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PLASTIC AXISYMMETRIC BUCKLING OF RING-STIFFENED CYLINDRICAL SHELLS FABRICATED FROM STRAIN-HARDENING MATERIALS AND SUBJECTED TO EXTERNAL HYDROSTATIC PRESSURE

by

Myron E. Lunchick, Ph.D.

[redacted]

STRUCTURAL MECHANICS LABORATORY
RESEARCH AND DEVELOPMENT REPORT

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1. Recent developments in metallurgy have resulted in new structural materials which can be used advantageously in submarine pressure hulls. All these materials exhibit strain-hardening stress-strain curves. In enclosure (1) theory is extended to provide procedures for the design of stiffened cylinders made from strain-hardening materials.

2. Failure of the shell between stiffeners by plastic axisymmetric buckling is analyzed in detail. Theoretical expressions for collapse pressure are derived for this mode of failure, and some limited experimental results are presented to confirm the theory.

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PLASTIC AXISYMMETRIC BUCKLING OF RING-STIFFENED CYLINDRICAL SHELLS FABRICATED FROM STRAIN-HARDENING MATERIALS AND SUBJECTED TO EXTERNAL HYDROSTATIC PRESSURE

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NOTATION

A_1, A_2, A_{12}	Plasticity coefficients
B	Axial rigidity
C, C_a	Coefficients in plastic buckling equations
c	Shell parameter $= \dfrac{L^2}{\pi^2 Rh}$
D	Flexural rigidity
E	Young's modulus
E_s	Secant modulus
E_t	Tangent modulus
e_i	Strain intensity
h	Shell thickness
H	Plasticity coefficient used to compute A's
k	Ratio of circumferential membrane stress to axial membrane stress
K^2	$1 - k + k^2$
L	Length of cylinder between transverse boundaries
M	Moment
N	Force
n	Number of axial half waves of buckling
p	Pressure
p_{cr}	Buckling pressure
p_∞	Elastic buckling pressure of an infinitely long cylinder
R	Radius of shell
u, v, w	Displacements in the axial, circumferential, and radial directions, respectively
x, y, z	Coordinates in the axial, circumferential, and radial directions, respectively
α	Wave parameter $= \sqrt[4]{\dfrac{3(1-\mu^2)}{R^2h^2}}$
γ	Shear strain
δ_{ij}	Kronecker delta
ϵ	Normal strain
μ	Poisson's ratio at any stress level

μ_e	Poisson's ratio in elastic range
θ, ϕ, ψ	Angles used in applying Budiansky's criterion
ξ, η	Coordinates of loading path diagram
σ	Normal stress
σ_i	Stress intensity
τ	Shear stress
χ	Curvature

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m	Midbay
mx,my	Membrane value in axial and circumferential directions, respectively
p	Plastic
$S\text{-}P$	Salerno and Pulos analysis
YP	Yield point
x,y	Axial and circumferential directions, respectively
xy	Twisting
	Primes refer to variations of the terms during the buckling process.

ABSTRACT

A cylindrical shell stiffened by uniformly spaced transverse rings and subjected to external hydrostatic pressure is analyzed for plastic buckling in the axisymmetric mode. In this analysis strain-hardening of the material is accounted for and Poisson's ratio is treated as a variable which varies from its value in the elastic state to an upper limit of 1/2 for incompressible material. The differential equation of equilibrium for the plastic range is derived for axisymmetric buckling. Expressions for the plastic axisymmetric buckling pressures are obtained. A criterion of length of shell for "long" and "short" cylinders is presented, below which the shell can buckle in only one half wave. As a special case of the problem considered, a solution for the plastic buckling of a long cylinder under end compression is obtained. This solution degenerates to that previously obtained by Gerard for a Poisson's ratio of 1/2 but represents a more rigorous solution in terms of variable Poisson's ratios. Finally, limited experimental evidence is presented in support of the theory.

INTRODUCTION

Recent developments in metallurgy have resulted in new structural materials which can be advantageously used in closed stiffened cylinders loaded by external hydrostatic pressure. Some examples are aluminum alloys, titanium alloys, and steel alloys with yield strengths above 125,000 psi. All these materials exhibit strain-hardening stress-strain curves. Hitherto, the theories for collapse pressure of stiffened cylinders loaded into the plastic range have been developed for elastic, perfectly plastic materials or ones exhibiting a plateau-type stress-strain curve. An extension of the theory is necessary to develop adequate procedures for the design of stiffened cylinders made from strain hardening materials.

In particular, a cylindrical shell made from a strain-hardening material stiffened by uniformly spaced transverse rings and subjected to external hydrostatic pressure will be treated. Failure of the shell between stiffeners by plastic *axisymmetric* buckling will be analyzed in detail. Theoretical expressions for collapse pressure will be derived for this mode of failure, and some limited experimental results will be presented to confirm the theory.

HISTORICAL DEVELOPMENT OF THEORY

Considerable study has been devoted to the failure of ring-stiffened cylindrical shells by the formation of an accordion pleat in the shell between stiffeners when the shells are loaded by external hydrostatic pressure. Salerno and Pulos[1] determined from the *elastic*

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[1]References are listed on page 31.

equations of equilibrium the pressure at which axisymmetric deflections of the shell would be infinite. They showed that this pressure was identical to the *elastic* buckling pressure derived by Timoshenko[2] for an infinitely long unstiffened cylinder loaded only by uniformly distributed compressive end loads (i.e., no pressure on the lateral surface of the cylinder).

Invariably, failure in the axisymmetric mode involves yielding of the shell material. Von Sanden and Gunther[3] derived expressions which predict failure on the basis of the onset of yielding by a maximum-principal-stress criterion. Investigators[4] at the David Taylor Model Basin extended this theory to include the Hencky-Von Mises criterion as a yield criterion and to allow for a measure of plastic reserve strength between the onset of yielding and final collapse. Various investigators at Brown University[5,6,7] and the Polytechnic Institute of Brooklyn[8,9] analyzed the problem using an incremental theory of plasticity together with an elastic, perfectly plastic material and a maximum-shearing-stress criterion of yielding. These last analyses did not adequately account for the elastic stiffening rings commonly used. Still another group of investigators[10,11,12] used an approach similar to that used in the limit design of beams and trusses. In effect, a three-hinge mechanism of collapse was used in conjunction with the Hencky-Von Mises criterion of yielding and, again, an elastic, perfectly plastic material was assumed. All these theories which are based on an elastic, perfectly plastic material are compared with experimental results in Reference 13.

None of the theories previously cited account for strain-hardening of the material. An entirely different school of investigators concentrated on the plastic buckling of structural elements composed of a strain-hardening material. Bijlaard[14] and Handelman and Prager[15] used incremental plasticity theory to obtain expressions for the plastic buckling loads of plates. Ilyushin,[16] Stowell,[17] and Bijlaard[18] obtained comparable solutions using the deformation theory of plasticity. Gerard[19] extended the inelastic (or plastic) equations of equilibrium derived by Stowell for flat plates to cylindrical shells. Gerard's inelastic differential equations degenerate to the *elastic* differential equations of equilibrium of Donnell[20] when the plastic moduli (tangent and secant) are set equal to Young's modulus E and when Poisson's ratio μ is assumed to be ½. Gerard[19, 21] used his equations to solve the following cases of plastic buckling:

1. The unstiffened circular cylinder subjected to uniform end load and failing in the axisymmetric mode.
2. The unstiffened circular cylinder subjected to pure torsion.
3. Moderately long $\left(100 \frac{h}{R} < \frac{L^2}{R^2} < 5Rh\right)$* circular cylinders subjected to lateral pressure only.
4. Long $\left(\frac{L^2}{R^2} > \frac{R}{h}\right)$ circular cylinders subjected to lateral pressure only.

---

*L, R, and h are the length, radius, and thickness of the cylinder, respectively.

In this report the work of Gerard will be extended to include the plastic (inelastic) *axisymmetric* buckling of circular cylinders subjected to both end loads and lateral pressure or, in other words, subjected to hydrostatic pressure. The theory will be applicable to cylinders shorter than those considered by Gerard which buckle only under lateral pressure. Furthermore, the solution for buckling pressure will not be confined to a value of ½ for Poisson's ratio but will be expressed explicitly in terms of Poisson's ratio.

THEORETICAL DEVELOPMENT OF ELASTIC AXISYMMETRIC BUCKLING PRESSURE

To understand better the buckling behavior of a stiffened cylinder in the plastic range, the elastic buckling pressure should first be examined. The expressions for the plastic buckling pressures should degenerate to those for the elastic case when the moduli used are set equal to Young's modulus E. Since stiffened cylinders have not been observed to fail *elastically* in the axisymmetric mode, no expressions for the buckling pressure have been reported, to the knowledge of the author, which take into account a shortening of the half buckling wave due to the presence of ring stiffeners. The expressions for the elastic buckling pressures will now be derived for a ring-stiffened cylindrical shell subjected to hydrostatic pressure p, assumed positive when external.

The notation for displacements is shown in Figure 1. Primes are used to denote variations of the displacements, loads, or forces developed during buckling. The equilibrium expressions of Donnell[20] during buckling are:

$$R\nabla^4 u' = -\mu \frac{\partial^3 w'}{\partial x^3} + \frac{\partial^3 w'}{\partial x\, \partial y^2} \qquad [1]$$

$$R\nabla^4 v' = -(2+\mu)\frac{\partial^3 w'}{\partial x \partial y} - \frac{\partial^3 w'}{\partial y^3} \qquad [2]$$

$$D\nabla^4 w' + \frac{B}{R}\left(\mu \frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{w'}{R}\right) + N_x \frac{\partial^2 w'}{\partial x^2} + 2N_{xy}\frac{\partial^2 w'}{\partial x\, \partial y} + N_y \frac{\partial^2 w'}{\partial y^2} - p' = 0 \qquad [3]$$

where

$$D = \frac{Eh^3}{12(1-\mu^2)} \qquad [4]$$

and

$$B = \frac{Eh}{1-\mu^2} \qquad [5]$$

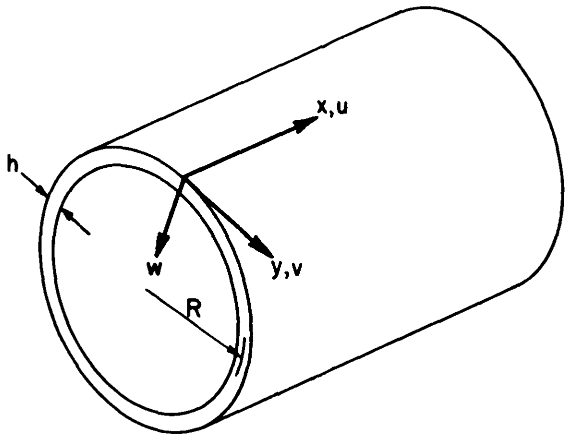


Figure 1 – Coordinates and Components of Displacement

For axisymmetric deflections

$$\frac{\partial^n}{\partial y^n} = 0 \qquad [6]$$

For hydrostatic pressure loading the axial load is

$$N_x = \frac{pR}{2} \qquad [7]$$

Substituting Equations [6] and [7] into Equations [1] and [3], we obtain

$$\frac{\partial^4 u'}{\partial x^4} = -\frac{\mu}{R}\frac{\partial^3 w'}{\partial x^3} \qquad [8]$$

and

$$D\frac{\partial^4 w'}{\partial x^4} + \frac{B}{R}\left(\mu\frac{\partial u'}{\partial x} + \frac{w'}{R}\right) + \frac{pR}{2}\frac{\partial^2 w'}{\partial x^2} - p' = 0 \qquad [9]$$

Applying $\frac{\partial^3}{\partial x^3}$ to Equation [9] and substituting Equations [4], [5], and [8] into the resulting expression, we obtain, after simplifying and expressing in terms of total derivatives of w,

$$\frac{d^7 w'}{\partial x^7} + 6(1-\mu^2)\frac{pR}{Eh^3}\frac{d^5 w'}{dx^5} + \frac{12}{R^2 h^2}(1-\mu^2)\frac{d^3 w'}{dx^3} = 0 \qquad [10]$$

LONG CYLINDERS

For the moment let us consider a "long" cylinder under hydrostatic loading. A "long" cylinder will be defined as one which buckles in more than one half wave. The criterion for "long" will be established in the analysis. A solution to Equation [10] can be written

$$w' = w_m' \sin\frac{n\pi x}{L} \qquad [11]$$

where L is the length of the cylinder. When Equation [11] and the appropriate derivatives are substituted into Equation [10], the expression for the buckling pressure in terms of the number n of half waves of buckling results:

$$p_{cr} = p_\infty\left[\left(\frac{\alpha L}{n\pi}\right)^2 + \frac{1}{4}\left(\frac{n\pi}{\alpha L}\right)^2\right] \qquad [12]$$

where

$$p_\infty = \frac{2}{\sqrt{3(1-\mu^2)}} E\left(\frac{h}{R}\right)^2 \qquad [13]$$

and

$$\alpha = \sqrt[4]{\frac{3(1-\mu^2)}{R^2 h^2}} \qquad [14]$$

The buckling curves represented by Equation [12] are shown in Figure 2.

If, for specified values of n, Equation [12] is differentiated with respect to αL and if the resulting expression is set equal to zero and then solved for αL, the value of αL at which the buckling pressure and $\frac{p_{cr}}{p_\infty}$ is a minimum for a given n is

$$(\alpha L)_{\min} = \frac{n\pi}{\sqrt{2}} \qquad [15]$$

From Figure 2 it can be seen that the minimum value of $\frac{p_{cr}}{p_\infty}$ at $(\alpha L)_{\min}$ given by Equation [15] is always unity.

The cusp points shown in Figure 2 where the curves for modes n and $n+1$ intersect can be obtained by equating Equation [12] to a similar equation where n has been replaced by $n+1$. In this manner the cusps are obtained at

$$(\alpha L)_{\text{cusp}} = \frac{\pi}{\sqrt{2}} \sqrt{n(n+1)} \qquad [16]$$

Equation [16] gives $\alpha L = \pi$ when $n = 1$. If a cylinder is such that αL is greater than π, it will buckle into two or more half waves and thus is defined as "long."

The values of $\frac{p_{cr}}{p_\infty}$ at which the cusps occur are found by substituting Equation [16] into Equation [12], thus

$$\left(\frac{p_{cr}}{p_\infty}\right)_{\text{cusp}} = \frac{2n^2 + 2n + 1}{2n^2 + 2n} \qquad [17]$$

From Equation [17] the buckling pressure p_{cr} of a cusp approaches p_∞ as n goes to infinity. The buckling pressure p_∞ expressed in Equation [13] is identical to that obtained by Timoshenko[2] for an infinitely long cylinder subjected to end forces only. This is not

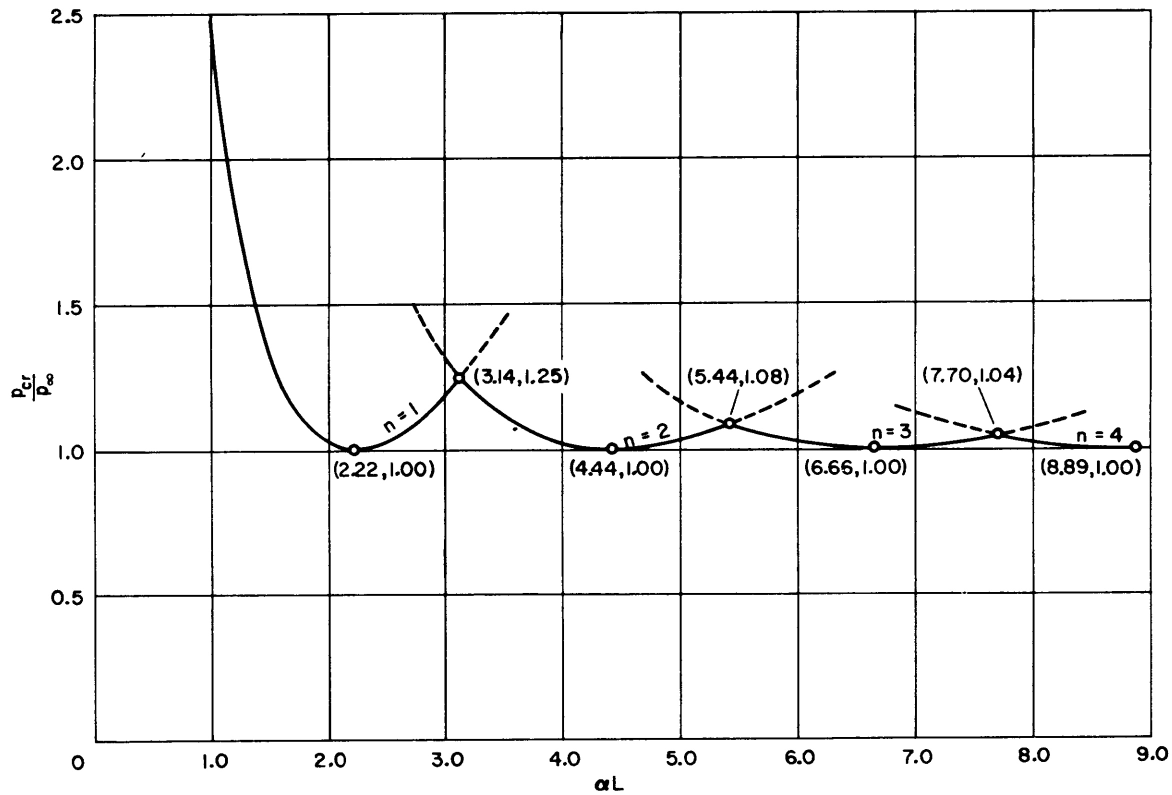


Figure 2 – Curves for Axisymmetric Buckling of Cylinders under External Hydrostatic Pressure

surprising if one considers that, when $\frac{\partial^n}{\partial y^n} = 0$ was applied to Equation [3], all the terms involving forces other than the axial force N_x vanish.

SHORT CYLINDERS

If αL is less than π, the cylinder will be denoted as "short." If a cylinder is stiffened by transverse rings uniformly spaced at distance L apart such that $\alpha L < \pi$, then the half wave of buckling can be taken as the distance between stiffening rings. For such "short" cylinders $n = 1$, and Equation [12] reduces to

$$p_{cr} = p_\infty \left[\left(\frac{\alpha L}{\pi} \right)^2 + \frac{1}{4} \left(\frac{\pi}{\alpha L} \right)^2 \right] \qquad [18]$$

Equation [18] can also be written as:

$$p_{cr} = 2\, E c^2 \left(\frac{h}{R} \right)^2 + \frac{1}{6(1-\mu^2)} \frac{E}{c^2} \left(\frac{h}{R} \right)^2 \qquad [19]$$

where

$$c^2 = \frac{L^2}{\pi^2 Rh} \qquad [20]$$

For "short" cylinders it should be noted from Figure 2 and Equation [15] that the buckling pressure is a minimum at

$$\alpha L = \frac{\pi}{\sqrt{2}} = 2.22 \qquad [21]$$

THEORETICAL DEVELOPMENT OF PLASTIC AXISYMMETRIC BUCKLING PRESSURE

Plastic equilibrium expressions have been derived for flat plates by Stowell,[17] whose work has been extended by Gerard[19] to circular cylindrical shells. These equilibrium expressions were derived assuming the material to be incompressible and Poisson's ratio to be ½. Equilibrium expressions will now be derived for circular cylindrical shells in which the deflections are axisymmetric and Poisson's ratio μ varies between its value for the purely elastic state and a value of ½ for the purely plastic incompressible state.

The deformation theory of plasticity will be used in the plastic analysis. This theory, in contrast to the incremental theory of plasticity, is valid only for a limited range of loading paths. It has long been recognized to be valid for proportional loading, that is, where the stresses increase linearly with an increase in load. Budiansky[22] has shown that loading paths other than proportional loading are admissible without violation of general requirements for the physical soundness of a plasticity theory. Budiansky presents a criterion for the extent of admissible variations from proportional loading. The following analysis will be applicable where Budiansky's criterion is satisfied for both the *prebuckling* and the *buckling* stresses. Later, when experimental results are presented for comparison with theory, it will be shown that Budiansky's criterion was satisfied for the stiffened cylinders tested.

In the deformation theory of plasticity the stress and strain intensities are defined by Ilyushin[16] as:

$$\sigma_i = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3\tau^2} \qquad [22]$$

and

$$e_i = \frac{2}{\sqrt{3}} \sqrt{\epsilon_x^2 + \epsilon_y^2 + \epsilon_x \epsilon_y + \frac{\gamma^2}{4}} \qquad [23]$$

where σ_x is the stress in the x-direction,

σ_y is the stress in the y-direction,

τ is the shear stress,

ϵ_x is the strain in the x-direction,

ϵ_y is the strain in the y-direction, and

γ is the shear strain.

The stress-strain relations consistent with Equations [22] and [23] are

$$\epsilon_x = \frac{\sigma_x - \frac{1}{2}\sigma_y}{E_s} = \frac{S_x}{E_s} \qquad [24]$$

$$\epsilon_y = \frac{\sigma_y - \frac{1}{2}\sigma_x}{E_s} = \frac{S_y}{E_s} \qquad [25]$$

$$\gamma = \frac{3\tau}{E_s} \qquad [26]$$

where

$$E_s = \frac{\sigma_i}{e_i} \qquad [27]$$

Stowell states "According to the fundamental hypothesis of the theory of plasticity, the intensity of strain σ_i is a uniquely defined single-valued function of the intensity of strain e_i for any given material if σ_i increases in magnitude (loading condition). If σ_i decreases (unloading condition), the relation between σ_i and e_i becomes linear as in a purely elastic case." In practice the relationship between σ_i and e_i is the stress-strain curve obtained in a uniaxial loading test. (The results in tension and compression are assumed to be the same.)

Equations [22] through [26] can be generalized to any value of Poisson's ratio as follows:

$$\epsilon_x = \frac{\sigma_x - \mu\sigma_y}{E_s} = \frac{S_x}{E_s} \qquad [28]$$

$$\epsilon_y = \frac{\sigma_y - \mu\sigma_x}{E_s} = \frac{S_y}{E_s} \qquad [29]$$

$$\gamma = \frac{2(1+\mu)\tau}{E_s} \qquad [30]$$

For principal stresses for which $\tau = \gamma = 0$

$$\sigma_i = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y} \qquad [31]$$

$$e_i = \frac{1}{1-\mu^2}\sqrt{(1-\mu+\mu^2)(\epsilon_x^2+\epsilon_y^2)+(4\mu-\mu^2-1)\,\epsilon_x\epsilon_y} \qquad [32]$$

Equation [32] is derived by solving Equations [28] and [29] for σ_x and σ_y substituting the expressions for the stresses in Equation [31] and then using the resulting expression for σ_i in Equation [27]. Equations [28] and [29] degenerate to Hooke's law when the secant modulus E_s equals Young's modulus E. Also, Equations [28] through [32] degenerate to Equations [22] through [27] for a Poisson's ratio of ½ and for principal stresses.

When buckling begins, the strains ϵ_x and ϵ_y will vary slightly from their prebuckling values. The variation ϵ_x' will arise partly from the variation of membrane strain and partly from bending strain; thus

$$\epsilon_x' = \epsilon_{mx}' - Z\,\chi_x' \qquad [33]$$

The variation ϵ_y' will result only from the variation in membrane strain since only axisymmetric deflections are considered; thus

$$\epsilon_y' = \epsilon_{my}' \qquad [34]$$

where ϵ_{mx}' is the membrane strain variation in the x-direction,

ϵ_{my}' is the membrane strain variation in the y-direction,

χ_x' is the change in curvature in the x-direction, and

Z is the distance out from the middle surface of the shell.

The corresponding variations S_x' and S_y' must be obtained. A procedure similar to that presented by Stowell will be used. From Equation [28]

$$S_x = E_s\,\epsilon_x$$

therefore

$$S_x' = E_s\ \epsilon_x' - \frac{\epsilon_x}{e_i}\left(\frac{\sigma_i}{e_i} - \frac{d\sigma_i}{de_i}\right)e_i' \qquad [35]$$

The variation of the strain intensity expressed in Equation [32] can be shown to be

$$e_i' = \frac{1}{2(1-\mu^2)\sigma_i}\Bigg\{\Big[(2-\mu)\,\sigma_x + (2\mu-1)\,\sigma_y\Big]\,\epsilon_x'$$

$$+\Big[(2\mu-1)\sigma_x + (2-\mu)\sigma_y\Big]\epsilon_y' + \Big[2(2-\mu)\,\sigma_x\sigma_y$$

$$-(1-2\mu)(\sigma_x^2 + \sigma_y^2)\Big]\frac{e_i}{\sigma_i}\mu'\Bigg\} \qquad [36]$$

Gerard and Wildhorn[23] have shown that for isotropic, plastically incompressible solids the following expression for Poisson's ratio in the inelastic region is applicable:

$$\mu = \frac{1}{2} - \left(\frac{1}{2} - \mu_e\right)\frac{E}{E_s} \qquad [37]$$

where μ_e is the value of Poisson's ratio in the *elastic* region. The first variation of Equation [37] is

$$\mu' = \frac{\left(\frac{1}{2} - \mu_e\right)}{E e_i}(E_s - E_t)\,e_i' \qquad [38]$$

Substitution of Equations [33], [34], and [38] into Equation [36] gives

$$e_i' = \frac{1}{2(1-\mu^2)H\sigma_i}\Bigg\{\Big[(2-\mu)\,\sigma_x + (2\mu-1)\,\sigma_y\Big]\epsilon_{mx}' + \Big[(2\mu-1)\,\sigma_x$$

$$+(2-\mu)\Big]\epsilon_{my}' - \Big[(2-\mu)\,\sigma_x + (2\mu-1)\,\sigma_y\Big]Z\,\chi_x'\Bigg\} \qquad [39]$$

where

$$H = \frac{\left(1 - \frac{E_t}{E_s}\right)}{4(1-\mu^2)\,\sigma_i^2}\Bigg\{\Big[(2-\mu)\,\sigma_x - (1-2\mu)\,\sigma_y\Big]^2 - 3(1-\mu^2)\,\sigma_x^2\Bigg\} \qquad [40]$$

Substitution of Equation [39] into Equation [35] gives

$$S_x' = E_s \epsilon_x' - \frac{\epsilon_x\left(\dfrac{\sigma_i}{e_i} - \dfrac{d\sigma_i}{de_i}\right)}{2(1-\mu^2)H\sigma_i e_i}\left\{\left[(2-\mu)\,\sigma_x + (2\mu-1)\,\sigma_y\right]\epsilon_{mx}' \right.$$

$$\left. + \left[(2\mu-1)\,\sigma_x + (2-\mu)\,\sigma_y\right]\epsilon_{my}' - \left[(2-\mu)\sigma_x + (2\mu-1)\sigma_y\right] Z\chi_x'\right\} \qquad [41]$$

Let the coordinate for which $e_i' = 0$ be $Z = Z_0$. The expression for Z_0 is obtained from Equation [39]:

$$Z_0 = \frac{\left[(2-\mu)\sigma_x + (2\mu-1)\sigma_y\right]\epsilon_{mx}' + \left[(2\mu-1)\sigma_x + (2-\mu)\sigma_y\right]\epsilon_{my}'}{\left[(2-\mu)\sigma_x + (2\mu-1)\sigma_y\right]\chi_x'} \qquad [42]$$

Substituting Equation [42] into Equation [41] and recognizing $\dfrac{\sigma_i}{e_i}$ as E_s and $\dfrac{d\sigma_i}{de_i}$ as the tangent modulus E_t, we have

$$S_x' = E_s\,(\epsilon_{mx}' - Z\chi_x') + \frac{\epsilon_x(E_s - E_t)}{2(1-\mu^2)H\sigma_i e_i}\left[(2-\mu)\sigma_x + (2\mu-1)\sigma_y\right][Z - Z_0]\chi_x' \qquad [43]$$

Similarly, it may be shown that

$$S_y' = E_s\,\epsilon_{my}' + \frac{\epsilon_y(E_s - E_t)}{2(1-\mu^2)\,H\sigma_i e_i}\left[(2-\mu)\sigma_x + (2\mu-1)\sigma_y\right][Z - Z_0]\,\chi_x'. \qquad [44]$$

The variations of the stresses σ_x' and σ_y' can be found using S_x' and S_y'. It can easily be shown that

$$\sigma_x = \frac{S_x + \mu S_y}{1-\mu^2} \qquad [45]$$

Taking the variation of σ_x as given by Equation [45] and treating Poisson's ratio μ as a variable, we have

$$\sigma_x' = \frac{S_x' + \mu S_y'}{1-\mu^2} + \frac{\left[(1+\mu^2) S_y + 2\mu S_x\right]\mu'}{(1-\mu^2)^2} \qquad [46]$$

Substituting Equations [43], [44], [38], and [39] into Equation [46] gives the expression for σ_x' in terms of the strain variations:

$$\sigma_x' = \frac{1}{1-\mu^2}\Bigg\{E_s(\epsilon_{mx}' - Z\chi_x' + \mu\epsilon_{my}') + \frac{(E_s - E_t)}{2(1-\mu^2)H\sigma_i e_i}\Big[(2-\mu)\sigma_x + (2\mu-1)\sigma_y\Big]$$

$$\Big[(Z - Z_0)(\epsilon_x + \mu\epsilon_y)\chi_x'\Big] + \frac{(\sigma_y + \mu\sigma_x)}{2(1-\mu^2)H\sigma_i e_i}\,\frac{\frac{1}{2} - \mu_e}{E}(E_s - E_t)\Bigg\{\Big[(2-\mu)\sigma_x$$

$$+ (2\mu-1)\sigma_y\Big](\epsilon_{mx}' - Z\chi_x') + \Big[(2\mu-1)\sigma_x + (2-\mu)\sigma_y\Big]\epsilon_{my}'\Bigg\}\Bigg\} \qquad [47]$$

Similarly, the variation of the circumferential stress is

$$\sigma_y' = \frac{1}{1-\mu^2}\Bigg\{E_s(\epsilon_{my}' + \mu\epsilon_{mx}') + \frac{E_s - E_t}{2(1-\mu^2)H\sigma_i e_i}\Big[(2-\mu)\sigma_x + (2\mu-1)\sigma_y\Big]\Big[(Z - Z_0)(\epsilon_y + \mu\epsilon_x)\chi_x'\Big]$$

$$+ \frac{(\sigma_y + \mu\sigma_x)}{2(1-\mu^2)H\sigma_i e_i}\Bigg\{\Big[(2\mu-1)\sigma_x + (2-\mu)\sigma_y\Big](\epsilon_{mx}' + \epsilon_{my}') - \Big[(2-\mu)\sigma_x + (2\mu-1)\sigma_y\Big]Z\chi_x'\Bigg\}\Bigg\}$$

[48]

The variation in the longitudinal moment is

$$M_x' = \int_{-h/2}^{h/2} \sigma_x' \, Z \, dZ \qquad [49]$$

Substituting Equation [47] into Equation [49], integrating, using Equation [42] for Z_0, and recognizing that $\chi_x' = \frac{d^2 w'}{dx^2}$, the second derivative of the variation of radial displacement w', we obtain

$$M_x' = - D_p \, A_1 \frac{d^2 w'}{dx^2} \tag{50}$$

where

$$D_p = \frac{E_s \, h^3}{12(1 - \mu^2)} \tag{51}$$

and

$$A_1 = 1 - \frac{\left(1 - \dfrac{E_t}{E_s}\right)}{4(1 - \mu^2) H \sigma_i^2} \left[(2 - \mu)\sigma_x - (1 - 2\mu)\sigma_y\right]^2 \tag{52}$$

The variation in the circumferential force is

$$N_y' = \int_{-h/2}^{h/2} \sigma_y' \, dZ \tag{53}$$

Substituting Equation [48] into Equation [53], integrating, and then using Equation [42] for Z_0, we get

$$N_y' = B_p \left[\mu A_{12} \frac{du'}{dx} + A_2 \frac{w'}{R}\right] \tag{54}$$

where

$$B_p = \frac{E_s h}{1 - \mu^2} \tag{55}$$

$$A_2 = 1 \frac{\left(1 - \dfrac{E_t}{E_s}\right)}{4(1 - \mu^2) H \sigma_i^2} \left[(1 - 2\mu)\sigma_x - (2 - \mu)\sigma_y\right]^2 \tag{56}$$

$$A_{12} = 1 + \frac{\left(1 - \dfrac{E_t}{E_s}\right)}{4\mu(1 - \mu^2) H \sigma_i^2} \left[(2 - \mu)\sigma_x - (1 - 2\mu)\sigma_y\right]\left[(1 - 2\mu)\sigma_x - (2 - \mu)\sigma_y\right] \tag{57}$$

and after recognizing that

$$\epsilon'_{mx} = \frac{du'}{dx}$$

$$\epsilon'_{my} = \frac{w'}{R}$$

Similarly,

$$N'_x = B_p \left[A_1 \frac{du'}{dx} + \mu A_{12} \frac{w'}{R} \right] \qquad [58]$$

The prebuckling equilibrium expression for radial forces acting on a circular cylinder undergoing axisymmetric deformations is

$$-\frac{d^2M_x}{dx^2} + \frac{N_y}{R} + N_x \frac{d^2w}{dx^2} - p = 0 \qquad [59]$$

To obtain the equilibrium expression during buckling, the variations of M_x, N_y, N_x, $\frac{d^2w}{dx^2}$, and p are added to their respective prebuckling values, and the sums, such as $(M_x + M'_x)$, are substituted into Equation [59] which then is subtracted. The following equilibrium expression for *buckling* results:

$$-\frac{d^2M'_x}{dx^2} + \frac{N'_y}{R} + N_x \frac{d^2w'}{dx^2} + N'_x \frac{d^2w}{dx^2} + N'_x \frac{d^2w'}{dx^2} - p' = 0 \qquad [60]$$

But

$$N'_x = \frac{R}{2} p' \qquad [61]$$

Thus, from Equation [58]

$$\frac{R}{2} p' = B_p \left[A_1 \frac{du'}{dx} + \mu A_{12} \frac{w'}{R} \right]$$

or

$$\frac{du'}{dx} = \frac{1}{A_1} \left[\frac{R}{2B_p} p' - \mu A_{12} \frac{w'}{R} \right] \qquad [62]$$

Equations [61], [62], [50], and [54] are substituted into Equation [60] to obtain

$$D_p A_1 \frac{d^4w'}{dx^4} + \frac{B_p}{R}\left(A_2 - \mu \frac{A_{12}^2}{A_1}\right) w' + N_x \frac{d^2w'}{dx^2} - \left[1 - \mu \frac{A_{12}}{A_1} - \frac{R}{2}\left(\frac{d^2w}{dx^2} + \frac{d^2w'}{dx^2}\right) p' = 0 \quad [63]$$

During the buckling process the variation in pressure p' is very small compared with N_x and with variations in deflection w' or curvature $\dfrac{d^2w'}{dx^2}$. Consequently, p is assumed to be zero. Then Equation [63]

$$D_p A_1 \frac{d^4w'}{dx^4} + \frac{B_p}{R}\left(A_2 - \mu^2 \frac{A_{12}^2}{A_1}\right) w' + N_x \frac{d^2w'}{dx^2} = 0 \quad [64]$$

Substituting Equations [51] and [55] into Equation [64], recognizing that $N_x = \dfrac{pR}{2}$, and rearranging terms, we have:

$$A_1 \frac{d^4w'}{dx^4} + 6(1-\mu^2)\frac{pR}{E_s h^3}\frac{d^2w'}{dx^2} + \frac{12}{R^2h^2}\left[A_2 - \mu \frac{A_{12}^2}{A_1}\right] w' = 0 \quad [65]$$

LONG CYLINDERS

As in the elastic solution let us first consider a long cylinder under hydrostatic loading. The definition of long will be determined from the analysis. A solution of Equation [64] can be written

$$w' = w_m' \sin \frac{n\pi x}{L} \quad [66]$$

The deflection function given by Equation [66] is based on the assumption that simple support exists at the stiffeners bounding the shell during buckling. When Equation [66] and its appropriate derivatives are substituted into Equation [65], the following expression in terms of n results:

$$p_{cr} = \frac{2}{\sqrt{3(1-\mu^2)}} E_s \left(\frac{h}{R}\right)^2 C\left[\left(\frac{\alpha_p L}{\pi n}\right)^2 + \frac{1}{4}\left(\frac{\pi n}{\alpha_p L}\right)^2\right] \quad [67]$$

where

$$C = \sqrt{\frac{A_1 A_2 - \mu^2 A_{12}^2}{1-\mu^2}} \quad [68]$$

and

$$\alpha_p = \sqrt[4]{\frac{3\left(\frac{A_2}{A_1} - \mu^2 \frac{A_{12}^2}{A_1^2}\right)}{R^2 h^2}} \qquad [69]$$

In a manner similar to that described for *elastic* buckling it can be shown that the number of half waves of buckling changes from $n = 1$ to $n = 2$ when $\alpha_p L = \pi$. Thus, if a cylinder is such that $\alpha_p L$ is greater than π, it will be denoted as "long." Also, in a manner similar to that described for the elastic case, it can be shown that as n goes to infinity, which would be the case for infinitely long cylinders, the buckling pressure can be expressed as

$$p_{cr} = C \frac{2}{\sqrt{3(1-\mu^2)}} E_s \left(\frac{h}{R}\right)^2 \qquad [70]$$

Again, if the moduli E_s and E_t are set equal to Young's modulus E, Equation [70] degenerates to the elastic solution given by Equation [13].

LONG CYLINDERS UNDER AXIAL COMPRESSION

Equation [70] is general enough to apply to the case of loading a very long cylinder by end compression as well as by an enveloping hydrostatic pressure field. Thus, for end compression only, $\sigma_y = 0$, $\sigma_i = \sigma_x$, and the plasticity coefficients reduce to

$$A_1 = 1 - \frac{\left(1 - \frac{E_t}{E_s}\right)}{4(1-\mu^2)H}(2-\mu)^2$$

$$A_2 = 1 - \frac{\left(1 - \frac{E_t}{E_s}\right)}{4(1-\mu^2)H}(1-2\mu)^2$$

$$A_{12} = 1 + \frac{\left(1 - \frac{E_t}{E_s}\right)}{4\mu(1-\mu^2)H}(1-2\mu)^2$$

$$H = 1 + \frac{\left(1 - \frac{E_t}{E_s}\right)}{4(1-\mu^2)}(1-2\mu)^2 \qquad [71]$$

When Equations [71] are substituted into Equation [70], the expression for the buckling pressure on the ends of the cylinder results:

$$p_{cr} = C_a \frac{2}{\sqrt{3(1-\mu^2)}} \sqrt{\frac{E_t}{E_s}}\, E_s \left(\frac{h}{R}\right)^2 \qquad [72]$$

Since

$$\sigma_{cr} = \frac{p_{cr} R}{2h}$$

then

$$\sigma_{cr} = C_a \frac{1}{\sqrt{3(1-\mu^2)}} \sqrt{\frac{E_t}{E_s}}\, E_s \left(\frac{h}{R}\right) \qquad [73]$$

where

$$C_a = \frac{1}{\sqrt{1 + \left(1 - \frac{E_t}{E_s}\right)\frac{(1-2\mu)^2}{4(1-\mu^2)}}} \qquad [74]$$

For

$$\mu = \mu_e = \frac{1}{2}, \quad C_a = 1$$

and

$$\sigma_{cr} = \frac{2}{3}\sqrt{\frac{E_t}{E_s}}\, E_s \left(\frac{h}{R}\right) \qquad [75]$$

Equation [75] is identical to that derived by Gerard[19] for $\mu = \frac{1}{2}$. Gerard, however, recommended the use of the following expression for values of μ other than $\frac{1}{2}$:

$$\sigma_{cr} = \frac{1}{\sqrt{3(1-\mu^2)}} \sqrt{\frac{E_t}{E_s}}\, E_s \left(\frac{h}{R}\right) \qquad [76]$$

It should be noticed that Equation [73] differs from Equation [76] only in the coefficient C_a. For values of μ between 0.3 and $\frac{1}{2}$, C_a is almost equal to one, and Equation [73] gives values of σ_{cr} which are less than 2 percent smaller than those given by Equation [76]. Thus Gerard's solution given in Equation [76] is a very good approximation to the more rigorous solution represented by Equation [73].

SHORT CYLINDERS

If $\alpha_p L$ is less than π, the cylinder will be denoted as "short" as in the elastic analysis. If the half-wave length of buckling is taken equal to the stiffener spacing, Equation [67] becomes, for $n = 1$,

$$p_{cr} = \frac{2}{\sqrt{3(1-\mu^2)}} E_s \left(\frac{h}{R}\right)^2 C \left[\left(\frac{\alpha_p L}{\pi}\right)^2 + \frac{1}{4}\left(\frac{\pi}{\alpha_p L}\right)^2\right] \qquad [77]$$

Equation [77] can be rewritten in another form, which may be more convenient to use, as follows:

$$p_{cr} = \frac{2}{(1-\mu^2)} \left(A_2 - \mu^2 \frac{A_{12}^2}{A_1}\right) c^2 E_s \left(\frac{h}{R}\right)^2 + \frac{1}{6(1-\mu^2)} \frac{A_1}{c^2} E_s \left(\frac{h}{R}\right)^2 \qquad [78]$$

The parameters and coefficients of Equations [77] and [78] are:

$$c^2 = \frac{L^2}{\pi^2 R h} \qquad [20]$$

$$\alpha_p = \sqrt[4]{\frac{3\left(\frac{A_2}{A_1} - \mu^2 \frac{A_{12}^2}{A_1^2}\right)}{R^2 h^2}} \qquad [69]$$

$$\mu = \frac{1}{2} - \left(\frac{1}{2} - \mu_e\right)\frac{E_s}{E} \qquad [37]$$

$$A_1 = 1 - \frac{\left(1 - \frac{E_t}{E_s}\right)}{4(1-\mu^2)K^2 H}\left[(2-\mu) - (1-2\mu)\,k\right]^2 \qquad [79]^*$$

$$A_2 = 1 - \frac{\left(1 - \frac{E_t}{E_s}\right)}{4(1-\mu^2)K^2 H}\left[(1-2\mu) - (2-\mu)k\right]^2 \qquad [80]$$

*Equations [79] through [82] are derived by substituting Equations [83] and [84] into Equations [40], [52], [56], and [57].

$$A_{12} = 1 + \frac{\left(1 - \frac{E_t}{E_s}\right)}{4\mu(1 - \mu^2) K^2 H} \left[(2 - u) - (1 - 2\mu) k\right]\left[(1 - 2\mu) - (2 - \mu) k\right] \quad [81]$$

$$H = 1 + \frac{\left(1 - \frac{E_t}{E_s}\right)}{4(1 - \mu^2) K^2} \left\{\left[(2 - \mu) - (1 - 2\mu) k\right]^2 - 3\,(1 - \mu^2)\right\} \quad [82]$$

$$k = \frac{\sigma_y}{\sigma_x} \quad [83]$$

$$K^2 = 1 - k + k^2 \quad [84]$$

PROCEDURE FOR COMPUTING PLASTIC BUCKLING PRESSURE

The application of the plastic buckling equation requires some practical judgment. In the buckling equation the moduli E_s and E_t are considered constant at all points throughout the shell, whereas these moduli actually vary along the length as well as through the thickness of the shell. An infinite number of buckling pressures p_{cr} can be obtained by substituting different values of the moduli into Equation [78], but the only p_{cr} that is a solution is that which satisfies both the buckling equation, Equation [78], and the prebuckling equilibrium condition of the shell.

At present there appears to be no rigorous theoretical solution for computing prebuckling stresses in the plastic range for the problem considered. A procedure will be followed analogous to that used in the past for elastic, perfectly plastic materials. For this case the membrane stresses at collapse are computed by using an elastic analysis, such as that of Salerno and Pulos. Furthermore, the stresses considered are those at midbay which is considered the critical point in the shell. Similarly, for a strain-hardening material the membrane stresses can be computed at midbay by the elastic analyses of Reference 1. Then σ_i can be computed for different values of pressure p_{S-P} . Consequently, a plot of p_{S-P} against σ_i is obtained.

From the stress-strain curve σ_i, E_s, and E_t can be obtained. The secant modulus is computed as simply total stress divided by total strain, and E_t is the slope of the stress-strain curve measured at a particular σ_i. For different σ_i, p_{cr} can be computed by substituting the values of E_s and E_t into Equation [78]. A plot of p_{cr} against σ_i can be obtained. The pertinent buckling pressure is obtained where the $p_{S-P} - \sigma_i$ and $p_{cr} - \sigma_i$ curves intersect.

For an elastic, perfectly plastic material $E_t = 0$ and E_s varies from a value of E to 0 for constant stress equal to the yield point σ_{YP}. Thus, when the plastic buckling equation is used, the plot of p_{cr} against σ_i is a vertical line at the abscissa σ_{YP}. The intersection of the $p_{S-P} - \sigma_i$ curve with the $p_{cr} - \sigma_i$ curves gives a buckling pressure which is equal to that obtained by satisfying the Hencky-Von Mises yield condition with the membrane stresses at midbay. This membrane-stress procedure has been shown[13] to give collapse pressures that agreed fairly well with those obtained in experiments with cylinders composed of steels with a plateau-type stress-strain curve. However, this method generally gives pressures above the experimental pressures since it does not adequately account for the plastic bending of the shell. Nevertheless, within certain limits discussed in Reference 12, this membrane-stress method is a good approximation. Thus the procedure for computing the plastic buckling pressure for a strain-hardening material degenerates to a known procedure for an elastic, perfectly plastic material.

COMPARISON OF THEORY WITH EXPERIMENT

At the present time the amount of experimental evidence which can be offered in support of the theory is very limited. Only three tests are presented; many more would be required to prove conclusively the validity of the theory.

Test results are presented for two cylinders of strain-hardening steel, Models 1 and 2, and a third cylinder of strain-hardening aluminum, Model 3. Nondimensionalized stress-strain curves of the materials are shown in Figure 3 which includes tables of tangent and

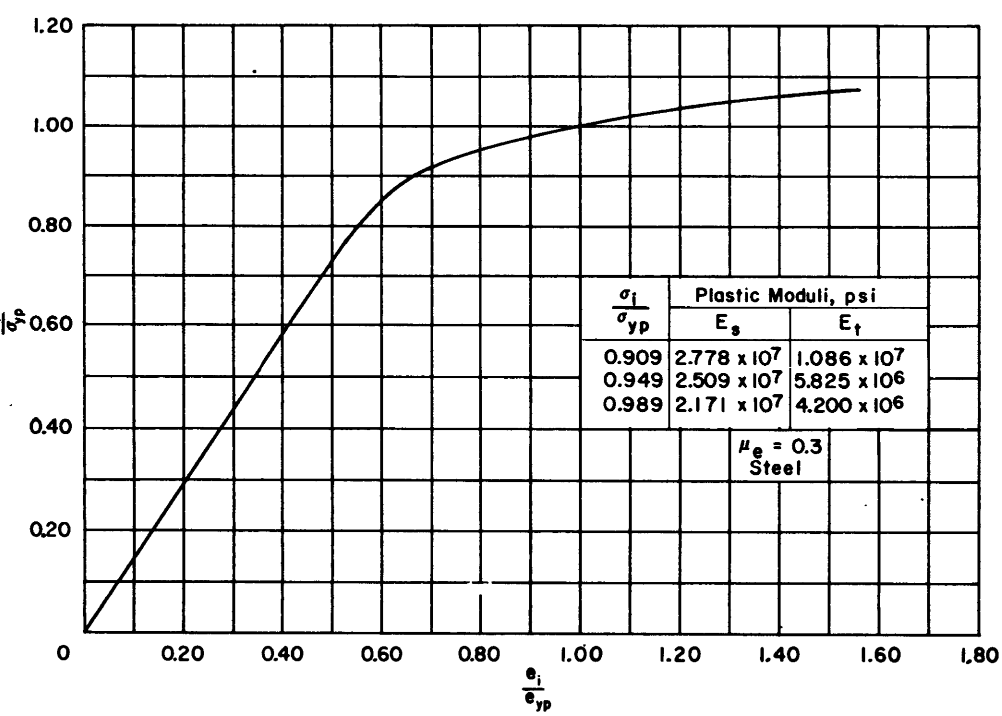


Figure 3a – Model 1

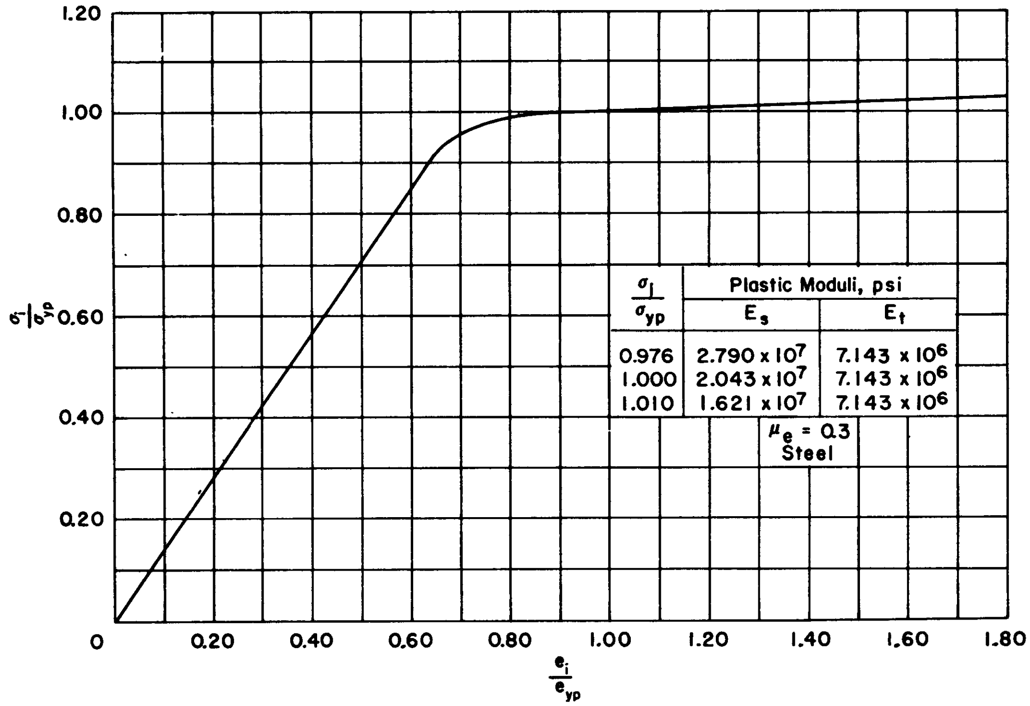


Figure 3b – Model 2

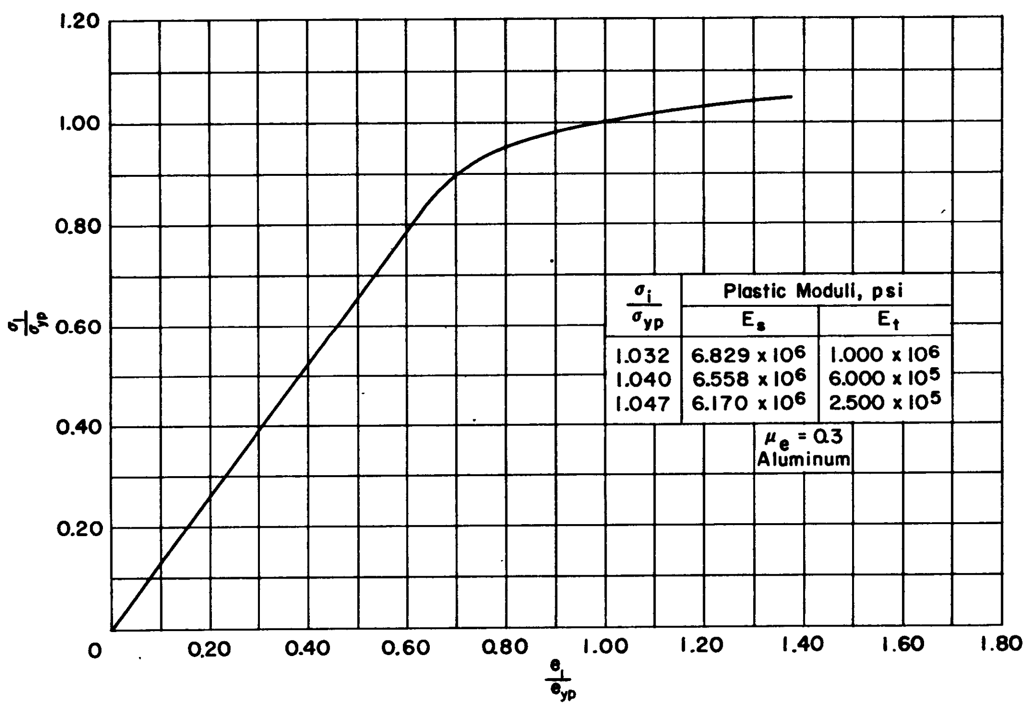
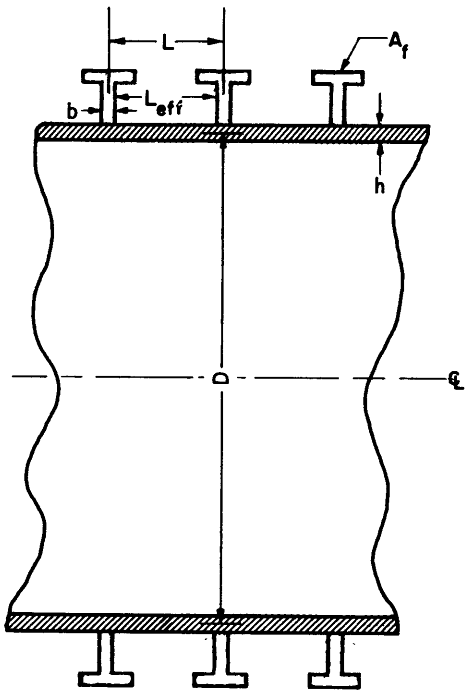


Figure 3c – Model 3

Figure 3 – Nondimensionalized Stress-Strain Curves



Geometric Parameter	Model 1	Model 2	Model 3
$\frac{\text{Shell Thickness}}{\text{Mean Diameter}} = \frac{h}{D}$	0.00629	0.00694	0.01389
$\frac{\text{Effective Frame Spacing}}{\text{Mean Diameter}} = \frac{L_{eff}}{D}$	0.0985	0.0783	0.1187
$\frac{\text{Frame Faying Width}}{\text{Ctr. to Ctr. Frame Spacing}} = \frac{b}{L}$	0.0250	0.0313	0.0601
$\frac{\text{Area of Frame}}{\text{Area of Shell}} = \frac{A_f}{hL}$	0.579	0.435	0.373

Figure 4 – Geometry of Models

secant moduli used to calculate theoretical buckling pressures and also to evaluate the *criterion* of Budiansky for ascertaining the validity of the deformation theory of plasticity. This criterion, determinations of which is presented in detail in the Appendix, was satisfied for the three cylinders.

The stiffened cylinders were made from thick forged tubes. The tubes were machined to form ring frames of T-section integrally attached to a thin shell. The geometries of the cylinders are shown in Figure 4.

The cylinders were tested under external hydrostatic pressure in a 20-in. diameter pressure tank at the David Taylor Model Basin. The general appearance of a collapsed model is shown in Figure 5.

The theoretical results are shown in nondimensional form in Figure 6. The intersection of the curve representing the prebuckling equilibrium state of the cylinder as determined by the analysis of Salerno and Pulos with the curve satisfying the plastic axisymmetric buckling equation is clearly shown as the theoretical buckling pressure. The membrane stress method is simply the calculation of that pressure at which the membrane stresses satisfy the Hencky-Von Mises criterion for the 0.2-percent-offset yield point of the material. The experimental results are also indicated.

The theoretical results compare very well with the experimental results. For the axisymmetric plastic buckling theory the calculated values were 2 percent higher than the experimental values for Model 1, 1 percent lower for Model 2, and 6 percent higher for Model 3.

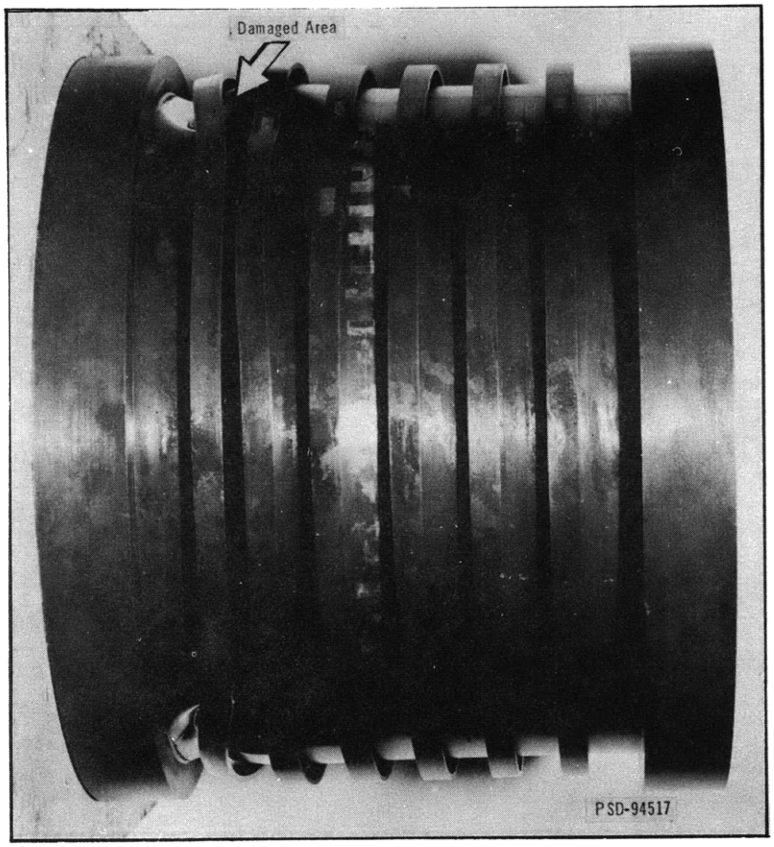


Figure 5 – Collapse of Model 1

For the membrane-stress method the calculated values were 4 percent higher than the experimental values for Model 1, 1 percent lower for Model 2, and 1 percent higher for Model 3. It should be noted that the results by both the buckling equation and the membrane-stress method agree very closely with one another for Model 2. This agreement is so close because the stress-strain curve for Model 2 is very nearly a plateau-type curve. The use of the elastic Salerno and Pulos analysis to compute the stress intensity in the *inelastic* range, while not strictly valid, apparently gives good agreement between theory and experiment, at least for these three models.

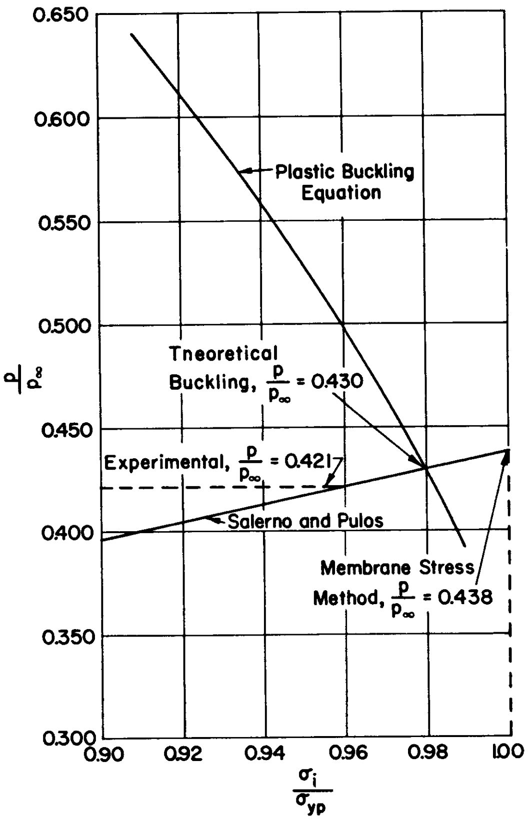


Figure 6a – Model 1

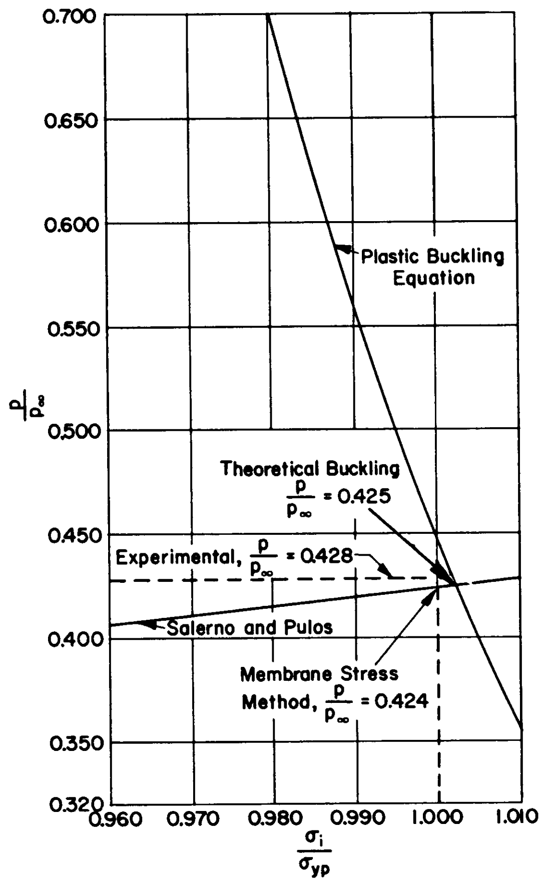


Figure 6b – Model 2

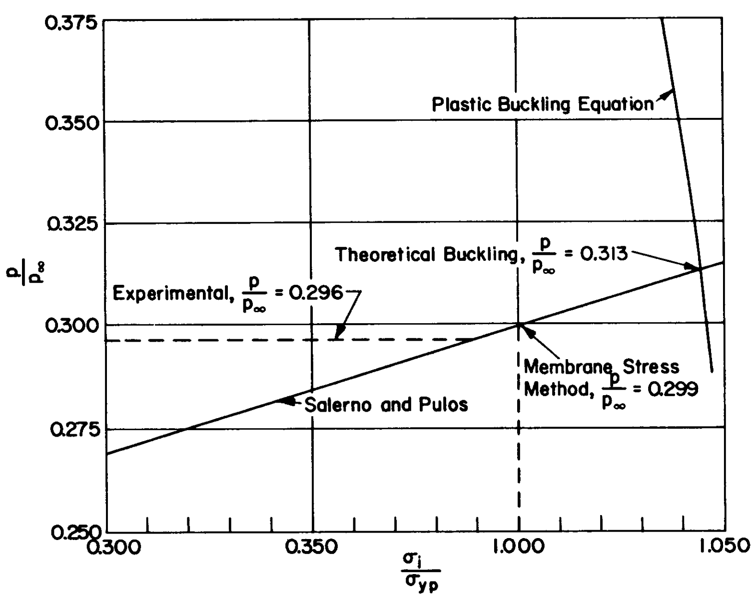


Figure 6c – Model 3

Figure 6 – Graphical Determination of Buckling Pressure

SUMMARY

The theoretical and experimental results are as follows:

1. The differential equation of equilibrium for a cylindrical shell buckling *axisymmetrically* under hydrostatic pressure has been derived for the plastic range and for *variations* in *Poisson's ratio.*

2. Expressions for the plastic buckling pressures for "long" and "short" cylinders have been derived. These expressions degenerate to those for the elastic buckling pressures.

3. The criterion of length of shell for "long" and "short" cylinders is presented for both elastic and plastic buckling, below which the shell can buckle in only one half wave.

4. Limited experimental evidence is in agreement with the theory, but more experimental evidence is needed before confirmation is conclusive.

5. A solution for the plastic buckling of a cylinder under end compression has been obtained as a special case of the problem considering hydrostatic loading. The solution degenerates to that previously obtained by Gerard for a Poisson's ratio of ½ but represents a more rigorous solution in terms of variable Poisson's ratio than that of Gerard.

RECOMMENDATIONS

A systematic series of stiffened cylinders designed to fail by axisymmetric plastic buckling under external hydrostatic pressure should be tested. In particular, the following effects should be studied:

1. The influence of strain-hardening on buckling pressure should be investigated.

2. The influence of geometry should be explored. The criterion between "long" and "short" cylinders should be established experimentally. In addition, the geometries delimiting axisymmetric and asymmetric (multilobe) buckling should be investigated.

3. The influence of residual stresses and imperfections on plastic axisymmetric buckling should be investigated.

ACKNOWLEDGMENTS

The author wishes to express his appreciation to Mr. M.A. Krenzke for the design of the test cylinders and direction of the test program. Mr. A.R. Willner procured the strain-hardening materials. Messrs R.F. Keefe and T.J. Kiernan were responsible for the instrumentation and testing of the cylinders. Mr. R.D. Short provided valuable service in the development and review of the theoretical analysis. In addition to personnel at the Taylo Model Basin, the author is also indebted to Professor S.R. Bodner of Brown University for his valuable comments.

APPENDIX

DETERMINATION OF CRITERION FOR VALIDITY OF DEFORMATION THEORY OF PLASTICITY

By Reference 22 the criterion to be satisfied for ascertaining the validity of the use of deformation theory is

$$\cos\psi \geq \left[\frac{1}{1+\dfrac{\dfrac{1}{E_t}-\dfrac{1}{E}}{\dfrac{1}{E_s}-\dfrac{1}{E}}}\right]^{1/2} \quad [85]$$

where

$$\cos\psi = \frac{S_{ij}\,\dot{S}_{ij}}{[S_{kl}S_{kl}]\,[\dot{S}_{pq}\dot{S}_{pq}]^{1/2}} \quad [86]$$

Equation [86] is expressed in terms of the stress deviation tensor

$$S_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{kk}\,\delta_{ij} \quad [87]$$

where δ_{ij} is the Kronecker delta. For biaxial compression

$$\left.\begin{aligned} \sigma_{33} &= \sigma_z = 0 \\ \sigma_{13} &= \tau_{xz} = 0 \\ \sigma_{23} &= \tau_{yz} = 0 \end{aligned}\right\} \quad [88]$$

Therefore

$$S_{13}\,S_{23} = 0 \quad [89]$$

and

$$\begin{aligned} S_{11} &= \sigma_{11} - \frac{1}{3}(\sigma_{11}+\sigma_{22}) = \sigma_x - \frac{1}{3}(\sigma_x+\sigma_y) \\ S_{22} &= \sigma_{22} - \frac{1}{3}(\sigma_{11}+\sigma_{22}) = \sigma_y - \frac{1}{3}(\sigma_x+\sigma_y) \quad [90] \\ S_{33} &= 0 - \frac{1}{3}(\sigma_{11}+\sigma_{22}) = -\frac{1}{3}(\sigma_x+\sigma_y) \\ S_{12} &= \sigma_{12} = \tau \end{aligned}$$

When the components of the tensor S_{ij} as expressed in Equations [90] are substituted into Equation [86] and the appropriate derivatives with respect to time are performed,

$$\cos \gamma = \frac{\dot{\sigma}_i}{\left\{\left[\frac{1}{2}(\dot{\sigma}_y + \dot{\sigma}_x)\right]^2 + \left[\frac{\sqrt{3}}{2}(\dot{\sigma}_y - \dot{\sigma}_x)\right]^2\right\}^{1/2}} \qquad [91]$$

Let

$$\xi = \frac{1}{2}(\sigma_y + \sigma_x)$$
$$\eta = \frac{\sqrt{3}}{2}(\sigma_y - \sigma_x) \qquad [92]$$

A plot of ξ versus η is shown in Figure 7. Equation [7] indicates that ψ is the angle between the radius vector σ_i, which has the value of stress intensity given by Equation [22], to any point and the tangent to the loading path ($\xi - \eta$ diagram) at that point. The angle ψ is clearly shown together with the angles θ and ϕ denoting the slopes of the radius vector σ_i and the tangent to the loading path respectively.

From Figure 7 it can be seen that

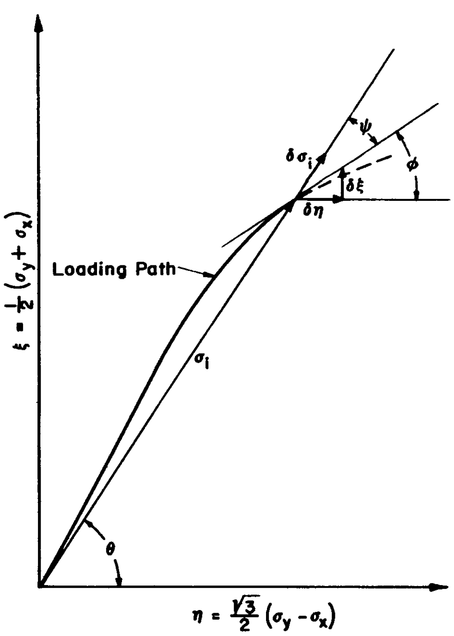


Figure 7 – Loading Path Diagram for Determination of Angle ψ

$$\psi = \theta - \phi \qquad [93]$$

where

$$\theta = \arctan \frac{\xi}{\eta} \qquad [94]$$

and

$$\phi = \arctan \frac{\delta\xi}{\delta\eta} \qquad [95]$$

After substituting Equations [92] into Equation [94] and using the relation $k = \sigma_y / \sigma_x$, we obtain

$$\theta = \frac{1}{\sqrt{3}}\left(\frac{k+1}{k-1}\right) \qquad [96]$$

The variations of Equations [92] are:

$$\delta\xi = \frac{h}{2}(N_y' + N_x') \qquad [97]$$

$$\delta\eta = \frac{\sqrt{3}}{2}(N_y' - N_x') \qquad [98]$$

Previously, it was found that

$$N_x' = B_p \left[A_1 \frac{du'}{dx} + \mu A_{12} \frac{w'}{R} \right] \tag{58}$$

and

$$N_y' = B_p \left[\mu A_{12} \frac{du'}{dx} + A_2 \frac{w'}{R} \right] \tag{54}$$

By symmetry, $\frac{du'}{dx'}$ must be zero at midbay; therefore, at midbay

$$\begin{aligned} N_x' &= \mu B_p \frac{A_{12}}{R} w_m' \\ N_y' &= B_p \frac{A_2}{R} w_m' \end{aligned} \tag{99}$$

where w_m' is the variation of the deflection at midbay.

After substituting Equations [99] into Equations [97] and [98], we obtain

$$\begin{aligned} \delta\xi &= \frac{h B_p}{2R} (A_2 + \mu A_{12}) w_m' \\ \delta\eta &= \frac{\sqrt{3}\, h B_p}{2R} (A_2 - \mu A_{12}) w_m' \end{aligned} \tag{100}$$

The slope of the tangent to the loading path is determined by substituting Equations [100] into Equation [95], thus

$$\phi = \arctan \frac{1}{\sqrt{3}} \frac{A_2 + \mu A_{12}}{A_2 - \mu A_{12}} \tag{101}$$

For the models tested, the angles θ, ϕ, and ψ were evaluated at midbay and at the computed buckling pressure. The angles θ and ϕ were determined from Equations [96] and [101]. Both equations depend on k, which was determined by the elastic theory of Salerno and Pulos at midbay and at the computed buckling pressure. As stated previously, an elastic theory is admittedly approximate but is used in lieu of a more rigorous plastic stress analysis. However, it is believed that the ratio of membrane stresses $k = \frac{\sigma_y}{\sigma_x}$ will not change too much after yielding begins. Indeed, for very long cylinders, k always equals two for both the elastic and plastic ranges. The angle ψ was then obtained from Equation [93] and the criterion of Equation [85] was evaluated for the values of E_t and E_s and used to compute the angles θ and ϕ.

Budiansky's criterion can be evaluated only with the foregoing procedure at midbay. As a rule, however, a point at midbay is considered to be the most critical in failure by axisymmetric yielding.

REFERENCES

1. Salerno, V.L. and Pulos, J.G., "Stress Distribution in a Circular Cylindrical Shell under Hydrostatic Pressure Supported by Equally Spaced Circular Ring Frames," Polytechnic Institute of Brooklyn Aeronautical Laboratory Report 171-A (1951).

2. Timoshenko, S., "Theory of Elastic Stability," McGraw-Hill Book Company, Inc., New York and London, first edition (1936), p. 439.

3. Von Sanden, K. and Gunther, K., "The Strength of Cylindrical Shells, Stiffened by Frames and Bulkheads, under Uniform Pressure on All Sides," (Über das Festigkeitsproblem querversteifter Hohlzylinder unter allseitig gleichmässigem Aussendruck), Werft und Reederei, Vol. 1, Nos. 8, 9, 10 (1920), and Vol. 2, No. 17 (1921). See also David Taylor Model Basin Translation 38 (Mar 1952).

4. Wenk, E., Jr., et al, "Tests of the Yield Strength of Ring-Stiffened Cylindrical Shells, Models BR-2 and BR-2A ($\lambda = 0.4$), Subjected to Hydrostatic Pressure," David Taylor Model Basin Report C-440 (Feb 1954) CONFIDENTIAL.

5. Drucker, D.C., "Limit Analysis of Cylindrical Shells under Axially-Symmetric Loading," Proceedings First Midwest Conference on Solid Mechanics (1953), pp. 158-163.

6. Onat, E.T., "The Plastic Collapse of Cylindrical Shells under Axially Symmetrical Loading," Brown University Report DA-19-020-798/9 (1954).

7. Onat, E.T. and Prager, W., "Limits of Economy of Material in Shells," Ingenieur, No. 10, Technisch Wetenschappelijk Onderzoek 4 (1955).

8. Hodge, P.G., "The Rigid-Plastic Analysis of Symmetrically Loaded Cylindrical Shells," Journal of Applied Mechanics, Vol. 21 (1954), pp. 336-342.

9. Paul, B., "Carrying Capacity of Elastic-Plastic Shells with Various End Conditions under Hydrostatic Pressure," Polytechnic Institute of Brooklyn Aeronautical Laboratory Report 420 (Apr 1958).

10. Schischka, E., "The Plasticity of the Pressure Hulls of U-Boats," (Zur Plastizitat des Druckkorpers von U-Booten), Forschung bericht Nr. 37, Wien (Jan 1945).

11. Kempner, J. and Salerno, V.L., "Analysis of the Inelastic Behavior of Transversely Reinforced Cylindrical Shells under Hydrostatic Pressure," Polytechnic Institute of Brooklyn Aeronautical Laboratory Report 172 (1950).

12. Lunchick, M.E., "Yield Failure of Stiffened Cylinders under Hydrostatic Pressure," Proceedings Third U.S. National Congress of Applied Mechanics (Jun 1958). See also David Taylor Model Basin Report 1291 (Jan 1959).

13. Lunchick, M.E., "A Survey of Procedures for Calculating the Yield Strength of Stiffened Cylindrical Shells under External Hydrostatic Pressure," David Taylor Model Basin Report C-1052 (Sep 1959) CONFIDENTIAL.

14. Bijlaard, P.P., "Theory and Tests on the Plastic Stability of Plates and Shells," Journal of Aeronautical Sciences, Vol. 16, No. 9 (1949), pp. 529-541.

15. Handelman, G.H. and Prager, W., "Plastic Stability of a Rectangular Plate under Edge Thrusts," National Advisory Committee for Aeronautics Report 946 (1949).

16. Ilyushin, A.A., "The Elasto-Plastic Stability of Plates," National Advisory Committee for Aeronautics Technical Memorandum 1188 (1947).

17. Stowell, E.Z., "A Unified Theory of Plastic Buckling of Columns and Plates," National Advisory Committee for Aeronautics Report 898 (1948).

18. Bijlaard, P.P., "On the Plastic Stability of Thin Plates and Shells," Verh. Koninklijke Nederlandsche Akademie van Wetenschappen, Vol. L, No. 7, (Sep 1947), pp. 765-775.

19. Gerard, G., "Compressive and Torsional Buckling of Thin-Wall Cylinders in Yield Region," National Advisory Committee for Aeronautics Technical Note 3726 (Aug 1956).

20. Donnell, L.H., "Stability of Thin-Walled Tubes in Torsion," National Advisory Committee for Aeronautics Report 479 (1933).

21. Gerard, G., "Plastic Stability Theory of Thin Shells," Journal of Aeronautical Sciences, Vol. 24 No. 4 (Apr 1957).

22. Budiansky, B., "A Reassessment of Deformation Theories of Plasticity," American Society of Mechanical Engineers Paper 58-A-54, Journal of Applied Mechanics, Vol 26, Series E, No. 2 (Jun 1959).

23. Gerard, G. and Wildhorn, S., "A Study of Poisson's Ratio in the Yield Region," National Advisory Committee for Aeronautics Technical Note 2561 (Jan 1952).

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