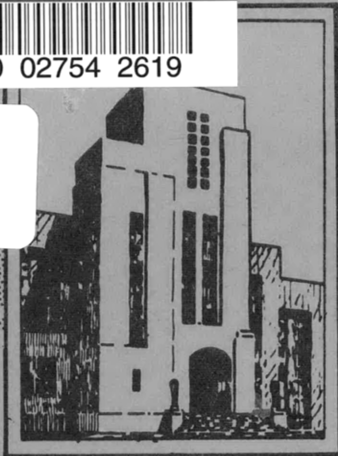

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THE THEORETICAL PREDICTION OF THE LONGITUDINAL MOTIONS OF HYDROFOIL CRAFT

by

T. Francis Ogilvie

HYDROMECHANICS LABORATORY

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NOTATION

A	Average of $e^{-k\zeta}$ over the foil span
A_0	First (constant) term in the expansion of A in powers of $(z + \delta l \psi)$
a	Amplitude of water waves
b	Foil chord
$C(K)$	Theodorsen function, of argument K, used in predicting unsteadiness effects
c	Velocity of water waves, with respect to a fixed observer
c_0	Lift coefficient of foil in steady flight in calm water
c'	Lift-curve slope, assumed constant
D	A function of frequency of encounter, used in predicting unsteadiness effects
d	Depth of apex of V-foil in steady flight below calm water surface
E	Function of frequency of encounter and of speed of craft with respect to velocity of waves, used in predicting unsteadiness effects
I	Moment of inertia of the craft about its center of gravity
$J_n(x)$	Bessel functions of the first kind, of order n and argument x
$K_n(ix)$	Modified Bessel functions of the second kind, of order n and imaginary argument ix
k	Wave propagation number = $2\pi/\lambda$
L	Lift force on a foil
L_{qs}	Lift force on a foil, calculated on assumptions of quasi-steady motion
l	Horizontal distance from craft center of gravity to center of foil
m	Mass of the craft
S	Horizontal projection of the wetted area of foil
s	Parameter replacing time in Laplace transforms
t	Time
U	Factor used in specifying the fluid velocity with respect to foil in calculations of unsteadiness effects
u_w	Horizontal component of wave orbital velocity
V	Constant forward speed of craft
v	Vertical velocity of foil
v_w	Vertical component of wave orbital velocity

W	Weight of craft
x	Horizontal distance measured from center of gravity of craft
x'	Horizontal distance measured from a fixed point
Z	Amplitude of heave in linear calculations
Z_0	Nondimensional steady heave in waves = z_0/a
Z_1	Heave amplification factor, that is, the nondimensional amplitude of fundamental heave oscillation in waves = z_1/a
Z_1, Z_2, Z_3, Z_4	Constant (real) coefficients of the terms in the heave transient response solutions
z	Heave position, with respect to height during steady flight in calm water, positive up
z_0	Steady component of heave for motion in waves
z_1	Amplitude of fundamental heave oscillation in waves
z_w	Instantaneous wave height
α	Instantaneous effective angle of attack, measured from calm water trim condition
β	Change of depth of the apex of a V-foil from calm water trim condition
δ	Special symbol always followed by l_f or l_a; equals + 1 when followed by l_f, − 1 when followed by l_a
ζ	Distance between the mean position of a water particle and the undisturbed surface level
η	Nondimensional time = $(2V/b)t$
θ	Angle with respect to the x-axis in the conformal mapping of a flat airfoil into a circle (see Figure 15)
λ	Wave length of waves on water surface
μ	Angle of dihedral of V-foil
ν	Circular frequency of encounter
ξ	Horizontal distance from center of foil
ρ	Density of water
σ	Roots of stability equations of craft
$\phi(\eta)$	Wagner function, giving unsteady lift on a foil following a step change in angle of attack
ϕ_{z_1}	Phase lead of heave fundamental oscillation peak with respect to wave peak at craft center of gravity·

ϕ_{ψ_1}	Phase lead of pitch fundamental oscillation peak with respect to wave peak at center of gravity of craft
Ψ	Amplitude of pitch in linear calculations
Ψ_0	Nondimensional steady component of pitch in waves $= l\psi_0/a$
Ψ_1	Pitch-amplification factor, that is, the nondimensional amplitude of fundamental pitch oscillation in waves $= l\psi_1/a$
$\Psi_1, \Psi_2, \Psi_3, \Psi_4$	Constant (real) coefficients of the terms in the pitch transient response solutions
ψ	Pitch angle, with respect to horizontal, positive for bow up
ψ_0	Steady component of pitch
ψ_1	Amplitude of fundambntal pitch oscillation in waves
ω	Circular frequency of waves, as seen by a fixed observer

Notes

(a)	Subscripts f and a refer the associated quantity to either the forward or aft foil, respectively
(b)	A bar over a letter indicates the Laplace transform of the quantity: $\bar{f}(s) = \mathcal{L}\{f(\eta)\}$
(c)	$\dot{z} = dz/dt$; $z' = dz/d\eta$; etc.
(d)	$z(0)$, $\dot{z}(0)$, $z'(0)$, etc., indicate initial values (at $t = \eta = 0$)
(e)	$\pm$ or $\mp$: The upper sign always refers to head seas, the lower sign to following seas

ABSTRACT

The nonlinear theory of Weinblum for predicting the longitudinal response of hydrofoil craft in waves is modified, and the results of analog computations based on this theory are presented for comparison with available experimental data. The complete nonlinear equations are used in the computations, and it is shown that the nonlinearities affect the oscillatory amplitudes only slightly but that they cause large steady components of heave and sometimes of pitch. The steady heave is often as large as the amplitude of oscillation, and is always downward, tending to cause the craft to crash. The effects of unsteadiness are investigated, and it is shown that the forces are reduced as much as 40 percent because of unsteadiness but that there is little net effect on the amplitudes of heave and pitch. Transient responses are calculated, consideration being given to nonlinearities and unsteadiness. From the experimental data, it is concluded that the theory gives good predictions of the amplitudes of heave and probably of pitch, and fair predictions of phase.

INTRODUCTION

As interest in hydrofoil-supported craft has grown, particularly in the last decade, there has arisen a need to predict the characteristics of hydrofoils and of hydrofoil craft. One of the problems which has been considered extensively but which still poses considerable difficulty is the dynamic response of a hydrofoil craft in waves. Some theoretical work has been done by several investigators, and one systematic experimental program has been reported. However, until now the only direct comparison between theoretical and experimental results has indicated wide discrepancies between the two. The present report shows how modifications to the available theory result in realistic predictions for the most important group of experiments reported.

Only one particular configuration will be considered here. The craft to be studied has two identical foils placed at equal distances fore and aft of the craft's center of gravity. The foils are of the dihedral, or area-stabilized, type. The section profiles are assumed to be constant across the span and the foils are assumed to have no twist. This configuration closely approximates to several practical designs which have been built in the last two decades.

Weinblum[1] has published an approximate analysis of the heaving and pitching, in regular shallow waves, of craft supported on area-stabilized hydrofoils. The configuration considered in the present report is included in his analysis.

[1]References are listed on page 51.

Leehey and Steele[2] performed a series of tests on several small hydrofoil configurations, measuring the heaving and pitching motions in a variety of regular sea conditions. Using the linearized version of Weinblum's equations, they also calculated the responses for their experimental conditions.

This report is a direct extension of the work of Weinblum and of Leehey and Steele. In the first section there is a brief derivation of Weinblum's equations, with slight modifications. The procedures for the solution of these equations are discussed; in the following section the results of actual analog computations are presented. Next the effects of unsteadiness are considered. Such effects were ignored in Weinblum's equations and in the analog solutions presented herein. It is shown that the net effect of unsteadiness on the amplitudes of heave and pitch motions is generally quite small, although the forces on the foils are reduced by as much as 40 percent in some head seas conditions. In the next section, the transient responses of the craft are calculated. Linear and nonlinear, quasi-steady solutions are discussed – also linear, unsteady solutions. It is found here that the most important components of the transient solutions are only slightly affected by either nonlinearities or unsteadiness. Next, the effects of the horizontal component of water orbital motion are discussed briefly, then the effects of downwash are considered. It is shown that the unsteady vorticity shed by the forward foil has negligible effect on the after foil.

The report concludes with a comparison of the present theoretical results and the experimental data of Leehey and Steele, followed by a statement of the conclusions concerning the validity of the theoretical considerations.

WEINBLUM'S EQUATIONS OF MOTION

Consider a hydrofoil-supported craft moving in the positive x'-direction with a constant forward velocity V. Let there be an infinite train of regular sinusoidal waves moving along the x'-axis toward either decreasing or increasing x', according to whether the craft is traveling in head or following seas, respectively.

The craft is as described in the Introduction and is shown in Figure 1. A perspective view of the craft and the experimental apparatus for measuring heave and pitch (from Reference 2) is shown in Figure 2.

The wave profile observed at a fixed point x' can be represented by the expression:

$$z_w = a \cos (\omega t \pm kx')$$

where ω is the circular frequency of waves,

k is the propagation number $= 2\pi/\lambda$, and

λ is the wave length.

The upper sign (see Notation, page vi) in the expression for z_w specifies a wave traveling in the negative x'direction, which is a head sea, and the lower sign specifies a following sea.

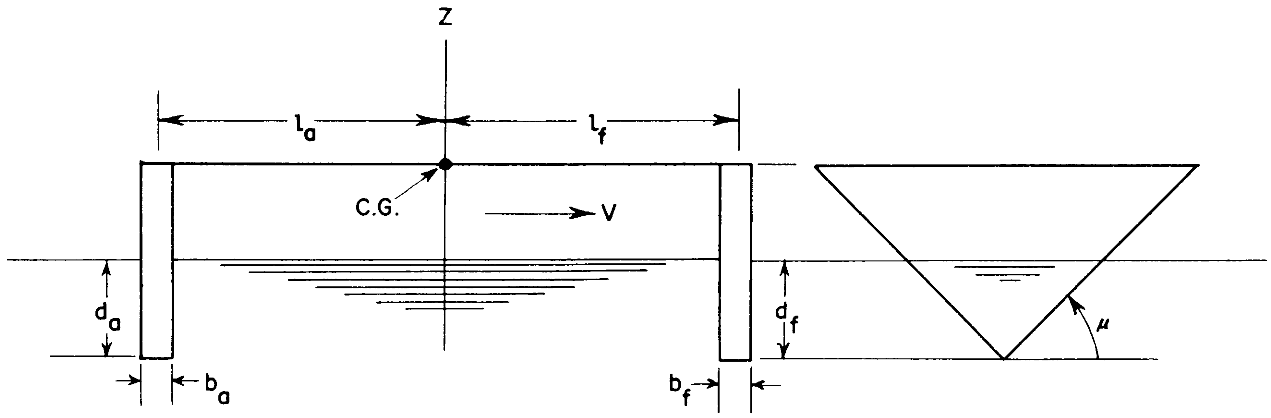


Figure 1 – Geometry of Craft

The craft is shown in its equilibrium position in calm water. Positive x is measured toward the right. The pitch angle ψ is measured with respect to the horizon, positive for bow up.

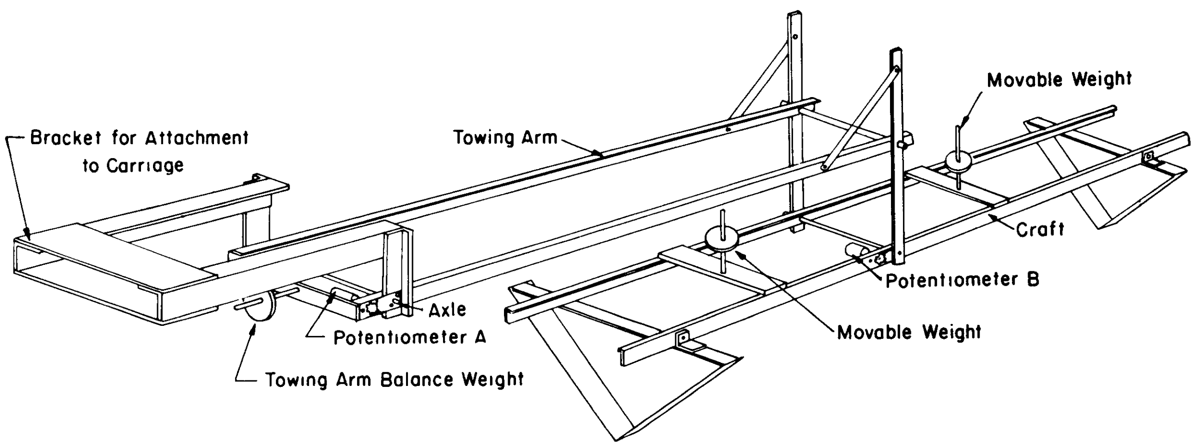


Figure 2 – Perspective View of Craft and Experimental Apparatus

Whenever a double sign appears hereafter, it will be understood that the upper sign applies to head seas and the lower to following seas.

The wave motion causes the water particles to have the following vertical velocities:

$$v_w = -a\,\omega\,e^{-k\zeta}\sin(\omega t \pm kx')$$

where ζ is the distance between the mean position of the water particle and the undisturbed surface level.

It will be desirable to express z_w and v_w with respect to a reference frame advancing with the craft. In particular, we select an origin located horizontally at the mean location of of the center of gravity of the craft, with a horizontal x-axis and a vertical z-axis (positive upward). Since the craft is assumed to be moving with a constant velocity V, we can transform from the coordinate x' to x by the substitution:

$$x' = x + Vt$$

Then we have for z_w and v_w:

$$z_w = a \cos(\nu t + kx)$$

$$v_w = \mp a \omega e^{-k\zeta} \sin(\nu t + kx)$$

where ν is the circular frequency of encounter.

We note that

$$\nu = kV \pm \omega = k(V \pm c) \qquad [1]$$

where c is the celerity of waves.

For hydrofoil craft it is generally true that $V > c$; therefore ν is always to be considered as positive.

The forward foil is located at $x = + l_f$ and the after foil at $x = - l_a$. Then the wave height and the vertical water particle velocity at each foil are:

$$z_{w_f} = a \cos(\nu t + kl_f)$$

$$z_{w_a} = a \cos(\nu t - kl_a)$$

$$v_{w_f} = \mp a \omega e^{-k\zeta_f} \sin(\nu t + kl_f)$$

$$v_{w_a} = \mp a \omega e^{-k\zeta_a} \sin(\nu t - kl_a)$$

We can simplify our equations here and later if we adopt the following special notation:

$$\delta = \begin{cases} +1 \text{ when followed by } l_f \\ -1 \text{ when followed by } l_a \end{cases}$$

Then we can write one equation to apply to both forward and after foils. Instead of the four equations above for z_w and v_w, we now have two:

$$z_w = a \cos(\nu t + k\delta l) \qquad [2]$$

$$v_w = \mp a \omega e^{-k\zeta} \sin(\nu t + k\delta l) \qquad [3]$$

It is understood that the appropriate subscripts will be attached to z_w, v_w, ζ, and l whenever these quantities are to be calculated for specific cases.

The heave z is measured from the steady flight level in calm water. During such steady flight we shall let d_f represent the submergence of the apex of the forward foil and d_a the submergence of the apex of the after foil.

The pitch ψ is measured with respect to the horizon, positive for bow up.

The lift on each foil is now calculated under several assumptions. Some of these assumptions are removed later.

1. The lift at any time is proportional to the instantaneous angle of attack times the instantaneous horizontal projection of the submerged foil area. This implies that unsteady effects as well as free surface effects are neglected. The effect of unsteadiness will be considered in detail later. The neglect of surface effects, which is maintained throughout this report, must be considered as an assumption that is justified only by the resulting agreement with experiments. It has often been assumed that the direct effect of the surface is negligible when the foil is submerged more than one chord length, and Kaplan has shown this for the rather complicated case of a two-dimensional foil moving in unsteady motion near a free surface.[3] However, a dihedral foil always has part of its span immediately adjacent to the surface and the condition is thus always violated. We can rationalize the assumption by asserting that the part of the foil very close to the surface is always a small fraction of the entire span and that the effect of the surface is therefore small.

2. The foil chord is much smaller than the wave lengths encountered, so the relative velocity between foil and water due to the wave motion may be considered uniform over the chord length.

3. The lift-curve slope is a constant for all motions and conditions considered.

4. Downwash and waves from the forward foil have negligible effect on the after foil.

5. The forward velocity of the craft is constant and is large enough that the horizontal water particle velocities caused by the waves will not affect the lift forces.

6. The foil chord is much smaller than the distance between craft center of gravity and either foil. This condition is necessary for two reasons.

a. There are two forces on the foils due to pitching rate: the force resulting from the vertical translational speed of the foils associated with pitching, and the force due to the angular velocity of the foils. The first is proportional to $(-\delta l \dot{\psi})$ and the second is proportional to $(b/4\,\dot{\psi})$. The latter is negligible compared with the former if this assumption is realized.

b. There are certain moments acting directly on the foils if they are not supported at their centers of pressure. If this assumption is valid, these moments are small compared with the moments caused by the lift forces acting through the long lever arms of l_f and l_a.

Under these assumptions the lift on each foil will be given by:

$$L = \left(\frac{1}{2}\rho V^2\right)\left(S\right)\left(c_0 + c'\alpha\right) \qquad [4]$$

where c_0 is the lift coefficient for steady flight in calm water,

c' is the lift-curve slope,

α is the instantaneous effective angle of attack, measured from still water trim condition, and

S is the instantaneous horizontal projection of the wetted area of the foil.

It is understood again that the appropriate subscript, f or a, will be applied to each quantity which may vary from one foil to the other.

From Figure 1 it is evident that the projected area S is:

$$S = 2\,b\,\cot\mu\,[d - z - \delta l \psi + a\cos(\nu t + k\delta l)]$$

Linear aerodynamic forces are assumed; therefore we may simply superpose the various contributions to the angle of attack. We have labeled the upward vertical velocity of the water particles v_w. If we let v represent the vertical velocity of a foil (positive upward), then the angle of attack is given by:

$$\alpha = \psi + \frac{v_w - v}{V}$$

$$= \psi - \frac{1}{V}\,[\dot{z} + \delta l \dot{\psi} \pm a\omega e^{-k\zeta}\sin(\nu t + k\delta l)]$$

We note that this angle of attack varies across the span, since the exponential factor depends on the depth of submergence and the depth varies across the dihedral foil. Before this expression can be introduced into the formula for lift, the explicit dependence on depth must be removed.

To do this we define a quantity A which is simply the mean value of the exponential factor over the depth of the foil:

$$A \equiv \overline{(e^{-k\zeta})} = \frac{\int_0^{(d-z-\delta l\psi)} e^{-k\zeta}\,d\zeta}{d - z - \delta l \psi} = \frac{1 - e^{-k(d-z-\delta l\psi)}}{k(d - z - \delta l\psi)}$$

We replace the exponential factor by this quantity in evaluating the angle of attack. If the sectional life-curve slope is reasonably uniform across the span, this implies the use of a "strip theory" for the aerodynamic forces on the wing and the assumption that surface effects are neglected. In practice, it will be necessary to simplify this expression since it would be quite difficult to produce such a factor with an analog computer. One can expand it into a series and retain only as many terms as the computer can conveniently handle:

$$A = A_0 \left[1 + p\,(z + \delta l \psi) + \ldots\right]$$

where $A_0 = \dfrac{1 - e^{-kd}}{kd}$, and p = constant, etc.

In the computations presented in the next section, this function is generally reduced to just the constant term. However, in those cases in which z has a large steady component, this latter part of z is added to d so that A finally represents the average of the exponential when the foil is at its mean position in waves.

The lift can now be written explicitly:

$$L = (\rho V^2)\,(b \cot \mu)\,[d - z - \delta l \psi + a \cos(\nu t + k \delta l)]\,\{c_0 + c' \psi - \frac{c'}{V}\,[\dot{z} + \delta l \dot{\psi} \pm a \omega A \sin(\nu t + k \delta l)]\} \qquad [5]$$

The two equations of motion which give the forces and moments in the heaving and pitching degrees of freedom are

$$m\ddot{z} = L_f + L_a - W$$

$$I\ddot{\psi} = l_f L_f - l_a L_a$$

where m and I represent the mass and moment of inertia of the craft and W is the weight of the craft. The moment of inertia is taken about the center of gravity. These equations imply two new assumptions:

1. Both m and I should include the effects of the added mass of the water. However, the assumptions previously stated included the neglect of unsteadiness effects and, if this is a realistic assumption, then added mass forces and moments will also be negligible for motion in waves. This will be demonstrated in the section on Unsteadiness Effects (page 21). For transient responses in calm water, added mass is actually more important than the effect of the wake (unsteadiness effect), and this will also be treated in detail (page 31). However, for the immediate problem we assume that such added mass forces can be disregarded. This limits the maximum accelerations that can be predicted with reasonable validity.

2. Horizontal forces, which would produce pitching moments, are neglected. This includes drag and also thrust of the propelling system (or force of the towing rig on a model). That portion of the drag which varies periodically with the motions and with the waves encountered is generally quite small, as can be shown by simple qualitative calculations. Also actual calculations of the response when reasonable varying drag is assumed show that the effect on predicted motions is quite negligible. There is an appreciable steady drag, to be sure, but it is assumed here that the steady angles of attack are adjusted slightly to offset the resulting steady pitching moment. This change in the steady lift coefficients causes only second-order effects on heaving and pitching amplitudes, since the time-dependent lift forces are not strongly dependent on these coefficients.

When the lift forces are substituted into the equations of motion, we have the following equations governing the longitudinal responses of the craft:

$$m\ddot{z} = (\rho V^2) \sum_{f,a} \Big[(b \cot \mu) \left[d - z - \delta l \psi + a \cos (\nu t + k \delta l) \right]$$

$$\left\{ c_0 + c' \psi - \frac{c'}{V} \left[\dot{z} + \delta l \dot{\psi} \pm a \omega A \sin (\nu t + k \delta l) \right] \right\} \Big] - W \qquad [6]$$

$$I\ddot{\psi} = (\rho V^2) \sum_{f,a} \Big[\delta l \, (b \cot \mu) \left[d - z - \delta l \psi + a \cos (\nu t + k \delta l) \right]$$

$$\left\{ c_0 + c' \psi - \frac{c'}{V} \left[\dot{z} + \delta l \dot{\psi} \pm a \omega A \sin (\nu t + k \delta l) \right] \right\} \Big] \qquad [7]$$

These are essentially Weinblum's equations, although his report does not give the second equation explicitly. The factors A_f and A_a are new here. The symbol $\sum_{f,a}$ indicates that the terms corresponding to the forward and after foils, respectively, are to be added. Of course, it is also implied that a subscript, f or a, is to be added to each quantity which may vary from one foil to the other.

(In the above form the equations can be extended to include more than two foils, provided each foil separately satisfies the conditions previously set forth in the calculation of lift. Also, the case of a flat foil can be handled easily. The following factors:

$$(b \cot \mu) \left[d - z - \delta l \psi + a \cos (\nu t + k \delta l) \right]$$

represent the instantaneous half-span. These factors can simply be replaced by the value of one-half the span. Such terms then become linear, so that this case is a simplification of the general problem.)

It can be shown that, generally, the effects of the wave motion are felt most strongly through the varying angle of attack resulting from the orbital motion. The effects of the varying span are almost insignificant unless the craft motions are extremely large. Therefore, the exact amplitude of the terms containing sin $(\nu t + k \delta l)$ is very important. Omission of the A-factors (that is, setting $A = 1.00$) increases these terms by a factor of about 2.0 in typical cases, and the resulting amplitudes of heave and pitch are increased in this same ratio.

Weinblum, in his original report, indicated that a linearization of these equations of motion was, at best, an expeditious procedure. Brief calculations can demonstrate that many of the discarded terms are comparable in size with the retained terms. This is particularly true for hydrofoil craft traveling in following seas, in which case the motions are often very large and nonlinear effects may be expected to become prominent. The results of such nonlinearities generally are the production of harmonics and of steady (d.c.) components of heave and pitch.

The experiments of Reference 2 showed no evidence of harmonic responses, and the analog computations of the next section, based on the complete nonlinear equations, also indicate that harmonics occur under only the most extreme conditions. Therefore the first consequence of dropping the higher degree terms is not realized: there are generally no harmonics to be lost.

However, there are often quite significant average values of heave and sometimes of pitch. Consider, for example, the special case in which the wave length of the seaway is equal to the distance between the foils. The craft which is considered in detail throughout this report has two identical foils, and in the assumed absence of drag, both foils will always encounter exactly the same conditions. Therefore there will be no pitch response at all and we can study the heave equation by itself.

The heave equation becomes, under these conditions:

$$m \ddot{z} = 2 \left(\frac{1}{2} \rho V^2\right) (2 b \cot \mu)(d - z - a \cos \nu t) \left[c_0 - \frac{c'}{V}(\dot{z} \mp a \omega A \sin \nu t)\right] - W$$

where $b \equiv b_f = b_a$; $\quad \mu \equiv \mu_f = \mu_a$; $\quad d \equiv d_f = d_a$; $\quad A \equiv A_f = A_a$;

$c_0 \equiv c_{0_f} = c_{0_a}$; $\quad c' \equiv c_f' = c_a'$.

If we assume a solution consisting of a steady component and a fundamental oscillation (remembering that the harmonics are generally negligible):

$$z = z_0 + z_1 \cos (\nu t + \phi_{z_1})$$

and if we substitute this solution into the simplified equation of motion above, we find the following relation between the steady component and the oscillatory solution:

$$z_0 = \frac{-c'}{2 V c_0} (\nu \mp \omega A)\, a\, z_1 \sin \phi_{z_1}$$

In general, if the amplitude of motion is not very great, the oscillatory amplitude z_1 will be proportional to the wave amplitude a, and the phase ϕ_{z_1} will be essentially independent of a. Therefore, z_0 will be proportional to the square of the wave amplitude. In the next section it will be shown that z_0 is generally large enough that it must be considered to be at least as important as the amplitude z_1 in estimating the seaworthiness of hydrofoil craft.

Since the principal effect of the nonlinearities is to change the mean flying position, it also changes the amplitude of orbital velocities encountered. This change is accounted for when A_f and A_a are allowed to be functions of z_0 and ψ_0. It reduces the expected oscillatory amplitudes slightly.

ANALOG COMPUTATIONS OF RESPONSES BASED ON NONLINEAR EQUATIONS

Hydrofoil craft usually experience very large motions when flying in following seas, and it has long been recognized that this is the condition which most severely limits their range of application. Since the amplitudes of motion are so large, it is also in this condition that one would expect the effects of nonlinearities to be most pronounced. Therefore it was undertaken to solve the complete equations of motion, in order that the solutions might be compared with the experimentally measured responses of Leehey and Steele[2]. No attempt was made to cover a large range of possible physical conditions; rather, attention was focused on the conditions for which corresponding data existed, and other conditions were studied only when slight extra effort sufficed to produce the solutions. Although following seas supply the most critical conditions, both head and following seas were studied since considerable data were available for both. Only the "tandem V-foil" configuration of Leehey and Steele was considered in these calculations.

A block diagram of the computer setup is included as an appendix to this report. The problem was initially set up on two different computers, one a Mid-Century computer (MIAC) the other a Reeves computer (REAC). Certain solutions from the two computers were superposed and they agreed within about 3 percent. The MIAC was used for most of the subsequent solutions. All of the results were recorded on a Reeves plotting board.

The computer setup was so arranged that one could obtain a trace of any of the following variables:

1. $\cos \nu t$, the wave height at the center of gravity;
2. z, the heave;
3. ψ, the pitch;

4. $\dot{z}$, the heave velocity;

5. $\dot{\psi}$, the pitch angular velocity;

6. $\alpha_f = \psi - 1/V\,[\dot{z} + l_f\dot{\psi} \pm a\,\omega\,A_f \sin(\nu t + kl_f)]$
   the effective angle of attack of the forward foil;

7. $\alpha_a = \psi - 1/V\,[\dot{z} - l_a\dot{\psi} \pm a\,\omega\,A_a \sin(\nu t - kl_a)]$
   the effective angle of attack of the after foil;

8. $\beta_f = -z - l_f\psi + a\cos(\nu t + kl_f)$
   the change of depth of the apex of the forward foil from calm water trim conditions;

9. $\beta_a = -z + l_a\psi + a\cos(\nu t - kl_a)$
   the change of depth of the apex of the after foil from calm water trim conditions.

Of course, z and ψ specify completely the longitudinal rigid body motions of the craft, and the other variables can be obtained from them. But the quantities designated β_f and β_a are particularly valuable in such convenient form, since they indicate directly how close either foil is to broaching or crashing.

On the actual craft it was necessary to adjust the steady angles of attack somewhat to compensate for the diving moment caused by drag. Since the equations do not allow for drag, this physical compensation had to be removed in the calculations. Specifically, it was assumed that c_{0_f} and c_{0_a} satisfied the following two equations:

$$\left(\frac{1}{2}\rho V^2\right)[c_{0_f}(2\,b_f d_f \cot\mu_f) + c_{0_a}(2\,b_a d_a \cot\mu_a)] - W = 0 \qquad [8]$$

$$\left(\frac{1}{2}\rho V^2\right)[l_f c_{0_f}(2\,b_f d_f \cot\mu_f) - l_a c_{0_a}(2\,b_a d_a \cot\mu_a)] = 0 \qquad [9]$$

These come directly from Equations [6] and [7] in the most degenerate case:

$$z = \dot{z} = \ddot{z} = \psi = \dot{\psi} = \ddot{\psi} = a = 0$$

They specify the conditions for longitudinal equilibrium of the craft flying level in calm water, without drag.

The values of the constant parameters in Equations [6] and [7] were taken from Reference 2 as follows:

$$m = (4.05\text{ lb})/(32.2\text{ ft/sec}^2) = 0.126\text{ slug}$$

$$I = 0.151\text{ slug-ft}^2$$

$$d_f = d_a = 0.550\text{ ft}$$

$$l_f = l_a = 1.50\text{ ft}$$

$$\rho = 1.94 \text{ slugs/ft}^3$$

$$\mu = 45 \text{ deg}$$

$$b = 2 \text{ in.} = 0.167 \text{ ft.}$$

The lift coefficient of each foil for the conditions of Equations [8] and [9] was taken as 0.456, and the corresponding lift-curve slopes were 4.41. The only forward velocity V used was 5.00 ft/sec.

Wave lengths from 2 to 5 ft were considered for head seas conditions, and from 2 to 4 ft for following seas conditions.

The following table indicates the values of the derived parameters used in the calculations:

λ	k	ω	ν		4_0
			Head Seas	Following Seas	
2.0 ft	3.14 ft^{-1}	10.05 sec^{-1}	25.76 sec^{-1}	5.66 sec^{-1}	0.476
2.5	2.51	8.99	21.56	3.58	0.542
3.0	2.09	8.21	18.68	2.27	0.594
3.5	1.80	7.60	16.57	1.38	0.636
4.0	1.57	7.11	14.96	0.75	0.670
4.5	1.40	6.70	13.68	–	0.698
5.0	1.26	6.36	12.64	–	0.722

The amplitude of the waves was varied over a large range, as shown in the following paragraphs.

RESULTS FOR HEAD SEAS

Calculations were made for seven wave lengths from 2.0 to 5.0 ft; the amplitude of the waves was varied from 0.05 to 0.50 ft. A typical set of solutions is shown in Figure 3, as produced directly by the analog computer. The heave response is shown for wave amplitudes of 0.05 to 0.50 ft; the wave length used in these calculations was 4.5 ft (head seas). The heave and pitch and their time-derivatives all had zero initial conditions, and the instantaneous wave height at the center of gravity at zero time was a.

It is apparent from these curves that the heave and pitch responses each consist essentially of two principal components, a zero shift and an oscillation at the fundamental frequency. Only when the amplitude of motion is extremely large does a higher harmonic response ever appear, and for the calculations shown it is never important over the range of amplitudes considered. Therefore we assume that the solutions are exactly of the following form:

$$z = z_0 + z_1 \cos (\nu t + \phi_{z_1})$$

$$\psi = \psi_0 + \psi_1 \cos (\nu t + \phi_{\psi_1})$$

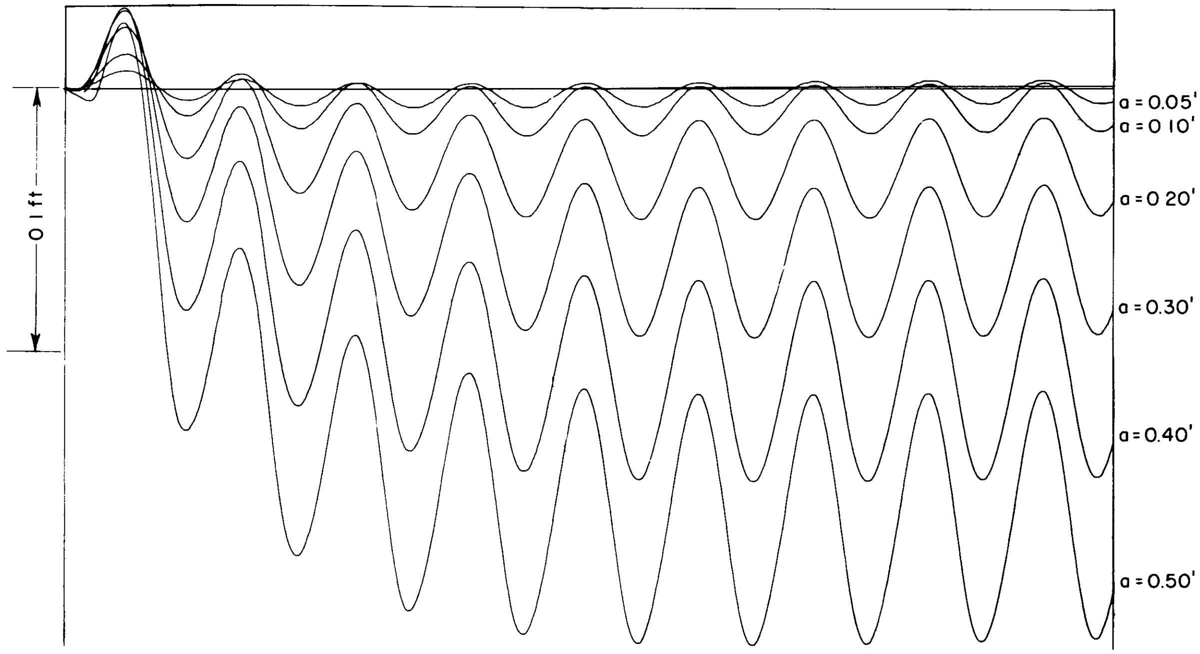


Figure 3 – Typical Set of Analog Solutions

The head-seas heave response is shown for waves of 4.5-ft wave length, amplitudes from 0.05 ft to 0.50 ft. The double zero trace shows the computer d.c. drift.

and we can describe the results by presenting the six real quantities: z_0, z_1, ψ_0, ψ_1, ϕ_{z_1}, and ϕ_{ψ_1}. Actually it is convenient to make the amplitudes nondimensional, as follows:

$Z_0 = z_0/a$ = nondimensional steady heave component,

$Z_1 = z_1/a$ = heave amplification factor,

$\Psi_0 = l\,\psi_0/a$ = nondimensional steady pitch component,

$\Psi_1 = l\,\psi_1/a$ = pitch amplification factor.

In these calculations l is taken as the distance between the center of gravity and either foil. (These two distances happen to be the same for the craft being considered.) Thus Ψ_0 and Ψ_1 represent vertical foil displacements in the same units as Z_0 and Z_1.

Zero phase leads are defined by the equation for the wave height at the center of gravity:

$$z_w/\text{c.g.} = a \cos \nu t.$$

This method of tabulating the computation results will lose some significance when appreciable harmonic responses exist. In such cases, Z_1 and Ψ_1 will simply represent one-half the peak-to-peak amplitude, regardless of harmonic content. Z_0 and Ψ_0 will represent the median of the peaks and troughs of the periodic oscillation.

For all of the head seas conditions for which calculations were made, the amplification factors Z_1 and Ψ_1 were practically independent of wave amplitude. This indicates that the linearized equations should be satisfactory for computing the fundamental oscillatory response. And, in fact, a direct comparison of the nonlinear analog-computed amplitudes with digital linear computations shows the difference in the response to be of the order of magnitude of the computer accuracy.

However, it is obvious from Figure 3 that the zero shift, or change of mean heave position in waves, cannot be ignored. In some cases it is larger than the amplitude of oscillation. This steady component of the motion results directly from the nonlinearity of the equations of motion and there is, of course, no way in which it could be predicted from a set of linear equations.

Generally there may also be a steady component of pitch. For head seas conditions, it is usually much less than the amplitude of pitch oscillation, so it is not considered further here.

The value of z_0 affects the values of the functions A_f and A_a. However, for the cases reported here, the effect is not noticeable in any of the amplitudes of motion, within the computer accuracy.

The heave-amplification factor Z_1 and the heave-phase lead ϕ_{z_1} are plotted in Figures 4 and 5, respectively, with wave length as abscissa. Figures 6 and 7 present the pitch-amplification factor Ψ_1 and the pitch-phase lead ϕ_{ψ_1} in the same kind of graphs. The points on the graphs are experimental results, which will be discussed presently.

In each of these four figures, the results could be shown by a single curve, because the calculated quantities were only slightly affected by the nonlinearities of the problem. However in Figure 8, where the nondimensional steady-heave component is shown for all of the calculated head-seas conditions, a separate curve must be presented for each wave amplitude, since this quantity depends on the nonlinearities for its existence. The curve of Figure 4, the heave-amplification factor, is repeated in Figure 8, so that the amplitude of oscillation can be compared directly with the steady-heave component for any amplitude of waves. Even for the smallest waves considered, the steady component is not trivial, and for the larger amplitudes it is much greater than the oscillatory amplitude.

It should be noted that the steady-heave displacement is always downward, tending to cause the craft to crash.

RESULTS FOR FOLLOWING SEAS

Calculations were made for five wave lengths from 2.0 to 4.0 ft, with amplitudes from 0.01 to 0.30 ft. The results are presented, as before, in Figures 9 through 14. The heave-amplification factors and phase leads are shown in Figures 9 and 10. The pitch-amplification factors and phase leads are shown in Figures 11 and 12. The steady-heave component is

(Text continued on page 18.)

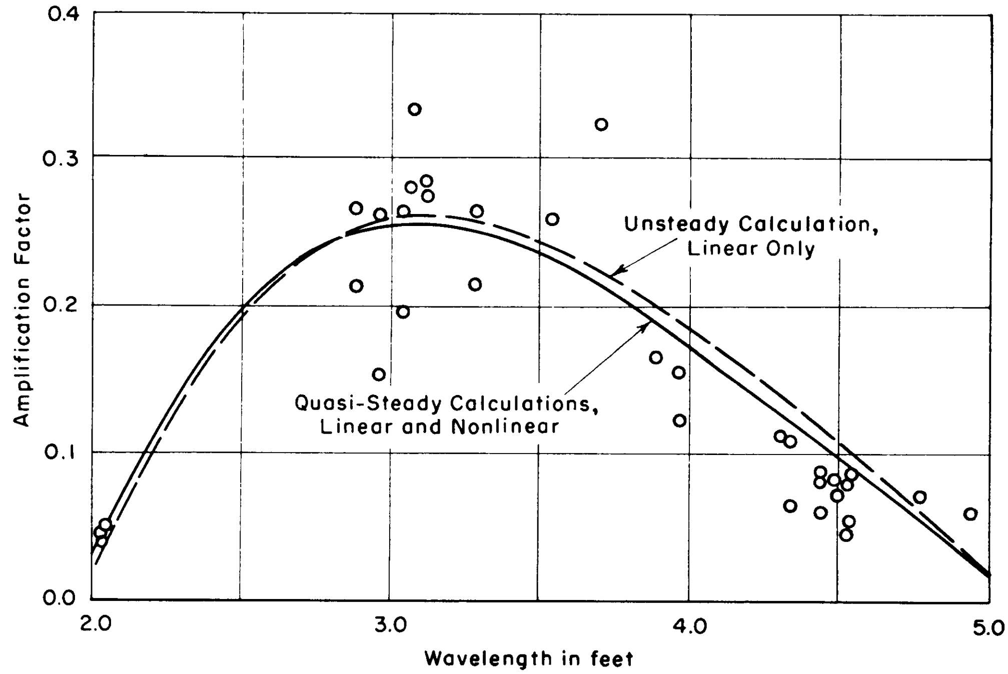


Figure 4 – Head-Seas Heave-Amplification Factor

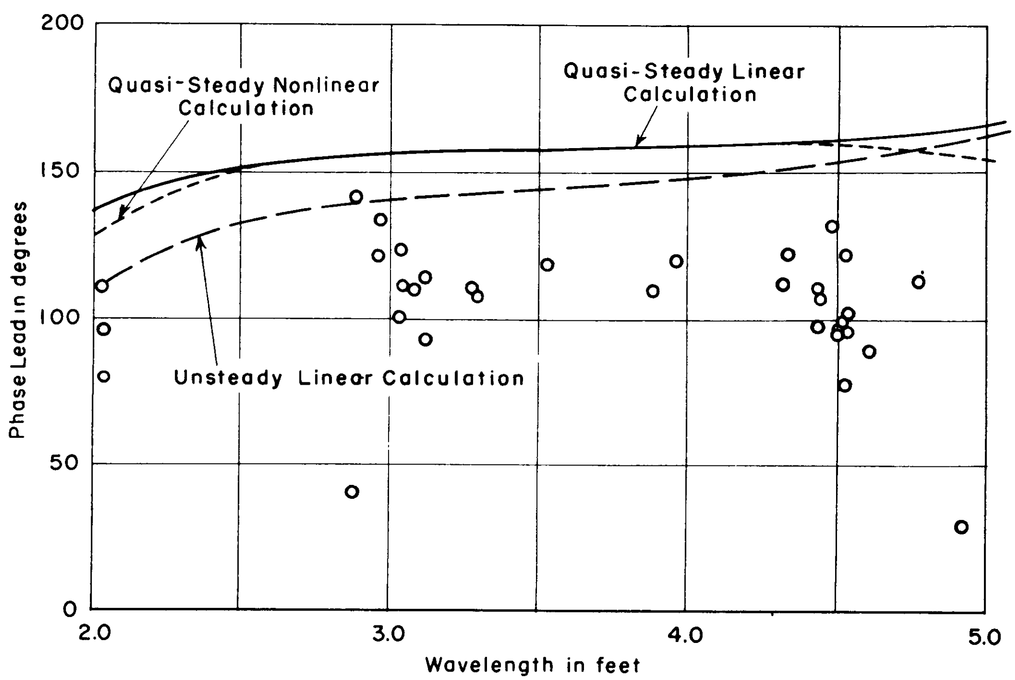


Figure 5 – Head-Seas Heave-Phase Lead

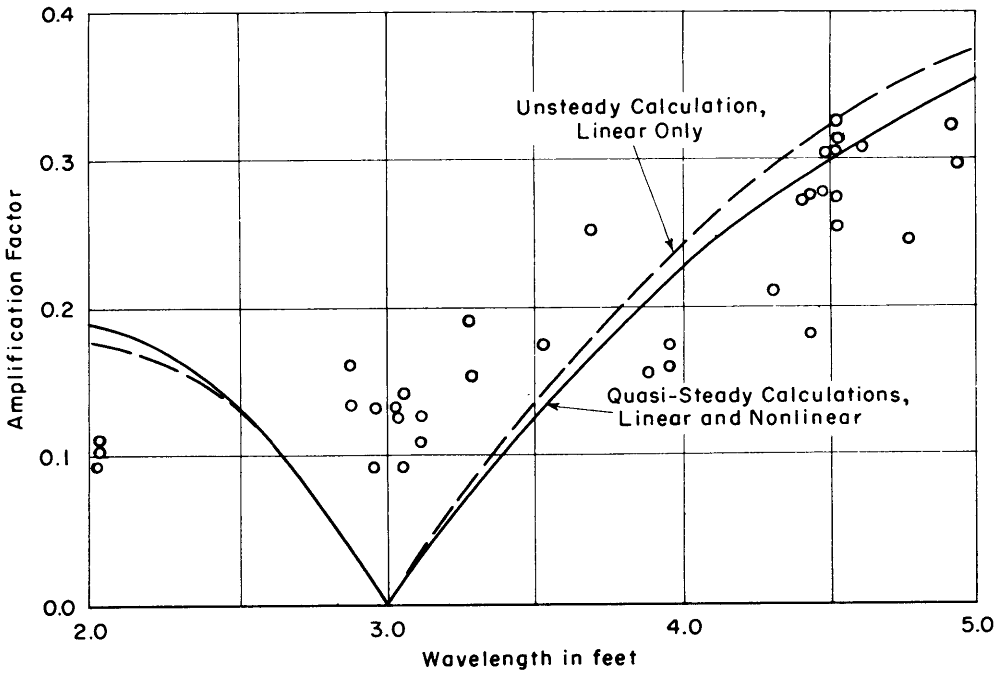


Figure 6 – Head-Seas Pitch-Amplification Factor

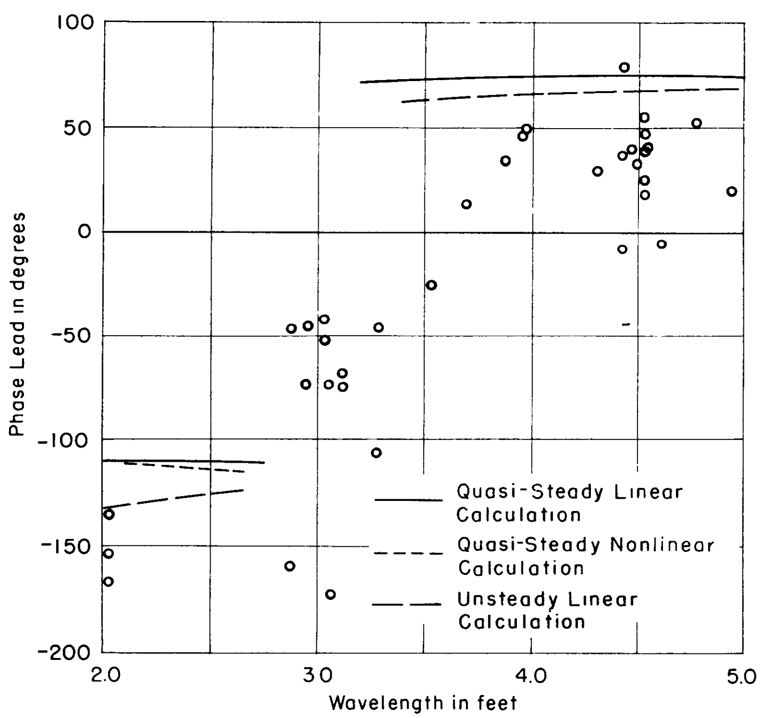


Figure 7 – Head-Seas Pitch-Phase Lead

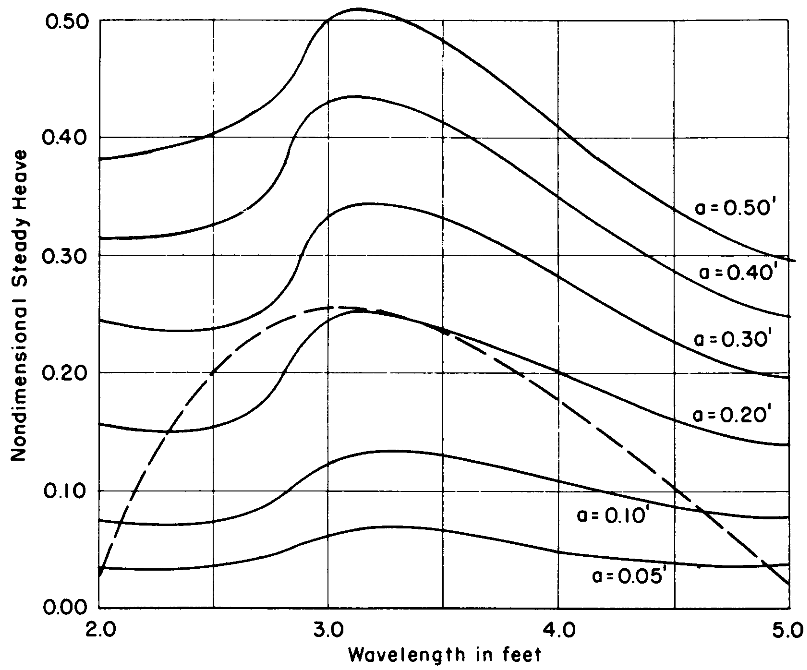


Figure 8 – Head-Seas Steady-Heave Component

Broken line shows head-seas heave-amplification factor from Figure 4. The steady heave component is always downward, tending to cause the craft to crash.

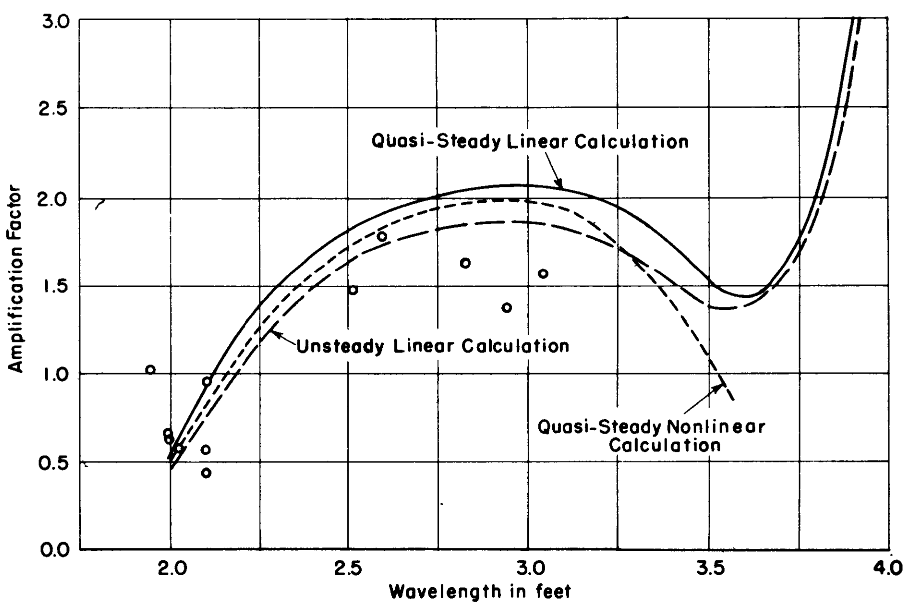


Figure 9 – Following-Seas Heave-Amplification Factor

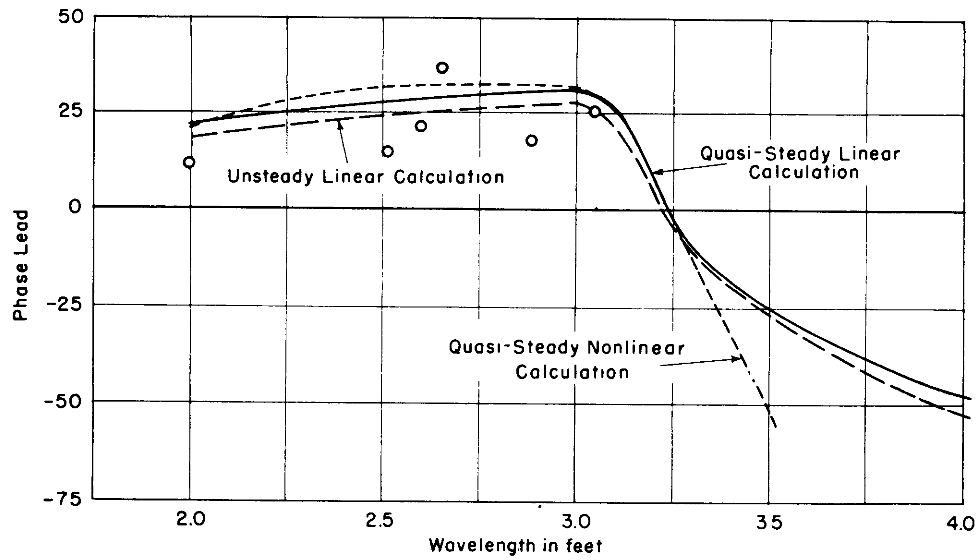


Figure 10 – Following-Seas Heave-Phase Lead

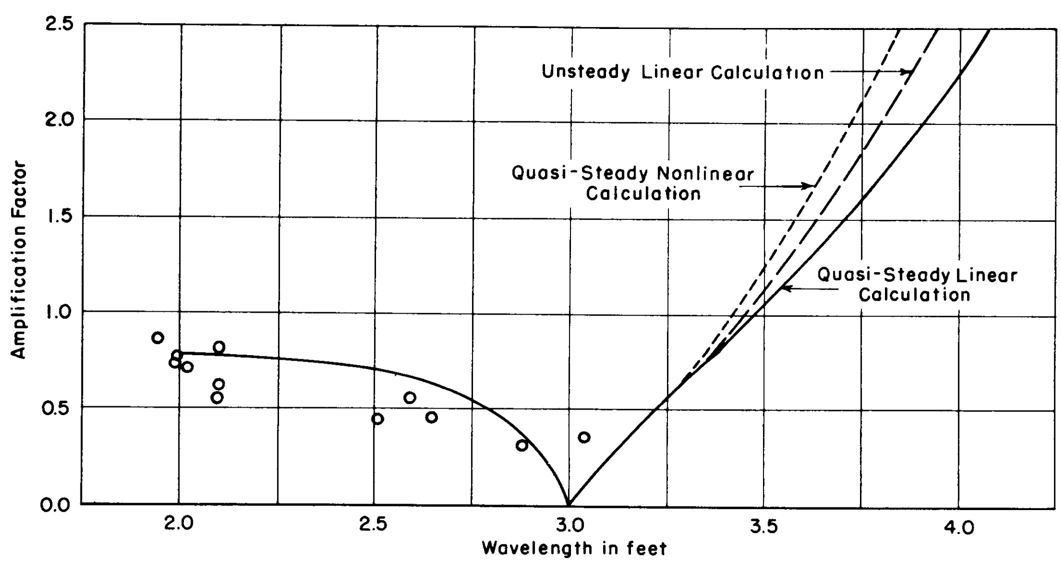


Figure 11 – Following-Seas Pitch-Amplification Factor

plotted in Figure 13a with wave length as abscissa and in Figure 13b with wave amplitude as abscissa. The steady pitch component is plotted in Figure 14, against wave length.

The steady-heave components were generally so large that it was no longer appropriate to use the *a priori* values of A_f and A_a, that is, A_0. The procedure used to find the solutions was an iterative one – a method particularly well adapted to the analog computer. In each

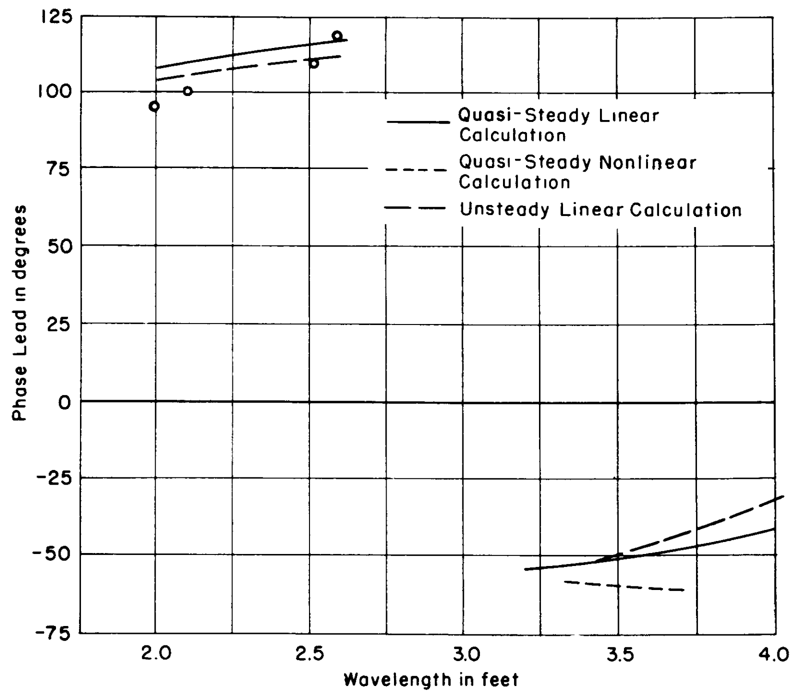


Figure 12 – Following-Seas Pitch-Phase Lead

case being considered, the heave response was first plotted with the function A approximated by the constant A_0. The steady heave response z_0 was measured and A was recalculated:

$$A \cong \frac{1 - e^{-k(d - z_0)}}{k(d - z_0)}$$

The computer was adjusted for this value and the heave response recomputed. This iteration was continued until successive solutions became identical (to within a few percent). Usually only two iterations were actually necessary.

In some cases the steady pitch level was such that an appreciable correction might have been made for it also. This would have necessitated using different values for A_f and A_a. This would not have been very difficult. In fact, the variables β_f and β_a, which were immediately available quantities, could have been used directly to supply such iterative corrections. Since this phenomenon appeared only for conditions under which flight would be impossible because of the severity of the oscillation, this refinement was not carried through.

The large magnitude of the steady-heave component means that the craft rides lower in the water where the orbital velocities are smaller than at the normal level of the craft. Thus a secondary result of the nonlinearities is that the amplitude of oscillation is somewhat smaller than predicted by a linear theory. This, of course, is associated with the variation of the quantity A, discussed previously. In Figures 9 and 11 the amplitudes of responses to

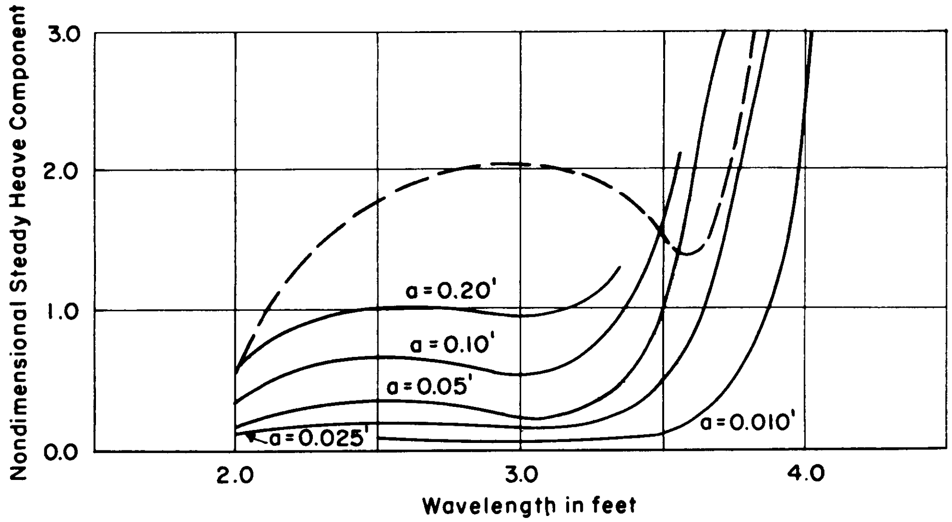


Figure 13a – Plotted versus Wave Length

The broken line shows the following-seas heave-amplification factor (linear calculation) from Figure 9. The steady-heave component is always downward, tending to cause the craft to crash.

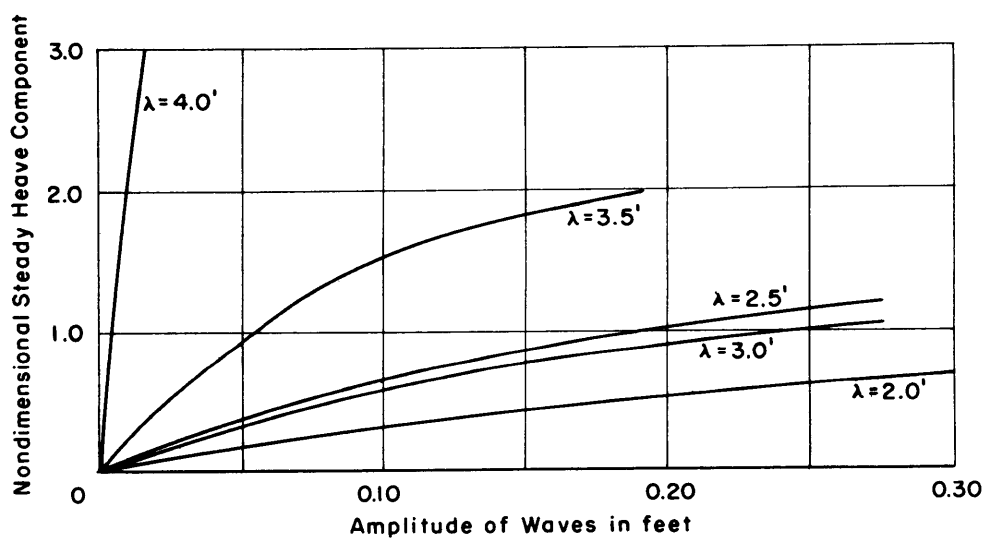


Figure 13b – Plotted versus Wave Amplitude

Figure 13 – Following-Seas Steady-Heave Component

infinitesimal wave amplitudes (the linear solutions) and to waves of 0.10-ft amplitude are shown. The dependence of heave and pitch amplification factors on wave height thus appears in the existence of separate curves for these two cases. In Figures 4 and 6, which presented the comparable results for head seas, no such distinction appeared.

The results of linear calculations for the amplification factors shown in Figures 9 and 11 were obtained with the functions A_f and A_a approximated by A_0. It is interesting to note

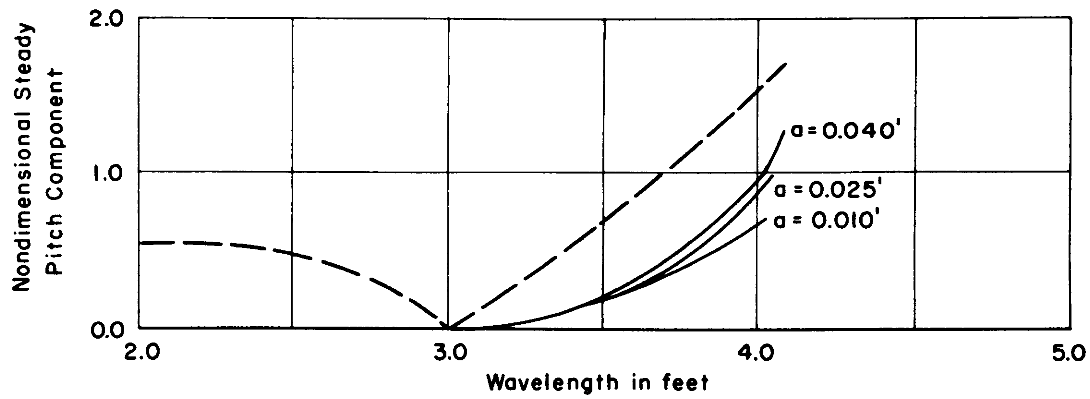


Figure 14 – Following-Seas Steady-Pitch Component

The broken line shows the following-seas pitch-amplification factor (linear calculation), from Figure 11. The steady-pitch is always bow down.

that if the actual values of A_f and A_a could be estimated, as by an extension of the approach in the previous section for the simple case of no pitching motion, and if these values were used in a linear calculation of the oscillatory amplitudes, the resulting amplification factors would generally be quite indistinguishable from the nonlinear results. Thus it appears that the only effect of the nonlinearities on the oscillatory amplitudes is through the change of mean level and trim condition.

It should be noted that the vertical scales in the graphs of following seas responses are quite different from the scales in the graphs of head seas responses.

The large magnitude of the steady responses in following seas can prohibit flight even in cases in which the oscillatory amplitudes are not too large. An explanation is thus provided for a qualitative experimental observation of Leehey and Steele. Whenever they attempted to fly their craft in following seas of 3.5-ft wave length, the craft pitched bow down and remained in that attitude until it crashed into the water. There seemed to be a large increase in drag. Actually, as shown in this section, the mean position which the craft tended to take was much lower than one would normally expect (and trimmed bow down), and as the craft moved toward that position it would appear to be experiencing a large drag moment. It is thus seen that this diving phenomenon is entirely attributable to the nonlinear lift forces.

UNSTEADINESS EFFECTS

Weinblum's analysis of the hydrofoil response problem was intended only as a first approximate solution. Several important factors are lacking, perhaps the most important being a consideration of effects of unsteadiness. In view of the conclusion of the last section, that oscillatory amplitudes are not greatly affected by nonlinearities, the effects of unsteadiness are studied here only with respect to linearized systems.

The linearized, quasi-steady expression for lift on a foil is obtained from Equation [5]

$$L_{qs} = (\rho V^2)(b \cot \mu) \left\{ c_0 d + \frac{c'd}{V} \left[V\psi - \dot{z} - \delta l \dot{\psi} \mp a\omega A \sin(\nu t + k\delta l) \right] - c_0 \left[z + \delta l \psi - a \cos(\nu t + k\delta l) \right] \right\} \quad [10]$$

(Several linear terms with time-dependent coefficients have also been dropped. These terms involve products such as (az), $(a\psi)$, etc., which are generally of the same order of magnitude as the nonlinear terms. This treatment of the equations has been followed by the previous investigators who "linearized" them. Since all of the ignored terms are of the same order of magnitude, it is consistent to drop both types together.)

We consider the terms of Equation [10] in four groups:

$$(\rho V^2)(b \cot \mu)(c_0 d) \quad [11]$$

$$(\rho V^2)(b \cot \mu) \left[\frac{c'd}{V} (V\psi - \dot{z} - \delta l \dot{\psi}) \right] \quad [12]$$

$$(\rho V^2)(b \cot \mu) \left\{ \frac{c'd}{V} \left[\mp a\omega A \sin(\nu t + k\delta l) \right] \right\} \quad [13]$$

$$(\rho V^2)(b \cot \mu) \{ - c_0 \left[z + \delta l \psi - a \cos(\nu t + k\delta l) \right] \} \quad [14]$$

The first, Expression [11], is the steady part, which will be ignored for the rest of this section. In the heave equation, it is canceled by the weight; in the pitch equation, the fore and aft steady lifts cancel. In any case there is no unsteadiness associated with it.

Expression [12] results from the variation of angle of attack associated with trim and motion of the foils vertically. For purposes of supplying an unsteadiness correction, we note that these terms have the same kind of quasi-steady velocity and vorticity distributions over the chord length, and flow unsteadiness will affect all three in an identical manner. Expression [13] also represents the effect of a change of angle of attack, but due to the orbital motions of the water particles. It is convenient now to relax the restriction that the chord length should be small compared with wave length, so that this term in the lift formula represents a fundamentally different distribution of velocity (and thus of vorticity) over the chord, as will be shown presently.

The unsteady aerodynamic forces associated with Expressions [12] and [13] will be computed on the assumption that the finite wing forces are modified by the same function of frequency that would be used to correct quasi-steady forces on a two-dimensional wing under similar conditions. The unsteady forces on the hypothetical two-dimensional wing are figured by the method of Von Kármán and Sears.[4,5]

Expression [14] results from the variation in area of the lifting surface. This is a problem without precedent in the aeronautical field: the unsteady flow due to a rapid variation in wing span. The picture is further complicated by the fact that this variation actually occurs at the free surface. It seems very unlikely that an accurate analysis of this problem is possible with available analytical techniques. Kaplan[3] has shown that even for the comparatively simple case of a two-dimensional airfoil in unsteady flow near the surface, the lift is a very complicated quantity to calculate. There is no justification for combining these lift terms with those due to varying angle of attack and for modifying them according to the plan outlined above. These terms are generally fairly small (although appreciable for following seas calculations), so we avoid the problem by keeping them in their quasi-steady form.

To obtain explicit expressions for the unsteady lift on a foil, we use a result developed by Sears.[5] Suppose a two-dimensional airfoil is moving toward negative ξ with velocity V. As shown in Figure 15a, we can represent each point on the foil in terms of a new variable θ:

$$\xi = b/2 \cos \theta, \qquad - b/2 < \xi < + b/2.$$

(Thin wing theory is assumed throughout this section, so that it does not matter if the wing has camber and thickness. Each point of the wing section is associated with its projection on the ξ-axis.) Now if the wing is oscillating, either in vertical translation or in pitch about its midpoint, the fluid has a vertical velocity with respect to the foil at each point of ξ for $- b/2 < \xi < + b/2$, which can be given as a function $v_w(\xi)$. Of course, it can be given just as well as a function of θ. Now Sears states that if this velocity of the fluid with respect to the foil can be expressed in the following way:

$$v_w = (B_0 + 2 \sum_{n=1}^{\infty} \cos n\,\theta)\, U\, e^{i\nu t} \tag{15}$$

then the lift per unit span of the two-dimensional wing is:

$$L' = \pi \rho b V U e^{i\nu t} \left[(B_0 + B_1)\, C\left(\frac{\nu b}{2V}\right) + \frac{i \nu b}{4V} (B_0 - B_2) \right] \tag{16}$$

where $C\left(\frac{\nu b}{2V}\right)$ is the Theodorsen function:

$$C\left(\frac{\nu b}{2V}\right) = \frac{K_1\left(\frac{i\nu b}{2V}\right)}{K_0\left(\frac{i\nu b}{2V}\right) + K_1\left(\frac{i\nu b}{2V}\right)}$$

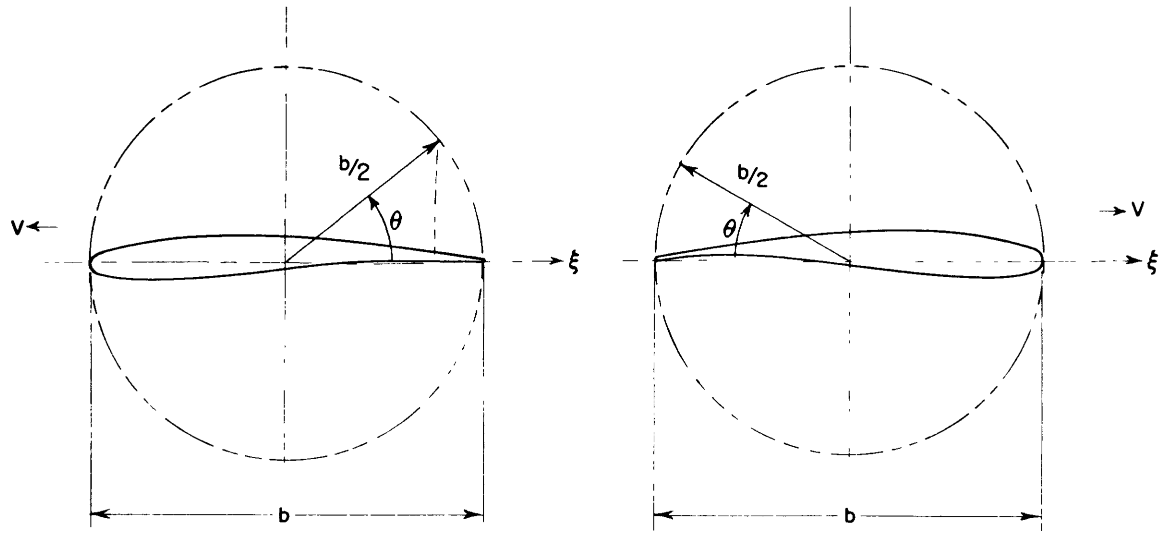


Figure 15a

As taken in Reference 6.

Figure 15b

As taken in this report.

Figure 15 – Geometry for Calculation of Unsteadiness Effects

and K_0 and K_1 are modified Bessel functions of the second kind, of order zero and one, respectively, and of argument $(i \nu b/2V)$. This is the result we wish to use.

Throughout this report we have assumed that the hydrofoil was moving toward positive x. It is convenient to continue this convention, and so we modify the above statements for the case that the foil is headed toward positive ξ. It is still necessary that the angle θ be measured upward from the trailing edge, since obviously the lift is unchanged if both the direction of motion and the fluid velocity distributions are reversed from right to left. From Figure 15b, it is evident that now:

$$\xi = -\, b/2 \cos \theta.$$

So if the distribution $v_w(\xi)$ is transformed to $v_w(\theta)$ by this formula, Sears' expressions above will be directly applicable. The only problem then will be to extract from Expressions [12] and [13] the appropriate vertical water velocities and to write them in the form of Equation [15].

It is now convenient to rewrite the quasi-steady lift in complex exponential form. Since we have linearized the system, it is permissible to do this. For physical quantities it is understood that the real part of any expression will be used. Thus the time-dependent part of the quasi-steady lift is:

$$L_{qs} = (\rho V^2)(b \cot \mu) \left\{ \frac{c'd}{V} [V \psi - \dot{z} - \delta l \dot{\psi} \pm i a \omega A e^{i(\nu t + k \delta l)}] \right.$$

$$\left. - c_0 [z + \delta l \psi - a e^{i(\nu t + k \delta l)}] \right\}$$

We shall assume sinusoidal solutions:

$$z = Z e^{i \nu t}$$

$$\psi = \Psi e^{i \nu t}$$

where generally Z and Ψ are now complex quantities. Then the quasi-steady lift becomes:

$$L_{qs} = (\rho V^2)(b \cot \mu) \left[\frac{c'd}{V} (V \Psi - i \nu Z - i \nu \delta l \Psi \pm i a \omega A e^{i k \delta l}) \right.$$

$$\left. - c_0 (Z + \delta l \Psi - a e^{i k \delta l}) \right] e^{i \nu t} \qquad [17]$$

The part of the lift given previously by Expression [12] is now:

$$(\rho V^2)(b \cot \mu) \left[\frac{c'd}{V} (V \Psi - i \nu Z - i \nu \delta l \Psi) \right] e^{i \nu t} \qquad [18]$$

This represents a quasi-steady lift on a wing of span ($2d \cot \mu$). The vertical velocity of the fluid (with respect to the foil) which corresponds to Sears' "gust" velocity is:

$$(V \Psi - i \nu Z - i \nu \delta l \Psi) e^{i \nu t}$$

We want to write this in the form of Equation [15] above. If we let U equal the quantity in parentheses here it is obvious that all of the constants B_n in Equation [15] are zero except B_0, which equals unity. Then the two-dimensional lift per unit span is, following Equation [16]:

$$\pi \rho b V e^{i \nu t} (V \Psi - i \nu Z - i \nu \delta l \Psi) \left[C \left(\frac{\nu b}{2V} \right) + \frac{i \nu b}{4V} \right]$$

The corresponding two-dimensional lift calculated on the quasi-steady basis would be:

$$\pi \rho b V e^{i \nu t} (V \Psi - i \nu Z - i \nu \delta l \Psi)$$

So the effect of unsteadiness on this part of the lift is to require that the quasi-steady two-dimensional lift be multiplied by the factor:

$$D = \left[C \left(\frac{\nu b}{2V} \right) + \frac{i \nu b}{4V} \right]$$

In accordance with our assumption that finite wing forces would be corrected for unsteadiness in the same way as the infinite (two-dimensional) wing forces, we multiply Expression [18] by the factor D, so that this contribution to the unsteady lift of the finite wing is taken to be:

$$(\rho V^2)(b \cot \mu) \left[\frac{c'd}{V} (V\Psi - i\nu Z - i\nu\delta l\Psi) \right] e^{i\nu t} \left[C \left(\frac{\nu b}{2V} \right) + \frac{i \nu b}{4V} \right] \qquad [19]$$

In Figure 16, the quantities C and D are plotted in their own complex plane. The dimensionless parameter $\nu b/2V$ varies along each curve, as required by the above expressions for C and D. These plots can be interpreted as follows. If L' is the lift of a two-dimensional wing calculated on quasi-steady assumptions and expressed in complex form then $L'C$ is the lift of the wing when the wake vorticity is considered, and $L'D$ is the lift when both wake vorticity and added mass are included. At frequencies of encounter approaching zero, both C and D approach real values of unity, indicating no effect of either wake or added mass. At larger values of the frequency of encounter, the effect of the wake is seen immediately as the difference between a vector drawn from the origin to the point (1,0) and a vector drawn from the origin to the appropriate point of the C-curve. The combined effect of wake vorticity and added mass is also seen qualitatively by a similar comparison of vectors drawn from the origin to (1,0) and to the appropriate point on the D-curve. At high frequencies of encounter the C-curve approaches the point (1/2,0) while the added mass effect causes the D-curve to extend upward without limit. Thus at extremely high frequencies of encounter, the wake vorticity ceases to cause any further change in lift, while the added mass effect continues to cause higher and higher forces. On the other hand, at small frequencies the added mass effect is small if the wake effect is small.

It is worth noting at this point that both of the quantities C and D depend only on the dimensionless frequency of encounter $\nu b/2V$. Whether the sea is from the bow or the stern does not matter. This is not the case with that portion of the lift considered next.

Expression [13], when written in the complex notation, becomes:

$$(\rho V^2)(b \cot \mu) \left\{ \frac{c'd}{V} [\pm i a \omega A e^{i(\nu t + k\delta l)}] \right\} \qquad [20]$$

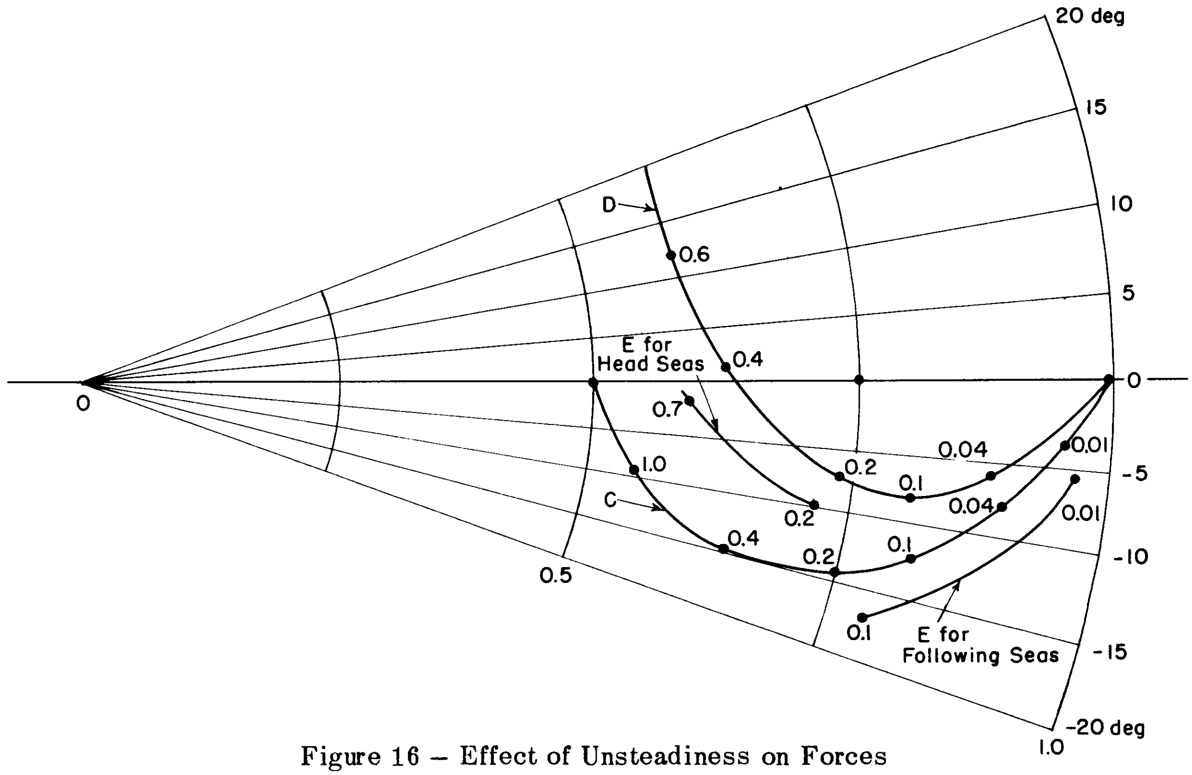


Figure 16 – Effect of Unsteadiness on Forces

See pages 26 and 29 for explanation of figure.

Again, this represents a uniform velocity over the chord. However, it is not difficult at this point to eliminate the requirement that chord length be small compared with the wave length, and an increase in generality of the result is thereby obtained. But when this is done, the vertical water velocity will not be constant over the chord.

For the geometry of Figure 1, the vertical orbital velocity at midchord of each foil (averaged spanwise at any instant) is given by:

$$v_w = \mp a\,\omega\,A \sin(\nu t + k\,\delta\,l)$$

or, in complex notation:

$$v_w = \pm\, i\,a\,\omega\,A\,e^{i(\nu t + k\,\delta\,l)}$$

If we again let ξ represent a coordinate parallel to x, with its origin at the center of either foil, then the vertical orbital velocity at any point of either foil can obviously be written:

$$v_w = \pm\, i\,a\,\omega\,A\,e^{i(\nu t + k\delta l + k\xi)} = \pm\, i\,a\,\omega\,A\,e^{ik\delta l}\,e^{i(\nu t + k\xi)}$$

We define U (see Equation [15] in this case as follows:

$$U = \pm\, i a \omega A e^{i k \delta l}$$

Then the water particle velocity is simply:

$$v_w = U e^{i \nu t} e^{i k \xi}$$

To obtain v_w in the desired form of Equation [15], we transform from ξ to θ:

$$v_w = U e^{i \nu t} e^{-i \left(\frac{kb}{2}\right) \cos \theta}$$

From the following two identities for Bessel functions:

$$e^{i z \cos \theta} = J_0(z) + 2 \sum_{n=1}^{\infty} (i)^n J_n(z) \cos n\theta$$

$$J_n(z) = (-1)^n J_n(-z)$$

we obtain the form required:

$$v_w = U e^{i \nu t} \left[J_0 \left(\frac{kb}{2}\right) + 2 \sum_{n=1}^{\infty} (-i)^n J_n \left(\frac{kb}{2}\right) \cos n\theta \right]$$

When we compare these coefficients with those in Equation [15], we can immediately write down the two-dimensional lift per unit span:

$$\pi \rho b V U e^{i \nu t} \left\{ \left[J_0 \left(\frac{kb}{2}\right) - i J_1 \left(\frac{kb}{2}\right) \right] C \left(\frac{\nu b}{2V}\right) + \frac{i \nu b}{4V} \left[J_0 \left(\frac{kb}{2}\right) + J_2 \left(\frac{kb}{2}\right) \right] \right\}$$

This can be somewhat simplified by use of the following relationships:

$$J_{n-1}(z) + J_{n+1}(z) = \frac{2n}{z} J_n(z)$$

$$\frac{i \nu b}{4V} \left[J_0 \left(\frac{kb}{2}\right) + J_2 \left(\frac{kb}{2}\right) \right] = \frac{i \nu b}{4V} \left[\frac{4}{kb} J_1 \left(\frac{kb}{2}\right) \right] = i \left(\frac{V \pm c}{V}\right) J_1 \left(\frac{kb}{2}\right)$$

The fact that $\nu = kV \pm \omega = k(V \pm c)$ has been used here.

The two-dimensional lift corresponding to Expression [13] is now:*

$$\pi \rho bVU e^{i\nu t} \left\{ \left[J_0 \left(\frac{kb}{2}\right) - i J_1 \left(\frac{kb}{2}\right) \right] C \left(\frac{\nu b}{2V}\right) + i \left(\frac{V \pm c}{V}\right) J_1 \left(\frac{kb}{2}\right) \right\}$$

We note that for the two-dimensional wing, with very low frequencies of encounter and very long wave length, the lift per unit span is:

$$\pi \rho \, bVU \, e^{i\nu t}$$

Unsteadiness and the nonuniformity of velocity over the chord are thus accounted for when the simple lift formula is multiplied by:

$$E = \left\{ \left[J_0 \left(\frac{kb}{2}\right) - i J_1 \left(\frac{kb}{2}\right) \right] C \left(\frac{\nu b}{2V}\right) + i \left(\frac{V \pm c}{V}\right) J_1 \left(\frac{kb}{2}\right) \right\}$$

Again, in accordance with our assumption that the finite wing is affected by unsteadiness (and obviously by the nonuniformity of velocity over the chord) in the same way as the infinite wing, we multiply the quasi-steady finite wing lift, Expression [20], by the factor E to obtain the more general unsteady lift contribution:

$$\pm (\rho V^2)(b \cot \mu) \frac{c' d i a \omega A e^{i(\nu t + k \delta l)}}{V}$$

$$\left\{ \left[J_0 \left(\frac{kb}{2}\right) - i J_1 \left(\frac{kb}{2}\right) \right] C \left(\frac{\nu b}{2V}\right) + i \left(\frac{V \pm c}{V}\right) J_1 \left(\frac{kb}{2}\right) \right\} \qquad [21]$$

Figure 16 shows the complex vector E, as well as C and D. The interpretation of curve E is entirely similar to that of curve D. It is evident that the difference between the two is the result of considering nonuniform velocity distributions over the chord; if this

---

*The two-dimensional unsteady lift formula obtained above differs from Sears' result for a similar case, inasmuch as the arguments of the Bessel functions and the Theodorsen function differ. If we let $K = \nu b/2V$ = the argument of the Theodorsen function, and $M = kb/2$ = the argument of the Bessel functions, we find the following relationship between the two arguments:

$$K = \frac{b}{2V} k (V \pm c) = M \left(\frac{V \pm c}{V}\right)$$

The difference between these quantities results, of course, from the fact that in the present problem the waves themselves have a velocity c, while in Sears' problem the waves were stationary. If we let c approach zero, the above result for unsteady lift becomes identical with Sears'.

generalization had not been made, the last lift contribution above would have reduced to the same form as the previous contributions. In this case there is a separate curve for head and following seas, even if the frequency of encounter should be the same for the two situations, since the curve depends on two parameters, $\nu b/2V$ and $V \pm c/V$. The curves shown in Figure 16 represent the conditions of the experiments of Reference 2, in which $V = 5$ ft/sec and the wave lengths varied from 2 to 5 ft for head seas and from 2 to 4 ft for following seas.

It is seen that the forces due both to foil motions and to water orbital motions are reduced as much as 40 percent in head seas and 15 percent in following seas. The phases are altered as much as 18 deg.

The complete expression for the time-dependent unsteady lift is obtained by adding Expressions [14] (with the sinusoidal solutions substituted), [19], and [21]:

$$L = (\rho V^2)(b \cot \mu)\left(\frac{c'd}{V}\right)\left[V\Psi - i\nu Z - i\nu\delta l\Psi\right]\left[C\left(\frac{\nu b}{2V}\right) + \frac{i\nu b}{4V}\right]e^{i\nu t}$$

$$\pm(\rho V^2)(b \cot \mu)\left(\frac{i a \omega A c'd}{V}\right)e^{ik\delta l}\left\{\left[J_0\left(\frac{kb}{2}\right) - iJ_1\left(\frac{kb}{2}\right)\right]C\left(\frac{\nu b}{2V}\right) + i\left(\frac{V \pm c}{V}\right)J_1\left(\frac{kb}{2}\right)\right\}e^{i\nu t}$$

$$-(\rho V^2)(b \cot \mu)(c_0)\left[Z + \delta l\Psi - a e^{ik\delta l}\right]e^{i\nu t} \qquad [22]$$

The sinusoidal solutions can now be substituted into the equations of motion, along with Equation [22] for the lift on each foil:

$$-m\ddot{z} + \sum_{f,\,a} L = 0$$

$$-I\ddot{\psi} + \sum_{f,\,a} \delta l L = 0$$

(The steady lift is still ignored; thus the weight of the craft does not appear in the heave equation.)

These equations have been solved digitally and the amplification factors and phases obtained for the same conditions as the experiments of Reference 2. In spite of the large effect of unsteadiness on the forces as indicated by Figure 16 the net effect on the solutions, as compared with the simple linearized quasi-steady Weinblum solutions, is quite small. In fact, in head seas (where the forces were reduced up to 40 percent) the amplification factors are changed negligibly, and in following seas they are reduced by 15 percent or less.

The small net effect of unsteadiness seems at first to be most remarkable, when we consider the large changes that actually occur in the forces. However, we can see qualitatively that this result could be reasonable. Four types of forces are involved: (1) wave forces depending on the orbital motion, (2) the forces due to varying trim attitude and vertical velocity of the foils, (3) the forces due to varying span, and (4) the inertial forces of the craft. If all four types were changed in the same ratio by unsteadiness, there would be no net effect on the resulting motions. In head seas, the responses are determined predominantly by forces of the first two types, and Figure 16 shows that they are altered in much the same manner, although there is a small phase difference. In following seas, the varying span forces are quite significant, and since these were not modified for unsteadiness the heave and pitch amplitudes do show a more marked change. It should be noted that, in view of the uncertainty about how to handle the forces due to varying span, these following seas predictions are less reliable than the head seas predictions.

It would be reasonable to assume that, in a situation in which the inertia were more important, there would be further changes. Such a situation would be most likely to occur at high speeds. In any particular problem the possibility of such changes should be carefully considered before unsteadiness is neglected, although such neglect will generally yield conservative estimates of craft response.

TRANSIENT RESPONSES

For the case of very small motions, the nature of transients can be found and described simply. Following the classical procedure, one sets the forcing functions in Equations [6] and [7] equal to zero (that is, all terms relating to wave motion of the water). Since the craft motions are assumed small, only the linear terms in z and ψ (and their derivatives) are retained. This does not alter the order of the equation, so the initial conditions can still be satisfied as generally as before.

One thus obtains two linear equations, homogeneous in z and ψ. The following assumed solutions are substituted into the equations:

$$z = Z\, e^{\sigma t}$$

$$\psi = \Psi\, e^{\sigma t}$$

yielding two algebraic equations:

$$(\sigma^2 + \alpha_1 \sigma + \alpha_2)\, Z + (\alpha_3 \sigma + \alpha_4)\, \Psi = 0 \qquad [23]$$

$$(\gamma_3 \sigma + \gamma_4)\, Z + (\sigma^2 + \gamma_1 \sigma + \gamma_2)\, \Psi = 0 \qquad [24]$$

The α's and γ's are constants.

For the existence of nontrivial solutions, it is necessary and sufficient that the determinant of the coefficients of Z and Ψ vanish:

$$\begin{vmatrix} (\sigma^2 + \alpha_1 \sigma + \alpha_2) & (\alpha_3 \sigma + \alpha_4) \\ (\gamma_3 \sigma + \gamma_4) & (\sigma^2 + \gamma_1 \sigma + \gamma_2) \end{vmatrix} = 0 \qquad [25]$$

This fourth-degree equation is then solved for the four roots of σ.

For the particular hydrofoil configuration considered throughout this report Equations [23], [24], and [25] are:

$$(\sigma^2 + 62.29\,\sigma + 58.55)\, Z + (-311.4)\, \Psi = 0$$

$$(\sigma^2 + 116.8\,\sigma + 109.8)\, \Psi = 0 \qquad [26]$$

$$\sigma^4 + 179\,\sigma^3 + 7440\,\sigma^2 + 13{,}680\,\sigma + 6430 = 0$$

The four roots are:

$$\sigma_{1,2} = -0.95$$

$$\sigma_3 = -115.9$$

$$\sigma_4 = -61.3$$

The existence of a double root is an accident, resulting from the special geometry of this craft. Generally there will be either four different real roots or two real roots and a complex pair. In the present case, the existence of the double root indicates that the assumed solution was not quite general enough and that there must also be terms of the following type:

$$Z_2\, t\, e^{\sigma_1 t} \;;\; \Psi_2\, t\, e^{\sigma_1 t}$$

This is of no general importance.

In the case of four distinct roots we can find a pair of constants, Z_i and Ψ_i, associated with each root. Thus there are eight constants. However, from Equation [26], four of these coefficients can be determined in terms of the other four. So there are four arbitrary constants in the solution, as one would expect from the nature of the differential equations. For the special case under consideration, we find:

$$\Psi_1 = 0.194\, Z_2$$

$$\Psi_2 = 0$$

$$\Psi_3 = 20.1\ Z_3$$

$$\Psi_4 = 0$$

The solutions are, explicitly:

$$z = (Z_1 + Z_2\, t)\, e^{-0.95\, t} + Z_3\, e^{-115.9\, t} + Z_4\, e^{-61.3\, t}$$

$$\psi = (0.194\, Z_2)\, e^{-0.95\, t} + (20.1\, Z_3)\, e^{-115.9\, t}$$

It is obvious that almost any initial condition will cause an exponential transient, decaying with a time constant of 1.05 sec. Only if Z_2 is much larger than Z_1 will this *not* be true. The terms containing Z_3 and Z_4 allow the initial conditions to be satisfied, but they decay so quickly that they will hardly be observable experimentally.

The nonlinear equations of Weinblum provide practically no extra general information. For any given set of initial conditions, the heave and pitch responses can be found with the analog computer, but because the equations are nonlinear the solutions so obtained cannot be superposed to yield other solutions.

However, the transient responses are not greatly altered by the existence of nonlinear forces. Figure 17 shows explicitly the nonlinearity effect in a simple case: a moderate initial heave displacement with all other initial conditions zero. The heave response is plotted for positive and negative initial heave, as predicted by both linear and nonlinear equations.

Other nonlinear transient responses were plotted with the analog computer. As was observed in the above case, they were quite similar to the linear responses, and since they cannot be superposed they are not reproduced here.

Finally, the effect of unsteadiness on the transient responses was considered. General conclusions can be reached from a study of the linear equations again, in view of the above result that the quasi-steady linear and nonlinear calculations do not differ greatly.

We start out immediately with the equations simplified for the symmetrical model considered throughout this report. For this model,Weinblum's equations become, after linearization:

$$m\ddot{z} - 2B'd\psi + \frac{2B'd}{V}\dot{z} + 2B_0 z = 0$$

$$I\ddot{\psi} + \frac{2B'd\,l^2}{V}\dot{\psi} + 2B_0 l^2 \psi = 0$$

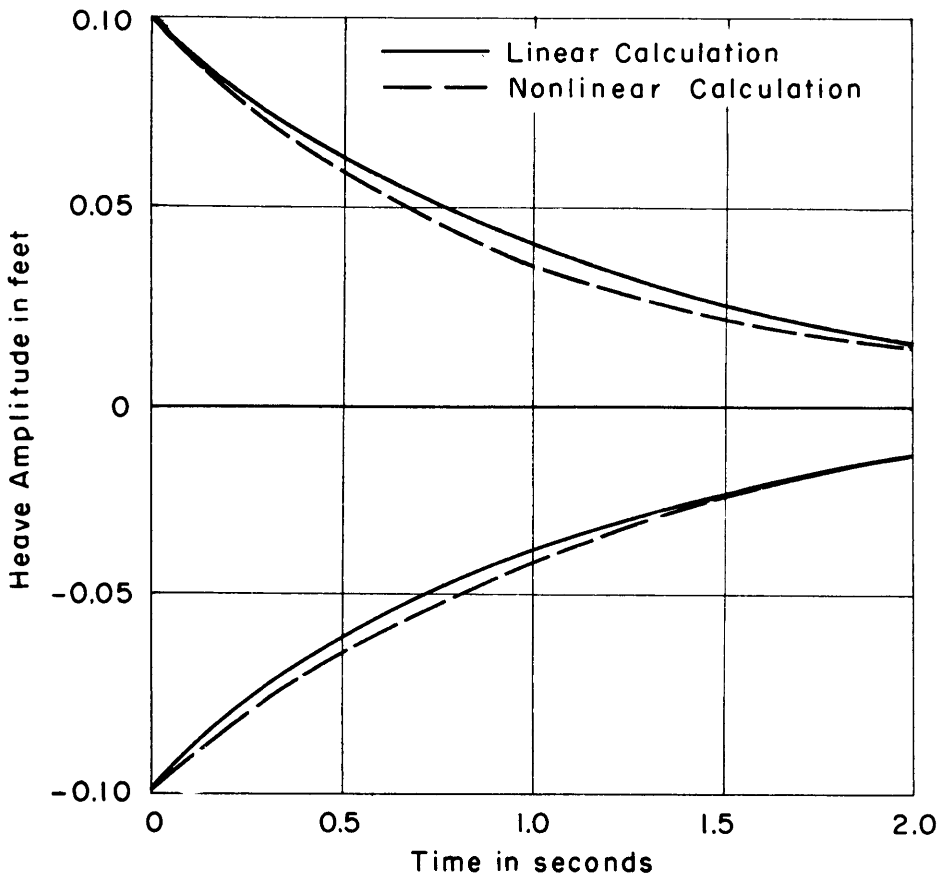


Figure 17a – Linear and Nonlinear Calculations

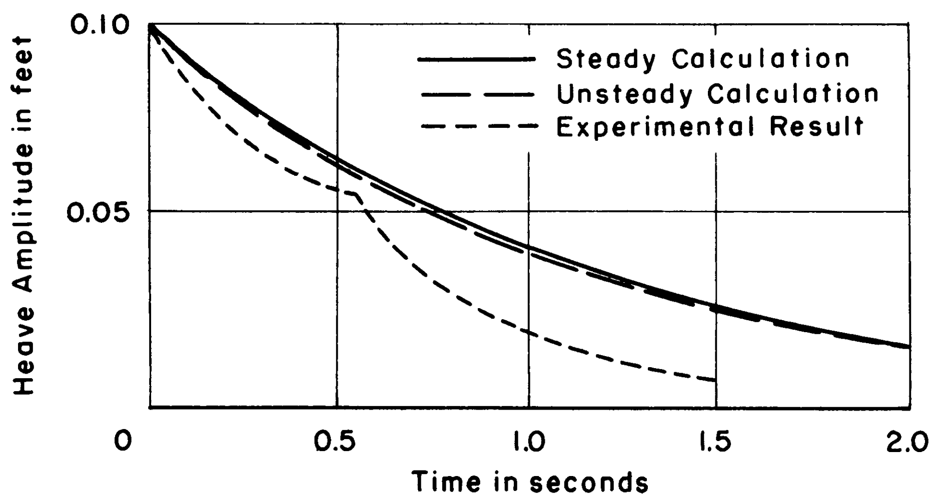


Figure 17b – Linear Steady and Unsteady Calculations and Experimental Result

Figure 17 – Initial Heave Transient Response

where $B' = (\rho V^2)(b \cot \mu)\, c'$, and $B_0 = (\rho V^2)(b \cot \mu)\, c_0$.
It is convenient to express these in terms of the nondimensional time,

$$\eta = \frac{2V}{b}\, t$$

Then the equations are:

$$\frac{4V^2 m}{b^2} z'' - 2B'd\psi + \frac{4B'd}{b} z' + 2 B_0 z = 0$$

$$\frac{4V^2 I}{b^2} \psi'' + \frac{4B'dl^2}{b} \psi' + 2 B_0 l^2 \psi = 0$$

As in the case of the wave responses, we correct the terms corresponding to varying angle of attack, but we retain in quasi-steady form the terms resulting from variation in span. We consider possible nonzero initial values of heave, heave velocity, pitch, and pitch velocity. We arbitrarily assume that at time $t = 0$ the lift forces corresponding to these conditions have reached their steady values. Then the equations can be written in the following form:

$$\frac{4V^2 m}{b^2} z'' - 2B'd\,[\psi - \psi(0)] + \frac{4B'd}{b}\,[z' - z'(0)] + 2B_0 z = 2B'd\psi(0) - \frac{4B'd}{b} z'(0) \qquad [27]$$

$$\frac{4V^2 I}{b^2} \psi'' + \frac{4B'dl^2}{b}\,[\psi' - \psi'(0)] + 2B_0 l^2 \psi = -\frac{4B'dl^2}{b} \psi'(0) \qquad [28]$$

The terms on the left of each equation represent the inertial forces and the time-dependent hydrodynamic forces, and the terms on the right specify the steady forces. The latter can be treated as step functions, for the purpose of the transient analysis.

Two changes must be effected to correct these equations for unsteadiness: (1) the mass and moment of inertia must be increased to include the added mass forces, and (2) the terms resulting from variation of the angle of attack must be changed to include wake effects.

The added mass is taken as the two-dimensional mass times the equilibrium span:

$$\Delta m = 2 \left[\pi \rho \left(\frac{b}{2}\right)^2\right](2d \cot \mu) = 0.093 \text{ slug}$$

(The extra factor of 2 enters because there are two identical foils.)
The total mass m' is then:

$$m' = m + \Delta m = 0.126 + 0.093 = 0.219 \text{ slug}$$

The moment of inertia becomes:

$$I' = I + \Delta I = 0.151 + (\Delta m)l^2 = 0.362 \text{ slug-ft}^2$$

In computing the added moment of inertia, the length l was used rather than the diagonal length from the center of gravity to the foils, because the added mass acts only for vertical translations.

The second and third terms in Equation [27] and the second term in Equation [28] result from the changing angle of attack. In unsteady flow theory, they become Duhamel integral terms. We now obtain these terms explicitly.

The lift on an airfoil following a unit step change in angle of attack is proportional to the Wagner function $\phi(\eta)$. This function has a value of 0.5 at $\eta = 0$ and it approaches unity at large times. It is derived by superposing the effects of all of the steady-state oscillations which combine in a Fourier integral representation of a step function. One fundamental formula for the function is:[6]

$$\phi(\eta) = \frac{2}{\pi} \int_0^\infty Re\,[C(K)]\, \frac{\sin K\eta}{K}\, dK$$

where $K = \nu b/2V$ and C = Theodorsen function.

A convenient approximation for $\phi(\eta)$ is:[6]

$$\phi(\eta) \cong 1 - 0.165\, e^{-0.0455\,\eta} - 0.335\, e^{-0.3\,\eta}$$

This approximation suffers in that it does not give a proper limiting behavior for large times, but it has the great advantage of having a simple Laplace transform. It should be noted that in true time the exponents are $-2.73\,t$ and $-18.0\,t$, respectively, in the approximation. These quantities indicate how quickly the actual circulatory lift approaches the quasi-steady lift predictions. Since it has already been found that the important transient in the quasi-steady linear analysis is an exponential $e^{-0.95\,t}$, it would not be expected that a lag in lift buildup specified by the exponential $e^{-2.73\,t}$ would have much effect. Solutions of the equations will confirm this prediction.

The actual lift on the foil is obtained through a Duhamel integral representation, with the function $\phi(\eta - \eta_1)$ serving as the indicial admittance. The terms of interest change as follows:

$$2B'd\,[\psi - \psi(0)] \rightarrow \int_0^\eta 2B'd\, \frac{d}{d\eta_1}\, [\psi - \psi(0)]\, \phi(\eta - \eta_1)\, d\eta_1$$

$$\frac{4B'd}{b}\, [z' - z'(0)] \rightarrow \int_0^\eta \frac{4B'd}{b}\, \frac{d}{d\eta_1}\, [z' - z'(0)]\, \phi(\eta - \eta_1)\, d\eta_1$$

$$\frac{4B'dl^2}{b}\left[\psi'-\psi'(0)\right] \rightarrow \int_0^{\eta} \frac{4B'dl^2}{b}\frac{d}{d\eta_1}\left[\psi'-\psi'(0)\right]\phi(\eta-\eta_1)\,d\eta_1$$

We make the following definitions and then write the complete equations for unsteady transient conditions:

$$A_1 \equiv \frac{4V^2m'}{b^2} = 788 \qquad C_1 \equiv \frac{4V^2I'}{b^2} = 1303$$

$$A_2 \equiv \frac{4B'd}{b} = 470 \qquad C_2 \equiv \frac{4B'dl^2}{b} = 1058$$

$$A_3 \equiv 2B_0 = 7.37 \qquad C_3 \equiv 2B_0 l^2 = 16.59$$

$$A_4 \equiv 2B'd = 39.2$$

$$A_1 z'' + \int_0^{\eta} \frac{d}{d\eta_1}\left\{A_2\left[z'-z'(0)\right] - A_4\left[\psi-\psi(0)\right]\right\}\phi(\eta-\eta_1)\,d\eta_1 + A_3 z =$$

$$A_4\psi(0) - A_2 z'(0)$$

$$C_1\psi'' + \int_0^{\eta} \frac{d}{d\eta_1}\left\{C_2\left[\psi'-\psi'(0)\right]\right\}\phi(\eta-\eta_1)\,d\eta_1 + C_3\psi = -C_2\psi'(0)$$

These are solved straightforwardly by the application of Laplace transforms, which are used particularly to render the convolution integrals tractable. The transform of the approximate Wagner function is:

$$\bar{\phi}(s) = \frac{1}{s} - \frac{0.165}{s+0.0455} - \frac{0.335}{s+0.3}$$

where s is the argument of the transforms, and the bar over the symbol $\phi(s)$ indicates the transform:

$$\mathcal{L}\left[f(\eta)\right] \equiv \bar{f}(s)$$

After being reorganized, the transform equations are:

$$\bar{z}(s)\,[A_1 s^2 + A_3 + A_2 s^2 \bar{\phi}(s)] + \bar{\psi}(s)\,(-A_4 s) =$$

$$z(0)\,[A_1 s + A_2 s \bar{\phi}(s)] + z'(0)\left[A_1 + A_2 \bar{\phi}(s) - \frac{A_2}{s}\right] + \psi(0)\left[-A_4 \bar{\phi}(s) + \frac{A_4}{s}\right]$$

$$\bar{z}(s)\,(0) + \bar{\psi}(s)\,[C_1 s^2 + C_3 + C_2 s^2 \bar{\phi}(s)] =$$

$$\psi(0)\,[C_1 s + C_2 s\, \bar{\phi}(s)] + \psi'(0)\left[C_1 + C_2 \bar{\phi}(s) - \frac{C_2}{s}\right]$$

Finally, with the numbers given above for the A's and C's, and with $\bar{\phi}(s)$ explicitly substituted in, we have:

$$\bar{z}(s)\,(788\, s^5 + 507\, s^4 + 150.9\, s^3 + 8.96\, s^2 + 0.1006\, s) +$$

$$\bar{\psi}(s)\,(-39.22\, s^4 - 13.55\, s^3 - 0.535\, s^2) =$$

$$z(0)\,(788\, s^4 + 507\, s^3 + 142.8\, s^2 + 6.42\, s) +$$

$$z'(0)(788\, s^3 + 37.25\, s^2 - 19.56\, s) + \psi(0)\,(19.61\, s^2 + 2.53\, s)$$

$$\bar{z}(s)\,(0) + \bar{\psi}(s)\,(1303\, s^5 + 979\, s^4 + 332\, s^3 + 20.2\, s^2 + 0.226\, s) =$$

$$\psi(0)\,(1303\, s^4 + 979\, s^3 + 315\, s^2 + 14.44\, s) + \psi'(0)\,(1303\, s^3 - 78.8\, s^2 + 50.5\, s)$$

If we make the following definitions:

$$\alpha_1(s) = 788\, s\,(s^3 + 0.643\, s^2 + 0.181\, s + 0.00815)$$

$$\alpha_2(s) = 788\, s\,(s^2 + 0.0473\, s - 0.0248)$$

$$\alpha_3(s) = 19.61\, s\,(s + 0.129)$$

$$\beta_3(s) = 1303\, s\,(s^3 + 0.751\, s^2 + 0.242\, s + 0.0111)$$

$$\beta_4(s) = 1303\, s\,(s^2 - 0.0605\, s + 0.0388)$$

We can write out the solutions explicitly:

$$\bar{z}(s) = \frac{\begin{vmatrix} [z(0)\,\alpha_1(s) + z'(0)\,\alpha_2(s) + \psi(0)\,\alpha_3(s)] & [-39.22\, s^2 (s^2 + 0.345\, s + 0.0136)] \\ [\psi(0)\beta_3(s) + \psi'(0)\beta_4(s)] & \begin{matrix}[1303\, s(s^4 + 0.751\, s^3 + 0.255\, s^2 \\ + 0.0155\, s + 0.000173)]\end{matrix} \end{vmatrix}}{D}$$

$$\bar{\psi}(s) = \frac{\begin{vmatrix} \begin{matrix}[788\, s\,(s^4 + 0.643\, s^3 + 0.191\, s^2 \\ + 0.00114\, s + 0.000128)]\end{matrix} & [z(0)\,\alpha_1(s) + z'(0)\,\alpha_2(s) + \psi(0)\,\alpha_3(s)] \\ 0 & [\psi(0)\beta_3(s) + \psi'(0)\beta_4(s)] \end{vmatrix}}{D}$$

where $D \equiv 1.027 \times 10^6 s^2 (s + 0.0147)^2 (s + 0.0583)^2 (s^2 + 0.678\, s + 0.205)(s^2 + 0.570\, s + 0.149)$.

Thus the transforms $\bar{z}(s)$ and $\bar{\psi}(s)$ have poles at:

$s = 0$ (double); -0.0147 (double); -0.0583 (double); $-0.285 \pm 0.260i$; $-0.339 \pm 0.300i$.

The inverse transforms are most easily found by writing both solutions in partial fractions. However, it is not necessary to evaluate the entire expressions to observe the nature of the solutions. The double pole at the origin would indicate a constant term and a term proportional to time. But these terms do not materialize, since this pole is canceled by the corresponding double zero at the origin in the numerator. Of the remaining terms, the most important will be proportional to $e^{-0.0147\eta}$ and $e^{-0.0583\eta}$. In real time, these decay exponentials are $-0.882\,t$ and $-3.50\,t$. The first of these is only slightly different from the quasi-steady solution, in which the exponent was $-0.95\,t$. It thus appears possible that unsteadiness will not alter the solutions greatly.

We now consider a special case to show that this prediction is actually true: the case in which there is an initial heave amplitude only. The solution for $\bar{z}(s)$ simplifies considerably:

$$\bar{z}(s) = z(0)\left[\frac{s^3 + 0.643\, s^2 + 0.181\, s + 0.00815}{(s + 0.0147)(s + 0.0583)(s^2 + 0.570\, s + 0.149)}\right]$$

When this is expanded into partial fractions, the inverse transforms taken, and the solution expressed in terms of real time t, we have our final result for this case:

$$z(t) = z(0)\left\{0.915\, e^{-0.882\, t} + 0.0798\, e^{-3.50\, t} + [0.000836 \cos(15.6\, t) + 0.0714 \sin(15.6\, t)]\, e^{-17.1\, t}\right\}$$

Thus it is actually found for this case that the wake effects are small, since this solution is quite similar to that found for the corresponding quasi-steady case. (See Figure 17b.) The components with fast decay are grossly different, to be sure, but they have little effect on the general nature of the response.

EFFECT OF HORIZONTAL ORBITAL MOTIONS

Equations [2] and [3] gave the wave height and vertical orbital velocities at each foil. To these equations we must add a third, the equation for the horizontal component of orbital velocity:

$$u_w = \mp a\,\omega\, e^{-k\zeta} \cos(\nu t + k\,\delta\, l)$$

The exponential factor can again be averaged over the span and replaced by A, its mean value.

The expression for lift, Equation [5], can be modified easily to include the effects of these horizontal velocities. The forward velocity is effectively changed, so that the lift can be written:

$$L = \frac{1}{2}\rho\,(V - u_w)^2\, S\,(c_0 + c'\alpha)$$

$$\cong \frac{1}{2}\rho\,(V^2 - 2\,u_w V)\, S\,(c_0 + c'\alpha)$$

if u_w/V is much less than unity. With this approximation (which is generally quite valid), the lift is:

$$L \cong \left(\frac{1}{2}\rho V^2\right) S \left\{ c_0 \left(1 - \frac{2\,u_w}{V}\right) + c' \left[\psi - \frac{1}{V}\,(\dot{z} + \delta\, l\,\dot{\psi} - v_w)\right]\right\}$$

Consider now only the orbital motion terms:

$$L_{o.m.} = \left(\frac{1}{2}\rho V^2\right) S \left[\frac{c' v_w}{V} - \frac{2\,c_0\,u_w}{V}\right]$$

The effect of the horizontal component of orbital velocity is obviously small only if the inequality,

$$2\,c_0 \left|\frac{u_w}{V}\right| << c' \left|\frac{v_w}{V}\right| \qquad [29]$$

is satisfied. It can be seen immediately that the inequality can be reduced to:

$$\frac{2\,c_0}{c'} \ll 1$$

Actually, for the case considered in this report, $2\,c_0/c' = 0.207$, which is not really small enough to be neglected.

The orbital motion lift forces can be expressed:

$$L_{o.m.} = \mp \left(\frac{1}{2}\,\rho V^2\right) \frac{S\,a\,\omega\,A}{V} \left[c' \sin\,(\nu t + k\,\delta\,l) - 2\,c_0 \cos\,(\nu\,t + k\,\delta\,l)\right]$$

$$= \mp \left(\frac{1}{2}\,\rho V^2\right) \frac{a\,\omega\,A\,S\,c'}{V} \sqrt{1 + 4\,(c_0/c')^2}\,\sin\,(\nu\,t + k\,\delta\,l - \gamma)$$

where $\gamma = \arctan\,(2\,c_0/c')$.

For our special case

$$\sqrt{1 + 4\,(c_0/c')^2} = 1.02$$

$$\gamma = 11.7 \text{ deg}$$

Thus the amplitude of the orbital motion forces is practically unchanged but the phase is retarded by about 12 deg. It has already been observed that the heave and pitch responses in head seas are controlled mostly by the orbital motion forces, and that a change in either magnitude or phase of these forces is reflected directly into the calculated magnification factors and phase leads. A sample calculation, for the condition that $\lambda = 3.0$ ft, does indeed yield this result; that is, that the heave phase is retarded by about 12 deg. Thus the effect of the horizontal components of velocity is not negligible, and the calculations of the previous sections would have been significantly improved had they been included.

Evidently, this effect decreases in importance as the velocity of advance increases.

EFFECT OF DOWNWASH

There are three components of the downwash behind an airfoil in unsteady motion: (1) the steady vorticity, which has its axis parallel to the direction of motion; (2) the unsteady vorticity in this same direction; and (3) the unsteady vorticity with axis parallel to the spanwise direction of the wing. This distinction is, of course, arbitrary, especially as it concerns the second and third components. But it is convenient to consider them individually in computing their effects on the after foil.

We will not consider the steady component inasmuch as it simply changes the lift coefficient of the after foil somewhat and we assume that this lift coefficient is adjusted further to compensate for it. This change in the lift coefficient causes some second-order effects, since it modifies the lift terms resulting from varying span, but the affected terms are fairly small generally and the change in these terms should be quite negligible.

We consider in detail the third component. It will be of interest to estimate the vertical velocities induced at the after foil by the distributed vorticity from the forward foil, and we make a first-order approximation of these induced velocities by assuming that whatever change of lift they may cause does not in turn affect the vorticity shed by the forward foil. The practical effect of this assumption is that we calculate the downwash from the forward foil as if it were alone in the fluid, and we assume that this downwash induces certain vertical velocities at the position of the after foil. These velocities are then used to compute any changes in the lift of the after foil.

From Reference 5 we quote the result that if the quasi-steady circulation about a foil is:

$$\Gamma = G\, e^{i\nu t}$$

then the vorticity distribution in the wake is:

$$\gamma(\xi) = -\frac{G}{K_0\left(\frac{i\nu b}{2V}\right) + K_1\left(\frac{i\nu b}{2V}\right)}\, e^{i\nu\left(t - \frac{\xi}{V}\right)} \qquad [30]$$

These formulas apply to a foil traveling in the negative ξ-direction. The trailing edge of the foil is located at $\xi = b/2$. Once we find the induced vertical velocity at the center of the after foil, it will not matter in which direction the foils are moving, so we do not bother to change any of the formulas before applying them to our foils, which we have taken to be traveling toward positive ξ.

We note that the exponential in Equation [30] above can be rewritten:

$$e^{i\nu\left(t - \frac{\xi}{V}\right)} = e^{i\left[\nu t - k\left(\frac{V \pm c}{V}\right)\xi\right]} = e^{i(\nu t - k'\xi)}$$

Thus the vortices will be shed with a wave length:

$$\lambda' = \lambda\left(\frac{V}{V \pm c}\right)$$

They stand still in space (except for a small vertical motion) and the second foil encounters them with a frequency:

$$\nu' = \frac{2\pi}{\lambda'} V = k(V \pm c) = \nu$$

This is expected, of course.

We next obtain G, for later use. We note that the lift per unit span of a two-dimensional ideal airfoil is $L' = \rho V \Gamma$. The average lift per unit span of our hydrofoil is approximately $L' = c'(1/2\,\rho V^2)\,b\alpha$. If we consider each spanwise section of the actual hydrofoil as part of a two-dimensional airfoil, we can consider the circulation about it to be given by:

$$\Gamma = \frac{c'\left(\frac{1}{2}\rho V^2\right) b\alpha}{\rho V} = \frac{1}{2} c' V b\alpha$$

(We are considering only the time-dependent part here, of course.)
Explicitly, we now have for Γ_f and $\gamma(\xi)$:

$$\Gamma_f = G_f e^{i\nu t} = \frac{c'}{2}\, b\, (V\Psi - i\nu Z - i\nu l_f \Psi \pm i a \omega A_f e^{ikl_f})\, e^{i\nu t} \qquad [31]$$

$$\gamma(\xi) \equiv g\, e^{i(\nu t - k'\xi)}$$

$$= -\frac{(c'/2)\,b}{K_0 + K_1} (V\Psi - i\nu Z - i\nu l_f \Psi \pm i a \omega A_f e^{ikl_f}) e^{i(\nu t - k'\xi)} \qquad [32]$$

Positive circulation is taken clockwise.

This would be the vorticity distribution if there were no dissipation in the wake. Actually the amplitude of the vorticity decays rapidly, and we must estimate this rate of decay before calculating the induced velocity at the after foil. From Reference 7 we find the rate of decay of energy of the discrete vortices shed from a cylinder to be given by:

$$\text{Energy density of vorticity} \propto e^{-x/12\,b}$$

where x is the distance downstream from the point of generation of the vortices. The vorticity described in Reference 7 is formed by a different mechanism than in the present problem, and the Karman vortex street is not a plane of vorticity, as is assumed to be the case with the hydrofoil wake. However, the data presented in the reference are the only known measurements concerning the rate of decay of vorticity in a wake of any kind, and it is assumed here that the result can be carried over to the present problem. The different generating mechanism should not matter greatly, and the fact that the Karman vortices are not in a plane is assumed not to be an important determining factor in their decay. In any case, the assumption of a flat wake for the hydrofoil is only a convenience.

The data of Reference 7 apply to Reynolds numbers up to 10^4. In the special case considered here, this parameter is about 5×10^4, so the results of the reference must be extrapolated somewhat. This is probably valid, in view of the fact that the dependence of the decay on Reynolds number is reported to be very weak in the range of large Reynolds numbers.

The energy of a vortex is proportional to the square of the vorticity, so we take the rate of decay of vorticity to be:

$$\gamma(\xi) \propto e^{-\left(\frac{\xi - b/2}{24\, b}\right)}$$

The vorticity is assumed to be generated at the trailing edge of the forward foil, which has the coordinate $\xi = b/2$. The exponential above is entered as a factor in $\gamma(\xi)$:

$$\gamma(\xi) = g\, e^{i(\nu t - k'\xi)}\, e^{1/48}\, e^{-\xi/24\, b}$$

We now let v_i be the vertical induced velocity at the after foil. From the Biot-Savart Law, it is given by:

$$v_i = -\frac{g}{2\pi}\, e^{1/48}\, e^{i\nu t} \int_{b/2}^{\infty} \frac{e^{-ik'\xi}\, e^{-\xi/24\, b}\, d\xi}{\xi_1 - \xi}$$

This integral has an infinite integrand at $\xi = \xi_1$, where ξ_1 is the distance between foil centers (equal to $2\, l$ in the present case). However, we note that the integrand can be rewritten so that its imaginary part is regular at this point, and the real part is odd with respect to $(\xi - \xi_1)$ for a small neighborhood of the point ξ_1. Therefore it may be expected that the principal value of the integral exists and that it represents the true physical conditions.

The induced velocity at the after foil was calculated numerically for a special case: $\lambda = 3.0$ ft, head seas. The integral above was found to equal $(-1.49 + 0.20\, i)$, so that the induced velocity is:

$$v_i = +\frac{g}{2\pi}(1.52 - 0.20\, i)\, e^{i\nu t}$$

The quantity g is evaluated from Equation [32]. It is necessary to use the relations for the Bessel functions, K_0 and K_1:

$$K_0(iz) = -\frac{\pi}{2} Y_0(z) - i\frac{\pi}{2} J_0(z)$$

$$K_1(iz) = -\frac{\pi}{2} J_1(z) + i\frac{\pi}{2} Y_1(z)$$

so that $(K_0 + K_1) = (1.05 - 5.42i)$. Finally, the velocity induced at the after foil by the distributed downwash from the forward foil is:

$$v_i = (0.00729 + 0.0219i)\, e^{i\nu t}(-V\Psi + i\nu Z + i\nu l_f \Psi + i a \omega A_f)$$

At the same time that this correction is made we should include the velocities induced at each foil by the vorticity around the other foil. Thus, associated with Γ_f at the forward foil (Equation [31]), there is an induced velocity at the after foil:

$$-\frac{G_f}{2\pi\xi_1}\, e^{i\nu t}$$

There is a similar induced velocity at the forward foil.

When these induced velocities are added to the other components of relative velocity at the after foil and the equations are solved for heave and pitch, it is found that there is negligible difference between this result and the previous calculation. Therefore we conclude that it is reasonable to ignore this component of downwash.

The unsteady components of downwash which consist of vortices parallel to the direction of motion should have an effect of the same order of magnitude as the vortices perpendicular to the direction of motion. This conclusion is based partly on the assumption that the orientation of the axis of a vortex does not greatly affect its rate of decay in a wake. A quantitative evaluation of the effect of this component would depend heavily on the three-dimensional properties of the hydrofoil, which are not well understood. Therefore, we do not consider it further, but assume that it is probably no more significant than the component considered in detail above.

COMPARISON OF THEORY AND EXPERIMENT

The experiments, which have been referred to several times, are described in detail in the report by Leehey and Steele.[3] Figure 2 is a simplified drawing of their apparatus for measuring heave and pitch.

The experimental data have been plotted on the same figures in this report which show the theoretical predictions. (See Figures 4 to 7, 9 to 12, and 17.) It can be seen that the agreement is good, with the following exceptions:

1. The experimental heave and pitch phase leads in head seas are about 30 deg less than the best theoretical predictions.

2. For the head seas wave length which equals the foil spacing, the prediction of very small pitch amplitude is not realized.

3. In following seas, the predicted heave amplitudes are too high by as much as 15 percent.

4. The observed response to a heave transient is damped much more quickly than is predicted.

A possible cause of the first difference is the neglect of waves generated by the forward foil, with their effect on the after foil. Since the craft is oscillating at the frequency of encounter, it will generate waves at this frequency. Therefore the after foil will experience periodic wave forces that are somewhat different from those used in the calculations. This difference in the after foil forces could possibly cause the observed phase angles. To the extent that this is true, the present theory must be considered as incomplete.

There is some ambiguity in the test results concerning the pitch amplitude for the condition that the wave length equals the foil spacing. The oscillograph records provide the data shown in Figure 6, but motion pictures of the tests show the existence of negligible pitch for this condition. It is noted that the pitch record was obtained by taking the difference of two electrical signals, both of which contained heave signal. Furthermore, the pitch signal for the condition in question is just 180 deg out of phase from the heave signal. Therefore it seems possible that the measurements are in error, and in view of the evidence provided by the motion pictures it is concluded that the data do not invalidate the theory for this condition.

The third point of disagreement, concerning heave amplitudes in following seas, may possibly be due to the existence of further nonlinearities in the system, which were not considered in this report. For example, the lift-curve slope is not actually constant, but depends in some manner on aspect ratio. Also, unsteadiness effects on the varying-span-forces were not calculated; this is a possible source of error.

It is also possible that the experimental results may again be at fault. At the low frequencies of encounter involved, a small amount of damping in the bearing pivots could cause a significant reduction in the measured amplitude. The experimenters have indicated that the craft constantly tended to yaw, and it might be expected that this would cause at least a small amount of binding in the bearings.

The reason for the failure of the theory to predict the transient response is not known. The theory for this type of problem is entirely similar to the theory for wave response, and its rather great success in the latter case indicates that the theory is probably not seriously in error.

CONCLUSIONS

It has been shown that the theory developed in this report can be applied to the conditions of the tests of Reference 2 with considerable confidence. It is believed with reasonable certainty that the theory can be applied to situations in which the conditions are quite different. Some restrictions have been noted.

In particular, the following points have been discussed:

1. A linear, quasi-steady theory, such as Weinblum's, gives very good predictions of heave amplitudes in head seas and fair predictions in following seas. The accuracy of pitch predictions is uncertain because of possible errors in the experimental data, but since the theory is so successful for heave predictions it is reasonable to assume that it will give valid results for pitch also. However, it must be emphasized that Weinblum's equations must be modified to include the effect of depth on the water orbital velocities. Without this modification, the predicted amplitudes of motion in typical cases are about twice as large as they should be.

2. The predicted heave and pitch phases are about 50 deg in error for head sea conditions, when calculated as described above. The cause is not entirely known. One possibility that was not considered is that the waves generated by the forward foil, acting on the after foil, are responsible. Such waves were not considered in the theory.

3. Neglect of the horizontal component of wave orbital motion, as in Weinblum's equations, causes about 12 deg phase error for the head seas conditions studied. This error should be smaller for following seas conditions. In general, it will be most significant in situations in which the lift coefficient is high and the lift-curve slope is small; that is, at low speeds.

4. It is necessary to use a nonlinear theory for predicting the important steady components of heave and pitch. According to the nonlinear equations studied, the steady component of heave which appears in both head and following seas is often quite comparable with the amplitude of oscillation, and sometimes is even larger. It is always downward, tending to cause the craft to crash. There is no experimental confirmation available for these predictions.

5. When there are large steady components of heave or pitch, the foils oscillate about mean positions which are quite different from their calm water equilibrium levels, and thus they encounter different orbital motions of the water. This results in a small decrease in the predicted amplitudes of motion, especially in following seas, where the amplitudes are usually large. This is generally the only way in which the nonlinearities of the system affect the oscillatory amplitudes.

6. Consideration of unsteadiness improves the phase lead predictions for head seas by about 10 deg in the cases studied. It causes practically no change in predicted amplitudes for head seas conditions, but reduces the amplitudes in following seas by as much as 10 percent. It is important to note that the major forces on the foils are reduced by as much as 40 percent in head seas when unsteadiness is considered, but because all of the forces change in approximately the same manner the net result on the motions is small. In situations in which the

inertia is much more important than in the experiments considered, there may be a significant reduction in motions because of unsteadiness. Neglect of unsteadiness should always produce conservative results.

7. Downwash from the forward foil causes negligible effects on the after foil.

Again it is pointed out that several factors were neglected in the theory:

1. The effects of generated waves were not considered.
2. Several sources of nonlinearity were not investigated.
3. The effect of the surface on the lift of the shallow part of a foil was ignored.
4. The effects of unsteadiness on the forces due to varying span were not calculated.

The most important lack of knowledge about the dynamic response of hydrofoil craft results from the paucity of experimental information. Reference 2 includes the only known systematic accumulation of data. In that project, Leehey and Steele were greatly handicapped by the limited speed range available with their carriage, especially in their investigation of following sea responses. A wave only slightly longer than their craft had a celerity as great as the maximum speed of the carriage. This is a condition which is very unlikely to occur with a prototype hydrofoil craft. Therefore it was not possible to investigate much of the range of following-sea conditions which is of interest. A higher available test speed would also be useful in determining the effect of the "mass ratio"; that is, the ratio of the mass of the craft to the added mass of the foils. A higher value would be possible only with higher forward speeds. It has been mentioned previously that unsteady effects might be more noticeable if such a variation were considered.

APPENDIX

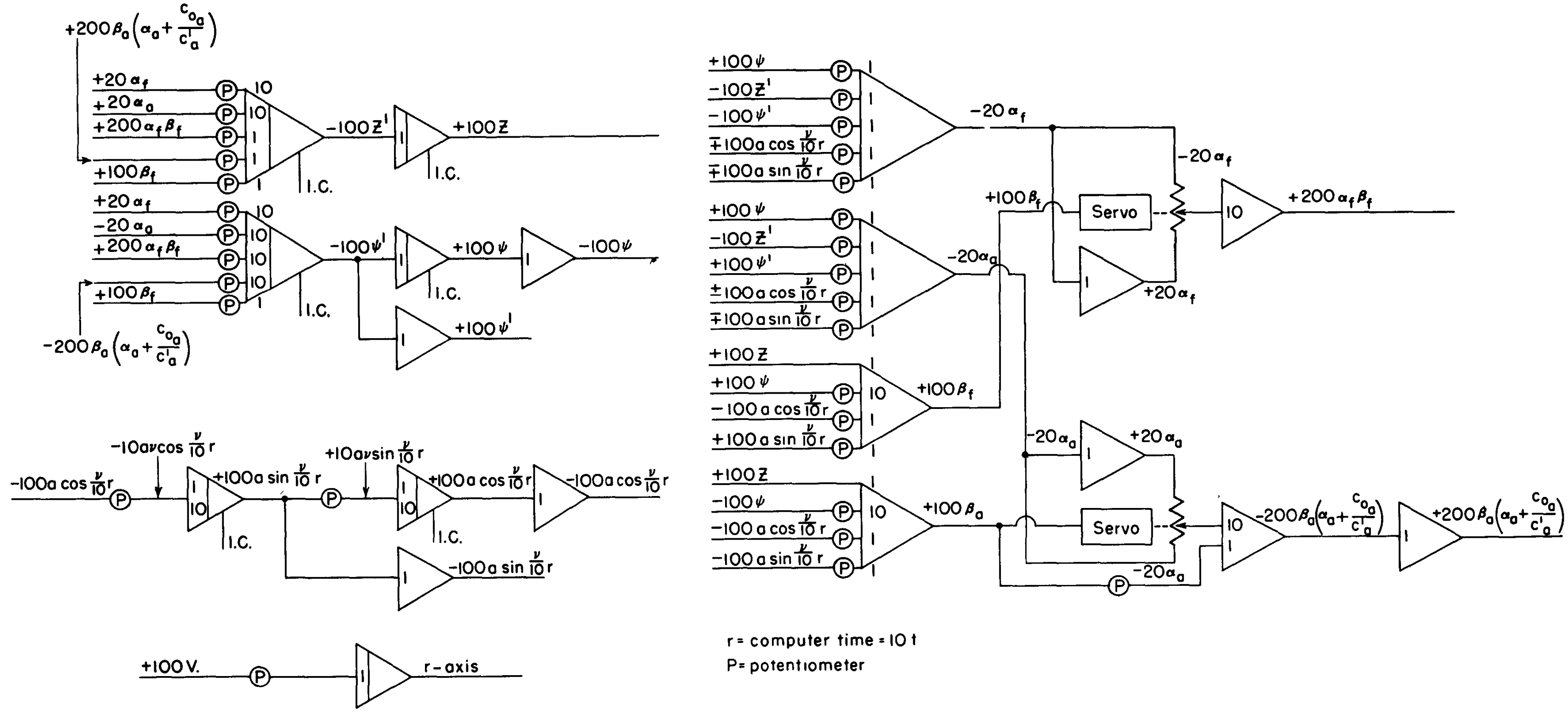
ANALOG COMPUTER SETUP

Figure 18 is a block diagram of the analog computer setup. The layout follows closely the form of Equations [6] and [7]. The elements which form the quantities α_f, α_a, β_f, and β_a are summed and then multiplied appropriately by the servos. Note that the output of the servos is equal to the product of the inputs divided by 100. The proper quantities are taken from the right-hand side of the diagram and summed at the left to form $\ddot{z}$ and $\ddot{\psi}$. These are integrated twice, thus becoming available for the first summation mentioned above.

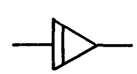
The forcing function, representing the wave amplitude and the orbital motions, is generated by the secondary circuit shown at the lower left.

Initial conditions can be inserted at each block marked "I.C."

The computer solutions were obtained at a speed 1/10 of the real craft rates; that is, 1 sec of real time was equal to 10 sec of computer time. This change was necessary because of the frequency response of the plotting board. It also simplified the problem of scaling forces for the computer, offsetting the factor of 1/100 introduced by the servo multiplication.



r = computer time = 10 t
P = potentiometer

 is a summer, sign inverter, and integrator; i. e. (output) = $-\int_0^r \Sigma$ (input) dr'

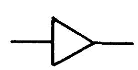
 is a summer and sign inverter; i. e. (output) = $-\Sigma$ (input).

Figure 18 – Analog Computer Block Diagram

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