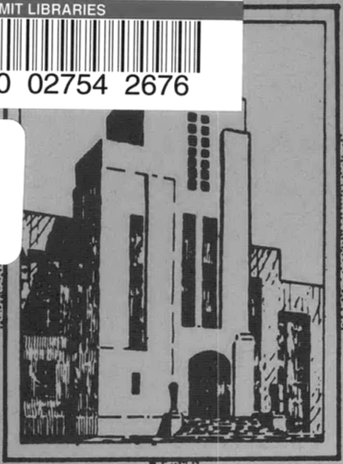
#2



NAVY DEPARTMENT

DAVID TAYLOR MODEL BASIN

[redacted]

HYDROMECHANICS

○

AERODYNAMICS

○

STRUCTURAL
MECHANICS

○

APPLIED
MATHEMATICS

BEHAVIOR OF CYLINDERS WITH INITIAL SHELL DEFLECTION

by

M.E. Lunchick and R.D. Short, Jr.

STRUCTURAL MECHANICS LABORATORY
RESEARCH AND DEVELOPMENT REPORT

July 1957

Report 1150

PRNC-TMB-648 (REV. 2-57)

BEHAVIOR OF CYLINDERS WITH INITIAL SHELL DEFLECTION

by

M.E. Lunchick and R.D. Short, Jr.

Reprint of paper presented at the Applied
Mechanics Division Summer Conference, Berkeley,
California, 13-15 June 1957, of The American
Society of Mechanical Engineers.

July 1957 Report 1150

Behavior of Cylinders With Initial Shell Deflection

By M. E. LUNCHICK[1] and R. D. SHORT, JR.,[2] WASHINGTON, D. C.

Evidence is presented which indicates that, for stiffened cylinders fabricated by welding, initial deflections exist for the shell between frames. The initial deflections of the shell of a stiffened cylinder were measured and found to be approximated closely by a second-degree curve. Using the mathematical expression for the initial deflection curve, a theoretical analysis was made for a cylinder subjected to external hydrostatic pressure, resulting in expressions for the additional deflections and stresses caused by the initial deflection of the shell. Since the initial deflections of the shell for internally and externally framed cylinders were found to be of opposite sign, the theory is offered as a possible explanation of the observed differences in behavior between internally and externally framed cylinders.

Introduction

The THEORETICAL analysis of closed stiffened cylinders subjected to external hydrostatic pressure has been treated by a number of investigators. Considerable progress has been achieved in that more refinements have been made to the theory. Continuous refinement is necessary and desirable in order to increase the accuracy with which stresses and collapse pressures can be predicted, thus resulting in more satisfactory designs of stiffened cylinders.

Von Sanden and Gunther (1)[3] obtained the basic solution for the deflections and stresses for a stiffened cylinder which was perfectly circular, had a shell of uniform diameter, and consisted of a material that deformed elastically at all times. Von Sanden and Gunther neglected, however, to account for the moments developed in the longitudinal direction due to the pressure on the ends of a closed cylinder. Salerno and Pulos (2) in their solution, however, accounted for the longitudinal moments due to the pressure on the ends of a closed cylinder. Bodner and Berks (3), Galletly and Bart (4), and Donnell (5) obtained expressions for the stresses developed in the shell of a stiffened cylinder resulting from out-of-roundness in the shell.

Another source of imperfection is the initial deflection of the shell between frames. Previously, measurements of this type of imperfection had been made at the Taylor Model Basin for externally framed cylinders. It was found that the shell between frames was initially convex toward the exterior of the cylinder, but no analysis had been made of the influence of these initial deflections on structural behavior.

Wu, Goodman, and Newmark (6) analyzed the problem of a cylindrical shell with a dent varying in depth along the longitudinal direction. They derived the governing displacement equations of equilibrium. However, they did not examine the type of imperfection which is axisymmetric in form, nor did they consider the influence of stiffening rings on the elastic behavior of the cylinder.

In the present paper the initial deflection of the shell is reported for an internally framed cylinder. It was found that, for this case, the shell was initially convex toward the interior of the cylinder. These initial deflections, when magnified, cause the cylinder to resemble an accordion; hence this type of imperfection will be referred to hereafter as initial accordion deflection.

It is conjectured that the initial accordion deflection is due mainly to the contraction of the welds between frame and shell. These contractions of the weld induce moments at the frames which bend the shell toward the frame. This results in convex curvature of the shell for external framing and concave curvature for internal framing, considering the curvature from the exterior of the cylinder.

An analytical solution will be derived for the additional deflections and stresses resulting from initial accordion deflection when the cylinder is subjected to external hydrostatic pressure. The resulting theoretical analysis is applicable to both internal and external framing. The theory will then be used to explain some of the differences in behavior between internally framed and externally framed cylinders.

Measurement of Initial Accordion Deflection

Initial longitudinal contours of an internally framed cylinder were measured at the David Taylor Model Basin. These measurements were taken along two generators of the cylinder 90 deg apart. The method of measurement was developed to provide not only the maximum deviation of the shell from that of a perfect cylinder but also the contours of the shell between frames.

The arrangement devised for measuring these contours is shown in Fig. 1. The cylinder was placed with its axis horizontal. A rod with an Ames dial gage (accurate to 0.001 in.) attached at one end was clamped to another horizontal rod graduated in inches. This graduated rod was placed in the radial plane containing a line scribed on the surface of the shell along one generator. As can be seen in Fig. 1, the graduated rod was supported on vertical stands. The Ames dial gage was moved horizontally along the line scribed on the cylinder, and deflection readings were taken at the frames and at five intermediate points. By this procedure measurements could be repeated to within 0.001 in. Plots of measured initial accordion deflections are shown in Fig. 2.

In Fig. 2 plots of the following equation also are shown

$$w_1 = \frac{4}{L}\left(x - \frac{x^2}{L}\right)\Delta \qquad [1]$$

where w_1 is the initial deflection at any point, x is the co-ordinate measured along a generator from an origin located at a frame, L is the frame spacing, and Δ is the maximum initial deflection of the shell. The terms just defined are indicated in Fig. 3. In plotting Equation [1] the maximum initial deflection Δ was set equal to

---

[1] Head, Structural Evaluation Section, David Taylor Model Basin.

[2] Structural Research Engineer, David Taylor Model Basin.

[3] Numbers in parentheses refer to the Bibliography at the end of the paper.

For presentation at the Applied Mechanics Division Summer Conference, Berkeley, Calif., June 13–15, 1957, of The American Society of Mechanical Engineers.

Discussion of this paper should be addressed to the Secretary, ASME, 29 West 39th Street, New York, N. Y., and will be accepted until one month after final publication of the paper itself in the Journal of Applied Mechanics.

Manuscript received by ASME Applied Mechanics Division, January 2, 1957. Paper No. 57—APM-35.

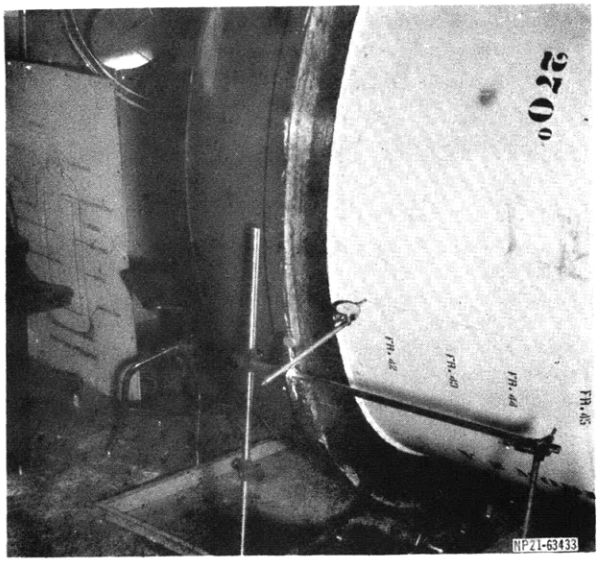


FIG. 1 ARRANGEMENT FOR MEASURING INITIAL ACCORDION DEFLECTION

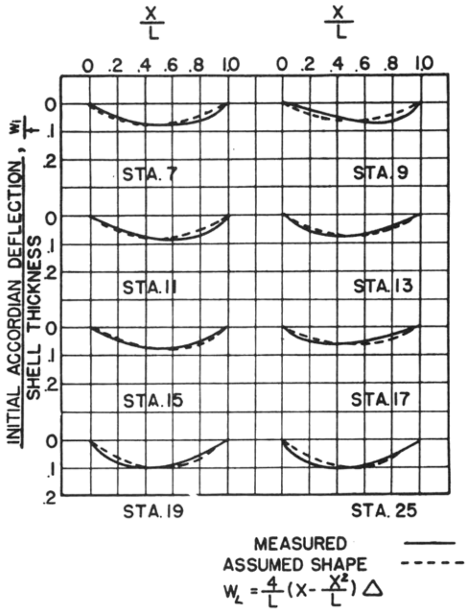


FIG. 2 COMPARISON OF MEASURED INITIAL ACCORDION DEFLECTION AND ASSUMED SECOND-DEGREE CURVE

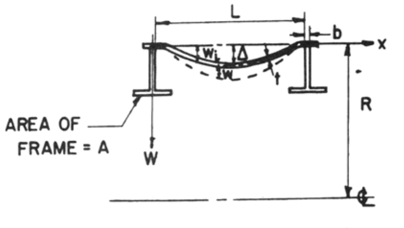


FIG. 3 NOTATION FOR ANALYSIS OF STIFFENED CYLINDERS WITH INITIAL ACCORDION DEFLECTION

the observed value at mid-bay, considered positive toward the axis of the cylinder. From Fig. 2 it can be seen that the measured deflection contours and the curves represented by Equation [1] are in fair agreement.

DERIVATION OF DIFFERENTIAL EQUATION OF EQUILIBRIUM OF A STIFFENED CYLINDER WITH INITIAL ACCORDION DEFLECTION

The moment M_E resulting from the pressure on the ends of a closed cylinder of radius R is

$$M_E = -N_x(w_i + w) \ldots\ldots\ldots\ldots\ldots\ldots [2]$$

where w_i is given by Equation [1], and $N_x = -pR/2$ (the axial load per inch of circumference), and w denotes the additional radial displacements from the initial deflection surface, taken positive when inward. The deflections w are caused by the applied pressure p which is considered positive when applied externally. If it is assumed that the initial accordion deflection contour does not change about the circumference of the cylinder, the moment M_E is obtained by combining Equations [1] and [2], thus

$$M_E = -N_x\left[\frac{4}{L}\left(x - \frac{x^2}{L}\right)\Delta + w\right] \ldots\ldots\ldots [3]$$

The differential shear resulting from the moment M_E can be obtained by taking the second derivative of Equation [3], thus

$$\frac{d^2M}{dx^2} = -N_x\left(\frac{d^2w}{dx^2} - \frac{8\Delta}{L^2}\right) \ldots\ldots\ldots\ldots [4]$$

The differential equation of equilibrium for a cylinder of perfectly uniform diameter $2R$ is as follows (7)

$$D\,\frac{d^4w}{dx^4} = p + \frac{1}{R}\,\mu N_x - \frac{w}{R^2}\,Et + N_x\,\frac{d^2w}{dx^2} \ldots\ldots [5]$$

where E is Young's modulus, μ is Poisson's ratio, t is the shell thickness, and

$$D = \frac{Et^3}{12(1 - \mu^2)}$$

The last term on the right side of Equation [5] is the differential shear resulting from the moment caused by pressure on the ends of the cylinder. When Equation [4] is substituted for the last term of Equation [5], the differential equation of equilibrium governing the deformation of a cylinder with an initial accordion deflection, expressed by Equation [1], is obtained as follows

$$D\,\frac{d^4w}{dx^4} = p + \frac{1}{R}\,\mu N_x - \frac{w}{R^2}\,Et + N_x\left(\frac{d^2w}{dx^2} - \frac{8\Delta}{L^2}\right) \ldots [6]$$

Equation [6] also can be written

$$\frac{d^4w}{dx^4} + 4\alpha^4\beta^2\,\frac{d^2w}{dx^2} + 4\alpha^4\left[w - \frac{pR^2}{Et}\left(1 - \frac{\mu}{2}\right) - \frac{8\beta^2}{L^2}\,\Delta\right] = 0 \ldots\ldots [7]$$

where

$$\alpha^4 = \frac{3(1 - \mu^2)}{R^2t^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots [8]$$

and

$$\beta^2 = \frac{pR^3}{2Et} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [9]$$

If a change in variable is made in Equation [7] as follows

$$Z = \frac{pR^2}{Et}\left(1 - \frac{\mu}{2}\right) + \frac{8\beta^2}{L^2}\Delta - w \ldots\ldots\ldots [10]$$

a simplified expression is obtained

$$\frac{d^4Z}{dx^4} + 4\alpha^4\beta^2\frac{d^2Z}{dx^2} + 4\alpha^4 Z = 0 \ldots\ldots\ldots\ldots [11]$$

SOLUTION OF DIFFERENTIAL EQUATION OF EQUILIBRIUM FOR A STIFFENED CYLINDER WITH INITIAL ACCORDION DEFLECTION

Equation [11] is a linear differential equation and its solution can be shown to be of the form

$$Z = C_1 \sinh K_1x \sin K_2x + C_2 \sinh K_1x \cos K_2x$$
$$+ C_3 \cosh K_1x \sin K_2x + C_4 \cosh K_1x \cos K_2x \ldots\ldots [12]$$

where

$$K_1 = \alpha\,(1 - \alpha^2\beta^2)^{1/2} \ldots\ldots\ldots\ldots\ldots [13]$$

and

$$K_2 = \alpha\,(1 + \alpha^2\beta^2)^{1/2} \ldots\ldots\ldots\ldots\ldots [14]$$

For the geometries generally used in practice, values of the parameters K_1 and K_2 are real and positive.

As a result of symmetry and continuity, the following boundary conditions are assumed to exist (see Fig. 3)
At

$$x = 0, \quad w = w_0 \ldots\ldots\ldots\ldots\ldots\ldots [15]$$

$$x = L, \quad w = w_0 \ldots\ldots\ldots\ldots\ldots\ldots [16]$$

$$x = 0, \quad \frac{dw}{dx} = 0 \ldots\ldots\ldots\ldots\ldots\ldots [17]$$

$$x = L, \quad \frac{dw}{dx} = 0 \ldots\ldots\ldots\ldots\ldots\ldots [18]$$

where w_0 is the deflection occurring at a frame. When the four boundary conditions of Equations [15] to [18] are substituted into Equation [12] and its first derivative, four equations are obtained which, when solved simultaneously, yield expressions for the four coefficients of Equation [12] as follows

$$C_1 = \left[\frac{K_2 \sin K_2L - K_1 \sinh K_1L}{K_2 \sinh K_1L + K_1 \sin K_2L}\right] Z_0 \ldots\ldots [19]$$

$$C_2 = \left[\frac{K_2\,(\cos K_2L - \cosh K_1L)}{K_2 \sinh K_1L + K_1 \sin K_2L}\right] Z_0 \ldots\ldots [20]$$

$$C_3 = \left[\frac{K_1(\cosh K_1L - \cos K_2L)}{K_2 \sinh K_1L + K_1 \sin K_2L}\right] Z_0 \ldots\ldots [21]$$

$$C_4 = Z_0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [22]$$

where

$$Z_0 = \frac{pR^2}{Et}\left(1 - \frac{\mu}{2}\right) + \frac{8\beta^2}{L^2}\Delta - w_0 \ldots\ldots [23]$$

When Equations [19] to [22] are substituted into Equation [12], the variable Z can be expressed in terms of its value Z_0 at a frame as

$$Z = \frac{f(x)}{G} Z_0 \ldots\ldots\ldots\ldots\ldots\ldots [24]$$

where

$$f(x) = K_2 \sinh K_1x \cos K_2(L - x)$$
$$+ K_1 \cosh K_1x \sin K_2(L - x)$$
$$+ K_1 \sin K_2x \cosh K_1(L - x)$$
$$+ K_2 \cos K_2x \sinh K_1(L - x) \ldots\ldots [25]$$

and

$$G = K_2 \sinh K_1L + K_1 \sin K_2L \ldots\ldots\ldots\ldots [26]$$

The value of the parameter Z at the frame denoted as Z_0 can be obtained by considering the equilibrium of radial forces at the frame, which is assumed to deform as a thin ring. Referring to Fig. 4, the summation of radial forces at a frame is as follows

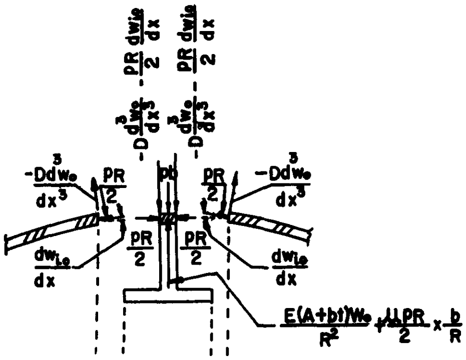


FIG. 4 FORCES AT A FRAME

$$2D\left.\frac{d^3w}{dx^3}\right|_{x=0} + \frac{E(A + bt)}{R^2} w_0 - pb + \mu\frac{pR}{2}\frac{b}{R}$$
$$+ 2\frac{pR}{2}\left.\frac{dw_i}{dx}\right|_{x=0} = 0 \ldots\ldots [27]$$

where the derivatives of the deflections are evaluated at the frame as indicated. In terms of the variable Z, Equation [27] can be expressed as

$$\left.\frac{d^3Z}{dx^3}\right|_{x=0} + 2\alpha^4\left(\frac{A + bt}{t}\right) Z_0 - \frac{4\alpha^4\beta^2}{Rt} A\left(1 - \frac{\mu}{2}\right)$$
$$- 16\alpha^4\beta^2\frac{(A + bt)}{tL}\frac{\Delta}{L} - 16\alpha^4\beta^2\frac{\Delta}{L} = 0 \ldots\ldots [28]$$

Substituting Equation [24] and its third derivative evaluated at $x = 0$ into Equation [28] and solving for Z_0 yields

$$Z_0 = \frac{2\alpha^2\beta^2G}{H}\left[\frac{A\left(1 - \frac{\mu}{2}\right)}{Rt} + \frac{4\Delta}{L}\left(1 + \frac{A + bt}{Lt}\right)\right] . [29]$$

where

$$H = \frac{\alpha^2}{t}(A + bt)G + 2K_1K_2(\cosh K_1L - \cos K_2L) .. [30]$$

When Equations [10] and [29] are substituted into Equation [24], the general expression for the deflection of a stiffened cylinder with the assumed form of initial accordion deflection results

$$w = \frac{\beta^2}{R}(2 - \mu) - \frac{2\alpha^2\beta^2}{H}\left[\frac{A\left(1 - \frac{\mu}{2}\right)}{Rt}\right] f(x)$$
$$+ \frac{8\beta^2\Delta}{L^2}\left[1 - \frac{\alpha^2}{H}\frac{(A + bt + tL}{t} f(x)\right] \ldots\ldots [31]$$

Let

$$w_c = \frac{\beta^2}{R}(2-\mu) - \frac{2\alpha^2\beta^2}{H}\left[\frac{A\left(1-\frac{\mu}{2}\right)}{Rt}\right] f(x) \ldots [32]$$

and

$$w_\Delta = \frac{8\beta^2\Delta}{L^2}\left[1 - \frac{\alpha^2}{H}\frac{(A+bt+tL)}{t} f(x)\right] \ldots\ldots [33]$$

Then

$$w = w_c + w_\Delta \ldots\ldots\ldots\ldots\ldots\ldots [34]$$

The deflection w_c is the deflection for a stiffened cylinder of uniform diameter. It is the solution for the same boundary conditions of the identical differential equation of equilibrium solved previously by Salerno and Pulos (2). The expression for the deflection w_c given by Equation [32] is in a different form than that presented by Salerno and Pulos and is considered to be simpler to evaluate. The deflection w_Δ represents the additional deflection due to an initial accordion deflection.

ADDITIONAL STRESSES AND STRAINS DUE TO INITIAL ACCORDION DEFLECTION

The engineer and designer are generally more concerned with stresses than deflections. Once the deflections are known, it is a relatively simple matter to determine stresses and strains in terms of deflection and its derivatives.

The expression for the circumferential membrane strain $\sigma_{m\phi}$ is well known as (7)

$$\sigma_{m\phi} = \frac{Ew}{R} + \mu\frac{pR}{2t} \ldots\ldots\ldots\ldots [35]$$

Introducing Equation [34] into Equation [35], the total membrane stress in the circumferential direction is

$$\sigma_{m\phi} = \frac{Ew_c}{R} + \mu\frac{pR}{2t} + \frac{Ew_\Delta}{R} \ldots [36]$$

The first two terms on the right side of Equation [36] represent that portion of the stress for a cylinder of perfectly uniform diameter. On the other hand, the last term is that portion of the total stress attributed to initial accordion deflection. Thus the additional membrane stress $\sigma_{m\phi\Delta}$ in the circumferential direction due to initial accordion deflection can be expressed as

$$\sigma_{m\phi\Delta} = \frac{Ew_\Delta}{R} \ldots\ldots [37]$$

where the additional deflection w_Δ is given by Equation [33].

The total bending stress σ_{bx} in the longitudinal direction on the surface of the cylinder is given by

$$\sigma_{bx} = \pm\frac{Et}{2(1-\mu^2)}\ddot{w} \ldots\ldots\ldots\ldots [38]$$

or since

$$w = w_c + w_\Delta \ldots\ldots\ldots\ldots [34]$$

$$\sigma_{bx} = \pm\frac{Et}{2(1-\mu^2)}\ddot{w}_c \pm \frac{Et}{2(1-\mu^2)}\ddot{w}_\Delta \ldots [39]$$

The component $\sigma_{bx\Delta}$, attributable to initial accordion deflection, of the total bending stress in the longitudinal direction is as follows

$$\sigma_{bx\Delta} = \pm\frac{Et}{2(1-\mu^2)}\ddot{w}_\Delta \ldots\ldots\ldots\ldots [40]$$

where $\ddot{w}_\Delta$ is the second derivative of Equation [33] which is found to be

$$\begin{aligned}\ddot{w}_\Delta = -\frac{16\alpha^4\beta^2}{HL^2}\Delta\left(\frac{A+bt+tL}{t}\right)&[K_1 \cosh K_1x \sin K_2(L-x) \\ &- K_2 \sinh K_1x \cos K_2(L-x) \\ &- K_2 \cos K_2x \sinh K_1(L-x) \\ &+ K_1 \sin K_2x \cosh K_1(L-x)] \ldots\ldots [41]\end{aligned}$$

The bending stress $\sigma_{b\phi\Delta}$ in the circumferential direction due to initial accordion deflection is then simply

$$\left.\begin{aligned}\sigma_{b\phi\Delta} &= \mu\sigma_{bx\Delta} \\ &\text{or} \\ \sigma_{b\phi\Delta} &= \pm\frac{\mu}{2}\frac{Et}{(1-\mu^2)}\ddot{w}_\Delta\end{aligned}\right\} \ldots\ldots [42]$$

The circumferential stress $\sigma_{\phi\Delta}$ on the surface of the cylinder attributable to initial accordion deflection is the sum of the additional membrane and additional bending stresses. Thus, adding Equations [37] and [42]

$$\sigma_{\phi\Delta} = \frac{Ew_\Delta}{R} \pm \frac{\mu}{2}\frac{Et}{(1-\mu^2)}\ddot{w}_\Delta \ldots\ldots\ldots\ldots [43]$$

where the terms w_Δ and $\ddot{w}_\Delta$ can be expressed by Equations [33] and [41], respectively.

The stress $\sigma_{x\Delta}$ in the longitudinal direction on the surface of the cylinder due to initial accordion deflection is simply the additional bending stress given by Equation [40], as the membrane stress in this direction is not altered by any initial accordion deflection. Thus

$$\sigma_{x\Delta} = \pm\frac{Et}{2(1-\mu^2)}\ddot{w}_\Delta \ldots\ldots\ldots\ldots [44]$$

The choice of signs in Equations [42] and [43] is dependent upon the location of the point at which the stress is determined. Keeping in mind that all radially inward deflections are considered positive, for cylinders subjected to external pressure the additional stresses due to initial accordion deflection on the exterior surface of the cylinders are

$$\sigma_{\phi\Delta_e} = \frac{Ew_\Delta}{R} - \frac{\mu}{2}\frac{Et}{(1-\mu^2)}\ddot{w}_\Delta \ldots\ldots\ldots [45]$$

and

$$\sigma_{x\Delta_e} = -\frac{Et}{2(1-\mu^2)}\ddot{w}_\Delta \ldots\ldots\ldots\ldots [46]$$

and on the interior surface are

$$\sigma_{\phi\Delta_i} = \frac{Ew_\Delta}{R} + \frac{\mu}{2}\frac{Et}{(1-\mu^2)}\ddot{w}_\Delta \ldots\ldots\ldots [47]$$

and

$$\sigma_{x\Delta_i} = \frac{Et}{2(1-\mu^2)}\ddot{w}_\Delta \ldots\ldots\ldots\ldots [48]$$

If the stresses in Equations [45] to [48] are expressed in terms of strain using Hooke's law for biaxial state of strain, the strain $\epsilon_{\phi\Delta}$ in the circumferential direction and the strain $\epsilon_{x\Delta}$ in the longitudinal direction, both attributable to initial accordion deflection, can be obtained as follows

$$\epsilon_{\phi\Delta} = \frac{w_\Delta}{R} \ldots\ldots\ldots\ldots [49]$$

and for a point on the exterior surface of the cylinder

$$\epsilon_{x\Delta_e} = -\frac{t}{2}\ddot{w}_\Delta - \mu\frac{w_\Delta}{R} \ldots\ldots\ldots\ldots [50]$$

and for a point on the interior surface

$$\epsilon_{x\Delta i} = \frac{t}{2}\ddot{w}_\Delta - \mu\frac{w_\Delta}{R} \ldots\ldots\ldots\ldots\ldots [51]$$

DISCUSSION

In order to examine the influences of initial accordion deflection on structural behavior, calculations were conducted for a hypothetical cylinder. The geometry selected was as follows

$$L/2R = 0.1$$

$$t/2R = 0.005$$

$$A/Lt = 0.5$$

$$b/L = 0.05$$

Pressure-strain curves were computed for this geometry and are presented in Figs. 5, 6, and 7. The strains in the circumferential direction and the strains in the longitudinal direction were investigated at a mid-bay point. These strains were computed by the linear theory of von Sanden and Gunther and the nonlinear theory for no accordion deflection and several ratios of initial accordion deflection to shell thickness. Figs. 5 to 7 were determined only for positive values of Δ or for internal framing.

It can be seen from Fig. 5 that for internal framing an initial accordion deflection increases the deflection (or circumferential strain) at mid-bay. On the other hand, a theoretical analysis would indicate that the deflection at a frame would be decreased owing to initial accordion deflection. The discrepancy between strains for a perfectly uniform cylinder and one with initial accordion deflection becomes much more pronounced for strains in the longitudinal direction. This is indicated in Figs. 6 and 7; in Fig. 7 it should be noticed that the initial accordion deflection can even cause the internal longitudinal strain to change sign from compression to tension.

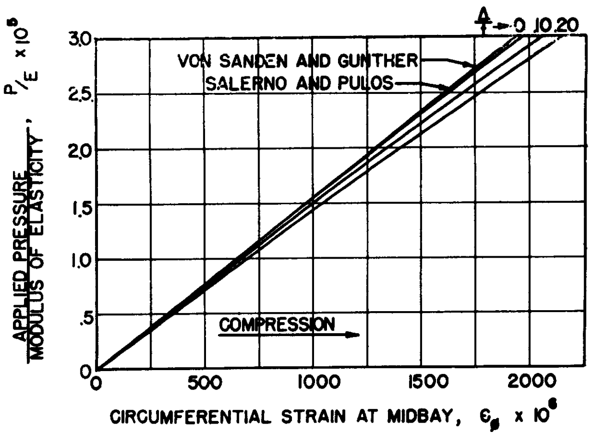


FIG. 5 VARIATION OF CIRCUMFERENTIAL STRAIN AT MID-BAY WITH PRESSURE

Of course, the additional strains resulting from initial accordion deflection increase as the magnitude of the initial deflection increases. Measurements on stiffened cylinders at the Taylor Model Basin have indicated that the ratio Δ/t generally varies between 0.1 and 0.2. The values of Δ/t shown in Figs. 5 to 7 are, therefore, realistic. It is evident from examining Figs. 5 to 7 that, when stiffened cylinders are designed to withstand higher pressures, the linear theory of von Sanden and Gunther becomes eventually inadequate. Recourse must then be had to a nonlinear theory, taking into account initial accordion deflection.

Figs. 5 to 7 show the variation of elastic strain with pressure for an internally framed cylinder. For an externally framed cylinder the additional strains due to initial accordion deflection are of opposite sense to those for an internally framed cylinder. This is true, as for an externally framed cylinder the initial accordion deflection is convex toward the exterior of the cylinder. On the other hand, the initial deflection is concave for the case of an internally framed cylinder. The differences in sign of the additional stresses between internally and externally framed cylinders suggest the possibility of differences in strength between internally framed and externally framed cylinders.

Calculations were made to ascertain whether the difference in strength between internal and external framing was significant. In particular, the strength of cylinders against axisymmetric yielding was examined. It was reasoned that the initial accordion deflection being axisymmetric would have the greatest influence on that mode of failure involving excessive axisymmetric deformations. On the other hand, a circumferential imperfection such as shell out-of-roundness would be expected to play a predominant role in failure by shell buckling during which numerous lobes form around the circumference of the cylinder. In actuality, welded stiffened cylinders have both types of imperfections, out-of-roundness superimposed on an initial accordion deflection. However, as the pressure for failure (of perfect cylinders) by axisymmetric yielding becomes smaller in relation to that by shell

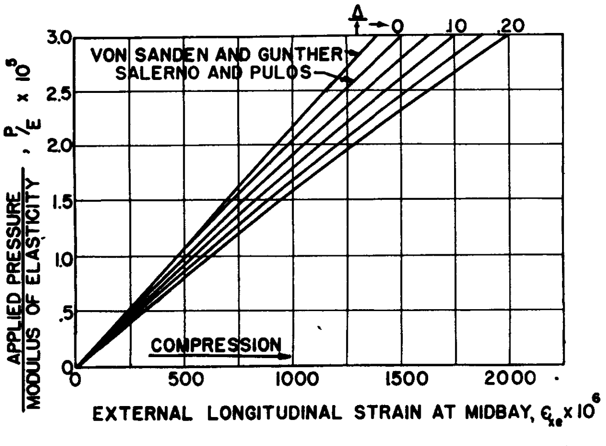


FIG. 6 VARIATION OF EXTERNAL LONGITUDINAL STRAIN AT MID-BAY WITH PRESSURE

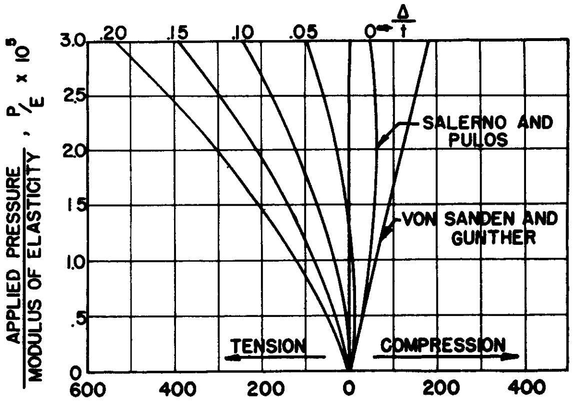


FIG. 7 VARIATION OF INTERNAL LONGITUDINAL STRAIN AT MID-BAY WITH PRESSURE

buckling, the initial accordion deflection would exert an increasingly greater influence on the collapse pressure. This being the case, the collapse pressures to be discussed are less than the shell-buckling pressures in order for the conclusions drawn to be valid.

In examining the strength of stiffened cylinders against axisymmetric yielding, the pressure at which yielding would occur at an exterior point at mid-bay was determined for the geometry previously selected under the assumption that the cylinder is initially stress-free. This particular point was chosen since failure by axisymmetric yielding is usually associated with a critical stress condition there. The criterion of yielding was taken as the octahedral shear stress or Hencky-von Mises criterion as follows

$$\sigma_y^2 = \sigma_x^2 + \sigma_\phi^2 - \sigma_x\sigma_\phi \ldots\ldots\ldots\ldots \quad [52]$$

where σ_y denotes the compressive yield strength of the material' The pressures p_y at which yielding would be initiated were determined for different values of yield strength. Actually, the calculations were made for various ratios of yield strength to modulus of elasticity, σ_y/E. In addition, the pressures p_y were determined for a range of values of the ratio of initial accordion deflection to shell thickness, Δ/t, and for positive values of Δ/t associated with internally framed cylinders and also negative values of Δ/t for externally framed cylinders.

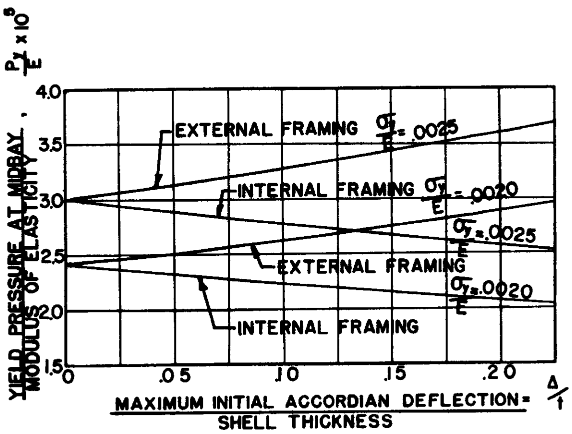


Fig. 8 Influence of Initial Accordion Deflection on Pressure to Initiate Yielding

Results of these calculations are shown in Fig. 8, which is a nondimensional plot of p_y/E versus Δ/t for two values of the ratio σ_y/E. In effect, the pressure for initiating yielding is plotted against initial accordion deflection for yield strengths of 60,000 and 75,000 psi in the case of steel. The pressures p_y are plotted in Fig. 8 for both internal and external framing. Fig. 8 vividly illustrates the large divergence in pressures to initiate yielding between the two types of framing as the initial accordion deflection increases. At $\Delta/t = 0.1$ and $\sigma_y/E = 0.0025$, the difference in p_y for the two types of framing is 18 per cent; for a value of $\Delta/t = 0.2$, it is 40 per cent.

The question arose as to whether the influence of initial accordion deflection was significant for cylinders of low yield strength. Accordingly, the ratio of $p_{y\Delta}$, the pressure to initiate yielding for a cylinder with a ratio $\Delta/t = 0.1$ to the value p_{yo}, the pressure at yielding for a cylinder with no initial accordion deflection, was computed for various values of σ_y/E. Again this was done for both internally framed and externally framed cylinders. The results are shown in Fig. 9, which indicates that the influence of initial accordion deflection on the onset of yielding can be almost as severe for low yield strength as for the higher yield strengths. Fig. 9 does indicate, however, that as the yield strength increases the influence of initial accordion deflections becomes gradually more severe.

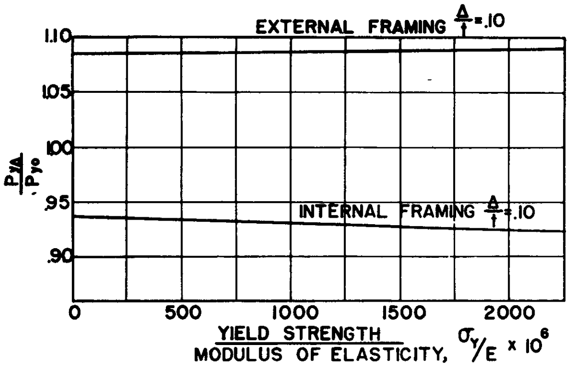


Fig. 9 Comparison of Pressures to Initiate Yielding Between Cylinders With and Without Initial Accordion Deflection

Summary and Conclusions

A theoretical analysis of a cylinder with initial accordion deflection indicates that the theory of von Sanden and Gunther may be inadequate for certain geometries and for sizable initial accordion deflections. The influence of initial accordion deflection is greater on the longitudinal strains than on the circumferential strains. Sizable initial deflections result in additional strains that cannot be neglected. If failure is associated with yielding of the shell at mid-bay, an internally framed cylinder may be appreciably weaker than an externally framed cylinder when the cylinders are fabricated by welding. Lastly, it was found that the influence of initial accordion deflection on failure by axisymmetric yielding cannot be neglected for low-yield-strength materials.

Acknowledgments

The authors wish to express their appreciation for the critical review of this paper given by Dr. E. H. Kennard, Chief Scientist, Structural Mechanics Laboratory, David Taylor Model Basin. In addition, the authors are indebted to Mr. M. A. Krenzke, who assisted in making measurements of initial accordion deflection.

BIBLIOGRAPHY

1 "The Strength of Cylindrical Shells Stiffened by Frames and Bulkheads, Under Uniform External Pressure on All Sides," by K. von Sanden and K. Gunther, Taylor Model Basin Translation 38, March, 1952.

2 "Stress Distribution in a Circular Cylindrical Shell Under Hydrostatic Pressure Supported by Equally Spaced Circular Ring Frames," by V. L. Salerno and J. G. Pulos, Polytechnic Institute of Brooklyn Aeronautical Laboratory Report No. 171-A, June, 1951.

3 "The Effect of Imperfections on the Stresses in a Circular Cylindrical Shell Under Hydrostatic Pressure," by S. R. Bodner and W. Berks, Polytechnic Institute of Brooklyn Aeronautical Laboratory Report No. 210, December, 1952.

4 "The Effects of Boundary Conditions and Initial Out-Of-Roundness on the Strength of Thin-Walled Cylinders Subject to External Hydrostatic Pressure," by G. D. Galletly and R. Bart, Journal of Applied Mechanics, Trans. ASME, vol. 78, 1956, pp. 351–358.

5 "Effect of Imperfections on Buckling of Thin Cylinders Under External Pressure," by L. H. Donnell, Journal of Applied Mechanics, Trans. ASME, vol. 78, 1956, pp. 569–575.

6 "Effect of Small Initial Irregularities on the Stresses in Cylindrical Shells," by T. S. Wu, L. E. Goodman, and N. M. Newmark, University of Illinois Structural Research Series No. 50, April, 1953.

7 "Theory of Elasticity Stability," by S. Timoshenko, McGraw-Hill Book Company, Inc., New York and London, first edition, 1936, pp. 424–425.

INITIAL DISTRIBUTION

Copies	
12	CHBUSHIPS, Library (Code 312) 5 Tech Library 1 Tech Asst to Chief (Code 106) 1 Prelim Des Br (Code 420) 1 Prelim Des Section (Code 421) 1 Ship Protection Section (Code 423) 1 Hull Des (Code 440) 1 Scientific and Research (Code 442) 1 Submarines (Code 525)
2	CHONR, Mech Br (Code 438)
2	OPNAV, Op 311
2	AFSWP
1	NAVSHIPYD PTSMH
1	NAVSHIPYD MARE
1	NAVSHIPYD NORVA, UERD (Code 270)
1	CDR, USNOL
1	DIR, USNRL
2	CDR, USNOTS, China Lake, Calif.
1	CO, USNUOS, Newport, R.I.
1	DIR, USNEES, Annapolis, Md.
2	CO, USN Admin Unit, MIT, Cambridge, Mass.
1	DIR, Aero Lab, Langley Field, Va.
1	General Dynamics Corp, Electric Boat Div, Groton, Conn.
2	SUPSHIPINSORD, Groton, Conn.

NAVY DPPO PRNC WASH D C

David Taylor Model Basin. Report 1150
BEHAVIOR OF CYLINDERS WITH INITIAL SHELL DEFLECTION, by M.E. Lunchick and R.D. Short. July 1957. 7p. illus., graphs, diagrs., refs. (Research and development report. ASME Paper 57-APM-35) UNCLASSIFIED

Evidence is presented which indicates that, for stiffened cylinders fabricated by welding, initial deflections exist for the shell between frames. The initial deflections of the shell of a stiffened cylinder were measured and found to be approximated closely by a second-degree curve. Using the mathematical expression for the initial deflection curve, a theoretical analysis was made for a cylinder subjected to external hydrostatic pressure, resulting in expressions for the additional deflections and stresses caused by the initial deflection of the shell. Since the initial deflections of the shell for internally and externally framed cylinders were

1. Cylinders (Stiffened) – Stresses – Mathematical analysis
2. Cylinders (Stiffened) – Strength – Mathematical analysis
I. Lunchick, Myron E.
II. Short, Robert D.

David Taylor Model Basin. Report 1150
BEHAVIOR OF CYLINDERS WITH INITIAL SHELL DEFLECTION, by M.E. Lunchick and R.D. Short. July 1957. 7p. illus., graphs, diagrs., refs. (Research and development report. ASME ~~Paper~~ 57-APM-35) UNCLASSIFIED

Evidence is presented which indicates that, for stiffened cylinders fabricated by welding, initial deflections exist for the shell between frames. The initial deflections of the shell of a stiffened cylinder were measured and found to be approximated closely by a second-degree curve. Using the mathematical expression for the initial deflection curve, a theoretical analysis was made for a cylinder subjected to external hydrostatic pressure, resulting in expressions for the additional deflections and stresses caused by the initial deflection of the shell. Since the initial deflections of the shell for internally and externally framed cylinders were

1. Cylinders (Stiffened) – Stresses – Mathematical analysis
2. Cylinders (Stiffened) – Strength – Mathematical analysis
I. Lunchick, Myron E.
II. Short, Robert D.

David Taylor Model Basin. Report 1150
BEHAVIOR OF CYLINDERS WITH INITIAL SHELL DEFLECTION, by M.E. Lunchick and R.D. Short. July 1957. 7p. illus., graphs, diagrs., refs. (Research and development report. ASME Paper 57-APM-35) UNCLASSIFIED

Evidence is presented which indicates that, for stiffened cylinders fabricated by welding, initial deflections exist for the shell between frames. The initial deflections of the shell of a stiffened cylinder were measured and found to be approximated closely by a second-degree curve. Using the mathematical expression for the initial deflection curve, a theoretical analysis was made for a cylinder subjected to external hydrostatic pressure, resulting in expressions for the additional deflections and stresses caused by the initial deflection of the shell. Since the initial deflections of the shell for internally and externally framed cylinders were

1. Cylinders (Stiffened) – Stresses – Mathematical analysis
2. Cylinders (Stiffened) – Strength – Mathematical analysis
I. Lunchick, Myron E.
II. Short, Robert D.

David Taylor Model Basin. Report 1150
BEHAVIOR OF CYLINDERS WITH INITIAL SHELL DEFLECTION, by M.E. Lunchick and R.D. Short. July 1957. 7p. illus., graphs, diagrs., refs. (Research and development report. ASME Paper 57-APM-35) UNCLASSIFIED

Evidence is presented which indicates that, for stiffened cylinders fabricated by welding, initial deflections exist for the shell between frames. The initial deflections of the shell of a stiffened cylinder were measured and found to be approximated closely by a second-degree curve. Using the mathematical expression for the initial deflection curve, a theoretical analysis was made for a cylinder subjected to external hydrostatic pressure, resulting in expressions for the additional deflections and stresses caused by the initial deflection of the shell. Since the initial deflections of the shell for internally and externally framed cylinders were

1. Cylinders (Stiffened) – Stresses – Mathematical analysis
2. Cylinders (Stiffened) – Strength – Mathematical analysis
I. Lunchick, Myron E.
II. Short, Robert D.

found to be of opposite sign, the theory is offered as a possible explanation of the observed differences in behavior between internally and externally framed cylinders.

found to be of opposite sign, the theory is offered as a possible explanation of the observed differences in behavior between internally and externally framed cylinders.

found to be of opposite sign, the theory is offered as a possible explanation of the observed differences in behavior between internally and externally framed cylinders.

found to be of opposite sign, the theory is offered as a possible explanation of the observed differences in behavior between internally and externally framed cylinders.

David Taylor Model Basin. Report 1150
BEHAVIOR OF CYLINDERS WITH INITIAL SHELL DEFLECTION, by M.E. Lunchick and R.D. Short. July 1957. 7p. illus., graphs, diagrs., refs. (Research and development report. ASME Paper 57-APM-35) UNCLASSIFIED

Evidence is presented which indicates that, for stiffened cylinders fabricated by welding, initial deflections exist for the shell between frames. The initial deflections of the shell of a stiffened cylinder were measured and found to be approximated closely by a second-degree curve. Using the mathematical expression for the initial deflection curve, a theoretical analysis was made for a cylinder subjected to external hydrostatic pressure, resulting in expressions for the additional deflections and stresses caused by the initial deflection of the shell. Since the initial deflections of the shell for internally and externally framed cylinders were

1. Cylinders (Stiffened) – Stresses – Mathematical analysis
2. Cylinders (Stiffened) – Strength – Mathematical analysis
I. Lunchick, Myron E.
II. Short, Robert D.

David Taylor Model Basin. Report 1150
BEHAVIOR OF CYLINDERS WITH INITIAL SHELL DEFLECTION, by M.E. Lunchick and R.D. Short. July 1957. 7p. illus., graphs, diagrs., refs. (Research and development report. ASME Paper 57-APM-35) UNCLASSIFIED

Evidence is presented which indicates that, for stiffened cylinders fabricated by welding, initial deflections exist for the shell between frames. The initial deflections of the shell of a stiffened cylinder were measured and found to be approximated closely by a second-degree curve. Using the mathematical expression for the initial deflection curve, a theoretical analysis was made for a cylinder subjected to external hydrostatic pressure, resulting in expressions for the additional deflections and stresses caused by the initial deflection of the shell. Since the initial deflections of the shell for internally and externally framed cylinders were

1. Cylinders (Stiffened) – Stresses – Mathematical analysis
2. Cylinders (Stiffened) – Strength – Mathematical analysis
I. Lunchick, Myron E.
II. Short, Robert D.

David Taylor Model Basin. Report 1150
BEHAVIOR OF CYLINDERS WITH INITIAL SHELL DEFLECTION, by M.E. Lunchick and R.D. Short. July 1957. 7p. illus., graphs, diagrs., refs. (Research and development report. ASME Paper 57-APM-35) UNCLASSIFIED

Evidence is presented which indicates that, for stiffened cylinders fabricated by welding, initial deflections exist for the shell between frames. The initial deflections of the shell of a stiffened cylinder were measured and found to be approximated closely by a second-degree curve. Using the mathematical expression for the initial deflection curve, a theoretical analysis was made for a cylinder subjected to external hydrostatic pressure, resulting in expressions for the additional deflections and stresses caused by the initial deflection of the shell. Since the initial deflections of the shell for internally and externally framed cylinders were

1. Cylinders (Stiffened) – Stresses – Mathematical analysis
2. Cylinders (Stiffened) – Strength – Mathematical analysis
I. Lunchick, Myron E.
II. Short, Robert D.

found to be of opposite sign, the theory is offered as a possible explanation of the observed differences in behavior between internally and externally framed cylinders.

found to be of opposite sign, the theory is offered as a possible explanation of the observed differences in behavior between internally and externally framed cylinders.

found to be of opposite sign, the theory is offered as a possible explanation of the observed differences in behavior between internally and externally framed cylinders.