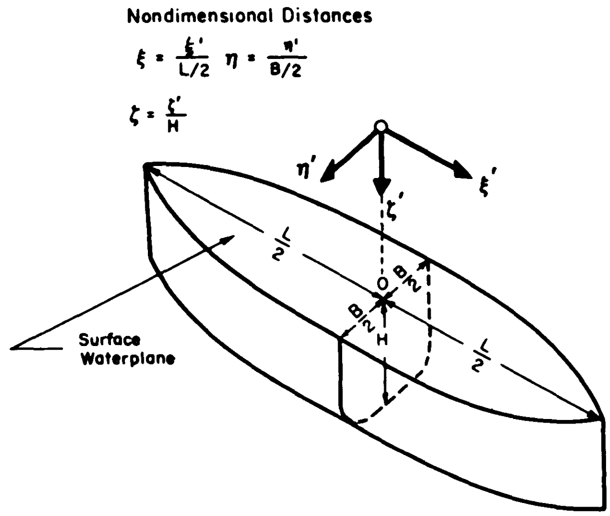


Fig. 1 Definition of dimensionless coordinates

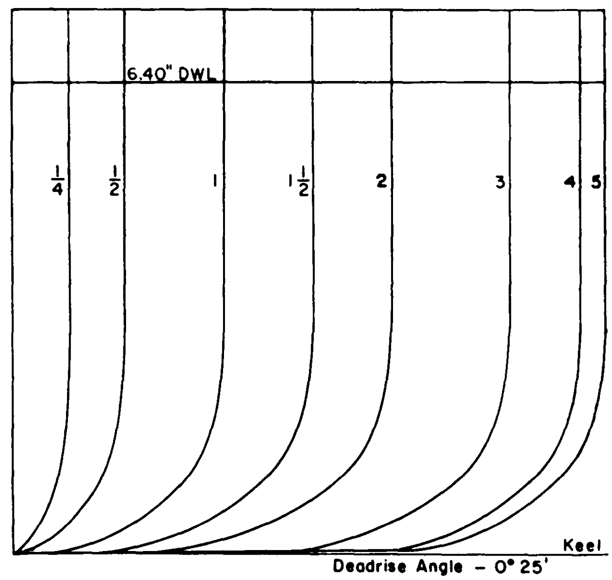


Fig. 2 Body plan of model

motion of a ship which is subjected to an arbitrary heaving force and pitching moment as follows:

$$a'\ddot{z} + b'\dot{z} + c'z + d'\ddot{\theta} + e'\dot{\theta} + f'\theta + \int_{-\infty}^{t} K_z (t-\tau)\, \dot{z}(\tau)\, d\tau + \int_{-\infty}^{t} K_{\theta z} (t-\tau)\, \dot{\theta}(\tau)\, d\tau = F(t) \qquad (2a)$$

and

$$A'\ddot{\theta} + B'\dot{\theta} + C'\theta + D'\ddot{z} + E'\dot{z} + F'z + \int_{-\infty}^{t} K_\theta (t-\tau)\, \dot{\theta}(\tau)\, d\tau + \int_{-\infty}^{t} K_{z\theta} (t-\tau)\, \dot{z}(\tau)\, d\tau = M(t) \qquad (2b)$$

where

$$a' = a(\infty) \qquad d' = d(\infty)$$
$$A' = A(\infty) \qquad D' = D(\infty)$$
$$c' = c, f' = f, C' = C, F' = F$$

and $b'$, $e'$, $B'$, $E'$ are constants independent of frequency; $K_z(t)$ and so on, are functions of time which are independent of frequency; $F(t)$ and $M(t)$ are arbitrary excitations of which the right-hand side of equations (1) are particular examples.

In either of the foregoing representations of the ship's motion, when the response characteristics of the ship are known, one still needs a knowledge of the excitation before the motions can be computed. Thus far, the major portion of the experimental research effort has been directed towards determining the coefficients on the left-hand side of equations (1). There is an extremely limited number of papers which present the results of direct measurement of the forcing functions. The papers published by Jinnaka [2] and Gerritsma [3,4] are the only ones which contain a significant amount of data. Jinnaka measured the exciting force for heaving and surging and the exciting moment for pitching and rolling on two models having different length-to-beam ratios. Gerritsma measured the exciting force for heaving and the exciting moment for pitching on Series 60 models having block coefficients of 0.60, 0.70 and 0.80. Both series of tests were conducted in head seas only.

In the present study measurements of the excitation in heave and pitch as well as the surging force have been made on a mathematical model in the Weinblum series. The tests were conducted in both head and following seas. In addition, use of three nominal wave heights permitted an extensive examination of the relation between the forces and moments and wave height.

Earlier tests were conducted on this model at the David Taylor Model Basin to evaluate the added mass and damping of the ship motion (for heave) as well as the pitching moments due to heaving velocity and acceleration [5].

The purpose of this paper is as follows:

1 To present the oscillatory exciting force and moment data with associated phases.

2 To discuss the validity of the assumption that the distortion of the water flow and of the pressure distribution in the wave can be neglected (Froude-Krylov hypothesis).[3]

3 To determine whether present methods of correcting for the influence of the ship on the wave (as sum-

[3] A more complete statement of the Froude-Krylov hypothesis is contained in a later section on the "Theoretical Calculation of Forces and Moments."

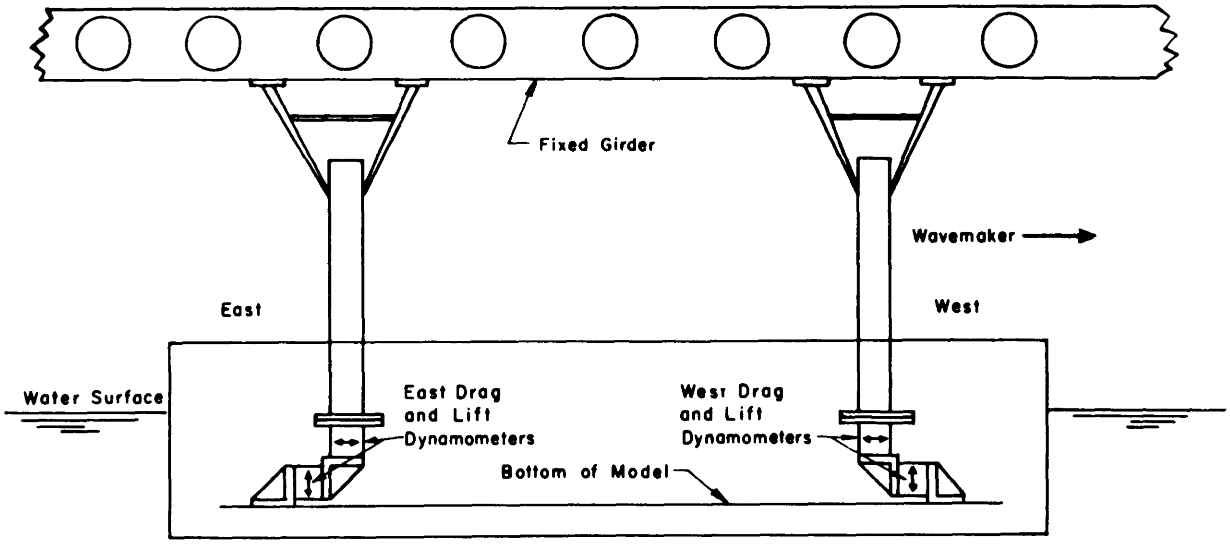


Fig. 3 Dynamometer and model assembly

Table 1 Model Constants

| | | |
|---|---|---|
| Length | 136 | inches |
| Beam | 16 | inches |
| Draft | 6.4 | inches |
| Displacement | 316.6 | pounds |
| Midship Area Coefficient | 0.964 | |
| Load Waterline Coefficient | 0.667 | |
| Prismatic Coefficient | 0.655 | |
| Block Coefficient | 0.632 | |

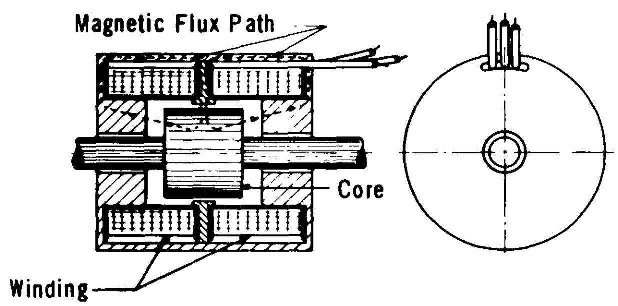


Fig. 4 Schematic of linear-displacement transducer

marized by Vossers [6]) substantially improve the prediction of forcing functions.

4 To discuss the significance of speed effect with regard to forces and moments.

5 To verify the assumption that the forces acting on a ship in a seaway vary linearly with wave height.

### Model Characteristics

The model used in these tests is one of a family of symmetrical, mathematical models whose lines were represented in polynomial form by G. P. Weinblum [7]. The general dimensionless expression for the model hull is given by $\eta = \eta(\xi,\zeta)$ where the dimensionless coordinates $\eta$, $\xi$, and $\zeta$ are defined in Fig. 1. The equation for the model which is of present interest is

$$\eta = \pm\ [(1 - \xi^2) - (\xi^2 - \xi^4)\,\zeta^{10}]\ [1 - 0.3\,\zeta^8 - 0.7\,\zeta^{150}]$$

The model possessed fore-and-aft symmetry and the body plan for the stations from bow to midship is shown in Fig. 2.

Table 1 is a listing of the various model constants. The model was not ballasted to the 6.4-in. waterline but was ballasted slightly heavy. It was then fixed to the towing struts so that it was at its design water line. This resulted in an initial tension load on the struts and provided stability under dynamic loading. This preloading of the dynamometers was compensated for by zeroing the recording equipment after the model was fixed in place.

### Instrumentation and Test Setup

Since the purpose of these experiments was the measurement of forces and moments acting on a model which is excited by wave forces, the body was restrained in all degrees of freedom and the force measurements made by means of displacement-type dynamometers. Accordingly two struts were attached to a fixed girder mounted on the carriage. The lower end of each of these struts was coupled to drag and lift dynamometers by means of steel brackets and the assembled unit rigidly joined to the model, Fig. 3.

The dynamometers were designed at DTMB and consist of an inductive displacement gage (magnigage) in combination with a flexure. The nucleus of the system

Fig. 5 Assembled modular force gage

is a transducer of the differential air-gap type which is shown schematically in Fig. 4. The winding is in two halves which set up the flux paths shown when energized. Displacement of the core results in the establishment of an impedance difference between the two windings with an associated alteration of the voltage drop across the windings. An output signal is then established. The assembled unit, transducer plus flexure, is in the form of a 4-in. cube, Fig. 5, and is sensitive to forces in one direction only. Accuracies of $\pm 1/2$ percent or better are obtainable.

The vertical forces were measured with two dynamometers located equidistantly from the center of the symmetrical model as shown in Fig. 3. Similarly, two drag dynamometers were used. The algebraic sum of the forces measured by the vertical-force gages is the heaving force, and the algebraic difference times one half of the distance between the gages is the pitching moment. Electrical circuits were used to perform these algebraic operations.

Wave height was measured by means of a capacitance-type wave probe mounted approximately 12.5 ft from the model centerline abreast of one end. Superposition of model-generated waves on the record of the incident-wave system is believed to be negligible. All transducer outputs were recorded on an 8-channel Sanborn recorder.

A sample Sanborn record is shown in Fig. 6. Proceeding from bottom to top of the record, the first four channels are the individual gage outputs. The next three channels are those obtained after the individual amplifier signals were passed through the add-subtract circuits. As exemplified by this record, the various signals contained high frequency "noise," largely due to carriage vibration. Since the surging forces are relatively small the signal-to-noise ratio in these traces was poorest. In a number of cases it was necessary to hand-fair the Sanborn traces to permit determination of the force magnitudes.

The tests were conducted in the Deep Water Basin at DTMB which is equipped with a pneumatic wavemaker for generating periodic, sinusoidal waves.

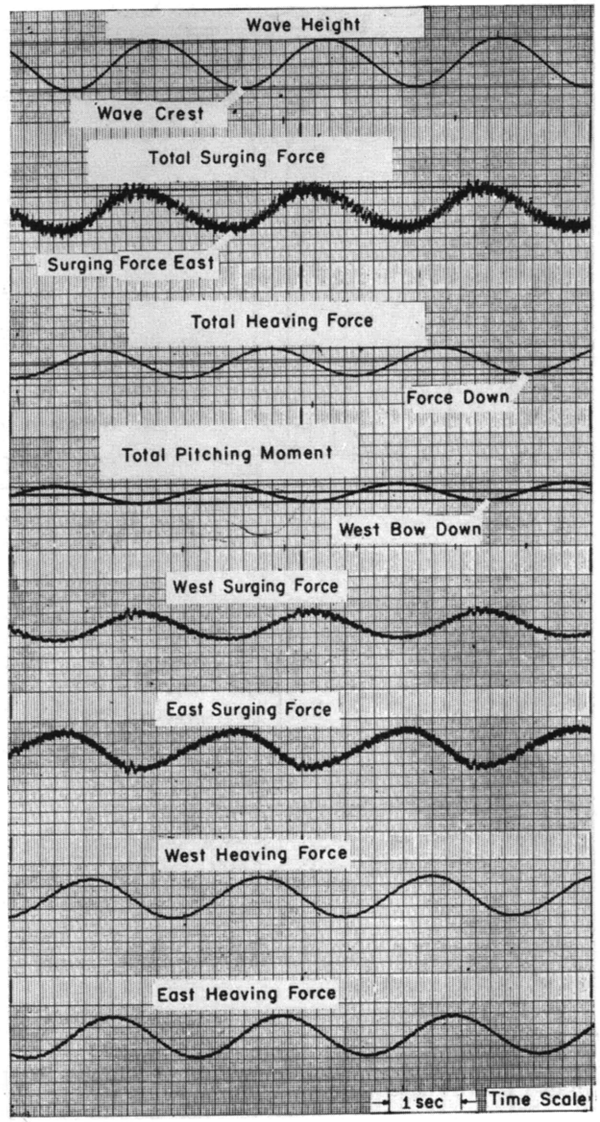


Fig. 6 Sample Sanborn record

## Test Program

The model tests were conducted in regular waves of the length and nominal wave height (distance from crest to trough) listed in Table 2. The model was towed in head and following seas at integer speeds ranging from 0 to 5 knots (Froude numbers 0 to 0.442). A wide range of conditions was used to establish adequately the relationship between the force and moment data and such experimental variables as wave length, wave height, and model speed.

Table 2 Model Speeds and Dimensions of Generated Waves

| Wavelength / Ship Length | Wavelength in feet | Nominal Wave Height, in inches | Wavelength / Wave Height |
|---|---|---|---|
| 1.5 | 17.00 | 2.5 | 81.6 |
| " | " | 5.0 | 40.8 |
| " | " | 7.0 | 29.1 |
| 2.0 | 22.67 | 2.5 | 108.8 |
| " | " | 5.0 | 54.4 |
| " | " | 7.0 | 38.9 |
| 2.5 | 28.33 | 2.5 | 136.0 |
| " | " | 5.0 | 68.0 |
| " | " | 7.0 | 48.6 |
| 3.0 | 34.00 | 2.5 | 163.2 |
| " | " | 5.0 | 81.6 |
| " | " | 7.0 | 58.3 |

| Model Speeds | | | | | | |
|---|---|---|---|---|---|---|
| Knots | 0 | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 |
| Froude Number | 0 | 0.088 | 0.177 | 0.265 | 0.354 | 0.442 |

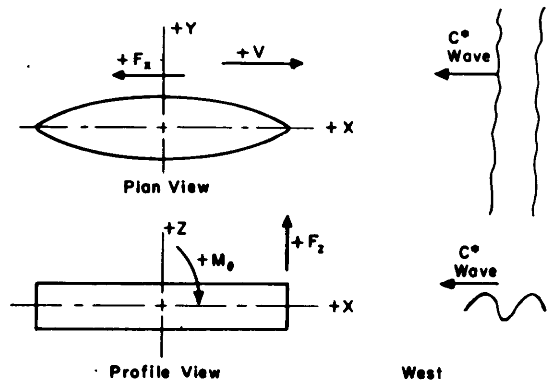

Fig. 7 Sign convention

### Theoretical Calculation of Forces and Moments

Before embarking on a discussion of the theoretical method used to compute the oscillatory forces and moments it is appropriate to define the sign convention used throughout this paper. The following statements apply to Fig. 7:

1 Model velocity is positive when the model heads towards the wavemaker and against the waves (i.e., in the head sea condition).

2 Heaving force is positive when acting upwards.

3 Pitching moment is positive when the effective bow of the model is forced down in head seas and up in following seas where the ship speed is less than the wave celerity.

4 Surging force is positive when it acts to impede the forward motion of the model.

5 Wave velocity, $C^*$, is always negative.

The frequency at which the model encounters the waves is given by

$$\omega = \frac{2\pi}{\lambda} (V - C^*)$$

where

$\lambda$ = wave length
$V$ = model velocity
$C^*$ = wave velocity

The Froude-Krylov hypothesis is the basic tool used here to determine theoretically the forcing function in the equations of motion. This hypothesis was presented by A. N. Krylov in his 1896 paper on the pitching of ships in waves [8] in which he used a simplifying assumption first made by Froude in his studies of ship rolling, viz., that in spite of the presence of the ship the pressure field in the wave and the water flow are not distorted. In addition, the Froude-Krylov hypothesis takes into account the dynamic pressure gradient (Smith effect) due to centripetal acceleration of water particles as they travel their circular orbits in a wave. The pressure due to this dynamic effect when superposed on the hydrostatic pressure subtracts from the latter under wave crests and adds to it under wave troughs. As a result of this, the pressure at a given depth under a wave crest is less than hydrostatic and under a wave trough it is greater than hydrostatic. One point at which the approach used here departs from that used by Krylov is that sinusoidal-wave theory will be applied to compute the pressures in the wave whereas Krylov used trochoidal-wave theory. The amount of work involved in computing the pressure distribution in a trochoidal wave is considerable and would be a formidable task for the number of wave conditions which we desired to investigate. In addition, since for small wave heights the trochoid is almost identical in shape with a sinusoid, it was felt that use of either of the two methods should produce the same results. This last assumption was checked by arbitrarily "placing" the ship model in a wave 22.7 ft in length and 7.0 in. in height and computing the total lift by means of both sinusoidal theory and a trochoidal-theory technique presented in the "Shipbuilding and Shipping Record" [9]. The results agree within 2.3 percent.

To recapitulate then, theoretical oscillatory forces and moments will be computed utilizing the Froude-Krylov hypothesis and sinusoidal-wave theory. Specifically, pressures will be computed at various points on the underwater portion of the model surface and integrations performed to yield forces and moments.

The pressure at any depth under the mean free surface of a sinusoidal wave with infinitesimal height is given by [10]

$$p = \rho g r e^{mz} \sin (mx - nt) - \rho g z \quad (3)$$

where

$p$ = pressure in fluid at depth $z$
$\rho$ = mass density of fluid
$g$ = acceleration due to gravity (32.2 ft/sec²)
$r$ = wave amplitude (zero point to crest or trough)
$m$ = wave number $(2\pi/\lambda)$
$n = 2\pi/\tau$
$\tau$ = wave period

It is important to note that $z$-values below the still-water surface should be taked as negative in this equation. The first term on the right-hand side of equation (3) therefore indicates an exponential decrease in the oscillatory pressure with depth.

The oscillatory heaving force at a specified time is then given by

$$F_z = \int_A p^* \cos(\hat{n},z)\, dA,$$

the oscillatory pitching moment by

$$M_\theta = \int_A p^* x \cos(\hat{n},z)\, dA,$$

and the oscillatory surging force by

$$F_x = \int_A p^* \cos(\hat{n},x)\, dA,$$

where

$p^*$ = oscillatory pressure obtained from equation (3) after removing the hydrostatic pressure term $\rho gz$
$\hat{n}$ = direction normal to model surface
$A$ = wetted surface

Maximum heaving force was obtained with the wave crest amidships, maximum pitching moment and surging force with wave nodal point amidships. All of the integrations were performed graphically.

Calculations were also made of oscillatory heaving force and pitching moment where the pressures were computed on the basis of hydrostatics with the head for any point on the hull taken with reference to the wave surface.

#### Dimensionless Representation of Theoretical and Experimental Oscillatory Forces and Moments

The single amplitudes of heaving force, surging force and pitching moment were obtained by reading peak-to-peak values on the Sanborn record, averaging the values from a number of successive cycles and dividing the result by 2. These oscillatory force and moment data and the values obtained theoretically were converted to dimensionless coefficients. The coefficients are defined as

$$\text{Heaving-force coefficient} = C_z = \frac{\bar{F}_z}{\rho g A_{wp} r}$$

$$\text{Pitching-moment coefficient} = C_\theta = \frac{\bar{M}_\theta}{\rho g L A_{wp} r}$$

$$\text{Surging-force coefficient} = C_x = \frac{\bar{F}_x}{\rho g m\, A_{wp} H r}$$

where $\bar{F}_z$, $\bar{M}_\theta$ and $\bar{F}_x$ are the single amplitude (i.e., maximum) values of oscillatory heaving force, pitching moment and surging force, respectively;

$\rho$ = mass density of fresh water (1.94 slugs/ft³)
$A_{wp}$ = water plane area (10.05 sq ft)
$L$ = model length between perpendiculars (11.33 ft)
$H$ = model draft (0.53 ft)

Superscripts will be added to the theoretical coefficients to distinguish between the various methods of calculation, as follows:

(0) for hydrostatic calculation assuming no disturbance of the wave
(1) for Froude-Krylov calculation (i.e., Smith effect included)
(2) extension of Froude-Krylov calculation to account for ship-wave interference.

The particular form of the coefficients was chosen after preliminary examination of the test results. It was found that the force and moment were approximately linearly related to wave height so that normalization with respect to wave height would cause the data to collapse. The use of the waterplane area in the coefficient results in a convenient asymptotic value of unity for $C_z$ at long wavelengths.

#### Heaving Force Coefficient

The heaving-force coefficient is presented as a function of Froude number in Fig. 8. Two theoretical values are shown for each wavelength. One represents the results of calculation following the Froude-Krylov hypothesis, $C_z^{(1)}$, as computed by the author. These values were then multiplied by a correction coefficient to account for the disturbance of the pressure field in the wave by the ship, this is indicated by $C_z^{(2)}$.

The paper by Vossers cited previously [6] presents a comprehensive survey of theoretical predictions of exciting forces and moments. In it, he discusses the results of investigations made by Grim [11], Korvin-Kroukovsky and Jacobs [12], and Haskind [13]. Each of these studies involved the use of different methods of calculating the vertical forces and moments inclusive of both Smith effect and ship-wave interference.

Vossers compares their results in a plot of the ratio of heaving-force coefficient for a two-dimensional circular section considering wave distortion, to that calculated by the aid of the Froude-Krylov hypothesis [i.e., $C_z^{(2)}/C_z^{(1)}$]. For the low values of draft to wavelength ratio at which the presently discussed experiments were conducted (<0.05), the three methods of calculation are in agreement.

Vossers also presents an equation for this ratio of exciting forces for a three-dimensional ship which was derived by Haskind [13]. It is applicable if the added mass for heaving, $m_z$, of the entire ship is known

$$\frac{C_z^{(2)}}{C_z^{(1)}} = 1 - \frac{m_z}{\rho\nabla}\,\frac{1-\kappa_2}{\kappa_2} \qquad (4)$$

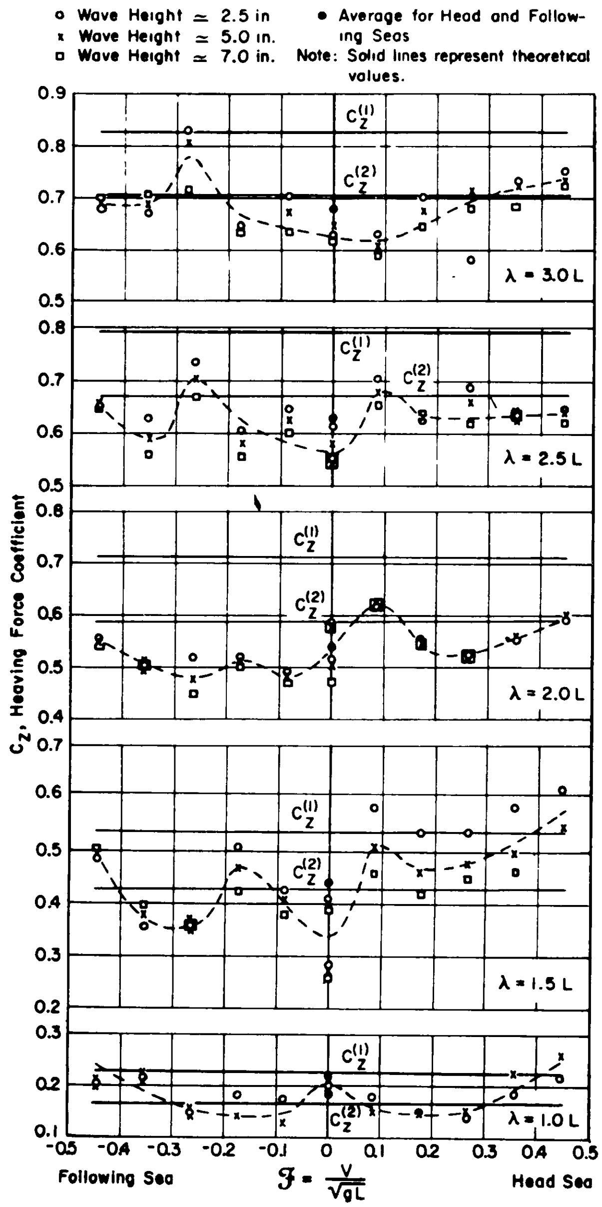


Fig. 8 Variation of heaving-force coefficient with Froude number

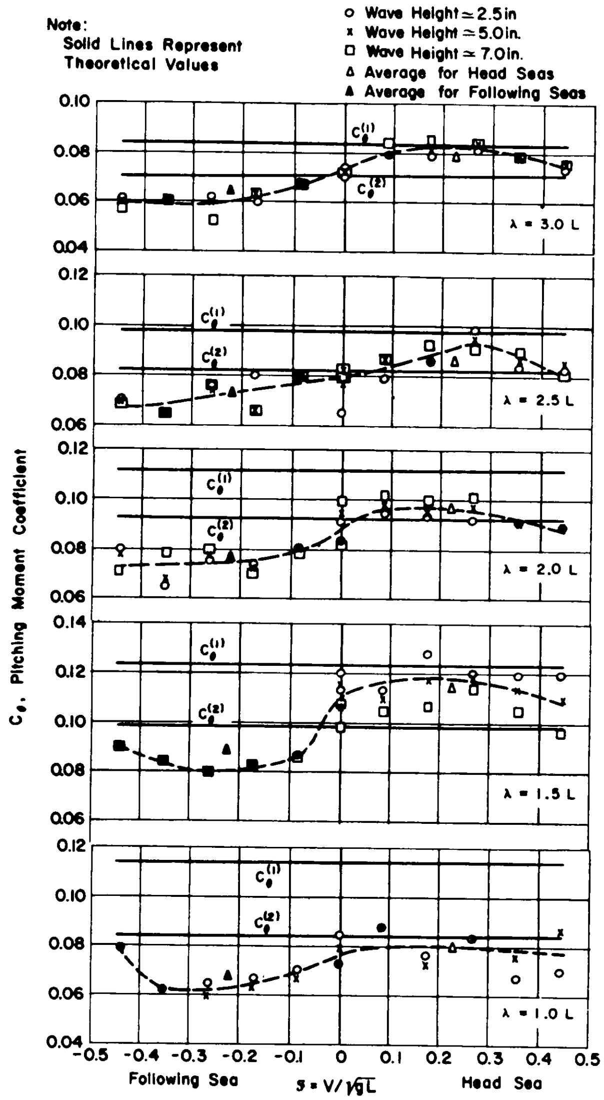


Fig. 9 Variation of pitching-moment coefficient with Froude number

where $\nabla$ is the volume displacement and $k_2$ is a coefficient which varies with $H/\lambda$ and vertical prismatic coefficient (see Fig. 58, reference [6]).

The line representing $C_z^{(2)}$ in Fig. 8 was calculated using equation (4). Of the two methods outlined as corrections for ship-wave interference, this three-dimensional solution yields results that agree better with experiment for the model involved. Although the force coefficients obtained from the other method are always greater, the difference is never more than 7 percent. The values of added mass used in the calculations were taken from reference [5].

As stated by Vossers, two attempts have been made theoretically to calculate the effect of head sea speed variation on the forces acting on surface craft. This work was done by Hanaoka [14] and Korvin-Kroukovsky and Jacobs [12]. Hanaoka predicts an increase of heaving force and pitching moment with forward speed for one ship form, while Korvin-Kroukovsky and Jacobs find for another ship form a decrease in heaving force and approximately constant pitching moment as speed increases. Because of what appear to be contradictions resulting from these studies no attempt was made to cor-

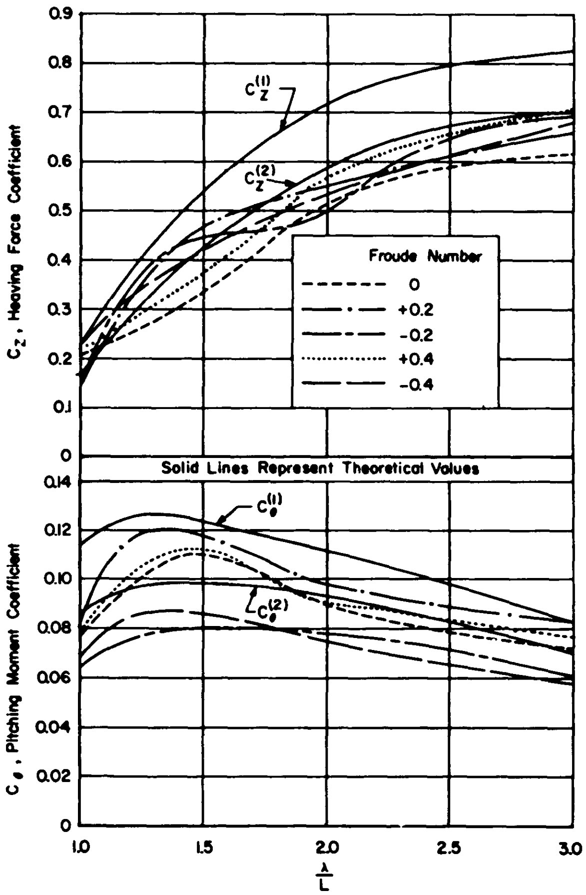


**Fig. 10 Heaving-force and pitching-moment coefficients as a function of wavelength to ship length ratio**

rect the theoretical curves for speed effect in the present investigation. The theoretical values in Fig. 8 have therefore been represented as applicable to all speeds although they are truly based on zero-speed calculations.

The experimental data indicate a complex relationship between force and speed with several peaks within the speed range shown. It is not to be denied that somewhat different curves could be faired through these same data points because of existing scatter. However, where double peaks are shown, the existence of these peaks is corroborated by the data given for the several wave heights.

It can be seen that the theoretical values, $C_z^{(2)}$, are generally in fairly good agreement with the experimental data since, for the most part, the lines are located in, or close to, the main body of data points. There is no question however, that the change in heaving force with speed (in both head and following seas) is significant, and here of course the theory is lacking. The improvement in prediction capability over that achieved with the Froude-Krylov theory *per se* [$C_z^{(1)}$] is obvious.

An average of the empirical data points was computed for each wavelength and are shown as solid circles. The correspondence between these averages and the $C_z^{(2)}$ lines is good with differences ranging from 2 to 8 percent and with a mean of 6 percent.

### Pitching Moment Coefficient

The variation of pitching-moment coefficient with Froude number is shown in Fig. 9. It is apparent that the moment acting on the model is greater in head seas than following seas. Following Vossers [6], $C_\theta^{(2)}$ was obtained from $C_\theta^{(1)}$ by means of the same correction coefficient applied to the heaving forces as derived by Haskind, equation (4). As before, the added mass for heaving was utilized. Implicit in this is the assumption that the error in computing pitching moment without accounting for wave distortion is the same as that for heaving forces. The results seem to encourage this assumption. Here too, the correction given for circular cylinders by Vossers (cf. section on heaving forces) would yield larger moment coefficients. Again, the theoretical values are represented as applicable to all speeds.

The prediction of zero-speed moment by means of $C_\theta^{(2)}$ is seen to be quite accurate. This, of course, is the only condition where great accuracy should be expected *a priori* since, as noted previously, the calculations are based on the assumption of zero forward speed. There are more significant discrepancies between theory and experiment as speed is increased in both head and following seas. In order to evaluate these differences average experimental values are presented for head and following seas independently. The theory predicts values for the head-sea condition which are, on the average, 7 percent different from the actual moment (range of 3 to 13 percent for the different wavelengths). In all cases except $\lambda = 1.0L$ the theory underestimates the true head-sea mean. For following seas it overestimates the actual moment by an average of 14 percent (range of 10 to 19 percent for the different wavelengths).

Correcting for ship-wave interference certainly yields better predictions of zero-speed moment than does direct application of the Froude-Krylov hypothesis. The same is true when underway in following seas. The importance of developing a correction for speed effect is most strikingly demonstrated in head seas where, for some wavelengths, $C_\theta^{(1)}$ actually is a better prediction than $C_\theta^{(2)}$. This occurs simply because the former overestimates the zero-speed moment and this fact certainly does not make it applicable for general use in predicting head-sea pitching moments.

It is evident that the moment coefficient varies quite linearly with respect to wave height (data for different wave heights superpose) so that a single curve adequately represents this relationship for at least those wave heights which do not exceed the maximum used in these experiments.

Gerritsma [3,4] has completed an extensive program designed to check the validity of equations (1) for determining ship motions. Three Series 60 models having block coefficients of 0.60, 0.70 and 0.80 were tested in head seas to obtain the hydrodynamic coefficients and exciting forces and moments in these equations. Then the motions were computed and the results compared with measured motions. The statement is made in reference [4] that it appears the influence of speed on the forcing functions is very small. The results presented herein contradict this, especially for heaving forces. Moreover, there is in evidence a considerable change in pitching moment as transition is made from the head sea to the following-sea regime.

### Relationship Between Coefficients and Wavelength

The magnitude of both $C_z$ and $C_\theta$ is dependent on the wavelength-to-ship length ratio as exhibited in Fig. 10. The curves for several representative speeds, which are well distributed over the test range, are shown. The form of the theoretical force and moment curves is in agreement with the experimental curves for both head and following seas.

It can be seen that the heaving force, both theoretical and experimental, is quite small at the value of $\lambda/L = 1.0$. This is in agreement with Vossers [6] who has demonstrated that for many hull forms the theoretical $C_z$ approaches zero at values of $\lambda/L$ between 0.5 and 1.0. Theoretically the coefficient should approach the value 1.0 as $\lambda \rightarrow \infty$; this trend is borne out by the experiments.

The coefficient $C_\theta$ also appears to be approaching small values at $\lambda/L < 1.0$. Reference [6] shows a similar trend with $C_\theta \rightarrow 0$ at $\lambda/L \simeq 0.7$.

A peak in the theoretical curves is indicated at $\lambda/L \simeq 1.3$ for $C_\theta^{(1)}$ and $\lambda/L \simeq 1.5$ for $C_\theta^{(2)}$. The existence of a maximum within this range is confirmed at all speeds by the experiments.

### Phase Angles

As a consequence of the Froude-Krylov assumptions several conclusions with regard to phase follow:

1 Maximum positive[4] heaving force occurs with wave crest amidships for the test conditions discussed in this paper.

2 Maximum positive[4] pitching moment lags wave crest amidships by 90 deg for all speeds in head seas, and in following seas it lags by 90 deg from zero speed to the point where model speed equals wave speed. When model speed is greater than wave speed it leads by 90 deg.

3 Maximum positive pitching moment referred to maximum positive heaving force follows from statements 1 and 2 above, i.e., for test conditions considered herein, the moment lags the force by 90 deg.

In the determination of phase angles from the graphical records difficulty was encountered because of high-frequency noise superposed on the main oscillatory trace. This noise, though not severe enough to interfere with the determination of the force and moment magnitudes, was a hindrance to the reading of zero crossings, the latter

[4] Following the sign convention defined in Fig. 7.

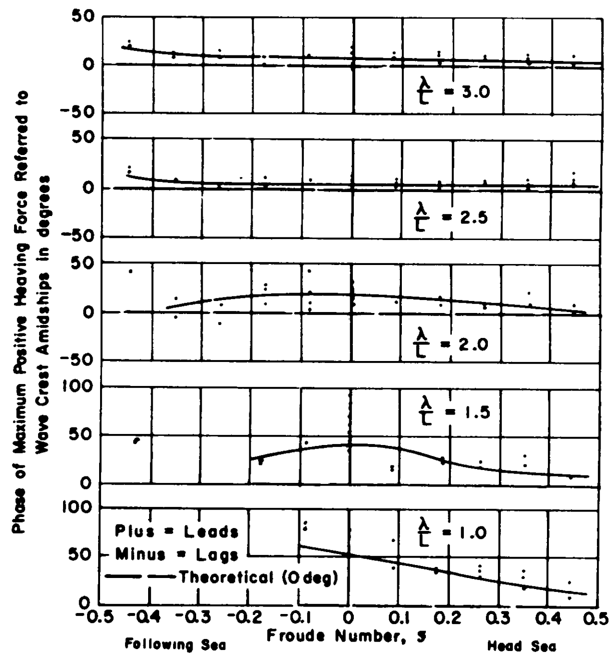


**Fig. 11 Phase of maximum positive heaving force as a function of Froude number**

being necessary in phase analysis. In addition, for the following-sea condition at high speed, since the frequency of encounter between model and wave is extremely low, the number of cycles obtained was quite limited. The first of these two facts is the primary cause of the scatter seen in the figures and the latter explains the relative paucity of data for following seas.

Since wave height was measured at the west bow and not amidships it was necessary to shift the wave trace on the Sanborn record in the increasing time direction by the angle

$$\varphi = \frac{360 \times d}{\lambda} \text{ in degrees}$$

where $d$ = distance between the bow and midships.

The Froude-Krylov theoretical value of 0 deg phase between maximum positive heaving force and wave crest amidships is shown in Fig. 11 for comparison with the experimental results. It can be seen that for the longer wavelengths (i.e., $\lambda/L = 2.0$, 2.5 and 3.0) the agreement is quite good and the actual lead angle is always less than approximately 20 deg. The phase is also nearly constant as model speed changes. For the shorter wavelengths, the differences are greater with the heaving force leading by as much as 60 deg at $\lambda/L = 1.0$ and $\mathfrak{F} = -0.1$, with indications of greater lead at higher speeds in following seas.

Fig. 12 is a plot of the phase between maximum pitching moment and wave crest amidships. It reveals that for the head-sea condition, operating in waves of $\lambda =$

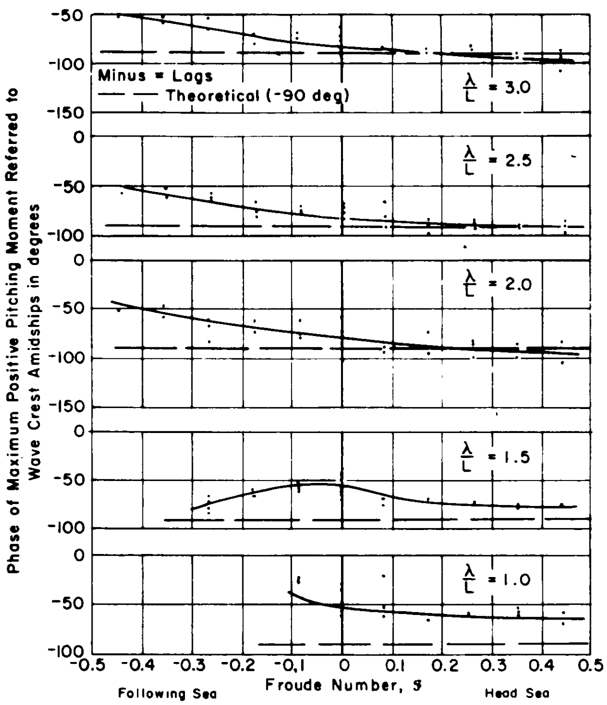


**Fig. 12 Phase of maximum positive pitching moment as a function of Froude number**

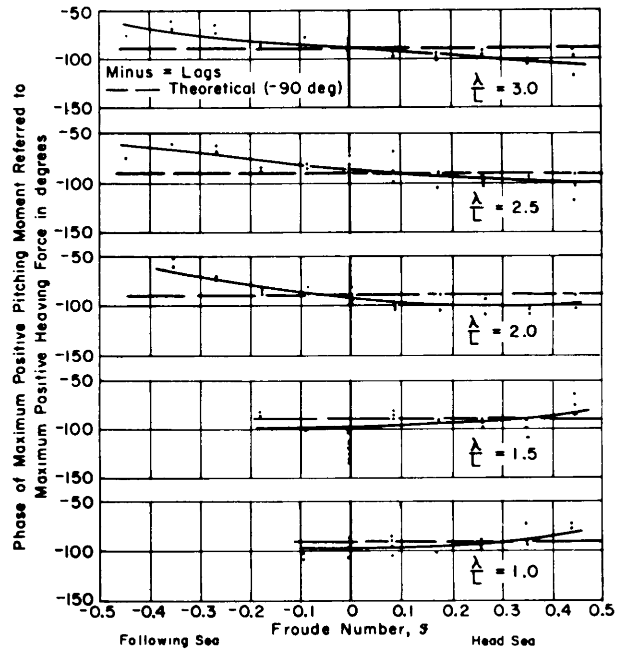


**Fig. 13 Phase between maximum positive pitching moment and maximum positive heaving force**

2.0$L$, 2.5$L$ and 3.0$L$, the theoretical value of $-90$ deg is a good estimate of the actual lag. Lags appreciably less than $-90$ deg are found for following seas at these wavelengths and throughout the represented speed range for the shorter wavelengths. In the short waves the theory is in error by as much as 50 deg.

A point-by-point addition of the values shown in Figs. 11 and 12 was performed and this yielded the phase between maximum pitching moment and maximum heaving force as shown in Fig. 13. The agreement between theory and experiment is good for all wavelengths and at all indicated speeds in both head and following seas, with the greatest difference being only 30 deg. For the longer waves there is an interesting tendency for the theory to overestimate the true lag in following seas and underestimate the true lag in head seas.

### Importance of Dynamic Pressure Gradient

Since inclusion of the dynamic pressure gradient (Smith effect) in the calculation of the force and moment acting on a model in waves increases the work involved manyfold, it is of interest to one performing such calculations to know if it significantly affects the results obtained. To answer this question, heaving-force and pitching-moment values were determined without the Smith effect being considered and are shown in Fig. 14 for comparison with results obtained with this effect included.

The maximum heaving force is determined with wave crest amidships and the maximum pitching moment with wave nodal point amidships. The dynamic pressure gradient in the wave decreases the pressure at any given depth under a crest and increases it under a trough (as compared to that which occurs hydrostatically). Therefore the theoretical heaving force and pitching moment are greater when the Smith effect is neglected than when it is taken into account.

It has been shown earlier that the Froude-Krylov theory, with distortion of the wave by the ship accounted for, better predicts the magnitude of the forces and moments acting on the model than does the uncorrected Froude-Krylov theory. As shown in Fig. 14 the magnitude of the Smith correction is of the same order as the ship-wave interference correction. Therefore, the Smith effect must be accepted as a necessary part of the force-moment theoretical calculation.

### Oscillatory Surging Force

Consideration of the forces causing the model to surge is of interest to complete the description of excitation causing motion in the longitudinal plane.

The oscillatory surging force data are presented in coefficient form in Fig. 15 where it can be seen that the force amplitude per unit wave amplitude is approximately constant as speed is varied in following seas. It increases without reaching any apparent peak as the gamut is run from high speed in following seas to high speed in head seas. The data points shown result from tests in waves of essentially three nominal heights and their reasonably

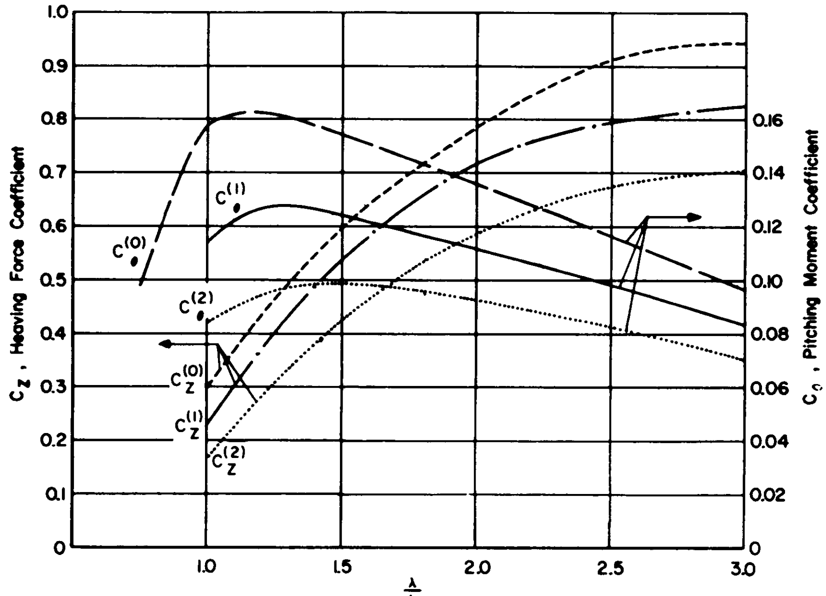


**Fig. 14 Comparison of the oretical lift and moment coefficients**

good superposition is evidence of the almost linear surge force-wave height relationship.

Theoretical forces were computed on the basis of the Froude-Krylov hypothesis as discussed earlier in the paper. The calculated value is in reasonable agreement with experiment at zero speed and in following seas. Since the true force increases appreciably with speed in the head-sea condition, the agreement is poor at the higher speeds. At present there is no coefficient available to correct surging force for the influence of the ship on the wave structure.

Surging force was defined herein as being positive when it acts to impede the forward motion of the model, and theoretically it is maximum when the wave nodal point is amidships. Thus, the theory states that maximum positive surging force leads wave crest amidships by 90 deg in head seas and lags by 90 deg in following seas. Fig. 16 reveals that, for the conditions represented, this assumption is a good one. Lack of data for the shorter wavelengths is due to the fact that of all recorded signals the surging force had the greatest amount of noise. This resulted in a prohibitive amount of scatter in the phase values for the short waves.

### Conclusions

The following statements summarize the principal findings of this study:

1 Application of the Froude-Krylov hypothesis with correction for ship-wave interference yields values of oscillatory heaving force which are in fairly good agreement with that actually existing. The predicted value for zero speed falls in, or near the grouping of experimental data points throughout the speed range in both head and following seas.

2 Values of oscillatory pitching moment calculated by means of the Froude-Krylov hypothesis with correction for ship-wave interference are in very good agreement with the actual zero-speed moment. The correlation with empirical results when the ship is underway in head or following seas is only reasonably good.

3 The change of heaving force and pitching moment with speed and heading is significant. The heaving force is a complicated function of model speed with several peaks occurring within the speed range, while pitching moment is greater in head seas than in following seas.

4 Heaving force and pitching moment are approximately proportional to wave height for the range of test wave heights.

5 The theoretical relationships between heaving force and wavelength and pitching moment and wavelength generally agree in form with the experimental curves. A peak in all moment curves is indicated in the range of $\lambda/L$ equal to 1.3 to 1.5.

6 The dynamic presure gradient in the wave (Smith effect) should be taken into account to obtain the best results possible.

7 The oscillatory surging force is approximately proportional to wave height for the range of wave heights used in these experiments. The force increases monotonically as the speed range is traversed from high speed in following seas to high speed in head seas.

8 Values of surging force computed by means of the Froude-Krylov hypothesis are in reasonable agreement with the true force at zero speed and in following seas.

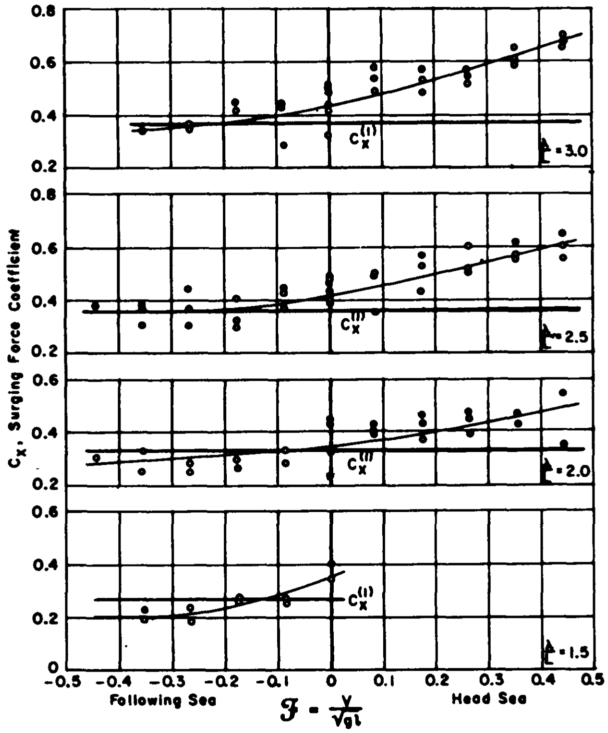


Fig. 15 Variation of surging-force coefficient with Froude number

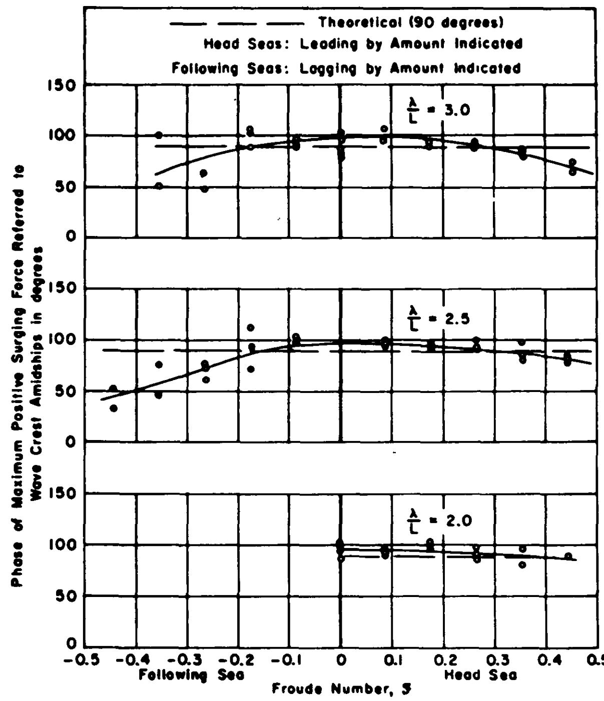


Fig. 16 Phase of maximum positive surging force as a function of Froude number

In head seas the agreement is poor, becoming progressively worse as speed increases.

9 The agreement between theoretical and actual phase data for heaving force, pitching moment, surging force, and wave elevation are, for the most part, good.

**Acknowledgment**

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