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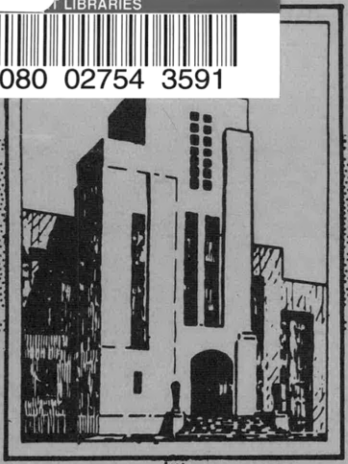

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DAVID TAYLOR MODEL BASIN

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RUDDER-EXCITED HULL VIBRATION ON USS FORREST SHERMAN (DD 931)

(A Problem in Hydroelasticity)

by

R.T. McGoldrick

[redacted]

STRUCTURAL MECHANICS LABORATORY
RESEARCH AND DEVELOPMENT REPORT

June 1960

Report 1431

DEPARTMENT OF THE NAVY
DAVID TAYLOR MODEL BASIN
WASHINGTON 7, D.C.

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Ser 7-149
1 August 1960

From: Commanding Officer and Director, David Taylor Model Basin
To: Chief, Bureau of Ships (335) (in duplicate)

Subj: Rudder-excited hull vibration on USS FORREST SHERMAN (DD 931); investigation of

Encl: (1) DATMOBAS Report 1431 entitled "Rudder-Excited Hull Vibration on USS FORREST SHERMAN (DD 931)" 3 copies

1. During the initial trials of USS FORREST SHERMAN (DD 931) a three-noded horizontal vibration of the hull was excited at a constant frequency over an extended range of speed. The Boston Naval Shipyard traced this vibration to the twin rudders and eliminated it by reversal of the rudder toe-angle setting.

2. The report describes both experimental and analytical investigations made in the attempt to explain the phenomenon, and advocates the hypothesis of a subcritical control-surface flutter condition.

3. The material given in this report was presented as a paper to the Annual Meeting of The Society of Naval Architects and Marine Engineers on 12-13 November 1959. It is reprinted as enclosure (1), together with the discussions and the author's closure.

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RUDDER-EXCITED HULL VIBRATION ON USS FORREST SHERMAN (DD 931)

(A Problem in Hydroelasticity)

by

R.T. McGoldrick

Reprint of Paper Presented at the
Annual Meeting, New York, New York
November 12–13, 1959, of The Society of
Naval Architects and Marine Engineers

June 1960 Report 1431

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. . . . (A Problem in Hydroelasticity)

By R. T. McGoldrick,[1] *Member*

Abstract

The vibration phenomenon encountered on USS *Forrest Sherman* (DD 931) was quite unlike the usual cases of ship vibration in that the hull was set into a three-noded horizontal vibration whose frequency remained constant over a considerable range of speed. The Boston Naval Shipyard traced this vibration to the twin rudders and eliminated it by reversal of the rudder toe-angle setting. No simple explanation of the phenomenon was apparent at the time of its occurrence, but it appeared that any mechanism producing such a condition would necessarily involve hydroelastic effects. This problem falls within the spheres of interest of both the Hydroelasticity Panel of the Hydrodynamics Committee and the Hydro-Structure Vibration Panel of the Hull Structure Committee. While officially the project was handled under strictly naval jurisdiction, these panels maintained an interest in its progress because of representation of the David Taylor Model Basin in their memberships. The author explores several conceivable explanations, and, while acknowledging contrary opinions, accounts for the phenomenon as due to a subcritical control-surface flutter condition.

Introduction

WITH the exception of slamming in a seaway, when severe hull vibration is encountered on ships, its frequency normally coincides either with the blade frequency (shaft RPM times number of blades per propeller) or the shaft frequency (shaft RPM). However, on the initial trials of USS Forrest Sherman (DD 931), there was encountered in the upper speed range a horizontal vibration of the entire hull at a frequency which remained constant at 4 cps regardless of shaft speed.

A comprehensive discussion of the early investigation of this vibration is given in reference [1].[2] It is shown there how the source of the vibration was traced to the twin rudders by the personnel of the Boston Naval Shipyard. Actually a correlation was discovered between the severity of the vibration and the toe-angle[3] setting of the twin rudders. Above all, the Boston Naval Shipyard personnel found a practical solution to the immediate problem by changing the toe-angle setting from a few degrees in to a few degrees out.

In spite of the elimination of the objectionable condition there remained the problem (equally important from the point of view of the Bureau of Ships) of explaining the phenomenon. A number of possible explanations were proposed and quickly eliminated during the acute phase of the initial investigation. Once it had been demon-

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[1] Physicist, Consultant to Director, David Taylor Model Basin, Navy Department, Washington, D. C.

[2] Numbers in brackets designate References at the end of the paper.

Presented at the Annual Meeting, New York, N. Y., November 12–13, 1959, of THE SOCIETY OF NAVAL ARCHITECTS AND MARINE ENGINEERS.

[3] For consistency with references [1] and [4] the toe angles are specified as "in" when the trailing edges are closer than the leading edges. The opposite convention is used in reference [12].

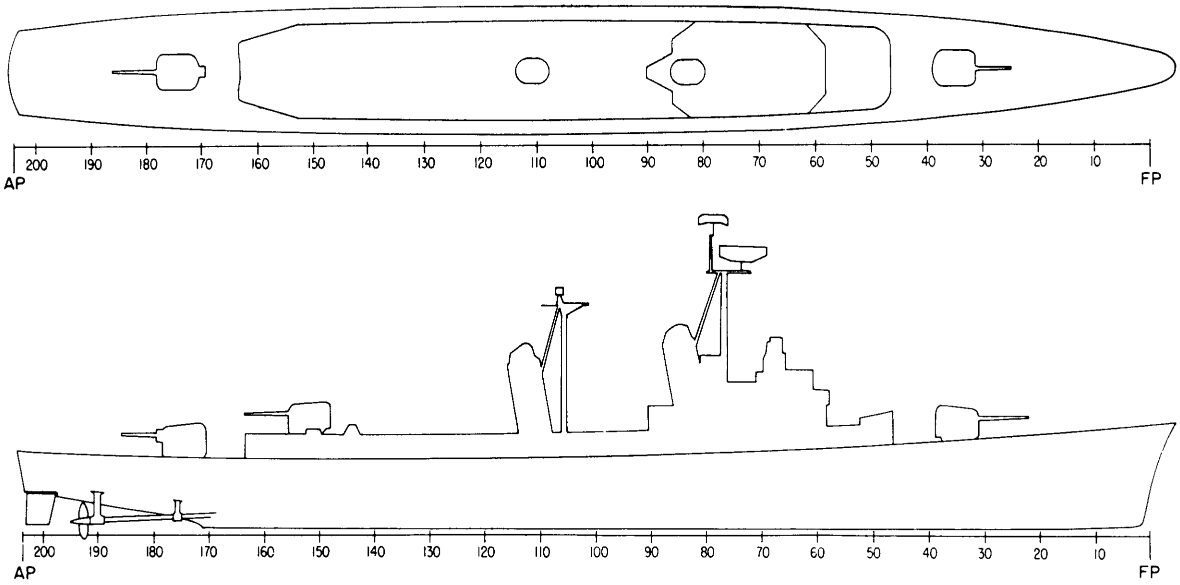

Fig. 1 Plan view and outboard profile of DD 931 Class destroyer

strated experimentally that the vibration came from the rudders, there remained for serious consideration only those hypotheses consistent with this fact. It is only with the latter that this paper is concerned.

The David Taylor Model Basin was authorized by the Bureau of Ships to make a general study of this particular hull-vibration phenomenon. It might have been expected that, once the source was located, the explanation would be obvious. This, however, was not the case. It appeared that more than one mechanism could produce the observed condition. In fact this proved to be a typical problem in "hydroelasticity," a field in which the hydrodynamic aspects and the structural or elastic aspects of problems are so interrelated that they cannot be treated independently.

Among other facts brought out in the surveys made by the Boston Naval Shipyard [1] was that vibration of the hull in the three-noded horizontal mode was accompanied by torsional oscillations of the rudders about the rudder-stock axes. A large 4-cps frequency component was present in the oscillograms of both hull vibration and torsional strain in the rudder stocks.

At the stage of the investigation with which this paper is concerned, the process of sifting hypotheses still seemed to admit at least the following possibilities:

(*a*) That random hydrodynamic forces acting on the rudders could excite the hull in the observed mode.

(*b*) That a self-excited or flutter type of oscillation, involving the coupling of torsional vibration of the rudders with flexural vibration of the hull, could be present.

(*c*) That periodic flow excitation not associated with flutter could produce the phenomenon.

Hence the program finally adopted by the David Taylor Model Basin involved several phases. Among these was a complete vibration survey of a ship of the class, including the use of the new three-mass, 40,000-lb vibration generator [2] designed by the Model Basin. This machine was installed on a sister ship, USS Decatur (DD 936). It was hoped that such a survey would indicate any unusual characteristics of the hull that would render it particularly sensitive to vibration in the observed mode.

Flow studies were initiated both on the ship and in the laboratory in order to reveal possible departures from the predicted flow in the vicinity of the twin rudders. In addition, a project was established to investigate the basic phenomenon of control-surface flutter.

The inclusion of the study of control-surface flutter in this program did not imply that the vibration problem on DD 931 had been identified as a flutter problem. As a matter of fact, although reference [3] indicates that rudder flutter was a recognized phenomenon at the time, a flutter condition had not previously been identified on United States naval ships. Since past experience had not indicated the necessity of considering the possibility of flutter in the design of control-surface systems, procedures for antici-

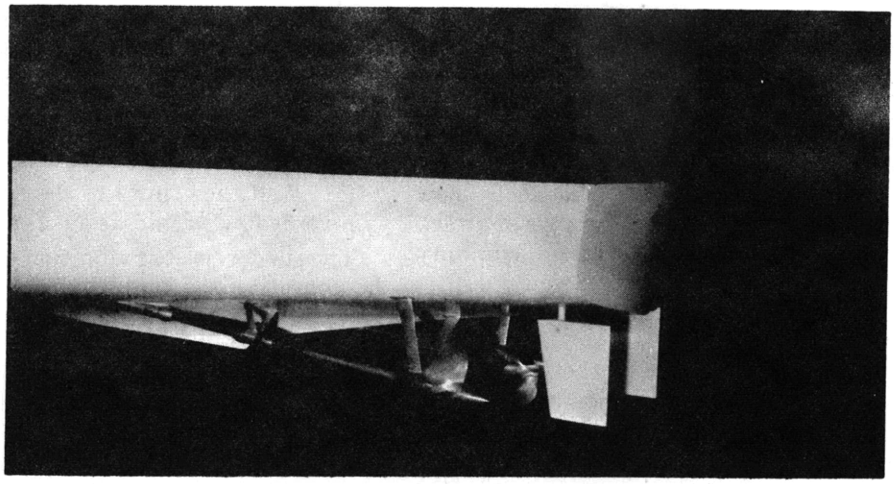

Fig. 2 View of model of DD 931 Class destroyer showing rudders and propellers. Fixed fairings at the top of the rudders were installed for later tests

pating such a condition were not available to designers. Although a vast amount of research on flutter had been conducted in the aeronautical field, and the general methods of aeroelasticity were believed applicable to its counterpart in naval architecture, namely, hydroelasticity, it was recognized that there were great differences in the magnitudes of the physical quantities involved in the two fields. A desired by-product of the flutter study was the effect of these differences.

The purpose of this paper is to summarize the results of the various phases of the investigation of the vibration phenomenon encountered, and to evaluate them to the extent possible in the present state of development of the field of hydroelasticity. With a view toward covering all aspects of the investigation, all the experimental procedures are presented under one main heading and all the analytical predictions under another heading.

Experiments Conducted by the David Taylor Model Basin

As indicated earlier, the investigation of rudder-excited hull vibration on the DD 931 Class involved a variety of experimental procedures covering flow studies, control-surface flutter studies, full-scale vibration-generator tests, and repetition of earlier underway hull-vibration surveys with the rudders at various toe-angle settings. Only brief mention is made of the details of these experimental procedures in this paper. Full details of the experiments will be found in the references. Fig. 1 shows a plan view and an outboard profile of the DD 931-Class destroyer and Fig. 2 shows the rudders and propellers of the original towing model.

Flow Studies

A distinction is made in this paper between "flow studies" and "control-surface flutter studies," although both involve flow. Use is made of the term "flow excitation" in a special sense to indicate any departure from steady flow that would result in time-varying lateral or lift forces on the rudders, but with the restriction that this condition is not itself created by torsional oscillations of the rudders. This covers a variety of possible conditions falling under such designations as flow separation, cavitation, vortex-shedding, venting, and turbulence, but does not include forces arising from control-surface flutter. The latter type of forces may exist under a uniform velocity field just forward of the control surface member once the oscillations have been started. The forces considered here under "flow excitation" could be of several different types. They could be periodic with one or more frequency components; they could be random and definable only in a statistical sense; or they could be considered as a succession of transient disturbances each approximating a single impulse.

To explain the observed hull vibration as due to flow excitation in the foregoing meaning of the term, requires showing not only that certain forces exist but that the structural characteristics of the ship are such that under these forces the hull would respond in the manner actually observed.

Hence it was necessary to carry out studies to determine the type of flow to rudders in the speed range in which vibration was observed and to determine experimentally or predict analytically the over-all response of the system to various types of forcing functions acting at the rudders.

As shown later, if the torsional natural frequency of the rudder system happens to fall close to the frequency of one of the horizontal modes of the hull, a normal mode of the entire system can involve angular displacements of the rudders about the rudder-stock axes. If the system were then set into vibration in such a mode by flow excitation, the resulting changes in angle of attack of the rudder would, in turn, set up vibratory lift forces. These forces, however, would not, in general, be in the phase required to sustain the vibration.

The flow studies undertaken by the Model Basin in connection with this investigation are discussed in detail in reference [4]. Full-scale observations were made to determine whether the rudders were completely immersed at high speed or whether venting or aeration were present. Temporary transom plates also were installed as shown in Fig. 2 of reference [4]. Since the latter had a negligible effect on the vibration, it was concluded that venting was not the cause of the phenomenon. Sampling-tube tests for aeration also gave negative results. It was, however, noted in dry dock that paint removal was much more pronounced on the outboard faces of the rudders than on the inboard faces. This was taken as an indication of cavitation on the suction faces, and subsequent flow studies at the Model Basin showed that with the initial toe-in angle setting of 3 deg the direction of flow would be such as to make the outboard faces the suction faces. On this basis cavitation was to be expected on the outboard faces, and a reversal of the toe angle was expected to reduce or eliminate the cavitation. As a result of these studies [4], it was considered possible that the cavitation could fluctuate in step with the oscillation of the rudders resulting in a self-sustaining hull vibration which would not be classified as flutter in the sense in which that term is used in this paper.

Control-Surface Flutter Studies

The term "flutter" appears in the technical literature under a variety of meanings, and it will not contribute to the understanding of the vibration phenomenon on DD 931 to inject a discussion in semantics at this point. As used in this paper the term "control-surface flutter" is a type of vibration that can exist in a mass-elastic system containing a control-surface member, moving at constant velocity in a fluid medium and involving angular oscillations of the control surface with accompanying variations in lift force. In this sense of the term such a phenomenon as the vortex-excited cantilever vibration of circular cylinders is not considered flutter. Even though vortices may be generated by the phenomenon the following conditions are considered essential to the flutter mechanism:

(*a*) Energy is derived from the alternating lift force.

(*b*) And the control of the flow of this energy is governed by the vibration of the system itself.

The rudder-excited vibration on DD 931 would be considered a control-surface flutter phenomenon if, when the hull was vibrating in its three-noded horizontal mode, the twin rudders were executing angular oscillations of the same frequency, and the phase relations were such that energy going into the hull and contributing to maintain the vibration was transmitted by the alternating component of the resulting lift forces.

It is important to observe that, although in the aircraft field the occurrence of flutter is usually accompanied by violent vibration resulting in damage or destruction of an airplane, this is not an essential feature of the basic flutter mechanism. The critical flutter speed occurs at the borderline between stable and unstable oscillations. The vibratory forces depend on the speed, but the stability of the oscillations depends also on the damping. The latter has both hydrodynamic and structural components. As the speed is increased, a point may be reached at which a balance of the energy dissipated by the system and that derived from the flow exists. If, as the speed increases, the input energy increases but the damping does not, then the oscillations will continue to increase in amplitude. However, conditions may be such that, at a still higher speed, energy input decreases, and then the system will return to a stable condition. It is also possible that the damping increases with amplitude thus limiting the buildup of vibration. Hence, while violence of the oscillations is a common characteristic of flutter, it is not an essential feature of the phenomenon as viewed here.

In fact it will be pointed out later that "subcritical flutter" or flutter below the region of actual instability may be very important in the field of naval architecture. This condition amounts to a reduction in damping of the over-all system due to the oscillations of the control surface. Here energy is still extracted from the flow but it is sufficient to account for only part of the energy dissipated, and the system has increased sensitivity to other sources of disturbance that may be present.

As in the case of flow excitation, in order to explore the flutter hypothesis it is essential to determine whether, under the observed conditions,

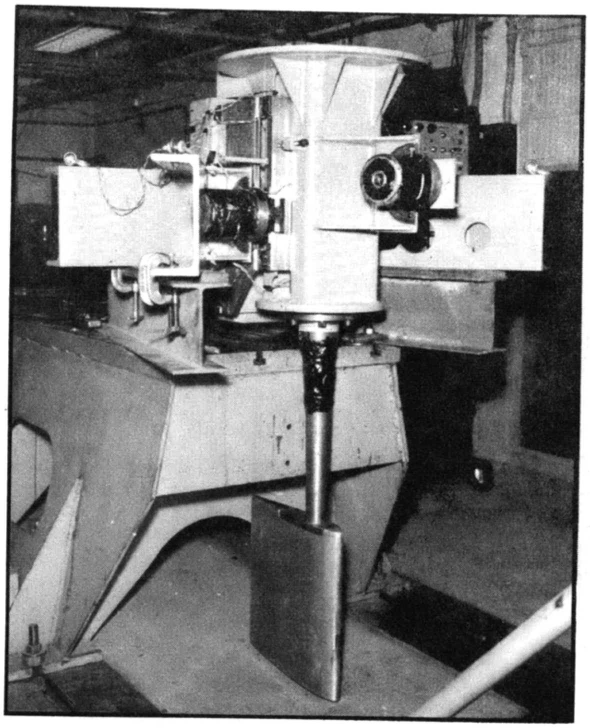

Fig. 3 The TMB flutter apparatus. This view shows apparatus set up in laboratory for checking instrumentation. Entire assembly to which hydrofoil is attached is free to rotate with it and carries a disk at top to which balance weights are attached. The two cylindrical members with horizontal axes are eddy-current dampers

there would exist exciting forces arising from the oscillations of the rudder in the field of flow and whether this particular hull actually has such structural characteristics that it would respond to these forces in the manner and to the degree actually observed. In other words, to substantiate the flutter hypothesis in this case it would be necessary to make a complete flutter analysis of the rudder-hull system and to show that the predicted flutter speed agreed with the observed critical speed.

The basic control-surface flutter study initiated at the Model Basin as a result of the vibration phenomenon on DD 931 is discussed in detail in reference [5]. The apparatus built for this purpose was not a model of the DD 931 rudder-hull system, and no assumption was made at the time of its design that the problem on that ship had been identified as one of control-surface flutter. However, it was hoped that, if a method of prediction could be developed which agreed with experimental results in the towing basin, the same general method could be applied to the ship case.

The TMB apparatus illustrated in Fig. 3 is based on the "classical" two-degree flutter system, and the basic constants used in the various analyses applied to it were determined experimentally in advance of the towing-carriage runs. An elementary analysis for the conditions of exact tuning of the two degrees of freedom, zero hydrodynamic moment and a lift force derived only from the steady-flow relation, predicted a flutter speed of 9.5 knots for the maximum (downstream) permissible mass unbalance. This analysis also predicted that instability would exist at all higher speeds with increasingly violent flutter.

The tests on the towing carriage actually indicated a condition at about this speed (speed for maximum vibration of the apparatus 9.3 knots) at which there was so large a magnification of the measured carriage vibration as to be inexplicable except in terms of subcritical flutter action. However, beyond this speed the system returned to a more highly damped condition, and a buildup to the amplitude level around 9.3 knots was not found at any higher speed up to the limit of 20 knots set for the apparatus.

During the test runs a steady angular displacement of the hydrofoil was observed which increased with speed, indicating that the foil was not hydrodynamically balanced (as had been assumed in its design). A hydrodynamic-moment term was, therefore, added to the equations used in the elementary analysis. The simple expedient was used of assigning a value to the distance from the axis (which in this case was at the forward quarter-chord point) to the center of lift which would account for the observed steady moment. The use of such a term in the vibration equations was equivalent to basing the oscillatory moment as well as the oscillatory lift force also on their steady values. When this was done, it was found from analog calculations that assigning to the lever arm of the center of lift a value larger than the value derived from steady deflection measurements would change the predicted characteristics of the system from one having a single point of transition from stability to instability to one exhibiting first stability, then instability, then stability again.

In view of known sources of carriage vibration at the frequency observed at 9.3 knots, it would have been imprudent to accept the observed buildup of vibration as being due to flutter action in the absence of independent evidence. Such evidence was furnished, (*a*) by a marked lead in phase of the angular displacement of the hydrofoil on the translational displacement (a necessary condition in the elementary theory in order to obtain a negative damping action from the flow) and, (*b*) by a series of impact test measurements

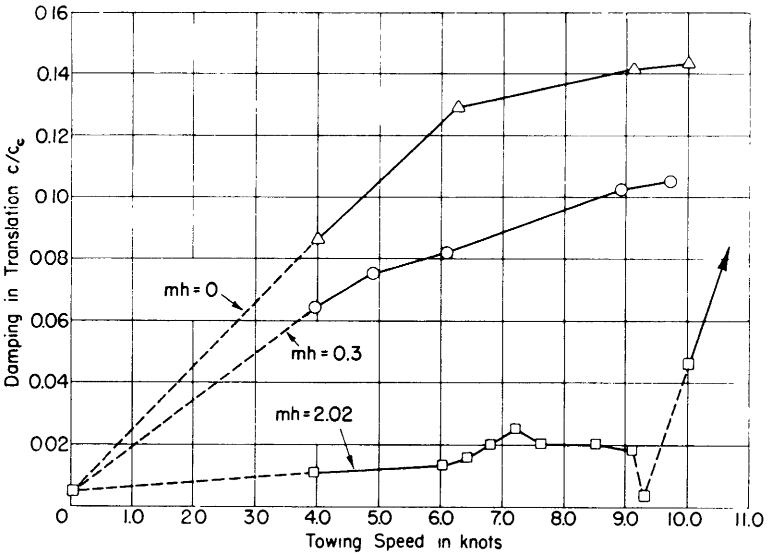

Fig. 4 TMB flutter apparatus-damping values derived from impact tests. Minimum value of C/C_c for $mh = 2.02$ at 9.3 knots was derived from observed magnification of carriage vibration

made at different settings of mass unbalance of the apparatus. From the latter it was possible to plot the damping of the system in translation as a function of carriage speed.

Fig. 4 based on reference [5] shows such a plot. The contrast between the damping characteristic of the system under a condition of mass balance and that of the system with maximum permissible mass unbalance is striking. This is discussed in detail in [5]. It is merely pointed out here that the structural damping could not change by an amount anywhere near large enough to account for the observed difference in over-all damping between the curve for the mass-balanced condition and the curve for the condition of maximum mass unbalance in the vicinity of 9 knots. This point is discussed further in this paper under the heading "Analytical Predictions."

The possibilities of the TMB flutter apparatus have not been explored exhaustively at this writing. Nevertheless, it is the belief of the author not only that the control-surface flutter problem has been brought into focus but, also that the basic vibration phenomenon originally existing on USS *Forrest Sherman* was reproduced in the towing basin with this apparatus, and that pertinent information has been obtained from these experiments for evaluating the role of control-surface flutter in the field of naval architecture.

It is indicated that, in the experiments with the TMB flutter apparatus just discussed, the marked decrease in the over-all damping of the system was due to "flutter action." By this is meant that, without any abnormal flow condition but merely as a result of a suitably phased alternating lift force, the over-all damping of the apparatus decreased to a minimum in a narrow speed range, and this minimum was of so low a level as to render the system unusually sensitive to external excitation. Since the oscillatory condition was still stable, this has been called a case of "subcritical flutter." At a slightly higher speed this system exhibited an increased margin of stability.

Vibration-Generator Tests on DD 936

At this point an entirely different phase of the program requires attention. The experiments conducted by the Model Basin to determine the vibration characteristics of the DD 931 hull are discussed in detail in reference [6]. The TMB three-mass 40,000-lb vibration generator illustrated in Fig. 7 of reference [2] was used in these experiments. This machine was installed directly over the propellers (Frame 192) on the main deck. While the test program included a complete survey of all hull modes that could be identified, this paper is concerned only with that part of the test program which contributed to the investigation of the rudder-excited hull vibration. This includes:

a) The experimental determination of the normal-mode pattern for the three-noded horizontal mode.

b) And the resonant response of the hull in the two and three-noded horizontal modes to known

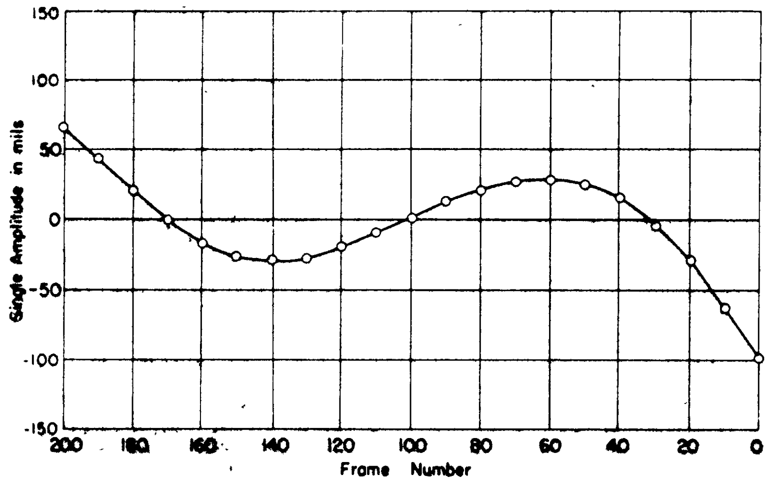

Fig. 5 Horizontal amplitude profile obtained on DD 936 in test with vibration generator at 234 cpm.; driving force 25,000 lb.

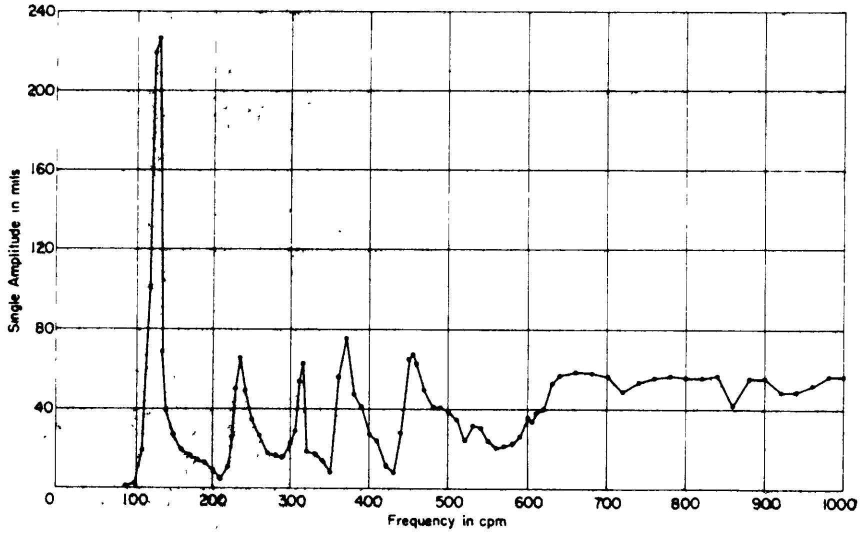

Fig. 6 Amplitudes measured at driving point in horizontal speed run with vibration generator on DD 936. Curve is plotted on basis of a driving force equal to 800 × $(\text{cpm}/41.9)^2$ lb, although at speeds above 320 rpm the eccentricity was actually reduced so as not to exceed the rated force of machine of 40,000 lb.

Table I Comparison of Peak Levels of Horizontal Hull Vibration in Underway Surveys on DD 936 with Rudders Toed In 3 Deg and Toed Out 1½ Deg

	Peak double amplitude of horizontal vibration at aft perpendicular, in.	
Horizontal hull mode	Rudder toed in 3 deg	Rudder toed out 1½ deg
2-noded.........	0.415	0.240
3-noded.........	0.159	0.026

sinusoidal forces applied at the main deck level.

The frequencies of the two and three-noded horizontal modes of DD 936 for approximate full-load condition were found to be 126 cpm and 234 cpm, respectively. The three-noded horizontal mode (plotted in Fig. 5) was found experimentally to have almost negligible torsion. On the other hand the next mode, with a natural frequency of 310 cpm, was found to be predominantly a one-noded torsional mode with negligible flexure. The test displacement on this occasion was 3600 tons, and the mean draft was 13 ft 8 in.

Fig. 6 shows the response at the driving point (main deck above the propellers, Frame 192) to sinusoidal horizontal forces increasing as the square of the frequency. It is particularly to be noted that the peak amplitude at the three-noded resonance is only about 30 per cent of that at the two-noded resonance.

Repetition of Underway Vibration Surveys

In addition to the vibration-generator test the Model Basin conducted vibration surveys on DD 936 under two different settings of the toe angles of the twin rudders. These settings corresponded with those existing during surveys previously made by the Boston Naval Shipyard on DD 931 and discussed in reference [1].

Table 1 shows the peak levels of horizontal vibration in the first two horizontal modes found at the aft perpendicular in the two underway vibration surveys. Here it is to be noted that, in the survey with the rudders in the toe-out condition, the peak level of first-mode horizontal hull vibration was 58 per cent of the value for the survey with the rudders in the toe-in condition, whereas in the case of the second horizontal mode (the mode under investigation) the corresponding ratio was only 16 per cent.

The agenda for the vibration tests on DD 936 also called for the experimental determination of the torsional natural frequencies of the rudders both in dry dock and in still water. As pointed out in reference [6] both of these attempts were hampered by the other modes of vibration that were excited, and it is not assumed in this paper that a direct experimental determination of these frequencies is available. However, the plot of the ratio of angular displacement of the rudder to rectilinear displacement of the hull at the rudder-stock location (Fig. 10 of reference [6]) shows a peak at 290 cpm. Moreover it is shown in the following section that the response of the hull to horizontal excitation in the vibration-generator tests can be used to obtain an estimate of the natural frequency of the rudders in still water.

Analytical Predictions

Hull Response Characteristics

It is now in order to inquire as to what light was shed on the phenomenon under discussion by analytical studies. It soon became apparent that, if the torsional oscillations of the rudders were tuned to a frequency near that of the second horizontal mode of vibration of the hull, this could not only produce conditions favorable for flutter (in the sense defined in this paper) but also could have a marked effect on the response of the hull itself to other sources of excitation.

UNIVAC calculations of the torsion-horizontal bending modes of the hull were carried out on the basis of the method applied to SS *Gopher Mariner* [7]. These indicated that there would be a pair of modes each having one node in torsion and three in horizontal flexure just as for *Gopher Mariner* (Figs. 33*a* and 33*b* of reference [7]), but, in contrast with the latter, the prediction for DD 931 was that torsion would be relatively small only in the lower of these modes, the higher being predominantly a one-noded torsional mode. For *Gopher Mariner*, the ratio of torsion to bending was of the same order for both modes of the pair, but the phase relations were reversed. The mode of most interest in the present problem is the lower one of the pair, which is predominantly a three-noded horizontal flexural mode. As previously stated, the experimental results verified that torsion was very small in this mode. Without torsion of the hull, however, there was no obvious explanation of the reduced amplitude response of the hull found in the second horizontal mode as contrasted with the first horizontal mode shown in Fig. 6.

In exploring the vertical modes of a hull with a vibration generator located at the aft perpendicular it is invariably found that, if the eccentric setting (mass unbalance) is held fixed, the amplitude produced at the driving point in the three-noded mode will be as large or larger than that produced in the two-noded mode. A similar situation would normally be expected for the horizontal modes if torsion-free. The first two horizontal modes of DD 936 had so little torsion that they could be considered practically torsion-free. However, in the vibration-generator test the response in the second horizontal mode was only about 30 per cent of that in the first horizontal mode. It should be noted that a similar situation was found in the vibration-generator tests on DD 865 (Fig. 17 of reference [8]).

In the absence of large torsion of the hull the only apparent explanation of this phenomenon is that the rudders, having a torsional natural frequency near the frequency of the second horizontal mode of the hull, act in the role of "sprung masses," and thus drastically modify the hull vibratory-response characteristics. It is not assumed here that only rudders can produce such effects. Theoretically any mass elastically attached to the hull could do so. Moreover, the mass need not be located in the vicinity of the driving force. In the DD 931 Class the only other structure that seemed to be a possible competitor with the rudders in this respect was the mainmast. However, as shown in reference [6] the critical horizontal frequencies of the mainmast were clear of the immediate vicinity of 240 cpm.

Only quite recently have attempts been made to calculate the effect of sprung masses on the normal modes of a hull. The ideal case of a single sprung mass attached to a uniform beam is discussed in reference [9], and methods of estimating the effect on actual ships are discussed in reference [10]. In order to make the rudders, which oscillate torsionally about vertical axes, tractable mathematically from the sprung-mass point of view, they were reduced to an "equivalent sprung mass" at the location of the rudder stocks by introducing an equivalent mass-spring combination which would have the required natural frequency. The mass in this case was taken as the mass of the two rudders plus the allowance for virtual mass in translation as used in the flutter analysis. This simplified approach neglects the moment applied to the hull when the rudder oscillates torsionally. A series of calculations was then made on UNIVAC of the horizontal modes (here considered torsion-free) with the equivalent sprung mass attached at the aft perpendicular. Each calculation was based on a particular value of the effective spring constant of the sprung-mass system, and thus each represented a particular tuning of the torsional oscillations of the rudders. Examples of such calculations are given in detail in Appendix 1.

Calculations of the driving-point impedance of entire hull derived from the UNIVAC calculations of the horizontal torsion-free normal modes by the formulas given in Appendix 1, yielded a set of values that would explain the response pattern obtained in the vibration-generator speed runs, Fig. 6, if the rudders were tuned either exactly to or slightly above the frequency of the three-noded mode of the hull as computed without the sprung-mass action. This effect of the rudders on the vibratory-response characteristics of the hull is of considerable interest aside from its relation to the rudder-excited vibration problem considered in this paper. The analytical study just mentioned combined with the experimentally determined hull-response characteristic provided substantial evidence that the natural frequencies of the rudders (with the ship dead in the water) were close to the frequency of the three-noded horizontal mode of the hull.

Flutter Speeds

The first attempt to calculate a flutter speed for USS *Forrest Sherman* was based on the elementary analysis given in reference [5]. This involved the following assumptions:

(*a*) The lift forces developed during oscillatory motion can be taken from the relation between lift force and angle of attack in steady flow.

(*b*) The rudders are hydrodynamically balanced at all angles of attack and all speeds, and the oscillatory moment can be neglected. (This analysis does, however, provide for the rotary-inertia effect of the surrounding water and an angular damping action to which the water may contribute.)

(*c*) The torsional natural frequency of the rudders coincides exactly with the frequency which the three-noded horizontal flexural mode of the hull would have if the rudders were locked.

(*d*) The twin rudders act in unison and, for the purpose of analysis, can be combined into a single equivalent rudder.

(*e*) Torsion of the hull in the horizontal mode having three flexural nodes is negligible.

(*f*) While the rudder may actually undergo combined torsion and bending, its mode in this case may be considered predominantly torsional.

(*g*) In any of its flexural modes of vibration the entire hull can be reduced to an effective system of one degree of freedom referred to an arbitrary driving point.

The direct equation for the flutter speed that can be derived under such simplifying assumptions is given in Appendix 2, where it is shown that this formula yielded flutter speeds from 32 to 54 knots for DD 931 for the range of parameters used. Here account was taken of the boost in inflow velocity to the rudders caused by the action of the propellers. The range of values corresponds to upper and lower limits in the estimate of the damping of the angular oscillations of the rudders.

As shown in reference [5], in the flutter analysis of the TMB apparatus, the procedure was to start with the simplest possible assumptions and to introduce additional terms in the simultaneous equations only as required to improve the correlation with experimental results.

Since it was observed with this apparatus that a steady angular displacement of the hydrofoil developed which increased with towing speed, a term was introduced into the equations to allow for a hydrodynamic moment. This yielded a pair of simultaneous equations (given in Appendix 2) from which a "flutter determinant" was readily derived. This determinant is formed from the coefficients of the rectilinear and angular displacements in the equations after they are converted from differential to algebraic form. Exploratory calculations of damping characteristics based on these equations have shown encouraging correlation with the behavior of the apparatus in the experiments that have so far been conducted in the towing basin. These are discussed in detail in reference [5].

According to the definition adopted earlier in this paper a condition in which the alternating lift force has the effect of reducing the over-all damping of the rudder-hull system without complete nullification of damping is a subcritical flutter condition. It is obvious that, if the damping of any mode of a mass-elastic system is drastically reduced when the speed at which the system is moving in a fluid medium reaches some definite value, the system may then execute relatively large vibrations due to external disturbances, even though this vibration may never build up to the level required for a structural failure. It is essential to note, however, that, if the system has such properties, a method of flutter prediction that explores only for zero or negative overall damping (unstable oscillations) will not detect such a subcritical condition no matter how closely the damping approaches zero.

A series of calculations was next made for DD 931 based on these simultaneous equations with various sets of values of the coefficients. These are summarized in Appendix 2 where mention is also made of some calculations based on the classical method of determining oscillatory lift forces and moments given by Theodorsen [11]. The former calculations indicate that the hydrodynamic moment may have a marked effect on the critical speed. The predicted speeds at which the over-all damping of the rudder-hull system is reduced to a very low value fall within the range of speeds in which rudder-excited hull vibration was actually encountered on DD 931 with the orginal toe-in rudder-angle setting.

The rudder toe angle does not appear explicitly in any of the equations given in Appendix 2. The correlation of flutter speeds (whether critical or subcritical) with toe angle requires nonlinear relations of lift force versus angle of attack or of hydrodynamic moment versus angle of attack. If these relations remained linear even under the complex velocity field in which these rudders operate, the critical speed would be independent of the toe-angle setting.

Appraisal of Suggested Hypotheses

The author is aware that there are differences of opinion as to the explanation of the rudder-excited hull vibration initially encountered on USS *Forrest Sherman*. This seems, however, all the more reason for bringing the subject to the attention of the Society at this time. Fortunately the consequences on this ship were not serious, and a practical solution was soon found. As a matter of fact the vibration difficulty on this particular class had already been eliminated before the investigation discussed in this paper was well under way. Hence, from the standpoint of making the ships of the class acceptable, the investigation was no longer urgent. Clearly, however, it was important from the naval architect's point of view to determine whether the phenomenon encountered need be considered in future designs, and this question is by no means restricted to naval ships.

The author believes that an explanation of the rudder-excited hull-vibration phenomenon on DD 931 must be sought among the four following possibilities three of which were suggested in the "Introduction":

(*a*) That the shedding of vortices from the trailing edges of the rudders at a frequency of 4 cps produced the observed hull vibration and this vortex shedding ceased with reversal of the toe-angle setting.

(*b*) That, under the initial 3-deg toe-in rudder-angle setting, cavitation developed at high ship speed and produced a random excitation at the rudders. The rudder-hull system responded to this excitation predominantly in the observed mode which involved three nodes in horizontal flexure. On reversal of the toe angle the cavitation disappeared and hence also the critical vibration.

(*c*) That rudder forces arising from cavitation fluctuating at a frequency of 4 cps produced the phenomenon. Since this cavitation disappeared on reversal of the toe angle, the critical vibration also disappeared.

(*d*) That, under the initial 3-deg toe-in rudder-angle setting, there developed at high speed a subcritical flutter condition involving the coupling of torsional oscillations of the rudders with the three-noded horizontal flexural oscillations of the hull. Cavitation may have been present. By lowering the over-all damping the flutter condition increased the sensitivity of the rudder-hull system in the observed mode to the excitation at the bow which is always present when a ship is underway even in calm seas. The tuning of the torsional oscillations of the rudders varies both with ship speed and mean angle of attack. On reversal of the toe angle the rudders were detuned sufficiently with respect to the natural frequency of the hull at high speed to eliminate the subcritical flutter condition. Hence the rudder-hull system was no longer sensitive in the mode previously excited to an objectionable degree and the amplitude of vibration dropped to a permissible level.

In choosing explanation (*d*) the author calls attention to the following points:

1 References [12] and [13] indicate that vortex-shedding frequencies for hydrofoils with such relatively sharp trailing edges as those of the DD 391 rudders would be about 50 times as high as the observed frequency of 4 cps. No detailed investigation of the vortex-shedding phenomenon was undertaken at the David Taylor Model Basin in connection with this problem since there was no indication from the full scale flow studies that vortex shedding was the cause of the phenomenon.

2 While random excitation due to cavitation at the rudders might have evoked greater response of the rudder-hull system in the three-noded horizontal mode than in the two-noded (on account of the sprung-mass effect of the rudders), a change of toe-angle should then have caused proportional decreases in the levels of amplitude in both modes. There has never been reported for the DD 931 Class any correlation between the level of the two-noded horizontal vibration and the toe-angle setting.

3 If cavitation, fluctuating at 4 cps, were the cause of the phenomenon, such fluctuations could be either coupled with the oscillations of the rudders or independent of these oscillations. In the latter case it is unlikely that the frequency of fluctuation would remain constant over any considerable range of ship speeds. Moreover, under such circumstances there would be no reason to expect the fluctuations at the two rudders to be in phase with each other, and the lift forces developed would be as likely to cancel each other as to reinforce. A fluctuating cavitation coupled with the oscillations would thus seem more likely than the uncoupled counterpart. However, in the coupled case, the resulting lift variations would be expected to peak when the hull was in the condition of maximum elastic deflection. Such a phase relation would not be favorable for maintaining the oscillations. Furthermore the angular amplitudes involved in this case were actually so small in comparison with the steady toe-angle changes required to produce the phenomenon (of the order of 1:30) that a fluctuation each cycle seems improbable (see 6 and 7 following).

4 Although the TMB flutter apparatus was not a model of the rudder-hull system of the ship, the two systems can be treated by the same basic differential equations when the hull is reduced to an equivalent lumped system for the mode of vibration in question. In the experiments in the towing basin a speed range was found in which the sensitivity of the apparatus to vibration was increased to a degree corresponding to that by which the sensitivity of the rudder-hull system of DD 931 increased in the three-noded horizontal mode at high ship speeds. Since it was demonstrated that the increased sensitivity of the TMB apparatus was due to subcritical flutter action, it is plausible that a similar condition existed on DD 931 under the initial toe-angle setting.

5 A simplified analysis applied to the TMB apparatus predicted a true flutter speed (unstable oscillations) very close to that at which a subcritical flutter condition (stable oscillations) was actually found. When it was assumed that under the initial toe-angle setting the rudders would be brought into close tuning with the three-noded mode of the hull at high speeds, this same analysis predicted a subcritical range for the ship close to that within which the objectionable vibration actually occurred.

6 As shown in Fig. 14 of reference [1] there was evidence during one of the trial runs that the natural frequency of the rudder-hull system of DD 931 involving three nodes in horizontal flexure was 222 cpm for one trial condition as against the 240-cpm frequency found for the critical vibration. In fact, the author of reference [1] called attention to this point. Such differences in frequency are characteristic of flutter as pointed out in reference [14]. Under the fluctuating-cavitation hypothesis the frequency would be expected to coincide with a natural frequency of the rudder-hull system (in this case 222 cpm).

7 Steering tests were conducted by the Boston Naval Shipyard [1] in which 3-deg right turns and 3-deg left turns were executed under the initial toe-angle setting (toe-angle 3 deg in). These tests showed that, whereas on the straight course both rudders oscillated in synchronism with the hull, on the 3-deg turns the rudder which became lined up with the longitudinal axis of the ship ceased to oscillate. This suggests that the latter rudder was then thrown well out of tune with the frequency of the hull vibration. This correlation of a variation of tuning of the rudder with angle of attack at high speeds is consistent with the flutter hypothesis. Reference [3] emphasizes the effect of "hydrodynamic stiffness" on both the tuning of rudders and the flutter speeds. The fluctuating-cavitation hypothesis, on the other hand, requires that the rudder which ceased to oscillate did so because cavitation ceased. However, under the fluctuating-cavitation hypothesis, since the rudder-hull system is then driven

at the same frequency by the other rudder, both rudders should continue to oscillate in accordance with the normal-mode pattern. In other words, when a normal mode of a mass-elastic system is excited, the response pattern does not change with a shift in the point of application of the driving force. Only the amplitude changes.

8 · The flutter hypothesis does not rule out the presence of cavitation. In fact, cavitation could contribute to producing flutter without fluctuating significantly simply by changing the slope of either the curve of lift force versus angle of attack or that of moment versus angle of attack.

Critical or subcritical flutter speeds vary with these slopes. Hence, if cavitation existed at high speeds under the initial toe-angle setting and the system were in a subcritical flutter condition, reversal of the toe angle could eliminate the vibration by raising the subcritical flutter speed beyond the range for this ship.

9 The fact that under the initial toe-angle setting the amplitude of the hull fluctuated widely (see page A5 of reference [1]) supports the subcritical flutter hypothesis. The vertical vibration of a hull in its fundamental mode caused by bow excitation usually varies in amplitude in a similar fashion. This supports the argument that in this case the role of the rudders was simply to make the three-noded horizontal mode the most sensitive mode as long as the ship speed was high and the toe-angle was in. The presence of two-noded horizontal hull vibration under either toe-angle condition confirms the presence of horizontal bow excitation.

Design Considerations

From the designer's point of view the most important aspect of the investigation of rudder-excited hull vibration on USS *Forrest Sherman* is the consideration of changes in design that would ensure against the recurrence of such a phenomenon. These changes are enumerated here without discussion of other factors which might hamper their adoption. The change of the toe-angle setting, of course, has been adopted for the DD 931 Class, but obviously it is desirable to be able to set the toe angle for minimum power without regard to the risk of hull vibration. Furthermore, the expedient of reversing the toe angle is restricted to twin-rudder designs.

The phenomenon encountered would not be likely to occur if the torsional natural frequency of the rudder could be kept well above the range of significant natural frequencies of the hull. If a lower limit of 600 cpm were selected as a natural-frequency criterion, this would mean, for the DD 931 Class, increasing the torsional stiffness of the rudder system by a factor of about 6. It is not clear at this writing why the torsional frequency was so low, but apparently the same condition exists in the DD 692 Class destroyers. This indicates much more compliance in the steering system than can be attributed to the torsional flexibility of the rudder stock itself. Moreover, the mode of the rudder is probably a torsion-bending mode in which bending flexibility also contributes to lowering the frequency. Since the mass unbalance provides the principal coupling between torsional oscillations of the rudder and the horizontal flexural vibration of the hull, mass balance, if attainable, would be expected to eliminate such a condition. However, on account of the relatively large virtual-mass effect of the water, it may be impossible to attain mass balance without a shift of the axis that would produce a large hydrodynamic unbalance.

Thus the basic conditions that would ensure against the recurrence of such a phenomenon appear difficult to realize in practice. Pending the accumulation of further information on the subject it would be prudent in design to apply a simplified analysis such as that given in Appendix 2 and discussed in more detail in reference [5]. The assumptions in this analysis are on the conservative side in the sense that they tend to underestimate critical speeds; and, if the predicted critical range were found to be well above the speed range of the ship, no necessity of pursuing the subject further would be indicated.

It is important for the designer to realize that the type of flutter considered in this paper is not similar to the torsion-bending flutter of aircraft wings so widely discussed in the literature. In the latter type of flutter the vibration of the hull would not play a significant role and the analysis would involve only the control-surface system itself, the hull being considered fixed.

Conclusions

In the author's opinion it is not safe for the naval architect to assume that control-surface flutter phenomena are out of the range of ship-operating speeds. This, however, does not imply that laborious calculations must be initiated in the design of all control-surface systems. At least for the type of flutter discussed in this paper simplified procedures can be used to determine the likelihood of occurrence. However, just as in the aircraft field, numerous other types of flutter (to which the analyses discussed in this paper are not directly applicable) are conceivable in the field of naval architecture. The fact that, with the exception of the singing propeller, other phenomena of this type have been rare in the marine field indicates the likelihood that the other critical speeds are quite high.

There is need for continuing research on both the hydrodynamic and elastic aspects of the basic flutter phenomenon. The forces and moments acting on oscillating hydrofoils should be explored further and the limits of frequency, amplitude, and Strouhal number over which simplified flutter analyses are feasible should be determined. The accumulation of the following types of experimental data would be helpful to the naval designer in this connection:

a) Oscillograms of torsional strain in rudder stocks obtained during full-scale trials and the analysis of these for the variation in torque for small angles of attack and an indication of the frequency of any torsional oscillations in the rudder stock.

b) Records of hull vibration in the lower horizontal flexural modes, especially when such vibrations occur under calm-sea conditions.

c) Experimental data on the relation between rudder lift force and angle of attack for small variations around the zero value of the latter.

d) Observation of cavitation at the surfaces of rudders in service and correlation of such cavitation with hull vibration as suggested in reference [4].

Aside from the possibility of rudder-excited hull vibration it is indicated that rudders can introduce marked effects on the vibratory-response characteristics of hulls in their horizontal modes. This is apparently due to their "sprung-mass" effect when their natural frequencies fall in the range of significant hull mode frequencies.

The need for expanded research in the general field of hydroelasticity, which until recently was largely neglected by the naval architect, is clearly indicated.

While, because of the ingenuity of the Boston Naval Shipyard personnel, a practical solution was found for the problem of rudder-excited vibration on the DD 931 Class, it should not be necessary to impose such restrictions as applied to these ships in order to avoid intolerable hull vibration.

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APPENDIX 1

Calculations Relating to Hull Response in the Second Horizontal Mode

In this appendix those calculations relating to the hull-vibration characteristics of the DD 931 Class which are particularly pertinent to the rudder-excited vibration problem are summarized. The normal-mode calculations were made by the method previously applied to SS *Gopher Mariner* and discussed in reference [7]. Some improvements were made in the evaluation of parameters for these calculations as pointed out in reference [6].

The DD 931 Class has the following principal dimensions: LBP 407 ft; beam 44.4 ft, depth 25 ft 3 in., and draft 14.7 ft. All UNIVAC calculations were made for the full-load displacement of 3800 tons. The displacement varied by as much as 5 per cent during the course of the tests.

Table 2 gives the UNIVAC calculations of the first three torsion-horizontal bending modes. Table 3 gives the first three horizontal modes treated as torsion-free. Table 4 gives a set of UNIVAC calculations in which the rudders were treated as an equivalent sprung mass (see reference [10]) tuned to the frequency of the second horizontal mode as computed without the sprung-mass effect. Table 5 gives a set of calculations similar to those given in Table 4 but with the rudders tuned to a frequency higher than that of the second mode as computed without the sprung-mass effect.

In predictions of hull response to a sinusoidal driving force as well as in some of the flutter calculations used was made of the concept of mechanical impedance.

The mechanical impedance Z of a system of one degree of freedom, if based on displacement, is given by the relation

Table 2 Torsion-Horizontal Bending Modes of DD 931 Class Hull as Computed on the UNIVAC

n	Y	γ	ϕ	V	M	T
$\omega = 12.09$ (115 cpm)						
00	−1.00000	0.01066	−0.00027	0	0	
01	−0.77694	0.01066	−0.00012	−405	−8751	504
02	−0.55226	0.01042	0.00016	−921	−28310	733
03	−0.33797	0.00980	0.00032	−1306	−55870	1832
04	−0.14136	0.00886	0.00041	−1615	−89927	2075
05	0.03101	0.00765	0.00046	−1782	−127669	1945
06	0.17174	0.00620	0.00045	−1735	−164355	2128
07	0.27878	0.00468	0.00039	−1496	−195757	2638
08	0.35051	0.00311	0.00033	−1147	−219700	3160
09	0.38947	0.00168	0.00028	−707	−234435	2949
10	0.39612	0.00027	0.00022	−156	−237694	3312
11	0.36973	−0.00114	0.00013	457	−228119	4286
12	0.30975	−0.00262	0.00003	960	−208049	4488
13	0.21715	−0.00408	−0.00008	1350	−179687	3537
14	0.09411	−0.00543	−0.00020	1622	−145556	1015
15	−0.05478	−0.00667	−0.00033	1772	−108058	59
16	−0.23102	−0.00802	−0.00050	1689	−72425	51
17	−0.43840	−0.00802	−0.00080	1446	−42117	268
18	−0.67381	−0.01081	−0.00130	1078	−19598	673
19	−0.93296	−0.01194	−0.00200	687	−5281	−116
20	−1.19849	−0.01254	−0.00251	251	0	−116
20A		−0.01254		0		0
$\omega = 23.9$ (228 cpm)						
00	−1.00000	0.01602	−0.01000	0	0	0
01	−0.64972	0.01602	−0.00922	−1563	−34695	−120
02	−0.29685	0.01505	−0.00770	−3232	−105068	−3080
03	0.01027	0.01276	−0.00649	−3981	−191043	−4786
04	0.24573	0.00955	−0.00562	−3929	−275977	−8282
05	0.39197	0.00583	−0.00492	−2827	−337875	−12246
06	0.43787	0.00201	−0.00427	−516	−350109	−19161
07	0.39848	−0.00124	−0.00373	1813	−312233	−20932
08	0.29525	−0.00374	−0.00324	3735	−233456	−23147
09	0.15419	−0.00525	−0.00272	5195	−123523	−28400
10	−0.00617	−0.00599	−0.00214	6040	4915	−34732
11	−0.16596	−0.00596	−0.00153	5985	132069	−40274
12	−0.30281	−0.00511	−0.00089	5102	239897	−42988
13	−0.39590	−0.00343	−0.00023	3602	315674	−40787
14	−0.42938	−0.00106	0.00019	1727	351669	−23481
15	−0.38291	0.00192	0.00045	−907	331598	−7618
16	−0.23426	0.00607	0.00102	−3164	263496	−7462
17	0.03929	0.01133	0.00210	−4135	175281	−5104
18	0.44176	0.01694	0.00410	−4016	90087	−4224
19	0.96434	0.02212	0.00746	−3002	26641	−1420
20	1.52868	0.02513	0.01017	−1248	−1	−192
20A		0.02513		0		0
$\omega = 29.3$ (280 cpm)						
00	−1.00000	0.03436	0.68090	0	0	0
01	−0.34098	0.03436	0.65966	−4458	−100004	222973
02	0.15723	0.03156	0.60734	−6988	−256662	626547
03	0.59751	0.02597	0.53559	−13297	−542444	1094609
04	0.88368	0.01686	0.46107	−11933	−798456	1522964
05	0.85735	0.00608	0.37106	−1282	−831385	2216530
06	0.63388	−0.00331	0.27169	6830	−688119	3041354
07	0.27666	−0.00971	0.17584	15353	−364224	3563640
08	−0.14392	−0.01262	0.08936	18864	33652	3880829
09	−0.54311	−0.01240	0.00832	17321	401040	4055252
10	−0.87889	−0.01000	−0.06861	12845	676288	4084193
11	−1.09325	−0.00594	−0.13591	4014	766000	3808890
12	−1.15946	−0.00096	−0.19720	−5019	664985	3479403
13	−1.12553	0.00368	−0.25504	−10110	455239	3008473
14	−1.11839	0.00710	−0.31089	−1036	425650	2468338
15	−1.07430	0.01072	−0.35700	5607	526264	1777876
16	−0.80613	0.01730	−0.39588	−4426	420326	1165247
17	−0.37948	0.02570	−0.43227	−5275	299148	751073
18	0.22070	0.03527	−0.46508	−4203	202755	427018
19	1.05438	0.04693	−0.50013	−6327	66509	259836
20	2.10401	0.05445	−0.51586	−3068	7	87668
20A		0.05445		0		0

Table 3 Horizontal Flexural Modes of DD 931 Class Hull

(Considered Torsion-Free as Computed on the UNIVAC)

n	Y	γ	ϕ	V	M	T
$\omega = 12.3$ (118 cpm)						
00	−1.00000	0.01065	0.00000	0	0	0
01	−0.77772	0.01065	0.00000	−420	−8562	0
02	−0.55410	0.01041	0.00000	−955	−28000	0
03	−0.33987	0.00980	0.00000	−1355	−55580	0
04	−0.14281	0.00886	0.00000	−1677	−89712	0
05	0.03022	0.00765	0.00000	−1853	−127421	0
06	0.17172	0.00621	0.00000	−1805	−164164	0
07	0.27947	0.00468	0.00000	−1559	−195893	0
08	0.35176	0.00312	0.00000	−1197	−220253	0
09	0.39105	0.00169	0.00000	−738	−235285	0
10	0.39780	0.00027	0.00000	−166	−238669	0
11	0.37127	−0.00115	0.00000	472	−229053	0
12	0.31094	−0.00264	0.00000	995	−208790	0
13	0.21787	−0.00410	0.00000	1407	−180137	0
14	0.09426	−0.00545	0.00000	1683	−145874	0
15	−0.05518	−0.00669	0.00000	1836	−108493	0
16	−0.23196	−0.00805	0.00000	1750	−72872	0
17	−0.43936	−0.00950	0.00000	1496	−42408	0
18	−0.67360	−0.01086	0.00000	1113	−19740	0
19	−0.92973	−0.01199	0.00000	710	−5288	0
20	−1.19232	−0.01259	0.00000	259	0	0
20A		−0.01259	0.00000	0		0
$\omega = 24.47$ (234 cpm)						
00	−1.00000	0.01607	0.00000	0	0	0
01	−0.65100	0.01607	0.00000	−1664	−33881	0
02	−0.30087	0.01512	0.00000	−3435	−103783	0
03	−0.00777	0.01286	0.00000	−4294	−191177	0
04	0.24648	0.00965	0.00000	−4265	−277977	0
05	0.39509	0.00590	0.00000	−3065	−340355	0
06	0.44201	0.00205	0.00000	−611	−352798	0
07	0.40209	−0.00122	0.00000	1898	−314172	0
08	0.29712	−0.00373	0.00000	3959	−233597	0
09	0.15390	−0.00525	0.00000	5491	−121843	0
10	−0.00879	−0.00598	0.00000	6382	8048	0
11	−0.17078	−0.00593	0.00000	6327	136804	0
12	−0.30902	−0.00505	0.00000	5374	246182	0
13	−0.40224	−0.00332	0.00000	3753	322568	0
14	−0.43376	−0.00009	0.00000	1739	357969	0
15	−0.38275	0.00213	0.00000	−1050	336593	0
16	−0.22834	0.00634	0.00000	−3422	266937	0
17	0.05038	0.01168	0.00000	−4410	177187	0
18	0.45347	0.01735	0.00000	−4236	90974	0
19	0.96613	0.02258	0.00000	−3161	26647	0
20	1.51838	0.02559	0.00000	−1309	0	0
20A		0.02559		0		0
$\omega = 39.07$ (373 cpm)						
00	−1.00000	0.02134	0.00000	0	0	0
01	−0.51011	0.02134	0.00000	−4244	−86372	0
02	−0.02935	0.01892	0.00000	−7780	−244698	0
03	0.33429	0.01358	0.00000	−7993	−407374	0
04	0.51904	−0.00674	0.00000	−4798	−505032	0
05	0.50225	−0.00007	0.00000	1643	−471583	0
06	0.31266	−0.00540	0.00000	9595	−276314	0
07	0.03886	−0.00797	0.00000	14120	11045	0
08	−0.22836	−0.00788	0.00000	14628	308741	0
09	−0.42470	−0.00587	0.00000	11626	545347	0
10	−0.51200	0.00260	0.00000	5356	654355	0
11	−0.46603	0.00132	0.00000	−2937	594581	0
12	−0.29740	0.00518	0.00000	−9561	400005	0
13	−0.04283	0.00798	0.00000	−13539	124484	0
14	0.24431	0.00891	0.00000	−14085	−162160	0
15	0.47035	0.00754	0.00000	−10079	−267280	0
16	0.54997	0.00294	0.00000	−2647	−421151	0
17	0.40683	−0.00547	0.00000	3415	−351652	0
18	−0.01605	−0.01672	0.00000	6992	−209348	0
19	−0.71724	−0.02876	0.00000	6895	−69019	0
20	−1.54269	−0.03656	0.00000	3391	0	0
20A		−0.03656	0.00000	0		0

$$Z = K - M\omega^2 + jC\omega$$

where

Z = mechanical impedance
K = spring constant
M = mass
C = viscous-damping constant
j = $\sqrt{-1}$
ω = circular frequency

When a normal-mode pattern of a hull is computed on UNIVAC on the basis of an equivalent lumped system (finite-difference approximation), the effective mass of the entire system at any driving point d and for the ith normal flexural mode is given by the relation

$$M_{di} = \frac{\Sigma m Y_i^2}{Y_{di}^2}$$

where Y_i is the displacement at any ship station obtained from the normal-mode calculation and m is the mass lumped at a station for the calculation. Then

Table 4 Horizontal Flexural Modes of DD 931 Hull Calculated on the UNIVAC With the Twin Rudders Treated As an Equivalent Sprung Mass Tuned to Frequency of Second Mode as Given in Table 3

n	Y	y Sprung	M	V	γ	n	Y	y Sprung	M	V	γ
			$\omega = 11.76$ (113 cpm)						$\omega = 27.73$ (264 cpm)		
0	1.0000(00)	1.2995(00)	0.0000(00)	0.0000(00)	−1.1536(−02)	0	1.0000(00)	−3.5467(00)	0.0000(00)	0.0000(00)	−1.0475(−02)
1	7.5601(01)	7.5601(−01)	1.4346(04)	7.0494(02)	−1.1536(−02)	1	8.0676(−01)	8.0676(−01)	−3.0964(04)	−1.5216(03)	−1.0475(−02)
2	5.1491(−01)	5.1491(−01)	3.8351(04)	1.1796(03)	−1.1134(−02)	2	5.6001(−01)	5.6001(−01)	−4.5962(03)	1.2957(03)	−1.1342(−02)
3	2.8878(−01)	2.8878(−01)	6.9268(04)	1.5193(03)	−1.0298(−02)	3	2.9064(−01)	2.9064(−01)	6.3584(04)	3.3504(03)	−1.1442(−02)
4	8.5376(−02)	8.5376(−02)	1.0527(05)	1.7693(03)	−9.1346(−03)	4	3.2508(−02)	3.2508(−02)	1.6024(05)	4.7498(03)	−1.0374(−02)
5	−8.8940(−02)	−8.8940(−02)	1.4323(05)	1.8653(03)	−7.7134(−03)	5	−1.8065(−01)	−1.8065(−01)	2.6104(05)	4.9531(03)	−8.2108(−03)
6	−2.2739(−01)	−2.2739(−01)	1.7860(05)	1.7377(03)	−6.09 9(−03)	6	−3.1686(−01)	−3.1686(−01)	3.3251(05)	3.5122(03)	−5.2611(−03)
7	−3.2900(−01)	−3.2900(−01)	2.0789(05)	1.4396(03)	−4.4340(−03)	7	−3.7049(−01)	−3.7049(−01)	3.5697(05)	1.2018(03)	−2.1687(−03)
8	−3.9305(−01)	−3.9305(−01)	2.2926(05)	1.0501(03)	−2.7708(−03)	8	−3.4747(−01)	−3.4747(−01)	3.3178(05)	−1.2376(03)	6.8700(−04)
9	−4.2295(−01)	−4.2295(−01)	2.4111(05)	5.8212(02)	−1.2806(−03)	9	−2.6625(−01)	−2.6625(−01)	2.5977(05)	−3.5388(03)	2.8436(−03)
10	−4.1968(−01)	−4.1968(−01)	2.4144(05)	1.6492(01)	1.6602(−04)	10	−1.4134(−01)	−1.4134(−01)	1.4746(05)	−5.5191(03)	4.4022(−03)
11	−3.8293(−01)	−3.8293(−01)	2.2925(05)	−5.9932(02)	1.6147(−03)	11	9.6176(−03)	9.6176(−03)	1.1668(04)	−6.6726(03)	5.2869(−03)
12	−3.1253(−01)	−3.1253(−01)	2.0702(05)	−1.0924(03)	3.1048(−03)	12	1.6233(−01)	1.6233(−01)	−1.2272(05)	−6.6037(03)	5.3628(−03)
13	−2.0986(−01)	−2.0986(−01)	1.7708(05)	−1.4710(03)	4.5539(−03)	13	2.9145(−01)	2.9145(−01)	−2.3484(05)	−5.5099(03)	4.5038(−03)
14	−7.7307(−02)	−7.7307(−02)	1.4221(05)	−1.7136(03)	5.8821(−03)	14	3.7453(−01)	3.7453(−01)	−3.0883(05)	−3.6358(03)	2.7424(−03)
15	8.0157(−02)	8.0157(−02)	1.0500(05)	−1.8284(03)	7.0909(−03)	15	3.8082(−01)	3.8082(−01)	−3.1985(05)	−5.4182(02)	1.1736(−04)
16	2.6385(−01)	2.6385(−01)	7.0129(04)	−1.7137(03)	8.4034(−03)	16	2.8342(−01)	2.8342(−01)	−2.6919(05)	2.4899(03)	−3.8809(−03)
17	4.7689(−01)	4.7689(−01)	4.0617(04)	−1.4502(03)	9.8060(−03)	17	5.7091(−02)	5.7091(−02)	−1.8649(05)	4.0638(03)	−9.2647(−03)
18	7.1552(−01)	7.1552(−01)	1.8835(04)	−1.0704(03)	1.1106(−02)	18	−3.0383(−01)	−3.0383(−01)	−9.8645(04)	4.3167(03)	−1.5232(−02)
19	9.7496(−01)	9.7496(−01)	5.0274(03)	−6.7851(02)	1.2189(−02)	19	−7.8621(−01)	−7.8621(−01)	−2.9632(04)	3.3913(03)	−2.0904(−02)
20	1.2405(00)	1.2405(00)	1.0000(−01)	−2.4704(02)	1.2757(−02)	20	−1.3147(00)	−1.3147(00)	1.0000(00)	1.4562(03)	−2.4253(−02)
20A				1.0000(−03)	1.2757(−02)	20A				2.0000(−02)	−2.4253(−02)
			$\omega = 22.16$ (212 cpm)						$\omega = 39.43$ (377 cpm)		
0	1.0000(00)	5.5085(00)	0.0000(00)	0.0000(00)	−2.4508(−02)	0	1.0000(00)	−6.2866(−01)	0.0000(00)	0.0000(00)	−2.0385(−02)
1	4.2894(−01)	4.2894(−01)	1.1235(05)	5.5209(03)	−2.4508(−02)	1	5.3870(−01)	5.3870(−01)	7.2189(04)	3.5474(03)	−2.0385(−02)
2	−8.5456(−02)	−8.5456(−02)	2.4416(05)	6.4772(03)	−2.1362(−02)	2	7.4613(−02)	7.4613(−02)	2.2174(05)	7.3490(03)	−1.8363(−02)
3	−4.8028(−01)	−4.8028(−01)	3.7190(05)	6.2770(03)	−1.6040(−02)	3	−2.8684(−01)	−2.8684(−01)	3.8255(05)	7.9022(03)	−1.3529(−02)
4	−7.2707(−01)	−7.2707(−01)	4.6959(05)	4.8005(03)	−9.7918(−03)	4	−4.8198(−01)	−4.8198(−01)	4.8656(05)	5.1111(03)	−7.1026(−03)
5	−8.1497(−01)	−8.1497(−01)	5.0821(05)	1.8978(03)	−3.4523(−03)	5	−4.8374(−01)	−4.8374(−01)	4.6662(05)	−9.7990(02)	−5.3399(−04)
6	−7.4967(−01)	−7.4967(−01)	4.6237(05)	−2.2525(03)	2.2904(−03)	6	−3.1445(−01)	−3.1445(−01)	2.8800(05)	−8.7773(03)	4.7388(−03)
7	−5.7019(−01)	−5.7019(−01)	3.4551(05)	−5.7425(03)	6.5904(−03)	7	−5.7561(−02)	−5.7561(−02)	1.5089(04)	−1.3411(04)	7.4172(−03)
8	−3.2041(−01)	−3.2041(−01)	1.7987(05)	−8.1393(03)	9.3545(−03)	8	1.9933(−01)	1.9933(−01)	−2.7341(05)	−1.4177(04)	7.5380(−03)
9	−4.3587(−02)	−4.3587(−02)	−1.3330(04)	−9.4941(03)	1.0524(−02)	9	3.9252(−01)	3.9252(−01)	−5.0762(05)	−1.1509(04)	5.7608(−03)
10	2.3103(−01)	2.3103(−01)	−2.1075(05)	−9.7010(03)	1.0444(−02)	10	4.8367(−01)	4.8367(−01)	−6.2177(05)	−5.6092(03)	2.7151(−03)
11	4.7306(−01)	4.7306(−01)	−3.8367(05)	−8.4973(03)	9.1792(−03)	11	4.4761(−01)	4.4761(−01)	−5.7359(05)	2.3676(03)	−1.0155(−03)
12	6.5092(−01)	6.5092(−01)	−5.1257(05)	−6.3345(03)	6.6854(−03)	12	2.9270(−01)	2.9270(−01)	−3.9358(05)	8.8453(03)	−4.7438(−03)
13	7.3798(−01)	7.3798(−01)	−5.8450(05)	−3.5344(03)	3.0974(−03)	13	5.2846(−02)	5.2846(−02)	−1.3248(05)	1.2831(04)	−7.4989(−03)
14	7.1559(−01)	7.1559(−01)	−5.9477(05)	−5.0468(02)	−1.2863(−03)	14	−2.2136(−01)	−2.2136(−01)	1.4261(05)	1.3518(04)	−8.4925(−03)
15	5.6299(−01)	5.6299(−01)	−5.2824(05)	3.2695(03)	−6.3419(−03)	15	−4.4023(−01)	−4.4023(−01)	3.4249(05)	9.8222(03)	−7.2803(−03)
16	2.5420(−01)	2.5420(−01)	−4.0347(05)	6.1309(03)	−1.2945(−02)	16	−5.2154(−01)	−5.2154(−01)	3.9824(05)	2.7397(03)	−2.9992(−03)
17	−2.3884(−01)	−2.3884(−01)	−2.6037(05)	7.0322(03)	−2.1014(−02)	17	−3.9153(−01)	−3.9153(−01)	3.3488(05)	−3.1135(03)	4.9656(−03)
18	−9.1104(−01)	−9.1104(−01)	−1.3101(05)	6.3566(03)	−2.9346(−02)	18	5.6940(−03)	5.6940(−03)	2.0018(05)	−6.6190(03)	1.5682(−02)
19	−1.7386(00)	−1.7386(00)	−3.7705(04)	4.5849(03)	−3.6879(−02)	19	6.6967(−01)	6.6967(−01)	6.6199(04)	−6.5840(03)	2.7193(−02)
20	−2.6202(00)	−2.6202(00)	1.0000(00)	1.8529(03)	−4.1140(−02)	20	1.4533(00)	1.4533(00)	0.0000(00)	−3.2530(03)	3.4673(−02)
20A			1.0000(00)	1.0000(−02)	−4.1140(−02)	20A				1.0000(00)	3.4673(−02)

Table 5 Horizontal Flexural Modes of DD 931 Hull Calculated on the UNIVAC With Rudders Treated As an Equivalent Sprung Mass Tuned to a Frequency Slightly Higher Than That of Second Mode As Given in Table 3 (tuning frequency ω = 26.3)

n		y Sprung	M	V	γ
		ω = 11.78 (113 cpm)			
0	1.0000(00)	1.2513(00)	0.0000(00)	0.0000(00)	−1.1510(−02)
1	7.5664(−01)	7.5664(−01)	1.4183(04)	6.9697(02)	−1.1510(−02)
2	5.1605(−01)	5.1605(−01)	3.8059(04)	1.1732(03)	−1.1113(−02)
3	2.9027(−01)	2.9027(−01)	6.8880(04)	1.5145(03)	−1.0283(−02)
4	8.7066(−02)	8.7066(−02)	1.0483(05)	1.7665(03)	−9.1263(−03)
5	−8.7196(−02)	−8.7196(−02)	1.4277(05)	1.8646(03)	−7.7111(−03)
6	−2.2572(−01)	−2.2572(−01)	1.7817(05)	1.7393(03)	−6.0978(−03)
7	−3.2749(−01)	−3.2749(−01)	2.0752(05)	1.4426(03)	−4.4408(−03)
8	−3.9177(−01)	−3.9177(−01)	2.2897(05)	1.0539(03)	−2.7806(−03)
9	−4.2194(−01)	−4.2194(−01)	2.4090(05)	5.8621(02)	−1.2923(−03)
10	−4.1895(−01)	−4.1895(−01)	2.4132(05)	2.0509(01)	1.5305(−04)
11	−3.8250(−01)	−3.8250(−01)	2.2919(05)	−5.9580(02)	1.6009(−03)
12	−3.1241(−01)	−3.1241(−01)	2.0702(05)	−1.0895(03)	3.0907(−03)
13	−2.1004(−01)	−2.1004(−01)	1.7713(05)	−1.4690(03)	4.5398(−03)
14	−7.7777(−02)	−7.7777(−02)	1.4228(05)	−1.7124(03)	5.8683(−03)
15	7.9416(−02)	7.9416(−02)	1.0507(05)	−1.8283(03)	7.0776(−03)
16	2.6286(−01)	2.6286(−01)	7.0187(04)	−1.7143(03)	8.3910(−03)
17	4.7568(−01)	4.7568(−01)	4.0655(04)	−1.4512(03)	9.7948(−03)
18	7.1412(−01)	7.1412(−01)	1.8855(04)	−1.0713(03)	1.1096(−02)
19	9.7339(−01)	9.7339(−01)	5.0332(03)	−6.7919(02)	1.2180(−02)
20	1.2388(00)	1.2388(00)	1.0000(−01)	−2.4732(02)	1.2749(−02)
20A				1.0000(−03)	1.2749(−02)
		ω = 22.58 (216 cpm)			
0	1.0000(00)	3.8192(00)	0.0000(00)	0.0000(00)	−2.2098(−02)
1	4.9171(−01)	4.9171(−01)	9.1017(04)	4.4726(03)	−2.2098(−02)
2	2.4872(−02)	2.4872(−02)	2.0519(05)	5.6103(03)	−1.9550(−02)
3	−3.4375(−01)	−3.4375(−01)	3.2058(05)	5.6707(03)	−1.5077(−02)
4	−5.8624(−01)	−5.8624(−01)	4.1367(05)	4.5741(03)	−9.6908(−03)
5	−6.8975(−01)	−6.8975(−01)	4.5732(05)	2.1451(03)	−4.1063(−03)
6	−6.5570(−01)	−6.5570(−01)	4.2679(05)	−1.5002(03)	1.0614(−03)
7	−5.1645(−01)	−5.1645(−01)	3.3179(05)	−4.6681(03)	5.0306(−03)
8	−3.0953(−01)	−3.0953(−01)	1.9095(05)	−6.9211(03)	7.6849(−03)
9	−7.3244(−02)	−7.3244(−02)	2.2466(04)	−8.2793(03)	8.9261(−03)
10	1.6644(−01)	1.6644(−01)	−1.5335(05)	−8.6403(03)	9.0609(−03)
11	3.8242(−01)	3.8242(−01)	−3.1088(05)	−7.7403(03)	8.1407(−03)
12	5.4607(−01)	5.4607(−01)	−4.3147(05)	−5.9258(03)	6.1200(−03)
13	6.3287(−01)	6.3287(−01)	−5.0245(05)	−3.4879(03)	3.0997(−03)
14	6.2520(−01)	6.2520(−01)	−5.1856(05)	−7.9154(02)	−6.6861(−04)
15	5.0296(−01)	5.0296(−01)	−4.6503(05)	2.6305(03)	−5.0763(−03)
16	2.4227(−01)	2.4227(−01)	−3.5751(05)	5.2835(03)	−1.0889(−02)
17	−1.8226(−01)	−1.8226(−01)	−2.3185(05)	6.1749(03)	−1.8039(−02)
18	−7.6689(−01)	−7.6689(−01)	−1.1708(05)	5.6399(−03)	−2.5458(−02)
19	−1.4907(00)	−1.4907(00)	−3.3801(04)	4.0922(03)	−3.2190(−02)
20	−2.2634(00)	−2.2634(00)	2.0000(00)	1.6611(03)	−3.6010(−02)
20A				1.0000(−02)	−3.6010(−02)

n	Y	y Sprung	M		γ
		ω = 28.99 (277 cpm)			
0	1.0000(00)	−4.6114(00)	0.0000(00)	0.0000(00)	−8.2216(−03)
1	8.7157(−01)	8.7157(−01)	−6.0397(04)	−2.9679(03)	−8.2216(−03)
2	6.6546(−01)	6.6546(−01)	−5.3143(04)	3.5644(02)	−9.9127(−03)
3	4.0721(−01)	4.0721(−01)	8.3795(03)	3.0232(03)	−1.1071(−02)
4	1.3364(−01)	1.3364(−01)	1.1348(05)	5.1648(03)	−1.0930(−02)
5	−1.1414(−01)	−1.1414(−01)	2.3716(05)	6.0776(03)	−9.3984(−03)
6	−2.9305(−01)	−2.9305(−01)	3.4061(05)	5.0833(03)	−6.7185(−03)
7	−3.8703(−01)	−3.8703(−01)	3.9656(05)	2.7493(03)	−3.5508(−03)
8	−3.9448(−01)	−3.9448(−01)	3.9586(05)	−3.4018(01)	−3.7838(−04)
9	−3.3076(−01)	−3.3076(−01)	3.3710(05)	−2.8875(03)	2.1947(−03)
10	−2.0926(−01)	−2.0926(−01)	2.2366(05)	−5.5746(03)	4.2174(−03)
11	−4.7767(−02)	−4.7767(−02)	7.2259(04)	−7.4399(03)	5.5593(−03)
12	1.2649(−01)	1.2649(−01)	−8.6746(04)	−7.8135(03)	6.0290(−03)
13	2.8363(−01)	2.8363(−01)	−2.2681(05)	−6.8826(03)	5.4218(−03)
14	3.9602(−01)	3.9602(−01)	−3.2633(05)	−4.8905(03)	3.7207(−03)
15	4.2478(−01)	4.2478(−01)	−3.5313(05)	−1.3172(03)	9.4695(−04)
16	3.3663(−01)	3.3663(−01)	−3.0477(05)	2.3764(03)	−3.4672(−03)
17	1.0094(−01)	1.0094(−01)	−2.1486(05)	4.4183(03)	−9.5627(−03)
18	−2.9148(−01)	−2.9148(−01)	−1.1501(05)	4.9068(03)	−1.6438(−02)
19	−8.2671(−01)	−8.2671(−01)	−3.4890(04)	3.9371(03)	−2.3051(−02)
20	−1.4172(00)	−1.4172(00)	1.0000(00)	1.7145(03)	−2.6994(−02)
20A				1.0000(−02)	−2.6994(−02)
		ω = 39.60 (378 cpm)			
0	1.0000(00)	−7.8631(−01)	0.0000(00)	0.0000(00)	−1.9940(−02)
1	5.5207(−01)	5.5207(−01)	6.5495(04)	3.2185(03)	−1.9940(−02)
2	9.5676(−02)	9.5676(−02)	2.1099(05)	7.1496(03)	−1.8106(−02)
3	−2.6491(−01)	−2.6491(−01)	3.7105(05)	7.8654(03)	−1.3506(−02)
4	−4.6502(−01)	−4.6502(−01)	4.7818(05)	5.2644(03)	−7.2725(−03)
5	−4.7546(−01)	−4.7546(−01)	4.6464(05)	−6.6532(02)	−8.1706(−04)
6	−3.1553(−01)	−3.1553(−01)	2.9373(05)	−8.3986(03)	4.4334(−03)
7	−6.6310(−02)	−6.6310(−02)	2.7344(04)	−1.3090(04)	7.1651(−03)
8	1.8601(−01)	1.8601(−01)	−2.5716(05)	−1.3981(04)	7.3838(−03)
9	3.7795(−01)	3.7795(−01)	−4.9054(05)	−1.1469(04)	5.7123(−03)
10	4.7101(−01)	4.7101(−01)	−6.0728(05)	−5.7363(03)	2.7690(−03)
11	4.3955(−01)	4.3955(−01)	−5.6450(05)	2.1019(03)	−8.7463(−04)
12	2.9085(−01)	2.9085(−01)	−3.9111(05)	8.5203(03)	−4.5439(−03)
13	5.7551(−02)	5.7551(−02)	−1.3640(05)	1.2516(04)	−7.2817(−03)
14	−2.1098(−01)	−2.1098(−01)	1.3366(05)	1.3271(04)	−8.3047(−03)
15	−4.2683(−01)	−4.2683(−01)	3.3140(05)	9.7170(03)	−7.1686(−03)
16	−5.0904(−01)	−5.0904(−01)	3.8814(05)	2.7881(03)	−3.0261(−03)
17	−3.8497(−01)	−3.8497(−01)	3.2757(05)	−2.9764(03)	4.7368(−03)
18	9.0100(−04)	9.0100(−04)	1.9623(05)	−6.4543(03)	1.5219(−02)
19	6.4856(−01)	6.4856(−01)	6.4996(04)	−6.4487(03)	2.6502(−02)
20	1.4140(00)	1.4140(00)	7.0000(00)	−3.1935(03)	3.3846(−02)
20A				1.0000(−01)	3.3846(−02)

$$K_{di} = M_{di}\omega_i^2$$

where ω_i is the circular frequency of the ith mode.

For the damping constant an empirical value has been used based on accumulated data from vibration-generator tests. This value is

$$C_{di} = 0.030 M_{di}\omega_i$$

The extension of the foregoing concepts to torsion-bending modes leads to the following relation:

$$M_{di} = \frac{\Sigma m Y_i^2 + \Sigma(I_{mx} - m\bar{z}^2)\phi_i^2}{(Y_{di} - z_d\phi_{di})^2}$$

where I_{mx} is the lumped value of mass moment of inertia with respect to the assumed longitudinal axis of the ship, $\bar{z}$ is the z-coordinate of the center of mass at a station, ϕ_i is the rotation (twist) with respect to a longitudinal axis obtained in the normal-mode calculation, and z_d is the z-coordinate of the driving point. The co-ordinate system used is illustrated in Fig. 2 of reference [7]. Also, as in the case of the pure flexural mode,

$$K_{di} = M_{di}\omega_i^2$$

and it was assumed that $C_{di} = 0.030 M_{di}\omega_i$

In the case of a flexural mode in which the rudders produce a significant sprung-mass effect the same basic formula for M_{di} as given previously applies. However, in this case $\Sigma m Y_i^2$ will include also the m and Y for the sprung mass. If the sprung-mass action causes a considerable change in the mode shape with suppression of the relative amplitude at some driving point d at which a vibration generator is installed, it will be found that M_{di} will be greatly increased and hence also C_{di}. In such a case, when the vibration generator is operated through its speed range, the peak at this mode is much lower than if the sprung mass were absent.

APPENDIX 2

Control-Surface Flutter Calculations

As shown in reference [5], under the simplifying assumptions of exact tuning of the control-surface member to a natural frequency of the hull, zero hydrodynamic moment, an alternating lift force derived from the steady linear relation, and damping in both degrees of freedom small enough to have a negligible effect on the critical frequencies, the flutter speed is given by the simple equation

$$C - \frac{AS^2mh}{c} + AS + \frac{m^2h^2\omega^2}{c} = 0$$

where

C = effective viscous damping constant of the entire hull in the mode in question, and referred to the rudder-stock location

A = lift force per unit velocity squared, per unit angle of attack

S = velocity of water in direction of longitudinal axis of ship and relative to rudder

m = mass of rudder including allowance for virtual mass,

h = distance from rudder-stock axis to center of mass of rudder (including effect of virtual mass) considered positive if downstream

c = effective viscous damping constant of the rudder system in angular motion

ω = circular frequency of oscillation

Table 6 Summary of Terms Applicable to DD 931 in Simple Flutter Speed Equation for c = 1,260,000 in lb sec and A = 2.48 lb sec²/in.²

S, knots[a]	C	$\frac{-AS^2mh}{c}$	AS	$\frac{m^2h^2\omega^2}{c}$	Σ
25	3470	−1720	1260	5680	8690
30	3470	−2450	1510	5680	8200
35	3470	−3340	1760	5680	7570
40	3470	−4360	2010	5680	6800
50	3470	−6810	2510	5680	4850
60	3470	−9810	3020	5680	2360
70	3470	−13350	3520	5680	−680
80	3470	−17440	4020	5680	−4270
For c = 126,000 in-lb-sec and A = 2.48 lb-sec²/in.²					
25	3470	−17020	1260	56790	44500
30	3470	−24520	1510	56790	37200
35	3470	−33370	1760	56790	28600
40	3470	−44590	2010	56790	18700
50	3470	−68110	2510	56790	−5340
For c = 126,000 in-lb-sec and A = 3.10 lb-sec²/in.²					
25	3470	−21300	1580	56790	40540
30	3470	−30600	1890	56790	31550
35	3470	−42000	2210	56790	20470
40	3470	−55000	2550	56790	7810
50	3470	−86000	3160	56790	−22580

[a] The speeds are given here in knots although the inch-pound-second system is actually used in the equation. S is velocity relative to rudder. Ship speed will be lower due to propeller jet effect.

In order to evaluate the constant C used in this equation it is necessary to determine the normal mode of the hull likely to be involved and to reduce the hull in this mode to an effective lumped system by the general theory given in reference [15]. This leads to the equations for M_{di} and C_{di} given in Appendix 1. The latter is the C used here.

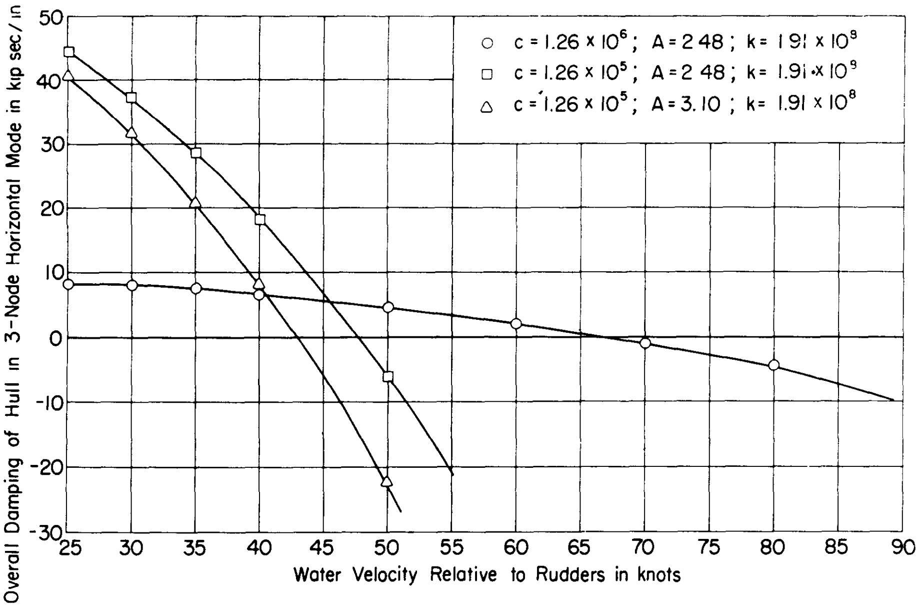

Fig. 7 Variation in damping of 3-node horizontal mode of the DD 931 hull at various ship speeds predicted on basis of simple flutter-speed equation. Owing to propeller jet effect ship speeds may be as much as 20 per cent lower than above-water velocities

The values of the parameters as derived initially for DD 931 and based on two rudders acting as one are as follows:

C = 3470 lb-sec/in.
A = 2.48 lb-sec^2/in.^{2}
m = 274 lb-sec^2/in.
h = 12.3 in.
c = 1,260,000 in-lb-sec
ω = 25.1 (for 4 cps)

Here the value of c was based on a resonance magnification factor of 6.

The values of the four terms in the equation for flutter speed are given in Table 6 for various fluid velocities relative to the rudder. The flutter speed is the speed at which the summation of terms in the table vanishes. Table 6 also includes a similar set of values obtained when c is reduced by a factor of 10.

The flutter speed thus predicted, however, is the speed of water just ahead of the rudder relative to the ship and not the speed of the ship. Hence, if in spite of the boundary-layer effect the propellers boost the velocity of water impinging on the rudders by 25 per cent, as indicated in reference [4], the predicted ship speed would be lowered accordingly. It can be seen that with the existing values of the other parameters the damping in the torsional degree of freedom has a large effect on the flutter speed. Fig. 7 presents graphically the prediction of over-all damping characteristics given by the simple flutter-speed equation.

The flutter speeds indicated in Fig. 7, even after the correction for propeller action, are in general higher than the speed at which the rudder-exited hull vibration first become noticeable on DD 931. However, it is maintained in this paper that actually only a subcritical condition was encountered on this ship. This should be expected to occur at a lower speed than a true critical speed. Furthermore it is indicated that reasonable changes in certain of the constants could bring even the critical flutter speed directly into range. The first of the latter is the constant A which is really proportional to the slope of the curve of lift force versus angle of attack. The assumed linearity of this relation is based on values measured at relatively large angles of attack. In the range of rudder angles between +5 and −5 deg a nonlinear relation might give slopes varying by a ratio of 1.25 or more.

As shown in reference [14], the general method used in calculating critical flutter speeds can be used to estimate the variation in damping at subcritical speeds. The system under consideration may have such characteristics that the damping in a certain mode may drop to a very low value at a certain speed but not reach zero and then start rising again as the speed increases. This would be a subcritical flutter condition in the sense defined in this paper. A method that searched only for

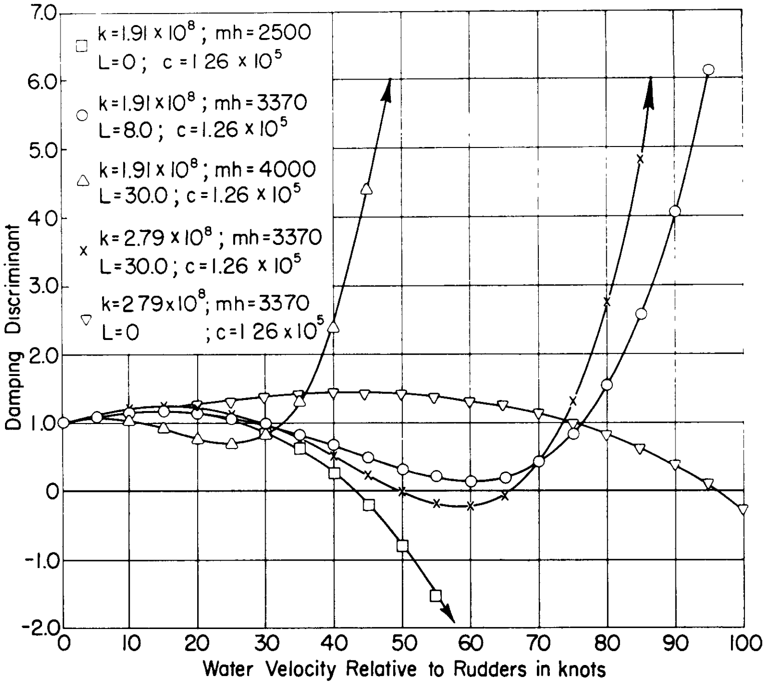

Fig. 8 Variation in damping of 3-node horizontal mode of the DD 931 hull at various water velocities relative to rudders as predicted by simultaneous equations. Owing to propeller jet effect ship speeds may be as much as 20 per cent lower than above-water velocities

speeds at which unstable oscillations begin would not reveal such a condition. However, if the simple equation yields a critical speed within or just above the running range the desirability of a more extensive analysis is indicated.

The next step taken in the flutter analysis was based on the solution of the two following simultaneous linear differential equations:

$$I\ddot{\theta} + c\,\dot{\theta} + (k - ALS^2)\theta - mh\ddot{Y} + \mathrm{ALS}\dot{Y} = 0$$

$$-mh\ddot{\theta} - AS^2\theta + (M + m)\ddot{Y} + (C + AS)\dot{Y} + KY = 0$$

In these equations, Y is the displacement of the rudder axis normal to the direction of the ship's velocity and θ is the rotation of the rudder about its axis.

The hydrodynamic moment is introduced by the parameter L, which is the distance from the rudder-stock axis to the location of a center of lift which would produce the required moment with the actual value of the lift force. In these equations L is considered positive when the center of lift is upstream. The real significance of L, however, is that it is proportional to the slope of the curve of hydrodynamic moment versus angular displacement of the rudder. This slope may be positive when the moment itself is negative, and vice versa.

Other terms appearing in the foregoing equations and not previously defined are as follows:

$(M+m)$ = effective mass of entire hull in horizontal mode involved with rudder locked against rotation and referred to rudder-stock location

K = effective stiffness of entire hull in horizontal mode involved, and referred to rudder-stock location

I = mass moment of inertia of rudder with respect to rudder stock axis, including allowance for inertia effect of water.

All quantities must be expressed in a consistent set of units. Hence, if the inch-pound-second system is used, S must be expressed in inches per second.

The values applicable to DD 931 not previously given (or in the revised units) are the following:

$(M+m)$ = 4620 lb-sec^2/in.
K = 2.92×10^6 lb/in.
I = 302,000 lb-sec^2-in.
k = 1.91×10^8 in-lb/rad

As elsewhere these values are based on combining the two rudders into one equivalent rudder.

These simultaneous equations can be solved by means of an analog computer such as REAC, or, if they are converted to algebraic form on the assumption of simple harmonic solutions, the determinant of the coefficients of θ and Y yields a polynomial in ω with S appearing in certain terms. Various ways of finding the roots of such a polynomial are discussed in reference [16]. Pure imaginary roots indicate neutral stability, whereas complex roots indicate either damped or unstable oscillations depending on the sign of the real component. The results of a series of calculations based on these equations are indicated in Fig. 8, in which the variation in the "damping discriminant" of the three-noded horizontal mode of the hull is plotted against ship speed for various sets of values of the parameters. For further discussion of the damping discriminant see reference [5].

It is possible to evaluate the driving-point impedance of the hull at any desired frequency without having made a prior computation of the normal modes. This can be done by computing the response to a unit driving force by a digital process as discussed in reference [7]. Such a calculation requires the introduction of damping terms and the use of a complex number notation. The rudders can then be considered attached to a system having the mechanical impedance characteristic thus derived. The over-all impedance of the combined system to external excitation will then vary with the ship's speed because of the hydrodynamic forces and moments acting on the rudders. At speeds and frequencies at which the over-all impedance vanishes a critical flutter condition is indicated. Up to the time of this writing this method was not explored in detail for DD 931.

The analyses discussed so far do not take account of the several effects which modify the lift force and moment developed with oscillating hydrofoils. In reference [11] Theodorsen gave a theoretical treatment of the forces and moments on a thin plate of infinite aspect ratio (having a hinged portion) in an ideal incompressible fluid. When the aileron of an aircraft is locked to the wing (hinged portion locked at zero angle), this treatment reduces to the case of a thin rudder if of uniform section and infinite aspect ratio. The treatment involves functions of the reduced frequency, $b\omega/v$, which itself involves the critical speed and frequency.

Up to the time of this writing only a preliminary exploration of Theodorsen's method has been made. Digital calculations have been carried out covering both the TMB flutter apparatus and the DD 931 rudder-hull system by a modification of this method as discussed in reference [5].

Discussion

Prof. F. M. Lewis, *Honorary Vice-President:* Rudder vibration is a rare phenomenon. Two years ago a case of such vibration on a tug came to my attention, and a few notes on the nature of this vibration and the means used to eliminate it may be of interest. The tug was of somewhat higher power and faster free-running speed than previous tugs of the same dimensions.

Running free and at nearly full speed a violent vibration of the rudder was generated. The frequency was constant independent of engine speed and of the order of 6 cps. The vibration occurred at any rudder position, including zero angle, but appeared to be more readily initiated with the rudder at an angle. With the rudder at zero angle it might remain quiescent for a minute or two and then build up in amplitude. The oscillation was of considerable violence so that the hydraulic cylinders at the point of attachment to the hull would show a movement of the order of 1/8 in. and threatened to tear loose from the hull.

The rudder was of the blunt-ended type preferred for tug service, and this immediately suggests that the trailing Karman vortices might be the exciting force. However, the manner in which the vibration built up with time made it appear more like a control-surface flutter. The elimination of the vibration was effected on the supposition that it was a flutter rather than a trailing-vortex phenomenon.

In a flutter oscillation, rotations and transverse motions are coupled in such phase that the motion produces forces equivalent to negative damping. With the addition of positive damping of sufficient magnitude to make the total damping positive the self-excited flutter will be not just reduced in magnitude but completely eliminated. With hydraulic steering gears this positive damping can be very readily introduced. In the case of the tug, a T-connection was inserted in each of the two pipes leading from the control valve to the pistons. A high-pressure air flask was attached to each with a valve interposed. The flasks and the system were completely filled with oil. With the valves closed the flasks were out of the system and the rudder oscillated as before. With the valves wide open it also oscillated, but for a range of intermediate openings there was no vibration of the rudder at all. This strengthens the assumption that a self-excited flutter rather than a trailing-vortex type of vibration existed for if it were the latter the vibration would have been merely diminished and not eliminated.

The insertion of the damping system had no effect upon the steering of the tug.

If further experimental work is to be carried out it would be of interest to determine if this same method of eliminating the vibration would be effective.

CDR. S. R. Heller, Jr., USN, *Member:* Over the years the Society has been singularly fortunate in having the author offer so many stimulating papers. This present effort is no exception. On the contrary, this paper may well be the most provocative. Although I shall refrain from technical comment on this paper because of my juxtaposition with the author during its preparation, I cannot withhold personal comments.

Precisely because of the inextricable interrelation between elastic and hydrodynamic aspects which characterizes hydroelasticity, this paper is almost certain to evoke protagonists of the dominance of each aspect. Such discussions are signs of a healthy and vigorous profession. Indeed, if this were the only contribution of the paper, it would be a most valuable one.

My chief point, however, is the stature of the man who authored this paper. How many men do you know who, realizing that a controversy was certain to be engendered, would unhesitatingly offer themselves and their judgment as targets? For this courage and unselfish promotion of the study of a branch of the profession heretofore but poorly understood, I commend the author—and, at the same time, applaud the Society for its support.

H. N. Abramson, *Member:* The paper under discussion constitutes not only a careful and detailed study of an intriguing problem—or perhaps I should say "mystery"—but constitutes also a striking example of a new problem type within the ever-expanding domain of hydroelasticity. I have recently predicted elsewhere[4,5] that naval architects and marine engineers would continually and with increasing frequency encounter such new problem types, as have their aeronautical brethren for the past 56 years—it is both gratifying and disturbing to see this prediction being fulfilled at so early a date. However, perhaps we can gain succor from the realization that we have now had our "fair warning" and recognize, in the author's words, "there is indicated a need for continuing research on both the hydrodynamic and elastic aspects of the basic flutter phenomenon . . ." and even more importantly that . . . "the need for expanded research in the general field of hydroelasticity . . . is clearly indicated."

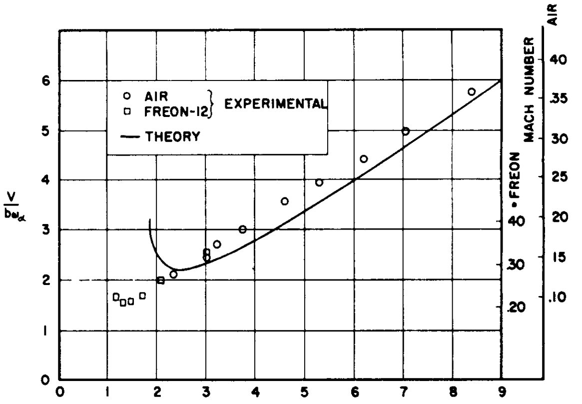

Fig. 9 Typical variation of flutter speed coefficient $V/b\omega_\alpha$ with relative density parameter $\sqrt{\mu}$ for NACA tests

My earlier use of the word "mystery" did perhaps smack of the dramatic, but I think the reader will agree that we still do not really understand the mechanism underlying the rudder-excited hull vibration observed on the *USS Forrest Sherman*. After a very careful and searching study, the author has certainly provided us with considerable insight into the problem and with at least four possible explanations of the phenomenon (one of which he himself strongly favors) from which we may choose—but, just as certainly, the real explanation remains a "mystery." I, of course, do intend to offer some thoughts of my own along these same lines, and perhaps even a fifth possible explanation, but allow me to preface those remarks with others of a more general nature.

Early in his paper the author mentions that while ". . . the general methods of aeroelasticity were believed applicable to its counterpart in naval architecture, namely hydroelasticity, it was recognized that there were great differences in the magnitudes of the physical quantities involved in the two fields." What is really the effect of these differences, at least as regards flutter? Unfortunately, the answer is not a simple one, and, I am afraid, depends rather heavily on extrapolation of data and on individual speculation. The

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[4] S. R. Heller and H. N. Abramson, "Hydroelasticity: A New Naval Science," First Symposium on Naval Structural Mechanics, Aug. 1958, and *Journal of the American Society of Naval Engineers, 71,* 2, pp. 205–209, May 1959.

[5] H. N. Abramson, and W. H. Chu, "A Discussion of the Flutter of Submerged Hydrofoils," Tech. Rept. No. 1, Contract Nonr 2470 (00), Southwest Research Institute, Aug. 1958 (to be published in the *Jour. of Ship Research*).

simple fact of the matter is that so little flutter information, within the range of parameters of direct and immediate interest to naval architects, is available that we are not at all sure that flutter—at least the classical type of flutter mostly studied by aeronautical engineers—can even occur in naval applications; at least one recent paper[6] holds to this view. But let me give a few brief facts.

Some years ago, the National Advisory Committee for Aeronautics conducted a series of experiments on the flutter of wings in which the density of the surrounding fluid medium was varied, with the typical result shown in Fig. 9 of this discussion. It is noted that as the fluid density is increased the flutter speed decreases until, at some particular value of the density, the theoretically calculated flutter speed rises abruptly but the experimentally determined flutter speed continues to decrease, at least for a time. While the trends between theory and experiment are certainly similar, it is clearly obvious that the use of theory, in this range of parameters, for design purposes would be foolhardy indeed. This fundamental inadequacy of conventional aerodynamic-flutter theory has been confirmed in many subsequent flutter investigations, both at the NACA and elsewhere. Suffice it to say, at this time, that even after extremely careful study,[5,6] we are still at an almost complete loss to explain the reasons for this situation, except to say possibly that flow nonlinearities must be the governing factor. The conclusion is, of course, that one might therefore do well to exhibit some hesitancy in applying conventional aerodynamic-flutter theory to naval problems. The author has wisely been quite cautious in this regard.

With the foregoing discussion in mind, and since the basic mechanism of flutter is rather well known,[7] at least to aeronautical engineers, and has been mentioned by the author, I shall pass immediately to the particular problem of control-surface flutter, which is of course the central problem of the paper being discussed, and shall speak, at first, of its occurrence in aeronautics. There are three somewhat distinct types of flutter to be considered, and these occur for wings as well as control surfaces—the distinguishing feature being that a predominantly single-degree-of-freedom type of motion is usually involved.

For certain ranges of Mach number (which governs, in part, the magnitude of the aerodynamic force and moment) and for certain locations of the axis of rotation, it has been found that flutter of a control surface, or even an entire wing, can occur in a single-degree-of-freedom mode—usually the pitching mode. In fact, unstable pitching oscillations are possible, even at very low speeds, if the axis of rotation is located well forward of the surface. This classical type of flutter is based entirely on potential-flow considerations and arises only as the result of the combination of certain rather unusual geometrical configurations and system elastic stiffnesses.

Frequently, as the angle of attack of a wing is increased to values near the stalling angle, it is observed that the flutter velocity is much lower than when the wing is near the zero-lift angle. The classical flutter theory based on potential flow cannot account for the effects of separation, and therefore flutter in this range of lift coefficients, even though the wing may not be completely stalled in the usual sense, is called "stall flutter." This type of flutter is usually predominantly a single-degree-of-freedom motion in the torsion mode. As the flutter speed is decreased (by increasing the mean angle of attack), a minimum value is reached and rises again as the surface becomes completely stalled. In this latter range, the motion more nearly corresponds to forced vibration than to self-excited flutter; the reason for this is that the wake frequency (Karman vortex street) becomes well defined, and the surface reacts to the changes in circulation which result from the vortex formation according to the Kutta-Joukowski law. The situation is not, however, the same as buffeting since there the vibration is forced by the vortices impinging on a surface located in the wake. In the completely stalled range of stall flutter, the amplitude is usually quite small compared with the large-amplitude violent flutter occurring in the range of partial stall. The phenomenon is essentially a nonlinear one, and consequently, analytical study of the problem is extremely difficult.

The final problem relating to control-surface phenomena that I would like to mention is that of *buzz*. The behavior is that of a very high-frequency oscillation of the surface about its hinge line and is probably a consequence of certain boundary layer mechanisms. Beyond that, we know very little.

Now, my reasons for going into this somewhat lengthy discussion of flutter, and particularly control-surface flutter, are: (*a*) to indicate the very great complexity and the many ramifications of the problem, (*b*) to emphasize that nonlinearities often play a governing role in determining the basic flutter mechanism involved, and (*c*) to

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[6] C. J. Henry, J. Dugundji, and H. Ashley, "Aeroelastic Stability of Lifting Surfaces in High Density Fluids," *Jour. of Ship Research, 2*, 4, pp. 10–21, March 1959.

[7] H. N. Abramson, "An Introduction to the Dynamics of Airplanes," The Ronald Press Company, New York, 1958.

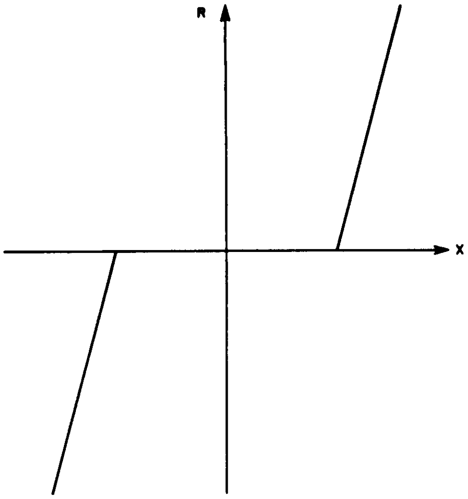

Fig. 10 Flat-spot type of structural nonlinearity

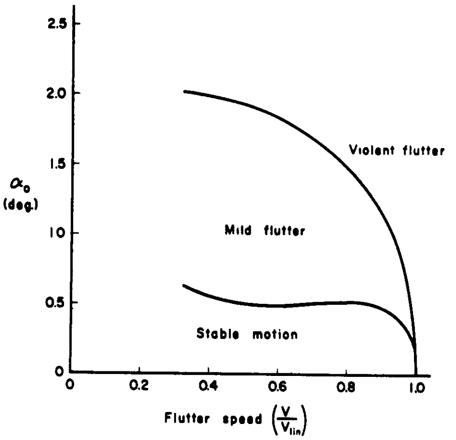

Fig. 11 Flutter regions with flat-spot nonlinearity

point out clearly and emphatically that our knowledge is indeed meager.

That some type of flutter mechanism was indeed responsible for the rudder-excited hull vibration observed on the USS *Forrest Sherman* seems to me, on the basis of the author's careful and systematic appraisal, a correct conclusion. I would, I think, object to his introduction of the term "subcritical flutter condition" on the basis that this could quite well add considerable confusion to a problem already fraught with misconceptions and misunderstanding. But, with his conclusion that this is flutter, I cannot disagree.

At the risk, on my own part, of contributing some additional confusion to the general picture, I would now like to offer some additional speculations relating to flutter on the *Forrest Sherman*, but which, I believe, may be of some consequence. Let me begin by pointing out two salient features concerning the observed phenomenon: (*a*) Paint removal was much more pronounced on the outboard faces of the rudder than on the inboard faces, indicating that some degree of cavitation, or flow separation, occurred; and (*b*) that changing the toe-angle setting from a few degrees in to a few degrees out provided an immediate cure for the problem.

Both of these observations seem quite closely related to certain aspects of the stall-flutter problem of aeronautics, and in fact I am rapidly coming to the view that at least the basic ingredients of that phenomenon are present in the problem of the *Forrest Sherman*.[8] But let me consider with you just one other interesting question.

Turning once again to aeronautical experience, we find that structural nonlinearities can also play an important role in the flutter problem.[9] The structure provides elastic restoring-force or moment terms in the equations of motion, and hence we may consider a nonlinear spring, characterized by the restoring force or moment as a function of displacement, as shown in Fig. 10. This flat-spot type of nonlinearity may, for example, represent free play in a hinge or linkage system of a control surface; e.g., a rudder or diving plane. The results of two-degree-of-freedom flutter calculations, based on a nonlinearity of this type, are shown in Fig. 11 with the interesting feature that the flutter speed decreases as the magnitude of the initial disturbance increases. Furthermore, the flutter speeds are well below those of the corresponding linear system and the flutter conditions reveal both a region of mild (limited amplitude) flutter and a region of violent flutter. The ordinate in Fig. 11, incidentally, corresponds to the initial angular displacement of the wing.

The results described in the preceding paragraph are, at least to me, very significant when viewed in light of the problem of the *Forrest Sherman*. For instance, the flutter "cure" achieved by altering the toe-in angles of the rudders might correspond to changing from the mild flutter region to the stable motion region of Fig. 11; other similar speculations and analogies might easily be made. In fact, could not this "mild flutter" be indeed

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[8] Incidentally, because of the flow curvature produced by the ship hull, what was the initial mean angle of attack of the rudders?

[9] D. S. Woolston, H. L. Runyan, and R. E. Anderson, "An Investigation of Effects of Certain Types of Structural Nonlinearities on Wing and Control Surface Flutter," *Jour. Aero. Sci.*, *24*, 1, pp. 57–63, January 1957.

the "subcritical flutter" which is used by the author.

The foregoing is contributed to the very excellent analysis and presentation made by the author to emphasize the difficulty of the problem and to provide additional speculations with which to incite and encourage further study of this intriguing new field of hydroelasticity.

Capt. H. E. Saunders, USN (Ret), ***Honorary Member*** **and Lieut. (jg) J. F. Hillman, USNR,** ***Associate Member:*** We wish to urge that, in his reply to the discussions, the author supplement the diagrams of Fig. 1 and the photograph of Fig. 2 with a plan view to large scale, indicating the relative positions and angles of the propellers and shafts with respect to the ship centerplane. Most important of all, the rudders should be shown in their proper horizontal positions relative to the propellers and to the ship hull. The rudders should lie at their original neutral or zero angles, with the modified neutral positions and the fully angled positions shown in broken lines or equivalent fashion.

Since when under way the flow through and abaft the propellers will take place in lines roughly paralleling the hull buttocks above, it would be helpful to supplement the plan view with an elevation of the stern, drawn to much larger scale than Fig. 1, and showing the position and slope of a buttock lying generally over the propeller and rudder on one side.

We look forward to the day when diagrams of this kind will show the flow patterns, as predicted from physical considerations or as observed on a self-propelled model in a circulating-water channel. After all, it is the nature of the flow which causes troubles of the kind described in this paper.

C. J. Henry, ***Member:*** The field of hydroelasticity is at present not replete with literature. As the speed of water vehicles increases, and as the weight and structural requirements become more stringent, problems involving both the elasticity of a structure and the hydrodynamic forces acting on it will become more vital to efficient and safe operation. This paper, along with the few other existing discussions in this field is, therefore, only the beginning of hydroelastic literature.

Each of the theoretical analyses described by the author are, to varying extents, simplifications of classical flutter theory. This classical theory is based on the most exact mathematical description available of the unsteady hydrodynamic forces acting on a lifting surface. It is surprising, therefore, that the author has not presented results based on this more exact theory for comparison with the results of the experiments and with the results of the simplified analyses. If such a comparison shows that the various results are in agreement, then the author would have a very strong argument in favor of the conclusions stated in this paper. If the results of classical flutter theory are at variance with the results of the simplified analyses and/or with the experimental results, then the author would have the added task of explaining any discrepancy. The explanation of such a discrepancy could be that the classical flutter theory is in error. This theory, however, is supported by many experiments reported in aeronautical literature. It is safe to assume, therefore, that the classical analysis gives correct results. It would be more reasonable to suspect the simplified analyses.

A cursory comparison between the results of classical flutter theory[10] and the theoretical and experimental results as presented by the author does in fact show several discrepancies.

1 According to classical flutter theory, the results are in no way dependent on the mean angle of attack. It is assumed that the foil is initially at a steady angle of attack and is in equilibrium with the restoring forces of the foil supports providing the necessary equal and opposite forces to the steady hydrodynamic lift and moment. These steady "equilibrium" forces and the initial steady angle of attack do not appear further in the classical analysis. The classical flutter theory, therefore, does not explain why the change in toe angle on DD 931 had any effect on the observed vibration.

2 It is reported by the author that the frequency of vibration observed on DD 931 was constant over a wide speed range. In classical flutter theory, in aeroelastic flutter experiments, and in aeronautical practice, it is found that the damping coefficient and the natural frequency of the mode leading to flutter both vary with speed. (The same may be said in fact for all natural modes of vibration.) This result can be explained as follows:

The natural frequency of a damped, spring-mass system is dependent on the damping coefficient, the spring rate and the mass. Each of these three over-all "properties," in the case of an oscillating lifting surface, include contributions which are specific functions of the forward speed. Hence, the natural frequency of a mode and the damping in that mode are a function of the forward speed. It is hard to believe, therefore, that

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[10] See, for example, Henry, Dugundji, Ashley, "Aeroelastic Stability of Lifting Surface in High Density Fluids," Journal of Ship Research, vol. 2, March, 1959.

the classical theory would predict large responses in one mode of vibration to an excitation at constant frequency over a large speed range as was observed aboard DD 931.

A third discrepancy between classical flutter-theory results and the experimental and analytical results described by the author is that the classical flutter theory predicts freedom from flutter at all finite speeds for all density ratios *below* 1.3, whereas, the reported analysis and experiments are at a density ratio of 0.5; i.e, less than one half of the critical density ratio. In all calculations known to the writer, based on classical flutter theory in which the density ratio is so far below the critical value (i.e., so far into the stable region), the theoretical hydrodynamic damping ratio indicates solid rock stability rather than the proximity of a critical flutter condition. It can be surmised, therefore, that if classical flutter theory were applied to the experimental setup of the DTMB flutter apparatus reported by the author, then the theoretical results would not agree with the data presented in Fig. 4 of the paper.

If these three discrepancies between the simplified analyses and the experimental observations presented in this paper on one hand and the results of the classical flutter theory on the other cannot be explained, then there is no alternative but to conclude that the observed vibrations on DD 931 and the DTMB flutter apparatus are not explained by classical flutter theory and that the simplified analyses are too idealized to give reliable results.

L. A. Becker,[11] ***Visitor:*** The author of this paper is to be congratulated on his excellent presentation of a very interesting problem. The control-surface-flutter explanation seems very plausible in light of the available data, particularly since this explanation contains the angle of attack. The practical solution to the problem was to change the initial toe angle and thus it appears that any theory which explains the phenomenon must contain angle of attack somewhere.

The flutter theory contains the angle of attack in terms of the slopes of the lift and moment curves. However, the theory states that flutter speed is a function of angle of attack only if the lift and moment curves are nonlinear with angle of attack. These nonlinear effects are present in the model studies, reference [5] of the paper, but until recently no full-scale data were available.

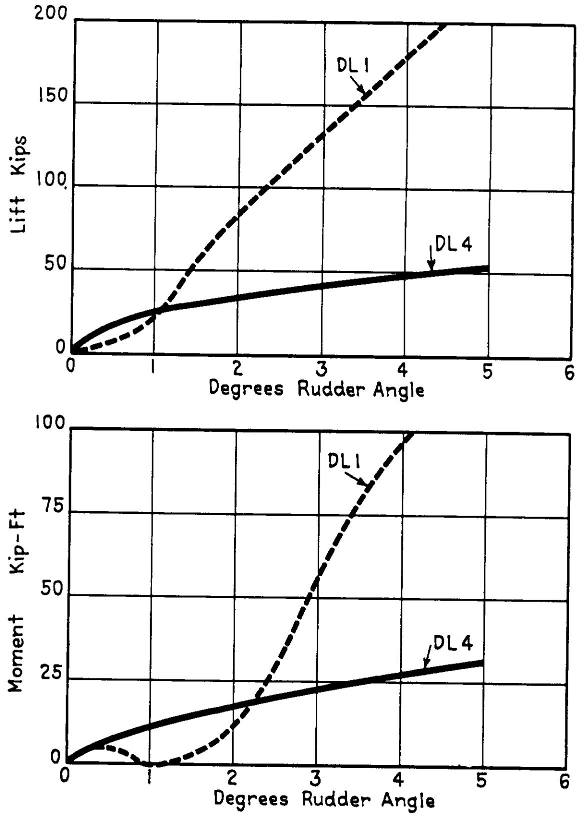

Fig. 12

The writer participated in two full-scale trials in which rudder forces and rudder-stock torques were measured. The results of one of these trials is presented in the author's bibliography [5] and the second is being prepared as a David Taylor Model Basin Report.[12] Several oscillograms were re-examined at the low rudder angles. (The rudder angle and the angle of attack are approximately equal at the low angles.) Curves of lift versus angles of attack and moment versus angle of attack are plotted in Fig. 12 of this discussion. The nonlinearity is clearly shown in all the curves at the very small angles. It must be remembered that the method used to find these curves is not too accurate at the very low angles so that values may be slightly in error but the accuracy is sufficient to indicate the nonlinear shape of the curves.

Further examination of the oscillograms from USS *Norfolk* (DL 1), bibliography [5], indicates vibrations in the rudder stock at a frequency of 872 cpm which is close to the blade frequency of 912 cpm.

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[11] David Taylor Model Basin, Washington, D. C.

[12] D. J. Duffy, L. A. Becker, and J. S. Brock, "Experimental Determination of Rudder Forces during Full-Scale Trials of USS Willis A. Lee (DL-4)," David Taylor Model Basin Report 1382 (in preparation).

LCDR. J. R. Baylis, USN, *Associate Member:* In studying this paper I was stimulated to consider a number of related problems in my own experience and was amazed at the number of times we suspected that vibrations or noise problems were hydrodynamically excited. Then, looking at the field of hydroelasticity as defined by Heller and Abramson in their 1958 paper (author's bibliography [1]), we see that most of the dynamics problems in ship design are included in the science of hydroelasticity. The author states that the need for expanded research in this field is clearly indicated. I think this is an understatement; the need for fully developed theory and techniques is already with us.

We now have a very small number of people competent in this new science. If we are to expand research, and staff ship-design organizations with trained people, we must find a source of such men. Graduate study in naval architecture and marine engineering now includes the study of hydrodynamic, elastic, and inertial forces but is not very complete in the study of the coupling between these forces. This is understandable since this is a new science. On the other hand, aeroelasticity is a few years older and is an established part of graduate study in aeronautical engineering.

I believe that we can fill the gap, while research is developing this new science, by encouraging graduate students in naval architecture to study the theory and methods of aeroelasticity. I offer this expedient now because I believe we need to apply this science now, and the need will not diminish.

D. A. Jewell,[13] *Visitor:* The author is to be commended for bringing this problem before the Society. He has shown that hydroelastic phenomena can have a deleterious effect on a ship's performance. Designers of high-speed vessels should note this paper.

The author stresses the importance of a subcritical flutter condition in his explanation of the cause of the vibration. The writer maintains that the role of cavitation in promoting this vibration should be emphasized.

Any explanation of the DD 931 vibration must show the amplitude dependence on the speed and rudder toe angle. Both the exciting force and the amplitude-response characteristics to such excitation must be given. The author, using the results of an elementary flutter analysis, argues that the second horizontal hull mode became so lightly damped at high speed that it responded with great magnification to small excitation at the bow. This is called subcritical flutter. The frequency of the rudders in rotation was shown to depend on the hydrodynamic moment, which varies with ship speed and mean rudder angle. When initially, the rudders were toed-in, their oscillatory frequency was nearly equal to the natural hull frequency. But at high speed, when the rudders were toed-out, their frequency was changed with respect to the hull. This hydroelastic detuning effect partly accounts for the decrease in the vibratory amplitude of the inboard rudder during turns. It also partly accounted for the disproportionate amplitude decrease in the first and second horizontal hull modes when the rudder angles were changed from toe-in to toe-out in the under-way vibration surveys.

The flutter analysis used by the author is not appropriate because of two unsupported assumptions. These are:

1 "The lift forces developed during oscillatory motion can be taken from the relation between lift force and angle of attack in steady flow."

2 "The twin rudders act in unison and, for the purpose of analysis, can be combined into a single equivalent rudder."

Aeroelastic studies (author's reference [14]) have shown that use of the steady lift and moment relations in a flutter analysis can lead to a marked underestimation of stability or damping. In these cases, use of the quasi-steady or unsteady relation provides a more reliable estimate. The importance of unsteadiness is indicated by the value of the Strouhal number based on the control-surface semi-chord length. The quasi-steady or unsteady relations are required when this value is above 0.1. Since the Strouhal number is about 2.3 for the DD 931 vibration, the steady relation is not applicable. The lift force is overestimated and the flutter speed is likely to be underestimated.

The equivalent rudder assumption implies an in-phase relation between the motions of the two rudders. However, simultaneous torsional strains in the DD 931 rudder stocks, as recorded by the Boston Naval Shipyard, appeared to have a random phase relation. This indicates partial but not complete cancellation of the lift forces on the two rudders which would also increase the flutter speed. An analysis based on an equivalent rudder assumption cannot support the subcritical-flutter explanation when the ship is turning and the two rudders have different frequencies of oscillation. Comparison with experiment [5] showed that the analysis used by the author

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[13] Physicist, David Taylor Model Basin, Washington, D. C.

generally predicted the vibrations of the TMB flutter apparatus to be less stable than was the actual case.

A flutter analysis using the quasi-steady lift and moment relations and the equivalent rudder concept has been made for the DD 931 by the writer. This quasi-steady analysis also predicts rudder detuning effects if the center of lift is not the control-surface axis. A calculation based on this analysis indicates that damping increases slightly with speed to a maximum at 15 knots, and then steadily decreases, yielding a 78-knot critical flutter speed. The most important result obtained was that the damping was nearly the same at 30 knots as it was at zero speed. This means that no energy contributing to maintain this vibration was transmitted to the hull by the alternating hydrodynamic forces. Therefore simple subcritical flutter is not the primary causative mechanism of the DD 931 hull vibration, although the hydroelastic detuning effects shown by the flutter analyses are required.

What then is the role of excitation at the bow in this vibration? The author states:

1 That excitation at the bow is always present when a ship is underway, even in calm seas, and

2 That two-noded horizontal hull vibration is present under either rudder toe-angle condition.

These arguments are not sufficient to identify bow excitation as the source of three-noded horizontal hull vibrations. It is, therefore, necessary to look further for the explanation.

A study [4] of flow conditions indicates not only that partial cavitation was present on the DD 931 rudders, but also that forces due to cavitation must be included in any appraisal of the cause of vibration. Time-dependent forces could arise from changes in cavity shape or location. First, consider cavity fluctuations coupled with the rudder oscillations. The author argues that the angular rudder amplitudes were so small in comparison with the steady toe-angle changes required to produce the phenomenon that a coupled fluctuation during each cycle seems improbable. If true, this argument would invalidate the entire flutter concept for which only infinitesimal amplitudes are required. An analytical method of predicting the forces for partial cavitation is not yet available. It can only be inferred, from an analysis[14] which applies to supercavitating foils, that the effect of partial cavitation would be to reduce the flutter speed. This is another reason for designers of high-speed vessels to be concerned with the possibilities of hydroelastic phenomena.

Now consider cavity fluctuations which do not depend on rudder oscillations. Time-dependent forces on partially cavitating foils have been observed by Kermeen,[15] who observed severe buffeting which increased with the steady foil angle. The buffeting was caused by cavity shedding; that is, a cavity would grow to about one foil length, then rapidly separate and flow downstream, leaving the foil nearly fully wetted. Another cavity would immediately begin to grow, thus restarting the cycle. The resultant alternating force appeared to excite foil vibration. Its magnitude was apparently of the order of the steady lift force, and the frequency has been estimated to be about 5 cps. Cavitation shedding on the DD 931 rudders could provide a sufficiently large alternating force to excite the hull at the observed frequency. If the cavitation-induced force was only 25 per cent of the lift on one rudder without cavitation, it would amount to 17,000 lb at 30 knots. In a full-scale vibration generator test of a sister ship, *Paladino* [6] found that an oscillatory force of 25,000 lb produced an amplitude of from 48 to 65 mils at Frame 192. Therefore only about 20,000 lb would be required to cause the 40-mil amplitude observed on the DD 931. This cavitation-shedding concept provides an explanation for the undiminished hull amplitude during turns, whereas the subcritical-flutter concept did not.

This discussion leads one to believe that cavitation shedding, occuring at high speeds with the rudders toed-in, is the most likely cause of the vibration of the DD 931 hull in its second horizontal mode. When the rudders are toed-out, cavitation shedding decreases and hydrodynamic detuning effectively uncouples them from the hull mode, resulting in a large reduction of the vibratory amplitude.

J. P. Craven, *Member:* This very stimulating paper sets forth a number of plausible hypotheses for the DD 931 rudder vibration and concludes that cavitation was not a primary factor in the phenomenon. With this the reviewer disagrees and expresses his own preference for the rudder torsional-three-noded hull flexural mode with the fluctuations from the noncavitating to cavitating state as the forcing function. This preference is based on the following evidence:

(*a*) Reference [4] of the paper demonstrates that incipient cavitation could occur for the

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[14] B. R. Parkin, "Fully Cavitating Hydrofoils in Nonsteady Motion," California Institute of Technology, Engineering Division Report No. 85-2, July 1957.

[15] R. W. Kermeen, "NACA 4412 and Walchner Profile 7 Hydrofoils in Noncavitating and Cavitating Flows," California Institute of Technology, Hydrodynamics Report 47-5, Feb. 1956.

"toe-in" setting in the observed speed range, but would not occur for the "toe-out" setting.

(*b*) The paint-erosion pattern coincides with the expected regions of cavitation.

(*c*) Buffeting of partially cavitating foils has been observed by Kermeen and others whenever the foil is at the transition from the cavitating to the supercavitating state, with accompanying large-scale changes in lift and moment.

The equations which should be employed for analyzing this kind of system are the same as those employed by the author for the noncavitating condition (except that there is a step function in the lift and moment values at the critical angle). Therefore the statements that the cavitation hypothesis "would be expected to coincide with a natural frequency of the rudder-hull system" and would have "a phase relation not favorable for maintaining the oscillations" do not appear to be justified.

In addition, during a turn, the proper rudder (the one whose angle of attack is reduced) ceases to vibrate when theory indicates that it would cease to cavitate. That this cessation of vibration resulted from removal of the forcing function on an otherwise weakly coupled system seems far more plausible than a detuning resulting from a hypothesized (but not physically explained) nonlinearity in the lift versus angle-of-attack relation.

The writer must agree, however, that the selection of any given hypothesis is, in view of the limited data, conjectural. It is indeed unfortunate that the rudders were not left at the original angle long enough to make a decisive observation through viewing ports or by underwater photography. There is no doubt in the writer's mind that the profession will happen on this phenomenon again when some designer overlooks hypothesis (*a*), (*b*), (*c*), or (*d*) in establishing his design.

F. E. Reed, *Member:* Ships are always raising new problems. This latest problem of self-excited vibration is an excellent example of a new phenomenon that requires careful measurements and thorough analysis to explain. There can be little doubt that the vibration exists because the rudders are effectively coupled to the hull. The thorough studies made by the author and the Model Basin staff explain many aspects of the problem. The perspicacity of the engineers in recognizing the difference in vibration between the two rudders when at slight angles and in carrying this observation to a successful solution is astonishing.

We all recognize that this problem has two questions that still have to be answered. (1) Why did the change of toe-in angle of the rudders change the vibration characteristics of the hull so drastically? (2) Why has this self-excited vibration not occurred on other ships? The conventional rudder might be expected to react in much the same way as the *Forrest Sherman* rudders, and surely, in the many ships that have been built, a coupling of the rudder with a main hull mode should be expected. Also the tests in the towing basin, although they showed a degree of self-excitation, did not show enough negative damping to excite a ship.

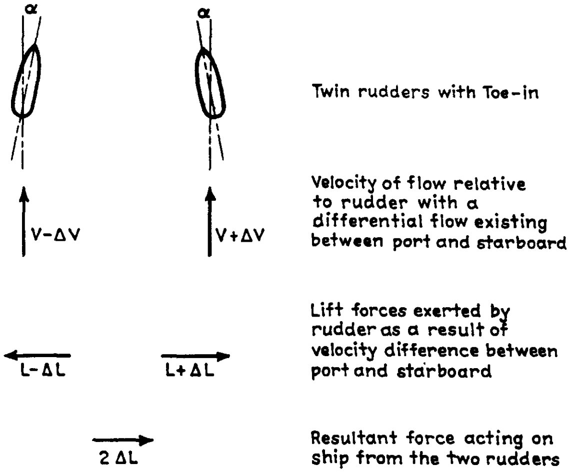
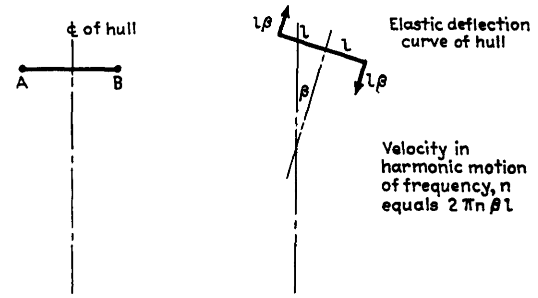


Fig. 13



(a) Location of rudders relative to hull

(b) Motion of rudders as a consequence of elastic hull deflection

Fig. 14

In discussing these problems with my colleagues, we evolved a partial explanation of how the twin rudders with toe-in might excite a self-sustained vibration. The explanation hinges upon the idea that toe-in or toe-out would be most apt to result in lateral forces if the longitudinal velocity is changed, and if the velocity relative to one rudder is increased and to the other rudder is decreased, a net transverse force will be generated (since the angles of attack of the rudders are opposite). This is shown in Fig. 13 of this discussion.

The next question concerns the magnitude of the force resulting from a small change in the velocity of the water flowing past the rudder. This is most easily obtained by differentiating the equation for the lift force on the rudder. Since

$$L = \tfrac{1}{2}\rho C_L A_R S^2$$

$$\Delta L = \rho C_L A_R S \Delta S$$

and it appears that significant transverse forces can be obtained by minor changes in velocity. An estimate of the changes in the flow velocity over the rudders that result from the transverse vibration of the hull can be made by the procedures outlined in Fig. 14. Specific values can be obtained from the horizontal amplitude profile given in Fig. 5 of the paper. Using this curve, with a frequency of 4 cps, and assuming that the rudders are 12 ft off the centerline of the ship, the harmonic change in velocity of the rudders is found to be about 4.25 ips. With this velocity, the value of AS as given in Table 6 of Appendix 2 of the paper and a 2½-deg toe-in angle, the exciting force at 30 knots water speed would be about 550 lb. This force is directly proportional to the rudder angle and, when the rudders are given a toe-in, gives a force that tends to increase the transverse vibration of the hull. When they are given a toe-out the force opposes the vibration of the hull.

This force is not large compared with that of the vibration generator that was required to establish the deflection curve but when combined with the inherent instability of the system described by the author, it gives a partial explanation of why the toe-in angle is significant; and, since it requires that the rudders be located off the centerline, also explains why most ships are free of this particular type of vibration.

E. D. Hoyt,[16] ***Visitor:*** Evidently the vibration observed on the *Forrest Sherman* is unusual in character and obscure in origin. The author has undertaken to explain its cause and rationalize its cure. This has proved to be no small task.

In view of the author's conclusion that the vibration results from ". . . the excitation at the bow which is always present when a ship is underway even in calm seas," one would be inclined to question the title of the paper. However, no direct evidence of such excitation has been presented. On the contrary, the author has ruled out, with reference to the rudders, the kind of flow instability which would be necessary at the bow to produce vibratory excitation in calm seas. In fact, the author has quite well disposed of all but two possible sources of the excitation: (1) random excitation from the waves and, (2) true flutter.

If waves are important, a change of course should have had some effect. Was this tried?

I think it would have helped a great deal if a sample of a typical vibration record had been shown; for not until Item 9 is it mentioned that the amplitude fluctuated widely. It is, of course, characteristic of randomly excited vibration that the response is highly irregular.

A self-excited vibration like flutter would be expected to be much more regular. However, the irregularity observed may have been due to the use of the rudders in steering, to impact with the waves, to varying rudder pressure associated with rolling which changed the backlash in the steering linkage. It does not seem that true flutter has been ruled out by any means.

If it had been possible to conduct vibration-generator tests with the ship underway, the effect of ship speed on the damping of the offending mode of vibration could have been exhibited directly. One wonders if this cannot be done.

The arguments against the influence of cavitation on the vibration are not very convincing. On the other hand I can think of no convincing arguments for its involvement.

I find most disappointing the lack of any plausible explanation for the influence of rudder toe-in. It is stated that the tuning is affected by the mean angle of attack and that this is consistent with the flutter hypothesis. But it is not clear that such mean angle of attack would do any more than add a constant lift and moment which, for twin rudders, would constitute a null system. If the author's hypothesis is true, it would seem to imply marked nonlinearity, perhaps associated with cavitation. It is possible that the opposed, constant moments on the two rudders take the slack out of the steering-gear linkage at one toe-in setting and not at another, thus changing the effective torsional stiffness.

Mention is made of a 240-cpm critical vibration. Yet the author's whole argument is aimed at proving the existence of a subcritical condition. How is this reconciled? And what is the evidence for a 222-cpm natural frequency when a 240-cpm critical vibration occurs?

The author has given a very illuminating glimpse of the complicated field of hydroelasticity. Fortunately in this case the effects were not calamitous. He has shown how serious they may be and amply justifies the effort required to master these problems.

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[16] Reed Research, Inc., Washington, D. C.

N. H. Jasper, Member: The author is to be congratulated for having called attention to a hydroelastic phenomenon which may well be responsible for occasional unusual sensitivity of ship structures to vibration excitation. This phenomenon relates to the phasing of the hydrodynamic forces relative to the nonhydrodynamic damping forces. When the phasing is such as to result in a rather "low" value of the effective dissipative forces applicable to a particular mode of vibration, we will have a relatively large response to exciting forces and this condition, a rather indefinite one, is termed subcritical flutter. Fig. 4 of the paper, and especially Fig. 5 of his reference [5] show that the effective damping may well be greater than at zero speed (positive hydrodynamic damping) over the entire speed range. Nevertheless, a speed range of pronounced decrease in effective damping is called a subcritical-flutter speed, though it is not a self-excited vibration as is flutter.

The natural frequencies and the magnitude of effective damping corresponding to any given speed of a hydroelastic system, such as the ship rudder system, may be obtained from a solution (a tedious one to be sure) of the governing differential equations usually set up for similar aircraft flutter problems. The author has particularized and simplified these equations in a way which permits direct solution for the flutter speed, and for the damping. Of course some accuracy is, at least theoretically, sacrificed for the sake of expediency.

The author, in Fig. 8 and in reference [5], plots the "damping discriminant" versus water speed and, in his discussion, utilizes the values of the discriminant as a direct indication of the magnitude of damping. Unless the damping is proportional to the value of the discriminant such inference is not valid.

Fig. 4 indicates the very great effect of mass unbalance on the effective damping. The damping values plotted for $mh = 2.0$ do show an appreciable decrease (subcritical flutter) near 9.3 knots. Comparison of Fig. 4 of the paper with Fig. 5 of reference [5], both of which presumably present the same results from the test data, do not indicate the same decrease in damping at 9.3 knots, in fact the writer could not infer a "subcritical-flutter" condition from the plot shown in Fig. 5 of reference [5].

The writer believes that the author's rationalization of the excessive vibration experienced on DD 931 class is well taken and that this condition may well have existed, unrecognized, in other ships. It would appear that a nonlinear curve of lift coefficient versus angle of attack, for small angles of attack, is essential if appreciable changes in level of vibrations are to be obtained by a change of rudder toe angles. The pronounced vibration difficulties experienced at the two critical speeds of 240 and 310 shaft rpm of the DD 692 class destroyers may also have been related to a "subcritical flutter" condition, although the "horn" type rudder installed on the latter vessels would seem to be inherently more stable than the hydrodynamically balanced spade-type rudder of the DD 931 class.

The author is to be congratulated for having called attention to a possibly important vibration phenomenon heretofore not recognized in the field of naval architecture.

W. E. Cummins, Member: I have the feeling that the author has violated one of the cardinal rules of this field of literature. He had selected his "rogue" very early in the investigation, and then has proceeded to stack the cards against him, stretching to the limit the clues which support his argument, and neglecting, or passing over with slight consideration those which are at variance with his desired conclusion. He may very well be right in this conclusion, but his argument is not fully convincing.

The villain is an hypothesized, flutter-like hydroelastic condition, arising from (*a*) the fact that the rudders have a natural period of oscillation very nearly equal to that of the three-noded transverse vibration of the hull, and (*b*) a reduction in damping of the hull-rudder system. The damping never reaches zero, or the case would have been conclusively solved by the self-destruction of the rudders, or worse.

Such a condition, as opposed to true flutter, cannot be more than an accessory to the crime, as it does not supply a source for the vibration, and the author recognizes this. The actual guilty party, according to the author, is "the excitation at the bow which is always present." In other words, the vibration is not rudder-excited at all, and the rudders take vibratory energy from the hull, rather than vice versa. If this is the case, the rudders are sort of heroes in disguise, who are prevented from performing their proper function by the reduction in damping.

This argument would be greatly strengthened, or possibly destroyed, if certain additional facts were given. For instance, the variation of the vibration with speed should be shown. If the vibration becomes perceptible as a certain speed is approached, and then increases and decreases as speed is increased, this might correspond to the variation in damping of the system, which apparently can both increase and decrease with speed. On the other hand, if the vibration either holds nearly steady, or increases uniformly with speed, it would seem more reasonable to attribute it to some regime of flow, rather than to "subcritical flutter." Unfortunately, no information of this sort is given, not even the speed at which the vibration starts.

Further, if the hull excites the rudders, rather than the contrary, it would seem a corollary that the vibrations of the two rudders would always be in phase. This could be shown by records of the stresses in the rudder stocks. Such data must have been taken, and they should be presented.

I would presume that changing the angle of a rudder might change the hydrodynamic damping, but it should not significantly change the natural period of the rudder. Therefore, coupling between the hull and rudder would still exist. If the hull is exciting the rudders, then during a 3-deg turn, both rudders should vibrate, even though perhaps with different amplitudes.

If the rudders are such important elements in the damping of vibration, coupling would ordinarily be expected to increase this damping. For other ships, then, three-noded vibrations must be much more serious, since they are not ordinarily coupled with the rudder system, and yet the bow excitation is always there.

Strong support for the author's hypothesis could possibly be obtained by vibration-generator tests under way. If marked variation of response with speed were observed, which could be correlated with theoretical calculations of the damping, the evidence might well be conclusive.

I do not follow the argument that random forces acting on the rudders could not be responsible for the three-noded vibration. Such forces would tend to excite the rudders themselves in their own natural frequency, which would then feed energy to the hull at this frequency. There is no need for these random forces on the rudder to cause all the vibrations of the hull which were observed.

The discussion has brought out a strong argument for further consideration of cavitation as the primary culprit, based upon the difference between the steady-state lift for the cavitating and non-cavitating regimes of flow. I would like to suggest a slightly different mechanism by which cavitation could be responsible. In Professor Abkowitz's paper on antipitching fins and in the discussion thereto, a serious transverse vibration of the hull was noted. This vibration has been very definitely traced to a hydrodynamic impact associated with a collapse of a cavity against the side of the bow. It is possible that a similar impact occurs when an intermittent cavitation bubble attached to the rudder of the DD 931 collapses. Such an impact could certainly produce forces of sufficient magnitude. This hypothesis seems quite consistent with all the facts given in this paper, particularly with the variation in amplitude of the vibration which is mentioned. It would be interesting to see traces of the rudder-stock stresses, to determine if they could be attributed to a shock, rather than to a periodic force.

B. B. Cook, Jr., ***Associate Member:*** Flutter problems have been extremely rare in the field of naval architecture; and, with the possible exception of the singing propeller, have been generally considered unlikely to occur in the marine field for surface vessels. In the field of submarines, with increased speed, the problem of fin flutter becomes a matter of great concern; since, if the unstable oscillation associated with fin flutter is permitted to exist, the submarine will be lost in a comparatively short time.

The paper presents a concise summary of the results of the various phases of the investigation of the vibration phenomenon encountered and what appears to be reasonable evaluations and conclusions. Of course, it is desirable to determine definitely the cause of the rudder-excited vibration on DD 931, and perhaps by presenting the paper at this time, conflicting ideas will be expressed by those familiar with the detailed data obtained from the various tests and trials, from which a more positive solution or agreement will be forthcoming.

In any event, it is believed that this paper should make it clear to naval architects that the phenomena of control-surface flutter should be considered in future designs. It was fortunate that in the subject class of vessels practical corrective measures could be accomplished so simply; but one must realize that the problem may not always lend itself to such a simple solution.

The writer is of the opinion that the subject of control-surface flutter is one that needs extensive investigation. We should immediately expand our research in the field of hydroelasticity so as to be in a position to avoid serious difficulties in the design stage rather than after the ship has been built.

Paul Kaplan, ***Associate Member:*** I am very happy to see that hydroelastic problems are now being recognized by the profession as a field of importance. The aim of continually higher speed, with greater structural loadings, accentuates this concern and points to a continued necessity in the future to take account of hydroelastic phenomena.

This is standard practice in the aeronautical field and may evolve into almost equal importance in the hydrodynamic field as well.

Turning now to the content of the paper, the author tends to relate the cause of the phenomenon on the DD 931 as the same as that indicated in the DTMB report by McGoldrick and Jewell, reference [5] of the paper. However, I intend to question the validity of this conclusion in the present case. Flow studies, reported in reference [4] of the paper, point out the occurrence of an irregular cavitation pattern. My intention is to discuss the effect of this cavitation on the observed behavior of the DD 931.

During the past year, Charles Henry and the writer carried out theoretical studies at the Davidson Laboratory on the flutter of supercavitating hydrofoils with large fully-developed cavities. The results of the study showed that severe flutter was possible at low-density ratios, in distinction to the results obtained for fully-wetted flow conditions. The present situation is different, since it is not a case where a long steady cavity is established, about which arbitrary disturbances occur. Nevertheless, certain similarities in behavior can be carried over. The hypothesis is as follows:

The cavitation pattern may be fluctuating and irregular, and hence contain a spectrum of frequencies. The rudder will respond mainly in the region near the natural frequency and this starts to drive the hull. The motion is maintained by this irregular cavity pattern, since it alternately forms and breaks down; i.e., it appears that perhaps a flutter starts and probably breaks down, but still drives the hull. This continues to occur as long as the cavitation is present.

The damping variation of supercavitating foils varies in a complicated manner as a function of frequency and cavitation number. In fact, the torsional damping for supercavitated foils is negative for a small frequency range. The forces and moments in the cavitated-foil case, especially for large reduced frequencies, which are greater than 2 for the present case, show large phase leads. This differs from the standard fully-wetted case and may be responsible for providing the proper phasing for flutter. The large response of the rudder will occur at frequencies near the natural frequency, but not exactly at this "resonance" condition, since the damping is highly frequency dependent.

As far as the result of reduced vibration of one rudder due to turning is concerned, this behavior is also supported by the fluctuating-cavity hypothesis, since cavitation is avoided. Without cavitation no reduced damping will occur and hence no vibration of that rudder. The author's concept of the flutter possible with cavitation is based on steady-state concepts while in actuality the flow is time dependent throughout and it is rapidly changing. The wide fluctuations of amplitude of the hull also may be supported by the fluctuating-cavity hypothesis because of the varying exciting and damping effects. This is a characteristic of the nature of flutter, but it can be shown to be due to the effects of cavitation, which induces the observed flutter.

The various possibilities for the cause of this hydroelastic phenomenon point out the difficulties in explaining the true cause. It is a fond hope that the provocative nature of this study will stimulate further work in this field. The author has certainly done the naval architectural profession a service by serving as a pioneer in confronting actual operational problems with a hydroelastic analysis.

T. M. Buermann, *Member:* Having been associated with the trials of the 931 class destroyers, especially those concerning vibration, I have reviewed this paper with great interest.

This problem was most interesting because even after its cure by resetting the rudder angles, the actual cause of the vibration remained unknown. Steps taken to uncover the underlying cause are reported in the paper and the primary possibilities are listed in the paper as follows:

(*a*) Random hydrodynamic forces.
(*b*) Flutter.
(*c*) Periodic flow excitation.

The author makes his case for flutter and "accounts" for it, although he makes it clear that he has not proved that flutter is the cause of vibration. This means that the other possibilities still remain.

In developing the "flutter" thesis, the author states, "The analytical study just mentioned combined with the experimentally determined hull-response characteristic provided substantial evidence that the natural frequencies of the rudders (with the ship dead in the water) were close to the frequency of the three-noded horizontal mode of the hull." There is also reason to believe that the rudders did not have a natural frequency of 4 cps. Fig. 15 of this discussion (Fig. 1, Appendix D of the Boston Naval Shipyard Evaluation Report, reference [1] in the paper) gives the response in air of the rudders and shows a small peak at 10 cps, and a large one at 23 cps. These results are qualified by the fact that there was play at the tiller-arm pin which would not be present with the ship underway, owing to the lateral forces on the rudder.

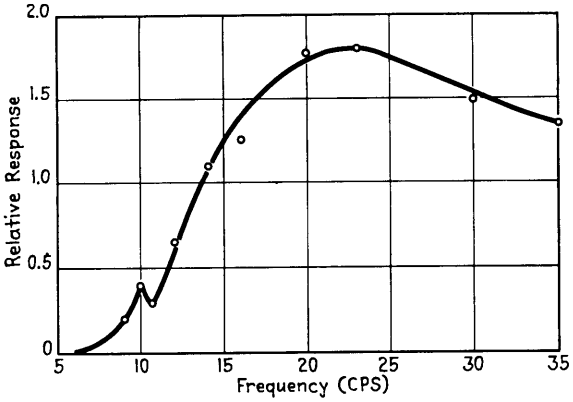

Fig. 15 Response of trailing edge of rudder during natural frequency investigation of USS Forrest Sherman in May 1956

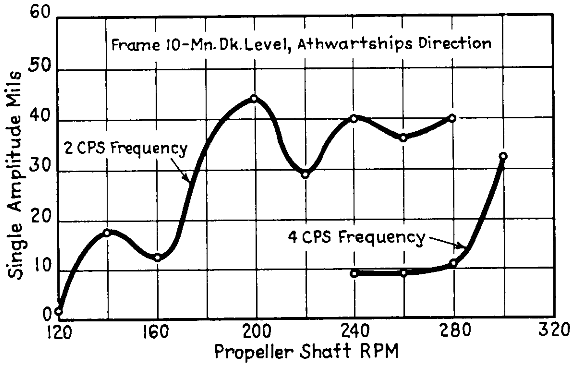

Fig. 16 From graphical summary of hull-vibration, USS Forrest Sherman

Calculation of the rudder frequency at Gibbs & Cox was complicated by the oil in the system, but resulted in a frequency of 20 cps for both bending and torsion.

The author gives a table showing that the amplitude of the 2-noded, 2-cps vibration is 3 times the amplitude of the 3-noded, 4-cps vibration with the original rudder setting.

The author points out that with the new setting, the 3-noded, 4-cps vibration is only 16 per cent of the original, while the 2-noded, 2-cps vibration is still 58 per cent of the original. The relationship not mentioned in the paper is that the 2-cps vibration has 10 times the amplitude of the 4-cps vibration with the new rudder setting.

A study of the Boston Naval Shipyard Evaluation Report [1] discloses that the filtering system used in recording the 4-cps vibration sometimes removed the 2-cps record. Even so, ample records of 2-cps vibrations are shown in reference [1], and indicate that the main hull structure at varying locations from bow to stern vibrates at 2 cps at shaft RPM from 160 to 280 with single amplitudes of 40 mils or more with the original rudder settings. An example from Enclosure 1, Appendix B of reference [1] is included in this discussion as Fig. 16. With the new rudder settings, the 2-cps vibration probably amounts to 20 mils single amplitude over the range just indicated.

It is surprising that no more attention has been paid to the cause of the 2-cps vibration. This vibration occurs at such low speed as to eliminate cavitation as a cause. If I understand the present paper correctly, the flutter theory cannot account for the 2-cps vibration in the same manner it accounts for the 4-cps vibration.

Of the causes of the vibration mentioned, only random flow excitation remains to explain the 2-cps vibration. The author seems to support this in discussing flutter. In this connection he states, "... in this case the role of the rudders was simply to make the three-noded horizontal mode the most sensitive mode as long as the ship speed was high and the toe-angle was in. The presence of two-noded horizontal hull vibration under either toe-angle condition confirms the presence of horizontal bow excitation."

A destroyer of necessity has a slender, flexible hull, easily excited at its natural frequencies, but can we conclude that the 2-cps vibrations are from random-flow excitation, or is there some other possiblity? If this is the cause of the 2-cps vibration, does it also affect the 4-cps vibration dealt with in the paper? It seems desirable that the study of the cause of the hull vibrations on the DD 931 class destroyers should continue.

CDR. Patrick Leehey USN,[17] ***Visitor:*** L. C. Woods[18] established certain analytic results concerning the unsteady oscillation of cavitating hydrofoils which shed some light on the principal unresolved problem; namely, the reason why a change of toe angle on the rudders eliminated the source of the vibration. Woods treated the case of a partially cavitating hydrofoil with infinite cavity undergoing small periodic oscillations. He found that the principal effect of a suction-side cavity was a marked reduction in the lift and pitching moment "damping" derivatives from their values for the corresponding classical unsteady-oscillation problems of Theodorsen. In particular, flutter was predicted for a certain range of

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[17] Bureau of Ships, Navy Department, Washington, D. C.

[18] Proceedings of the Royal Society of London, Series A, vol. 239, 1957, pp. 328–337.

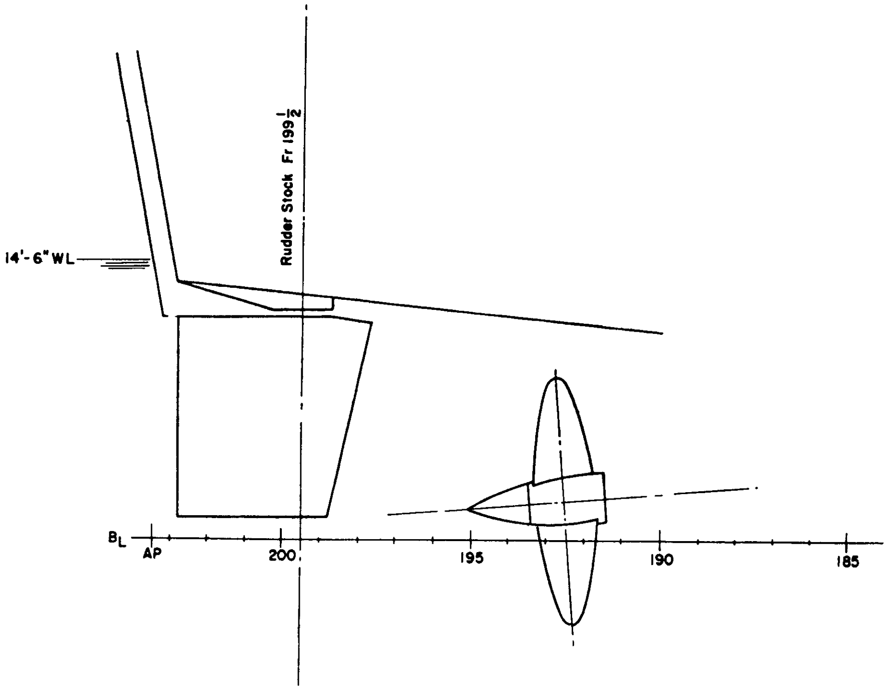


Fig. 17 Profile view of rudders and propellers of DD 931 class destroyer

Strouhal numbers for the cavitating-hydrofoil case where no flutter would occur for the corresponding noncavitating case. Unfortunately, his analysis required that the point of flow separation be known in advance, hence no immediate test of the applicability of his results to the problem at hand is available. It appears a reasonable conjecture, however, that the change from a toe-in to a lesser toe-out angle eliminated outboard (suction-side) cavitation and increased the pitching moment damping of the rudders sufficiently to prevent flutter and consequent lateral hull vibration.

Author's Closure

The author takes this opportunity of thanking those who have contributed to the discussion of this paper. Their varied points of view are invaluable. It is interesting to find such a diversity of opinion and this applies both to those who advocate an explanation in terms of a flutter mechanism and those who advocate an explanation in terms of exciting forces arising from cavitation. At the risk of some repetition the author has attempted to answer individually all those discussers who have questioned the validity of various arguments presented in the paper.

First it will be helpful to note the additional views of the rudders and propellers inserted here at the suggestion of Captain Saunders and Lieutenant Hillmann, Figs. 17 and 18. A photograph of the towing model taken from astern, Fig. 19, is also included.

Dr. Jasper's first point is that the term "subcritical flutter" has been applied in ranges of speed in which the reduction in damping due to flutter action may still leave an over-all damping as large or larger than the zero-speed damping value. It is quite true that, while the critical flutter speed can be defined sharply as the speed of null damping, any subcritical speed range is quite arbitrary. This, however, should not be allowed to obscure the fact that the oscillations of the hydrofoil can bring about a very marked reduction in the damping that would otherwise exist. For example, in the case of the TMB flutter apparatus, if the hydrofoil were locked in rotation, the damping at 9 knots would be about 15 times as great as the zero-speed damping. Thus, at this speed, flutter action has reduced the damping to only about 7 per cent of the value which would otherwise prevail.

The author agrees with Dr. Jasper that the calculation of the actual damping constant would be preferable to the evaluation of the damping discriminant given in Fig. 8 of the paper. Logarithmic decrements were obtained by the use of

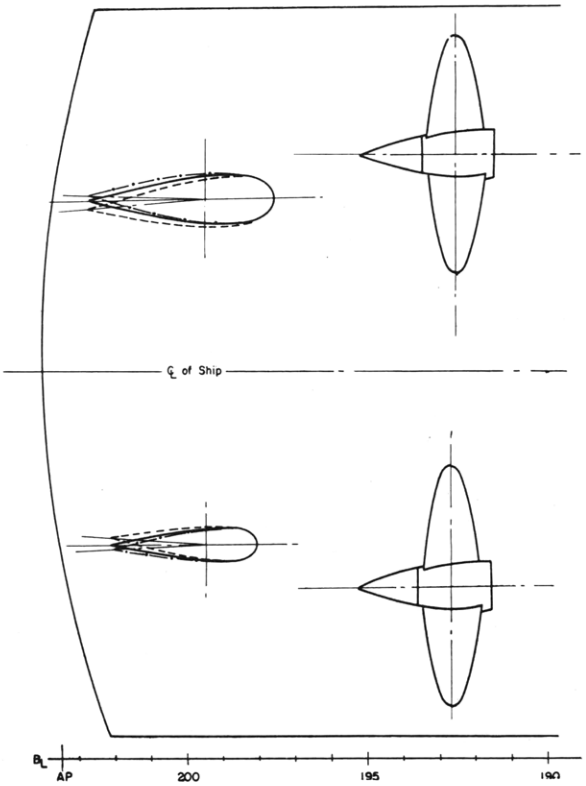


Fig. 18 Plan view of rudders and propellers of DD 931 class destroyer

Solid lines indicate zero toe-angle setting. Dashed lines indicate rudders in 3 deg at trailing edge. Dot-dash lines indicate rudders out 1½ deg at trailing edge. Port section is at top of rudder. Starboard section is at level of propeller center

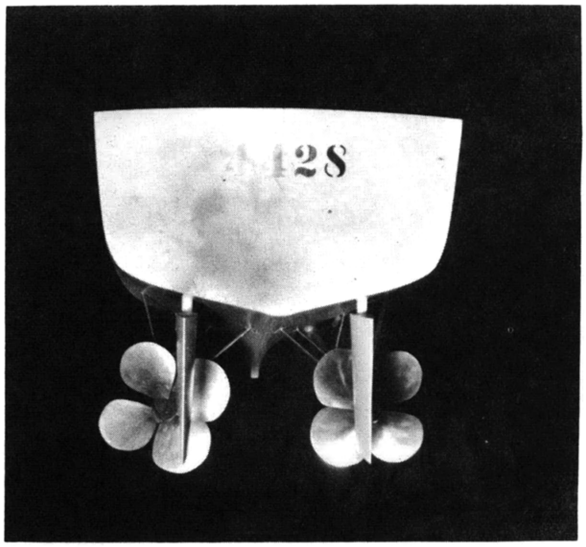


Fig. 19 Astern view of model of DD 931 class destroyer

analog computers in one set of calculations and the aim in the future will be to obtain the complex roots of the polynomial which will yield the actual damping values in such analyses. However, it should be noted that the discriminant curves and the actual damping curves must cross the axis at the same speeds.

Dr. Jasper calls attention to the difference between Fig. 5 of reference [5] and Fig. 4 of the paper, both presenting damping values for the DTMB flutter apparatus. Both figures show damping values derived from decay rates observed in impact tests. In the figure in the paper, however, the author inserted a value at 9.3 knots derived from the selectivity of the apparatus response. It would have been impossible to establish a decay rate here since it was found that excitation at a frequency which was a harmonic of the towing-carriage drive-wheel frequency coincided with the apparatus frequency at this speed. The series of runs on which the paper is based extended only to 10 knots. In reference [5] of the paper, which was prepared at a later date, all data available up to 20 knots were plotted. The curves given in the paper were drawn by the author. In the reference it was not attempted to plot curves but only to indicate specific values. The rate of increase of damping above 10 knots indicated by the extension of the curve in the paper is less than indicated in the reference, but both figures show a distinct rise in damping above 9 knots.

While Dr. Jasper is well justified in calling attention to this, the author's argument for a subcritical-flutter condition on DD 931 does not rest on the indication in the paper that a sharp drop in damping in the apparatus occurred around 9.3 knots. It is sufficient to establish that the minimum damping was far below the value that would have existed without flutter action. An effect that could reduce the damping by a factor of the order of more than 10 could be expected to reduce it to zero or even to negative values for only relatively small changes in the design parameters of the rudder-hull system.

The full-scale lift and moment data furnished by Mr. Becker for spade rudders are quite pertinent to the DD 931 hull vibration problem. The symbol L appearing in the equations in the paper is the ratio of the moment to the lift force. If both are directly proportional to the angle of attack, L is a constant, and no changes in the over-all damping characteristics of the system are to be expected with a change in mean angle of attack. The curves presented by Mr. Becker show marked nonlinearities. This indicates that, in the case of a spade rudder, not only does L vary over a wide range as the mean angle of attack is changed, but the coefficient A in the equations on page 358, which depends on the slope of the lift-force curve, also changes considerably. This indicates that only a relatively small change in the toe-angle setting can cause the damping characteristics of the rudder-hull system to change radically.

The author agrees with Mr. Henry's statement: "It can be surmised, therefore, that if classical flutter theory were applied to the experimental setup of the DTMB flutter apparatus reported by the author, then the theoretical results would not agree with the data presented in Fig. 4 of this paper." Since Mr. Henry accepts the classical or exact flutter theory, this implies that he rejects the experimental data presented. However, he offers no criticism of the experimental technique. On the contrary he calls for an explanation of three discrepancies between the exact theory and observations on DD 931 and the DTMB flutter apparatus.

The author is glad to report that, owing to the recent extension at the Model Basin of the project under which this paper was written, it will be possible in the near future to carry out calculations according to the more elaborate mathematical analysis advocated by Mr. Henry. In fact, Mr. Jewell's discussion refers to a step that has already been taken in that direction.

While Mr. Henry's discussion implies that there is great confidence in the exact theory in the aircraft field, the author infers from reference [14] of the paper that in that field the exact theory has been experimentally verified only up to a Strouhal number of about 0.4. For DD 931 the Strouhal number is 2.3. However, the exact theory was by no means ignored in the DTMB investigation.

The paper mentions three analyses that were applied to the hydrofoil having two degrees of freedom; namely, the modified Theodorsen analysis, the simplified analysis using simultaneous equations, and the still simpler analysis which yields a direct formula for the critical flutter speed. In fact, the latter was used as a basis for the design of the DTMB flutter apparatus. It is true that this formula proved to be incapable of forecasting critical flutter, but it was the erroneous prediction of the exact theory that hydrodynamic balance would exist if the axis of the hydrofoil were set at the forward quarter chord position that suggested this simplified analysis in the first place. The equations given in the paper to which Mr. Henry refers indicate that when the axis is so located, and when the system is locked in translation, equilibrium is possible under zero torque at any preset angle of attack at any speed. If it is impossible to manufacture a hydrofoil that has such properties, is it not reasonable to ask how the exact theory applies to real hydrofoils? It was not until the moment term (which according to the exact theory should have been zero) was included in the analysis of the DTMB apparatus that it was possible to derive curves exhibiting damping characteristics anything like those actually found experimentally.

While no results of calculations by the exact theory are given in the paper, the modified Theodorsen analysis discussed in reference [5] is a closer approximation to it than the other analyses presented. This was applied to both the DTMB apparatus and to the rudder-hull system of DD 931. Results have been obtained so far only in terms of the Routh discriminant which is used as a basis for the damping discriminant. In the case of the TMB apparatus they indicated higher speeds for both critical and subcritical flutter conditions than indicated by the simplified analysis as shown by Fig. 7 of reference [5]. In the case of the DD 931, the calculations so far made by the modified Theodorsen analysis have indicated neither critical nor subcritical conditions below 100 knots, whereas the results plotted in Fig. 8 of the paper show subcritical conditions close to the top speed of the ship.

As to the first discrepancy for which Mr. Henry requests an explanation, the exact theory shows no dependence on mean angle of attack because of the linearity of the lift force and moment terms in the equations. Both the experiments in the towing basin and the full-scale data for spade rudders furnished by Mr. Becker show that real hydrofoils, and especially spade rudders have distinctly nonlinear characteristics. In the case of DD 931 it must be realized that nonuniformity of the velocity field ahead of the rudders and the possibility of cavitation contribute to this nonlinearity. Under the circumstances it would be surprising if the flutter characteristics of the rudder-hull system did not vary with the mean angle of attack.

The second discrepancy to which Mr. Henry refers is the reported constancy of the frequency of the critical vibration on DD 931. Mr. Henry apparently concludes, on the basis of the exact theory and on aircraft experience, that the constancy of the frequency rules out flutter as the explanation of the phenomenon. If the constant frequency reported in reference [1] is to be taken literally, flutter is ruled out not only by the exact theory but by all analyses mentioned in the paper except the one that yields the simple flutter-speed formula. Even as regards the latter, however, it is pointed out in reference [5] that the formula is considered valid only when the proximity to flutter reduces the damping to such a low level that it has a negligible effect on the frequency.

In the early investigation of the vibration of DD 931 the fact that the frequency did not correlate with the shaft speed was the signal that an unusual vibration problem existed. Although the probable situation was that the frequency varied slightly, the early reports stated that the frequency was constant, chiefly to emphasize that it was independent of the shaft speed. Both reference [1] and the paper do state that the frequency was 4 cps. However, the use of a single figure in such a case indicates that it is only an average value and not a precise one. In certain places the frequency is given to three significant figures and then the value given is based on a closer measurement. In the paper the author actually used as an argument supporting flutter the observation by the Boston Naval Shipyard that in one of the trial runs with a toe-out rudder-angle setting an apparent resonance was passed through at 222 cpm whereas under the toe-in rudder-angle setting critical vibration appeared at a frequency of 240 cpm. The argument is that the 240-cpm frequency is the flutter frequency, while the 222-cpm frequency is a natural frequency of the rudder-hull system.

The author believes that a narrower range of frequencies is to be expected in the ship case than in the aircraft case because of the great difference in the ratios of the torsional to the flexural parameters in the two cases. If the rudder becomes detuned to an appreciable degree it is no longer able to exert a noticeable effect on the hull. In the aircraft case the torsional frequency is directly observable whereas in the ship case there can be observed directly only the vibration of the hull and one can determine how the rudders are performing only by using special measuring techniques.

The third discrepancy pointed out by Mr. Henry involves the density ratio. This is a question of evaluation of parameters. In arriving at a density ratio of ½, Mr. Henry takes simply the ratio of mass per unit length of the hydrofoil to the added mass of water per unit length. This procedure is not valid for either DD 931 or the DTMB apparatus since in neither case can the hydrofoil move in translation independently of other masses moving with the axis. The author has estimated the density ratio applicable to the apparatus as 2.5. In the case of the ship it is necessary to derive an effective or generalized mass for the entire hull applicable to the rudder-stock location. The value of the effective mass thus derived is given in the paper. On the basis of this the author estimated a density ratio for DD 931 of 36 which is actually in the range of aircraft values.

It is important to note that subcritical conditions are possible in systems in which critical flutter will not occur at any speed. Here the damping drops to a minimum with increasing speed and then rises again, never reversing thereafter. Those who disparage the simplified analyses in favor of the exact flutter theory should realize that there is another side of the coin. Thus the modified Theodorsen analysis predicts that at 9 knots the apparatus damping would be at least 10 times the zero-speed damping. Experimentally this damping was found to be actually less than the zero-speed damping. What would the advocates of the exact theory say if it predicted that the DD 931 rudder-hull system would be so heavily damped in its upper speed range that vibration should have been negligible?

The author agrees with Dr. Abramson that the term "subcritical flutter" can introduce some confusion, but there seems to be a need for a clear recognition of the fact that a condition may exist in a control-surface system at a speed far below a critical flutter speed at which serious trouble may be encountered. At this stage it seems more important to recognize the condition than to provide a suitable term for it.

While the types of nonlinearities pointed out by Dr. Abramson would invalidate the analyses so far applied to DD 931, it must be remembered that the DTMB flutter apparatus was designed especially to be free of such conditions. The author believes that the phenomenon occurring on DD 931 was essentially reproduced in the towing basin, but no attempt is made to conceal the fact that many simplifying assumptions were made in idealizing the rudder-hull system of DD 931 for the purpose of analysis. While aircraft flutter analysts apparently also make linear approximations, it is welcome to have Dr. Abramson call attention to special types of nonlinearities found in aircraft control-surface systems. However, the twin-rudder arrangement with toe angle should take up any backlash in the steering system.

Since the publication of reference [5] Mr. Jewell has made flutter calculations for the DD 931 rudder-hull system by the quasi-steady analysis frequently used in aeroelasticity. This is in line with the recommendations in the paper, but the author does not agree that the results presented by Mr. Jewell furnish a sound basis for rejecting the subcritical flutter hypothesis for this ship. Instead of making similar calculations for the DTMB apparatus and showing his analysis to be in better agreement with the experiment than the analysis presented by the author, Mr. Jewell bases this claim on the evidence in the aircraft field that the quasi-steady analysis is more accurate at high Strouhal numbers. One of the main questions requiring clarification is whether results in aeroelasticity are directly transferable to hydroelasticity. As Dr. Abramson points out, there is considerable evidence to the contrary. It should be noted that whereas in the aircraft field high Strouhal numbers imply high frequency, in the ship case the high Strouhal number is due to the low velocity, the frequency itself being relatively low.

However, even these recent results of Mr. Jewell seem to the author to leave the flutter hypothesis still very much in the picture. They still indicate that the over-all damping drops to the zero speed level at 30 knots. Presumably this is without correction for the propeller jet effect which could reduce it to 25 knots. When, in addition to this one considers that not one but a series of such calculations should be made among which would be included one based on the maximum value of lift slope indicated in such curves as furnished by Mr. Becker, it appears that they could well show even a critical flutter speed not far above the top speed of this ship.

Dismissing the flutter hypothesis on the basis of this latest analysis, Mr. Jewell then considers the possible role of cavitation in producing the phenomenon. First he discusses cavitation which is coupled with the rudder oscillations, thus fluctuating each cycle. He refers to the author's statement that such a phenomenon would require rudder angular amplitudes of the same order of magnitude as the change in mean angle of attack or toe angle required to develop the cavitating condition. He then states that, if the author's argument on this point were sound, flutter also would be ruled out since flutter theory is developed from considerations applicable to infinitely small amplitudes. While the author has stressed the importance of nonlinearities in this problem, the contention has been that linear approximations are valid in the range of small oscillations. In fact, this is assumed in most vibration analyses in engineering. In the author's view, when changes of toe-angle setting are made, the nonlinearities effectively change the coefficients in the equations which, however, still remain valid for small variations of angle of attack about the new mean value. The author regards the collapse of a cavity as introducing a discontinuity which no longer permits such linear approximations, the argument being that if the amplitudes remain small, collapse of the cavity will not occur but cavitation may still play a role in lowering the flutter speed by changing the system parameters.

However, Mr. Jewell does not pursue the argument for coupled cavitation but goes on to uncoupled cavitation and finally advocates the hypothesis of periodic cavity shedding. His argument here is that this phenomenon is independent of the rudder oscillations and furnishes the 4-cps lift-force variation which vibrates the hull. Tacitly admitting that there is no direct evidence that this occurred on the ship, he bases this on the experiments reported by R. W. Kermeen.[15] The reference reports hydrofoil experiments in a water tunnel in which periodic formation and shedding of cavities occurred at a frequency of about 5 cps. Mr. Jewell deduces from those experiments that the lift-force variations would be of the right order to vibrate the hull of DD 931 at the observed amplitude and that these forces would not diminish during such turning tests as were conducted by the Boston Naval Shipyard on DD 931.

Mr. Jewell rejects the author's deduction from the Boston Naval Shipyard records that when the ship is on straight course and the amplitude builds up to a peak the rudders come into phase, and hence, he concludes that the author's treatment of the twin rudders as a single equivalent rudder is invalid. Thus, while affirming his belief that the phase relation between the angular oscillations of the rudders is random, he apparently believes that they shed cavities in a reinforcing phase relation when the ship is on a straight course.

With regard to the treatment of the two rudders as a single equivalent rudder it should be pointed out that when the ship is on a straight course the rudder-hull system is analogous as a vibratory system to a geared-turbine propulsion system in torsional vibration in which two turbine branches connected to a common reduction gear are tuned to the same frequency (See "Dynamic Effects" by F. M. Lewis, chapter 2, vol. 2 of *Marine Engineering*, published by SNAME, 1944). Such a design is called a "nodal drive" and the mode in which the two branches oscillate in opposite phase with the gear not oscillating (hence a node) is said to be "tuned out." In the case of the rudder-hull system, while theoretically a mode exists in which the two rudders oscillate in opposite phase, the hull remains at rest in this mode and thus it is also "tuned out." In the modes in which the hull also vibrates the rudders oscillate in phase with each other. On turns, however, the rudders are subjected to different hydrodynamic tuning effects and then this situation does not apply.

Cavitation shedding is surely a phenomenon about which very little is known at the present time. While it would be unreasonable to call for a mathematical analysis of this phenomenon under these circumstances, it is not unreasonable to ask how the shedding frequency is transferred from the water-tunnel experiments reported in Mr. Jewell's reference[15] to the DD 931. Mr. Jewell states that this mechanism could maintain undiminished hull amplitude during the turning maneuvers whereas the subcritical flutter mechanism could not. The author can just as well argue that in these maneuvers, in which the rudder that lines up with the ship's longitudinal axis ceases to oscillate, the other rudder is subject to greater lift-force variations due to the change in mean angle of attack.

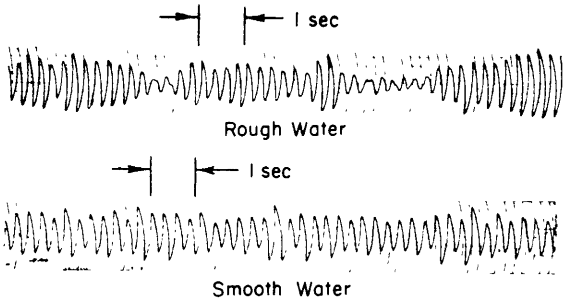
At this point a basic difference in the arguments of Mr. Jewell and the author should be noted. Mr. Jewell argues that, if the flow were normal, the subcritical flutter speeds would be out of range. Then he states that cavitation is known to be present. He then maintains that since flutter is not possible a source of excitation must be found. He finds this source in cavity shedding; but, if cavitation had been present, the values of the parameters which he used in his flutter analysis would not have been applicable. The author's argument is that, even if the flow were normal, subcritical flutter conditions would prevail near the top speed of the ship under the initial toe-angle setting. The author then contends that if cavitation develops it can contribute further to the subcritical flutter condition by changing the slope of the lift-force curve without fluctuating significantly.

Both Mr. Jewell and Mr. Hoyt state that the author has furnished little evidence of the real source of excitation under the subcritical flutter hypothesis. The author acknowledges that no quantitative data of this kind are available. The argument is that any ship underway is subject to transient excitation under any sea conditions but excitation varying in intensity according to the sea state. The 2-node vertical vibration of any ship underway can be recorded if the instrumentation has a sufficient range of sensitivity. While the level may fall below the sensitivity of common instruments, the vibration is present nevertheless. In fact this vibration has even been recorded on ships tied up at dockside. The term "bow excitation" is admittedly vague but it is used because under rough sea conditions the wave forces are known to act at the bow. Since Mr. Jewell's contention is that the cavitation shedding frequency is 4 cps, he must agree that there exists some transient disturbance causing the observed 2-node horizontal hull vibration whose frequency is around 2 cps. The author contends simply that this same disturbance also evokes response in the 3-node horizontal mode which becomes noticeable only when this merges into the subcritical flutter mode. A point in this connection that so far has not been sufficiently emphasized is that in the flutter mode the sprung-mass effect of the rudders increases the ratio of bow to stern amplitude. This accentuates the effect of bow forces in this mode in contrast with other modes. This is a feature of the influence function of the mode over and above the damping considerations involved in the flutter analysis.

Mr. Hoyt inquires as to the effect of waves and change of course on the vibration on DD 931. Originally wave action was thought to be the direct cause of the observed hull vibration and it was believed that some peculiarity in the hull construction accentuated the response in the second horizontal mode. In the early trials, runs were made not only at different headings but in both calm and rough seas. As shown in reference [1] the vibration was practically as severe in calm seas as in rough seas. Under calm-sea conditions the amplitude fluctuated in a random fashion. In rough seas there was a noticeable correlation between the amplitude level and the rolling of the ship. Under the subcritical flutter hypothesis the author explains the failure of the vibration to build up to substantially larger amplitudes in rough seas than in calm seas as due to the nonlinearity of the structural damping and to the lower ship speeds for the same shaft RPM in rough seas.

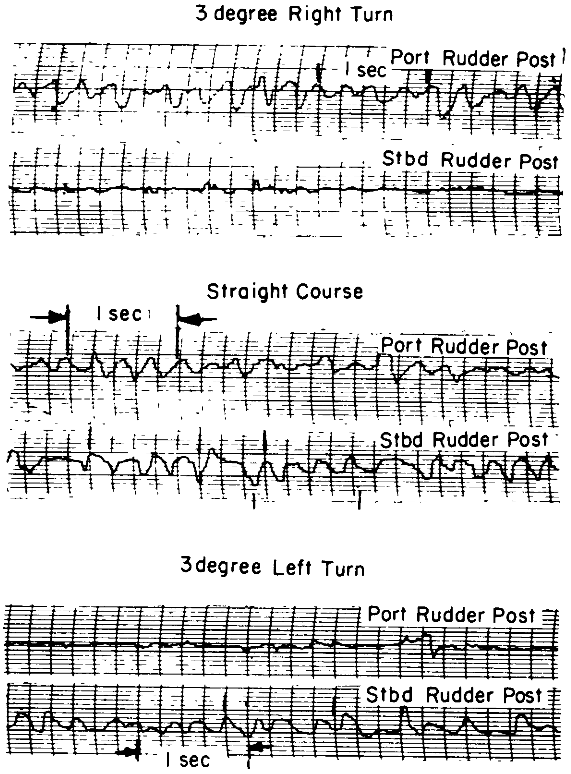
Mr. Hoyt apparently views the condition as nearer to one of true flutter than to subcritical flutter. The author's justification for retaining the term "subcritical flutter" is the belief that bow excitation was present at all times and, of course, the fact that oscillatory instability was never actually reached.

Mr. Hoyt's suggestion to run the vibration generator while the ship is underway is in line with aircraft flight testing practice. The shipboard vibration machines, however, are large and difficult to operate even with the ship dead in the water. Great caution would have to be used in approaching the critical frequency in such an experiment, but, by starting with very small eccentricities one could guard against excessive vibration. Such a test should enable one to distinguish between cavitation shedding and subcritical flutter conditions. In the latter case the amplitude should always build up as the critical frequency is approached since the damping is reduced. If cavitation shedding of the type


Rough Water

Smooth Water

Fig. 20 Oscillograms of hull vibration, reproduced from reference [1]


3 degree Right Turn

Straight Course

3degree Left Turn

Fig. 21 Oscillograms of torsional strain in rudder stocks, from reference [1]

discussed by Mr. Jewell is the culprit, since its frequency remains constant, it should be possible to produce beats between the 4 cps and a frequency near this at which the vibration generator could be kept running. In such a test it should also be possible to distinguish between the frequency of the normal mode of the rudder-hull system and the critical frequency. If further shipboard tests are run, clearly an attempt should be made to observe the flow at the rudders directly, difficult as this experimental observation might be.

Mr. Hoyt's remark that the paper attributes to the change in toe-angle setting only a change in the natural frequency of the rudders is inaccurate. It is also stated in the paper that the effect of this change in eliminating the vibration requires nonlinear lift and moment versus angle-of-attack characteristics.

With regard to Mr. Hoyt's objection to the simultaneous use of the terms "critical vibration" and "subcritical flutter" the author intended to mean by the former term only that the vibration was severe. Critical flutter would be unstable. The evidence for the 222-cpm natural frequency and the 240-cpm vibration requested by Mr. Hoyt will be found on page 25 of reference [1]. Mr. Hoyt's statement that the lift and moment due to toe-angle in a twin-rudder design constitutes a null system is applicable to the steady values, but the oscillatory values are reinforcing when the rudders move in phase. At Mr. Hoyt's suggestion there are reproduced here sample oscillograms of hull vibration, Fig. 20, and torsional strain in the rudder stocks, Fig. 21, taken from reference [1] of the paper.

Mr. Reed's discussion illustrates how many aspects may be involved in a hydroelastic problem. He calls attention to the effect of the rotation of the hull at the stern in the mode of flexural vibration involved. There are at least two components of this rotational effect of which he mentions only one. The rotation of the stern with respect to a vertical axis will cause a slight change in angle of attack as well as a slight fore-and-aft displacement of the rudder stocks. Both of these motions are tied to the normal mode pattern of the hull and hence are locked in phase with the horizontal displacement of the hull.

The change in angle of attack had actually been considered at the Model Basin. Since this change would be negative for a positive hull displacement at the rudder-stock location it would produce a component of lift force opposite in direction to the displacement and hence assignable to stiffness in the force equation (the second equation on page 360). In this category its magnitude was estimated to be negligible.

The effect pointed out by Mr. Reed is equivalent to a sinusoidal variation in the velocity field ahead of each rudder. Where the angular amplitude of the rudder is small relative to the mean angle of attack (as is true in this case on the basis of the experimental evidence) the chief component of this involves $\dot{y}$ and the mean angle of attack and not the terms containing θ in the equations mentioned. Thus it can yield either a positive or negative damping action depending on the sign of the toe angle.

It appears, however, on the basis of values derived from the normal mode pattern, as given in Table 3 of the paper, that Mr. Reed has overestimated the magnitude of this effect and that the present state of development of flutter analysis in naval architecture would not warrant taking it into account.

In terms of the metaphor of Dr. Cummins the author acknowledges many shortcomings in the role of a Sherlock Holmes. However, to have jumped at a conclusion at an early stage of this investigation would not only have violated one of the cardinal rules of detective story fiction but would have been highly unscientific as well. It is clearly stated in the last paragraph on page 342 of the paper that, in initiating the control-surface flutter study, no assumption was made as to the explanation of the phenomenon on DD 931.

As Dr. Cummins points out, if a range of severe vibration had been passed through on DD 931 and the vibration had died out at top speed, the case for subcritical flutter would have been much stronger. All that is known is that the vibration kept building up as shown in Fig. 22, herewith, which is reproduced here from reference [1]. While the TMB investigation indicates that subcritical conditions may be a prelude either to critical flutter at higher speeds or to a minimum damping condition beyond which highly stable conditions will prevail, all that can be said at this stage is that the analysis applied to DD 931 and shown graphically in Fig. 8 of the paper indicates the possibility of either of these situations. The rate of increase of amplitude for the 4½-deg toe-in angle setting, however, shown in Fig. 22, strongly suggests that critical flutter would have occurred below 300 rpm at that setting. If, as Dr. Cummins suggests, flow excitation would be expected to yield a steady amplitude or at most an amplitude increasing uniformly with speed, this evidence certainly favors subcritical flutter against fluctuating cavitation excitation.

Dr. Cummins assumes that if conditions are subcritical the rudders take energy from the hull rather than from the flow. This is not necessarily the case. In general the subcritical condition would involve a level of over-all damping lower than the zero-speed damping wherein some negative damping or positive energy is extracted from the flow by the rudders. Only if the damping were greater than the zero-speed damping, would the rudders necessarily be absorbing part of the input energy from the external source and transmitting it to the water. It is to be noted that in aeroelasticity the critical flutter speed is sometimes defined as that speed at which only the aerodynamic damping has been nullified and the structural damping is treated independently. Thus reference [11] is concerned only with aerodynamic forces.

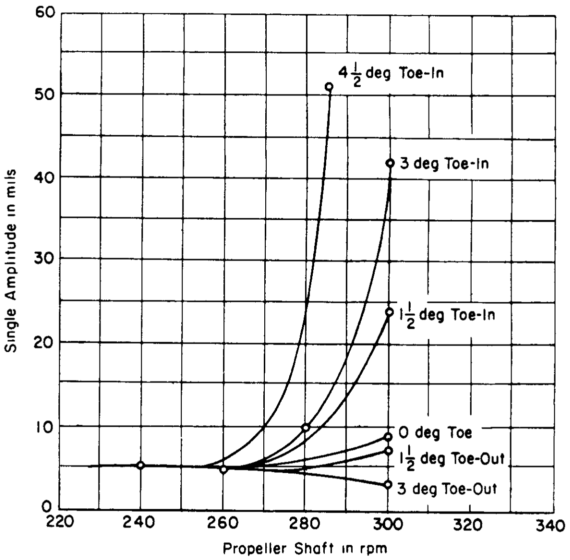


Fig. 22 Athwartship vibration of DD 931 observed at frame 69 under various toe-angle settings, from reference [1]

As shown in Fig. 21 the rudders come essentially into phase in the 4-cps component when the vibration builds up. Other frequency components are also present in these strain records, however, and the in-phase condition is not sustained; but neither is the hull vibration. Even when the ship is on a straight course the rudders must operate occasionally to correct changes in the ship's heading.

The author believes that the tuning of the DD 931 rudders can change appreciably with the toe-angle setting due to the hydrodynamic unbalance at small angles of attack. As indicated by the curves given in Mr. Becker's discussion, the parameter L shifts drastically with a change in the rudder orientation. While the toe-angle settings are definite in relation to the longitudinal axis of the ship the real angles of attack for DD 931 are uncertain because of the complexity of the flow pattern.

Dr. Cummins' statement with regard to 3-node horizontal vibration on other ships seems to be based on a misinterpretation of the argument used in the paper. The author's contention is that it is a rare occurrence for the rudders to be in tune with a horizontal hull mode. When the rudders are not in tune they produce a positive hydrofoil damping action in addition to the structural or zero-speed damping to which vertical modes are subject. In the majority of ships, bow excitation is relatively ineffective beyond the first horizontal mode. When, however, the rudders are in tune with a particular horizontal mode, negative damping can then be developed from the alternating lift force and then the bow forces are able to produce a large hull response in the flutter mode which involves that particular mode of the hull.

Dr. Cummins' suggestion with regard to future vibration-generator tests under way is in line with Mr. Hoyt's. The author agrees that such a test could yield valuable information.

As Dr. Cummins points out, random forces on the rudders could excite the 3-noded horizontal mode of the hull to a greater degree than the 2-noded mode. However, in order for the level of vibration in the 3-noded mode to decline 84 per cent on removal of these forces, the rudder forces would have to be the chief source of random excitation of the entire rudder-hull system. Hence, there should be expected a much larger reduction in the 2-noded vibration than observed when this random source is removed. In fact, the early investigators should then have reported that both 2 and 3-node horizontal hull vibration developed at high speed and that both modes of vibration disappeared on reversal of the toe-angle setting.

Dr. Cummins' final point is that shock loads on the rudders due to collapse of cavitation at presumably nonuniformly spaced intervals could have produced the observed vibration. Both the strain and vibration records do indicate the possibility of this general type of transient loading which the author visualizes as acting at the bow. While the collapse of a cavity might cause a more intense local impact pressure than a wave impact at the bow, the over-all effect on the rudder-hull system could be very much the same. Then the question to be answered is: If this type of loading were applied first at the rudders, then at the bow, which would be more likely to produce the observed response? Here again the author maintains that under such loading at the rudders, changing the toe-angle setting should decrease the response in both the 2 and 3-node modes proportionally whereas with this type of loading acting at the bow, and with the damping characteristics in the 3-noded mode dependent on the toe-angle setting, the change of toe angle would cause a much greater reduction in amplitudes in the 3-node mode than in the 2-node mode.

Dr. Craven's discussion suggests that cavitation would impose a step loading in both lift force and moment on the rudders (this loading evidently being triggered by the oscillation of the rudders themselves). While the duration of the step load is not specified in his discussion, presumably the lift force would be sustained for about a half cycle so that, acting in the same direction as the hull velocity (first on one rudder, then on the other) it could yield a net energy input per cycle which would account for the energy dissipated per cycle by the damping forces.

The author agrees that under the present limited state of knowledge of forces arising from cavitation one can only assume certain standard types of forcing functions, but once this has been done the mechanism should be consistent with experimental observations. If such a mechanism were to cause a buildup of vibration to a certain amplitude level, what would cause it to diminish thereafter as long as the ship's speed was then maintained and steering action did not take place?

Under this assumption the effect of change of toe angle does not involve rudder tuning but simply a cessation of cavitation. This then fails to explain the cessation of the oscillation of the rudder that becomes lined up with the ship's axis in the turning maneuvers, for this rudder (still being in tune with the hull) should then be driven by the hull.

The author believes that an explanation which takes no account of such nonlinearities of lift force and moment as indicated in the curves furnished by Mr. Becker cannot be considered realistic.

The case of apparent rudder flutter cited by Prof. F. M. Lewis is most interesting and particularly the method by which it was eliminated. While it is not clear in this case to what extent hull vibration was involved, the calculations made for DD 931 indicated that the angular damping constant, *c*, had a very pronounced effect on the predicted flutter speeds. This is shown in Fig. 7 of the paper. It seems possible in a hydraulic steering system of the type mentioned by Professor Lewis to produce not only a change in angular damping but also a change in angular stiffness that can throw the critical flutter speed well out of the operating speed range.

Mr. Buermann's discussion involves two points of paramount importance in considering the DD 931 hull-vibration problem. The first is the question as to what was the torsional natural frequency of the rudders. The paper mentions the fact that attempts were made to determine this frequency by direct experiment. Both the Boston Naval Shipyard and the David Taylor Model Basin conducted experiments for that purpose. Recognizing that this was a very difficult experimental undertaking, the author considered both of these attempts inconclusive. It is true that, if a determination could be made of the natural frequency of the rudders in air, a fair estimate of the value for the submerged condition could then be made by allowing for the inertia effect of the water. However, when a small vibration generator is used for such a determination (as in the case of the experiments at the Boston Naval Shipyard cited by Mr. Buermann) there are great difficulties in insuring that the desired mode of vibration has been produced. In many instances the small vibration generator will not develop sufficient driving force to excite the lower modes of vibration which will hence escape detection. Note that Mr. Buermann's Fig. 15 shows no measurable response below 6 cps.

The belief that the torsional natural frequency of the rudders when submerged was in the vicinity of 4 cps is based on the tests made by the Model Basin on DD 936 with the DTMB 3-mass 40,000-lb vibration generator. This machine was capable of vibrating the entire ship. A plot of the ratio of the angular amplitude of the rudder to the rectilinear amplitude developed in the hull while operating this machine at a fixed eccentricity setting gave a sharp peak at 290 cpm, reference [6]. This is inexplicable unless the torsional frequency of the rudders (with the ship dead in the water) is near this frequency since such a plot automatically corrects for any hull-resonance effects. Furthermore, the peculiar hull response in the test with this machine pointed out in the paper (Fig. 6 of the paper) could not be explained unless the rudders played the role of tuned vibration neutralizers in the vicinity of the second resonance. Otherwise the second peak in this plot should have been comparable with the first. One has only to compare the response in the first two vertical modes with that in the first two horizontal modes in this experiment [6] to become convinced that the rudders are playing a vital role in the horizontal vibratory response characteristics of the rudder-hull system. This role requires that they be tuned to a frequency near that of the 3-node horizontal mode of the hull when the ship is traveling on a steady course in its upper speed range.

Although the Boston report [1] states that the indication from the rudder test in air was that the torsional frequency in water would be higher than 4 cps, it also points out (first paragraph on page 21 of reference [1]) the inconclusive nature of that test.

The author agrees with Mr. Buermann that the response of DD 931 in the 2-node horizontal mode is quite pertinent to the present problem. Mr. Buermann's Fig. 16 seems to indicate that the 2-cps vibration amplitude does not correlate with ship speed at least in calm seas and this is in line with the author's contention that the 2-cps vibration is due to transient excitation at the bow. Unfortunately the figure does not record the level of 2 cps at speeds beyond which the 4-cps vibration began to build up.

If it could be established that the 2-cps vibration did not build up under conditions in which cavitation at the rudders may have been present, this would, in the author's view, rule out the type of cavitation excitation suggested by Dr. Cummins, that is the impulse or shock type of excitation.

The levels of 2 cps and 4-cps horizontal vibration given in Table 1 of the paper are peak levels measured in the underway surveys made by the Model Basin on DD 936. The amplitudes of both fluctuated widely and the peak values were reported merely to show qualitatively the contrast between the reduction in 4-cps vibration and the reduction in 2-cps vibration occurring after reversing the toe-angle setting of the rudders. The only significance that the author attaches to the reduced level of 2-cps vibration under the toe-out condition given in this table is that it is not comparable with the reduction shown for the 4-cps vibration. The author did not assume that any definite correlation between the 2-cps vibration and the toe-angle setting was indicated by this table, and, by the same token, could not consistently have maintained that such a correlation existed for the 4-cps vibration without abundant independent evidence from the tests run by the Boston Naval Shipyard.

The fact that even after reversal of the toe angle there remains a relatively high level of 2-node horizontal vibration is not surprising to the author. The fundamental vertical vibration of most ships is also relatively high. The 2-node vertical and horizontal modes of a hull are the modes that are the most readily excited at the bow. Their frequency is low and hence the amplitudes, although large, usually do not involve objectionable accelerations. If excessive 4-cps horizontal vibration had not developed, it seems quite likely that no notice would have been taken of the 2-cps horizontal vibration at all. If the 2-cps vibration existing after correction of the 4 cps had been considered excessive a further investigation would have been called for.

The discussions of both Commander Leehey and Dr. Kaplan indicate that true flutter is possible under cavitating flow, the cavitation, in effect, modifying the coefficients which appear in the equations used in the flutter analysis. This modification is in the direction of lowering the critical flutter speeds. In fact, Commander

Leehey's discussion is essentially in agreement with the concept of the possible role of cavitation presented in the paper and presupposes a steady cavity or flow separation.

Dr. Kaplan, however, as regards the particular case of DD 931, adopts the view that a steady, fully developed cavity would not exist, but rather a cavity that fluctuates so as to yield a complex spectrum of excitation. Under this concept of fluctuating cavitation the response of the hull at 4 cps is due to the width of a continuous input spectrum and not to a predominant 4-cps component as suggested in reference [4] of the paper.

This would not be a flutter mechanism from the point of view of the paper but it would still be a hydroelastic phenomenon and one strongly influenced by the tuning of the torsional degree of freedom of the rudders.

Dr. Kaplan contends that this explanation of the phenomenon would also account for the cessation of the oscillation of one rudder during the Boston turning maneuvers. He bases this contention, however, on the fact that cavitation is no longer present on the rudder that ceases to oscillate. This leaves unanswered the contention in the paper that such a hypothesis requires that both rudders continue to oscillate in accordance with the modal pattern of the rudder-hull system. Whether one or two sources of excitation are present will not determine the response pattern in this mode although it will determine the magnitude of the over-all response.

Dr. Kaplan's discussion clearly emphasizes the growing importance in the field of naval architecture of the type of studies to which he refers.

The discussions have indicated a considerable body of opinion that cavitation at the rudders was the essential cause of the vibration phenomenon encountered on DD 931. As has been shown, the explanation proposed by the author in terms of subcritical flutter action does not require the presence of cavitation but is believed to be valid if steady cavitation is present. It is unfortunate that the question of cavitation has beclouded the larger issue as to whether control-surface flutter phenomena are possible in the range of ship operating speeds.

If further full-scale observations are made it is important to keep in mind that merely verifying the presence of cavitation does not prove that cavitation collapse is the essential cause of the phenomenon. Difficult as the experimental technique might be, it would be necessary to confirm either by underwater photography or by recording pressure variations at the rudders that the cavitation produces exciting forces large enough to account for the observed vibration. It would also be of considerable value to establish the phase relation between rudder angular displacement and hull rectilinear displacement at the rudder stock location during the vibratory condition. This can only be done by a phase calibration of the recording system which must include any filter units used in the entire recording system.

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