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THE EFFECTS OF MACHINE AND FOUNDATION RESILIENCE AND OF WAVE PROPAGATION ON THE ISOLATION PROVIDED BY VIBRATION MOUNTS

by

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The Effects of Machine and Foundation Resilience and of Wave Propagation on the Isolation Provided by Vibration Mounts

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ABSTRACT

THE effects on the transmission of vibration through isolation mounts of machine and foundation resilience, and of wave propagation are investigated. The prediction of the effectiveness of mounts is discussed, and curves are presented for estimating their effectiveness under certain conditions.

A number of conclusions are drawn relevant to the problems of mount design and selection.

IN the simplest approach to the problem of estimating the effectiveness of a vibration mount[1], it is customary to assume that the machine or equipment to be isolated is a rigid mass M_M, that the mount is a massless mechanically paralleled spring and dashpot of stiffness K_m and resistance R_m, and that M_M, K_m, and R_m are constants independent of frequency.[2]

For the case of foundation-excited machine vibration,[3] one assumes that the vibratory foundation velocity is not effected by the machine, whether rigidly or resiliently attached.[4] The ratio of the vibratory velocity amplitude of the isolated machine to the vibratory velocity amplitude of the foundation is calculated; and this ratio, *transmissibility,* is used as a criterion for isolation prediction.

The complex transmissibility of a rigid machine, massless spring-dashpot mount system is given by:[5]

$$T = \frac{R_m - \dfrac{jK_m}{\omega}}{R_m + j\left(\omega M_M - \dfrac{K_m}{\omega}\right)} \tag{1}$$

where:

$j = \sqrt{-1}$

$\omega = 2\pi f$, the frequency of the exciting disturbance in radians per sec

The magnitude or modulus of T can be written:

$$|T| = \left\{ \frac{1 + \left[\dfrac{\omega}{\omega_o}\dfrac{\omega_o R_m}{K_m}\right]^2}{\left[\left(\dfrac{\omega}{\omega_o}\right)^2 - 1\right]^2 + \left(\dfrac{\omega}{\omega_o}\dfrac{\omega_o R_m}{K_m}\right)^2} \right\}^{\frac{1}{2}} \tag{2}$$

where:

$\omega_o = 2\pi f_o = \sqrt{\dfrac{K_m}{M_M}}$, the natural frequency of the machine-mount system in radians per sec

$\dfrac{\omega R_m}{K_m}$ = Loss factor of the mount at the circular frequency ω

Transmissibility modulus in db is plotted in Fig. 1 as a function of frequency ratio $\dfrac{\omega}{\omega_o}$ with $\dfrac{\omega_o R_m}{K_m}$ as a parameter.

$$|T|_{db} = 20 \log_{10}|T| \tag{3}$$

From Equation 2 and Fig. 1, one can conclude that:

1. A mount magnifies the vibratory velocity of the machine for frequency ratios $\dfrac{\omega}{\omega_o} < \sqrt{2}$.

2. A mount reduces the vibratory velocity of the machine for frequency ratios $\dfrac{\omega}{\omega_o} > \sqrt{2}$.

3. For frequency ratios greater than $\dfrac{\omega}{\omega_o} = \sqrt{2}$, the isolation a mount provides increases with increasing $\dfrac{\omega}{\omega_o}$; for $\dfrac{\omega_o R_m}{K_m} = 0$, the isolation increases in proportion to $\left(\dfrac{\omega}{\omega_o}\right)^2$.

4. Increasing the resistance in a mount will decrease its effectiveness for frequency ratios $\dfrac{\omega}{\omega_o} > \sqrt{2}$ and increase it for $\dfrac{\omega}{\omega_o} < \sqrt{2}$.

These conclusions and the assumptions made in deriving them are not always correct, however.

The assumptions that R_m and K_m are constants independent of frequency are wrong for many rub-

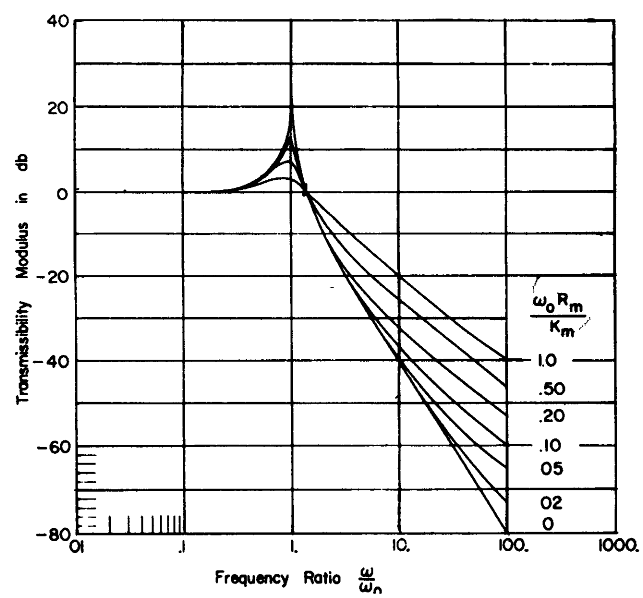

Fig. 1 – Transmissibility modulus in db versus frequency ratio, R_m and K_m are constants independent of frequency

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[1] This paper considers only steady state vibration problems in which all vibratory forces and velocities are colinear.

[2] Throughout the rest of the text, the word machine will be used to describe whatever equipment is to be isolated.

[3] The problem of reducing machine-excited foundation vibration has been discussed by the author in a parallel paper, "Theory of Sound Transmission in Isolation Mounts," presented in March, 1957, at the Office of Naval Research sponosred symposium, "A Decade of Basic and Applied Science in the Navy." It was published by ONR in October, 1957.

[4] This assumption is equivalent to assuming the foundation is infinitely stiff and massive.

[5] Pp. 176-186 of "Vibration and Shock Isolation," by Charles E. Crede. Pub. by John Wiley and Sons, New York, 1951.

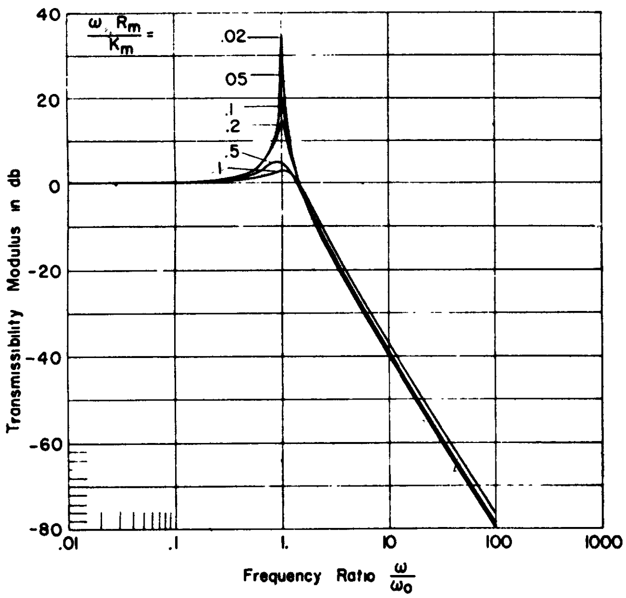

Fig. 2 – Transmissibility modulus in db versus frequency ratio, loss factor $\omega R_m/K_m$ is a constant independent of frequency

ber mounts.[6] At frequencies above a few hundred cps, machines, particularly large ones, do not act like rigid masses; foundation structures (for example, air frames or ships' hulls) are neither infinitely stiff nor massive; and wave effects occur in the mounts.

Isolation of Nonrigid Machines from Nonrigid Foundations

Since transmissibility, as defined by Equation 1, cannot be used for predicting the isolation a mount will provide when isolating a nonrigid machine from a nonrigid foundation, a new criterion must be selected.[7]

Such a criterion is effectiveness.[8] This quantity can be defined in terms of two hypothetical experiments illustrated in Fig. 3. In the first experiment, the machine is attached directly to the vibrating foundation and the resulting vibratory velocity amplitude $\dot{x}_{12}$ of the machine-foundation juncture is measured. In the second experiment, the machine is attached to the same point on the vibrating foundation by a resilient mount, and the vibratory velocity amplitude $\dot{x}_{22}$ of the machine-mount juncture is measured. The ratio of the first to the second velocity amplitude defines the effectiveness of the mount. In the notation of Fig. 3, the complex effectiveness of a mount is:

$$E = \frac{\dot{x}_{12}}{\dot{x}_{22}} \tag{4}$$

It is the magnitude of E which is of interest.[9] If it is greater than unity, the mount reduces the machine vibration; if it is less than unity, the machine vibration is increased.

Effectiveness of Massless Spring-Dashpot Mounts Isolating Nonrigid Machines from Nonrigid Foundations—The effectiveness of a mount can be calculated most easily by the application of mechanical network theory.[10]

Consider the first experiment illustrated in Fig. 3. By Norton's theorem:[11]

$$\dot{x}_{12} = \frac{Z_F}{Z_M + Z_F}\dot{x}_{12}{}' \tag{5}$$

where:

$\dot{x}_{12}$ = Vibratory velocity amplitude of the machine-foundation juncture when the machine is directly attached to the foundation

$\dot{x}_{12}{}'$ = Vibratory velocity amplitude of the foundation at the point at which the machine is to be attached, before the machine is attached

Z_F = Mechanical impedance of the foundation, looking into it at the point of attachment of the machine

Z_M = Mechanical impedance of the machine, looking into it at the point of attachment to the foundation.[12]

Now consider the second experiment as illustrated in Fig. 4.

By Norton's theorem and the rules for combining mechanical impedances:

$$\dot{x}_{23} = \dot{x}_{12}{}' \frac{Z_F}{\dfrac{Z_m Z_M}{Z_m + Z_M} + Z_F} \tag{7}$$

where:

Z_m = Mechanical impedance of the mount

Since:

$$\frac{\dot{x}_{22}}{\dot{x}_{23}} = \frac{Z_m}{Z_m + Z_M} \tag{8}$$

it follows that:

$$\dot{x}_{22} = \frac{Z_m}{Z_m + Z_M}\dot{x}_{23} = \frac{Z_m}{Z_m + Z_M}\frac{Z_F}{\dfrac{Z_m Z_M}{Z_m + Z_M} + Z_F}\dot{x}_{12}{}' \tag{9}$$

Consequently, the effectiveness E of the mount is

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[6] For rubber mounts, R_m usually decreases with frequency and K_m increases. For some mounts, the lost factor $\frac{\omega R_m}{K_m}$ is nearly constant independent of frequency. The assumption loss factor constant independent of frequency has been used with Equation 2 to plot Fig. 2, which shows that the transmissibility of mounts with constant loss factors is essentially independent of loss factor at frequency ratios $\frac{\omega}{\omega_o} > \sqrt{2}$, and that the loss factor of the mount is important only in that it determines the transmissibility at the natural frequency f_o of the machine-mount system. (See Additional References.)

[7] The words, machine and foundation, are used loosely to refer to whatever structures are attached at the machine and foundation ends of the mount. It is assumed that the metal components of the mount can be considered massless and infinitely stiff.

[8] Effectiveness in mechanical network theory corresponds to insertion loss in electrical network theory.

[9] Effectiveness has been defined in terms of foundation-excited machine vibration. It might also have been defined in terms of machine-excited foundation vibration. It has been proved that the effectiveness of a mount at a given frequency is the same whether isolating foundation vibration from the machine or machine vibration from the foundation. (See Additional Reference 3.)

[10] Mechanical network theory as used in this paper is reviewed in Additional Reference 3.

[11] Norton's theorem in mechanical terminology states that if $\dot{x}_p{}'$ is the vibratory velocity amplitude at any point p on a vibrating structure, the vibratory velocity amplitude $\dot{x}_p$ at that point after attaching an additional mechanical structure is given by:

$$\dot{x}_p = \dot{x}_p{}' \frac{Z_p}{Z_p + Z_a} \tag{6}$$

where:

Z_p = Mechanical impedance of the original vibrating structure looking into it at p

Z_a = Mechanical impedance of the added structure looking into it at its point of attachment to p

[12] Mechanical impedance as used in this paper is defined as the quotient of vibratory force and velocity amplitudes, taking account of phase. Mechanical resistance is indicated by R, mechanical reactance by X. The convention chosen is that masslike reactance is positive, springlike reactance negative.

given by:

$$E = \frac{\dot{x}_{12}}{\dot{x}_{22}} = \frac{Z_M + Z_F\left(1 + \frac{Z_M}{Z_m}\right)}{Z_M + Z_F} \qquad (10)$$

If the foundation impedance is much larger than the machine impedance, that is, $Z_F \gg Z_M$, Equation 10 reduces to:

$$E = 1 + \frac{Z_M}{Z_m} \qquad (11)$$

The effectiveness in this case is independent of foundation impedance and, if the machine acts like a rigid mass, is equal to the reciprocal of the transmissibility (Equation 1). The mount will be effective if its impedance is much less than the machine impedance.

If the machine impedance is much larger than the foundation impedance:

$$E = 1 + \frac{Z_F}{Z_m} \qquad (12)$$

The effectiveness in this case is independent of machine impedance. The mount will be effective if its impedance is much less than the foundation impedance.

The machine, mount, and foundation impedances, expressed in terms of their resistive and reactive components, are given by:

$$Z_M = R_M + jX_M \qquad (13)$$

$$Z_m = R_m + jX_m = R_m - j\,\frac{K_m}{\omega} \qquad (14)$$

$$Z_F = R_F + jX_F \qquad (15)$$

These quantities can be nondimensionalized by dividing them by the magnitude of the mount reactance $|X_m|$:

$$\frac{Z_M}{|X_m|} = \frac{R_M + jX_M}{|X_m|} = r_M + jx_M = z_M \qquad (16)$$

$$\frac{Z_m}{|X_m|} = \frac{R_M + jX_M}{|X_m|} = r_m - j = z_m \qquad (17)$$

$$\frac{Z_F}{|X_m|} = \frac{R_F + jX_F}{|X_m|} = r_F + jx_F = z_F \qquad (18)$$

If Equation 10 is nondimensionalized by dividing numerator and denominator by $|X_m|$ and is then expressed in terms of z_M, z_m and z_F, one can show that the effectiveness modulus is:

$$|E| = \left[\frac{[r_m(r_M + r_F) + x_M + x_F + r_M r_F - x_M x_F]^2}{[r_m(r_M + r_F) + x_M + x_F]^2 + [r_m(x_M + x_F) - r_F - r_M]^2} + \frac{[r_m(x_M + x_F) - r_M - r_F + x_M r_F + x_F r_M]^2}{[r_m(r_M + r_F) + x_M + x_F]^2 + [r_m(x_M + x_F) - r_F - r_M]^2}\right]^{\frac{1}{2}} \qquad (19)$$

where:

$$r_m = \frac{\omega R_m}{K_m}, \text{ loss factor of the mount}$$

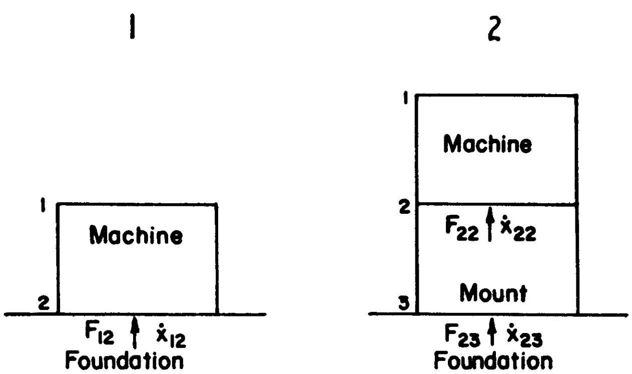
This equation can be used to estimate the effectiveness of any massless spring-dashpot mount isolating any nonrigid machine from any nonrigid foundation, if the system of interest can be represented by Fig. 4 and if the mechanical impedances of the machine, mount, and foundation are known.

Since Equation 19 is unchanged if machine and foundation are interchanged, it follows that subscript M can refer to either the machine or foundation, likewise subscript F. For this reason, subscripts M and F will be replaced by 1 and 2 in subsequent discussion, and it should be understood that if, for example, subscript 1 refers to the machine, subscript 2 refers to the foundation. With this notation, Equations 16 and 18 become:

$$z_1 = \frac{R_1 + jX_1}{|X_m|} = r_1 + jx_1 \qquad (20)$$

$$z_2 = \frac{R_2 + jX_2}{|X_m|} = r_2 + jx_2 \qquad (21)$$

Effectiveness calculations have been made from Equation 19 for r_m constant independent of fre-



Effectiveness $E = \frac{\dot{x}_{12}}{\dot{x}_{22}} = \frac{F_{12}}{F_{22}}$

$|E|$ in db $= 20 \log_{10} \frac{|\dot{x}_{12}|}{|\dot{x}_{22}|}$

Fig. 3 – Definition of effectiveness

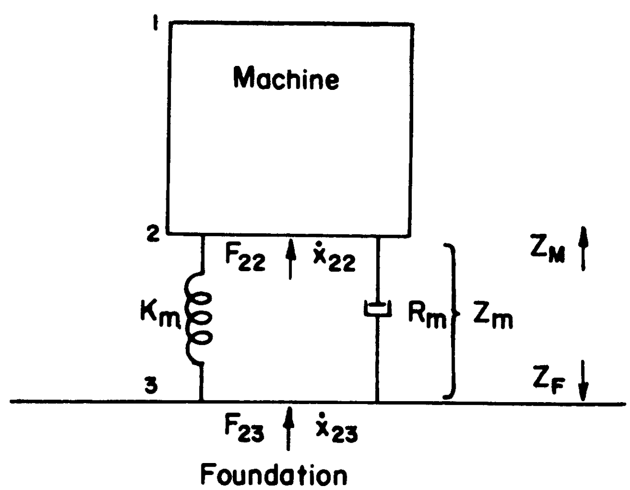


Fig. 4 – Machine spring-dashpot mount foundation system

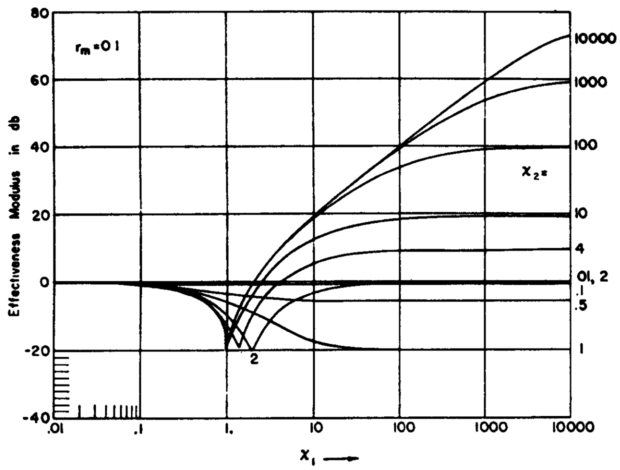


Fig. 5 – Effectiveness modulus in db versus x_1 with x_2 as a parameter; x_1 and x_2 masslike

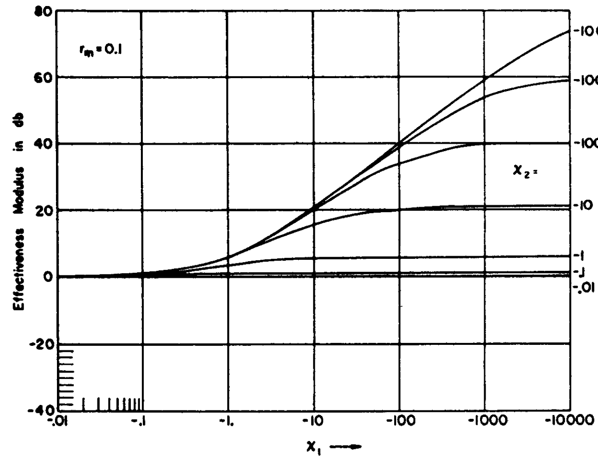


Fig. 6 – Effectiveness modulus in db versus x_1 with x_2 as a parameter; x_1 and x_2 springlike

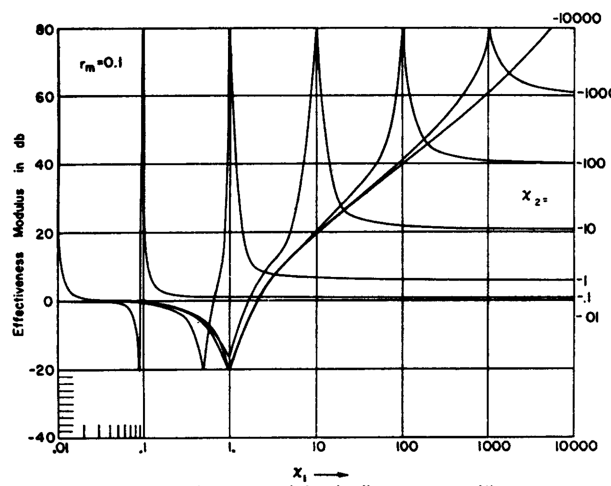


Fig. 7 – Effectiveness modulus in db versus x_1 with x_2 as a parameter; x_1 masslike, x_2 springlike

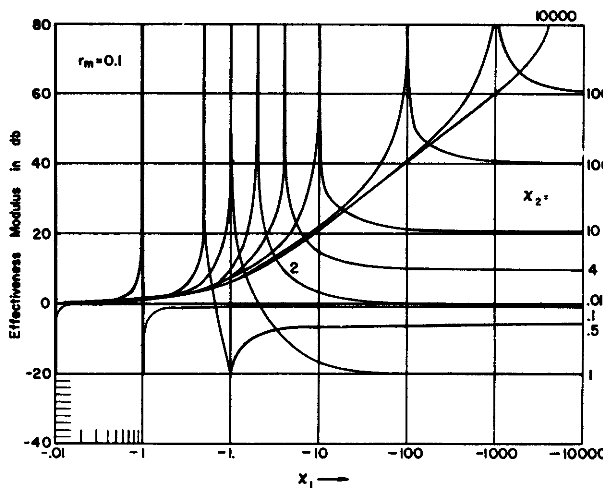


Fig. 8 – Effectiveness modulus in db versus x_1 with x_2 as a parameter; x_1 springlike, x_2 masslike

quency ($r_m = 0.1$) for a variety of machine and foundation conditions, namely:

1. z_1 and z_2 reactive:
   - x_1 and x_2 masslike
   - x_1 and x_2 springlike
   - x_1 masslike, x_2 springlike
   - x_1 springlike, x_2 masslike
2. z_1 reactive and z_2 resistive:
   - x_1 masslike, r_2 resistive
   - x_1 springlike, r_2 resistive
3. z_1 and z_2 resistive.

The results of these calculations are presented in Figs. 5–11 where effectiveness modulus in db $|E|_{db}$ is plotted versus z_1 with z_2 and r_m as parameters.

$$|E|_{db} = 20 \log_{10} |E| \tag{22}$$

Two conclusions valid for every case studied are:

1. The softer a mount is, (that is, the larger z_1 and z_2 are) the more effective the mount is provided that z_1 and z_2 are both greater than about 2.[13]

2. For z_1 and z_2 both much greater than unity, and for z_1 much greater than z_2, the effectiveness of the mount will be determined by z_2; in fact:

$$|E| \cong |z_2| \tag{23}$$

For z_1 and z_2 respectively both masslike, masslike and springlike, or masslike and resistive, depending on their magnitudes, use of a mount can cause vibration magnification. (See Figs. 5, 7–9). The minimum in effectiveness—corresponding to the maximum magnification that can occur—is controlled by the loss factor of the mount used, and is given by:

$$|E|_{min} \cong r_m \tag{24}$$

If z_1 and z_2 respectively are both masslike, or masslike and springlike, the effectiveness minimum occurs near the values of x_1 and x_2 for which:

$$x_1 + x_2 - x_1 x_2 = O \tag{25}$$

provided that $r_m \ll 1$. For z_1 and z_2 both masslike, the minimum corresponds to resonance of the mount with the equivalent masses of x_1 and x_2. For z_1 masslike and z_2 springlike, the minimum corresponds to resonance of the equivalent mass of X_1, with com-

[13] A trivial exception occurs for z_1 masslike and z_2 springlike, in which case, for $|z_1| = |z_2|$, the effectiveness is infinite without regard to mount stiffness.

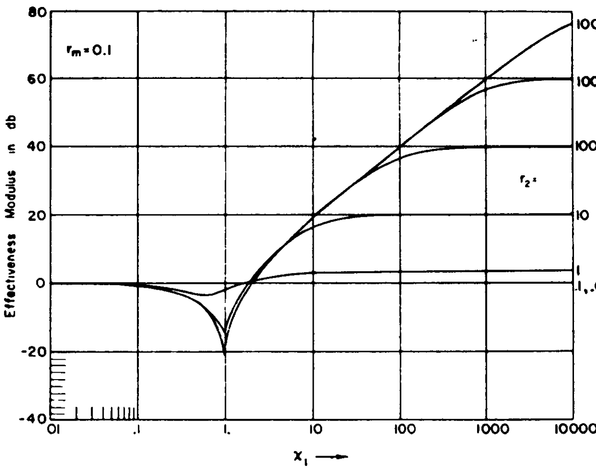


Fig. 9 – Effectiveness modulus in db versus x_1 with r_2 as a parameter; x_1 masslike, r_2 resistive

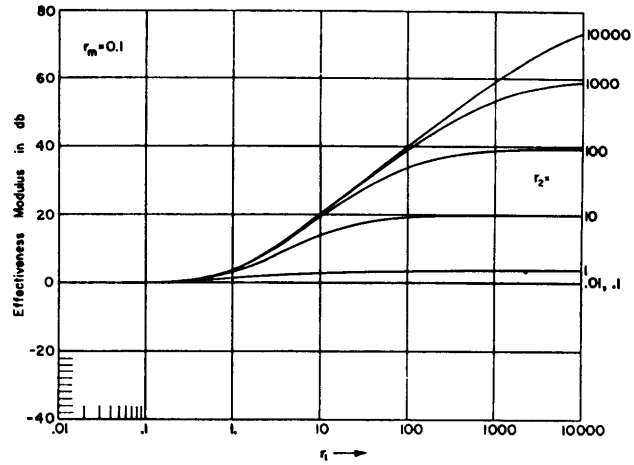


Fig. 10 – Effectiveness modulus in db versus x_1 with r_2 as a parameter; x_1 springlike, r_2 resistive

bined stiffness of the mount and X_2 mechanically in series.

If z_1 is masslike, z_2 is resistive, and $x_1 \ll r_2$, the minimum occurs near $x_1 = 1$. This minimum corresponds to resonance of the equivalent mass of X_1 with the mount.

For z_1 and z_2 respectively both springlike, springlike and resistive, or both resistive, the effectiveness of a mount is always unity or greater, and magnification of the vibration does not occur. (See Figs. 6, 10, and 11.)

For z_1 masslike and z_2 springlike, an infinity in effectiveness occurs for $x_1 = |x_2|$. This reflects the fact that when a mount is used, the machine and foundation no longer resonate for $X_1 = |X_2|$ as they do if directly connected. In a practical case, the effectiveness is not infinite because both machine and foundation are somewhat damped. From Equation 19, the effectiveness for $x_1 = |x_2|$ is given by:

$$|E| = \left\{ \frac{\left[r_m + \frac{r_1 r_2}{r_1 + r_2} + \frac{x_1^2}{r_1 + r_2} \right]^2 + \left[-1 + \frac{x_1 (r_2 - r_1)}{r_1 + r_2} \right]^2}{1 + r_m^2} \right\}^{\frac{1}{2}} \tag{26}$$

For $r_m \ll 1$ and for r_1, $r_2 \ll x_1$ this equation reduces approximately to:

$$|E| = \frac{x_1^2}{r_1 + r_2} = \frac{X_1}{|X_m|} \frac{X_1}{R_1 + R_2} \tag{27}$$

It follows that for $x_1 = |x_2|$ the softer the mount is, the more effective it is.

Effectiveness of Massless Spring-Dashpot Mounts Isolating Rigid Machines from Nonrigid Foundations—Although the curves of Figs. 5–11 are valid for both nonrigid and rigid machines, the isolation of rigid machines will now be given further consideration since a different and more familiar representation of the effectiveness data is possible.

If the machine is rigid, its mechanical impedance is given by:

$$Z_M = j\omega M_M \tag{28}$$

where:

M_M = Its mass

Substituting Equations 14 and 28 in Equation 10, defining some new symbols, and taking the modulus

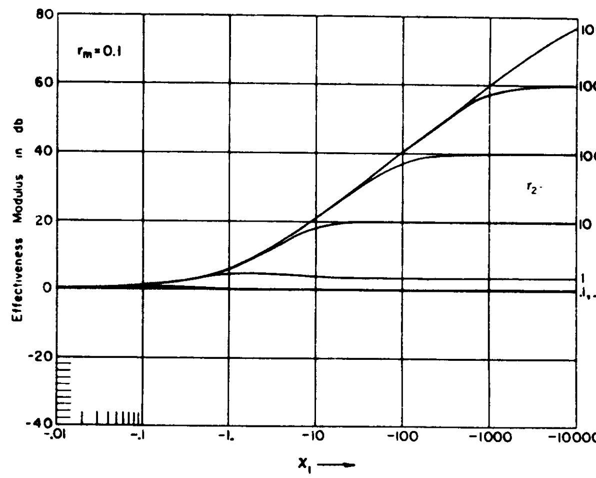


Fig. 11 – Effectiveness modulus in db versus r_1 with r_2 as a parameter; r_1 and r_2 resistive

of the resulting expression, one can show that:

$$|E| = \left[\frac{[f_r^2(1 - f_r x_{FO}) + f_r r_{FO} r_m + f_r x_{FO}]^2}{[f_r r_{FO} r_m + f_r^2 + f_r x_{FO}]^2 + [r_m(f_r^2 + f_r x_{FO}) - f_r r_{FO}]^2} + \frac{[r_m(f_r^2 + f_r x_{FO}) + f_r r_{FO}(f_r^2 - 1)]^2}{[f_r r_{FO} r_m + f_r^2 + f_r x_{FO}]^2 + [r_m(f_r^2 + f_r x_{FO}) - f_r r_{FO}]^2} \right]^{\frac{1}{2}} \tag{29}$$

where:

$$f_r = \frac{\omega}{\omega_o}$$

$$\omega_o = \sqrt{\frac{K_m}{M_M}}$$

$$x_{FO} = \frac{\omega_o X_F}{K_m}$$

$$r_m = \frac{\omega R_m}{K_m}$$

$$r_{FO} = \frac{\omega_o R_F}{K_m}$$

This equation has been plotted in Figs. 12–14 as

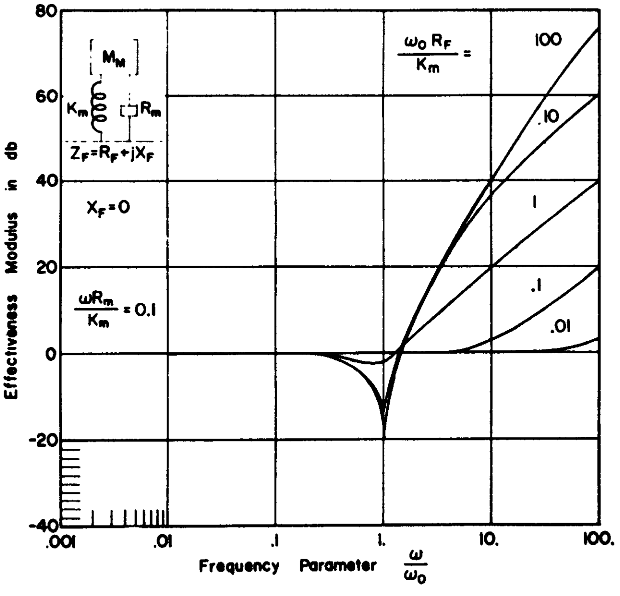


Fig. 12 – Effectiveness modulus in db versus frequency ratio ω/ω_0; rigid machine, spring-dashpot mount, resistive foundation

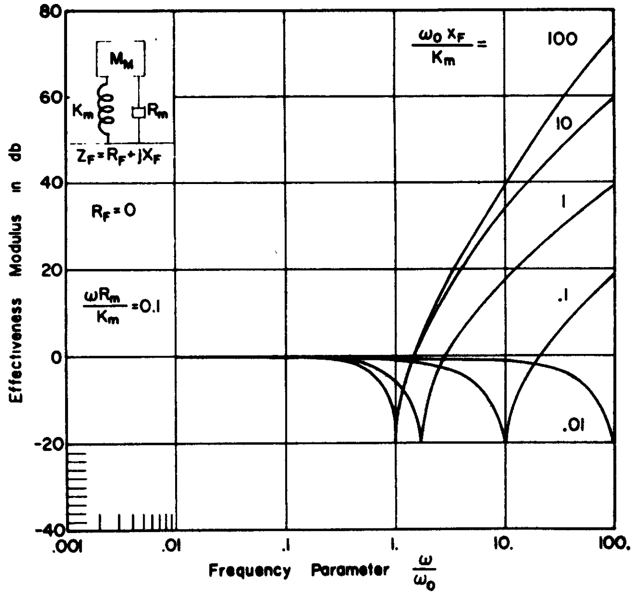


Fig. 13 – Effectiveness modulus in db versus frequency ratio ω/ω_0; rigid machine, spring-dashpot mount, masslike foundation

a function of frequency ratio $\frac{\omega}{\omega_o}$, with the parameter r_m constant independent of frequency ($r_m = 0.1$), for resistive, masslike, and springlike foundations.

From Fig. 12 it is evident that for $\frac{\omega}{\omega_o} \gg 1.5$, the greater the foundation resistance, the more effective a mount will be. For $\frac{\omega_o R_F}{K_m} \gg 1$, the effectiveness is minimum near the natural frequency of the machine-mount system ($\frac{\omega}{\omega_o} = 1$); at the minimum, the effectiveness is controlled by the damping factor of the mount, and is given by Equation 24.

From Figs. 13 and 14 for reactive foundations, it is evident that an effectiveness minimum occurs for each value of the parameter $\frac{\omega_o X_F}{K_m}$. This minimum corresponds to the natural frequency of a system composed of the rigid machine, spring-dashpot mount, and a hypothetical foundation whose reactance X_F is constant independent of frequency. For $R_F \ll X_F$, the frequency ratio of the effectiveness minimum is given by:

$$\left(\frac{\omega}{\omega_o}\right)_{\min} = \frac{\frac{1}{x_{FO}} + \sqrt{\left(\frac{1}{x_{FO}}\right)^2 + 4\left(\frac{r_{FO} r_m + x_{FO}}{x_{FO}}\right)}}{2} \qquad (30)$$

The effectiveness at $\left(\frac{\omega}{\omega_o}\right)_{\min}$ is determined by the loss factor of the mount and is given once again by Equation 24.

For $r_m \ll 1$, it can be shown from Equation 30 that for masslike foundations:

$$\left(\frac{\omega}{\omega_o}\right)_{\min} \geqq 1 \qquad (31)$$

and that the smaller $|x_{FO}|$, the higher $\left(\frac{\omega}{\omega_o}\right)_{\min}$ will be; for springlike foundations:

$$\left(\frac{\omega}{\omega_o}\right)_{\min} \leqq 1 \qquad (32)$$

and the smaller $|x_{FO}|$, the lower $\left(\frac{\omega}{\omega_o}\right)_{\min}$ will be.

For masslike foundations, the mount will isolate only for $\left(\frac{\omega}{\omega_o}\right)$ greater than about 1.5 $\left(\frac{\omega}{\omega_o}\right)_{\min}$. For springlike foundations, the mount will isolate only near $\left(\frac{\omega}{\omega_o}\right)_\infty$ given by:[14]

$$\left(\frac{\omega}{\omega_o}\right)_\infty = -x_{FO} \qquad (33)$$

and for frequency ratios $\frac{\omega}{\omega_o}$ greater than 1 or 2.

Effectiveness of Continuous Mounts Isolating Rigid Machines from Nonrigid Foundations**—The use of a massless spring-dashpot model for an isolation mount is possible only at frequencies for which inertia forces associated with the mount can be neglected. At higher frequencies, the mount must be considered an extended mechanical system with distributed mass and elasticity, through which mechanical energy can be propagated by elastic waves.**

The effects of wave propagation on the transmissibility of mounts were investigated several years ago at David Taylor Model Basin.[15, 16, 17] The trans-

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[14] $\left(\frac{\omega}{\omega_o}\right)_\infty$ corresponds to resonance of a rigid machine and a foundation whose purely reactive impedance, specified by x_{FO}, is constant independent of frequency.

[15] DTMB Report 766 (Revised), August, 1952: "Wave Effects in Isolation Mounts," by M. Harrison, A. O. Sykes, and M. Martin. David Taylor Model Basin.

[16] DTMB Report 818, February, 1952: "Use of Helical Springs as Isolation Mounts," by A. O. Sykes. David Taylor Model Basin.

[17] DTMB Report 845, October, 1953: "Study of Compression Isolation Mounts Constructed from Cylindrical Samples of Various Natural and Synthetic Rubber Materials," by A. O. Sykes. David Taylor Model Basin.

missibility of compression and shear mounts with simple shapes was determined both theoretically and experimentally, and it was demonstrated that:

1. The transmissibility of mounts having certain simple configurations can be accurately predicted if the properties of the mount material are known.
2. The transmissibility of a mount can be increased at standing wave resonances by 5–20 or more db over what lumped spring-dashpot mount theory predicts.
3. For minimum transmissibility, the wave propagation velocity (the phase velocity) in a mount should decrease with frequency.
4. For minimum transmissibility at standing-wave resonant frequencies, the damping factor for a mount should be high.

Recently, the effects of wave propagation on the effectiveness of mounts have been investigated.

Consider the mechanical system of Fig. 15. The machine and foundation are considered nonrigid and are characterized respectively by the impedances Z_M and Z_F looking into them from their junctures with the mount.

If the mount, pictured as a vertical cylinder, is a long thin rod[18] or a thin disc[19] in tension or compression or torsion, a shear sandwich, or a helical spring in tension or compression, it can be shown that the approximate equation of motion of a cross-section of the mount which is normal to the direction of wave propagation is:[20]

$$G\frac{\delta^2 x(y)}{\delta y^2} + \mu\frac{\delta^3 x(y)}{\delta^2 y \delta t} = \rho\frac{\delta^2 x(y)}{\delta t^2}, \tag{34}$$

where:

$x(y)$ = Instantaneous displacement of the cross-section from its equilibrium position
y = Coordinate of the cross-section measured from one end of the mount
G = Real part of the appropriate elastic modulus of the mount
μ = Appropriate viscosity coefficient
ρ = Density of the mount material

If Equation 34 is solved for $\dot{x}(y)$—the dot indicates the time derivative of $x(y)$—subject to the boundary conditions existing at the junctures of the machine and foundation with the mount, the velocity amplitude $\dot{x}_{22}$ (See Fig. 3) at the machine-mount juncture can be computed.

If this result and Equation 5 for $\dot{x}_{12}$ are substituted in Equation 4, one can show that the complex effectiveness of the mount is given by:

$$E = \cosh \gamma l + \left(\frac{SZ_o + \dfrac{Z_M Z_F}{SZ_o}}{Z_M + Z_F}\right)\sinh \gamma l \tag{35}$$

where:

$\gamma = \alpha + j\beta$, the complex progagation constant of the mount

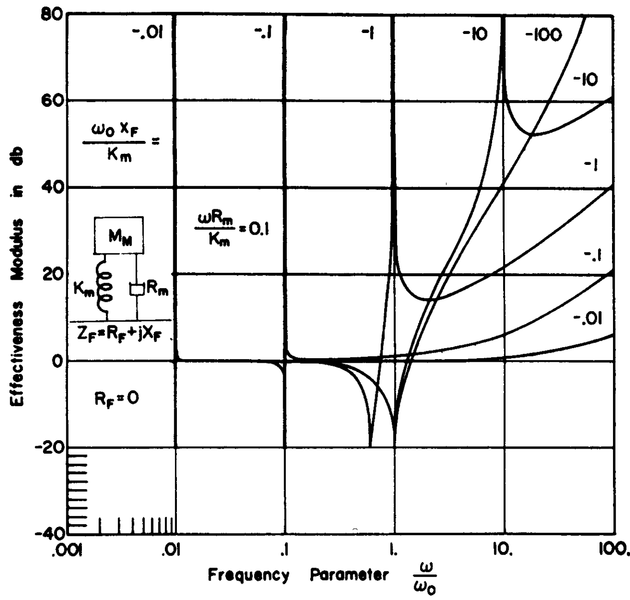

Fig. 14 – Effectiveness modulus in db versus frequency ratio ω/ω_0; rigid machine, spring-dashpot mount, springlike foundation

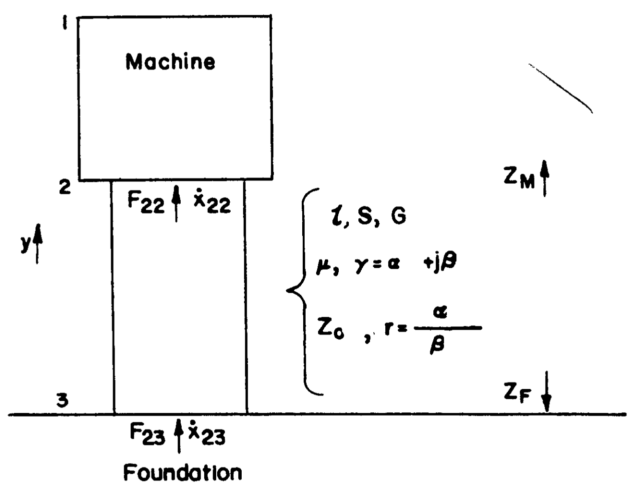

Fig. 15 – Machine continuous mount foundation system

α = Attenuation per unit length in the mount
$\beta = \frac{\omega}{c}$, the phase constant of the mount
$j = \sqrt{-1}$
$\omega = 2\pi f$
f = Frequency in cps
c = Phase velocity in the mount
l = Length of the mount
S = Cross-sectional area of the mount
Z_o = Complex characteristic impedance per unit area of the mount (SZ_o is referred to as the characteristic impedance)
Z_M = Complex machine impedance looking toward the machine from the machine-mount juncture
Z_F = Complex foundation impedance looking toward the foundation from the

[18] By a thin rod is meant a rod whose diameter is much smaller than the wave length of the vibration in the rod at the highest frequency of interest.
[19] By a thin disc is meant a disc whose edges are constrained so as to prevent motion perpendicular to the direction of wave propagation.
[20] The damping stresses in the mount have been assumed proportional to the time rate of strain $\frac{\delta}{\delta t}\left(\frac{\delta x(y)}{\delta y}\right)$.

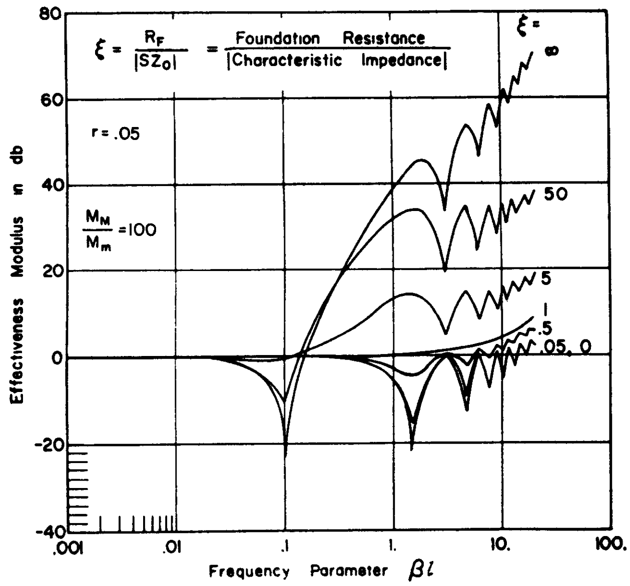

Fig. 16 – Effectiveness modulus in db versus frequency parameter βl; rigid machine, continuous mount, resistive foundation

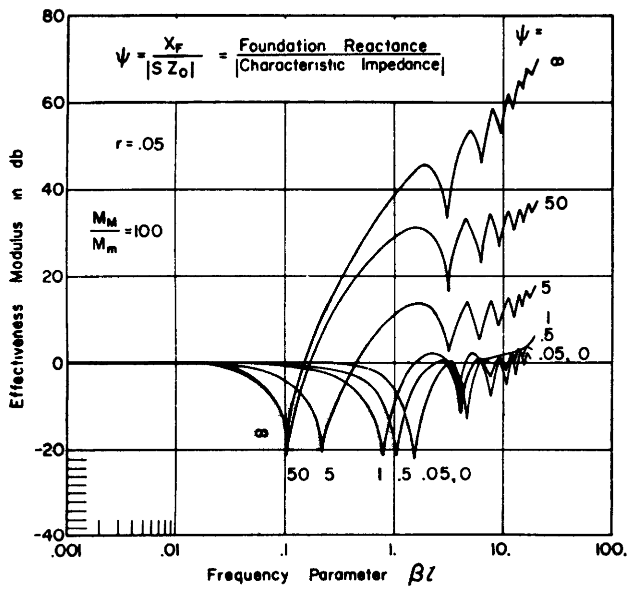

Fig. 17 – Effectiveness modulus in db versus frequency parameter βl; rigid machine, continuous mount, masslike foundation

foundation-mount juncture

The modulus of this equation can be written:

$$|E| = [(1 + A_1)\sinh^2 r\beta l + \cos^2 \beta l + A_1 \sin^2 \beta l + A_2 \sin 2\beta l + A_3 \sinh 2r\beta l]^{\frac{1}{2}} \quad (36)$$

where:

$$r = \frac{\alpha}{\beta}, \text{ the damping factor of the mount}$$

If the machine is rigid, its impedance is given by Equation 28, in which case:

$$A_1 = \frac{1 + \frac{4r\xi m\beta l}{\sqrt{1+r^2}} - \frac{2(1-r^2)m\psi\beta l}{\sqrt{1+r^2}} + (\xi^2+\psi^2)(m\beta l\sqrt{1+r^2})^2}{\xi^2 + (\psi + m\beta l\sqrt{1+r^2})^2}$$

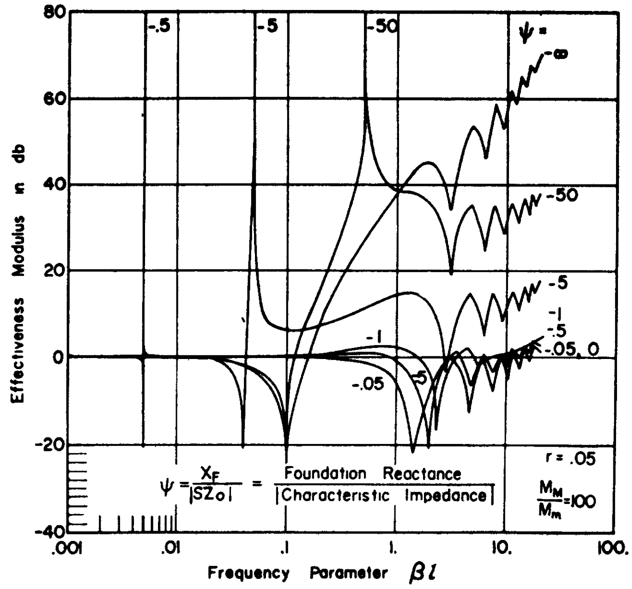

Fig. 18 – Effectiveness modulus in db versus frequency parameter βl; rigid machine, continuous mount, springlike foundation

$$A_2 = \frac{m\beta l\sqrt{1+r^2} + \psi - r\xi + (m\beta l\sqrt{1+r^2})^2(r\xi - \psi)}{[\xi^2 + (\psi + m\beta l\sqrt{1+r^2})^2]\sqrt{1+r^2}} - \frac{(\xi^2+\psi^2)m\beta l}{\xi^2 + (\psi + m\beta l\sqrt{1+r^2})^2}$$

$$A_3 = \frac{\xi + r(m\beta l\sqrt{1+r^2} + \psi) + (\xi + r\psi)(m\beta l\sqrt{1+r^2})^2}{[\xi^2 + (\psi + m\beta l\sqrt{1+r^2})^2]\sqrt{1+r^2}} + \frac{rm\beta l(\xi^2+\psi^2)}{\xi^2 + (\psi + m\beta l\sqrt{1+r^2})^2}$$

where:

$$m = \frac{M_M}{M_m}$$

$$\xi = \frac{R_F}{|SZ_o|}$$

$$\psi = \frac{X_F}{|SZ_o|}$$

M_m = Mass of the mount

$Z_F = R_F + jX_F$, the mechanical impedance of the foundation

$|SZ_o| = \frac{\rho cS}{\sqrt{1+r^2}}$, the magnitude of the characteristic impedance of the mount

Examination of Equation 36, for $Z_F \to \infty$ and $r^2 \ll 1$, shows that an effectiveness minimum will occur near:

$$\beta l = \sqrt{\frac{M_m}{M_M}} \overset{\text{def}}{=} (\beta l)_o \quad (37)$$

if $\beta l \ll 1$. This minimum corresponds to resonance of the machine and mount at the natural frequency f_o of the machine-mount system. Additional minima will occur for:

$$\beta l \cong n\pi$$

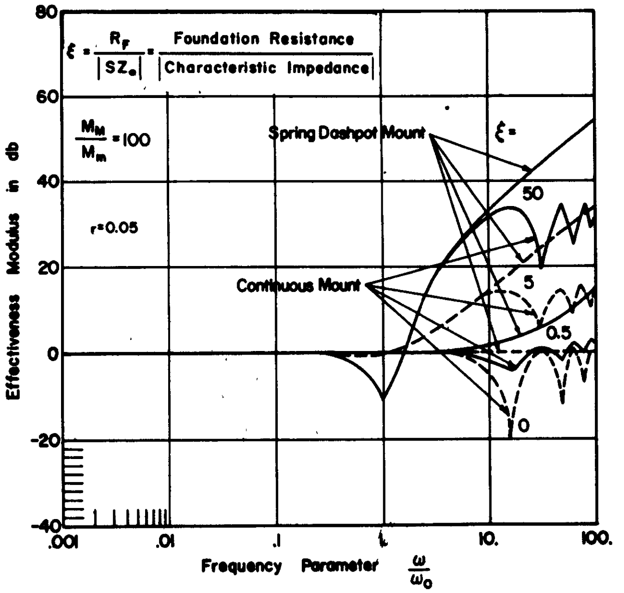

Fig. 19 – Effectiveness modulus in db versus frequency ratio ω/ω_0; rigid machine, spring-dashpot and continuous mounts, resistive foundation

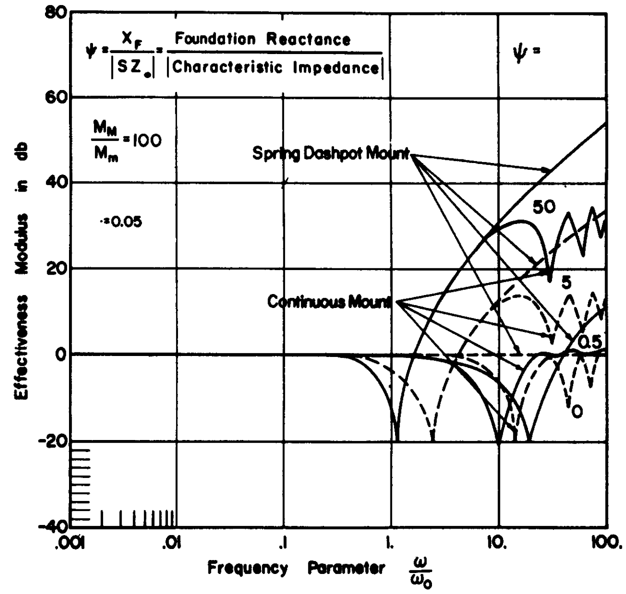

Fig. 20 – Effectiveness modulus in db versus frequency ratio ω/ω_0; rigid machine, spring-dashpot and continuous mounts, masslike foundation

where:

$$n = 1, 2, 3 \tag{38}$$

These minima occur at the standing-wave resonant frequencies f_n of the mount:

$$f_n = \frac{nc}{2\iota} \tag{39}$$

for which the length of the mount is an integral number of half wavelengths.

Examination of Equation 36, for $Z_F \to 0$ and $r^2 \ll 1$, shows that effectiveness minima will occur for:

$$\beta\iota = (2s+1)\frac{\pi}{2}$$

where:

$$s = 0, 1, 2 \tag{40}$$

These minima occur at the standing-wave resonant frequencies f_s of the mount:

$$f_s = \frac{(2s+1)}{4}\frac{c}{\iota} \tag{41}$$

for which the length of the mount is an odd integral number of quarter wavelengths.

Equation 36 has been plotted in Figs. 16–18 as a function of the frequency parameter $\beta\iota$ for r constant independent of frequency ($r = 0.05$), for a machine-mount mass ratio $\frac{M_M}{M_m} = 100$, for resistive, masslike, and springlike foundations.

These continuous-mount effectiveness curves have been compared with spring-dashpot mount curves in Figs. 19–21 by making use of the transformations:

$$\frac{\omega}{\omega_o} = \frac{1+r^2}{\sqrt{1-r^2}}\sqrt{\frac{M_M}{M_m}}\,\beta\iota \tag{42}$$

$$\frac{\omega_o R_F}{K_m} = \sqrt{\frac{(1+r^2)}{(1-r^2)}\frac{M_m}{M_M}}\,\frac{R_F}{|SZ_o|} \tag{43}$$

$$\frac{\omega_o X_F}{K_m} = \sqrt{\frac{(1+r^2)}{(1-r^2)}\frac{M_m}{M_M}}\,\frac{X_F}{|SZ_o|} \tag{44}$$

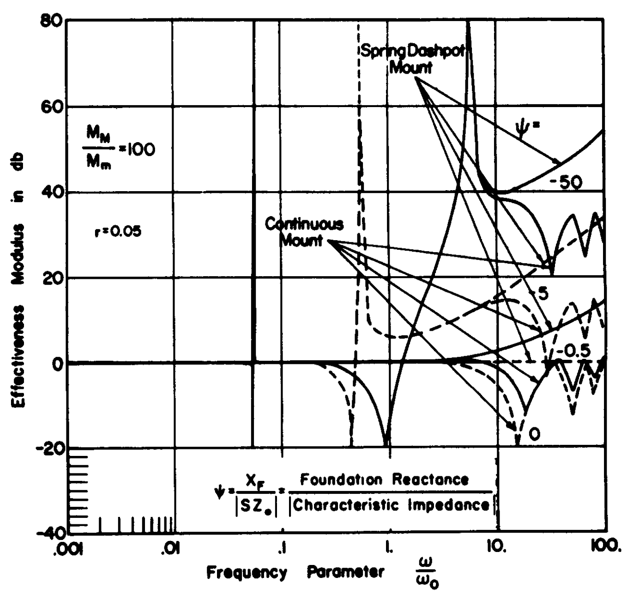

Fig. 21 – Effectiveness modulus in db versus frequency ratio ω/ω_0; rigid machine, spring-dashpot and continuous mounts, springlike foundation

$$\frac{\omega R_m}{K_m} = \frac{2r}{1-r^2} \tag{45}$$

From these curves and Equations 38–41, it follows that the behavior of continuous mounts is quite accurately described by spring-dashpot mount theory, if the wavelength in the mount of the exciting vibration is approximately 10 or more times the length of the mount. On the other hand, if the wavelength in the mount of the exciting vibration is less than about six times the mount length, the mount effectiveness can be considerably lower than spring-dashpot mount theory predicts. Particularly large reductions in effectiveness occur at the lower standing-wave resonances of the mount, irrespective of the magnitude of the foundation impedance and whether it is resistive or reactive. In fact, if the foundation impedance is predominantly resistive or reactive, and if the impedance ratios $\frac{R_F}{|SZ_o|}$ and $\frac{X_F}{|SZ_o|}$ are less than unity at any of the exciting frequencies, the mount will magnify the vibration at those frequencies.

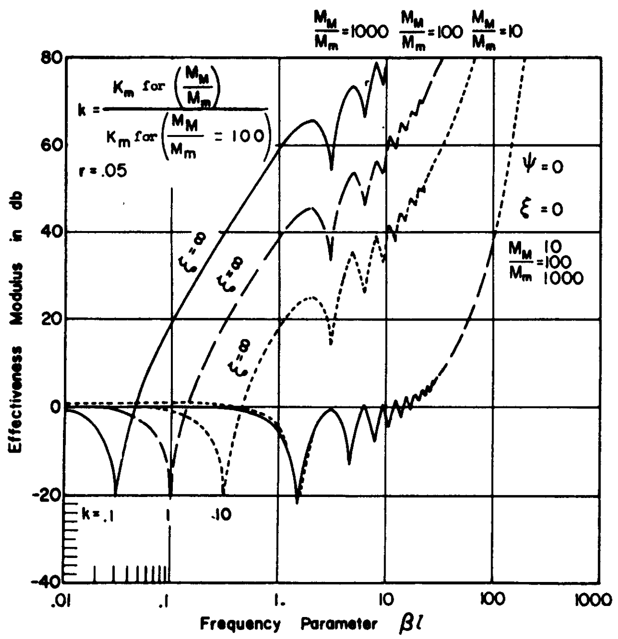

Fig. 22 – Effectiveness modulus in db versus frequency parameter βl; rigid machine, continuous mount, zero and infinite impedance foundations; mount length fixed, cross-sectional area varied

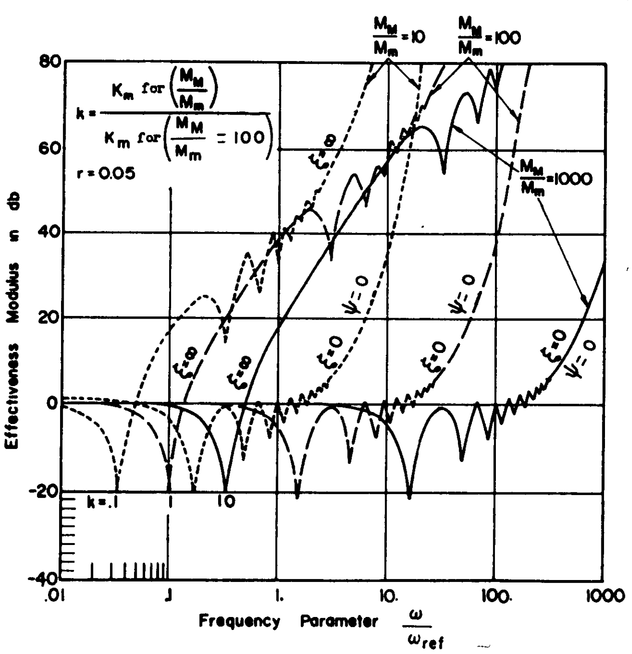

Fig. 23 – Effectiveness modulus in db versus frequency ratio ω/ω_{ref}; rigid machine, continuous mount, zero and infinite impedance foundations; mount length varied, cross-sectional area fixed

For ξ and $|\psi| > 1$, the standing-wave resonant frequencies are more or less independent of foundation impedance, and are given approximately by Equation 39. If the foundation impedance is resistive and is of such magnitude that $\xi < 1$, the standing-wave resonant frequencies are given approximately by Equation 41. If the foundation impedance is reactive such that $|\psi| < 1$, the standing-wave resonant frequencies will depend on the magnitude of X_F and on whether it is masslike or springlike. If $|\psi| \ll 1$, the standing-wave resonant frequencies will be near the quarter-wave frequencies given by Equation 41. If the foundation is masslike, the lowest resonant frequency will decrease as ψ increases from zero, until for $\psi \gg 1$ it will coincide with f_o, the natural frequency of the machine-mount system; the higher resonant frequencies will decrease as ψ increases, until for $\psi > 1$ they nearly coincide with the half-wave resonant frequencies (Equation 39). If the foundation is springlike, the natural frequency of the machine-mount-foundation system will increase from zero as $|\psi|$ increases from zero, until for $|\psi| \gg 1$ it also coincides with f_o; the frequencies of the standing-wave resonances will increase as $|\psi|$ increases from zero, from the quarter-wave resonant frequencies given by Equation 41 to the half-wave resonant frequencies given by Equation 39.

Influence of Mount Characteristics on Effectiveness

Spring-dashpot mount theory describes a mount in terms of its stiffness K_m and resistance R_m. Experiment has shown that this description is reasonably adequate for most mounts provided that the dynamic strains in the mount are small enough for it to act as a linear spring, and the frequencies of the exciting forces are low enough for wave effects in the mount to be unimportant.

Continuous-mount theory (for small strains) can describe a mount in terms of its propagation constant γ, length ι and characteristic impedance SZ_o (Equation 35), or in terms of various combinations of related properties. It can be shown from Equations 50, 52, 57, and 62 that a mount can be described by specifying its damping factor r, and any two of the quantities mount stiffness K_m, mass M_m, frequency constant $\frac{c}{\iota}$, and characteristic impedance SZ_o, which themselves can be expressed in terms of the density ρ of the mount material, the phase velocity c in the mount, the damping factor r, the length ι, and cross-sectional area S.

These physical characteristics are determined by various factors:

ρ depends only on the mount material.

c and r depend not only on the material but on what isolation mechanism is chosen, that is, compression, shear, dilation, or some combination thereof and, to a small degree, on the shape (c and r may be somewhat different for thin round rods and thin square rods).

ι and S are determined by design requirements such as, for example, static load.

The effects on effectiveness of varying certain mount characteristics will now be investigated. It will be assumed that the machine to be isolated acts as a rigid mass M_M, and that the behavior of the mount is described by Equation 36.

For the following discussion, it will be assumed also that the mount material, mechanism of isolation, and shape have been selected, and that this selection has fixed the phase velocity c in the mount and its damping factor r.

Mount Dimensions and Effectiveness—Suppose, for example, that a thin cylindrical rod has been selected as a compression mount. Excluding density, phase velocity, and damping factor—which have been fixed by selection of the mount material, isolation mechanism, and shape—the mount characteristics important in determining its effectiveness can be changed by changing its length, or cross-sectional area, or both.

1. Mount length fixed and cross-sectional area varied.

Fixing the length of the mount fixes its frequency constant, since the phase velocity in the mount has been fixed. Changing its cross-sectional area changes its stiffness, mass, and characteristic impedance in direct proportion.

The effects on effectiveness of varying mount cross-sectional area, with mount frequency constant fixed, are illustrated in Fig. 22 where effectiveness in db is plotted versus $\beta\iota$ for three mount cross-sectional areas, for $r = 0.05$, $\xi = \infty$ and $\xi, \psi = 0$.[21] The area changes are indicated by the changes in mass ratio $\frac{M_M}{M_m}$, and stiffness ratio k defined by:

$$k = \frac{K_m \text{ for } \left(\frac{M_M}{M_m}\right)}{K_m \text{ for } \left(\frac{M_M}{M_m} = 100\right)} \tag{46}$$

For $k = 0.1$, the cross-sectional area of the mount has been reduced by a factor of 10 from the area for $k = 1$; for $k = 10$, the area has been increased by a factor of 10.

For $\xi = \infty$, it is evident that the mount is effective for all exciting frequencies $(\beta\iota)_i$ greater than about 1.5 $(\beta\iota)_o$ (Equation 37); further, if a mount is effective at a particular frequency, a mount with smaller cross-section—a softer mount—will be more effective.

For ξ and $\psi = 0$, the mount magnifies the vibration except for $\beta\iota \gg \frac{\pi}{2}$; its effectiveness is essentially independent of cross-section.

2. Mount length varied and cross-sectional area fixed.

Fixing the cross-sectional area of the mount fixes its characteristic impedance, since the mount's density and phase velocity have been fixed. Changing its length changes its mass in direct proportion, and its stiffness and frequency constant in inverse proportion.

The effects on effectiveness of varying mount length, with mount characteristic impedance fixed, are illustrated in Fig. 23 where effectiveness in db is plotted versus $\frac{\omega}{\omega_{ref.}}$ for three mount lengths, for $r = 0.05$, $\xi = \infty$, and $\xi, \psi = 0$. $w_{ref.}$ is defined by:

$$\omega_{ref.} = \frac{c}{\iota_{100}} \tag{47}$$

where:

$$\iota_{100} = \text{Length of the mount for } \frac{M_M}{M_m} = 100$$

The length changes are indicated by changes in $\frac{M_M}{M_m}$ and k. For $k = 0.1$, the mount length is 10 times that for $k = 1$; for $k = 10$, the length is 1/10 that for $k = 1$.

Both for $\xi = \infty$ and $\xi, \psi = 0$, the effectiveness is length dependent. Increasing the length of the mount decreases both the frequency at which the mount first begins to provide isolation, and the standing wave resonant frequencies; it may either increase or decrease effectiveness depending on the exciting frequencies.

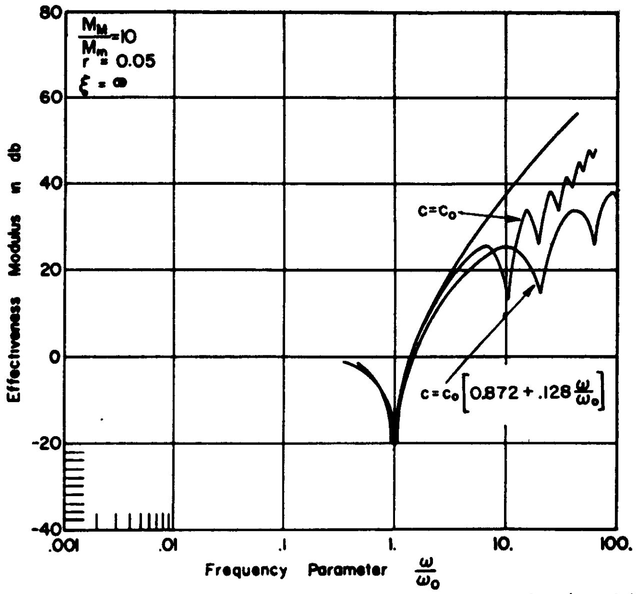

Fig. 24 – Effectiveness modulus in db versus frequency ratio ω/ω_0; rigid machine, continuous mount, infinite foundation impedance; mount phase velocity increasing with frequency. For comparison the unidentified curve shows the effectiveness of an undamped massless spring with stiffness constant independent of frequency

Phase Velocity and Effectiveness—It has been established experimentally that the dynamic elastic moduli of many viscoelastic materials are frequency dependent. (See Additional Reference 2.) Below a few hundred cps, they usually increase with frequency. The increase in elastic modulus results from an increase in phase velocity in the mount.

The effects on effectiveness of phase velocity increasing with frequency are illustrated in Fig. 24 for $\frac{M_M}{M_m} = 10$, $r = 0.05$, $\xi = \infty$.

Increasing phase velocity with frequency results in reduced effectiveness of the mount, and increases the standing-wave resonant frequencies.

Damping Factor and Effectiveness—The effects on

[21] If the foundation impedance is infinite, the effectiveness of a mount is the same whether the foundation impedance is resistive, reactive, or resistive and reactive.

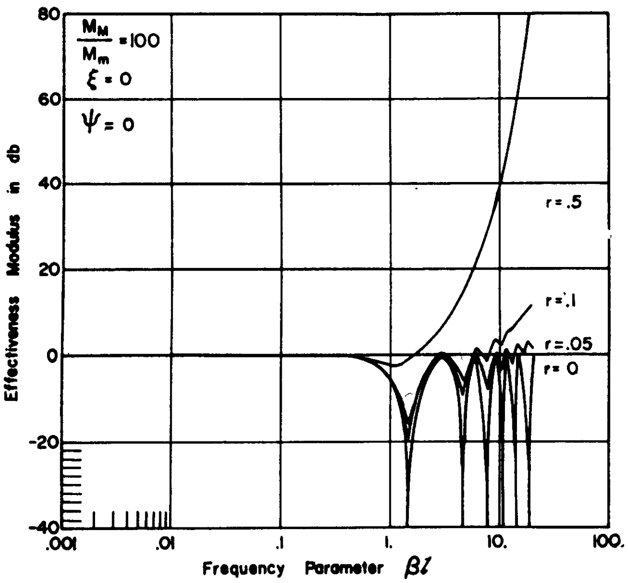

Fig. 25 – Effectiveness modulus in db versus frequency parameter βl; rigid machine, continuous mount, zero foundation impedance; mount damping factor r as a parameter

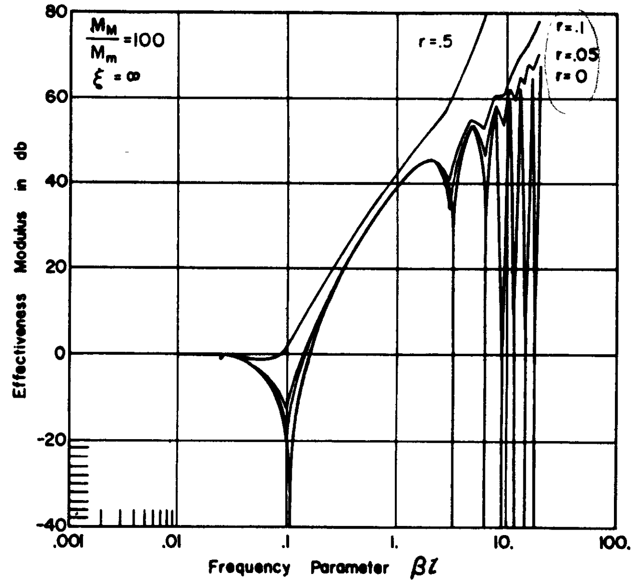

Fig. 26 – Effectiveness modulus in db versus frequency parameter βl; rigid machine, continuous mount, infinite foundation impedance; mount damping factor r as a parameter

effectiveness of mount damping factor are illustrated in Figs. 25 and 26 for:

$$\frac{M_m}{M_M} = 100$$
$$r = 0,\ 0.05,\ 0.1,\ 0.5$$
$$\xi = \infty$$
$$\xi,\ \psi = 0$$

For both $\xi = \infty$ and ξ, $\psi = 0$, increasing r increases the mount effectiveness at effectiveness minima and at very high frequencies for which the mount length is many wavelengths.

One can infer that increasing damping factor with frequency should increase the effectiveness of a mount. That this inference is correct is demonstrated in Fig. 27.

Characteristic Impedance and Effectiveness—The characteristic impedance of a mount can be changed by changing the mount material, isolation mechanism, or mount geometry. If mount mass and frequency constant are allowed to change, mount characteristic impedance can be changed without changing the mount stiffness.[22]

Effectiveness curves have been computed for continuous mounts, having the same stiffness but different characteristic impedances, isolating a rigid machine from a resistive foundation. The results are presented in Fig. 28.

It is evident that increases or decreases in characteristic impedance can either increase or decrease effectiveness depending on the frequency ratio. What characteristic impedance is best can not be decided without knowledge of the machine vibratory output spectrum.

Mount Design

Design Parameters—As mentioned in the previous section, a mount can be described in terms of its damping factor r and any pair of the quantities mount stiffness K_m, mass M_m, frequency constant $\frac{c}{l}$ and characteristic impedance SZ_o.

The most convenient pair for design purposes are stiffness and frequency constant, since the mount stiffness determines its behavior over the frequency range for which it can be treated as a massless spring-dashpot, while its frequency constant is indicative of the frequency at which wave effects become important.

Design Procedure—Considering only vibration isolation and excluding problems involving shock, static load, drift, and the like, to design a mount for a specific application, one should know:

1. The blocked force or free velocity frequency spectrum of the exciter (whether machine or foundation).

2. The machine and foundation impedances, as functions of frequency, looking into them at the terminals which attach to the mount.

3. Acceptable vibration levels.

From the first two can be calculated the vibratory velocity amplitude as a function of frequency that would occur at the point of contact of machine and foundation if they were directly connected. These data, with the third, define what mount effectiveness is required.

If the exciting frequencies are low enough, it may be possible to design a mount which acts as a massless spring-dashpot over the entire frequency range. The mount stiffness required can be determined from the effectiveness requirement and the machine and foundation impedances, by using Equations 19 or 29 or one of Figs. 5–14, whichever is appropriate. The frequency constant required can be determined from the blocked force or free velocity frequency

---

[22] For $r^2 \ll 1$, it can be shown from Equations 50, 52, and 57 that:

$$M_m = \frac{|SZ_o|^2}{K_m} \tag{48}$$

$$\frac{c}{l} = \frac{K_m}{|SZ_o|} \tag{49}$$

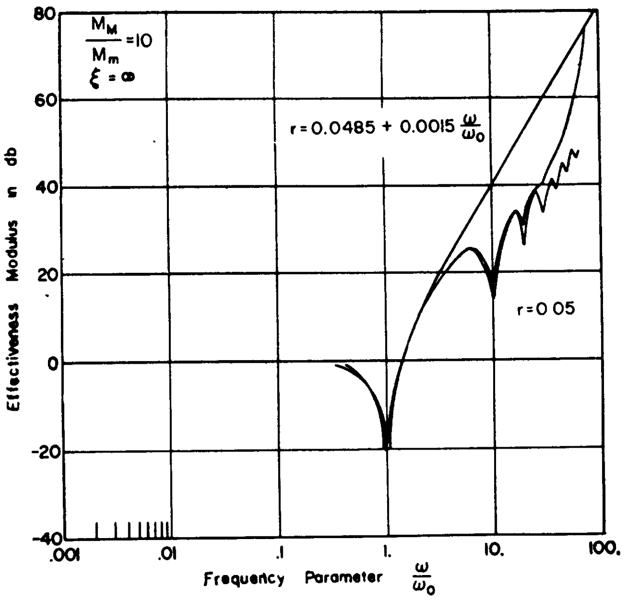

Fig. 27 – Effectiveness modulus in db versus frequency ratio ω/ω_0; rigid machine, continuous mount, infinite foundation impedance; mount damping factor increasing with frequency. For comparison the unidentified curve shows the effectiveness of an undamped massless spring with stiffness constant independent of frequency

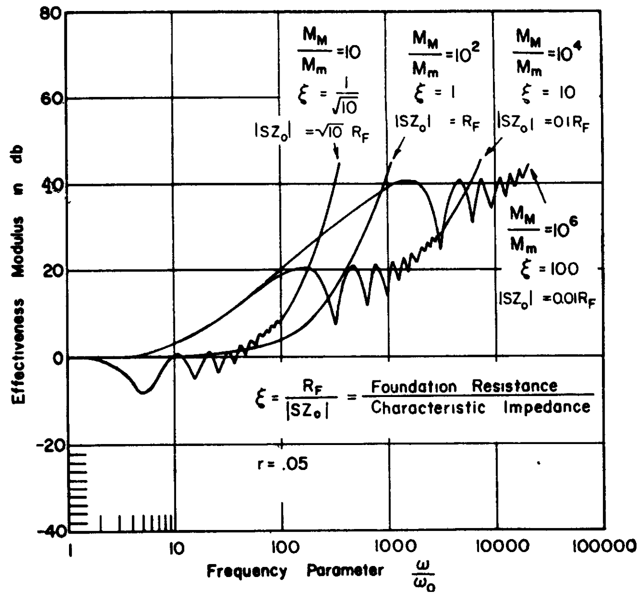

Fig. 28 – Effectiveness modulus in db versus frequency ratio ω/ω_0; rigid machine, continuous mount, resistive foundation; machine mass constant, mount stiffness constant, effectiveness varied by varying mount characteristic impedance

spectrum of the exciter, since to assure spring-dashpot mount performance, the frequency constant should be about 10 times the highest exciting frequency. Once these numbers are known, the design can be started. Mount material, isolation mechanism, and mount dimensions should be selected to meet the stiffness and frequency constant requirements, in addition to those of static load, deflection under shock, drift, temperature, oil resistance, and the like.

If the frequency constant requirement for spring-dashpot mount performance can not be met in a realistic design, an attempt should be made to choose $\frac{c}{\iota}$ to avoid coincidences between exciting and standing-wave resonant frequencies.[23]

If the exciting frequencies are all very high, it may be possible to use the high-frequency, high-effectiveness range of a mount. (See curves in Fig. 22 for $\frac{M_M}{M_m} = 10$ for $\beta\iota > 20$.) In this case, the mount should be designed so that $\frac{c}{\iota}$ is much lower than the lowest exciting frequency.

In either case, the mount damping factor should be as high as possible, commensurate with mount phase velocity fairly constant or decreasing with frequency.

Conclusion

The evaluation of massless mechanically paralleled spring-dashpot mounts isolating rigid and nonrigid machines from nonrigid foundations has been investigated using the concept of mount effectiveness.

Curves have been presented for estimating the effectiveness of spring-dashpot mounts isolating rigid and nonrigid machines from nonrigid foundations, provided that:

1. The machine-mount-foundation system is such that it can be represented by the model in Fig. 4, where all vibratory forces and velocities are colinear.

2. The machine, mount, and foundation impedances are known as functions of frequency.

3. The machine and foundation impedances are predominantly resistive or reactive.

4. The mount loss factor $\frac{\omega R_m}{K_m}$ is near 0.1.

Analytical expressions for computing effectiveness in cases for which the curves are inadequate are also presented (Equations 13–33).

The effectiveness of certain types of continuous mounts (Equation 34) isolating rigid machines from nonrigid foundations has also been investigated.

The spring-dashpot mount investigations show that:

1. The softer a mount is, the more effective it will be, if coincidences (or near coincidences) do not occur between exciting frequencies and the natural frequencies of the machine-mount-foundation system.

2. If the machine and foundation impedances are predominantely resistive or reactive and if the magnitude of one is much greater than the magnitude of the other, the effectiveness of the mount will be determined by the ratio of the smaller impedance to the magnitude of the mount reactance, if this ratio

[23] The most important factor in determining standing-wave resonant frequencies is the mount's frequency constant $\frac{c}{\iota}$; however, machine and foundation impedances influence these frequencies and must be considered in computing them.

is greater than about 4. (See Figs. 5–11 and Equation 23).

3. Vibration can be magnified by a mount if machine and foundation impedances are both masslike, or if one is masslike, and the other springlike or resistive (Figs. 5, 7–9); vibration is never magnified by a mount if both machine and foundation impedances are springlike or resistive, or if one is springlike and the other resistive (Figs. 6, 10, and 11).

4. The maximum magnification of vibration that a mount can cause is controlled by its loss factor (Equations 19, 24, and 29; Figs. 5, 7–9, and 12–14).

The investigations of continuous mounts show that:

1. A continuous mount acts like a massless mechanically paralleled spring-dashpot mount at frequencies whose wavelength in the mount is 10 or more times its largest physical dimension.

2. The effectiveness of a continuous mount at its lower standing-wave resonant frequencies can be 5–20 or more db less than spring-dashpot theory predicts, depending on the mount's damping factor; the higher the damping factor, the more effective the mount.

3. The effectiveness of a continuous mount at its lower standing-wave resonant frequencies is less than spring-dashpot mount theory predicts, whether the foundation impedance is large or small relative to the mount characteristic impedance.

4. A continuous mount will either provide no isolation or will magnify the vibration in the frequency range containing its lower standing-wave resonant frequencies, if the foundation impedance is resistive or reactive and if its magnitude is less than the magnitude of the mount characteristic impedance.

5. The effectiveness of a continuous mount, depending on its damping factor, can be much higher than spring-dashpot mount theory predicts at exciting frequencies much higher than the mount frequency constant. (See curves in Fig. 22 for $\frac{M_M}{M_m} = 10$).

6. The standing-wave resonant frequencies of a machine-mount foundation system depend not only on the characteristics of the mount, but also on the machine and foundation impedances. For a rigid machine isolated from a resistive or reactive foundation by a continuous mount, whose characteristic impedance is much lower than the foundation impedance, the standing-wave resonant frequencies will be those frequencies for which the mount length is an integral number of half wavelengths; if the mount characteristic impedance is much larger than the foundation impedance, the standing-wave resonant frequencies will be those frequencies for which the mount length is an odd number of quarter wavelengths. Other relationships between foundation impedance and mount characteristic impedance give intermediate standing-wave resonant frequencies.

The problem of mount design has also been discussed briefly and the need for specifying, in addition to mount stiffness and damping factor, a quantity which determines at what frequency wave effects become important has been pointed out. The use of frequency constant $\frac{c}{l}$, the ratio of phase velocity in the mount to mount length, has been suggested for this purpose for the simple types of mounts discussed here.

Acknowledgment

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APPENDIX I

Summary of Mount Formulas

A number of equations, most of which are not given in the text, but which are useful in dealing with mounts described by Equation 34 are listed below.[17]

1. Real part G of the elastic modulus:

$$G = \rho c^2 \frac{1 - r^2}{[1 + r^2]^2} \tag{50}$$

2. Viscosity coefficient μ:

$$\mu = \frac{\rho c^2}{\omega} \frac{2r}{[1 + r^2]^2} \tag{51}$$

3. Mount stiffness K_m:

$$K_m = G \frac{S}{l} \tag{52}$$

4. Mount resistance R_m:

$$R_m = \mu \frac{S}{l} \tag{53}$$

5. Complex elastic modulus $\overline{G}$:

$$\overline{G} = G + j\omega\mu \tag{54}$$

6. Magnitude of the complex elastic modulus G:

$$|G| = \sqrt{G^2 + (\omega\mu)^2} = \frac{\rho c^2}{1 + r^2} \tag{55}$$

7. Complex characteristic impedance per unit area:

$$Z_o = -j\frac{\overline{G}\gamma}{\omega} = \frac{\rho c}{1+r^2}(1+jr) \quad (56)$$

8. Magnitude of the complex characteristic impedance per unit area:

$$|Z_o| = \frac{\rho c}{\sqrt{1+r^2}} \quad (57)$$

9. The driving point impedance Z_D of a mechanical transmission line of complex characteristic impedance SZ_o and complex propagation constant γ terminated by a mechanical impedance Z_T:

$$Z_D = SZ_o \frac{SZ_o \sinh \gamma\iota + Z_T \cosh \gamma\iota}{SZ_o \cosh \gamma\iota + Z_T \sinh \gamma\iota} \quad (58)$$

10. The complex transmissibility T_m of a continuous mount:

$$T_m = \frac{1}{\cosh \gamma\iota} \quad (59)$$

11. The complex transmissibility T of a machine-continuous mount system:

$$T = \frac{1}{\cosh \gamma\iota + \frac{Z_M}{SZ_o}\sinh \gamma\iota} \quad (60)$$

12. The magnitude of T:

$$|T| = [(1+B_1)\sinh^2 r\beta\iota + \cos^2 \beta\iota + B_1 \sin^2 \beta\iota + B_2 \sin 2\beta\iota + B_3 \sinh 2r\beta\iota]^{-\frac{1}{2}} \quad (61)$$

where:

$$B_1 = \frac{|Z_M|^2}{|SZ_o|^2}$$

$$B_2 = \frac{SZ_o^{\,I} R_M - SZ_o^{\,R} X_M}{|SZ_o|^2}$$

$$B_3 = \frac{SZ_o^{\,R} R_M + SZ_o^{\,I} X_M}{|SZ_o|^2}$$

$$SZ_o = S(Z_o^{\,R} + jZ_o^{\,I})$$

$$Z_M = R_M + jX_M$$

13. Mount mass M_m:

$$M_m = \rho S\iota \quad (62)$$

14. Complex effectiveness E:

$$E = \cosh \gamma\iota + \left(\frac{SZ_o + \frac{Z_M Z_F}{SZ_o}}{Z_M + Z_F}\right)\sinh \gamma\iota \quad (35)$$

15. Magnitude of E:

The magnitude of E is given by:

$$|E| = [(1+B_1)\sinh^2 r\beta\iota + \cos^2 \beta\iota + B_1 \sin^2 \beta\iota + B_2 \sin 2\beta\iota + B_3 \sinh 2r\beta\iota]^{\frac{1}{2}} \quad (63)$$

where:

$$B_1 = \frac{1 + \frac{4r}{1+r^2}(\phi\psi + \xi\sigma) + 2\left(\frac{1-r^2}{1+r^2}\right)(\phi\xi - \sigma\psi)}{(\phi+\xi)^2 + (\sigma+\psi)^2} + \frac{(\xi^2+\psi^2)(\phi^2+\sigma^2)}{(\phi+\xi)^2 + (\sigma+\psi)^2}$$

$$B_2 = \frac{\sigma + \psi - r(\phi+\xi) + (\phi^2+\sigma^2)(r\xi - \psi)}{[(\phi+\xi)^2 + (\sigma+\psi)^2]\sqrt{1+r^2}} + \frac{(\xi^2+\psi^2)(r\phi - \sigma)}{[(\phi+\xi)^2 + (\sigma+\psi)^2]\sqrt{1+r^2}}$$

$$B_3 = \frac{\phi + \xi + r(\sigma+\psi) + (\xi + r\psi)(\phi^2+\sigma^2)}{[(\phi+\xi)^2 + (\sigma+\psi)^2]\sqrt{1+r^2}} + \frac{(\phi + r\sigma)(\xi^2+\psi^2)}{[(\phi+\xi)^2 + (\sigma+\psi)^2]\sqrt{1+r^2}}$$

16. Mount frequency constant $\frac{c}{\iota}$ in terms of mount stiffness K_m and M_m mass:

$$\frac{c}{\iota} = \left[\frac{(1+r^2)^2}{1-r^2}\frac{K_m}{M_m}\right]^{\frac{1}{2}} \quad (64)$$

17. Ratio of the frequency constant $\frac{c}{\iota}$ to the natural frequency f_o of the machine-mount system:

$$\frac{\frac{c}{\iota}}{f_o} = 2\pi\sqrt{\frac{M_M}{M_m}\frac{1+r^2}{1-r^2}} \quad (65)$$

where:

ρ = Density of the mount material
c = Phase velocity in the mount
$\gamma = \alpha + j\beta$, the propagation constant of the mount
$j = \sqrt{-1}$
α = Attenuation per unit length in the mount
$\beta = \frac{\omega}{c}$, the phase constant of the mount
$r = \frac{\alpha}{\beta}$, the damping factor of the mount
ι = Length of the mount
S = Cross-sectional area of the mount
$Z_M = R_M + jX_M$, the machine impedance looking toward the machine at the machine-mount juncture
$Z_F = R_F + jX_F$, the foundation impedance looking toward the foundation at the foundation-mount juncture,

$$\phi = \frac{R_M}{|SZ_o|}$$

$$\sigma = \frac{X_M}{|SZ_o|}$$

$$\xi = \frac{R_F}{|SZ_o|}$$

$$\psi = \frac{X_F}{|SZ_o|}$$

APPENDIX II

Effectiveness in Terms of Mobility

Equation 10, the effectiveness equation for a spring-dashpot mount isolating a nonrigid machine from a nonrigid foundation, was derived using the concept of mechanical impedance. Mobility, that is, reciprocal impedance, might have been used as well.

Defining the mobilities of the machine, mount, and foundation respectively by:

$$Y_M = \frac{1}{Z_M} \tag{66}$$

$$Y_m = \frac{1}{Z_m} \tag{67}$$

$$Y_F = \frac{1}{Z_F} \tag{68}$$

one can show from Equation 10 that the effectiveness in terms of mobility is given by:

$$E = \frac{Y_M + Y_m + Y_F}{Y_M + Y_F} \tag{69}$$

Equation 69 may be simpler to use for computational purposes than Equation 10.

APPENDIX III

Notation

c = Phase velocity in the mount
E = Mount effectiveness
f = Frequency in cps
f_o = Natural frequency of machine-mount system;
G = Real part of the elastic modulus
$\overline{G}$ = Complex elastic modulus;
$j = \sqrt{-1}$
K_m = Mount stiffness
k = Stiffness ratio
ι = Mount length
M_M = Machine or equipment mass
M_m = Mount mass, that is, mass of resilient element in mount
m = Mass ratio, $m = \frac{M_M}{M_m}$
R = Mechanical resistance
R_F = Foundation resistance
R_M = Machine resistance
R_m = Mount resistance
$r = \frac{\alpha}{\beta}$ = Mount damping factor
r_F = Ratio of the foundation resistance to the magnitude of the mount reactance, $r_F = \frac{R_F}{|X_m|}$
r_M = Ratio of the machine resistance to the magnitude of the mount reactance, $r_M = \frac{R_M}{|X_m|}$
r_m = Ratio of the mount resistance to the magnitude of the mount reactance, $r_m = \frac{R_m}{|X_m|} = \frac{\omega R_m}{K_m}$ = Mount loss factor
r_1, r_2 = Ratio of the machine or foundation resistance to the magnitude of the mount reactance
S = Mount cross-sectional area
T = Transmissibility of a machine-mount system
T_m = Transmissibility of a mount
X = Mechanical reactance
X_F = Foundation reactance
X_M = Machine reactance
X_m = Mount reactance
x_F = Ratio of the foundation reactance to the magnitude of the mount reactance, $x_F = \frac{X_F}{|X_m|}$
x_M = Ratio of the machine reactance to the magnitude of the mount reactance, $x_M = \frac{X_M}{|X_m|}$
x_1, x_2 = Ratio of the machine or foundation reactance to the magnitude of the mount reactance
$x(y)$ = Instantaneous displacement of the mount cross-section from its equilibrium position
Y_F = Foundation mobility
Y_m = Mount mobility
Y_M = Machine mobility
y = Coordinate of the mount cross-section measured from one end of the mount
Z = Mechanical impedance, $Z = R + jX$
Z_F = Foundation impedance
Z_M = Machine impedance
Z_m = Mount impedance
Z_o = Mount characteristic impedance per unit area
z_F = Ratio of the foundation impedance to the magnitude of the mount reactance, $z_F = \frac{R_F + jX_F}{|X_m|}$
z_M = Ratio of the machine impedance to the magnitude of the mount reactance, $z_M = \frac{R_M + jX_M}{|X_m|}$
z_m = Ratio of the mount impedance to the magnitude of the mount reactance, $z_m = r_m - j$
z_1, z_2 = Ratio of the machine or foundation impedance to the magnitude of the mount reactance
α = Attenuation per unit length in the mount
β = Phase constant, $\beta = \frac{\omega}{c}$
γ = Propagation constant; $\gamma = \alpha + j\beta$
μ = Viscosity coefficient
ξ = Ratio of the foundation resistance to the magnitude of the mount characteristic impedance, $\xi = \frac{R_F}{|SZ_o|}$
ρ = Density of the mount material
σ = Ratio of the machine reactance to the magnitude of the mount characteristic impedance, $\sigma = \frac{X_M}{|SZ_o|}$
ϕ = Ratio of the machine resistance to the magnitude of the mount characteristic impedance, $\phi = \frac{R_M}{|SZ_o|}$
ψ = Ratio of the foundation reactance to the magnitude of the mount characteristic impedance, $\psi = \frac{X_F}{|SZ_o|}$
ω = Frequency in radians per sec, $\omega = 2\pi f$
ω_o = Natural frequency of machine-mount system, $\omega_o = \sqrt{\frac{K_m}{M_M}}$

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