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CALCULATED HYDRODYNAMIC LOADS ON AN OSCILLATING HYDROFOIL

by

Thomas J. Langan and David W. Coder

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NOTATION

a	Nondimensional location of axis of rotation from midchord of foil
b	Semichord length of foil
$C(k)$	Theodorsen function
C_{LP}	Unsteady reduced lift coefficient due to sinusoidal pitching motion
C_{LT}	Unsteady reduced lift coefficient due to sinusoidal heaving motion
C_{MP}	Unsteady reduced moment coefficient due to sinusoidal pitching motion
C_{MT}	Unsteady reduced moment coefficient due to sinusoidal heaving motion
$\exp(t)$	Exponential function of t
F	Real part of the Theodorsen function
G	Imaginary part of the Theodorsen function
h	Heaving displacement
h_o	Amplitude of heaving oscillation
k	Reduced frequency $\frac{\omega b}{U}$
L	Lift force
L_{UP}	Time dependent part of the lift due to sinusoidal pitching motion
L_{UT}	Time dependent part of the lift due to sinusoidal heaving motion
L.E.	Leading edge of foil
M	Moment about the axis of rotation
M_{UP}	Time dependent part of the moment due to sinusoidal pitching motion
M_{UT}	Time dependent part of the moment due to sinusoidal heaving motion
T.E.	Trailing edge of foil
t	Time
U	Free stream velocity

α	Angle of attack measured from mean chord line
α_o	Amplitude of pitching oscillation
π	3.1416 in computations
ρ	Density of fluid
φ_{LP}	Phase angle of the unsteady lift for a sinusoidal pitching motion
φ_{LT}	Phase angle of the unsteady lift for a sinusoidal heaving motion
φ_{MP}	Phase angle of the unsteady moment for a sinusoidal pitching motion
φ_{MT}	Phase angle of the unsteady moment for a sinusoidal heaving motion
φ_{TP}	Phase angle by which the pitching oscillation leads the heaving oscillation
ω	Circular frequency of oscillation

ABSTRACT

Formulas for the amplitude and phase angle of the two-dimensional unsteady lift and moment on an oscillating foil are derived together with formulas for the reduced lift and moment coefficients. Numerical values of the reduced force coefficients and associated phase angles are presented in graphs and tables. A sample calculation is included to demonstrate the use of the formulas and graphs.

ADMINISTRATIVE INFORMATION

By Bureau of Ships letter S-F013 02 01 Serial 420-255 of 3 October 1960, the David Taylor Model Basin was assigned the responsibility of measuring the unsteady hydrodynamic forces on two-dimensional oscillating hydrofoils under the Hydrofoil Accelerated Research Program of the Bureau of Ships. The work covered by this report was supported by the David Taylor Model Basin Fundamental Hydrodynamic Research Program under Project SS 600-000, Task 1703.

INTRODUCTION

Under the Hydrofoil Accelerated Research Program of the Bureau of Ships, the unsteady hydrodynamic forces on two-dimensional oscillating hydrofoils was measured at the David Taylor Model Basin. A foil with a 24-in. chord was forced to oscillate sinusoidally in either heave or pitch while being towed in water at speeds up to 45 knots. Design criteria for the foil and auxiliary equipment were obtained from Theodorsen's expressions for the forces and moments on two-dimensional oscillating foils.[1] Formulas for the amplitude and phase angles of the unsteady lift and moment of a flat plate foil in heave and pitch were derived from Theodorsen's expressions and were programmed for the IBM 7090 Digital Computer.

This report presents the derivation of the formulas for obtaining the amplitude and phase angles of the unsteady oscillatory loads on a fully wetted, two-dimensional, thin foil moving in an incompressible, inviscid

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[1] References are listed on page 30.

fluid of infinite extent, with a mean angle of attack, small compared with the stall angle. The resulting data have been reduced to nondimensional form and are presented in tables and graphs in which reduced lift and reduced moment coefficients and phase angles are plotted against the reduced frequency. Although similar graphs were published by Halfman,[2] his data do not extend into the range of high values of the reduced frequency which are of particular value to hydrodynamicists.

THEORETICAL ANALYSIS

Formulas for the amplitudes and phase angles of the unsteady lift and moment on a sinusoidally oscillating foil are derived from Theodorsen's expressions.[1] These formulas are separated into two groups: the first group applies to the case of a foil oscillating in pure heave; the second to the case of a foil oscillating in pure pitch. The amplitudes of the forces are expressed in both dimensional and nondimensional form. The nondimensional amplitude of the lift or moment is referred to, respectively, as the reduced lift or reduced moment coefficient. Finally, the lift and moment on a foil, which is oscillating sinusoidally in a combination of pitch and heave, are presented as functions of reduced coefficients, phase angles, and amplitudes of the motion.

The coordinate system used in the analysis is shown in Figure 1. In this system, the direction of fluid velocity U at infinity is parallel to the mean position of the foil chord. The foil is free to translate in a direction perpendicular to the fluid velocity and to rotate around the spanwise axis located at distance ab from the midchord of the foil, where b is the semichord length and a is the nondimensional location of the axis of rotation measured positively down stream. The translational motion is referred to as heave, and its corresponding displacement h is measured positive downward. The angular displacement α is measured in a clockwise direction from the mean chord line. In accordance with the convention used by Theodorsen,[1] the positive direction of lift L is opposite to the positive direction of the heaving coordinate h; the moment M is positive in a clockwise direction.

Theodorsen's expressions for the unsteady lift and moment on a sinusoidally oscillating foil are:[1]

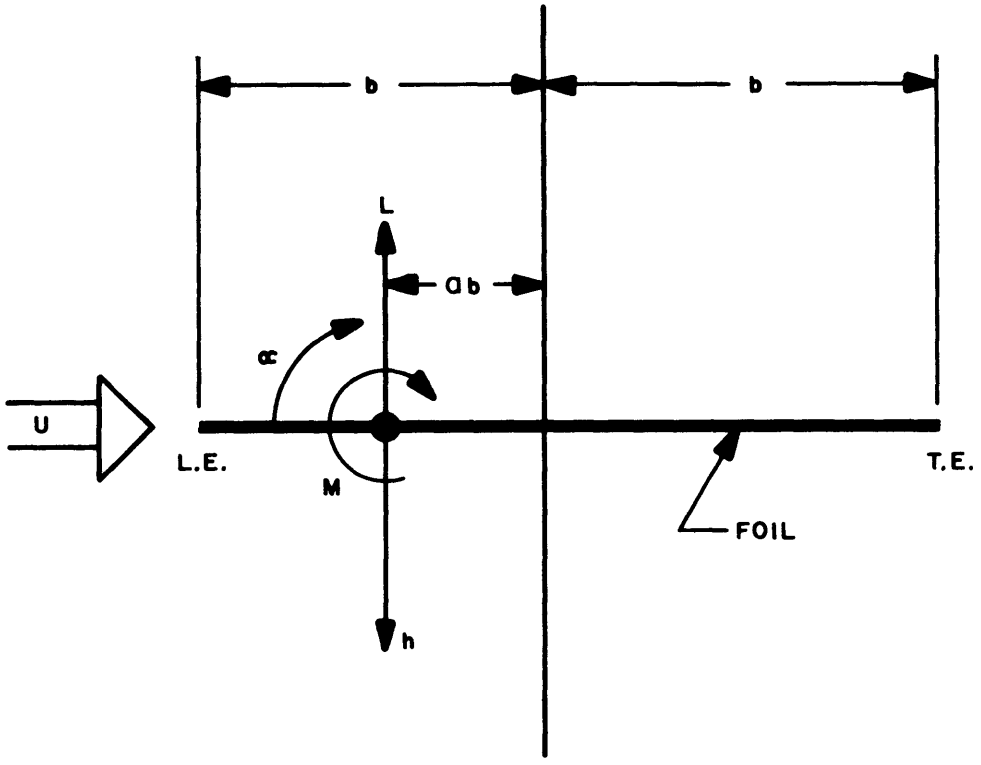

Figure 1 - Coordinate System

$$L(t) = \pi\rho b^2 [\ddot{h} + U\dot{\alpha} - ab\ddot{\alpha}] + 2\pi\rho bU\, C(k)\, [\dot{h} + U\alpha + (\tfrac{1}{2} - a)\, b\, \dot{\alpha}] \qquad [1]$$

$$M(t) = \pi\rho b^3 [a\,\ddot{h} - (\tfrac{1}{2} - a)\, U\dot{\alpha} - (\tfrac{1}{8} + a^2) b\ddot{\alpha}] + 2\pi\rho b^2 U\, C(k)\, (\tfrac{1}{2} + a)$$
$$[\dot{h} + U\,\alpha + (\tfrac{1}{2} - a)\, b\dot{\alpha}] \qquad [2]$$

where C(k) is the complex Theodorsen function. This is a function of the reduced frequency k. If its real part is denoted by F(k) and its imaginary part by G(k),

$$C(k) = F(k) + i\, G(k) \qquad [3]$$

where $k = \omega b/U$. Since Equations [1] and [2] are linear in heave and pitch, the two modes of operation may be analyzed separately and then combined into a general expression.

HEAVING MOTION

If the foil is oscillating sinusoidally in pure heave,

$$h = h_o \exp(i\omega t)$$
$$\alpha = 0$$

where h_o is the amplitude of the heaving motion and ω is the circular frequency. Then the unsteady lift and moment are*

$$L_{UT} = \pi\rho b^2\omega^2\, k^{-1}\, h_o\, [-k + 2i\, C(k)]\exp(i\omega t) \qquad [4]$$

$$M_{UT} = \pi\rho b^3\omega^2\, k^{-1}\, h_o\, [-ak + 2i\, C(k)\, (a + \tfrac{1}{2})]\exp(i\omega t) \qquad [5]$$

These complex expressions may be written in terms of their absolute values and phase angles:

$$L_{UT} = |L_{UT}|\exp[i(\omega t + \phi_{LT})] \qquad [6]$$

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*To obtain the total lift and moment on the foil with nonzero angle of attack, it is necessary to add the steady lift and moment to the unsteady forces.

$$M_{UT} = |M_{UT}| \exp [i (\omega t + \phi_{MT})] \quad [7]$$

Amplitudes of the unsteady lift and moment for oscillatory heaving motion are

$$|L_{UT}| = \pi \rho b^2 \omega^2 k^{-1} h_o [(k + 2G)^2 + 4 F^2]^{1/2} \quad [8]$$

$$|M_{UT}| = \pi \rho b^3 \omega^2 k^{-1} h_o \left\{ [ak + 2G (a + \tfrac{1}{2})]^2 + 4 F^2 (a + \tfrac{1}{2})^2 \right\}^{1/2} \quad [9]$$

The phase angles by which the lift and moment lead the displacement are

$$\phi_{LT} = \arctan [2F/-(k + 2G)] \quad [10]$$

$$\phi_{MT} = \arctan \{ 2F (a + 1/2)/-[ak + 2G (a + 1/2)]\} \quad [11]$$

The amplitudes of the unsteady loads may be expressed in terms of reduced lift and moment coefficients which are functions only of the reduced frequency k and the point of application of the load a:

$$C_{LT} = \frac{|L_{UT}|}{\pi \rho b^3 \omega^2} \left(\frac{b}{h_o} \right) = k^{-1} \left[(k + 2G)^2 + 4 F^2 \right]^{\frac{1}{2}} \quad [12]$$

$$C_{MT} = \frac{|M_{UT}|}{\pi \rho b^4 \omega^2} \left(\frac{b}{h_o} \right) = k^{-1} \left\{ [ak + 2G (\tfrac{1}{2} + a)]^2 + 4 F^2 (\tfrac{1}{2} + a)^2 \right\}^{\frac{1}{2}} \quad [13]$$

PITCHING MOTION

If there is no heaving motion and the foil oscillates sinusoidally about its axis of rotation,

$$h = \text{constant}$$
$$\alpha = \alpha_o \exp (i\omega t)$$

where α_o is the amplitude of the angular motion. For this motion the unsteady lift and moment are

$$L_{UP} = \frac{\pi \rho b^3 \omega^2 \alpha_o}{k^2} \left\{ ik + ak^2 + 2 C(k) [1 + i (\tfrac{1}{2} - a)k] \right\} \exp (i\omega t) \quad [14]$$

$$M_{UP} = \frac{\pi\rho b^4\omega^2\alpha_o}{k^2}\left\{-i\left(\frac{1}{2} - a\right) k + \left(\frac{1}{8} - a^2\right) k^2 + 2\,C(k)\left(\frac{1}{2} + a\right)\left[1 + i\left(\frac{1}{2} - a\right) k\right]\right\} \quad [15]$$

These complex expressions may also be written in terms of their absolute value and phase angles:

$$L_{UP} = |L_{UP}| \exp\left[i\left(\omega t + \phi_{LP}\right)\right] \quad [16]$$

$$M_{UP} = |M_{UP}| \exp\left[i\left(\omega t + \phi_{MP}\right)\right] \quad [17]$$

The amplitudes of the unsteady lift and moment in oscillatory pitching motion are

$$|L_{UP}| = \frac{\pi\rho b^3\omega^2\alpha_o}{k^2}\left\{\left[a\,k^2 + 2\,F + 2\,G\,k\left(a - \frac{1}{2}\right)\right]^2 + \left[2\,G + k - 2\,F\,k\left(a - \frac{1}{2}\right)\right]^2\right\}^{1/2} \quad [18]$$

$$|M_{UP}| = \frac{\pi\rho b^4\omega^2\alpha_o}{k^2}\left\{\left[k^2\left(\frac{1}{8} + a^2\right) - 2\left(\frac{1}{2} + a\right)(G\,k - F) + 2\left(\frac{1}{2} + a\right)^2 G\,k\right]^2 + \left[k\left(\frac{1}{2} - a\right) - 2\,F\left(\frac{1}{2} + a\right) k - 2\,G\left(\frac{1}{2} + a\right) + 2\left(\frac{1}{2} + a\right)^2 Fk\right]^2\right\}^{1/2} \quad [19]$$

and the phase angles by which the lift and moment lead the displacement are

$$\phi_{LP} = \arctan\left\{\left[2\,G + k - 2\,Fk\left(a - \frac{1}{2}\right)\right] / \left[a\,k^2 + 2\,F + 2\,G\,k\left(a - \frac{1}{2}\right)\right]\right\} \quad [20]$$

$$\phi_{MP} = \arctan\left\{\frac{-\left[k\left(\frac{1}{2} - a\right) - 2\,F\left(\frac{1}{2} + a\right) k - 2\,G\left(\frac{1}{2} + a\right) + 2\left(\frac{1}{2} + a\right)^2 Fk\right]}{k^2\left(\frac{1}{8} + a^2\right) - 2\left(\frac{1}{2} + a\right)(G\,k - F) + 2\left(\frac{1}{2} + a\right)^2 G\,k}\right\} \quad [21]$$

Once again, expressing the amplitudes of the unsteady loads in terms of reduced lift and moment coefficients, we have

$$C_{LP} = \frac{|L_{UP}|}{\pi\rho b^3\omega^2}\left(\frac{1}{\alpha_o}\right) = k^{-2}\left\{\left[ak^2 + 2\,F + 2\,G\,k\left(a - \frac{1}{2}\right)\right]^2 + \left[2\,G + k - 2\,F\,k\left(a - \frac{1}{2}\right)\right]^2\right\}^{1/2} \quad [22]$$

$$C_{MP} = \frac{|M_{UP}|}{\pi\rho b^4 \omega^2}\left(\frac{1}{\alpha_o}\right) = k^{-2}\left\{\left[k^2\left(\tfrac{1}{8} + a^2\right) - 2\left(\tfrac{1}{2} + a\right)(G\,k - F) + 2\left(\tfrac{1}{2} + a\right)^2 G\,k\right]^2\right.$$

$$\left. + \left[k\left(\tfrac{1}{2} - a\right) - 2F\left(\tfrac{1}{2} + a\right)k - 2G\left(\tfrac{1}{2} + a\right) + 2\left(\tfrac{1}{2} + a\right)^2 Fk\right]^2\right\}^{1/2} \qquad [23]$$

COMBINED HEAVING AND PITCHING MOTIONS

Theodorsen's formulas for the unsteady lift and moment on a foil oscillating sinusoidally both in heave and pitch may be written in terms of the reduced force coefficients and phase angles for the two separate modes of oscillation as follows:

$$L = \pi\rho\, b^3\omega^2\left\{\frac{h_o}{b} C_{LT} \exp\left[i\left(\omega t + \phi_{LT}\right)\right]\right.$$

$$\left. + \alpha_o\, C_{LP} \exp\left[i\left(\omega t + \phi_{LP} + \phi_{TP}\right)\right]\right\} \qquad [24]$$

and

$$M = \pi\rho b^4\, \omega^2\left\{\frac{h_o}{b} C_{MT} \exp\left[i\left(\omega t + \phi_{MT}\right)\right]\right.$$

$$\left. + \alpha_o\, C_{MP} \exp\left[i\left(\omega t + \phi_{MP} + \phi_{TP}\right)\right]\right\} \qquad [25]$$

where ϕ_{TP} is the phase angle by which the pitching motion leads the heaving motion, an angular displacement specified by the forcing mechanism.

NUMERICAL RESULTS

The formulas derived in the preceding section were programmed in FORTRAN for the IBM 7090 Digital Computer. The program includes the option of obtaining dimensional loads for a particular set of parameters or the reduced lift and moment coefficients in heave and pitch with their appropriate phase relations.

Nondimensional unsteady load data for an oscillating foil are presented in Tables 1 through 7 and Figures 2 through 9 as functions of the reduced frequency k and the location of the axis of rotation a. In the tables, a was varied between -1.0 and 1.0 in increments of 0.1 and k was varied between 0.004 and 100.0. Data for selected values of a are presented in the graphs for this same range of k.

The computed phase angles in Figures 3, 5, 7, and 9 agree with those obtained by Halfman[2] in a range of $k < 0.6$. Halfman's lift and moment coefficients were defined differently and are not directly comparable. The appendix presents a sample calculation to show how the data in the figures may be used to compute the unsteady loads on a hydrofoil.

TABLE 1

Reduced Lift Coefficient and Phase Angle of the Lift for a Foil Oscillating in Heave versus Reduced Frequency for $0.004 \leq k \leq 100$

k	C_{LT}	ϕ_{LT}	k	C_{LT}	ϕ_{LT}
0.004	496.8	1.550	0.620	1.941	1.866
0.006	330.0	1.543	0.740	1.655	1.988
0.008	246.7	1.536	0.940	1.393	2.1628
0.010	196.6	1.529	1.24	1.212	2.3618
0.012	163.3	1.524	1.50	1.138	2.4853
0.016	121.6	1.501	1.86	1.086	2.6067
0.020	96.59	1.515	2.24	1.057	2.6956
0.026	73.51	1.491	2.48	1.045	2.7382
0.038	49.21	1.470	2.98	1.031	2.8055
0.048	38.27	1.457	3.72	1.019	2.8724
0.062	28.89	1.446	4.96	1.011	2.9398
0.076	23.02	1.434	5.60	1.008	2.9629
0.094	18.06	1.426	7.50	1.005	3.0082
0.120	13.59	1.423	11.0	1.002	3.0507
0.150	10.41	1.428	15.0	1.001	3.0749
0.190	7.807	1.444	20.0	1.001	3.0916
0.250	5.562	1.484	40.0	1.000	3.1166
0.380	3.327	1.606	50.0	1.000	3.1216
0.460	2.661	1.692	100	1.000	3.1406

TABLE 2

Reduced Moment Coefficient for Heave versus Reduced Frequency (for $0.004 \leq k \leq 100$ and $-1 \leq a \leq 1$)

k	C_{MT} a = -1.0	C_{MT} a = -0.9	C_{MT} a = -0.8	C_{MT} a = -0.7	C_{MT} a = -0.6	C_{MT} a = -0.5	C_{MT} a = -0.4	C_{MT} a = -0.3	C_{MT} a = -0.2	C_{MT} a = -0.1	C_{MT} a = 0.0	C_{MT} a = 0.1	C_{MT} a = 0.2	C_{MT} a = 0.3	C_{MT} a = 0.4	C_{MT} a = 0.5	C_{MT} a = 0.6	C_{MT} a = 0.7	C_{MT} a = 0.8	C_{MT} a = 0.9	C_{MT} a = 1.0
0.004	248.4	198.7	149.0	99.34	49.67	0.5000	49.69	99.36	149.0	198.7	248.4	298.1	347.7	397.4	447.1	496.8	546.4	596.1	645.8	695.5	745.1
0.006	165.0	132.0	99.00	65.99	32.99	↓	33.02	66.02	99.02	132.0	165.0	198.0	231.0	264.0	297.0	330.0	363.0	396.1	429.1	462.1	495.1
0.008	123.3	98.65	73.99	49.32	24.66	↓	24.69	49.36	74.02	98.69	123.4	148.0	172.7	197.4	222.0	246.7	271.4	296.0	320.7	345.4	370.0
0.010	98.30	78.64	58.98	39.31	19.65	↓	19.69	39.35	59.02	78.68	98.35	118.0	137.7	157.3	177.0	196.7	216.3	236.0	255.7	275.3	295.0
0.012	81.63	65.30	48.97	32.64	16.31	↓	16.36	32.69	49.02	65.35	81.67	98.00	114.3	130.7	147.0	163.3	179.7	196.0	212.3	228.6	245.0
0.016	60.78	48.62	36.46	24.30	12.14	↓	12.20	24.36	36.51	48.67	60.83	72.99	85.15	97.31	109.5	121.6	133.8	146.0	158.1	170.3	182.4
0.020	48.26	38.61	28.95	19.29	9.638	↓	9.706	19.36	29.02	38.67	48.33	57.99	67.65	77.31	86.97	96.3	106.3	115.9	125.6	135.3	144.9
0.026	36.72	29.37	22.02	14.67	7.327	↓	7.407	14.75	22.10	29.45	36.80	44.15	51.50	58.85	66.20	73.55	80.90	88.25	95.60	102.9	110.3
0.038	24.56	19.64	14.72	9.805	4.896	↓	4.996	9.905	14.82	19.74	24.66	29.58	34.50	39.42	44.34	49.26	54.18	59.11	64.03	68.95	73.87
0.048	19.08	15.26	11.43	7.613	3.803	↓	3.915	7.726	11.55	15.37	19.20	23.02	26.85	30.67	34.50	38.33	42.15	45.98	49.81	53.63	57.46
0.062	14.39	11.51	8.620	5.738	2.870	↓	2.993	5.862	8.744	11.63	14.52	17.41	20.29	23.18	26.07	28.96	31.85	34.74	37.63	40.52	43.40
0.076	11.45	9.152	6.855	4.562	2.288	↓	2.421	4.698	6.991	9.289	11.59	13.89	16.19	18.49	20.79	23.09	25.39	27.69	30.00	32.30	34.60
0.094	8.974	7.171	5.370	3.575	1.804	↓	1.943	3.718	5.514	7.315	9.118	10.92	12.73	14.53	16.34	18.14	19.95	21.76	23.56	25.37	27.17
0.120	6.738	5.384	4.033	2.690	1.377	↓	1.515	2.834	4.179	5.530	6.885	8.240	9.597	10.95	12.31	13.67	15.03	16.39	17.74	19.10	20.46
0.150	5.158	4.123	3.092	2.071	1.089	↓	1.218	2.210	3.233	4.264	5.300	6.337	7.376	8.415	9.454	10.49	11.53	12.57	13.61	14.65	15.70
0.190	3.872	3.099	2.332	1.578	0.8722	↓	0.9789	1.699	2.456	3.224	3.998	4.773	5.550	6.328	7.107	7.886	8.665	9.445	10.22	11.00	11.78
0.250	2.782	2.238	1.700	1.179	0.7150	↓	0.7794	1.258	1.783	2.322	2.868	3.417	3.968	4.520	5.073	5.627	6.181	6.736	7.291	7.845	8.401
0.380	1.754	1.438	1.132	0.8461	0.6102	↓	0.5908	0.8182	1.101	1.405	1.720	2.041	2.365	2.691	3.018	3.347	3.676	4.006	4.336	4.667	4.998
0.460	1.477	1.229	0.9917	0.7729	0.5940	↓	0.5374	0.6850	0.8895	1.120	1.364	1.615	1.870	2.127	2.387	2.648	2.910	3.172	3.435	3.699	3.963
0.620	1.214	1.038	0.8706	0.7164	0.5865	↓	0.4810	0.5367	0.6481	0.7920	0.9539	1.126	1.304	1.486	1.671	1.858	2.046	2.235	2.425	2.616	2.807
0.740	1.127	0.9779	0.8352	0.7025	0.5868	↓	0.4588	0.4751	0.5439	0.6486	0.7749	0.9137	1.060	1.212	1.366	1.524	1.682	1.842	2.003	2.164	2.326
0.940	1.060	0.9334	0.8110	0.6950	0.5892	↓	0.4378	0.4149	0.4375	0.4995	0.5885	0.6943	0.8102	0.9326	1.059	1.189	1.320	1.453	1.587	1.722	1.857
1.24	1.024	0.9110	0.8006	0.6937	0.5924	↓	0.4225	0.3693	0.3516	0.3746	0.4318	0.5117	0.6056	0.7078	0.8153	0.9261	1.039	1.154	1.270	1.387	1.505
1.50	1.012	0.9045	0.7982	0.6944	0.5943	↓	0.4157	0.3485	0.3100	0.3108	0.3508	0.4188	0.5037	0.5982	0.6986	0.8025	0.9089	1.017	1.126	1.236	1.346
1.86	1.006	0.9012	0.7977	0.6957	0.5960	↓	0.4104	0.3322	0.2755	0.2549	0.2787	0.3375	0.4168	0.5071	0.6034	0.7033	0.8054	0.9091	1.014	1.119	1.225
2.24	1.003	0.9000	0.7978	0.6967	0.5971	↓	0.4072	0.3225	0.2540	0.2176	0.2291	0.2829	0.3604	0.4495	0.5446	0.6430	0.7434	0.8451	0.9476	1.051	1.154
2.48	1.002	0.8997	0.7980	0.6972	0.5976	↓	0.4059	0.3185	0.2448	0.2007	0.2061	0.2580	0.3354	0.4246	0.5195	0.6176	0.7175	0.8186	0.9204	1.023	1.126
2.98	1.001	0.8995	0.7984	0.6979	0.5983	↓	0.4041	0.3129	0.2317	0.1754	0.1705	0.2205	0.2990	0.3891	0.4844	0.5824	0.6819	0.7823	0.8833	0.9847	1.086
3.72	1.000	0.8995	0.7989	0.6986	0.5989	↓	0.4027	0.3083	0.2208	0.1524	0.1358	0.1856	0.2669	0.3589	0.4551	0.5534	0.6527	0.7528	0.8533	0.9541	1.055
4.96	1.000	0.8997	0.7993	0.6992	0.5993	↓	0.4015	0.3047	0.2119	0.1318	0.1014	0.1537	0.2395	0.3340	0.4314	0.5302	0.6297	0.7297	0.8299	0.9303	1.031
5.60	1.000	0.8997	0.7995	0.6993	0.5995	↓	0.4012	0.3037	0.2094	0.1256	0.0897	0.1437	0.2315	0.3269	0.4248	0.5238	0.6234	0.7233	0.8234	0.9237	1.024
7.50	1.000	0.8998	0.7997	0.6996	0.5997	↓	0.4007	0.3021	0.2053	0.1149	0.0669	0.1261	0.2180	0.3152	0.4139	0.5133	0.6130	0.7130	0.8131	0.9132	1.013
11.0	1.000	0.8999	0.7998	0.6998	0.5999	↓	0.4003	0.3010	0.2025	0.1072	0.0455	0.1129	0.2085	0.3071	0.4065	0.5062	0.6061	0.7060	0.8061	0.9061	1.006
15.0	1.000	0.9000	0.7999	0.6999	0.5999	↓	0.4002	0.3005	0.2013	0.1039	0.0334	0.1071	0.2046	0.3038	0.4035	0.5033	0.6033	0.7032	0.8033	0.9033	1.003
20.0	1.000	0.9000	0.8000	0.6999	0.6000	↓	0.4001	0.3003	0.2007	0.1022	0.0250	0.1041	0.2026	0.3022	0.4020	0.5019	0.6018	0.7018	0.8018	0.9019	1.002
40.0	1.000	0.9000	0.8000	0.7000	0.6000	↓	0.4000	0.3001	0.2002	0.1006	0.0125	0.1010	0.2007	0.3005	0.4005	0.5005	0.6005	0.7005	0.8005	0.9005	1.000
50.0	1.000	0.9000	0.8000	0.7000	0.6000	↓	0.4000	0.3000	0.2001	0.1004	0.0100	0.1007	0.2004	0.3003	0.4003	0.5003	0.6003	0.7003	0.8003	0.9003	1.000
100	1.000	0.9000	0.8000	0.7000	0.6000	0.5000	0.4000	0.3000	0.2000	0.1001	0.0050	0.1002	0.2001	0.3001	0.4001	0.5001	0.6001	0.7001	0.8001	0.9001	1.000

TABLE 3

Phase Angle of the Moment for a Foil Oscillating in Heave versus Reduced Frequency for $0.004 \leq k \leq 100$ and $-1 \leq a \leq 1$

k	ϕ_{MT} a = -1.0	ϕ_{MT} a = -0.9	ϕ_{MT} a = -0.8	ϕ_{MT} a = -0.7	ϕ_{MT} a = -0.6	ϕ_{MT} a = -0.5	ϕ_{MT} a = -0.4	ϕ_{MT} a = -0.3	ϕ_{MT} a = -0.2	ϕ_{MT} a = -0.1	ϕ_{MT} a = 0.0	ϕ_{MT} a = 0.1	ϕ_{MT} a = 0.2	ϕ_{MT} a = 0.3	ϕ_{MT} a = 0.4	ϕ_{MT} a = 0.5	ϕ_{MT} a = 0.6	ϕ_{MT} a = 0.7	ϕ_{MT} a = 0.8	ϕ_{MT} a = 0.9	ϕ_{MT} a = 1.0
0.004	-1.590	-1.589	-1.588	-1.587	-1.582	0.0	1.540	1.545	1.547	1.548	1.548	1.549	1.549	1.549	1.549	1.549	1.549	1.550	1.550	1.550	1.550
0.006	-1.596	-1.595	-1.594	-1.592	-1.584		1.528	1.535	1.538	1.539	1.540	1.540	1.541	1.541	1.541	1.541	1.541	1.541	1.542	1.542	1.542
0.008	-1.602	-1.601	-1.599	-1.596	-1.586		1.516	1.526	1.529	1.531	1.532	1.532	1.533	1.533	1.534	1.534	1.534	1.534	1.534	1.534	1.534
0.010	-1.608	-1.606	-1.604	-1.600	-1.587		1.504	1.517	1.521	1.523	1.524	1.525	1.526	1.526	1.527	1.527	1.527	1.527	1.527	1.528	1.528
0.012	-1.612	-1.611	-1.608	-1.603	-1.588		1.493	1.508	1.513	1.516	1.517	1.518	1.519	1.520	1.520	1.520	1.521	1.521	1.521	1.521	1.521
0.016	-1.621	-1.619	-1.616	-1.609	-1.588		1.472	1.492	1.499	1.503	1.505	1.506	1.507	1.508	1.508	1.509	1.509	1.509	1.510	1.510	1.510
0.020	-1.628	-1.626	-1.622	-1.613	-1.587		1.452	1.477	1.486	1.490	1.493	1.495	1.496	1.497	1.498	1.498	1.499	1.499	1.499	1.500	1.500
0.026	-1.638	-1.634	-1.629	-1.617	-1.583		1.423	1.457	1.468	1.474	1.477	1.479	1.481	1.482	1.483	1.484	1.484	1.485	1.485	1.486	1.486
0.038	-1.651	-1.646	-1.638	-1.621	-1.569		1.371	1.420	1.437	1.445	1.450	1.454	1.456	1.458	1.459	1.460	1.461	1.462	1.463	1.463	1.464
0.048	-1.659	-1.652	-1.641	-1.619	-1.553		1.330	1.393	1.414	1.425	1.431	1.436	1.439	1.441	1.443	1.444	1.446	1.447	1.447	1.448	1.449
0.062	-1.662	-1.653	-1.639	-1.610	-1.522		1.279	1.361	1.389	1.403	1.412	1.417	1.421	1.424	1.427	1.429	1.430	1.432	1.433	1.434	1.434
0.076	-1.665	-1.654	-1.636	-1.599	-1.489		1.228	1.328	1.363	1.381	1.391	1.398	1.403	1.407	1.410	1.412	1.414	1.416	1.417	1.419	1.420
0.094	-1.660	-1.647	-1.623	-1.577	-1.437		1.169	1.293	1.337	1.359	1.372	1.381	1.387	1.392	1.396	1.399	1.402	1.404	1.405	1.407	1.408
0.120	-1.646	-1.627	-1.596	-1.534	-1.351		1.090	1.248	1.304	1.333	1.351	1.363	1.371	1.378	1.383	1.387	1.390	1.393	1.395	1.397	1.399
0.150	-1.618	-1.594	-1.553	-1.473	-1.242		1.009	1.202	1.274	1.311	1.334	1.349	1.360	1.369	1.375	1.380	1.385	1.388	1.391	1.394	1.396
0.190	-1.569	-1.537	-1.483	-1.378	-1.093		0.913	1.148	1.241	1.290	1.320	1.340	1.355	1.366	1.374	1.381	1.387	1.391	1.395	1.399	1.402
0.250	-1.478	-1.433	-1.360	-1.221	-0.887		0.791	1.077	1.201	1.268	1.310	1.338	1.358	1.374	1.386	1.395	1.403	1.410	1.416	1.421	1.425
0.380	-1.247	-1.181	-1.079	-0.904	-0.576		0.598	0.949	1.134	1.242	1.311	1.358	1.393	1.419	1.439	1.456	1.469	1.481	1.490	1.498	1.506
0.460	-1.107	-1.034	-0.926	-0.753	-0.461		0.514	0.881	1.099	1.232	1.319	1.379	1.423	1.456	1.482	1.503	1.520	1.534	1.547	1.557	1.566
0.620	-0.871	-0.797	-0.694	-0.545	-0.322	0.0	0.396	0.764	1.035	1.217	1.340	1.426	1.490	1.538	1.575	1.605	1.630	1.650	1.757	1.682	1.694
0.740	-0.737	-0.668	-0.575	-0.446	-0.261		0.336	0.691	0.988	1.205	1.356	1.463	1.541	1.601	1.646	1.683	1.712	1.737	1.815	1.775	1.790
0.940	-0.577	-0.518	-0.442	-0.339	-0.197		0.267	0.591	0.915	1.183	1.380	1.522	1.626	1.702	1.761	1.807	1.843	1.874	1.899	1.920	1.938
1.24	-0.429	-0.384	-0.325	-0.248	-0.144		0.203	0.480	0.814	1.144	1.410	1.605	1.743	1.842	1.916	1.973	2.017	2.053	2.082	2.106	2.126
1.50	-0.350	-0.312	-0.264	-0.201	-0.117		0.168	0.410	0.737	1.106	1.430	1.670	1.835	1.950	2.034	2.096	2.143	2.181	2.211	2.236	2.257
1.86	-0.279	-0.248	-0.210	-0.160	-0.093		0.135	0.340	0.647	1.052	1.452	1.751	1.949	2.080	2.171	2.236	2.285	2.323	2.353	2.377	2.397
2.24	-0.229	-0.204	-0.172	-0.131	-0.076		0.112	0.287	0.569	0.994	1.469	1.829	2.054	2.195	2.288	2.354	2.401	2.438	2.466	2.489	2.508
2.48	-0.206	-0.184	-0.155	-0.118	-0.069		0.101	0.261	0.527	0.958	1.477	1.874	2.113	2.258	2.351	2.415	2.461	2.496	2.523	2.545	2.563
2.98	-0.171	-0.152	-0.128	-0.098	-0.057		0.084	0.219	0.456	0.887	1.491	1.962	2.221	2.368	2.458	2.519	2.561	2.593	2.618	2.637	2.653
3.72	-0.136	-0.121	-0.102	-0.078	-0.045		0.067	0.177	0.377	0.792	1.506	2.074	2.351	2.493	2.576	2.630	2.667	2.695	2.716	2.733	2.746
4.96	-0.101	-0.090	-0.076	-0.058	-0.034		0.050	0.133	0.291	0.662	1.521	2.229	2.508	2.635	2.705	2.750	2.780	2.802	2.819	2.832	2.842
5.60	-0.090	-0.080	-0.067	-0.051	-0.030		0.045	0.118	0.260	0.608	1.527	2.296	2.569	2.687	2.752	2.792	2.820	2.840	2.855	2.867	2.876
7.50	-0.067	-0.060	-0.050	-0.038	-0.022		0.033	0.089	0.197	0.484	1.538	2.453	2.698	2.796	2.847	2.878	2.900	2.915	2.926	2.935	2.943
11.0	-0.046	-0.040	-0.034	-0.026	-0.015		0.023	0.061	0.135	0.347	1.548	2.637	2.831	2.902	2.939	2.961	2.976	1.589	2.994	3.001	3.006
15.0	-0.033	-0.030	-0.025	-0.019	-0.011		0.017	0.044	0.100	0.260	1.554	2.759	2.911	2.965	2.992	3.009	3.020	3.028	3.033	3.038	3.042
20.0	-0.025	-0.022	-0.019	-0.014	-0.008		0.013	0.033	0.075	0.197	1.558	2.850	2.968	3.009	3.029	3.042	3.050	2.285	3.060	3.064	3.067
40.0	-0.013	-0.011	-0.009	-0.007	-0.004		0.006	0.017	0.037	0.100	1.565	2.993	3.054	3.075	3.085	3.092	3.096	2.713	3.101	3.103	3.104
50.0	-0.010	-0.009	-0.008	-0.006	-0.003		0.005	0.013	0.030	0.080	1.566	3.022	3.072	3.088	3.097	3.102	3.105	2.799	3.109	3.111	3.112
100	-0.005	-0.004	-0.004	-0.003	-0.002	0.0	0.003	0.007	0.015	0.040	1.568	3.082	3.107	3.115	3.119	3.122	3.123	2.970	3.125	3.126	3.127

TABLE 4

Reduced Lift Coefficient for Pitch versus Reduced Frequency for $0.004 \leq k \leq 100$ and $-1 \leq a \leq 1$

k	C_{LP} a = -1.0	C_{LP} a = -0.9	C_{LP} a = -0.8	C_{LP} a = -0.7	C_{LP} a = -0.6	C_{LP} a = -0.5	C_{LP} a = -0.4	C_{LP} a = -0.3	C_{LP} a = -0.2	C_{LP} a = -0.1	C_{LP} a = 0.0	C_{LP} a = 0.1	C_{LP} a = 0.2	C_{LP} a = 0.3	C_{LP} a = 0.4	C_{LP} a = 0.5	C_{LP} a = 0.6	C_{LP} a = 0.7	C_{LP} a = 0.8	C_{LP} a = 0.9	C_{LP} a = 1.0
0.004	124,200	124,200	124,200	124,200	124,200	124,200	124,200	124,200	124,200	124,200	124,200	124,200	124,200	124,200	124,200	124,200	124,200	124,200	124,200	124,200	124,200
0.006	55,010	55,010	55,010	55,010	55,010	55,010	55,010	55,010	55,010	55,010	55,010	55,010	55,010	55,010	55,010	55,010	55,010	55,010	55,010	55,010	55,010
0.008	30,840	30,840	30,840	30,840	30,840	30,840	30,840	30,840	30,840	30,840	30,840	30,840	30,840	30,840	30,840	30,840	30,840	30,840	30,840	30,840	30,840
0.010	19,670	19,670	19,670	19,670	19,670	19,670	19,670	19,670	19,670	19,670	19,670	19,670	19,670	19,670	19,670	19,670	19,670	19,670	19,670	19,670	19,670
0.012	13,610	13,610	13,610	13,610	13,610	13,610	13,610	13,610	13,610	13,610	13,610	13,610	13,610	13,610	13,610	13,610	13,610	13,610	13,610	13,610	13,610
0.016	7,603	7,602	7,602	7,602	7,602	7,601	7,601	7,601	7,601	7,601	7,601	7,601	7,601	7,601	7,601	7,601	7,601	7,601	7,601	7,601	7,601
0.020	4,832	4,831	4,831	4,831	4,831	4,830	4,830	4,830	4,830	4,830	4,830	4,830	4,830	4,830	4,830	4,830	4,830	4,830	4,830	4,830	4,830
0.026	2,830	2,829	2,829	2,829	2,829	2,828	2,828	2,828	2,828	2,828	2,828	2,828	2,828	2,828	2,828	2,828	2,828	2,828	2,828	2,828	2,828
0.038	1,298	1,297	1,297	1,297	1,297	1,296	1,296	1,296	1,296	1,296	1,296	1,296	1,296	1,296	1,296	1,296	1,296	1,296	1,296	1,296	1,296
0.048	799.8	799.5	799.3	799.0	798.8	798.6	798.5	798.3	798.2	798.1	797.9	797.9	797.8	797.7	797.7	797.7	797.7	797.8	797.8	798.0	798.0
0.062	468.5	468.3	468.0	467.8	467.6	467.4	467.2	467.1	467.0	466.8	466.7	466.7	466.6	466.5	466.5	466.5	466.5	466.5	466.6	466.7	466.7
0.076	305.3	305.1	304.8	304.6	304.4	304.2	304.1	303.9	303.8	303.7	303.6	303.5	303.4	303.4	303.4	303.4	303.4	303.4	303.4	303.5	303.6
0.094	194.6	194.3	194.1	193.9	193.7	193.5	193.4	193.2	193.1	193.0	192.9	192.8	192.8	192.7	192.7	192.7	192.7	192.7	192.7	192.8	192.9
0.120	115.5	115.3	115.1	114.9	114.7	114.5	114.4	114.2	114.1	114.0	113.9	113.8	113.8	113.8	113.7	113.7	113.7	113.7	113.8	113.8	113.9
0.150	71.64	71.42	71.21	71.02	70.84	70.68	70.53	70.40	70.29	70.18	70.10	70.03	69.97	69.93	69.91	69.90	69.91	69.93	69.97	70.02	70.09
0.190	43.22	43.01	42.82	42.64	42.48	42.32	42.18	42.06	41.95	41.86	41.77	41.71	41.65	41.62	41.59	41.58	41.59	41.61	41.65	41.70	41.76
0.250	24.24	24.05	23.88	23.72	23.56	23.43	23.30	23.18	23.08	22.99	22.92	22.86	22.81	22.77	22.75	22.74	22.75	22.77	22.80	22.85	22.91
0.380	10.51	10.35	10.21	10.07	9.945	9.828	9.721	9.625	9.539	9.464	9.400	9.347	9.306	9.277	9.260	9.254	9.261	9.280	9.310	9.352	9.406
0.460	7.410	7.273	7.142	7.019	6.905	6.799	6.701	6.613	6.535	6.467	6.409	6.361	6.325	6.299	6.285	6.282	6.290	6.309	6.340	6.381	6.433
0.620	4.547	4.431	4.321	4.218	4.121	4.031	3.948	3.873	3.807	3.750	3.702	3.663	3.635	3.617	3.609	3.611	3.624	3.648	3.681	3.724	3.777
0.740	3.530	3.426	3.326	3.232	3.143	3.061	2.986	2.919	2.859	2.808	2.766	2.733	2.710	2.697	2.695	2.702	2.719	2.747	2.784	2.830	2.885
0.940	2.622	2.528	2.438	2.353	2.273	2.199	2.132	2.072	2.020	1.977	1.942	1.917	1.902	1.897	1.902	1.917	1.942	1.977	2.021	2.073	2.133
1.24	1.977	1.889	1.805	1.725	1.650	1.581	1.519	1.464	1.418	1.380	1.353	1.336	1.329	1.334	1.350	1.376	1.412	1.458	1.511	1.573	1.641
1.50	1.689	1.602	1.519	1.440	1.366	1.297	1.236	1.182	1.137	1.102	1.078	1.066	1.065	1.077	1.101	1.135	1.180	1.233	1.295	1.363	1.437
1.86	1.463	1.375	1.291	1.210	1.134	1.064	1.001	0.9462	0.9011	0.8673	0.8461	0.8386	0.8450	0.8652	0.8981	0.9425	0.9967	1.059	1.129	1.204	1.284
2.24	1.326	1.237	1.151	1.068	0.9893	0.9162	0.8501	0.7925	0.7456	0.7114	0.6918	0.6881	0.7004	0.7281	0.7694	0.8223	0.8847	0.9547	1.031	1.112	1.197
2.48	1.269	1.179	1.091	1.007	0.9263	0.8513	0.7830	0.7235	0.6750	0.6401	0.6210	0.6192	0.6349	0.6668	0.7127	0.7702	0.8368	0.9106	0.9900	1.074	1.161
2.98	1.189	1.097	1.007	0.9196	0.8358	0.7569	0.6844	0.6205	0.5684	0.5313	0.5126	0.5142	0.5361	0.5758	0.6301	0.6955	0.7691	0.8490	0.9333	1.021	1.112
3.72	1.123	1.029	0.9360	0.8454	0.7576	0.6739	0.5958	0.5260	0.4681	0.4271	0.4079	0.4137	0.4435	0.4929	0.5568	0.6308	0.7117	0.7975	0.8866	0.9783	1.072
4.96	1.070	0.9735	0.8781	0.7841	0.6920	0.6029	0.5181	0.4403	0.3738	0.3255	0.3044	0.3158	0.3567	0.4186	0.4935	0.5765	0.6645	0.7558	0.8493	0.9443	1.040
5.60	1.055	0.9580	0.8617	0.7665	0.6731	0.5820	0.4948	0.4138	0.3435	0.2917	0.2692	0.2832	0.3290	0.3958	0.4747	0.5607	0.6510	0.7440	0.8388	0.9348	1.032
7.50	1.031	0.9326	0.8348	0.7376	0.6415	0.5470	0.4550	0.3675	0.2886	0.2273	0.2006	0.2213	0.2791	0.3564	0.4430	0.5346	0.6288	0.7247	0.8217	0.9195	1.018
11.0	1.014	0.9152	0.8163	0.7177	0.6196	0.5223	0.4264	0.3330	0.2451	0.1712	0.1365	0.1675	0.2399	0.3273	0.4205	0.5163	0.6135	0.7116	0.8101	0.9091	1.008
15.0	1.008	0.9082	0.8088	0.7095	0.6106	0.5121	0.4144	0.3181	0.2253	0.1427	0.1001	0.1404	0.2223	0.3150	0.4111	0.5088	0.6073	0.7062	0.8055	0.9049	1.004
20.0	1.004	0.9046	0.8049	0.7054	0.6060	0.5068	0.4081	0.3103	0.2146	0.1258	0.0750	0.1243	0.2128	0.3085	0.4063	0.5050	0.6041	0.7035	0.8031	0.9028	1.003
40.0	1.001	0.9012	0.8012	0.7013	0.6015	0.5017	0.4021	0.3026	0.2037	0.1070	0.0375	0.1066	0.2033	0.3021	0.4016	0.5012	0.6010	0.7009	0.8008	0.9007	1.001
50.0	1.001	0.9007	0.8008	0.7009	0.6010	0.5011	0.4013	0.3017	0.2024	0.1046	0.0300	0.1043	0.2021	0.3014	0.4010	0.5008	0.6007	0.7006	0.8005	0.9004	1.000
100	1.000	0.9002	0.8002	0.7002	0.6002	0.5003	0.4003	0.3004	0.2006	0.1012	0.0150	0.1011	0.2005	0.3003	0.4003	0.5002	0.6002	0.7001	0.8001	0.9001	1.000

TABLE 5

Phase Angle of the Lift for a Foil Oscillating in Pitch versus Reduced Frequency for $0.004 \leq k \leq 100$ and $-1 \leq a \leq 1$

k	ϕ_{LP} a = -1.0	ϕ_{LP} a = -0.9	ϕ_{LP} a = -0.8	ϕ_{LP} a = -0.7	ϕ_{LP} a = -0.6	ϕ_{LP} a = -0.5	ϕ_{LP} a = -0.4	ϕ_{LP} a = -0.3	ϕ_{LP} a = -0.2	ϕ_{LP} a = -0.1	ϕ_{LP} a = 0.0	ϕ_{LP} a = 0.1	ϕ_{LP} a = 0.2	ϕ_{LP} a = 0.3	ϕ_{LP} a = 0.4	ϕ_{LP} a = 0.5	ϕ_{LP} a = 0.6	ϕ_{LP} a = 0.7	ϕ_{LP} a = 0.8	ϕ_{LP} a = 0.9	ϕ_{LP} a = 1.0
0.004	-0.014	-0.015	-0.015	-0.016	-0.016	-0.016	-0.017	-0.017	-0.018	-0.018	-0.018	-0.019	-0.019	-0.020	-0.020	-0.020	-0.021	-0.021	-0.022	-0.022	-0.022
0.006	-0.019	-0.020	-0.020	-0.021	-0.021	-0.022	-0.023	-0.023	-0.024	-0.024	-0.025	-0.026	-0.026	-0.027	-0.027	-0.028	-0.029	-0.029	-0.030	-0.030	-0.031
0.008	-0.023	-0.024	-0.025	-0.025	-0.026	-0.027	-0.028	-0.029	-0.029	-0.030	-0.031	-0.032	-0.033	-0.033	-0.034	-0.035	-0.036	-0.037	-0.037	-0.038	-0.039
0.010	-0.026	-0.027	-0.028	-0.029	-0.030	-0.031	-0.032	-0.033	-0.034	-0.035	-0.036	-0.037	-0.038	-0.039	-0.040	-0.041	-0.042	-0.043	-0.044	-0.045	-0.046
0.012	-0.029	-0.030	-0.032	-0.033	-0.034	-0.035	-0.036	-0.038	-0.039	-0.041	-0.041	-0.042	-0.044	-0.045	-0.046	-0.047	-0.048	-0.050	-0.051	-0.052	-0.053
0.016	-0.034	-0.036	-0.037	-0.039	-0.040	-0.042	-0.044	-0.045	-0.047	-0.048	-0.050	-0.052	-0.053	-0.055	-0.056	-0.058	-0.060	-0.061	-0.063	-0.064	-0.066
0.020	-0.038	-0.040	-0.042	-0.044	-0.046	-0.047	-0.050	-0.052	-0.054	-0.056	-0.058	-0.060	-0.062	-0.064	-0.066	-0.068	-0.070	-0.072	-0.074	-0.076	-0.078
0.026	-0.041	-0.044	-0.046	-0.049	-0.052	-0.054	-0.057	-0.059	-0.062	-0.065	-0.067	-0.070	-0.072	-0.075	-0.078	-0.080	-0.083	-0.085	-0.088	-0.091	-0.093
0.038	-0.043	-0.047	-0.051	-0.055	-0.059	-0.062	-0.066	-0.070	-0.074	-0.078	-0.081	-0.085	-0.089	-0.093	-0.097	-0.100	-0.104	-0.108	-0.112	-0.116	-0.119
0.048	-0.042	-0.046	-0.051	-0.056	-0.061	-0.065	-0.070	-0.075	-0.080	-0.085	-0.089	-0.094	-0.099	-0.104	-0.109	-0.113	-0.118	-0.123	-0.128	-0.133	-0.137
0.062	-0.032	-0.038	-0.044	-0.051	-0.057	-0.063	-0.069	-0.075	-0.081	-0.088	-0.094	-0.100	-0.106	-0.112	-0.119	-0.125	-0.131	-0.137	-0.143	-0.150	-0.156
0.076	-0.023	-0.031	-0.038	-0.046	-0.053	-0.061	-0.069	-0.076	-0.084	-0.091	-0.099	-0.106	-0.114	-0.122	-0.129	-0.137	-0.144	-0.152	-0.160	-0.167	-0.175
0.094	-0.004	-0.014	-0.023	-0.032	-0.041	-0.051	-0.060	-0.069	-0.079	-0.088	-0.097	-0.107	-0.116	-0.125	-0.135	-0.144	-0.153	-0.163	-0.172	-0.182	-0.191
0.120	0.030	0.019	0.007	-0.005	-0.017	-0.028	-0.040	-0.052	-0.064	-0.076	-0.088	-0.099	-0.114	-0.123	-0.135	-0.147	-0.159	-0.171	-0.183	-0.195	-0.207
0.150	0.077	0.063	0.049	0.035	0.020	0.006	-0.009	-0.024	-0.038	-0.053	-0.068	-0.083	-0.098	-0.113	-0.127	-0.142	-0.157	-0.172	-0.187	-0.202	-0.217
0.190	0.149	0.132	0.114	0.096	0.078	0.060	0.042	0.024	0.005	-0.013	-0.032	-0.050	-0.069	-0.088	-0.107	-0.125	-0.144	-0.163	-0.182	-0.200	-0.219
0.250	0.267	0.245	0.223	0.201	0.178	0.155	0.132	0.108	0.085	0.061	0.037	0.013	-0.012	-0.036	-0.060	-0.085	-0.109	-0.134	-0.158	-0.182	-0.207
0.380	0.528	0.500	0.471	0.441	0.410	0.378	0.346	0.313	0.280	0.246	0.211	0.176	0.141	0.105	0.069	0.033	-0.003	-0.039	-0.074	-0.110	-0.145
0.460	0.680	0.649	0.617	0.583	0.549	0.513	0.477	0.439	0.400	0.361	0.320	0.279	0.238	0.196	0.154	0.111	0.069	0.027	-0.015	-0.056	-0.097
0.620	0.948	0.914	0.877	0.839	0.798	0.756	0.712	0.666	0.619	0.570	0.519	0.468	0.415	0.362	0.308	0.254	0.201	0.148	0.096	0.044	-0.005
0.740	1.119	1.082	1.043	1.001	0.957	0.911	0.862	0.811	0.758	0.702	0.644	0.585	0.525	0.464	0.403	0.341	0.281	0.221	0.163	0.106	0.051
0.940	1.354	1.314	1.271	1.225	1.176	1.123	1.067	1.007	0.944	0.878	0.809	0.738	0.665	0.592	0.519	0.446	0.375	0.306	0.240	0.177	0.117
1.24	1.675	1.582	1.534	1.482	1.425	1.364	1.296	1.224	1.146	1.064	0.977	0.888	0.797	0.706	0.617	0.530	0.447	0.368	0.295	0.227	0.165
1.50	1.804	1.759	1.709	1.654	1.592	1.524	1.448	1.365	1.275	1.178	1.076	0.970	0.863	0.758	0.655	0.558	0.468	0.384	0.308	0.240	0.178
1.86	1.996	1.951	1.900	1.842	1.775	1.700	1.614	1.518	1.411	1.294	1.169	1.041	0.912	0.787	0.670	0.562	0.465	0.379	0.302	0.235	0.176
2.24	2.151	2.107	2.056	1.996	1.928	1.847	1.754	1.646	1.522	1.385	1.237	1.084	0.932	0.789	0.659	0.544	0.444	0.358	0.284	0.221	0.166
2.48	2.230	2.186	2.137	2.078	2.008	1.926	1.829	1.715	1.582	1.432	1.269	1.100	0.935	0.782	0.646	0.528	0.427	0.342	0.271	0.210	0.158
2.98	2.362	2.321	2.274	2.217	2.148	2.065	1.963	1.839	1.689	1.514	1.319	1.118	0.926	0.755	0.609	0.489	0.390	0.309	0.243	0.188	0.141
3.72	2.502	2.466	2.423	2.370	2.305	2.224	2.121	1.988	1.819	1.610	1.369	1.121	0.893	0.702	0.550	0.432	0.340	0.266	0.208	0.160	0.120
4.96	2.652	2.622	2.586	2.542	2.485	2.411	2.313	2.178	1.990	1.735	1.420	1.094	0.818	0.611	0.462	0.354	0.273	0.212	0.164	0.126	0.095
5.60	2.705	2.678	2.646	2.604	2.552	2.483	2.388	2.256	2.064	1.790	1.437	1.073	0.775	0.568	0.424	0.322	0.247	0.191	0.148	0.113	0.085
7.50	2.813	2.792	2.766	2.733	2.690	2.633	2.552	2.431	2.240	1.930	1.471	1.000	0.668	0.464	0.336	0.251	0.191	0.147	0.113	0.086	0.065
11.0	2.916	2.901	2.883	2.859	2.829	2.786	2.725	2.628	2.460	2.128	1.503	0.862	0.514	0.339	0.240	0.177	0.133	0.102	0.078	0.060	0.045
15.0	2.976	2.965	2.951	2.933	2.910	2.878	2.831	2.755	2.615	2.298	1.521	0.727	0.400	0.257	0.179	0.131	0.099	0.075	0.058	0.044	0.033
20.0	3.017	3.009	2.998	2.985	2.968	2.943	2.907	2.847	2.735	2.453	1.533	0.598	0.310	0.196	0.136	0.099	0.075	0.057	0.044	0.033	0.025
40.0	3.079	3.075	3.070	3.063	3.054	3.042	3.023	2.992	2.932	2.759	1.552	0.335	0.161	0.099	0.069	0.050	0.037	0.029	0.022	0.017	0.012
50.0	3.092	3.088	3.084	3.079	3.072	3.062	3.047	3.022	2.973	2.831	1.556	0.272	0.129	0.080	0.055	0.040	0.030	0.023	0.017	0.013	0.010
100	3.117	3.115	3.113	3.110	3.107	3.102	3.094	3.082	2.293	2.983	1.563	0.139	0.065	0.040	0.027	0.020	0.015	0.011	0.009	0.007	0.005

TABLE 6

Reduced Moment Coefficient for Pitch versus Reduced Frequency for $0.004 \leq k \leq 100$ and $-1 \leq a \leq 1$

k	C_{MP} a = -1.0	C_{MP} a = -0.9	C_{MP} a = -0.8	C_{MP} a = -0.7	C_{MP} a = -0.6	C_{MP} a = -0.5	C_{MP} a = -0.4	C_{MP} a = -0.3	C_{MP} a = -0.2	C_{MP} a = -0.1	C_{MP} a = 0.0	C_{MP} a = 0.1	C_{MP} a = 0.2	C_{MP} a = 0.3	C_{MP} a = 0.4	C_{MP} a = 0.5	C_{MP} a = 0.6	C_{MP} a = 0.7	C_{MP} a = 0.8	C_{MP} a = 0.9	C_{MP} a = 1.0
0.004	62,090	49,670	37,250	24,830	12,420	250.0	12,430	24,840	37,260	49,680	65,100	74,520	86,940	99,360	111,800	124,200	136,600	149,000	161,500	173,900	186,300
0.006	27,500	22,000	16,500	11,000	5499	166.7	5507	11,010	16,510	22,010	27,510	33,010	38,510	44,010	49,510	55,010	60,510	66,010	71,510	77,010	82,510
0.008	15,420	12,330	9248	6165	3082	125.0	3090	6172	9255	12,340	15,420	18,510	21,590	24,670	27,760	30,840	33,920	37,010	40,090	43,170	46,260
0.010	9831	7864	5898	3931	1966	100.0	1973	3938	5904	7871	9837	11,800	13,770	15,740	17,700	19,670	21,640	23,600	25,570	27,540	29,500
0.012	6803	5442	4081	2720	1360	83.33	1367	2727	4087	5448	6809	8169	9530	10,890	12,250	13,610	14,970	16,330	17,700	19,060	20,420
0.016	3799	3039	2279	1519	759.8	62.50	765.8	1525	2284	3044	3804	4564	5324	6084	6844	7605	8365	9125	9,885	10,650	11,410
0.020	2414	1931	1448	964.9	483.0	50.00	488.4	970.3	1453	1936	2419	2902	3385	3868	4351	4834	5317	5800	6,283	6,766	7,249
0.026	1413	1130	847.3	564.7	283.1	38.46	287.9	569.5	851.9	1134	1417	1700	1983	2265	2548	2831	3114	3396	3,679	3,962	4,245
0.038	647.6	517.8	388.2	258.8	130.4	26.32	134.3	262.7	391.8	521.2	650.7	780.2	909.7	1039	1169	1298	1428	1558	1687	1,817	1,947
0.048	398.9	318.9	239.1	159.5	80.91	20.84	84.23	162.8	242.2	321.8	401.5	481.2	560.9	640.7	720.4	800.2	880.0	959.9	1040	1,120	1,200
0.062	233.7	186.8	140.1	93.66	48.18	16.13	50.77	96.25	142.5	189.0	235.6	282.1	328.7	375.4	422.0	468.7	515.4	562.1	608.8	655.6	702.4
0.076	152.3	121.8	91.37	61.27	32.12	13.16	34.24	63.43	93.38	123.5	153.8	184.0	214.3	244.6	275.0	305.3	335.7	366.1	396.5	427.0	457.5
0.094	97.20	77.74	58.44	39.42	21.33	10.64	22.90	41.04	59.93	79.01	98.17	117.4	136.6	155.8	175.1	194.4	213.7	233.0	252.3	271.7	291.2
0.120	58.00	46.46	35.06	23.96	13.72	8.342	14.68	24.98	35.96	47.15	58.41	69.71	81.03	92.37	103.7	115.1	126.5	137.9	149.4	160.9	172.4
0.150	36.33	29.19	22.20	15.48	9.523	6.677	9.987	15.97	22.57	29.37	36.24	43.16	50.10	57.06	64.03	71.03	78.05	85.09	92.16	99.26	106.4
0.190	22.41	18.12	13.96	10.06	6.769	5.277	6.817	10.05	13.82	17.78	21.82	25.90	30.01	34.14	38.28	42.44	46.63	50.84	55.07	59.33	63.63
0.250	13.20	10.81	8.540	6.470	4.806	4.018	4.537	6.008	7.901	9.969	12.12	14.31	16.53	18.77	21.03	23.30	25.60	27.92	30.27	32.64	35.04
0.380	6.570	5.538	4.585	3.742	3.067	2.658	2.614	2.931	3.499	4.210	5.000	5.835	6.699	7.586	8.492	9.416	10.36	11.32	12.31	13.32	14.37
0.460	5.037	4.301	3.626	3.028	2.539	2.206	2.081	2.183	2.472	2.886	3.374	3.908	4.473	5.061	5.668	6.294	6.938	7.604	8.291	9.002	9.740
0.620	3.530	3.059	2.627	2.242	1.913	1.656	1.492	1.438	1.496	1.647	1.866	2.130	2.426	2.745	3.085	3.443	3.820	4.217	4.635	5.075	5.540
0.740	2.943	2.562	2.213	1.900	1.627	1.402	1.239	1.149	1.140	1.208	1.336	1.509	1.714	1.943	2.193	2.462	2.749	3.056	3.384	3.733	4.105
0.940	2.369	2.066	1.790	1.539	1.318	1.128	0.9760	0.8693	0.8158	0.8188	0.8737	0.9706	1.099	1.253	1.428	1.622	1.835	2.067	2.319	2.591	2.886
1.24	1.917	1.667	1.439	1.233	1.049	0.8894	0.7549	0.6498	0.5793	0.5488	0.5600	0.6092	0.6897	0.7952	0.9220	1.068	1.233	1.418	1.622	1.846	2.092
1.50	1.699	1.471	1.263	1.076	0.9101	0.7649	0.6416	0.5422	0.4703	0.4307	0.4269	0.4581	0.5195	0.6058	0.7134	0.8407	0.9870	1.153	1.338	1.543	1.769
1.86	1.519	1.306	1.114	0.9410	0.7884	0.6555	0.5425	0.4504	0.3811	0.3383	0.3259	0.3453	0.3934	0.4660	0.5599	0.6735	0.8064	0.9586	1.131	1.323	1.535
2.24	1.407	1.202	1.018	0.8532	0.7084	0.5830	0.4769	0.3904	0.3246	0.2822	0.2668	0.2804	0.3216	0.3867	0.4729	0.5789	0.7043	0.8492	1.014	1.199	1.403
2.48	1.359	1.157	0.9759	0.8146	0.6729	0.5507	0.4475	0.3637	0.2998	0.2582	0.2422	0.2540	0.2925	0.3546	0.4377	0.5407	0.6632	0.8052	0.9669	1.149	1.351
2.98	1.291	1.093	0.9161	0.7588	0.6213	0.5032	0.4043	0.3244	0.2639	0.2241	0.2080	0.2178	0.2530	0.3112	0.3902	0.4891	0.6076	0.7458	0.9038	1.082	1.280
3.72	1.233	1.039	0.8649	0.7106	0.5762	0.4614	0.3659	0.2895	0.2322	0.1949	0.1796	0.1885	0.2214	0.2765	0.3522	0.4479	0.5633	0.6984	0.8534	1.028	1.223
4.96	1.187	0.9948	0.8226	0.6705	0.5382	0.4258	0.3329	0.2593	0.2050	0.1703	0.1565	0.1653	0.1966	0.2495	0.3227	0.4158	0.5288	0.6617	0.8145	0.9872	1.180
5.60	1.174	0.9822	0.8105	0.6589	0.5272	0.4153	0.3231	0.2504	0.1969	0.1631	0.1499	0.1588	0.1898	0.2420	0.3145	0.4070	0.5194	0.6516	0.8038	0.9759	1.168
7.50	1.153	0.9616	0.7907	0.6399	0.5090	0.3980	0.3068	0.2353	0.1833	0.1511	0.1391	0.1483	0.1788	0.2300	0.3014	0.3928	0.5041	0.6354	0.7866	0.9577	1.149
11.0	1.138	0.9474	0.7771	0.6267	0.4963	0.3859	0.2953	0.2247	0.1737	0.1426	0.1316	0.1412	0.1714	0.2220	0.2926	0.3833	0.4939	0.6244	0.7750	0.9455	1.136
15.0	1.132	0.9417	0.7715	0.6213	0.4911	0.3809	0.2906	0.2202	0.1697	0.1391	0.1286	0.1383	0.1685	0.2188	0.2891	0.3794	0.4898	0.6201	0.7704	0.9407	1.131
20.0	1.129	0.9388	0.7687	0.6186	0.4884	0.3783	0.2882	0.2180	0.1677	0.1373	0.1270	0.1369	0.1669	0.2171	0.2873	0.3775	0.4877	0.6179	0.7680	0.9382	1.128
40.0	1.126	0.9359	0.7659	0.6159	0.4859	0.3758	0.2858	0.2157	0.1657	0.1356	0.1255	0.1355	0.1655	0.2155	0.2856	0.3756	0.4857	0.6157	0.7658	0.9358	1.126
50.0	1.126	0.9356	0.7656	0.6156	0.4856	0.3755	0.2855	0.2155	0.1654	0.1354	0.1253	0.1353	0.1653	0.2153	0.2854	0.3754	0.4854	0.6155	0.7655	0.9355	1.126
100	1.125	0.9352	0.7651	0.6151	0.4851	0.3751	0.2851	0.2151	0.1651	0.1351	0.1251	0.1351	0.1651	0.2151	0.2851	0.3751	0.4851	0.6151	0.7651	0.9351	1.125

TABLE 7

Phase Angle of the Moment for a Foil Oscillating in Pitch versus Reduced Frequency for $0.004 \leq k \leq 100$ and $-1 \leq a \leq 1$

k	ϕ_{MP} a = -1.0	ϕ_{MP} a = -0.9	ϕ_{MP} a = -0.8	ϕ_{MP} a = -0.7	ϕ_{MP} a = -0.6	ϕ_{MP} a = -0.5	ϕ_{MP} a = -0.4	ϕ_{MP} a = -0.3	ϕ_{MP} a = -0.2	ϕ_{MP} a = -0.1	ϕ_{MP} a = 0.0	ϕ_{MP} a = 0.1	ϕ_{MP} a = 0.2	ϕ_{MP} a = 0.3	ϕ_{MP} a = 0.4	ϕ_{MP} a = 0.5	ϕ_{MP} a = 0.6	ϕ_{MP} a = 0.7	ϕ_{MP} a = 0.8	ϕ_{MP} a = 0.9	ϕ_{MP} a = 1.0
0.004	-3.152	-3.151	-3.150	-3.147	-3.138	-1.569	-0.037	-0.027	-0.024	-0.023	-0.022	-0.022	-0.022	-0.022	-0.022	-0.022	-0.023	-0.023	-0.023	-0.023	-0.024
0.006	-3.155	-3.154	-3.152	-3.147	-3.133	-1.569	-0.053	-0.038	-0.034	-0.032	-0.031	-0.031	-0.031	-0.031	-0.031	-0.031	-0.031	-0.032	-0.032	-0.033	-0.033
0.008	-3.157	-3.155	-3.153	-3.147	-3.127	-1.568	-0.068	-0.049	-0.043	-0.040	-0.039	-0.039	-0.038	-0.038	-0.039	-0.039	-0.039	-0.040	-0.040	-0.041	-0.042
0.010	-3.158	-3.156	-3.153	-3.146	-3.121	-1.567	-0.083	-0.059	-0.051	-0.048	-0.047	-0.046	-0.046	-0.046	-0.046	-0.046	-0.047	-0.048	-0.048	-0.049	-0.050
0.012	-3.159	-3.157	-3.155	-3.144	-3.114	-1.566	-0.097	-0.068	-0.059	-0.055	-0.054	-0.053	-0.052	-0.053	-0.053	-0.053	-0.054	-0.055	-0.056	-0.056	-0.057
0.016	-3.159	-3.157	-3.151	-3.139	-3.100	-1.565	-0.125	-0.086	-0.074	-0.069	-0.066	-0.065	-0.065	-0.065	-0.066	-0.066	-0.067	-0.068	-0.069	-0.070	-0.071
0.020	-3.159	-3.155	-3.149	-3.133	-3.084	-1.563	-0.152	-0.103	-0.088	-0.081	-0.078	-0.077	-0.076	-0.076	-0.077	-0.078	-0.079	-0.080	-0.081	-0.083	-0.084
0.026	-3.156	-3.151	-3.143	-3.123	-3.057	-1.561	-0.190	-0.127	-0.107	-0.098	-0.094	-0.092	-0.092	-0.092	-0.093	-0.094	-0.095	-0.097	-0.098	-0.100	-0.102
0.038	-3.145	-3.138	-3.125	-3.095	-2.998	-1.557	-0.263	-0.170	-0.141	-0.128	-0.122	-0.119	-0.117	-0.118	-0.119	-0.121	-0.123	-0.125	-0.127	-0.130	-0.133
0.048	-3.131	-3.123	-3.106	-3.067	-2.943	-1.553	-0.319	-0.203	-0.166	-0.149	-0.141	-0.137	-0.136	-0.136	-0.137	-0.139	-0.142	-0.145	-0.148	-0.151	-0.155
0.062	-3.105	-3.094	-3.071	-3.020	-2.858	-1.548	-0.391	-0.243	-0.194	-0.173	-0.162	-0.157	-0.155	-0.155	-0.157	-0.159	-0.162	-0.166	-0.170	-0.174	-0.179
0.076	-3.079	-3.064	-3.036	-2.972	-2.774	-1.542	-0.461	-0.284	-0.224	-0.197	-0.184	-0.178	-0.175	-0.175	-0.177	-0.180	-0.183	-0.188	-0.192	-0.198	-0.203
0.094	-3.036	-3.018	-2.982	-2.901	-2.662	-1.536	-0.541	-0.330	-0.256	-0.222	-0.205	-0.197	-0.193	-0.193	-0.195	-0.198	-0.203	-0.208	-0.214	-0.220	-0.227
0.120	-2.967	-2.943	-2.895	-2.791	-2.506	-1.526	-0.642	-0.391	-0.297	-0.253	-0.230	-0.219	-0.214	-0.213	-0.215	-0.219	-0.225	-0.231	-0.238	-0.246	-0.255
0.150	-2.879	-2.847	-2.787	-2.661	-2.345	-1.515	-0.740	-0.454	-0.338	-0.282	-0.252	-0.237	-0.230	-0.229	-0.231	-0.236	-0.242	-0.250	-0.259	-0.268	-0.279
0.190	-2.754	-2.714	-2.639	-2.492	-2.169	-1.500	-0.842	-0.528	-0.385	-0.314	-0.275	-0.255	-0.245	-0.242	-0.244	-0.249	-0.257	-0.266	-0.277	-0.288	-0.301
0.250	-2.565	-2.516	-2.429	-2.272	-1.976	-1.477	-0.951	-0.622	-0.447	-0.352	-0.300	-0.271	-0.256	-0.251	-0.252	-0.257	-0.266	-0.277	-0.290	-0.305	-0.321
0.380	-2.209	-2.156	-2.073	-1.939	-1.731	-1.429	-1.077	-0.769	-0.554	-0.419	-0.336	-0.287	-0.260	-0.247	-0.245	-0.250	-0.260	-0.274	-0.291	-0.310	-0.331
0.460	-2.036	-1.988	-1.915	-1.803	-1.636	-1.400	-1.113	-0.831	-0.607	-0.452	-0.353	-0.291	-0.256	-0.238	-0.233	-0.237	-0.247	-0.262	-0.281	-0.302	-0.325
0.620	-1.771	-1.738	-1.687	-1.611	-1.499	-1.342	-1.139	-0.909	-0.688	-0.510	-0.381	-0.296	-0.244	-0.216	-0.205	-0.206	-0.216	-0.232	-0.252	-0.276	-0.301
0.740	-1.621	-1.598	-1.561	-1.504	-1.420	-1.300	-1.138	-0.941	-0.732	-0.545	-0.400	-0.299	-0.236	-0.200	-0.184	-0.182	-0.191	-0.207	-0.228	-0.253	-0.279
0.940	-1.425	-1.416	-1.397	-1.364	-1.312	-1.232	-1.117	-0.963	-0.779	-0.590	-0.427	-0.304	-0.222	-0.174	-0.152	-0.147	-0.154	-0.170	-0.191	-0.215	-0.242
1.24	-1.209	-1.214	-1.212	-1.202	-1.179	-1.136	-1.064	-0.957	-0.808	-0.631	-0.454	-0.308	-0.206	-0.144	-0.113	-0.104	-0.110	-0.125	-0.145	-0.169	-0.194
1.50	-1.067	-1.079	-1.087	-1.089	-1.081	-1.058	-1.012	-0.932	-0.809	-0.647	-0.468	-0.309	-0.194	-0.123	-0.088	-0.078	-0.083	-0.097	-0.117	-0.139	-0.162
1.86	-0.915	-0.932	-0.948	-0.960	-0.966	-0.962	-0.938	-0.885	-0.790	-0.647	-0.472	-0.305	-0.179	-0.102	-0.064	-0.053	-0.057	-0.071	-0.089	-0.110	-0.130
2.24	-0.792	-0.812	-0.831	-0.850	-0.865	-0.872	-0.865	-0.831	-0.757	-0.632	-0.465	-0.295	-0.165	-0.085	-0.047	-0.036	-0.040	-0.053	-0.070	-0.088	-0.107
2.48	-0.729	-0.749	-0.770	-0.791	-0.810	-0.822	-0.821	-0.797	-0.734	-0.618	-0.457	-0.288	-0.157	-0.077	-0.040	-0.029	-0.033	-0.045	-0.061	-0.079	-0.096
2.98	-0.624	-0.644	-0.666	-0.689	-0.711	-0.730	-0.739	-0.728	-0.682	-0.583	-0.433	-0.270	-0.141	-0.064	-0.029	-0.019	-0.023	-0.034	-0.048	-0.064	-0.079
3.72	-0.512	-0.531	-0.552	-0.575	-0.599	-0.622	-0.638	-0.639	-0.608	-0.528	-0.394	-0.243	-0.122	-0.051	-0.019	-0.011	-0.014	-0.024	-0.037	-0.050	-0.062
4.96	-0.392	-0.408	-0.427	-0.448	-0.470	-0.493	-0.513	-0.522	-0.505	-0.444	-0.333	-0.203	-0.099	-0.038	-0.012	-0.005	-0.008	-0.016	-0.026	-0.036	-0.046
5.60	-0.349	-0.364	-0.381	-0.401	-0.422	-0.444	-0.464	-0.475	-0.462	-0.408	-0.306	-0.185	-0.089	-0.034	-0.009	-0.003	-0.007	-0.014	-0.023	-0.032	-0.040
7.50	-0.263	-0.275	-0.289	-0.305	-0.323	-0.342	-0.360	-0.371	-0.365	-0.325	-0.244	-0.146	-0.069	-0.025	-0.006	-0.001	-0.004	-0.010	-0.016	-0.023	-0.030
11.0	-0.181	-0.189	-0.199	-0.210	-0.224	-0.238	-0.252	-0.262	-0.260	-0.232	-0.174	-0.104	-0.048	-0.017	-0.004	0.000	-0.002	-0.006	-0.011	-0.016	-0.020
15.0	-0.133	-0.139	-0.147	-0.155	-0.165	-0.176	-0.187	-0.195	-0.194	-0.174	-0.130	-0.077	-0.036	-0.012	-0.003	0.000	-0.002	-0.004	-0.008	-0.011	-0.015
20.0	-0.100	-0.105	-0.110	-0.117	-0.124	-0.133	-0.141	-0.147	-0.147	-0.132	-0.099	-0.059	-0.027	-0.009	-0.002	0.000	-0.001	-0.003	-0.006	-0.009	-0.011
40.0	-0.050	-0.052	-0.055	-0.059	-0.062	-0.067	-0.071	-0.074	-0.074	-0.066	-0.050	-0.030	-0.014	-0.005	-0.001	0.000	-0.001	-0.002	-0.003	-0.004	-0.006
50.0	-0.040	-0.042	-0.044	-0.047	-0.050	-0.053	-0.057	-0.059	-0.059	-0.053	-0.040	-0.024	-0.011	-0.004	-0.001	0.000	0.000	-0.001	-0.002	-0.003	-0.004
100	-0.020	-0.021	-0.022	-0.023	-0.025	-0.027	-0.028	-0.030	-0.030	-0.027	-0.020	-0.012	-0.005	-0.002	-0.001	0.000	0.000	-0.001	-0.001	-0.002	-0.002

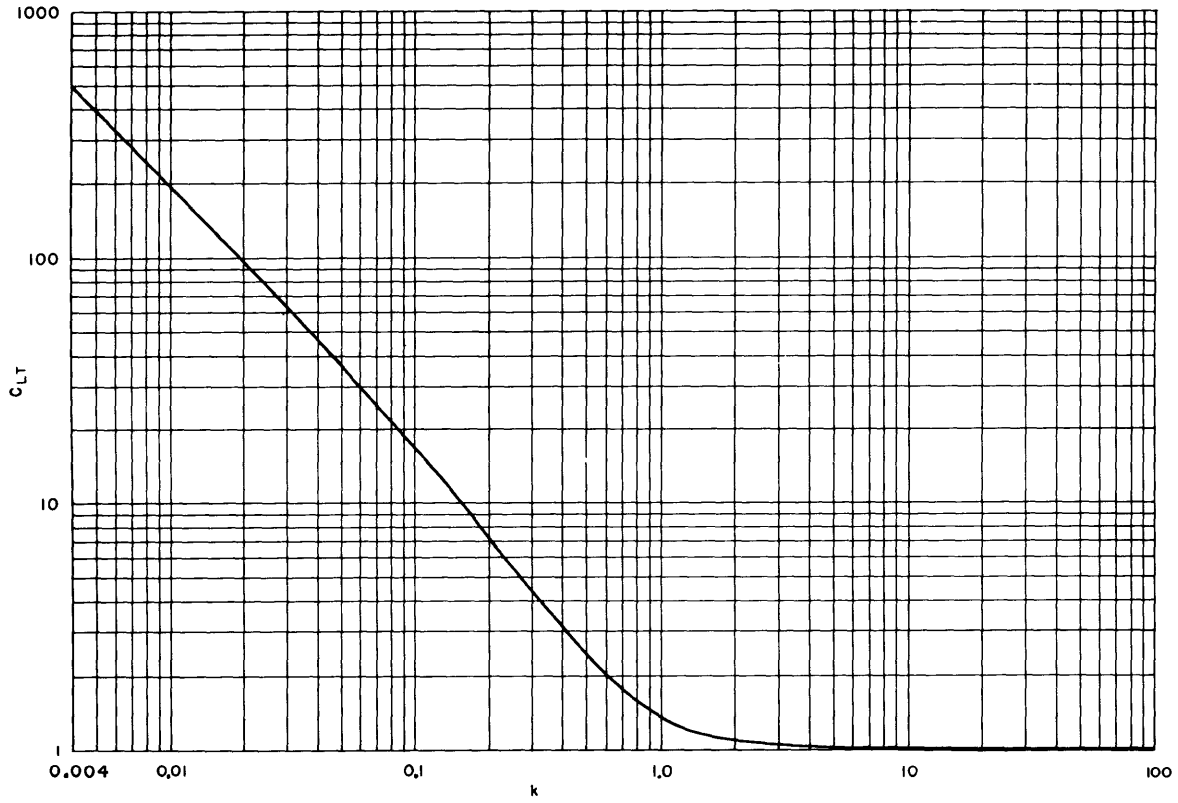

Figure 2 - Reduced Lift Coefficient for Heave versus Reduced Frequency for $0.004 \leq k \leq 100$

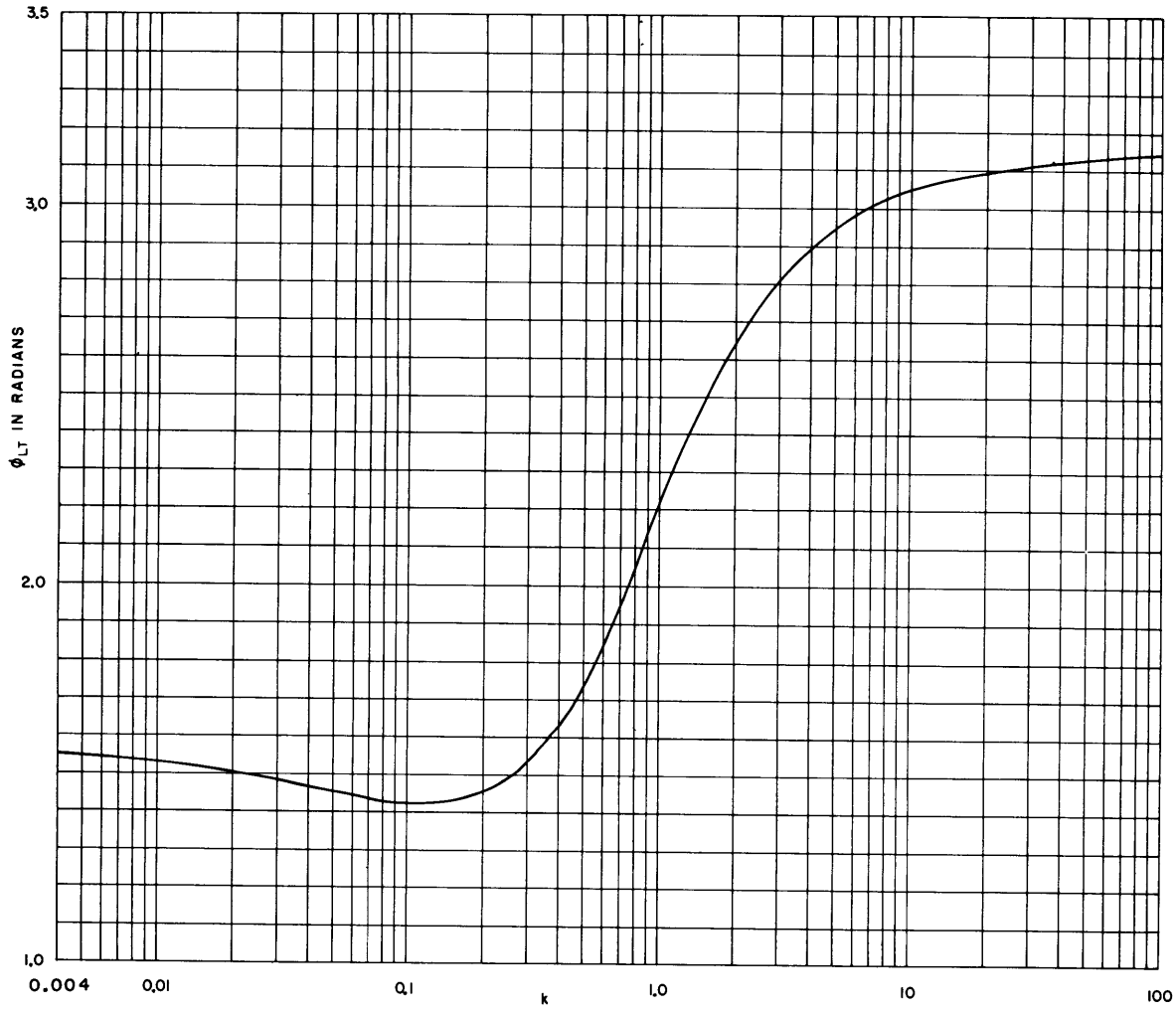

Figure 3 - Phase Angle of the Lift for a Foil Oscillating in Heave versus Reduced Frequency for $0.004 \leq k \leq 100$

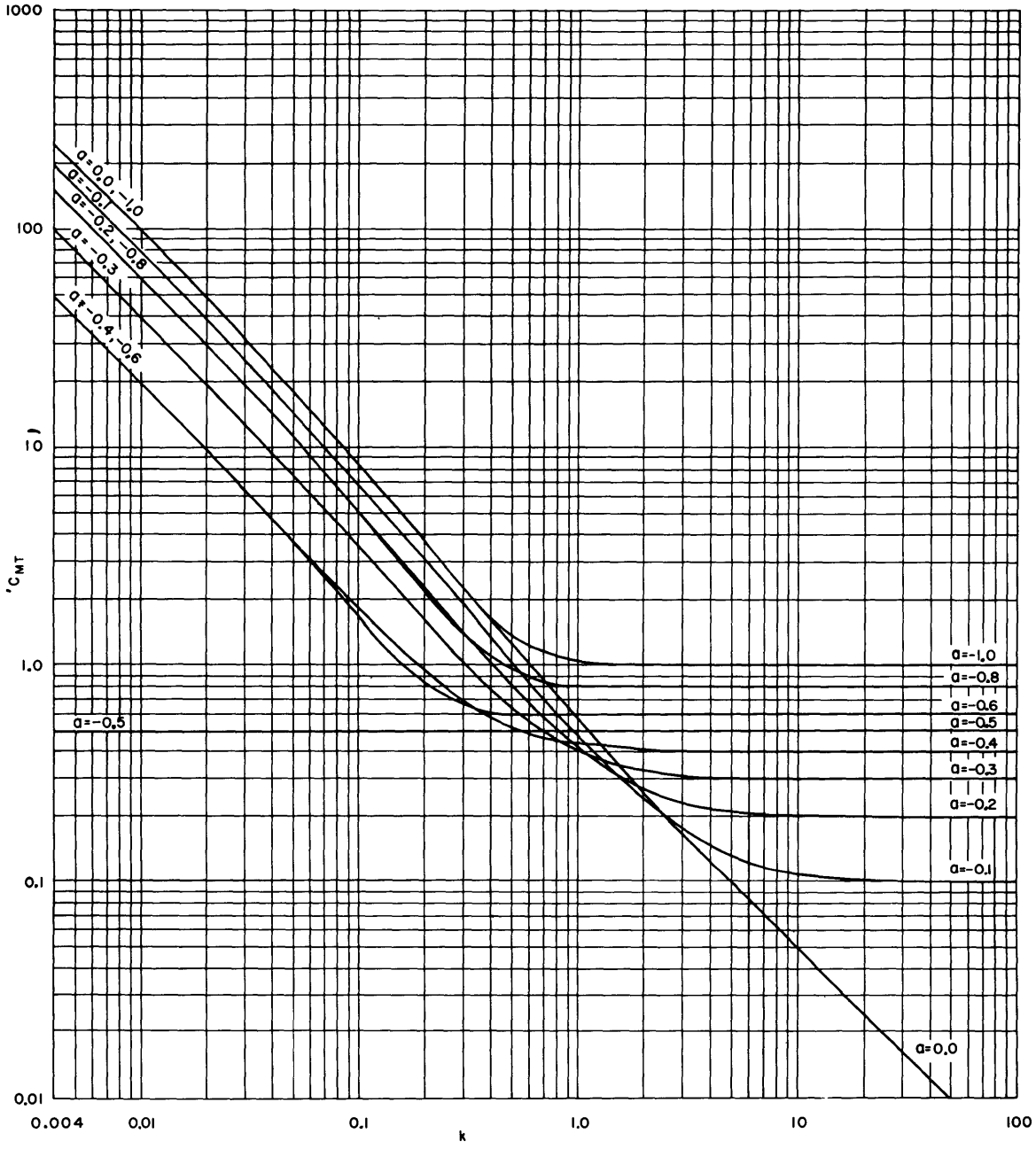

Figure 4a - Reduced Moment Coefficient for Heave versus Reduced Frequency for $0.004 \leq k \leq 100$, $-1 \leq a \leq 0$

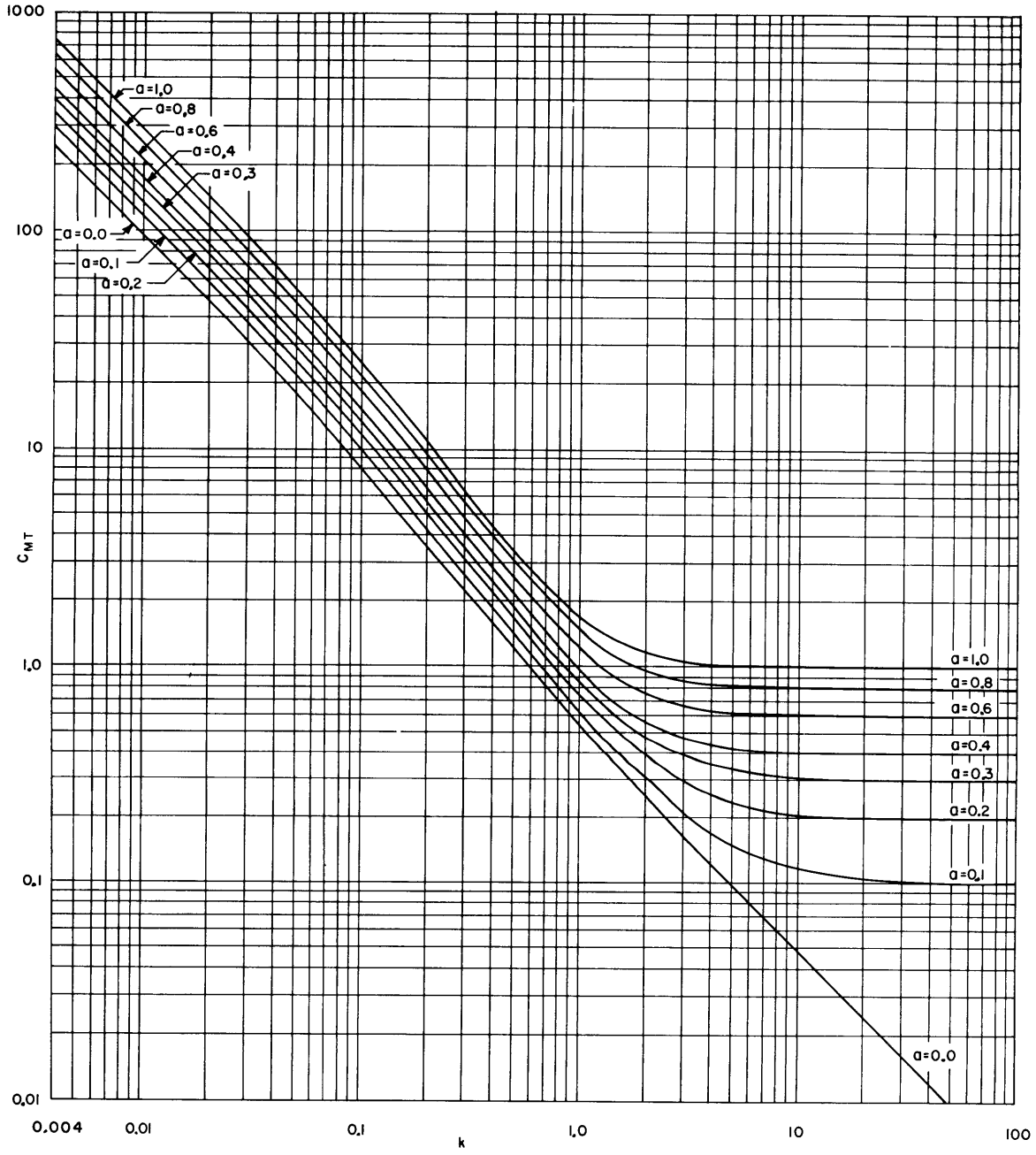

Figure 4b - Reduced Moment Coefficient for Heave versus Reduced Frequency for $(0.004 \leq k \leq 100)$ $0 \leq a \leq 1$

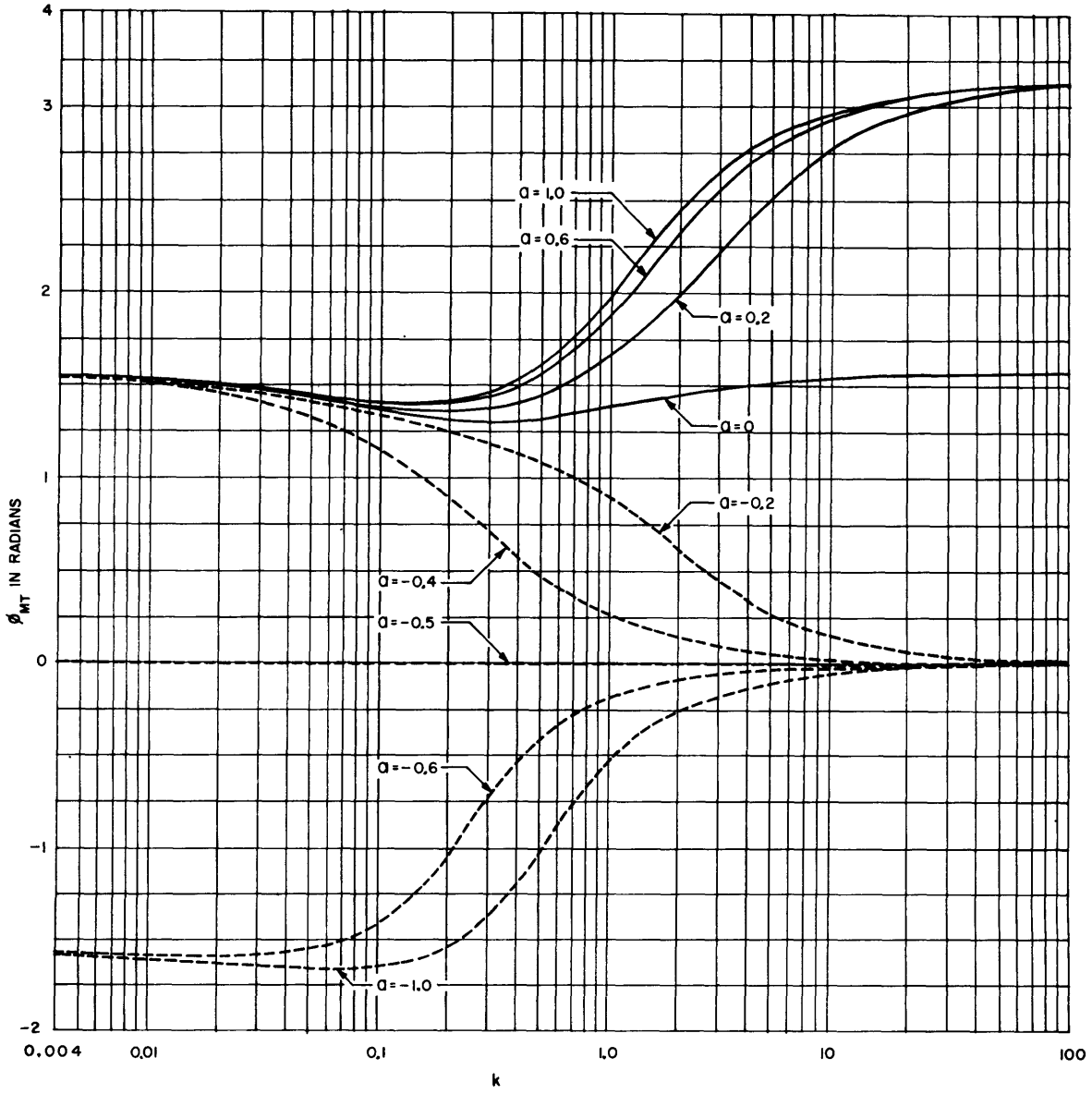

Figure 5 - Phase Angle of the Moment for a Foil Oscillating in Heave versus Reduced Frequency for $0.004 \leq k \leq 100$ and for $-1 \leq a \leq 1$

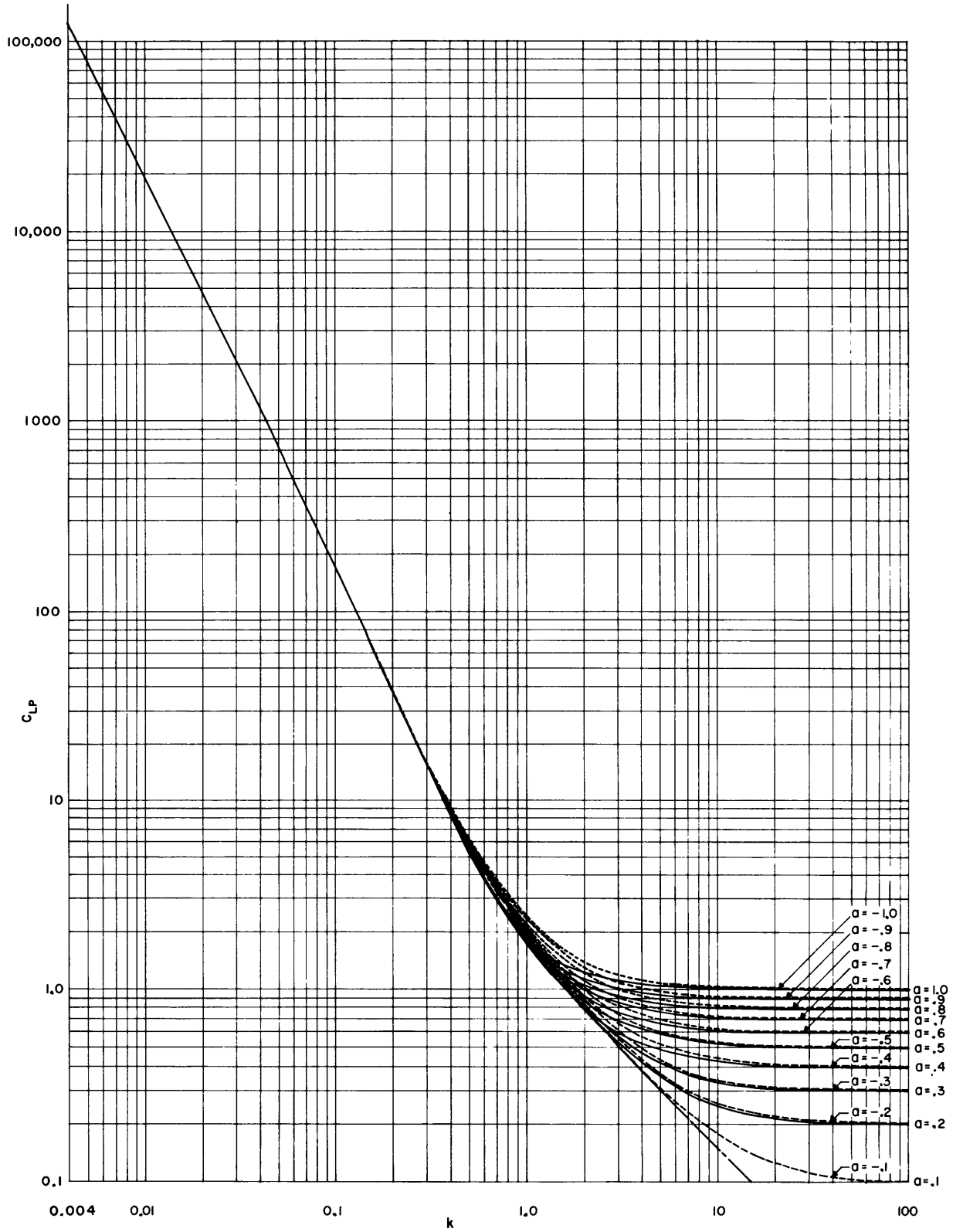

Figure 6a - Reduced Lift Coefficient for Pitch versus Reduced Frequency for $0.004 \leq k \leq 100$, $-1 \leq a \leq 1$

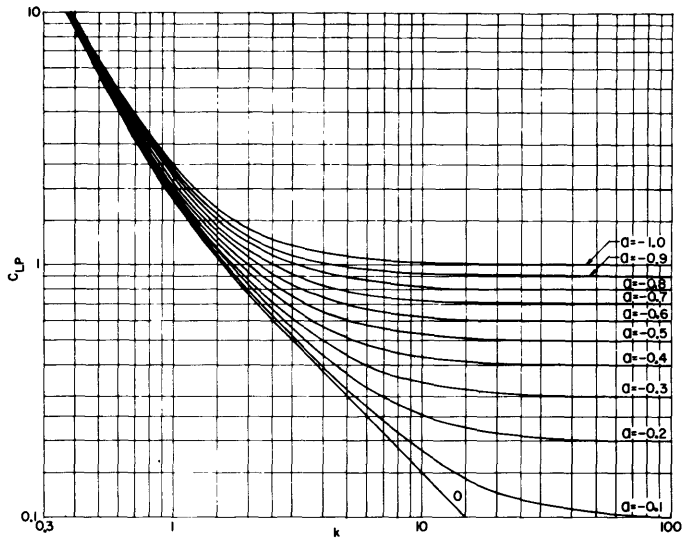

Figure 6b - Reduced Lift Coefficient for Pitch versus Reduced Frequency for $0.4 \leq k \leq 100$ and for $-1 \leq a \leq 0$

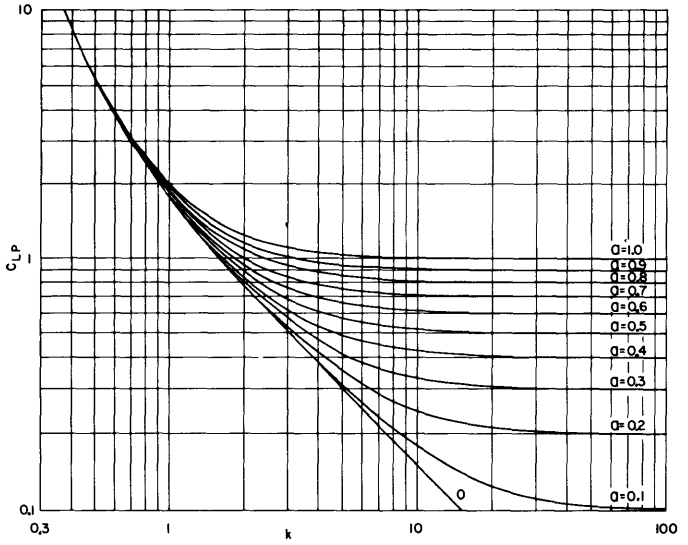

Figure 6c - Reduced Lift Coefficient for Pitch versus Reduced Frequency for $0.4 \leq k \leq 100$ and for $0 \leq a \leq 1$

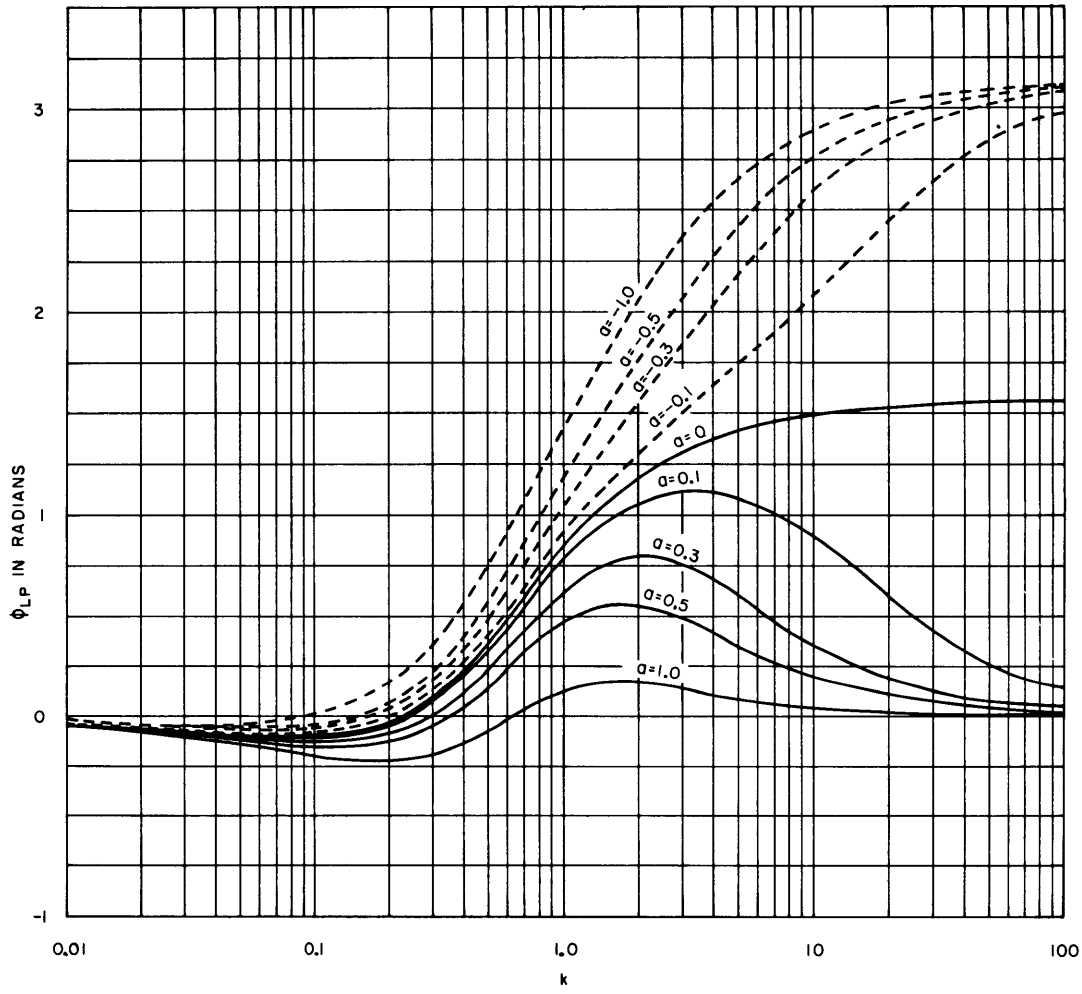

Figure 7 - Phase Angle of the Lift for a Foil Oscillating in Pitch versus Reduced Frequency for $0.01 \leq k \leq 100$ and $-1 \leq a \leq 1$

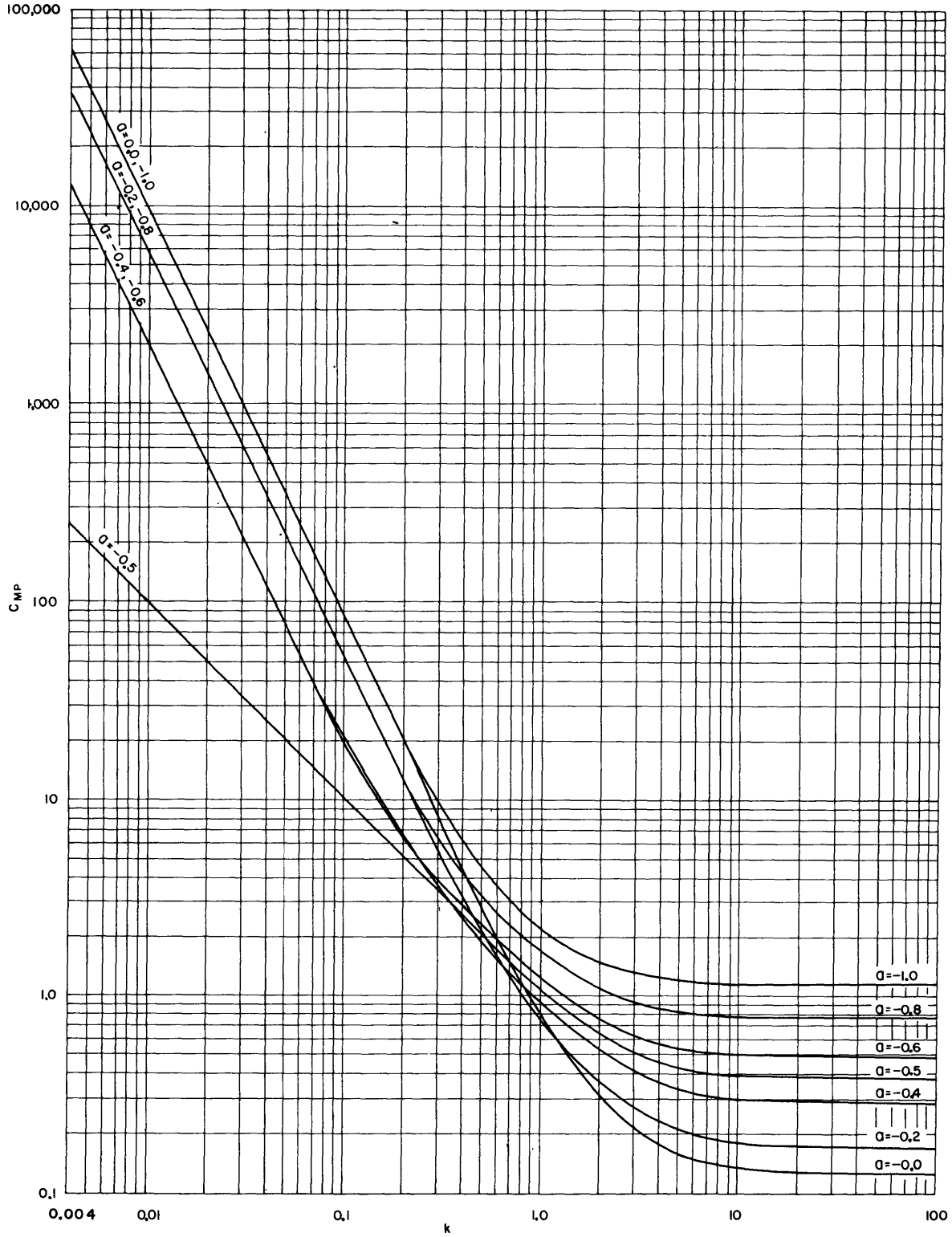

Figure 8a - Reduced Moment Coefficient for Pitch versus Reduced Frequency for ($0.004 \leq k \leq 100$) and for $-1 \leq a \leq 0$

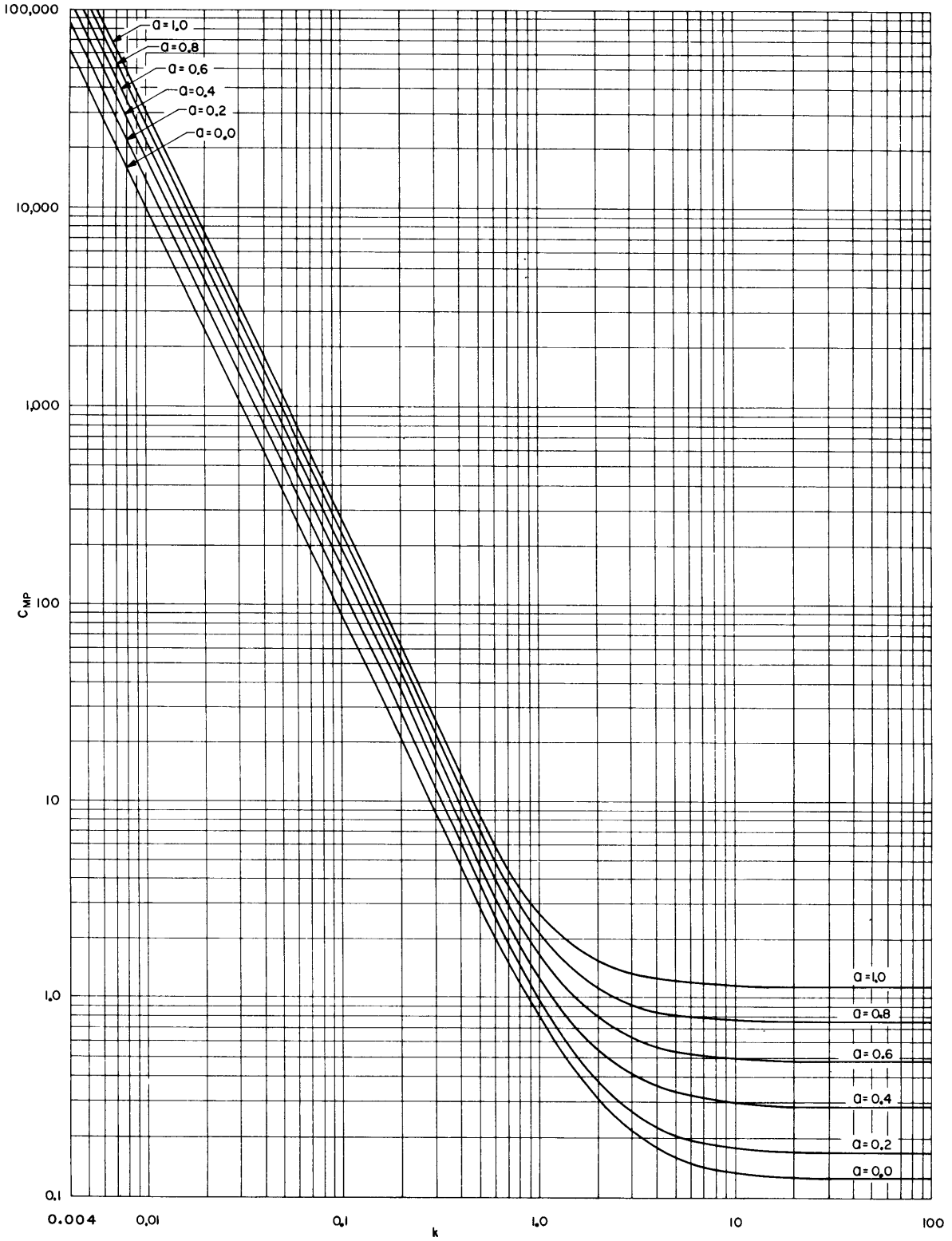

Figure 8b - Reduced Moment Coefficient for Pitch versus Reduced Frequency for ($0.004 \leq k \leq 100$) and for $0 \leq a \leq 1$

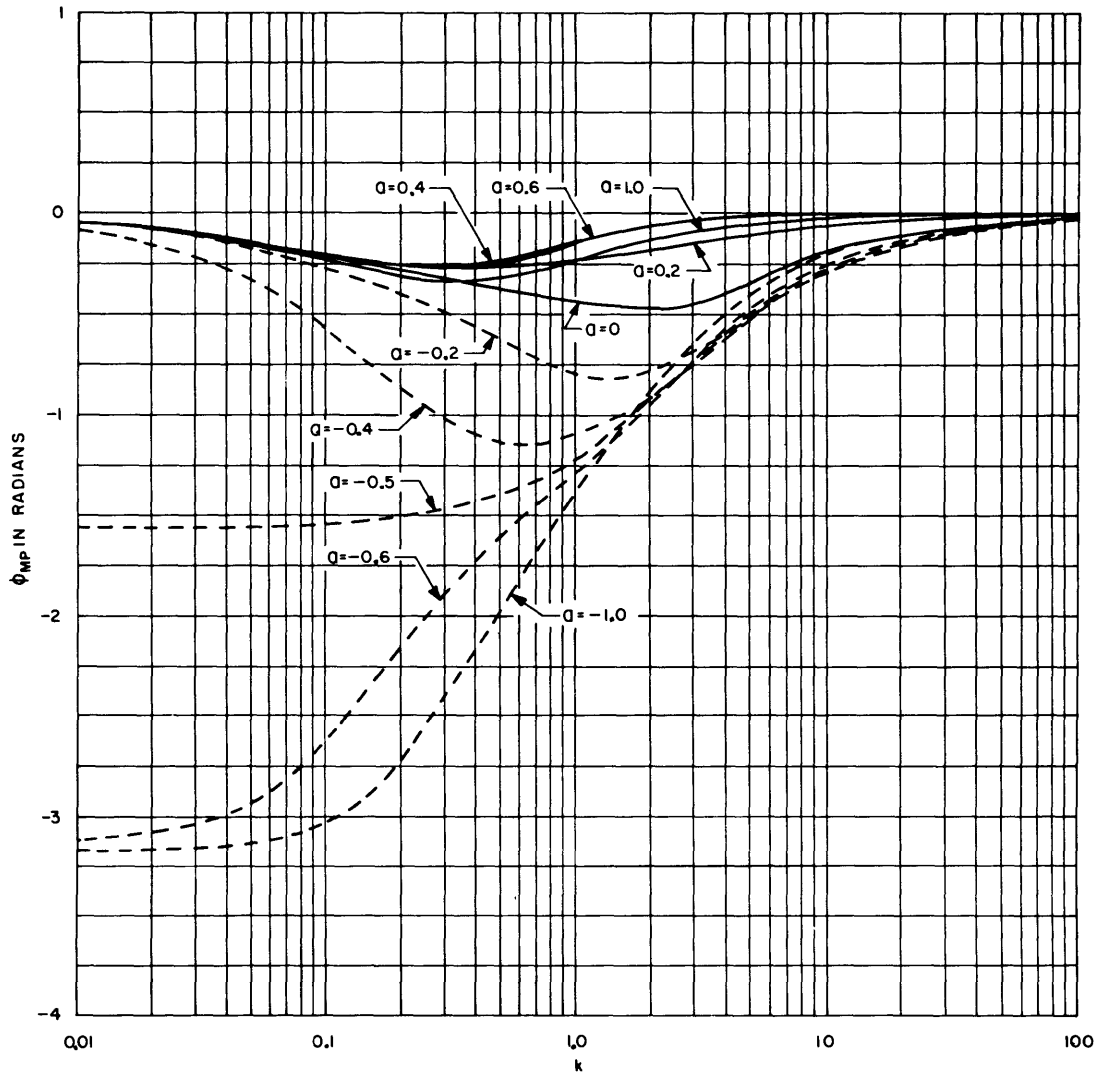

Figure 9 - Phase Angle of the Moment for a Foil Oscillating in Pitch versus Reduced Frequency for $0.01 \leq k \leq 100$ and $-1 \leq a \leq 1$

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APPENDIX

SAMPLE CALCULATION

To illustrate the use of the formulas and graphs in this report, the unsteady lift and moment of an oscillating, fully wetted hydrofoil in an infinite fluid will be computed for a particular set of conditions. Consider a thin hydrofoil with a 7-in. chord and an axis of rotation 2.1 in. aft of the loading edge. Then

$$b = 3.5 \text{ in.}$$

$$a = (2.1 - 3.5)/3.5 = -0.4$$

Let the foil velocity be 600 in./sec and the density of water be 9.35×10^{-5} slugs/in.^{3} The heaving and pitching motions will be determined separately and then combined into a general expression.

HEAVING MOTION

If the hydrofoil oscillates sinusoidally in heave with a frequency of 10 cps and an amplitude of 0.5 in.,

$$h_o = 0.5 \text{ in.}$$

$$\omega = 62.8 \text{ rad/sec}$$

$$k = 0.37$$

From Figures 2, 3, 4a, and 5, the reduced lift and moment coefficients and associated phase angles for this k and a are

$$C_{LT} = 3.43; \quad \phi_{LT} = 1.60 \text{ rad}$$

$$C_{MT} = 0.60; \quad \phi_{MT} = 0.62 \text{ rad}$$

The amplitudes of the lift and moment per unit span are obtained from Equations [8] and [9]:

$$|L_{UT}| = \pi \times 9.35 \times 10^{-5} \times (3.5)^2 \times (62.8)^2 \times 0.5 \times 3.43$$

$$= 24.4 \text{ lb/in.}$$

$$|M_{UT}| = \pi \times 9.35 \times 10^{-5} \times (3.5)^3 \times (62.8)^2 \times 0.5 \times 0.60$$
$$= 14.9 \text{ in-lb/in.}$$

PITCHING MOTION

If the hydrofoil oscillates sinusoidally in pitch with a frequency of 10 cps and an amplitude of 1 deg,

$$\omega = 62.8 \text{ rad/sec}$$
$$k = 0.37$$
$$\alpha_o = 1 \text{ deg} = 0.0174 \text{ rad}$$

From Figures 6a, 7, 8a, and 9, the reduced lift and moment coefficients and associated phase angles for this k and a are:

$$C_{LP} = 10.5; \quad \phi_{LP} = 0.32 \text{ rad}$$
$$C_{MP} = 2.70; \quad \phi_{MP} = -1.08 \text{ rad}$$

The amplitudes of the lift and moment per unit span are obtained from Equations [18] and [19]:

$$|L_{UP}| = \pi \times (9.35) \times 10^{-5} \times (3.5)^3 \times (62.8)^2 \times (0.0174) \times (10.5)$$
$$= 9.16 \text{ lb/in.}$$

$$|M_{UP}| = \pi \times 9.35 \times 10^{-5} \times (3.5)^4 \times (62.8)^2 \times (0.0174) \times (2.70)$$
$$= 82.5 \text{ in-lb/in.}$$

COMBINED MOTION

If the hydrofoil executes the two preceding modes of motion simultaneously and if the pitching motion leads the heaving by 90 deg,

$$\phi_{TP} = 1.57 \text{ rad}$$

The use of Equations [24] and [25] yields

$$L = \{24.4 \exp(1.60i) + 9.16 \exp[i(0.32 + 1.57)]\} \exp(62.8\,it)$$

$$= 33.3 \exp[i(62.8t + 1.68)] \text{ lb/in.}$$

$$M = \{14.9 \exp(0.62i) + 82.5 \exp[i(-1.03 + 1.57)]\} \exp(62.8\,it)$$

$$= 97.5 \exp[i(62.8t + 0.57)] \text{ in-lb/in.}$$

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