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THE PITCHING PERFORMANCE OF THE SS SILVER MARINER IN A STATE 5 SEA

by

Wilbur Marks

[redacted]

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ABSTRACT

Continuous time histories of pitch experienced by the SS SILVER MARINER in a State 5 sea were recorded. The measurements, each at least 28 minutes long, were taken at various ship speeds in head seas, and at various relative headings for the top operating speed. The resultant information was transformed into energy spectra, and a number of statistics that describe the ship's pitching performance were computed. It is shown that variation of heading is more influential in pitching behavior than is variation of ship speed. For this ship, bow seas (rather than head seas) and moderately high speeds (18 knots) at this heading are shown to produce maximum pitching. Prediction of maximum pitch angle from the computed spectra agrees well with observed maximum pitch angle.

INTRODUCTION

The MARINER-type ship (Figure 1) is being considered for certain operations which require a substantial reduction of pitching motion from that ordinarily experienced in a particular seaway. As a first step in the investigation of this problem, a model of the MARINER class, both with and without antipitching fins,[1] was tested at the David Taylor Model Basin. Further experiments were carried out, full scale, on the SS SILVER MARINER, without antipitching fins, to determine her normal pitching performance characteristics. At the conclusion of these trials, a report was issued on the general behavior of the SILVER MARINER with respect to a number of motion parameters.[2] Because the results of the trials were required as soon as possible, the analysis was somewhat primitive and the presentation abbreviated.

In this report the results of a more comprehensive analysis of the pitching performance of the SILVER MARINER are presented for a variety of speeds and headings in what is usually called a "State 5" sea,[3,4] or, more precisely, a fully developed sea appropriate to a wind speed of 21 knots.[5] For this comprehensive study, the pitching records were subjected to energy spectrum analysis, and from these results certain statistics were determined as descriptive of the population from which the sample records were taken.

The results obtained by spectrum analysis of the pitching records are compared with the results obtained in the model tests. The statistics determined from the observed spectra are compared further with those given in the preliminary report on this trial[2] in order to fill gaps in the results in some cases and modify the results in others.

In addition to the quantitative analyses, moving pictures of the tests were made and a technical film report was prepared for the Bureau of Ships.[6]

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[1]References are listed page 19.

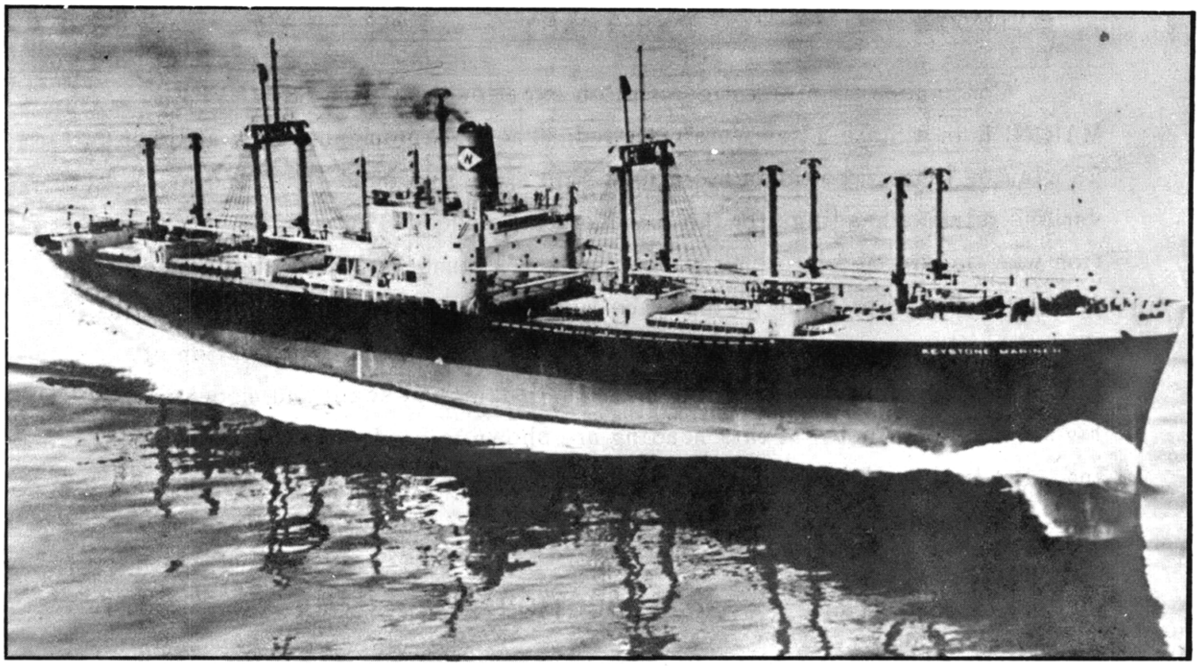

Figure 1 – A Ship of the MARINER Class

NATURE OF THE EXPERIMENT

The trial of the SILVER MARINER was essentially conducted for the purpose of recording the ship's general behavior in a State 5 sea, and more particularly for determining her pitching characteristics under various conditions of speed and heading. Pertinent hull characteristics of the SILVER MARINER are as follows:

LBP	528 ft
Beam	76 ft
Draft (design)	29 ft 10 in.
Draft during tests:	
Forward	18 ft
Aft	25 ft
Displacement	22,560 tons
Block coefficient	0.613
Waterplane coefficient	0.724

The experimental procedure was straightforward. The state of the sea was monitored by visual observation from the ship and by weather charts which were plotted daily. Valuable wave forecasts were sent periodically to the ship by the U.S. Navy Hydrographic Office. When it was considered that the sea was in a State 5 condition and would remain unidirectional and stationary for at least eight hours, the tests were begun. The experimental constants are listed in Table 1. The following parameters were recorded on chart paper:

1. Pitch angle
2. Roll angle
3. Linear acceleration normal to the deck (at a point 8 ft forward of the c.g.)
4. Roll acceleration (angular)
5. Pitch acceleration (angular),

The motion recording instrumentation is described in Reference 7.

The state of the sea remained reasonably invariant during the course of the tests, which lasted about eight hours. A brief discussion of sea conditions will be given in the next section.

Since the results of spectrum analysis become more reliable the greater the length of the record, or rather the greater the number of oscillations of the parameters measured, it was felt that the length of run should be extended as the relative heading of the ship approached 180 degrees. However, toward the end of the experiment, light began to fail and there was some indication that the sea might soon decay (as indeed it did) so the length of runs 8 and 9 was shorter than intended.

TABLE 1

Experimental Constants

Run	Shaft rpm	Approximate Speed knots	Relative Heading of Ship to Waves* degrees	Length of Run minutes
1	45	10	0	28
2	59	13	0	30
3	72	16	0	30
4	86	19	0	30
5	94	21	0	30
6	94	21	45	35
7	94	21	90	40
8	94	21	135	35
9	94	21	180	37

*Zero degree refers to head seas.

STATE OF THE SEA

The SILVER MARINER was on a routine cargo run out of Brooklyn, N.Y. bound for Bremerhaven, Germany. A double low-pressure system that had preceded the departure of the ship began to slow down and at 0030 Z, 2 May 1956, was situated as shown in Figure 2a. Curved lines on the chart indicate constant pressure in bars. The symbol is a wind vector where the barbs define wind speed in knots (a long barb, 10 knots; a short barb, 5 knots) and the shaft indicates the direction from which the wind blows. The sample wind vector indicates that the wind is from the northeast at 25 knots. The rectangle superimposed on the pressure distribution shows the closest fetch (wave-generating area) which produced waves in the environment of the ship. The winds in the fetch were generally from the WNW and were somewhat variable.

Twelve hours later (Figure 2b) the low began to consolidate about one center located 55° N, 42° W. The system also showed signs of stagnation as the SILVER MARINER entered the periphery of the wave field generated by the winds in this storm. During the next twelve hours (Figure 2c) the low moved 4 degrees southward and the SILVER MARINER was in the fetch area which showed some winds in excess of 30 knots. Since the wind field had only lately invaded this area, the state of the sea did not reflect the high winds blowing over it. The wave heights were visually estimated at 3 to 4 feet for the highest waves.

By 1230 Z on 3 May the SILVER MARINER was deep in the fetch area (Figure 2d). The sea had picked up noticeably, and waves 10 to 12 feet high were not uncommon. Many whitecaps were visible and the decks were repeatedly wetted down by spray coming over the bow. At about 1400 the exercises outlined in Table 1 were begun. They lasted about eight hours, during which time the sea remained reasonably stationary although the wind showed a definite decline. Fortunately, the decline of the seaway lagged the decline of the wind so that all tests were completed before any appreciable change in the state of the sea was noticed. The film of these tests[6] verifies qualitatively the constancy of the test conditions. It is interesting to note that during the buildup the winds in the fetch were around 30 knots, but as the seaway increased, the wind began to decrease. Evaluation of such a state of sea involves consideration of "piecewise steady state conditions"; that is, as the wind grows (or decays) it is assumed to remain steady for short periods so that changes are considered to be step-wise rather than continuous. Changes in the seaway are therefore also considered to be step-wise and small. By following certain procedures given in Reference 5, it was determined that a particular state of sea existed, during most of the experiment, which would have been developed by a constant wind of 22 knots blowing for at least 12 hours over a fetch of at least 100 nautical miles. Stated another way, the seaway was equivalent to a fully developed sea generated by a 22-knot wind. The characteristics of such a seaway are:

Wind Speed	22 knots
$\overline{H}$ (average height)	6.4 feet
$\overline{H}_{1/3}$ (average height of one-third highest waves)	10.2 feet

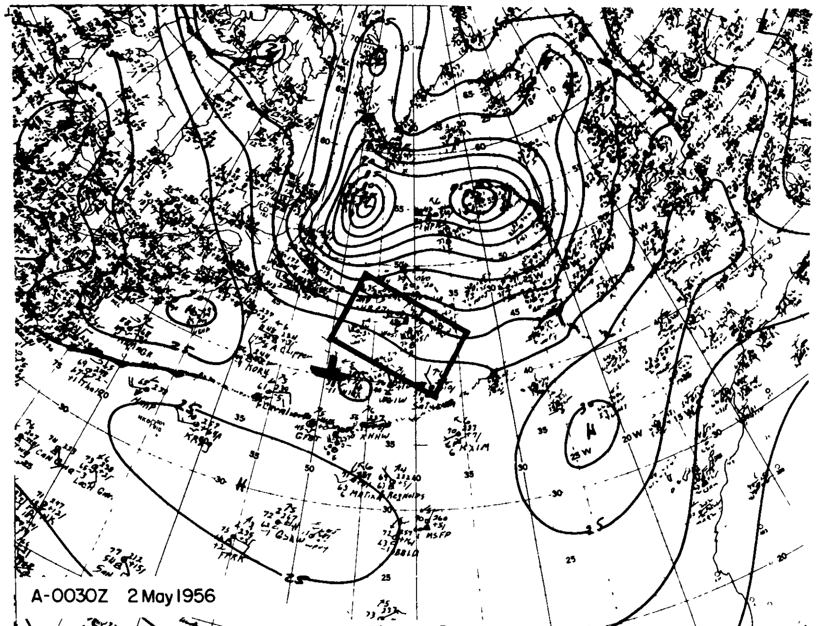

Figure 2a

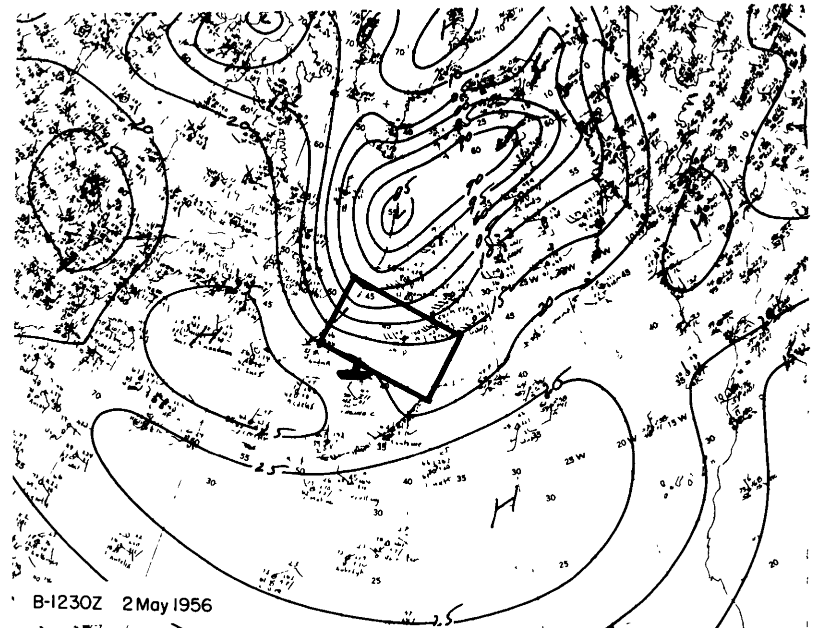

Figure 2b

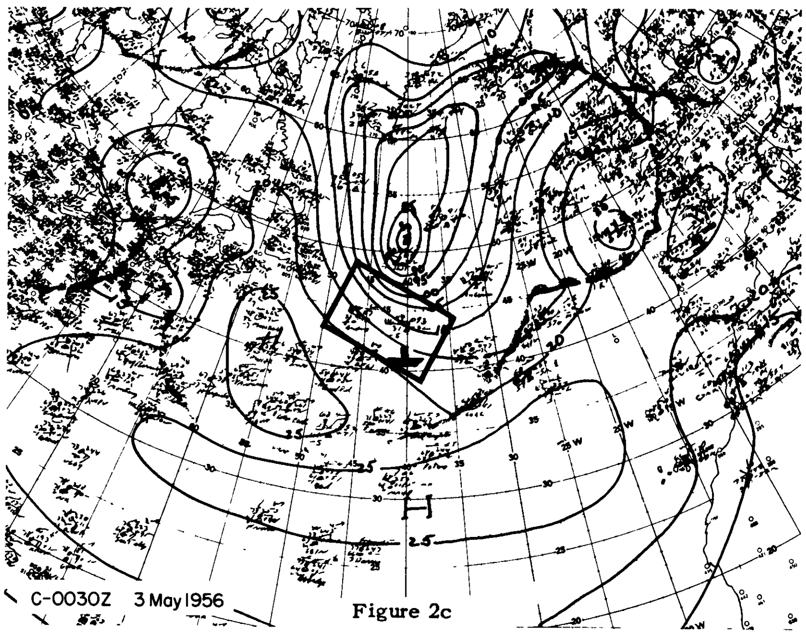

Figure 2c

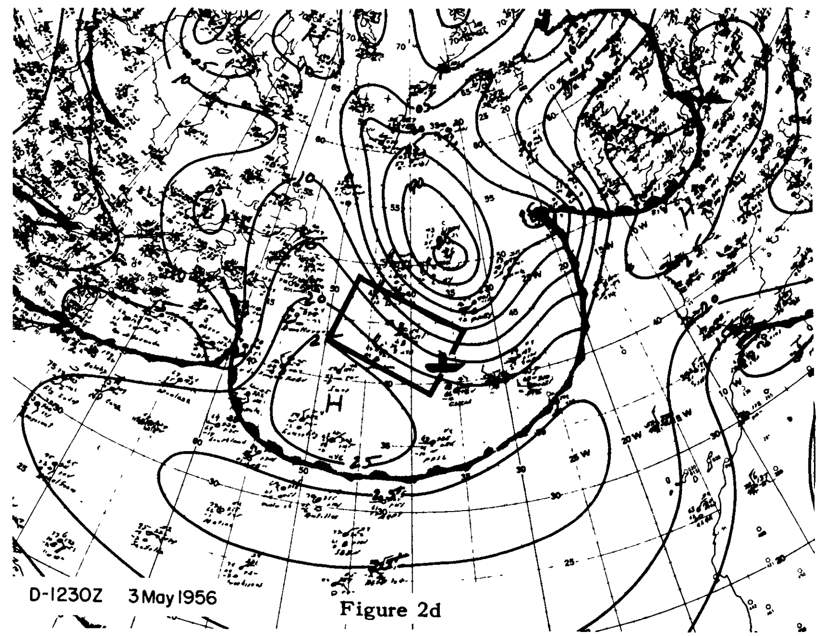

Figure 2d

Figure 2 – North Atlantic Surface Weather Charts

$\bar{H}_{1/10}$ (average height of one-tenth highest waves)	12.8 feet
$T_L - T_U$ (period range; represents 90 percent of total energy in seaway)	3.4 – 12.2 seconds
$\bar{T}$ (average period)	6.3 seconds
T_{max} (period of maximum energy)	8.9 seconds

The characteristics listed above agree reasonably well with visual observations made during the tests and with film records examined later. A low swell was running at the time in the same direction as the sea. Since the height was hardly discernible, and since the swell was of very low frequency and traveled in the same general direction as the seaway, it was considered to have had little or no effect on pitch and was ignored.

PRELIMINARY ANALYSIS

The first analysis of the pitch amplitude data was made aboard ship after the tests. The records were examined for certain qualitative information—in particular, the trend in pitching amplitude and pitching periods. Some samples of time histories of pitch amplitude are shown in Figure 3 where the ship speed is varied for several head seas runs. Figure 4 shows some time histories of pitch amplitude for constant ship speed and different headings. For the purpose of all further considerations in this paper, the experiment will be considered as consisting of two separate and independent parts:

1. Runs in head seas (Runs 1 – 5, Table 1).
2. Runs at constant speed (Runs 5 – 9).

In head seas (Figure 3) the records indicate that at the highest ship speed (21 knots) the pitch amplitude is generally smaller than at any other speed. At the lowest speed (10 knots) pitching motion is also relatively low. It is evident from these records that the pitching motion of the SILVER MARINER increases with speed until a critical speed is reached and then decreases with increasing speed. Changes in pitching period are not immediately evident. At constant speed (Figure 4), the effect of heading manifests itself by a pronounced increase in pitch amplitude (angle) at 45 degrees compared with 0 degrees (head seas). At 90 degrees (beam seas) there is a definite drop in amplitude, and at 135 degrees and 180 degrees this decrease is even more evident. It is also apparent that in changing the relative direction of ship to waves from 0 to 180 degrees a general shift to lower pitching frequencies is experienced.

The second step in the analysis of the data (also done aboard ship) was quantitative but rather crude. The peak-to-peak pitch amplitudes were measured by eye and grouped in class intervals of 0.633 degrees. Where double peaks occurred, these were measured separately, if they were prominent (as *a* in Figure 3 C) or ignored if not prominent (as *b* in Figure 4 B). These readings are then quite subjective, the eye acting as a filter to smooth the records. The resulting discrete distributions can then be expressed in the form

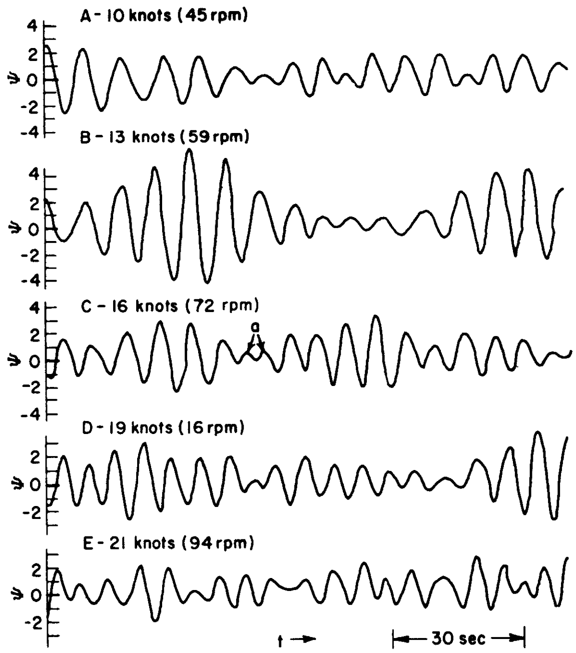

Figure 3 – Time Histories of Pitch, in Head Seas, for Different Ship Speeds

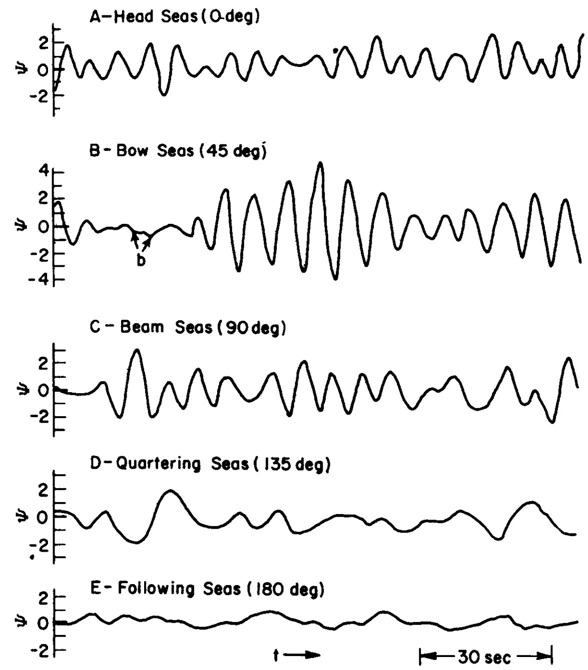

Figure 4 – Time Histories of Pitch for a Ship Speed of 21 Knots (94 RPM) and for Different Relative Headings of Ship to Seaway

$$P(\psi \leq \psi_i) = \frac{1}{N}\sum_{n=1}^{i} f_n \qquad [1]$$

which is read: the probability that a given (peak-to-peak) pitch angle ψ is less than a specific pitch angle ψ_i equals the total frequency of occurrence of all observed pitch angles f_n up to and including ψ_i divided by the total number of pitch angles measured N. It has been found that the distribution given by Equation [1] has in general a density function associated with it which is of the Rayleigh type, for the kind of data recorded here. That is,

$$f(\psi) \sim \frac{2\psi}{E}\, e^{-\psi^2/E} \qquad [2]$$

where E is the mean square value of ψ, the variance. A particular distribution of the type f_n is shown in Figure 5 and superimposed on it is the Rayleigh distribution that has the same variance. The agreement is good.

A more useful presentation is the empirical cumulative distribution function shown in Figures 6 and 7 in the continuous form and represented by

$$P(\psi \leq \psi_i) = \int_0^{\psi_i} f(\psi)\, d\psi \qquad [3]$$

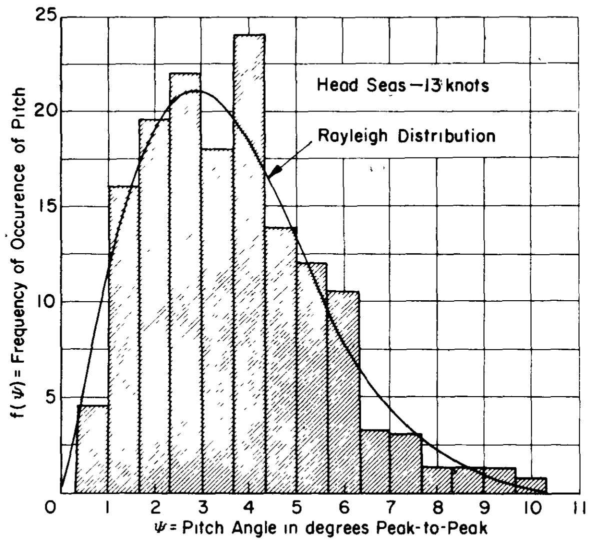

Figure 5 – Rayleigh Distribution Superimposed on Empirical Probability Distribution Function of Frequency of Occurrence of Pitch Angle

which is read: the probability that the pitch angle ψ will not exceed any specified pitch angle ψ_i is given by the integral in Equation [3].

The graphs in Figures 6 and 7 are very useful as crude predictors of pitch peak-to-peak variation. They also contain the information in Figure 5. To illustrate the use of the cumulative distribution function, consider Figure 6 and a hypothetical problem which specifies that a peak-to-peak pitch angle of 4 degrees is the maximum tolerable for a particular operation. A pitch angle greater than 4 degrees might be expected to occur 47 percent of the time at 13 knots but only 25 percent of the time at 21 knots. In fact, at 21 knots, 4 degrees is least likely to be exceeded and this speed would be recommended for the operation.

In a qualitative way, Figure 6 shows that in head seas maximum pitch response is reached at a speed of about 13 knots. Whereas the response at 16 knots and 19 knots is still quite high, the response at 21 knots is far less than at any other speed. In the case of variation in heading, at constant speed (21 knots), it is seen from comparison of Figure 6 with Figure 7 that change in heading is far more effective in reducing pitch than is change in speed. With respect to the hypothetical problem above, it is seen that at a relative heading of 0 degree the expectation of 4 degrees being exceeded at 21 knots is 26 percent which increases to 46 percent at 45 degrees and then decreases to 10 percent at 90 degrees and goes to zero very quickly as the relative heading goes beyond 90 degrees. Based on the limited data, it can be said that at 21 knots for any relative heading between 120 degrees and 180 degrees the expectation of exceeding 4 degrees pitch is zero. This is the most promising condition for the operation.

ENERGY SPECTRUM ANALYSIS

The preliminary analysis made aboard ship revealed some features of the pitching behavior. Further application of the technique of measuring bumps to learn of amplitudes and/or counting zero-crossings to learn of periods will yield very little for the great amount of labor that must be expended. There is, however, much more information that is recoverable from the original records. The key to unlocking the secrets of the pitch records lies in the *energy spectrum*; i.e., the separation of amplitudes (squared) according to frequency.

Once the energy spectrum is obtained, the original record is lost because phase relationships between the different frequencies are not retained during the transformation from the

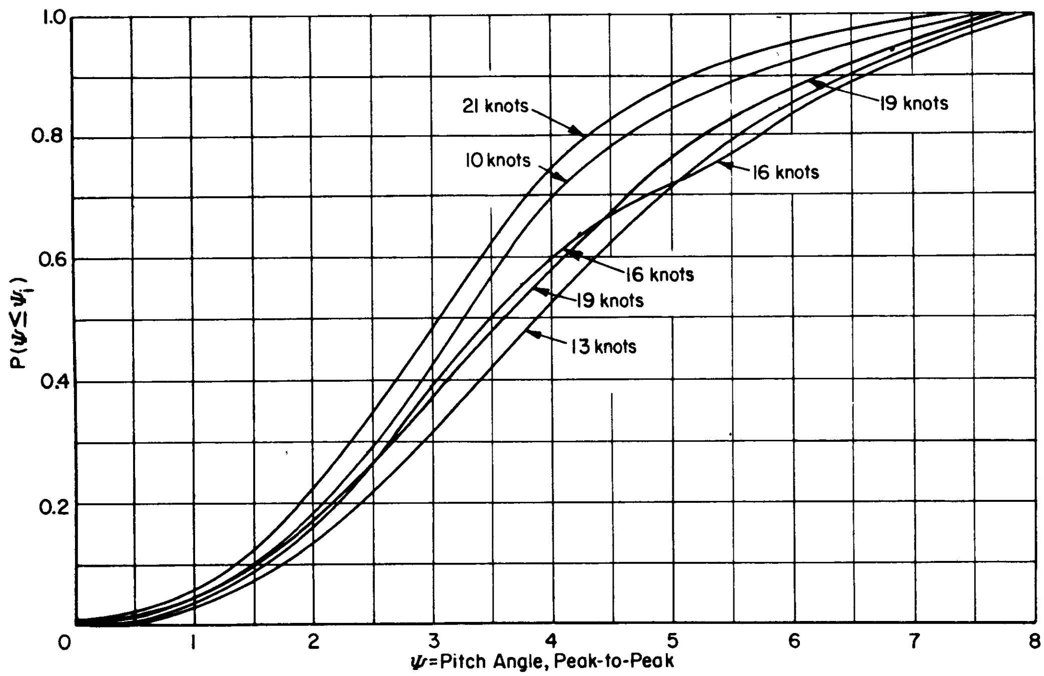

Figure 6 – Empirical Cumulative Relative Frequency of Occurrence of Pitch Angle as a Function of Ship Speed in Head Seas

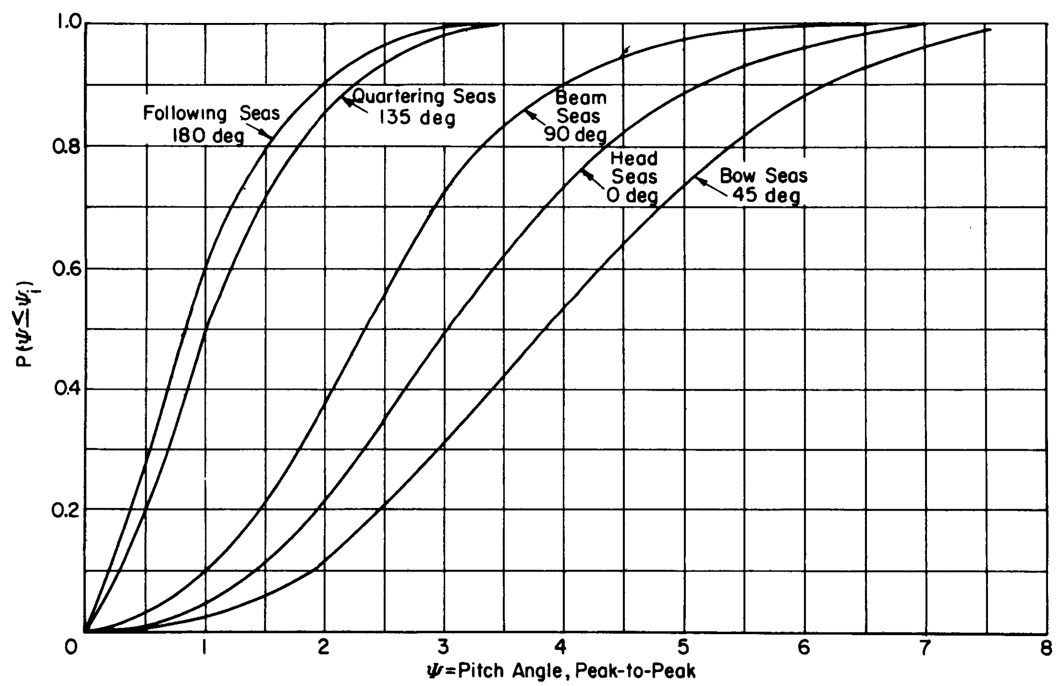

Figure 7 – Empirical Cumulative Relative Frequency of Occurrence of Pitch Angle as a Function of Heading for a Constant Ship Speed of 21 Knots

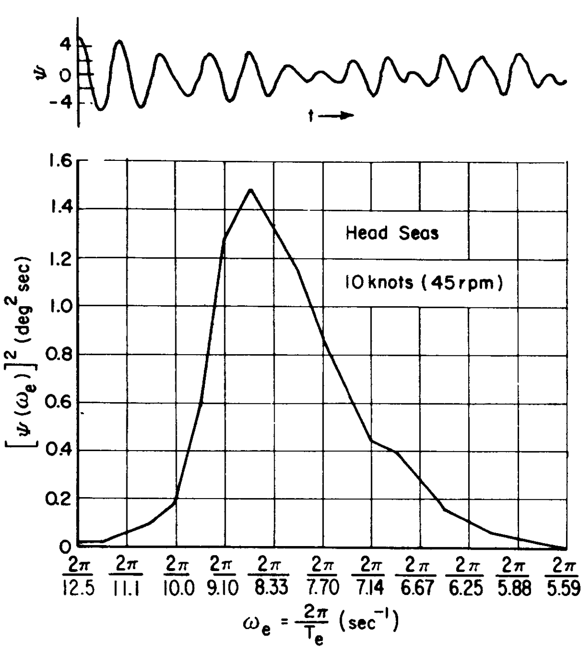

Figure 8 – Sample of a Computed Pitch Spectrum and a Portion of the Record from Which the Spectrum Was Computed

time base of the original record to the frequency domain. Figure 8 shows a portion of a pitch record and the energy spectrum computed from the entire record. The notation ω_e relates to the pitching frequencies which result from the encounter of the ship with the apparent modified wave form due to heading and speed. It is seen that under the conditions of the test, the pitching motion of the ship is concentrated in a particular band of frequencies and is essentially zero outside of this band. There is a frequency where the energy in the pitching record is a maximum; the energy content decreases to both sides of this frequency. Figure 8 represents, in general, the type of energy spectra that may be expected of ship motions, although exceptions with double peaks and other anomalies have been observed.

Certain statistics that describe the pitching performance of the ship under different conditions are obtained from the pitch spectra. The definition of these statistics, along with the method of computation for these tests, is given in the next section.

The pitch spectra were computed according to the formulas of Tukey[8] which prescribe a Fourier cosine transformation of the autocorrelation function of the original records plus a smoothing operation in order to realize the final form of the spectrum. All calculations were made on the TMB UNIVAC.

The computed spectra and the resultant statistics derived therefrom depend on the validity of certain *à priori* assumptions in terms of the original experiment. It is necessary to establish the validity of these assumptions in order to properly interpret the pitch spectra.

It is assumed that any pitch spectrum represents a stationary process; that is, if heading and speed remain constant, then the statistical properties of the motion will remain constant, so long as the statistical properties of the seaway do not change. It is most profitable to measure the state of the sea mechanically with a sea-state meter. Since no sea-state meter was available, visual observations plus hindcasts of the waves (see section on "State of the Sea") confirmed the stationary assumption.

It is assumed that a sufficiently long sample record is analyzed to adequately characterize the pitching performance for each set of experimental constants. If the record is infinitely long, it will of course define the pitching performance exactly. Such a record is impossible to obtain and would in any event be impractical to analyze. On the other hand,

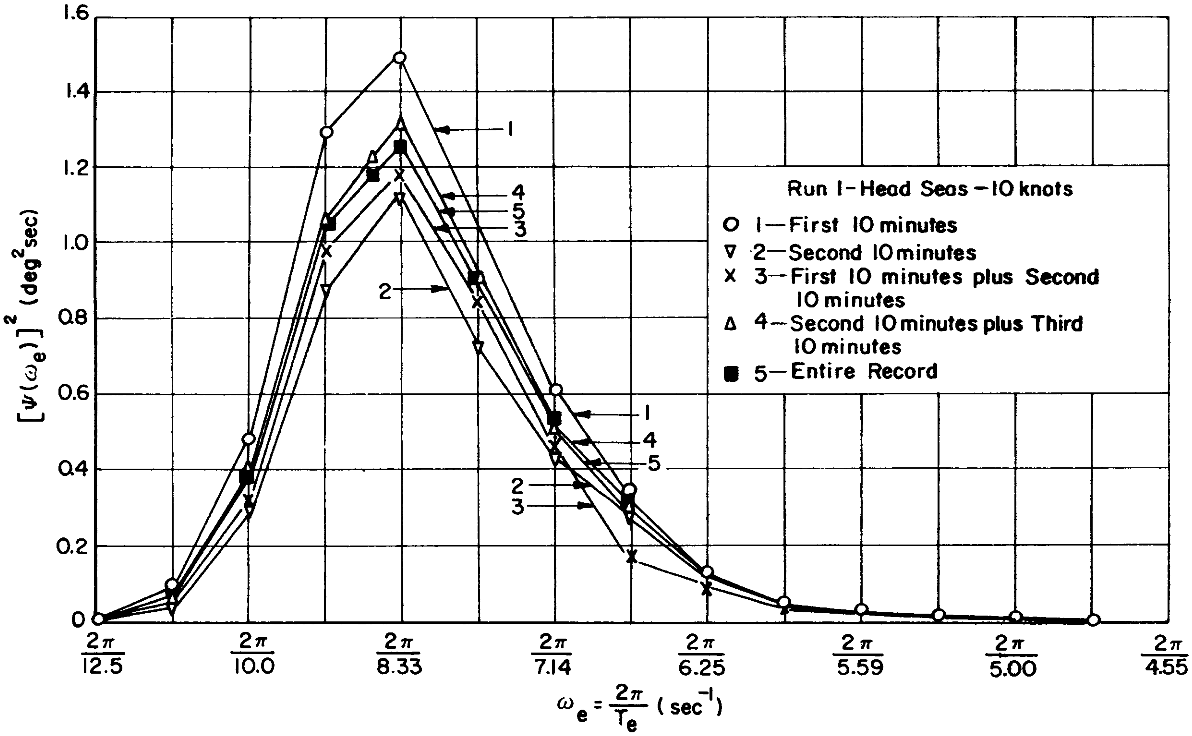

Figure 9 – Computed Pitch Spectra for Various Lengths of Record

it is evident that a very short record, say one or two cycles, would hardly be adequate to define the pitching performance. Consequently, an optimum record must be sufficiently long to adequately define the characteristics of pitch and yet be short enough to allow for all the runs to be made during the stationary life of the seaway. The lengths of records made during the experiment are given in Table 1.

In order to ascertain whether the lengths of record analyzed are sufficiently long to comply with the assumption, a simple experiment was performed. Each record was divided into several lengths and these were spectrum-analyzed separately and in combination. An example is shown in Figure 9. The numbers 1 and 2 represent successive 10-minute lengths from the same record, and it is seen that these two spectra have appreciable differences. The spectrum of the first 20 minutes of recording is labeled 3, and the spectrum of the last 20 minutes of recording is labeled 4. These are seen to be far less variable than the spectra of the shorter records. The spectrum of the entire record labeled 5 is between 3 and 4 and differences between them are small. It is likely that increasing the length of the record will not result in a spectrum that differs appreciably from 5, and hence the records analyzed here are considered to have sufficient length to properly characterize the ship's pitching performance.

It is assumed that randomly selected points on the records, in terms of their deviations from the mean, are normally distributed (Gaussian). This is considered to be a weak assumption and it is only necessary to show that it is nearly Gaussian. The chi-squared test, which, in effect, compares the observed frequency distribution with a theoretical normal frequency

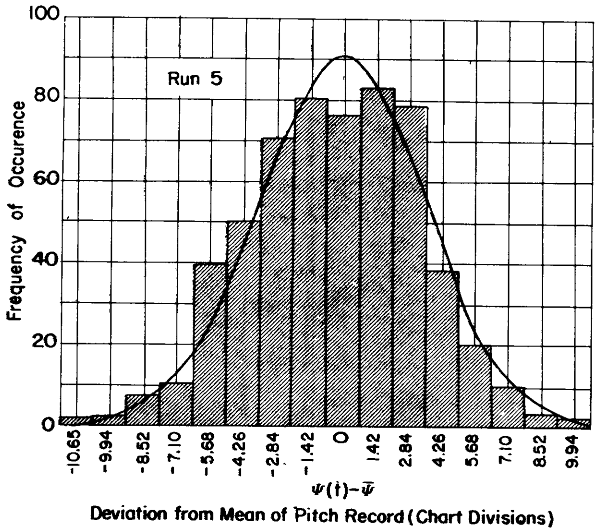

Figure 10 – Theoretical Normal Curve Superimposed on Observed Distribution (Histogram) of Pitch Angles

distribution with the same mean and standard deviation, will suffice to test the assumption. All of the records were so tested and Figure 10 shows the theoretical normal curve superimposed on the observed distribution of pitch angles for Run 5. The fit appears to be quite good and calculation of the probabilities that the observed distributions came from populations that are normally distributed indicate that the assumption is reasonable.

It is assumed that the energy spectrum embodies a "narrow" band of frequencies. This assumption applies to the calculation of statistical averages from the spectrum. If the spectrum is not narrow, a correction term[9] based on the spectrum and its second and fourth moments must be incorporated into estimates of the amplitude distribution of pitch. Investigation of the nine pitch spectra showed that no correction for pitch was necessary; the narrow-band spectrum assumption is satisfied.

The numerical analysis of these records includes in the basic system of equations a provision for estimating the error of the computed spectral densities. This error takes into account the length of the record being analyzed and the desired resolution of each spectral ordinate. As a consequence of this, each energy spectrum is equipped with confidence limits which define the probable range of each ordinate. That is, it may be said that the true spectrum lies between two imaginary spectra (defined by the confidence limits) with a given probability. This probabilistic range of spectral values will be very large if the record is short and if high resolution is desired. In such cases, the entire analysis may have been wasted effort. In the case of the work presented here, it has already been shown that the length of record is adequate to describe the pitch population. In addition, high resolution was not sought. In view of this, the confidence limits were not imposed on the pitch spectra.

The purpose of this section is to explain, in a general way, the type of analysis performed and to validate the results in view of the assumptions upon which the calculations are based. It has been shown that restrictions need not be imposed on the results because of the length of sample, resolution of spectral estimates, or the width of the spectrum. The results then will be taken at face value.

RESULTS

The results of this investigation appear in Figures 11 and 12 and in Table 2. The figures and the table complement each other and will be referred to repeatedly in the following discussion.

Consider Figure 11, which contains all of the pitch-angle spectra for head seas. It should be noted that the statistics of pitch-angle amplitude (peak-to-peak) are directly related to the total mean energy in the pitch spectrum, which in turn is given by the areas under the curves in Figure 11. By inspection, the shape and position of the spectrum curves will show some features of pitching performance as a function of speed. There is an obvious shift toward higher frequencies (lower periods) with increasing speed. This is to be expected in head seas. The areas under the curves indicate that the greatest pitch amplitudes will occur at a ship speed of 13 knots and the least at 21 knots. The general trend, therefore, indicates agreement with the preliminary analysis made aboard ship (Figure 6). In the case where heading is the experimental variable and speed is constant (Figure 12) the shift is toward lower frequencies as heading changes from head seas to following seas, again expected, and the areas under the curves appear to bear out the results shown in Figure 7.

There is probably little else than can be learned from these figures without resorting to quantitative measurements, so that is the next step. The statistics of pitch are tabulated in Table 2, and the explanation of these numbers follows:

A. E The total energy in the spectrum curve

$$E = \int_0^\infty [\psi(\omega_e)]^2 \, d\omega_e$$

$[\psi(\omega_e)]^2$ is the density function which defines the pitch spectra in Figures 11 and 12.

B. $\overline{\psi}$ The average peak-to-peak pitch angle

$$\overline{\psi} = 1.772\sqrt{E}$$

C. $\overline{\psi}_{1/3}$ The average of the one-third highest peak-to-peak pitch angles

$$\overline{\psi}_{1/3} = 2.83\sqrt{E}$$

D. $\overline{\psi}_{1/10}$ The average of the one-tenth highest peak-to-peak pitch angles

$$\overline{\psi}_{1/10} = 3.60\sqrt{E}$$

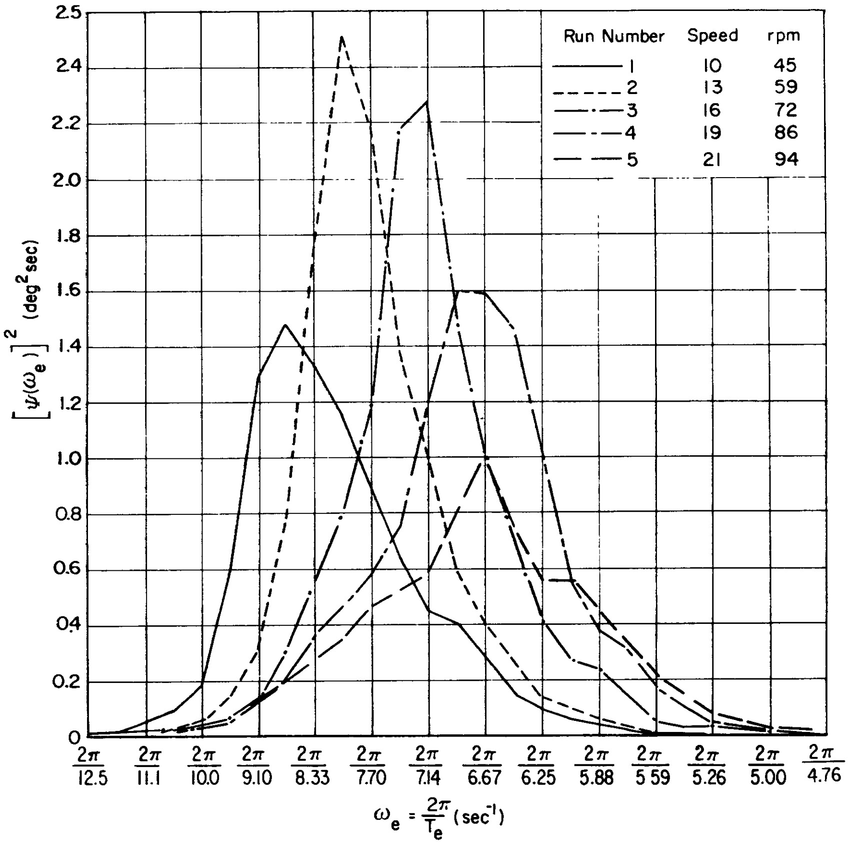

Figure 11 – Pitch Spectra for Different Ship Speeds in Head Seas

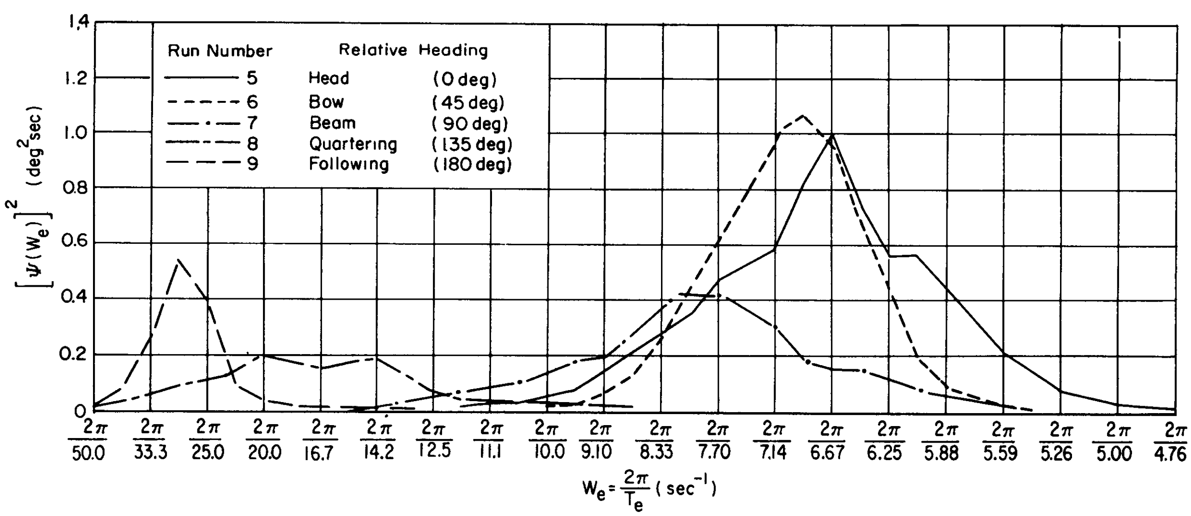

Figure 12 – Pitch Spectra for Different Relative Headings at an Approximate Speed of 21 Knots

TABLE 2

Results Obtained from the Pitch Spectra of the SS SILVER MARINER

Run	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*	*L*	*M*	*N*	*O*
	E (deg^2)	$\bar{\psi}$ (deg)	$\bar{\psi}_{1/3}$	$\bar{\psi}_{1/10}$	ψ_{50}	ψ_{100}	ψ_{200}	ψ_{500}	ψ_{1000}	T_U (sec)	T_m (sec)	T_L (sec)	ψ_0 (sec)	$\bar{T}_\psi$ (sec)	ψ_m (sec^{-1})
1	1.95	2.48	3.96	5.04	5.94	6.38	6.76	7.28	7.65	10.00	8.70	6.54	0.255	7.85	0.310
2	3.26	3.19	5.09	6.48	7.63	8.21	8.69	9.36	9.83	9.26	8.00	6.54	0.263	7.52	0.326
3	2.65	2.89	4.61	5.87	6.91	7.43	7.82	8.48	8.90	8.93	7.15	5.82	0.298	6.71	0.346
4	2.37	2.73	4.36	5.54	6.53	7.02	7.44	8.01	8.41	8.73	6.67	5.59	0.318	6.29	0.400
5	1.68	2.30	3.68	4.68	5.51	5.93	6.28	6.76	7.10	8.70	6.67	5.53	0.323	6.70	0.407
6	2.61	2.87	4.58	5.83	6.87	7.39	7.82	8.42	8.85	8.85	6.90	5.88	0.312	6.61	0.371
7	1.01	1.77	2.83	3.60	4.24	4.56	4.83	5.20	5.46	10.52	8.00	6.17	0.265	7.55	0.321
8	0.38	1.10	1.75	2.23	2.63	2.83	2.99	3.22	3.39	37.05	*	11.77	0.124	16.20	0.155
9	0.35	1.04	1.67	2.12	2.50	2.69	2.85	3.07	3.22	45.50	28.50	20.00	0.078	25.50	0.096

* 14.3 – 20.0 (Flat spectrum, peak not discernible).

E. to *I*. ψ_n The highest expected peak-to-peak pitch angle during n pitch cycles[10]

n	ψ_n
50	$4.24\sqrt{E}$
100	$4.56\sqrt{E}$
200	$4.86\sqrt{E}$
500	$5.20\sqrt{E}$
1000	$5.46\sqrt{E}$

J. T_U The highest period in the pitch spectrum*

K. T_m The period in the pitch spectrum at which maximum energy occurs.

L. T_L The lowest period in the pitch spectrum*

M. ψ_0 The expected number of zero crossings per second

$$\psi_0 = \frac{1}{\pi}\left[\frac{\int_0^\infty \omega_e^2\,[\psi(\omega_e)]^2\,d\omega_e}{\int_0^\infty [\psi(\omega_e)]^2\,d\omega_e}\right]^{1/2}$$

*The values T_U and T_L and cut-offs chosen such that approximately 95 percent of the energy in the spectrum is contained in the periods (frequencies) between them.

N. $\overline{T}_\psi$ The average time interval between zero up-crossings; a measure of pitching periods (seconds)

$$\overline{T}_\psi = 2\pi \left[\frac{\int_0^\infty [\psi(\omega_e)]^2 \, d\omega_e}{\int_0^\infty \omega_e^2 \, [\psi(\omega_e)]^2 \, d\omega_e} \right]^{1/2}$$

O. The number of maxima to both sides of the record mean per second[11]

$$\psi_m = \frac{1}{\pi} \left[\frac{\int_0^\infty \omega_e^4 \, [\psi(\omega_e)]^2 \, d\omega_e}{\int_0^\infty \omega_e^2 [\psi(\omega_e)]^2 \, d\omega_e} \right]^{1/2}$$

The statistics on amplitude of pitch angle (peak-to-peak) as given in Table 2, columns *B*, *C*, and *D*, show the general effects of speed and heading on pitch in a State 5 sea. These results cannot be appreciated unless taken in conjunction with the information on the pitching periods given in columns *J*, *K*, *L*, and *N* and with the information on the period distribution of the seaway given in the section on the state of the sea. When the periods of wave encounter coincide with the period band of maximum energy in the seaway spectrum (after transformation for speed and direction) and also with the natural pitching period of the ship, then the maximum pitching for that seaway will be experienced. In these tests this condition was met at a ship speed of 13 knots in head seas.

The predicted values of pitch maxima as a function of the number of pitching cycles experienced is given in Table 2, columns *E* to *I*. Since there were about 300 cycles in each record, columns *G* and *H* are particularly pertinent to these tests. Comparison of expected pitch maxima for 300 cycles ψ_{300} with measured maxima (from Reference 2) ψ_m is shown in Table 3.

The expected maximum pitch angle for the lengths of record involved is consistently lower than the observed maxima but sufficiently close to validate ψ_n (columns $E - I$ of Table 2) for prediction of maxima from the computed spectrum.

The quantities ψ_0 and ψ_m in columns *M* and *O* of Table 2 may be useful in certain military problems where the number of maxima or zeros of pitch per unit time is an important consideration.

Figure 1 of Reference 1 shows the results of model tests of the MARINER-Class ship in head seas. The range of observed pitch angles, for the wavelengths tested, is 0.1 – 0.325 degrees per foot of wave height. An attempt was made to verify the model tests by computing the transfer function in pitch for the head seas tests. The formula

$$[T(\omega_e)]^2 = \frac{[\psi(\omega_e)]^2}{[R(\omega_e)]^2},$$

TABLE 3

Expected Pitch Maxima for 300 Cycles of Record ψ_{300} Compared with Highest Observed Pitch Angle ψ_m

Run	ψ_m deg	ψ_{300} deg
1	7.6	7.1
2	10.5	9.0
3	8.7	8.1
4	9.6	7.7
5	7.1	6.5
6	9.0	8.1
7	5.9	5.0
8	3.9	3.1
9	3.2	3.0

where $[R(\omega_e)]^2$ is the seaway spectrum adjusted for ship speed, produced results which gave very poor agreement with Figure 1 of Reference 1. Recent investigation[12] indicates that the statistical variability in measuring $[\psi(\omega_e)]^2$ and $[R(\omega_e)]^2$ can easily account for conflicting results. Simultaneous measurements of $[\psi(\omega_e)]^2$ and $[R(\omega_e)]^2$ and their cross spectra are required for the accurate determination of $[T(\omega_e)]^2$.

It has been shown in Figure 7 and Table 2, column A, that a relative heading of 45 degrees produces the maximum pitch effects. This is interesting in view of Figure 13, taken from Reference 13, which shows that in 45-degree seas (bow), the ship speed is most adversely affected. Indeed, the manner in which ship speed is reduced as a function of heading corresponds to the way in which pitching motion was observed to increase.

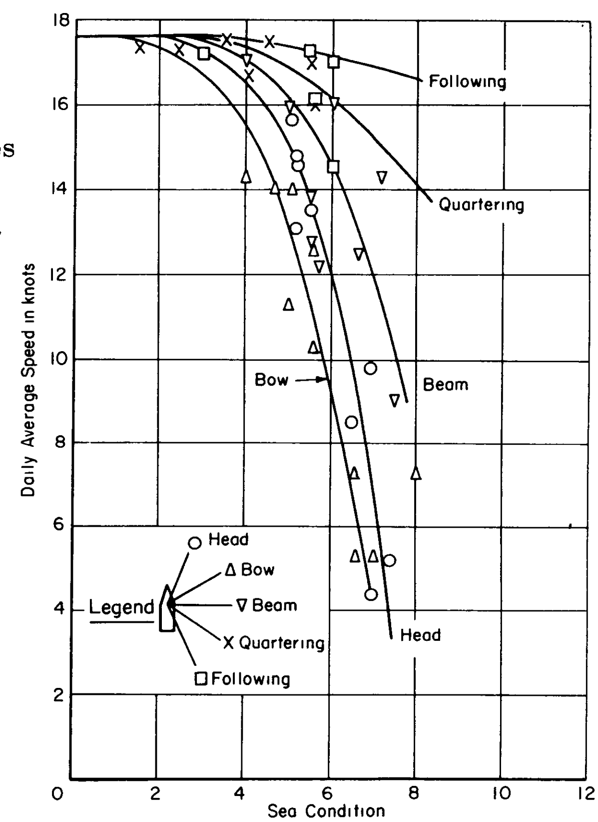

Figure 13 – Average Ship Speed versus Sea State for Different Relative Headings

(After Lewis and Morrison)

CONCLUSIONS

1. In head seas and, in particular, State 5 seas, the SILVER MARINER pitches most violently at around 13 knots and least at her top speed of 21 knots.

2. Bow seas (45 degrees) produce more serious pitching motions than head seas. Following seas show least evidence of pitching activity.

3. In general, change of heading will influence pitching motion to a much greater extent than will change of speed, in head seas.

4. Since certain merchant vessels reduce speed with heading inversely as pitching motion varies with heading in these tests, it is believed that there is a strong relationship between maintenance of sea speed and pitching behavior.

5. In view of Conclusions 1 to 4 it is believed that this experiment, designed to determine the most violent pitching behavior in a State 5 sea, was poorly designed. It would probably have been more profitable to vary ship speed in the 45-degree sea rather than head seas.

6. The technique of energy spectrum analysis yields results which are compatible with those obtained by measuring peak-to-peak oscillations and in addition provides much information which is not otherwise obtainable.

7. The 30-minute length of record appeared to be a more than adequate sample. Probably, 20-minute records would do as well.

8. In view of the results in Table 2, it is expected that the worst pitching conditions will occur in bow seas at about 18 knots.

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Reed Research, Inc. prepared the original data for UNIVAC and plotted the final results. U.S. Navy Hydrographic Office provided wave forecasts during the trials.

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