

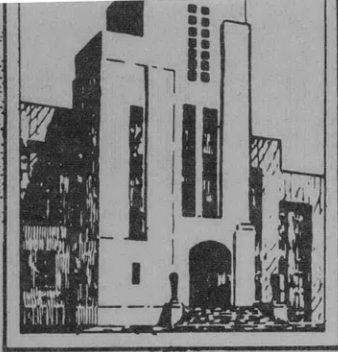
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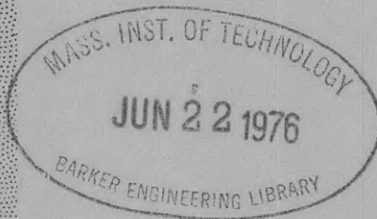


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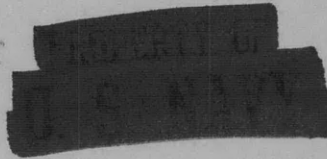
EFFECT OF INITIAL DEFLECTIONS AND RESIDUAL WELDING
STRESSES ON ELASTIC BEHAVIOR AND COLLAPSE PRESSURE
OF STIFFENED CYLINDERS SUBJECTED TO EXTERNAL
HYDROSTATIC PRESSURE

AERODYNAMICS



by

Martin A. Krenzke



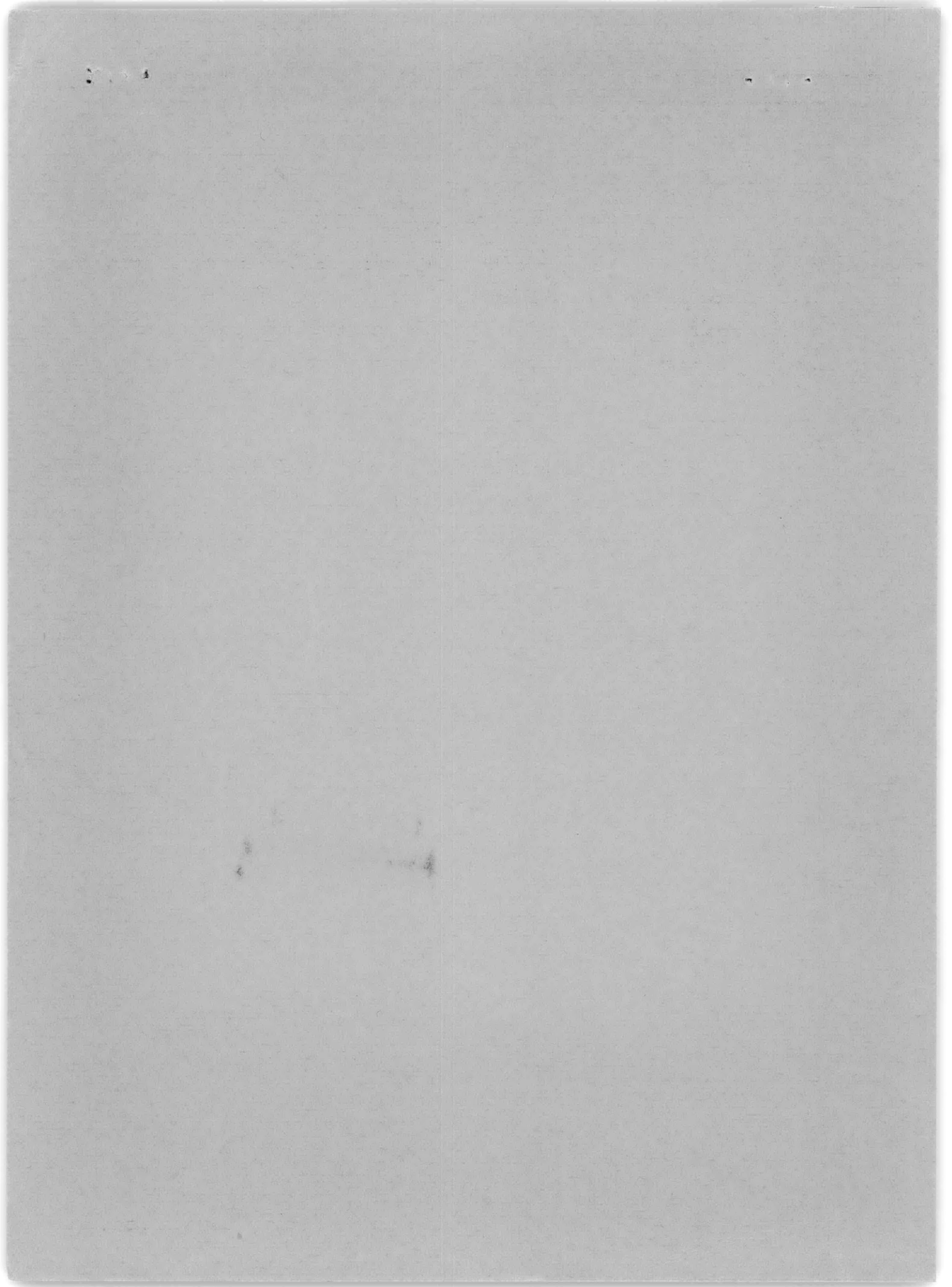
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7 June 1960


From: Commanding Officer and Director, David Taylor Model Basin
To: Chief, Bureau of Ships (335) (in duplicate)

Subj: Strength of stiffened cylinders; effect of initial
deflections and residual welding stresses on

Encl: (1) DATMOBAS Report 1327 entitled "Effect of Initial
Deflections and Residual Welding Stresses on
Elastic Behavior and Collapse Pressure of
Stiffened Cylinders Subjected to External
Hydrostatic Pressure" 3 copies

1. As part of Project SF013 0302 the David Taylor Model Basin has been studying the effect of initial axisymmetric deflections between frames on the elastic stress distribution and collapse strength of stiffened cylinders. In enclosure (1) are reported hydrostatic-pressure tests of four stiffened cylinders fabricated by welding.

2. Strains measured in the cylinders indicated that initial deflections between frames affect the elastic behavior of a stiffened cylinder. The tests also showed that initial deflections and residual welding stresses must be considered in calculating the collapse strength of welded stiffened cylinders.


E.E. JOHNSON
By direction

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**EFFECT OF INITIAL DEFLECTIONS AND RESIDUAL WELDING
STRESSES ON ELASTIC BEHAVIOR AND COLLAPSE PRESSURE
OF STIFFENED CYLINDERS SUBJECTED TO EXTERNAL
HYDROSTATIC PRESSURE**

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Martin A. Krenzke

April 1960

**Report 1327
S-F013 03 02**

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ABSTRACT

Four stiffened cylinders, fabricated by welding, were subjected to external hydrostatic pressure to determine how initial axisymmetric deflections between frames and residual stresses in the shell affect the elastic behavior and the collapse strength.

Measured strains indicated that initial deflections between frames affect the elastic behavior of a stiffened cylinder. These strain measurements were in good agreement with strains calculated by the theory of Lunchick and Short, an analysis which considers the influence of initial axisymmetric deflections.

The tests also showed that initial deflections and residual stresses must be considered in calculating the collapse strength of welded stiffened cylinders.

INTRODUCTION

When stiffeners are welded to a cylindrical shell to increase strength, the shell acquires an initial deflection between frames when the welds contract upon cooling. This deflection is inward for internally framed cylinders and outward for externally framed cylinders. The magnitude of these deflection depends on weld size relative to shell thickness and on welding procedures. An analysis of the effect of these deflections on the elastic stress distribution in a closed stiffened cylinder subjected to external hydrostatic pressure has been developed by Lunchick and Short.¹

The results of that analysis led to a preliminary experiment at the David Taylor Model Basin to study the effect of initial deflections between frames on the collapse strength of a stiffened cylinder. An externally stiffened cylinder, fabricated by welding, collapsed at an external pressure of 585 psi. The damaged portion was removed, and the remainder of the model was subjected to an internal pressure of 870 psi, thus placing an additional permanent outward deflection between frames beyond what may have existed originally. External pressure was again applied to the model; it then failed at a pressure of 694 psi, an increase in collapse strength of 19 percent over that of the initial test. The results of those tests indicated that a more systematic study of the effects of initial deflections between stiffeners and of residual stresses (resulting from welding) on the elastic strains, as well as on the collapse pressure, should be pursued.

Four models, fabricated by welding and designed to study the effects of initial deflection, were tested to failure under external hydrostatic pressure. In this report the fabrication procedures are described, the measurements of the initial contours of the shells are presented, the measured elastic strains and collapse pressures are compared with those calculated by available theories, and the effects of initial deflections and residual stresses on collapse pressure are discussed.

¹References are listed on page 37.

DESCRIPTION OF MODELS

Models M-1, M-2, M-3, and M-4 were stiffened cylindrical shells with the following identical geometric characteristics:

$$\begin{aligned}
 2R &= 26.000 \text{ in.} & b &= 0.300 \text{ in.} \\
 L_f &= 3.692 \text{ in.} & d &= 1.040 \text{ in.} \\
 L &= 3.225 \text{ in.} & A_f &= 0.312 \text{ sq in.}
 \end{aligned}$$

Here

- $2R$ is the diameter to the midsurface of the shell,
- L_f is the center-to-center distance between adjacent rectangular frames for the three middle bays,
- L is the length L_f less the width b of the stiffening ring and two-thirds of each weld which joins the stiffener to the shell,
- d is the depth of the stiffener, and
- A_f is the cross-sectional area of the stiffener.

The first two bays at each end of the model were shorter than the remaining bays, to force the failure to occur in a bay whose strength was not influenced by the rigidity of the closure plates. In Table 1 are listed the average measured thicknesses h and the yield strengths of the shell material. Young's modulus E and Poisson's ratio are assumed to be 30×10^6 psi and 0.3, respectively. A schematic drawing of the models is presented in Figure 1.

All models were manufactured by a process designed to minimize out-of-roundness. The shells were machined from 3/8-in.-thick cylinders which had been formed by rolling a flat plate of HY-80 steel to the proper diameter and joining the ends together by a single longitudinal weld. Each cylinder was stress-relieved, and each seam was X-rayed before machining. The frames were formed by flame-cutting 180-deg segments out of a plate with a nominal thickness of 0.300 in. These segments were welded together and machined to the proper diameter. The welding of the frame to the shell was performed in an identical manner for each model.

TABLE 1

Measured Thicknesses and Yield Strengths of Shell Material

Model	Average Measured Shell Thickness in.	Yield Strength of Shell Material psi
M-1	0.169	82,500
M-2	0.169	78,350
M-3	0.172	82,500
M-4	0.167	55,850

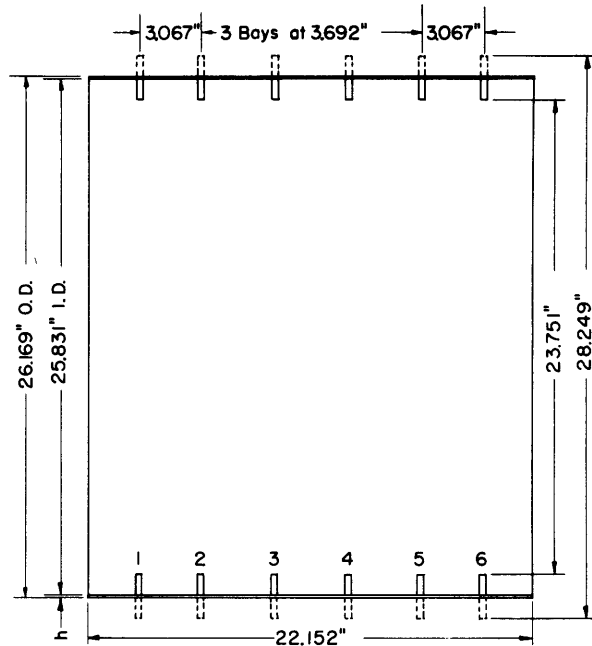
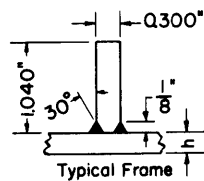


Figure 1 - Schematic Drawing of Models

Models M-1, M-3, and M-4 had internal frames; Model M-2 had external frames. Models M-3 and M-4 had machined outward deflections with maximum values before welding of 0.0635 in. in bays 2½, 3½, and 4½, and 0.0484 in. in bays 1½ and 5½.

Considerable care was taken to obtain reliable values of the yield strength of the material. The yield strength of the frames was found to be 84,000 psi by averaging the strengths of 24 compression specimens taken from the original sheet. The layout of specimens and the compressive yield strengths of the shell material are shown in Figure 2. The yield strength finally adopted was obtained by averaging the results of specimens taken from the shell adjacent to the failure after collapse and are given in Table 1.

Model M-1 was internally stiffened. The contraction of the welds which joined the frames to the cylindrical shell caused an initial inward deflection between frames.

Model M-2 was similar to Model M-1 except that the frames were external. For external framing, the contraction of the welds caused an initial outward deflection between frames.

Model M-3 was an internally stiffened model similar to Model M-1 except that there was an initial outward deflection between frames. This deflection was produced by machining an outward deflection in the shell between frames and then welding the internal stiffeners in place. The desired magnitude and shape of the initial deflection were those which theoretically produce pure membrane stresses throughout the shell; see Appendix A.

Model M-4 was an internally stiffened model fabricated by the same procedure used for Model M-3. However, Model M-4 was stress-relieved after the frames were in place so that it would be free of any residual welding stresses. During this final stress-relieving process,

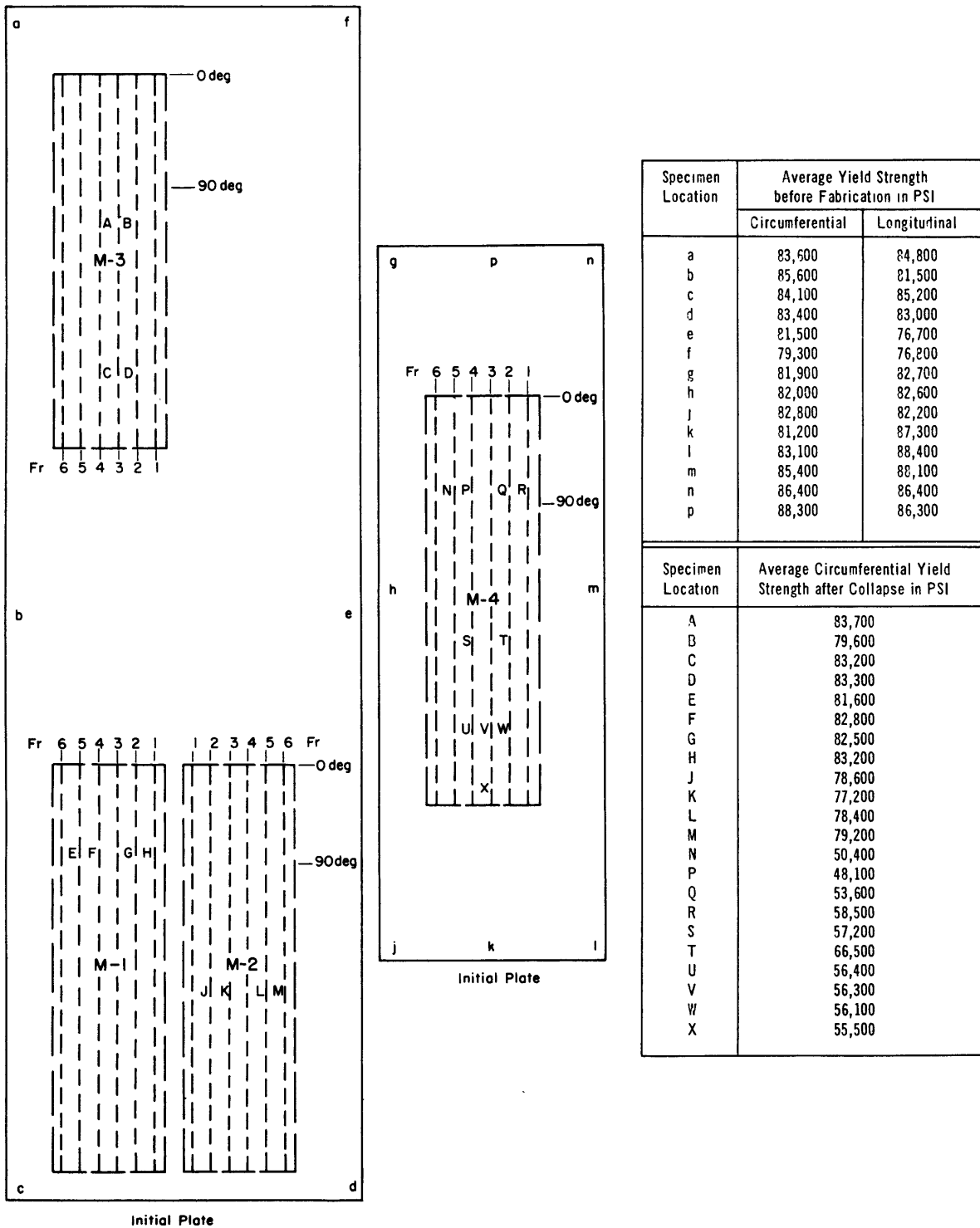


Figure 2 – Layout and Measured Yield Strengths of Shell Material

the furnace temperature exceeded the critical stress-relieving temperature; consequently, the yield strength of the material was lowered considerably.

TEST PROCEDURE

The initial circumferential contours at the middle of the bays and at the frames were recorded by an automatic recording deflectometer.² Initial circumferential contours for Bay 3½ and the adjacent frames are given in Figure 3 for each model. The average initial deflections at Bay 3½ relative to Frames 3 and 4 and the average difference between the radius at Bay 3½ of the welded model and the corresponding radius of the machined shell before fabrication are presented in Table 2. Inward deflections are assumed positive.

The method used to measure the longitudinal contours of the initial deflections is illustrated in Figure 4. A vernier height gage with an Ames dial gage (graduated in 0.0001 in.) attached to it was clamped to the closure plate. The dial gage was moved vertically, and readings were made between Frames 3 and 4 of each model. Typical contours for each model are shown in Figure 5.

The models were instrumented with SR-4 electric wire-resistance strain gages. Instrumentation was concentrated in Bay 3½ because it had the boundary conditions which most closely duplicated those assumed by theory; that is, a cylinder with evenly spaced stiffeners. Orientations were chosen to include those of maximum and minimum initial deflections.

The models were tested in the 37-in.-diameter pressure tank. Oil was used as a pressure medium. At least two pressure runs were made on each model to minimize the nonlinearity in strain data.

TABLE 2

Average Measured Initial Deflections

Model	Relative Deflection of Midbay to Frames in.	Residual Deflections or Change in Radius at Midbay after Welding in.
M-1	+0.007	+0.006
M-2	-0.003	+0.004
M-3	-0.057	+0.007
M-4	-0.055	

(Text continued on page 13.)

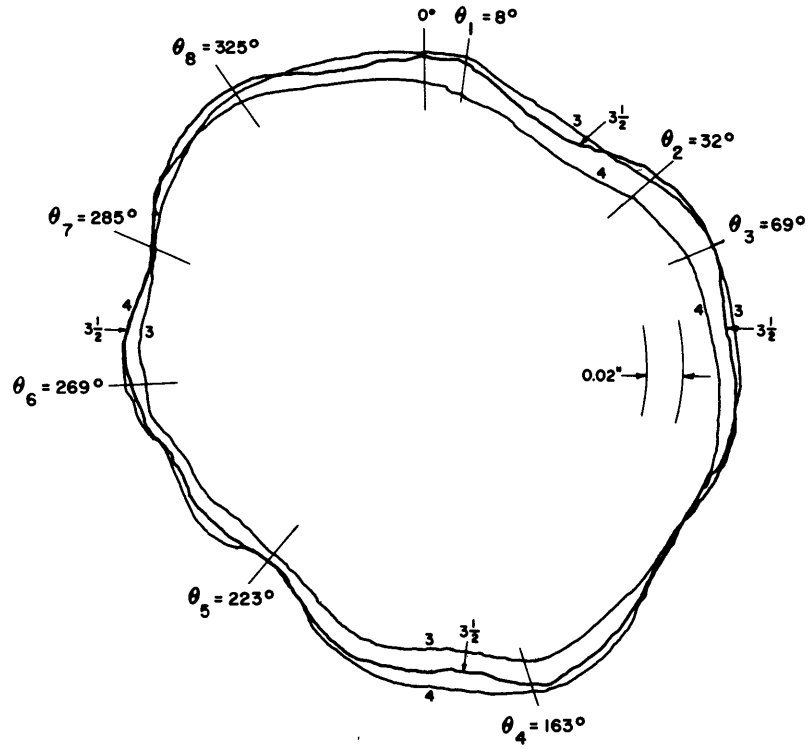


Figure 3a — Model M-1

Contours recorded with deflectometer probe on exterior surface of shell; therefore the average midbay radius is actually smaller than that of the frames.

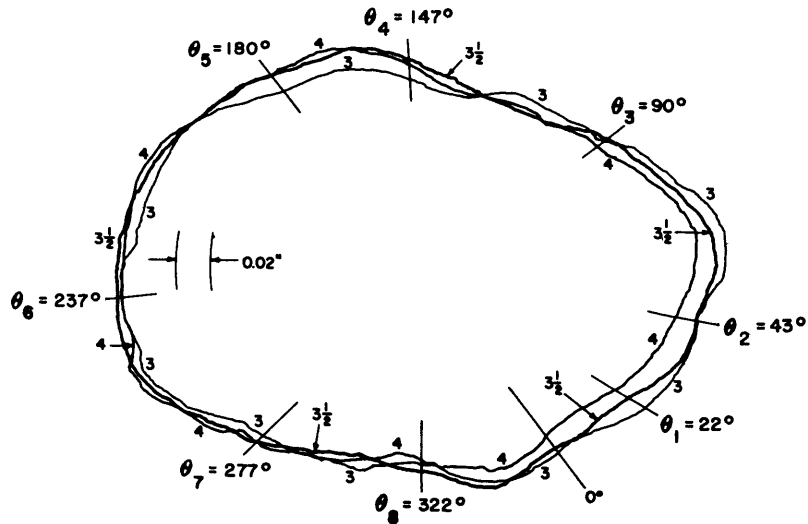


Figure 3b — Model M-2

Contours recorded with deflectometer probe on interior surface of shell; therefore relative position of midbay with respect to frames is as shown.

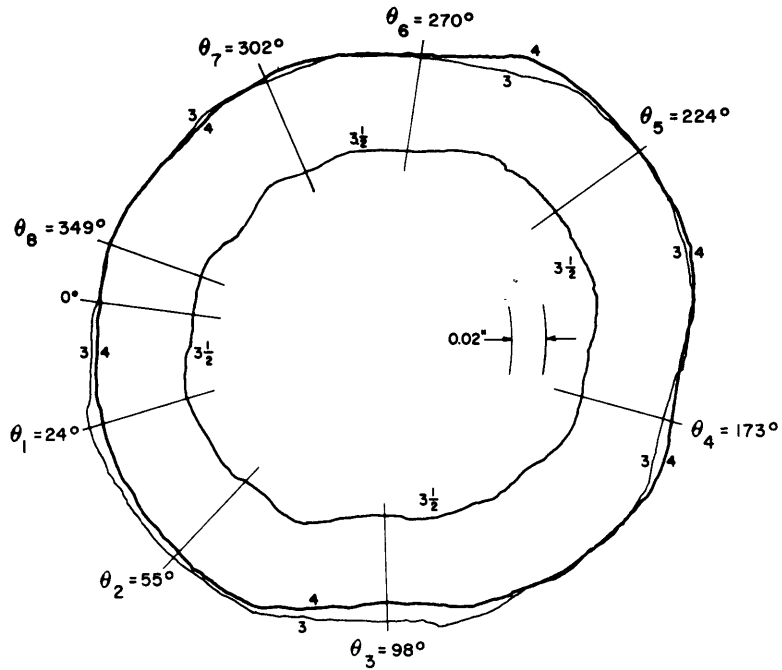


Figure 3c - Model M-3

Contours recorded with deflectometer probe on exterior surface of shell; therefore the midbay radius is actually larger than that of the frames.

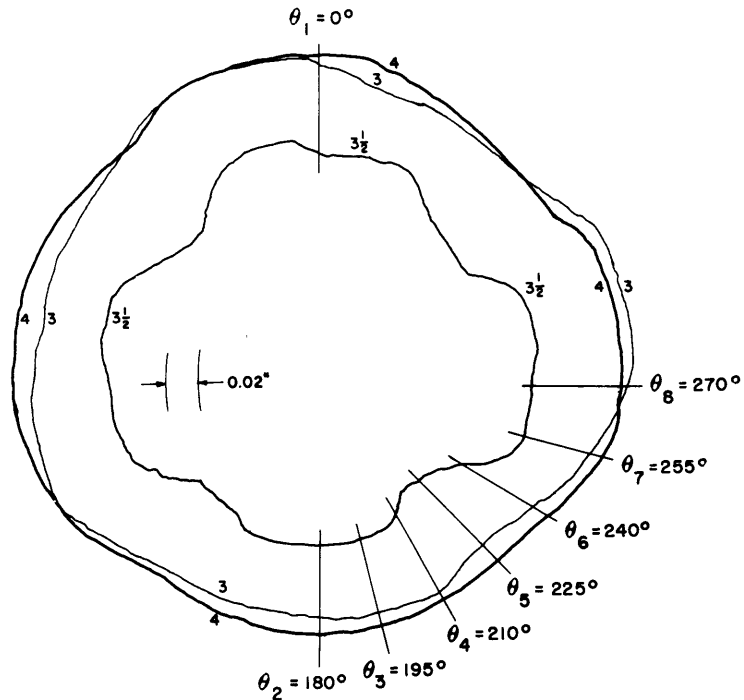


Figure 3d - Model M-4

Contours recorded with deflectometer probe on exterior surface of shell; therefore the midbay radius is actually larger than that of the frames.

Figure 3 - Initial Circumferential Contours

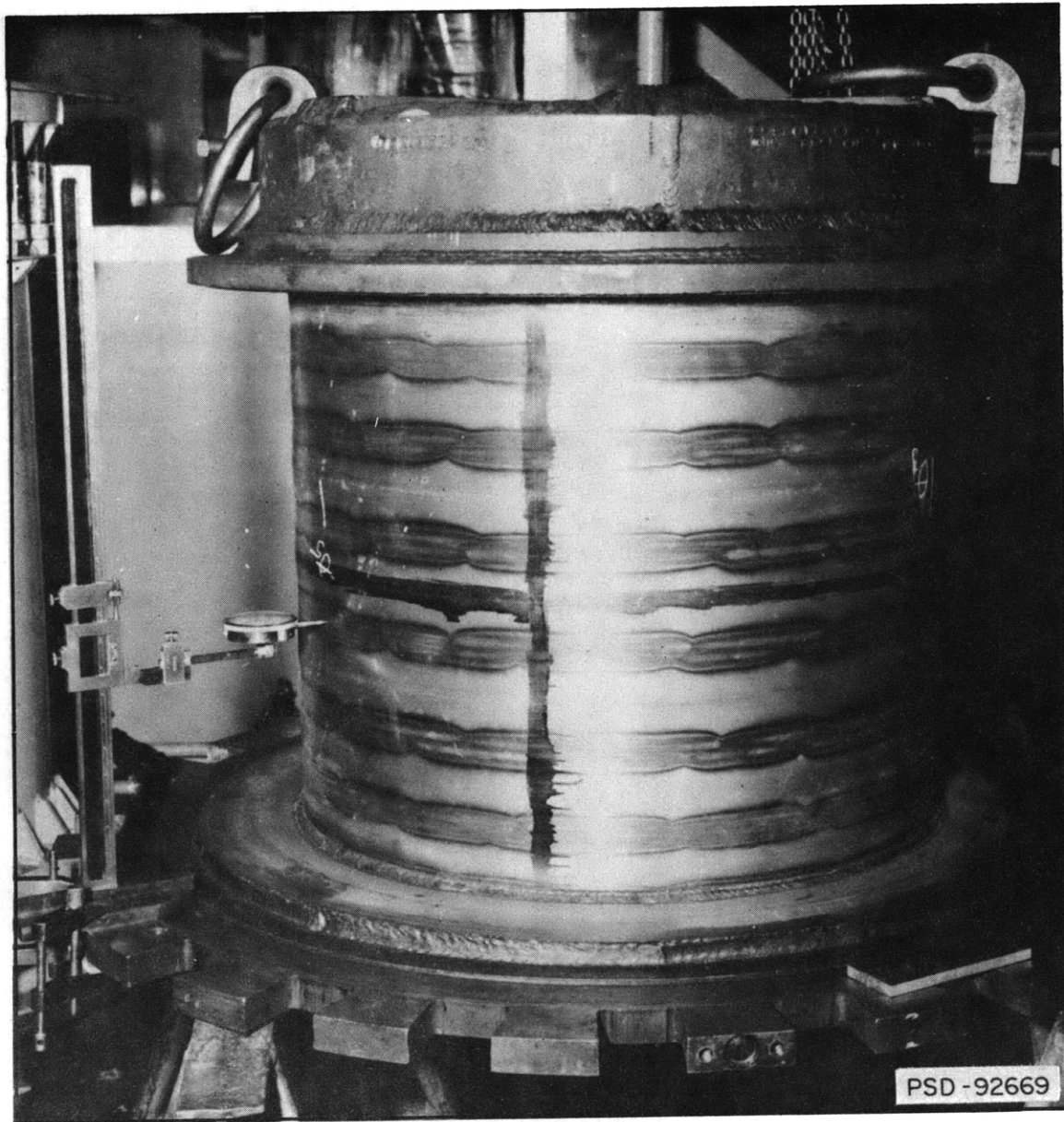


Figure 4 – Method Used to Measure Initial Longitudinal Contours

Figure 5 – Initial Longitudinal Contours

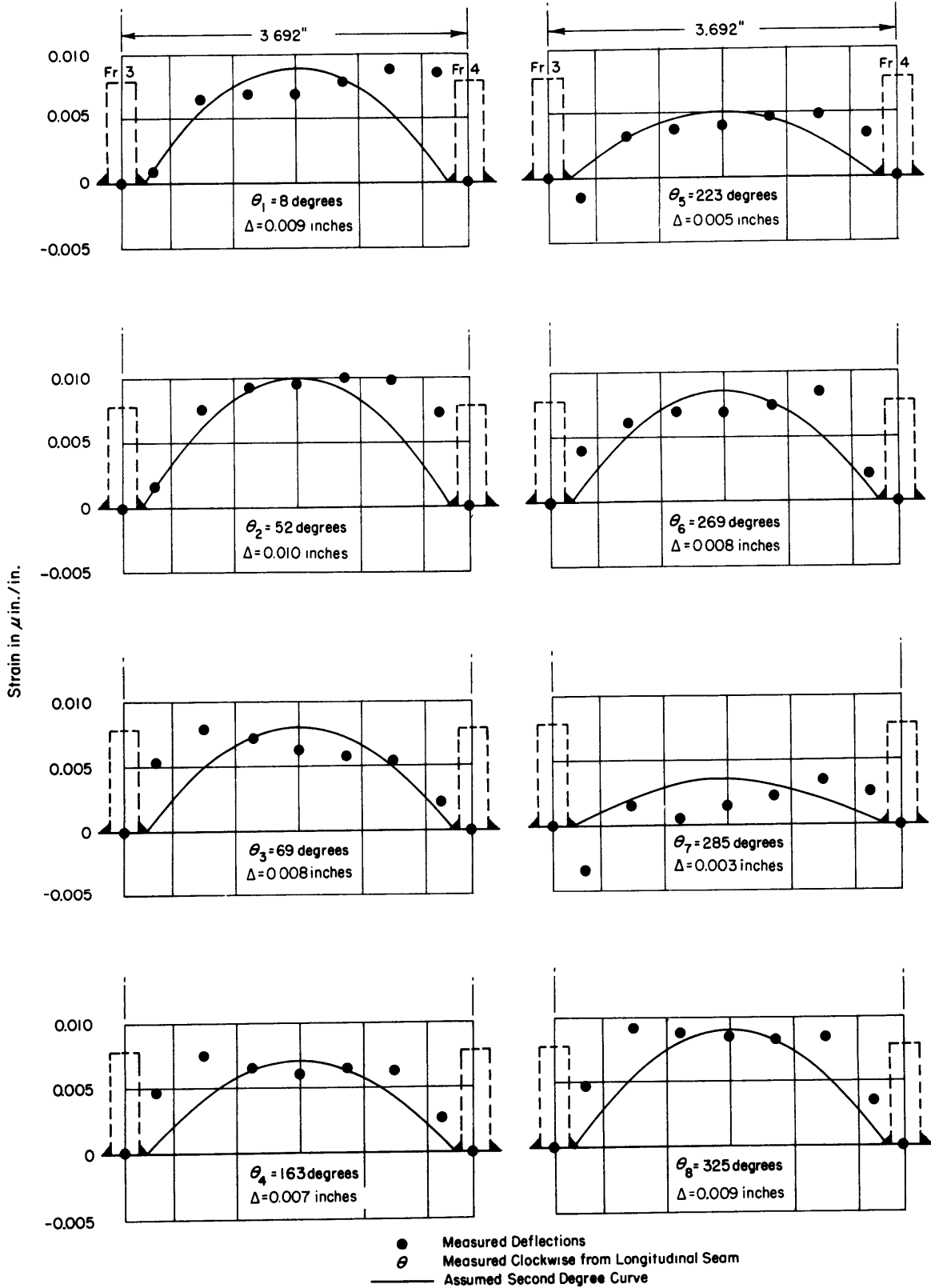


Figure 5a – Model M-1

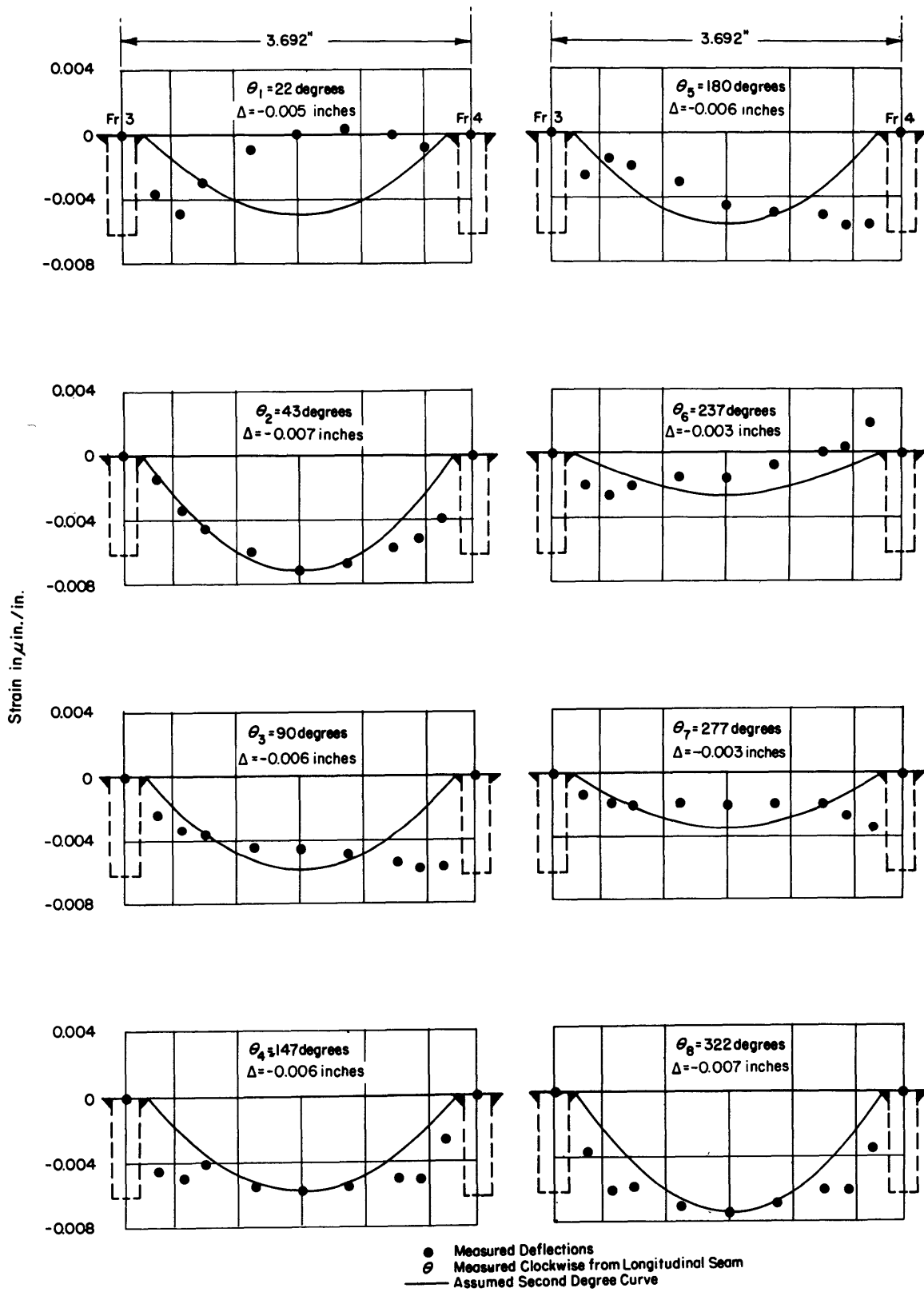


Figure 5b — Model M-2

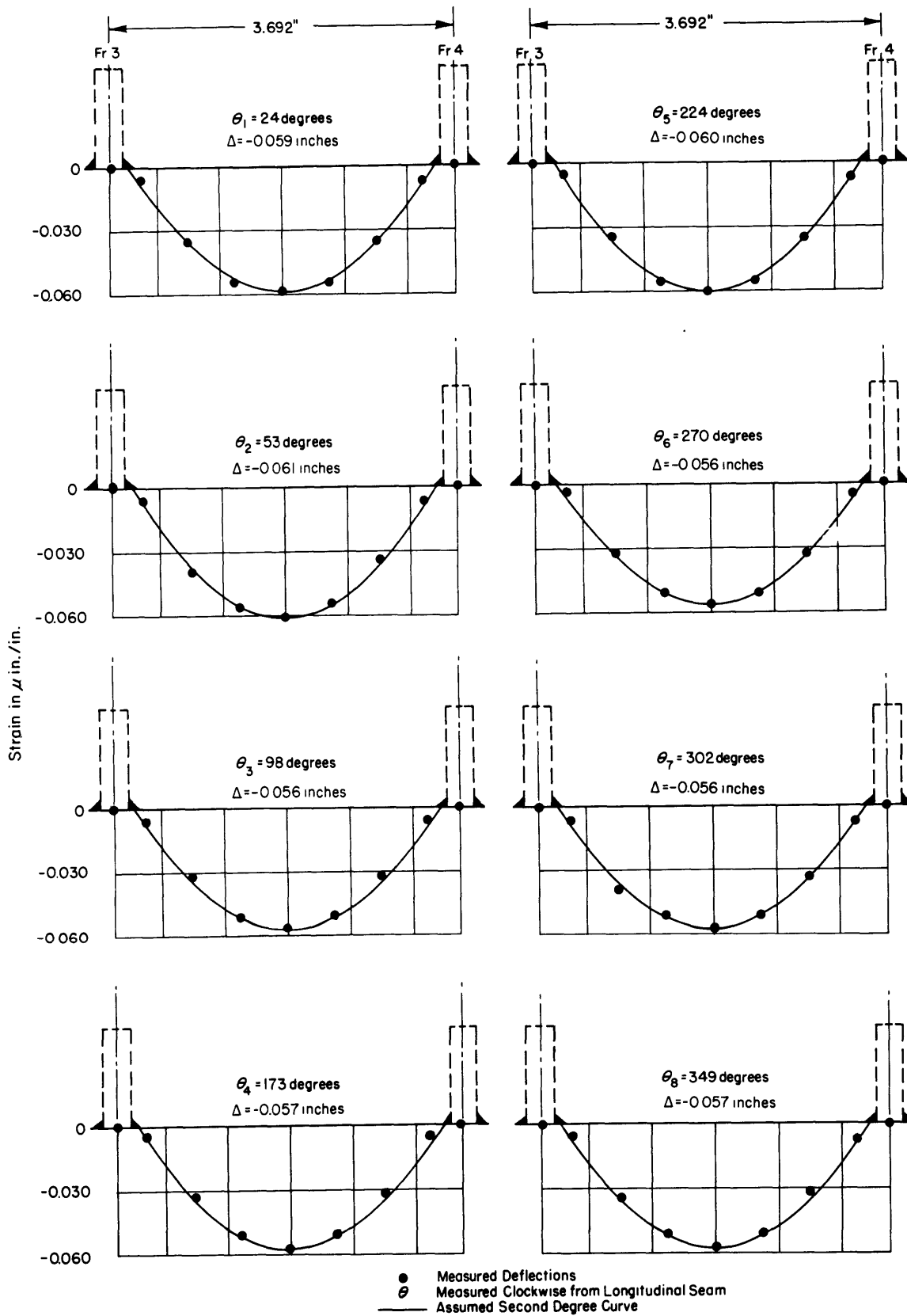


Figure 5c - Model M-3

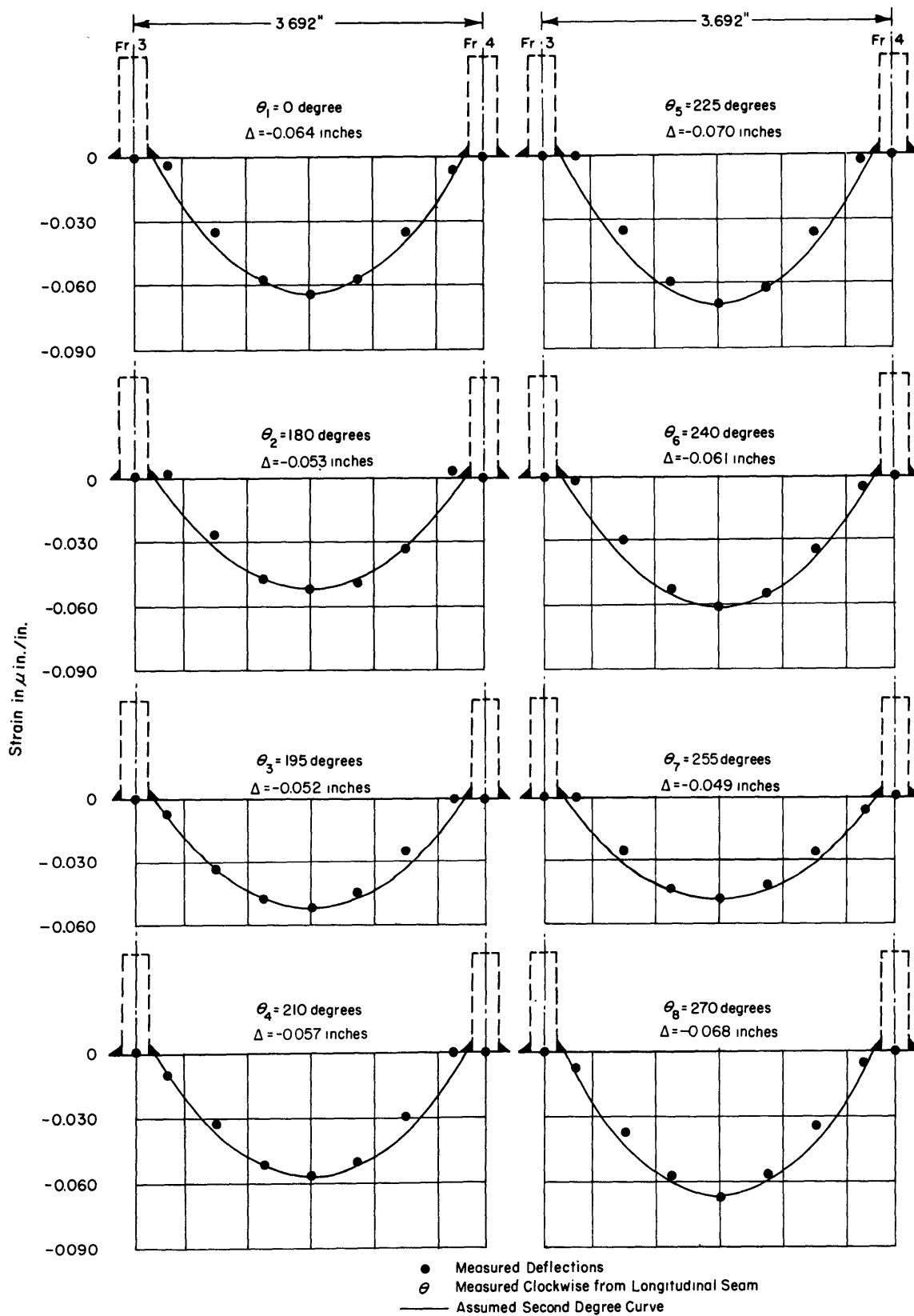


Figure 5d — Model M-4

TEST RESULTS

Models M-1, M-2, M-3, and M-4 failed at pressures of 1250 psi, 1225 psi, 1535 psi, and 1120 psi, respectively. Models M-1 and M-2 failed by the formation of inward pleats in the shell; the pleats extended for approximately 270-deg between Frames 3 and 4. Model M-3 failed by overall collapse of both the stiffeners and the shell over a 90-deg arc. Model M-4 failed by the formation of several pleats in the shell. Photographs of the collapsed models are presented in Figure 6.

Typical plots of pressure versus strain are presented in Figure 7 for Models M-1, M-2, and M-3. Except for slight nonlinearity of strains in the longitudinal weld of the shell, all data recorded from Model M-4 was linear.



Figure 6a — Model M-1 at 90-Degree Orientation

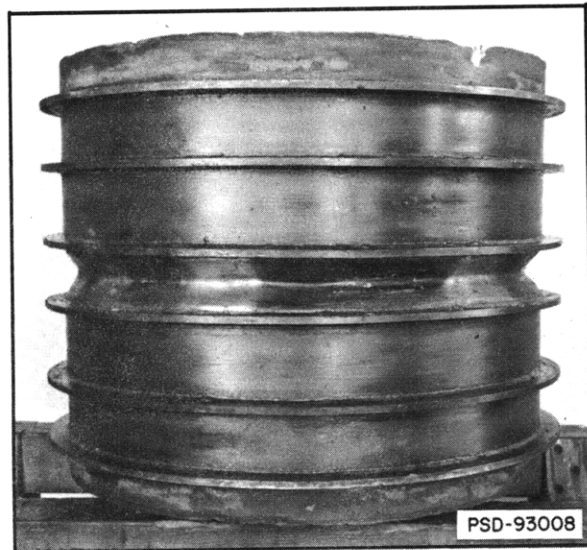


Figure 6b — Model M-2 at 210-Degree Orientation

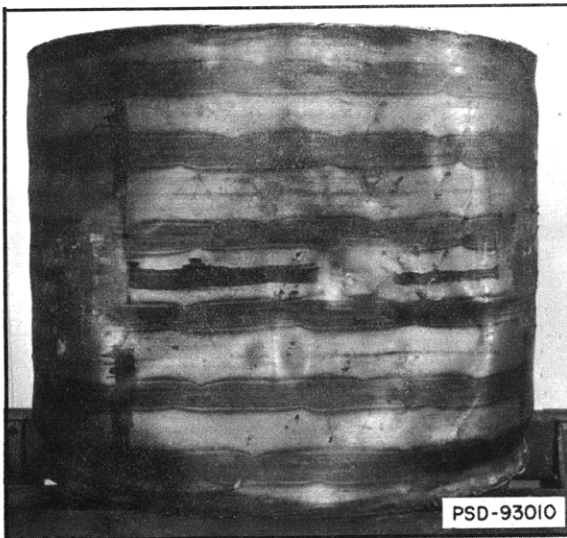


Figure 6c — Model M-3 at 210-Degree Orientation

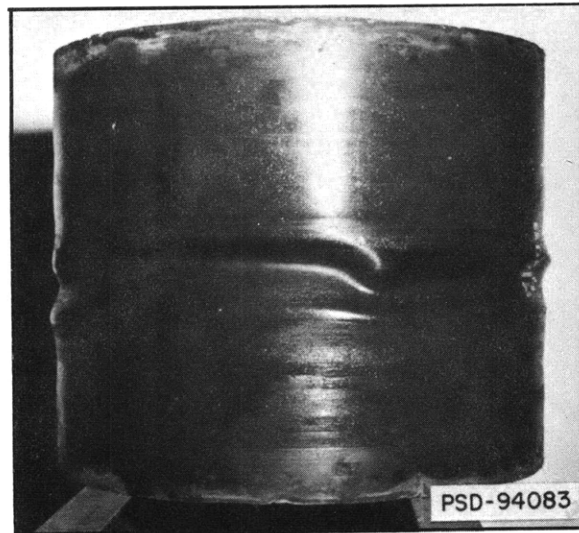


Figure 6d — Model M-4 at 110-Degree Orientation

Figure 6 — Photographs of Collapsed Models

Figure 7 – Typical Plots of Pressure Versus Strain

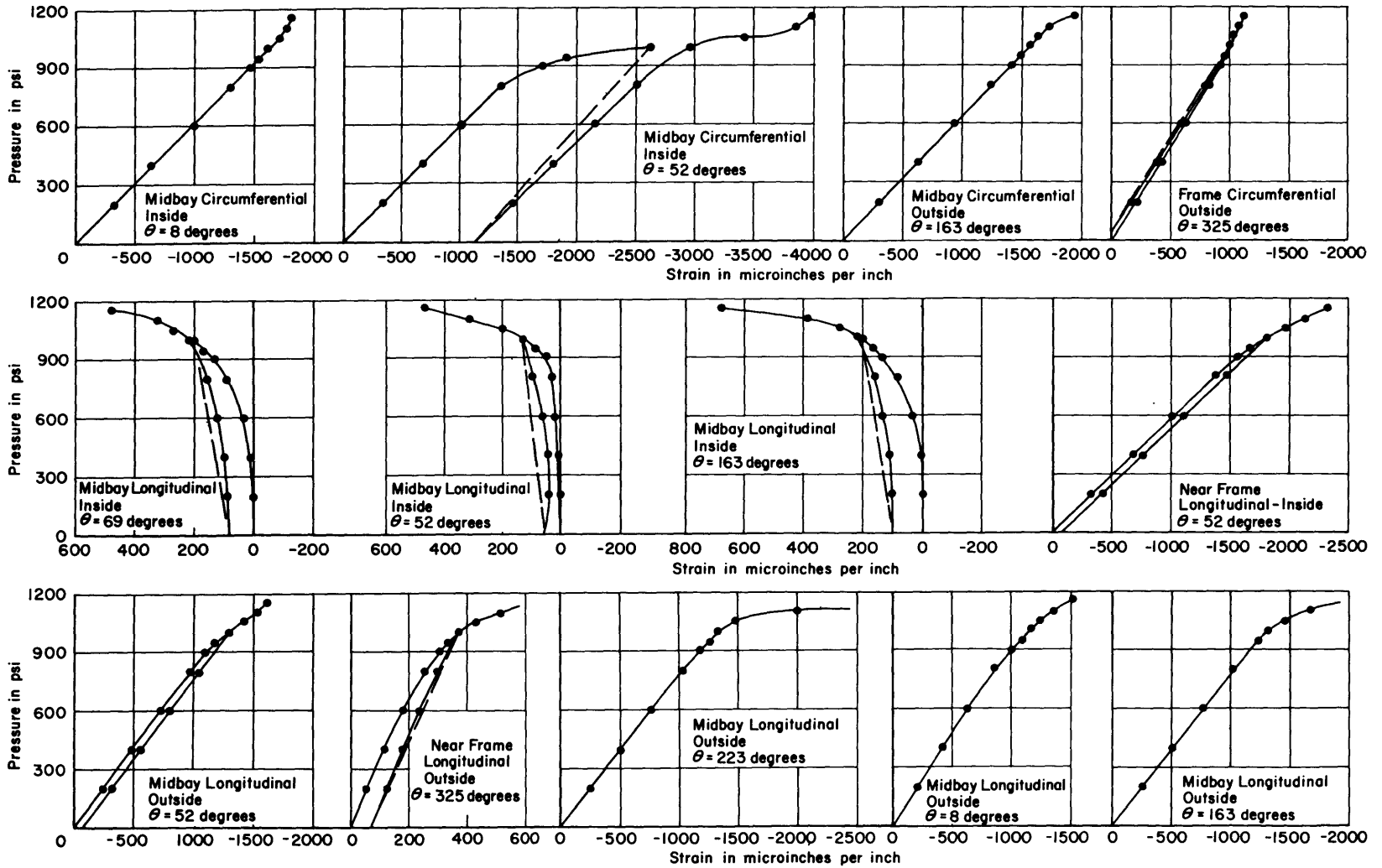


Figure 7a – Model M-1

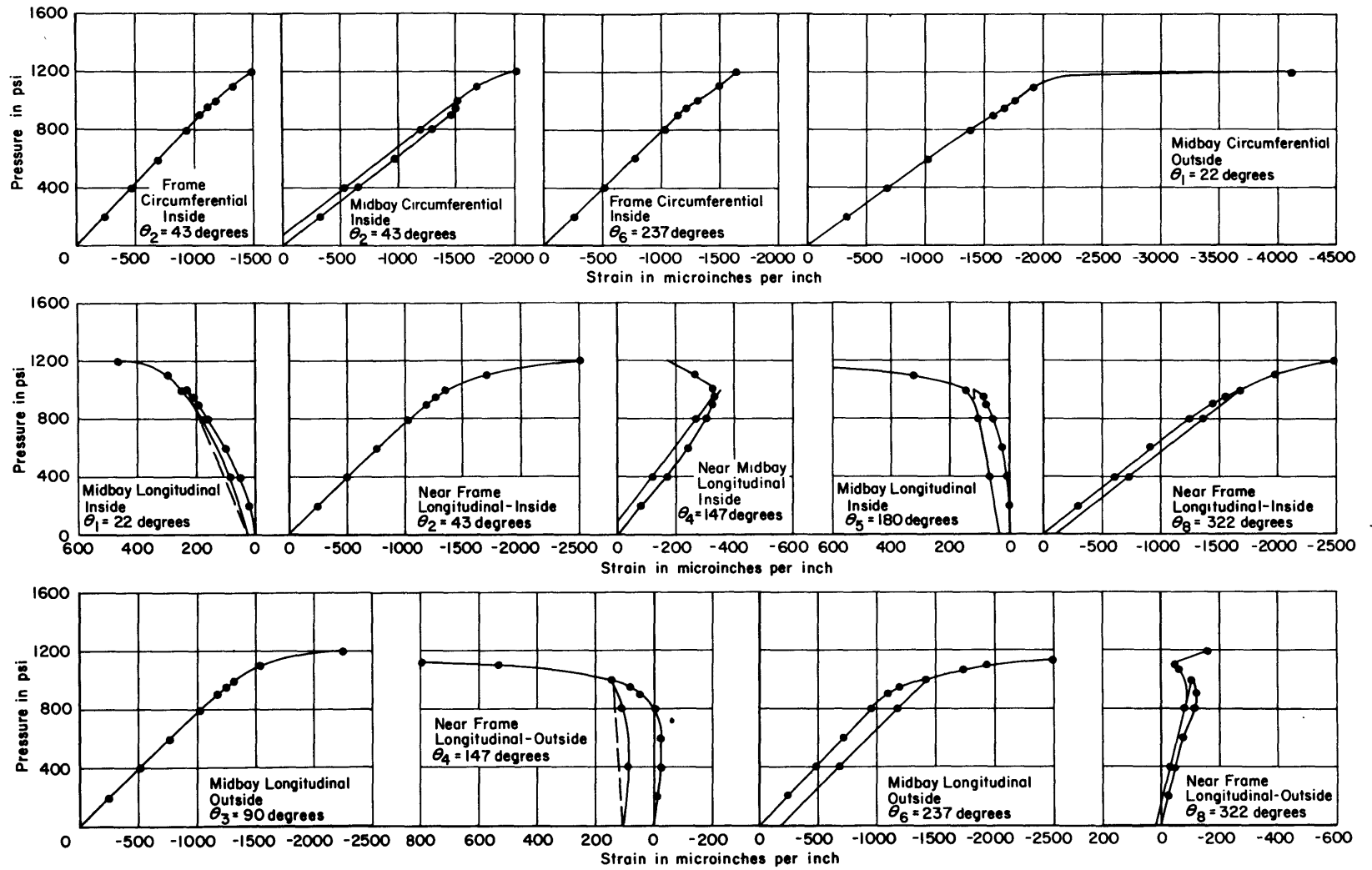


Figure 7b - Model M-2

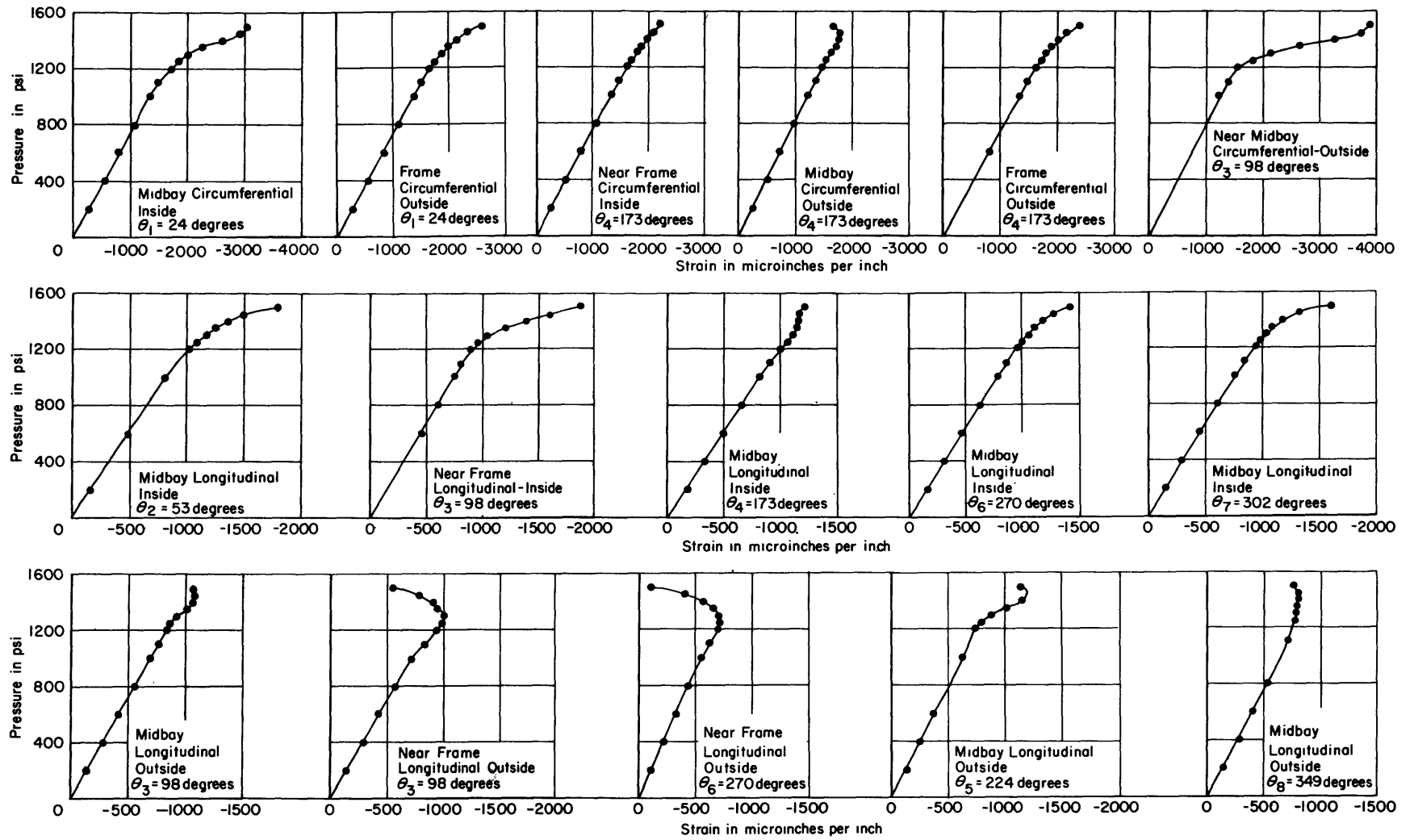


Figure 7c - Model M-3

DISCUSSION

The magnitude of the initial deflections in Models M-1 and M-2 were small and varied from zero to approximately twice the average values presented in Table 2. The initial deflection of Model M-3 was within 5 percent of 0.058 in., the deflection which will produce pure membrane stress in the shell as obtained from Equation [3] (Appendix A). The initial deflection of Model M-4 varied from 0.049 in. to 0.070 in. This nonuniformity is attributed to stress-relieving.

The shape of the initial longitudinal contour, if caused only by contractions of the welds, cannot generally be closely approximated by a second-degree curve for this particular series of models. The longitudinal contours of Models M-3 and M-4 agree well with the assumed second-degree curve because deflections were primarily introduced by machining and were large compared with deflections caused by contraction of the welds.

Measured strains are compared with theoretical strains in Figures 8 and 9. The experimental strains for Models M-3 and M-4 agreed best with strains computed by the theory of Lurchick and Short.¹ These models had initial deflections of approximately one-third of a shell thickness; therefore, the theories of Von Sanden and Gunther³ and Salerno and Pulos,⁴ in which these deflections are ignored, cannot be expected to predict their behavior accurately. There was very little difference in the theoretical strains obtained from these three analyses for Models M-1 and M-2, since their initial deflections were relatively small. It is noted that for many of the circumferential gages the measured strains were lower than those predicted by any of the theories. Later comparisons of strains measured by metaelectric and optical strain gages on a compression specimen with similar smoothness of surface showed a tendency toward lower readings from the metaelectric gages, indicating a need for extreme care when applying these gages to exceptionally smooth surfaces.

The highest attainable collapse pressure for a given stiffened cylinder which fails by yielding occurs when the stresses are constant throughout the shell. An initial deflection which produces this condition of membrane stresses is described in Appendix A. Expressions for the stresses in the shell for this condition are given in Equations [4] and [10] (Appendix B). Model M-4 was a stress-free stiffened cylinder whose shape approximated the shape required to eliminate bending and whose mode of collapse was by yielding of the shell material. Table 3 indicates that its collapse strength may be accurately calculated on the basis of the Hencky-Von Mises theory of yielding.⁵

At least two distinct features can cause internally stiffened cylinders, fabricated by welding, to collapse at pressures below those at which machined, stress-free, stiffened cylinders and externally stiffened, welded-cylinders of the same geometry collapse. They are:

1. The initial inward deflection which produces additional moments resulting from the end load; these moments increase the magnitude of the maximum elastic stresses present in a perfect cylinder, whereas an initial outward deflection reduces the maximum elastic stresses in the shell.

TABLE 3

Comparison of Theoretical Yield Pressures and Experimental Collapse Pressure of Model M-4

Theory of Failure	Design Equation	Ratio of Theoretical Membrane Yield Pressures to Experimental Collapse Pressure
Maximum Principal Stress	$\sigma = \sigma_{\max}$	0.90
Maximum Shear Stress	$\sigma = \sigma_{\max} - \sigma_{\min}$	0.90
Maximum Strain	$\sigma = \sigma_1 - \nu \sigma_2$	1.16
Maximum Total Energy	$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\nu\sigma_1\sigma_2}$	0.87
Hencky-Von Mises	$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2}$	1.02

2. Residual stresses which are produced in the shell to balance the forces produced by the contraction of the welds (and their heat-affected zone) upon cooling.

The residual membrane stresses produced at midbay are not the same for internally and externally framed cylinders. For internally framed cylinders, the moments produced by the contraction of the welds at the frames bow the shell in between frames, thus leaving a residual compressive stress in the circumferential direction which is a maximum at midbay. For externally framed cylinders, the initial deflection is outward; therefore, the circumferential residual stress at midbay is a minimum.

The presence of residual stresses, which are superimposed on the elastic stresses produced by hydrostatic loading, causes internally framed, welded cylinders to yield initially at pressures below those at which externally framed, welded cylinders or machined cylinders yield.

There appears to be no general method of rigorously predicting the effect of initial deflections and residual stresses on the collapse strength. As stated previously, the shape and magnitude of the initial deflection are functions of the welding procedure as well as geometric characteristics. Each cylinder would have to be analyzed separately to predict the effect of these initial conditions on the collapse strength.

One method of evaluating the effect of initial deflections and residual stresses for this series of tests is presented in Table 4, where the experimental collapse pressures are compared with the maximum pressures obtainable by the Hencky-Von Mises yield criterion.

An initial outward deflection resulting only in membrane stresses produced an increase in collapse pressure of 21 percent; compare the pressure ratios for Models M-1 and M-3. It is assumed that the residual stress patterns for both models were the same because identical welding procedures were used.

(Text continued on page 31.)

Figure 8 – Comparison of Experimental and Theoretical Elastic Strains at Midbay

The abbreviation S&P stands for Salerno and Pulos; S&G for Von Sanden and Gunther; L & S, for Lunchick and Short.

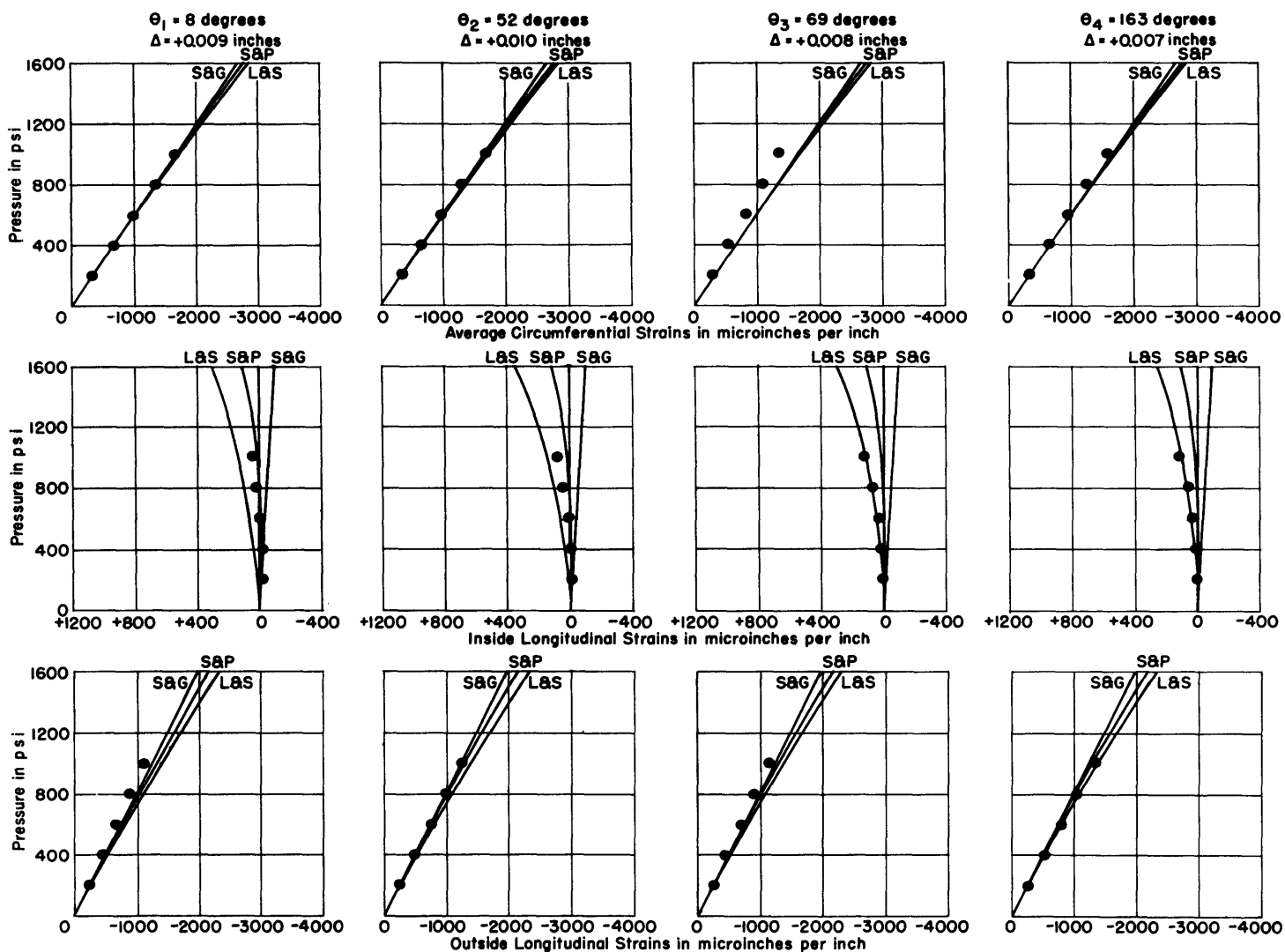


Figure 8a – Model M-1

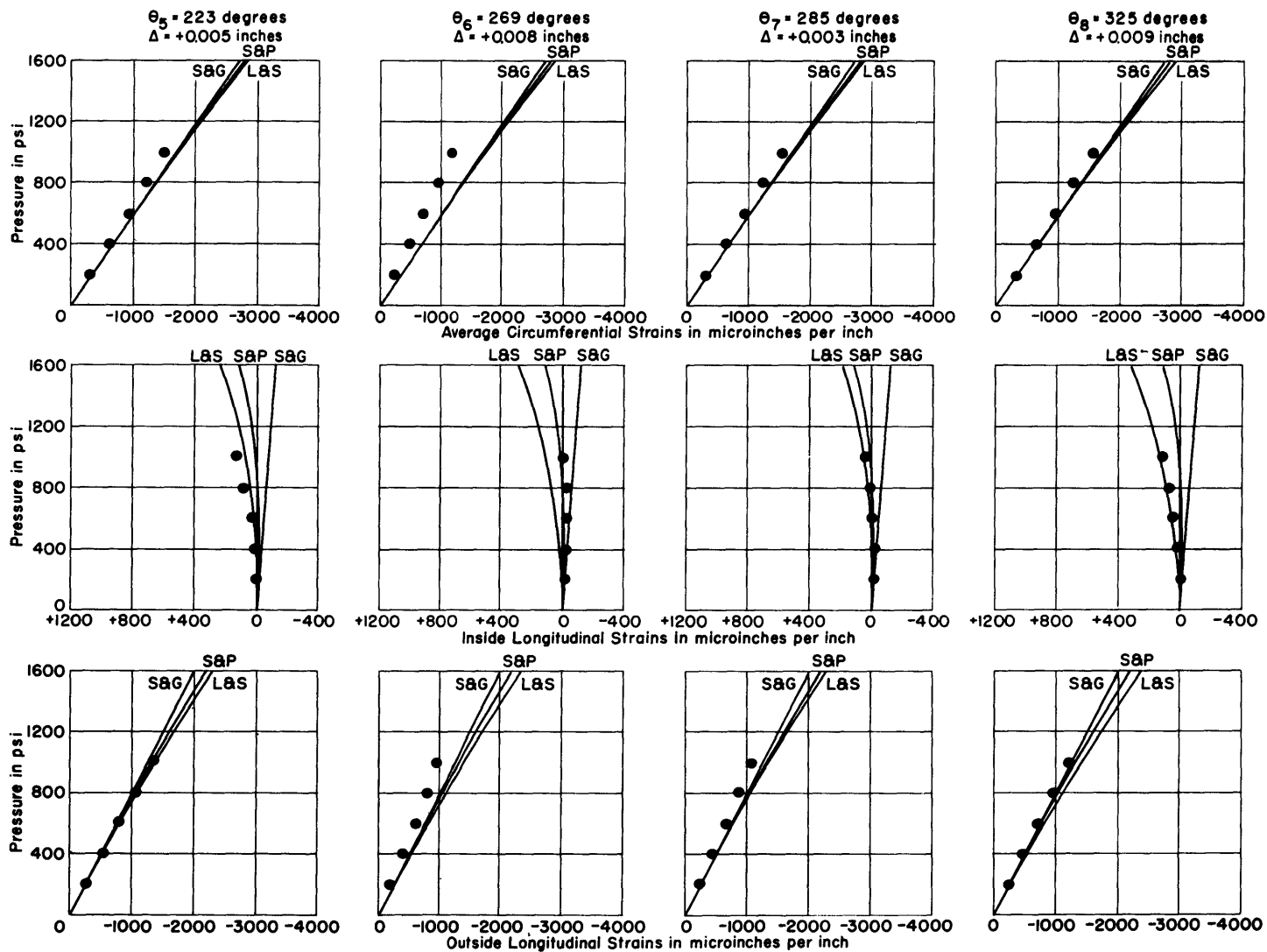


Figure 8a - Model M-1 (Continued)

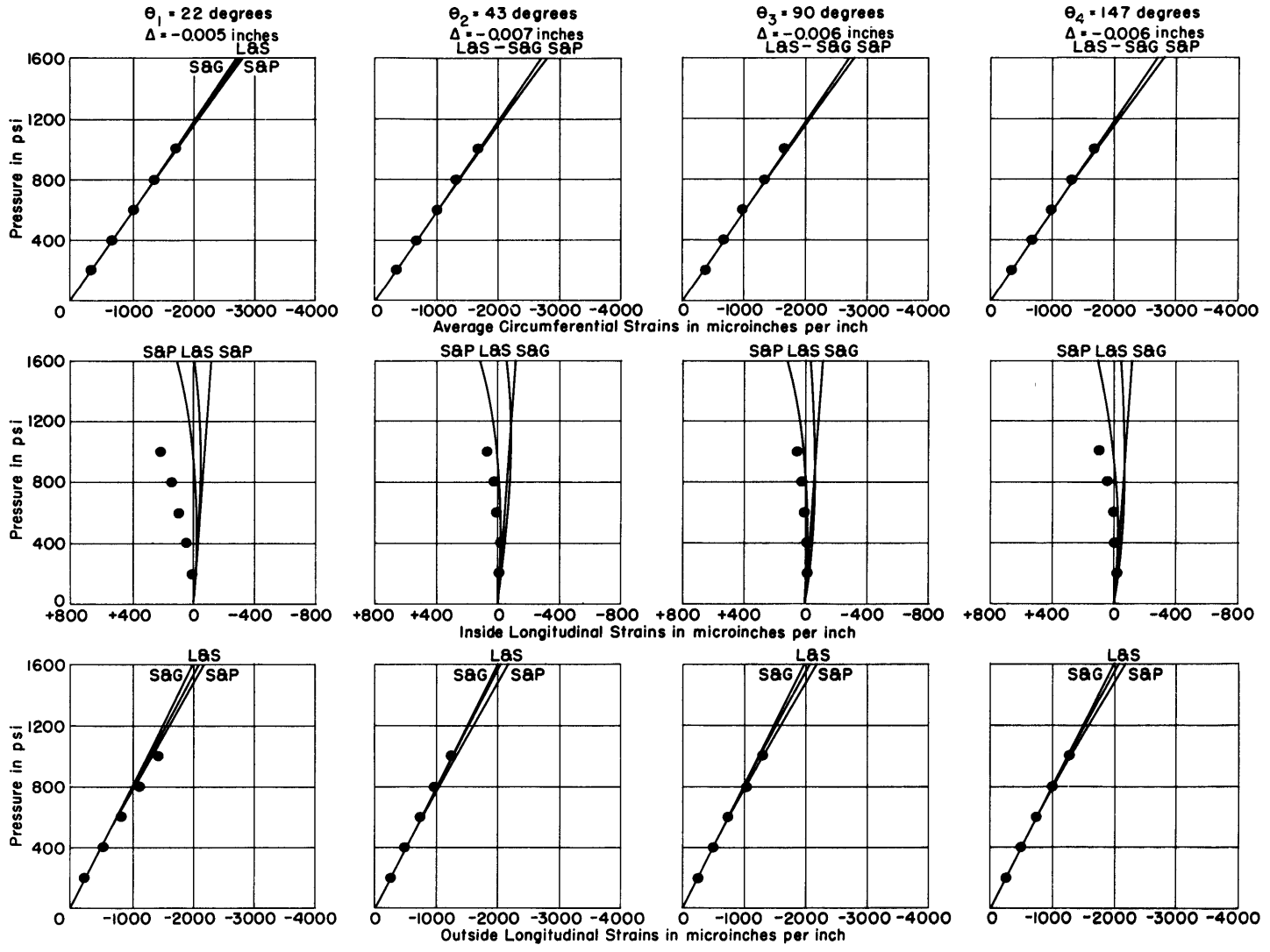


Figure 8b – Model M-2

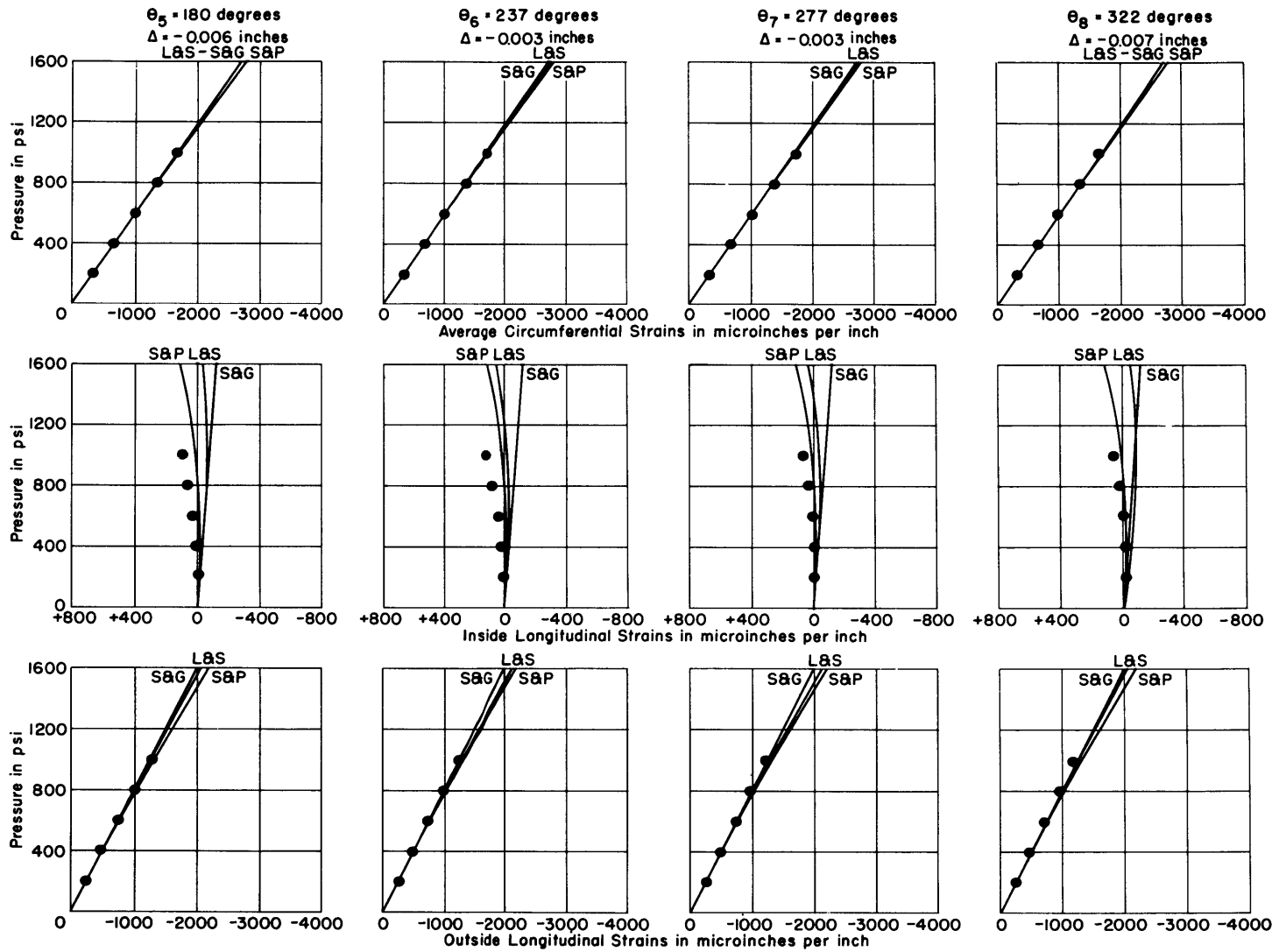


Figure 8b – Model M-2 (Continued)

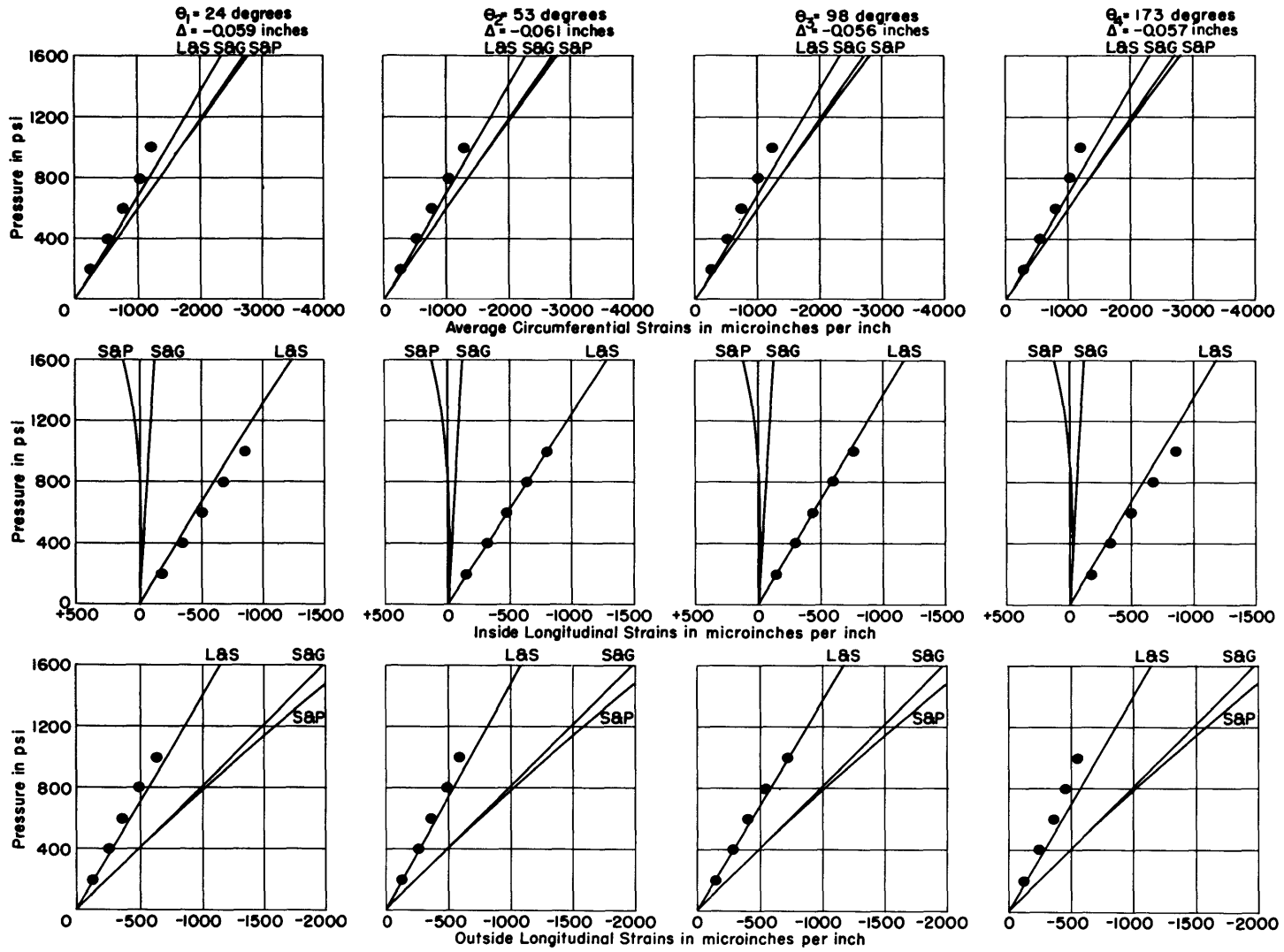


Figure 8c — Model M-3

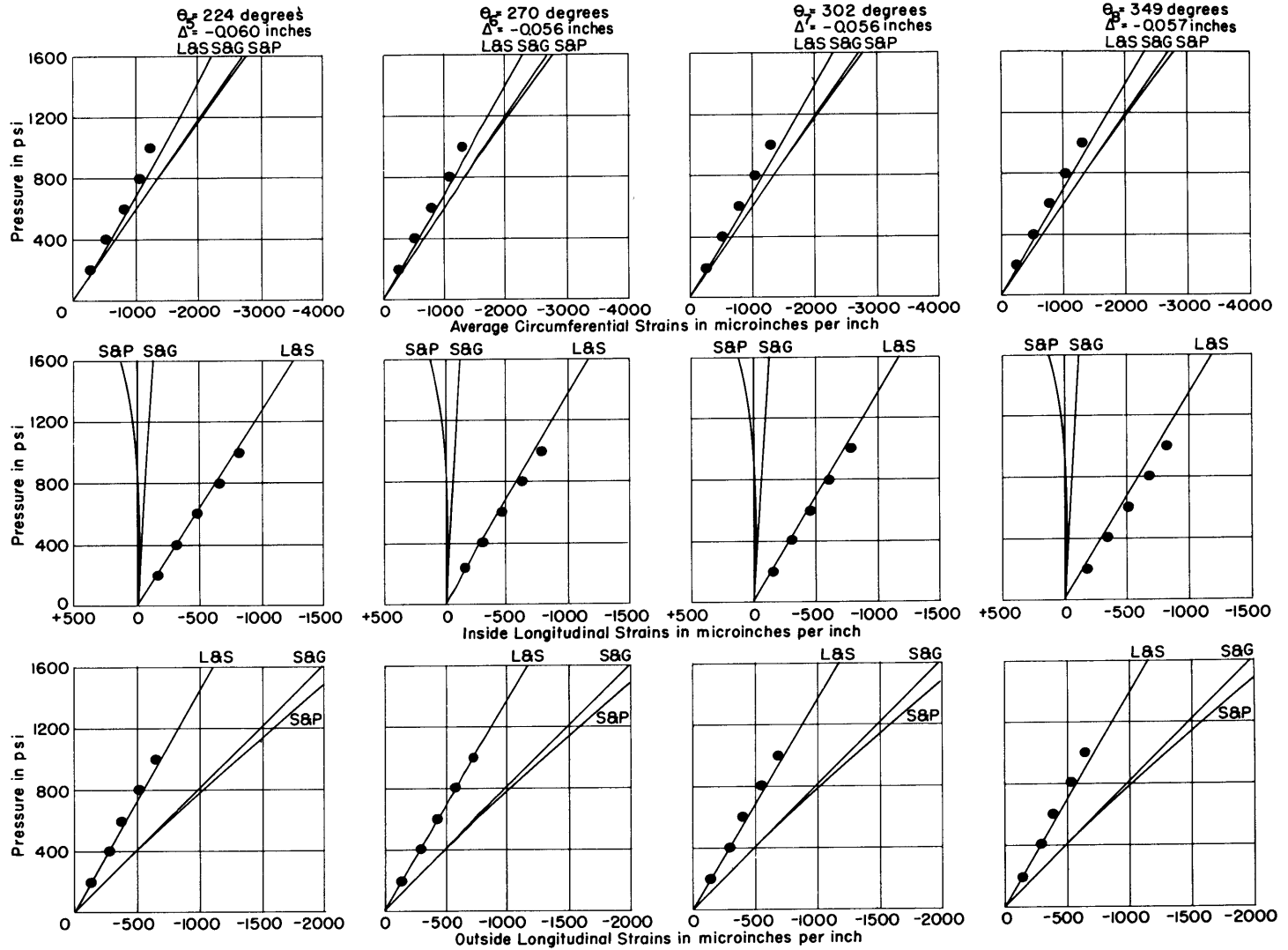


Figure 8c – Model M-3 (Continued)

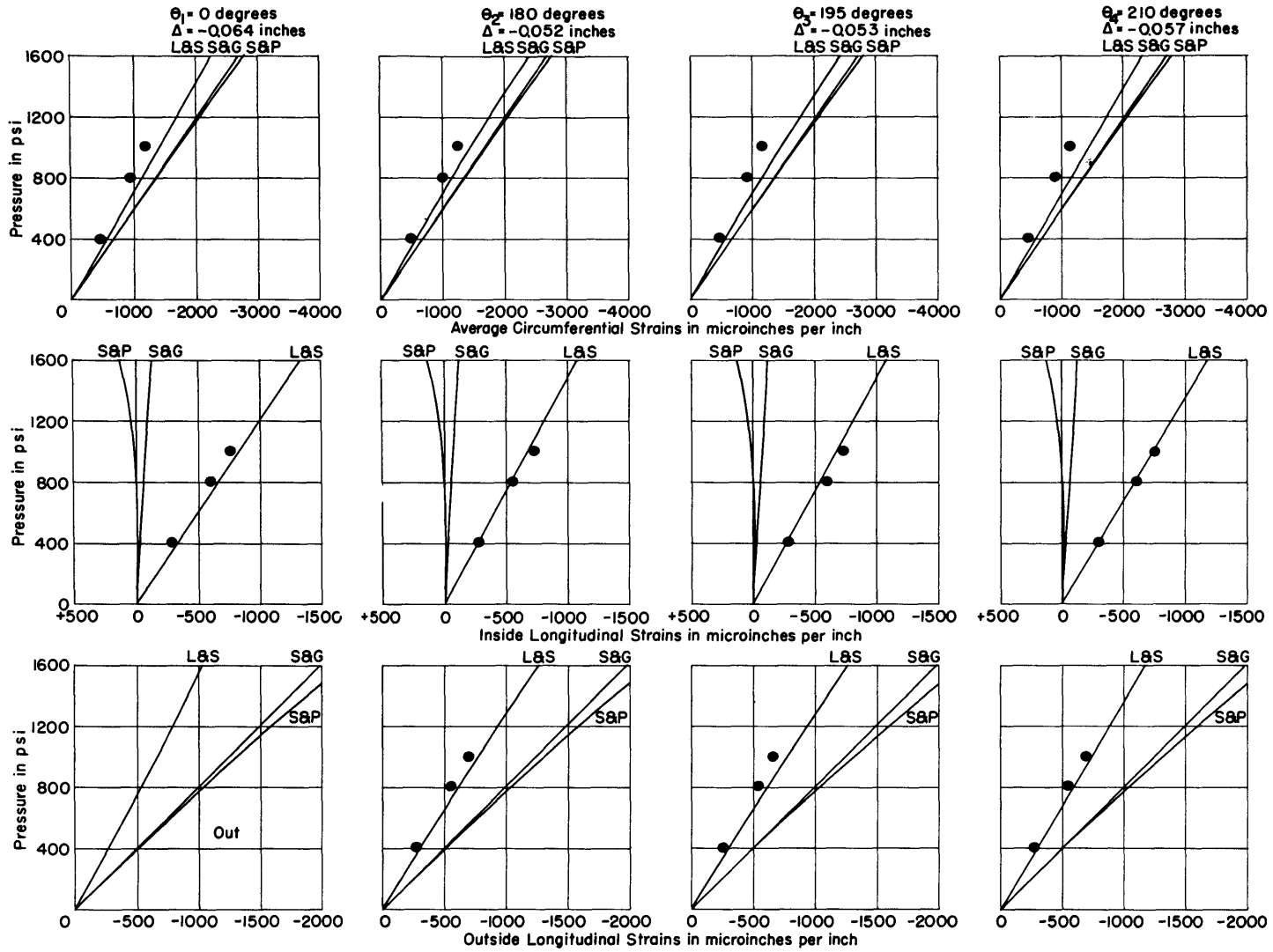


Figure 8d — Model M-4

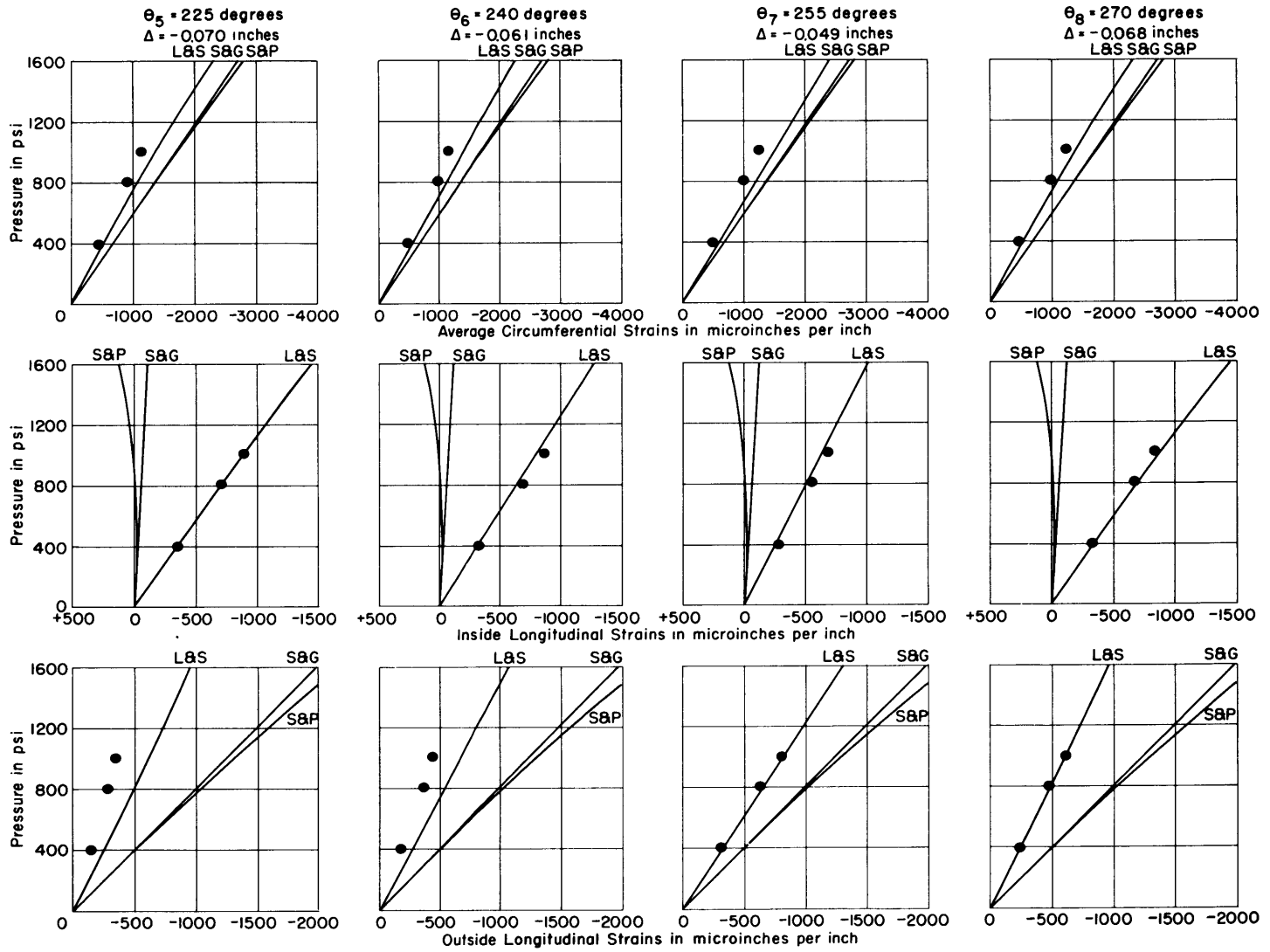


Figure 8d - Model M-4 (Continued)

Figure 9 – Comparison of Experimental and Theoretical Elastic Strain Distributions at 1000 PSI

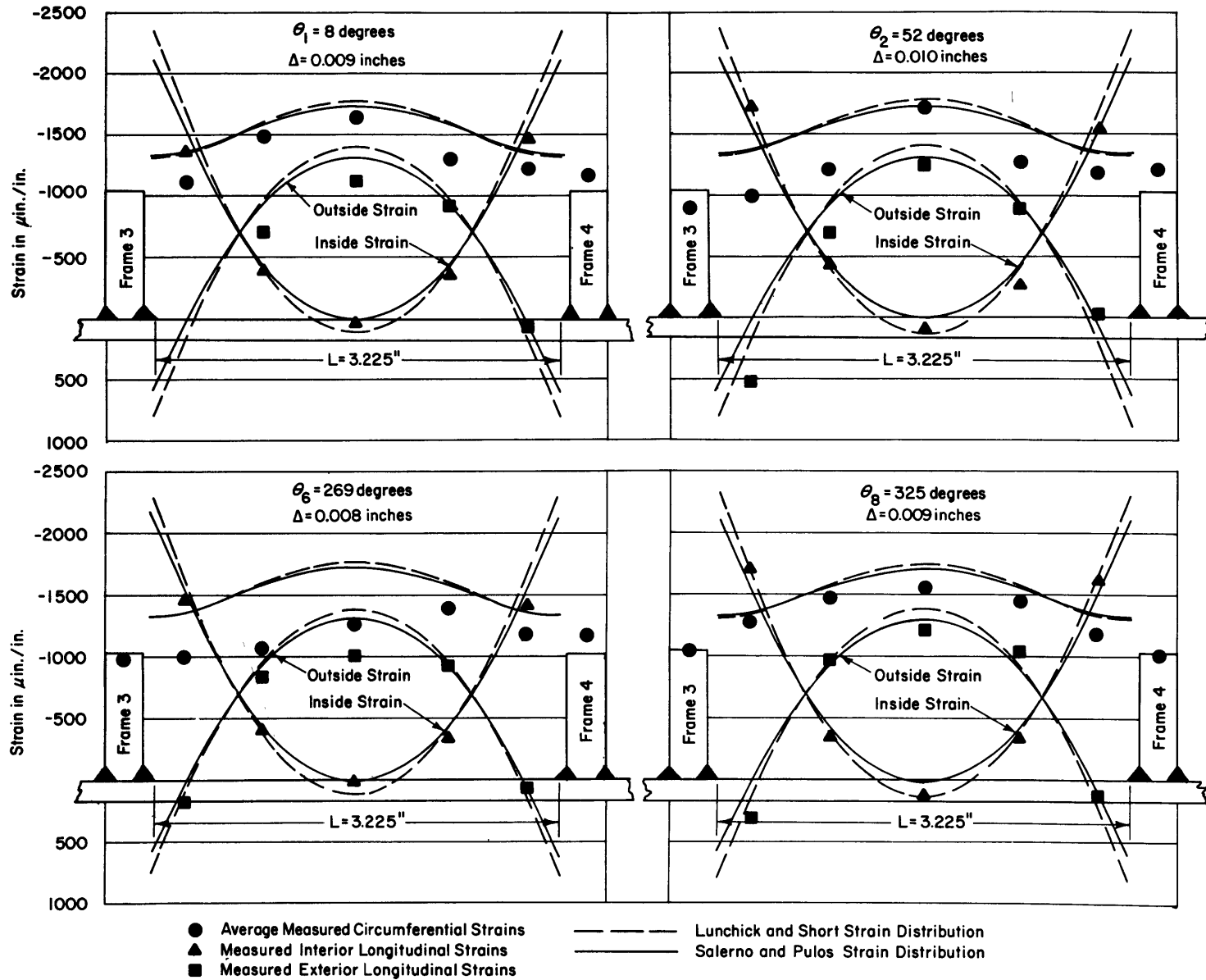


Figure 9a – Model M-1

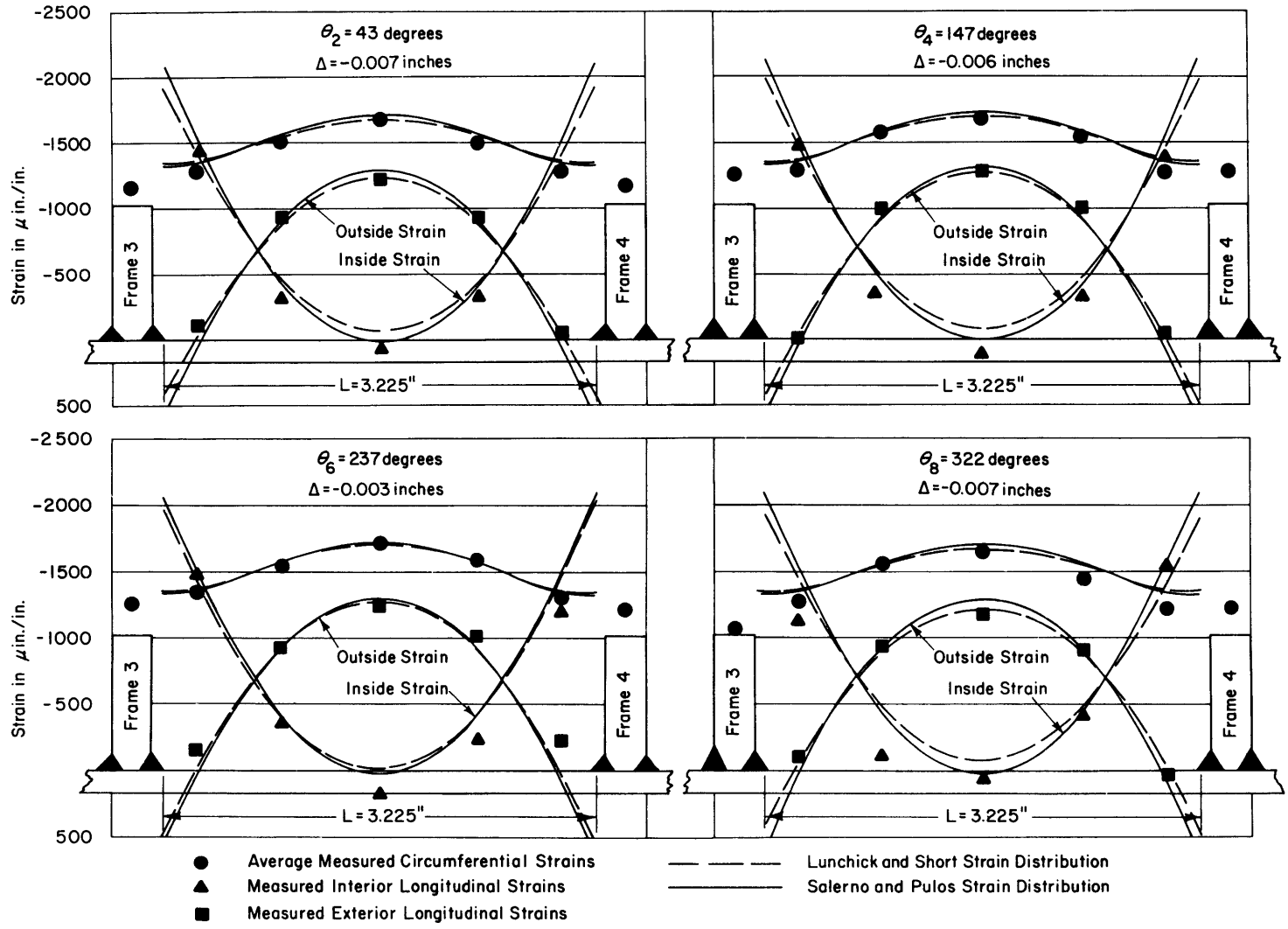


Figure 9b – Model M-2

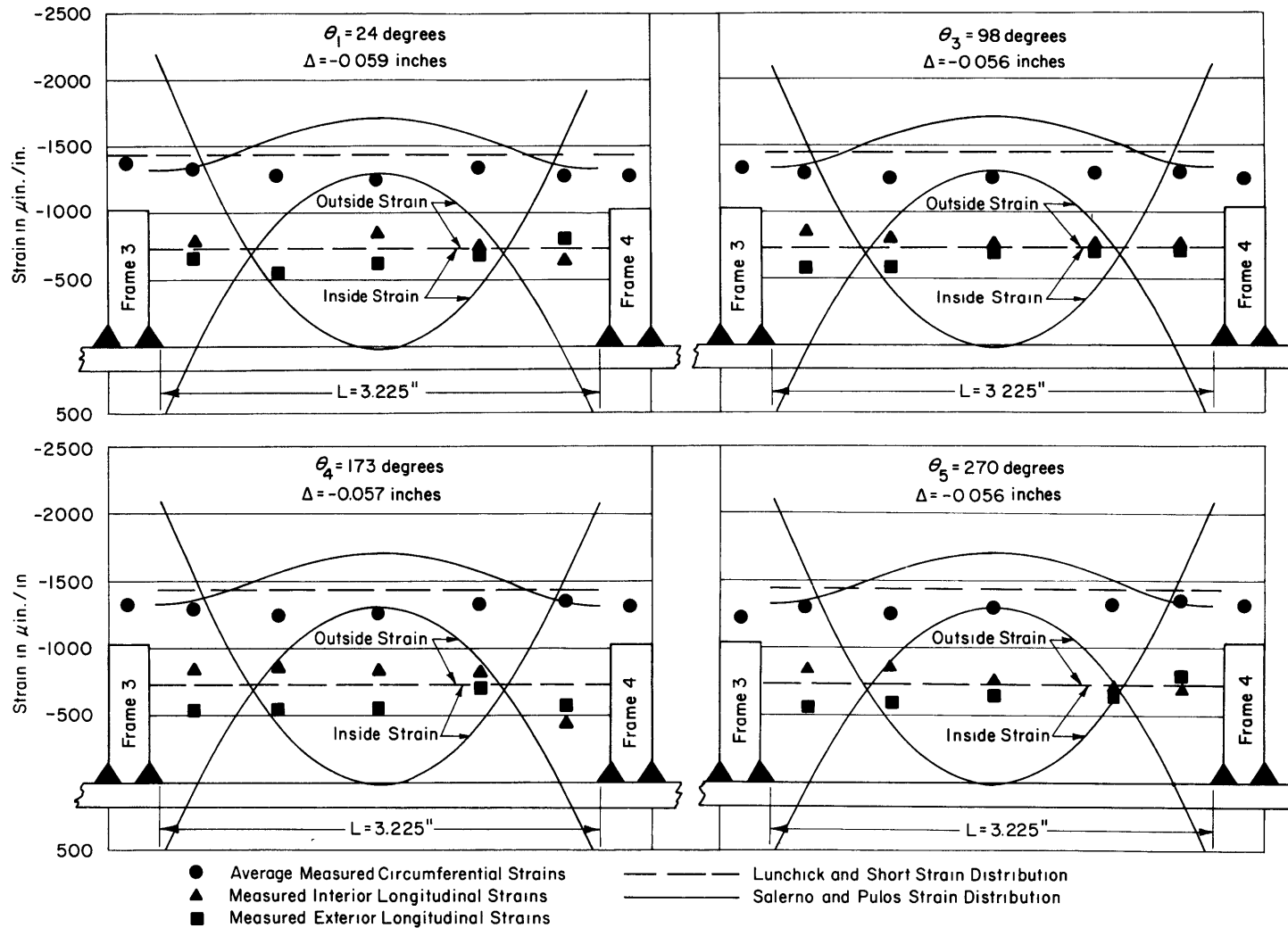


Figure 9c – Model M-3

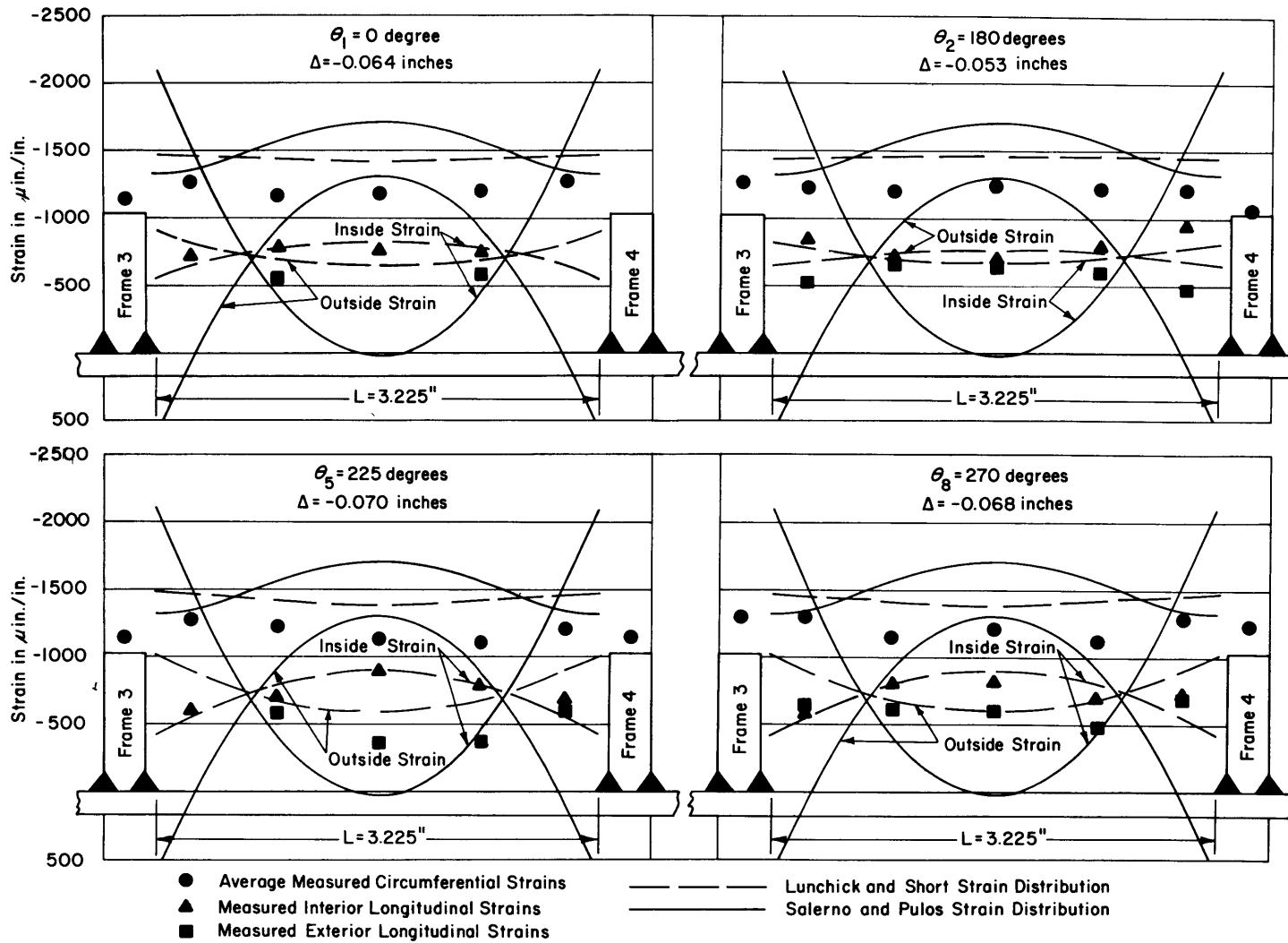


Figure 9d - Model M-4

TABLE 4

Comparison of Experimental Collapse Pressure with Hencky-Von Mises Yield Pressure for Membrane Shape

Models	M-1	M-2	M-3	M-4
Average Measured Shell Thickness, in.	0.169	0.169	0.172	0.167
Yield Strength of Shell Material, psi	82,500	78,350	82,500	55,850
Experimental Collapse Pressure, psi	1,250	1,225	1,535	1,120
Hencky-Von Mises Yield Pressure for Membrane Shape, psi	1,702	1,584	1,726	1,149
Ratio of Experimental Pressure to Pressure for Membrane Shape	0.734	0.773	0.890	0.975

Eliminating residual stresses resulted in a further increase, which is shown by comparing the pressure ratios of Models M-3 and M-4. Model M-4, which was stress-free, had a pressure ratio 10 percent higher than that of the similar Model M-3. Unfortunately, since the yield strength of Model M-4 was reduced appreciably during the stress-relieving operation, there is some doubt about the exact magnitude of this increase.

One method of evaluating the effect of placing stiffeners either internally or externally is based on the initial measurements presented in Table 2. Assuming that the effects of initial deflections and residual stresses are roughly proportional to the average measured deflections leads to the following observations:

1. The difference in deflections at midbay relative to those at the frames of Models M-2 and M-1 increases the strength of Model M-2 about 3 percent over that of Model M-1.
2. The smaller residual deflection (decrease in radius at midbay after welding) of Model M-2 increases its strength about 4 percent over that of Model M-1.

Adding these two corrections indicates that Model M-2 is 7 percent stronger than Model M-1. This is very close to the observed 5-percent difference in the ratios of collapse pressure to membrane pressure as given in Table 4.

Table 5 presents the ratio of the theoretical collapse pressures to experimental collapse pressures for this series of tests and for machined cylinders previously tested at the Model Basin.^{6,7} These additional models also failed by yielding of the shell material. The purpose of presenting the results of these models is only to support the following statements, which are based on the results of the series of tests discussed in this report.

1. Von Sanden and Gunther's Formulas [92] and [92A] give collapse pressures which are consistently below the experimental collapse pressures of initially perfect stiffened cylinders.

TABLE 5

Ratio of Theoretical Collapse Pressures to Experimental Collapse Pressures

Theory of Collapse	Experimental Models					
	M-1	M-2	M-3	M-4	EB-11 and EB-12*	BR-7M
Von Sanden and Gunther Formula [92]	0.84	0.81	0.69	0.63	0.88	0.82
Von Sanden and Gunther Formula [92A]	0.96	0.93	0.80	0.72	0.85	0.88
Salerno and Pulos Hencky-Von Mises Midbay-Outside	1.00	0.96	0.82	0.74	0.92	0.93
Salerno and Pulos Hencky-Von Mises Midbay-Midplane	1.20	1.16	0.99	0.89	1.02	1.04
Lunchick and Short Hencky-Von Mises Midbay-Midplane	1.18	1.17	1.12	1.02	1.02	1.04
Hodge Rigid Plastic	1.11	1.08	0.92	0.84	1.00	0.98
Paul Elastic Plastic	1.06	1.03	0.88	0.81	0.98	0.96
Lunchick Plastic Hinge	1.12	1.09	0.93	0.85	0.99	1.01
Modified Lunchick Plastic Hinge	0.96	1.00	1.00	1.02	0.99	1.01
*Average results of two identical models.						

2. A theory of collapse must be used which considers initial deflections, residual stresses, the correct yield criterion, and plastic reserve strength, in order to predict accurately the collapse strength of stiffened cylinders which fail by shell yielding.

Formulas [92] and [92A] of Von Sanden and Gunther give collapse pressures below the experimental collapse pressure for all models considered in Table 5. The pressure at which yielding first occurs on the outside surface at midbay, according to the Hencky-Von Mises criterion in conjunction with the Salerno and Pulos analysis, is also below the experimental collapse pressures for the machined, perfectly cylindrical models. This shows that stiffened cylinders which fail by yielding of the shell do not collapse when yielding first occurs.

The Salerno and Pulos analysis gives collapse pressures which are very close to the experimental collapse pressures of the machined, perfect cylinders when applying the Hencky-Von Mises yield criterion to the elastic stresses at midbay and midplane. This indicates that a plastic reserve strength is present when initial yielding takes place. The collapse pressures obtained from the plastic analyses of Lunchick,⁸ Hodge,⁹ and Paul¹⁰ are also in good agreement with the observed collapse pressures.

Results from an empirical method of modifying the plastic-hinge analysis⁸ to account for the initial deflections and the residual stresses are also presented in Table 5. All modifications are based on the average measured values of midbay deflections presented in Table 2. The analysis was simply adjusted for the elastic stress distribution as predicted by the

Lunchick and Short analysis¹ for a cylinder with an initial deflection. It is also assumed that the effect of residual stresses on the collapse strength is proportional to the residual midbay deflections, and that the effect of residual stresses on the collapse strength of Model M-3, which was designed to eliminate bending, is known. Therefore, the experimental collapse pressures were adjusted in the following manner:

$$P_m = P_{a-i} - \frac{\Delta_i}{\Delta_3} [P_{t-3} - P_{c-3}] \quad [1]$$

where P_m is the modified plastic-hinge collapse pressure,
 P_{a-i} is the Lunchick plastic-hinge pressure based on the Lunchick and Short elastic analysis,
 P_{t-3} is the theoretical membrane yield pressure of Model M-3 based on the Hencky-Von Mises yield criterion, 1726 psi,
 P_{c-3} is the experimental collapse pressure of Model M-3, 1535 psi,
 Δ_i is the average initial residual deflection at midbay from Table 2, and
 Δ_3 is the average initial residual deflection at midbay of Model M-3, + 0.007 in.

This analysis is presented in an effort to demonstrate that these factors (initial relative deflection between frames and midbay and initial residual deflection at midbay resulting from welding), along with plastic reserve strength, must be considered to predict accurately the collapse pressure of stiffened cylinders fabricated by welding. It will be seen from Table 5 that when corrections are made for the above-named factors the modified plastic-hinge analysis gives calculated pressures that agree with observed pressures within 4 percent for this series of models.

CONCLUSIONS

The following facts can be concluded for stiffened cylinders subjected to external pressure which fail by axisymmetric yielding of the shell material.

1. The analysis of Lunchick and Short adequately predicts the elastic behavior of a stiffened cylinder with initial deflections between frames which may be approximated by a second-degree curve.
2. The collapse strength may be influenced appreciably by the presence of initial deflections between stiffeners; inward deflections decrease the strength whereas outward deflections increase the strength.
3. The residual stresses in an internally stiffened cylinder fabricated by welding lower the collapse strength.
4. For the material used in these tests, yielding of the shell material may be accurately predicted by applying the Hencky-Von Mises yield criterion.

5. Collapse of an initially perfect cylinder is not associated with that pressure at which yielding first occurs.

6. An initial deflection between frames which eliminates bending produces the maximum obtainable strength for a given stiffened cylinder.

7. For this series of tests, the Lurchick plastic-hinge analysis is in agreement with experimental results when modified to consider initial deflections and residual stresses.

ACKNOWLEDGMENTS

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APPENDIX A

A METHOD OF PRODUCING PURE MEMBRANE STRESSES IN A STIFFENED CYLINDER

The general expression developed by Lurchick and Short¹ for the deflection of a closed stiffened cylinder subjected to hydrostatic pressure and having an initial axisymmetric deflection between stiffeners which may be approximated by a second-degree curve is

$$W = \frac{B^2}{R} (2 - \nu) - \frac{2\alpha^2 B^2}{H} \left[\frac{A_f \left(1 - \frac{\nu}{2}\right)}{Rh} \right] f(x) \quad [2]$$

$$+ \frac{8B^2 \Delta}{L^2} - \frac{8B^2 \Delta \alpha^2}{L^2 H} \left[\frac{A_f + bh + Lh}{h} \right] f(x)$$

where W is the radial deflection,

H is a deflection coefficient defined in Reference 1,

x is the coordinate measured along a generator from an *origin* located at a frame,

Δ is the maximum initial deflection of the shell,

$$B^2 = \frac{PR^3}{2Eh}, \text{ and}$$

$$\alpha^4 = \frac{3(1 - \nu^2)}{R^2 h^2}$$

Equating the second term on the right-hand side of Equation [2] to the negative value of the fourth term and solving for Δ produces

$$-\Delta = \frac{L^2}{4R} \left[\frac{A_f \left(1 - \frac{\nu}{2}\right)}{A_f + hL_f} \right] \quad [3]$$

This is the initial deflection which produces pure membrane stresses throughout the shell within the limitations of the assumptions used in the theory.¹

APPENDIX B

DERIVATION OF STRESS-STRAIN RELATIONSHIP IN A STIFFENED CYLINDER WITH AN INITIAL DEFLECTION WHICH ELIMINATES BENDING

The longitudinal stresses σ_x and the circumferential stresses σ_ϕ in a stiffened cylinder with an initial deflection between frames which eliminates bending may be expressed as follows:

$$\sigma_{x_{\text{shell}}} = \frac{PR}{2h}, \quad [4]$$

$$\sigma_{x_{\text{frame}}} = 0, \quad [5]$$

$$\sigma_{\phi_{\text{shell}}} = E\epsilon_\phi + \nu\sigma_{x_{\text{shell}}}, \quad [6]$$

where ϵ_ϕ is the circumferential strain, and

$$\sigma_{\phi_{\text{frame}}} = \sigma_{\phi_{\text{shell}}} - \nu\sigma_{x_{\text{shell}}}. \quad [7]$$

From equilibrium

$$PRL_f = \sigma_{\phi_{\text{shell}}} hL_f + \left[\sigma_{\phi_{\text{shell}}} - \nu \frac{PR}{2h} \right] A_q \quad [8]$$

where

$$A_q = A_f \frac{R_{\text{shell}}}{R_{\text{frame}}}$$

Then

$$\sigma_{\phi_{\text{shell}}} = \frac{PRL_f \left[1 + \nu \frac{A_q}{2L_f h} \right]}{L_f h + A_q} \quad [9]$$

or

$$\sigma_{\phi_{\text{shell}}} = \frac{PR}{h} \left[\frac{A_{\text{shell}}}{A_{\text{shell}} + A_q} \right] + \nu \frac{PR}{2h} \left[\frac{A_q}{A_{\text{shell}} + A_q} \right] \quad [10]$$

Therefore

$$E\epsilon_\phi = \frac{PRA_{\text{shell}}}{h(A_{\text{shell}} + A_q)} \left[1 - \frac{\nu}{2} \right] \quad [11]$$

and

$$E\epsilon_{\text{shell}} = \frac{PR}{2h} \left[1 - 2\nu \frac{A_{\text{shell}}}{A_{\text{shell}} + A_q} - \nu^2 \frac{A_q}{A_{\text{shell}} + A_q} \right] \quad [12]$$

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