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DETERMINATION OF INFLUENCE COEFFICIENTS AS APPLIED TO CALCULATION OF CRITICAL WHIRLING SPEEDS OF PROPELLER-SHAFT SYSTEMS

by

B.M. Wigle and N.H. Jasper, Dr. Eng.

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NOTATION

a, b, c, d	Lengths; see Figures 1 and 2
a_0, b_0	Lengths; see Figure 2
B	EI of shaft
f_N	Fundamental forward whirling frequency of shaft-disk system in cpm
G	$\tau_d(1-kh)$, an effective inertia
h	ω/Ω_N, dimensionless ratio of angular velocity of shaft to the whirling circular frequency (the "order" of the vibration is $1/h$)
I	Area moment of inertia
k	$\tau_p/\tau_d \approx 2$ for propellers in air
K_L	Linear stiffness obtained by combining K_L^1 and K_L^2
K_L^1	Total linear stiffness of bearing staves
K_L^2	Total linear stiffness of flexible mounting ring
K_R	Rotatory stiffness obtained by combining K_R^1, K_R^2, and K_R^3
K_R^1	Effective rotatory stiffness of bearing staves
K_R^2	Effective rotatory stiffness of flexible mounting ring
K_R^3	Effective rotatory stiffness of barrel support
l	Length; see Figure 2
m_{es}	Effective mass of shaft
m_p	Mass of propeller, including virtual mass of water when appropriate
m_s	Mass of shaft of length l
M_1, M_2, M_3	Moments; see Figure 1
M_0	Unit static moment applied at the propeller
P_0	Unit static load applied at the propeller
R_1, R_2, R_3	Bearing reactions; see Figure 1
x_0, x_1, x_2, x_3	Lengths; see Figures 1 and 2
y_i, y_i'	Linear displacements (further defined in text)
γ_i, γ_i'	Angular displacements (further defined in text)

δ_i	$y_i + y_i'$
δ_1	$y_1 + y_1'$
δ^M	Transverse displacement of propeller due to a unit static moment applied at the propeller
δ^P	Transverse displacement of propeller due to a unit static load applied at the propeller
θ_i	$\gamma_i + \gamma_i'$
θ_1	$\gamma_1 + \gamma_1'$
θ^M	Rotation of propeller about a transverse axis due to a unit static moment applied at the propeller
θ^P	Rotation of propeller about a transverse axis due to a unit static load applied at the propeller
μ	Mass of shaft per unit length including virtual mass of water when appropriate
τ_d	Mass moment of inertia of the propeller about a diameter including allowance for water when applicable
τ_p	Mass moment of inertia of the propeller about a polar axis including allowance for water when applicable
ω	Angular velocity of shaft
Ω_N	Whirling circular frequency of shaft

ABSTRACT

Formulas developed at the David Taylor Model Basin for computing the critical frequencies of whirling vibration of propeller shafting systems require the determination of influence coefficients in their application. This report describes methods for determining the necessary influence coefficients for use with these formulas and tabulates, for purposes of comparison, computed and experimentally determined natural frequencies.

INTRODUCTION

In Taylor Model Basin Reports 827[1] and 890[2] theoretical methods were derived for computing the natural frequencies of whirling vibration of shaft-disk systems. The general formula, Equation [5] of Reference 1, for the case of a massless shaft with a single disk contains four influence coefficients, δ^P, θ^P, δ^M, and θ^M. A method is given in this report by which these influence coefficients can be determined. Since their determination is, in general, a lengthy procedure, a simplified form of the general formula was derived in Reference 2 which contains only the influence coefficient δ^P. Hence δ^P has been chosen for the sample calculation which is given in this report to illustrate the method. Computed natural frequencies and experimentally determined natural frequencies are tabulated for purposes of comparison.

GENERAL CONSIDERATIONS

The general formula, Equation [5] of Reference 1, is

$$\Omega_N{}^2 = \frac{(m_p\delta^P + \theta^M G) \pm \sqrt{(m_p\delta^P + \theta^M G)^2 - 4\,m_p G\,(\delta^P\theta^M - \delta^M\theta^P)}}{2\,m_p G\,(\delta^P\theta^M - \delta^M\theta^P)}$$

where Ω_N is the natural whirling frequency of the symmetrical shaft-disk system consisting of a massless shaft with a single disk, in radians per second,

m_p is the mass of the propeller,*

δ^P is the transverse displacement of the propeller due to a unit static load applied at the propeller,

δ^M is the transverse displacement of the propeller due to a unit static moment applied at the propeller,

θ^P is the rotation of the propeller about a transverse axis due to a unit static load applied at the propeller,

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[1] References are listed on page 13.

*Estimates of allowance for the virtual mass effect of the entrained water are given in Reference 2.

θ^M is the rotation of the propeller about a transverse axis due to a unit static moment applied at the propeller,

G is an effective inertia equal to $\tau_d (1 - kh)$,

τ_d is the mass moment of inertia of the propeller about a diameter,*

k $= \tau_p / \tau_d$

τ_p is the mass moment of inertia of the propeller about the polar axis,* and

h is equal to the angular spin velocity ω of the shaft divided by the angular whirling velocity Ω_N of the shaft.

The influence coefficients are δ^P, θ^P, δ^M, and θ^M.

The two values of Ω_N obtained from the general formula give the natural frequencies for the two lowest modes of vibration corresponding to a given value of h. The lowest frequency is obtained when the minus sign in front of the radical is used, but this would require taking the difference of numbers of which the first few significant figures might be identical. Therefore, when the minus sign is to be used, multiply both the numerator and denominator of the general equation by the conjugate of the numerator. When this is done, the equation for the lowest mode frequency of vibration takes the form

$$\Omega_N{}^2 = \frac{2}{(m_p \delta^P + \theta^M G) + \sqrt{(m_p \delta^P + \theta^M G)^2 - 4 m_p G (\delta^P \theta^M - \delta^M \theta^P)}} \qquad [1]$$

When, in Formula [1], G is taken equal to zero

$$\Omega_N = \sqrt{\frac{1}{\delta^P m_p}} \qquad [2]$$

Setting G equal to zero is equivalent to the assumption that the propeller acts as if it were a point mass, that is, $\tau_d = 0$. From another point of view, it would be equivalent to considering the propeller as a thin disk ($k = 2$), which is whirling at twice the shaft rpm, that is, $h = \frac{\omega}{\Omega_N} = \frac{1}{2}$. Formula [2] gives an underestimate of the first-order forward whirl, which will be on the side of safety.

The influence of the mass of the shaft on the critical frequency may be approximated by adding an effective mass m_{es} to the mass of the propeller. Equation [2] would then take the form[2]

$$\Omega_N = \sqrt{\frac{1}{\delta^P (m_p + m_{es})}} \qquad [3]$$

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*Estimates of allowance for the virtual mass effect of the entrained water are given in Reference 2.

The effective mass of the shaft, m_{es}, is a mass, which, if assumed to vibrate with the center of gravity of the propeller, will have a maximum kinetic energy equal to the maximum kinetic energy of the shaft when it is vibrating in the particular mode under consideration. Estimates of m_{es}, determined by the use of the lowest mode shapes found for several propeller-shaft systems, both by experiment and by computation, have fallen within the range $0.10\ m_s < m_{es} < 0.40\ m_s$ for the lowest mode of vibration.

GENERAL METHOD FOR DETERMINING INFLUENCE COEFFICIENTS FOR OVERHANGING PROPELLERS

Figure 1 represents a propeller-shaft system with the propeller located at $x = x_0$. The influence coefficients δ^P, δ^M, θ^P, and θ^M represent linear and angular displacements at $x = x_0$ for an application at $x = x_0$ of a unit static load $P_0 = 1$ and a unit moment $M_0 = 1$, respectively.

The method used for determining the influence coefficients will be to treat the shaft as a cantilever beam and replace the shaft supports, except for the built-in end of the cantilever, with equivalent reactions. The procedure will then be to evaluate the displacements at $x = x_0$, x_1, x_2, and x_3 due to P_0 and M_0, and the displacements at $x = x_0$, x_1, x_2, and x_3 due to the reactions at the three flexible supports. This may be conveniently done by use of the formulas for cantilever beams given in handbooks such as Reference 3. Next, the equations of equilibrium are applied at each flexible support, i.e., the sum of the displacements due to the external loads and to the spring reactions are equal to the displacements at that support, which, in turn, must be equal to the reaction divided by the spring constant. In this way, six equations in the six unknown reactions R_1, R_2, R_3, M_1, M_2, and M_3 are obtained. With the reactions known, the displacements, and thus the influence coefficients, can be determined directly.

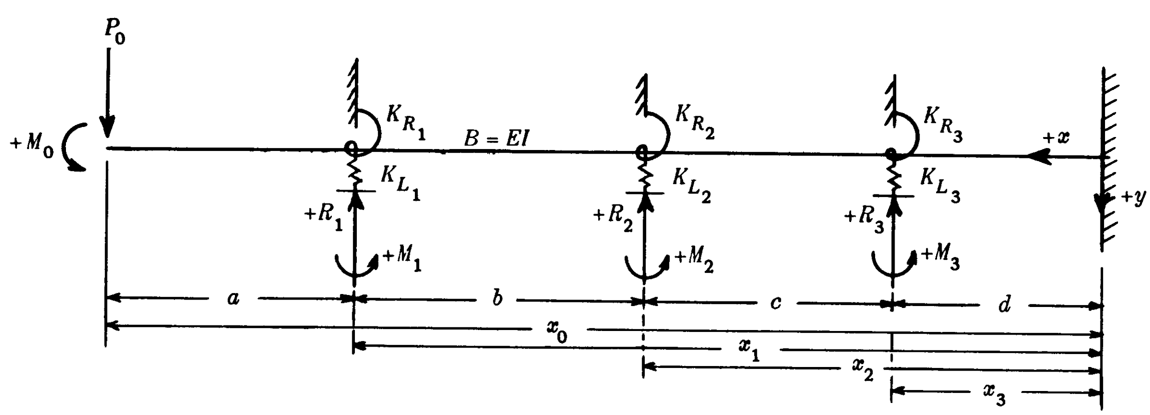


Figure 1 - Schematic Sketch of Propeller-Shaft System for Determining Influence Coefficients

Let y_i be the linear displacement at $x = x_i$ due to P_0 and M_0 that would exist if there were no bearing reactions,

γ_i be the angular displacement at $x = x_i$ due to P_0 and M_0 that would exist if there were no bearing reactions,

y_i' be the linear displacement at $x = x_i$ due only to the reactions $\sum_{j=1}^{n} R_j + \sum_{j=1}^{n} M_j$, and

γ_i' be the angular displacement at $x = x_i$ due only to the reactions $\sum_{j=1}^{n} R_j + \sum_{j=1}^{n} M_j$.

The total linear displacement at x_i is $y_i + y_i' = \delta_i$. The total angular displacement at x_i is $\gamma_i + \gamma_i' = \theta_i$. Then, with $B = EI$,

$$By_0 = \frac{1}{2} M_0 x_0^2 + \frac{1}{3} P_0 x_0^3 \quad [4]$$

$$By_1 = \frac{1}{2} M_0 x_1^2 + \frac{1}{6} P_0 x_1^2 (2x_0 + a) \quad [5]$$

$$By_2 = \frac{1}{2} M_0 x_2^2 + \frac{1}{6} P_0 x_2^2 (2x_0 + a + b) \quad [6]$$

$$By_3 = \frac{1}{2} M_0 x_3^2 + \frac{1}{6} P_0 x_3^2 (2x_0 + a + b + c) \quad [7]$$

and

$$\begin{aligned} By_0' = &-\frac{1}{2} R_1 x_1^2 \left(\frac{2}{3} x_1 + a\right) - \frac{1}{2} R_2 x_2^2 \left(\frac{2}{3} x_2 + a + b\right) \\ &-\frac{1}{2} R_3 x_3^2 \left(\frac{2}{3} x_3 + a + b + c\right) + M_1 x_1 \left(\frac{1}{2} x_1 + a\right) \\ &+ M_2 x_2 \left(\frac{1}{2} x_2 + a + b\right) + M_3 x_3 \left(\frac{1}{2} x_3 + a + b + c\right) \end{aligned} \quad [8]$$

$$\begin{aligned} By_1' = &-\frac{1}{3} R_1 x_1^3 - \frac{1}{2} R_2 x_2^2 \left(\frac{2}{3} x_2 + b\right) - \frac{1}{2} R_3 x_3^2 \left(\frac{2}{3} x_3 + b + c\right) \\ &+ \frac{1}{2} M_1 x_1^2 + M_2 x_2 \left(\frac{1}{2} x_2 + b\right) + M_3 x_3 \left(\frac{1}{2} x_3 + b + c\right) \end{aligned} \quad [9]$$

$$\begin{aligned} By_2' = &-\frac{1}{6} R_1 x_2^2 \, (2x_1 + b) - \frac{1}{3} R_2 x_2^3 - \frac{1}{2} R_3 x_3^2 \left(\frac{2}{3} x_3 + c\right) \\ &+ \frac{1}{2} M_1 x_2^2 + \frac{1}{2} M_2 x_2^2 + M_3 x_3 \left(\frac{1}{2} x_3 + c\right) \end{aligned} \quad [10]$$

$$\begin{aligned} By_3' = &-\frac{1}{6} R_1 x_3^2 (2x_1 + b + c) - \frac{1}{6} R_2 x_3^2 (2x_2 + c) - \frac{1}{3} R_3 x_3^2 \\ &+ \frac{1}{2} M_1 x_3^2 + \frac{1}{2} M_2 x_3^2 + \frac{1}{2} M_3 x_3^2 \end{aligned} \quad [11]$$

By an analogous process, equations for the y_i's and y_i''s can be derived.

At $x = x_1$

$$\delta_1 = \frac{R_1}{K_{L_1}} \quad \text{and} \quad \theta_1 = -\frac{M_1}{K_{R_1}}$$

where K_{L_1} is the combined linear stiffness for the entire bearing and K_{R_1} is the combined rotatory stiffness. Since $\delta_1 = y_1 + y_1'$, we obtain, by combining Equations [5] and [9],

$$\frac{BR_1}{K_{L_1}} = \frac{1}{2} M_0 x_1^2 + \frac{1}{6} P_0 x_1^2 (2x_0 + a) - \frac{1}{3} R_1 x_1^3 - \frac{1}{2} R_2 x_2^2 \left(\frac{2}{3} x_2 + b\right)$$
$$- \frac{1}{2} R_3 x_3^2 \left(\frac{2}{3} x_3 + b + c\right) + \frac{1}{2} M_1 x_1^2 + M_2 x_2 \left(\frac{1}{2} x_2 + b\right) \qquad [12]$$
$$+ M_3 x_3 \left(\frac{1}{2} x_3 + b + c\right)$$

By a similar process it can be shown that

$$-\frac{BM_1}{K_{R_1}} = M_0 x_1 + M_1 x_1 + M_2 x_2 + M_3 x_3 + \frac{1}{2} P_0 \ (x_0^2 - a^2)$$
$$- \frac{1}{2} R_1 x_1^2 - \frac{1}{2} R_2 x_2^2 - \frac{1}{2} R_3 x_3^2 \qquad [13]$$

At $x = x_2$, by combining Equations [6] and [10], we obtain

$$\frac{BR_2}{K_{L_2}} = \frac{1}{2} M_0 x_2^2 + \frac{1}{6} P_0 x_2^2 (2x_0 + a + b) - \frac{1}{6} R_1 x_2^2 (2x_1 + b) - \frac{1}{3} R_2 x_2^3$$
$$- \frac{1}{2} R_3 x_3^2 \left(\frac{2}{3} x_3 + c\right) + \frac{1}{2} M_1 x_2^2 + \frac{1}{2} M_2 x_2^2 + M_3 x_3 \left(\frac{1}{2} x_3 + c\right) \qquad [14]$$

By a similar process it can be shown that

$$-\frac{BM_2}{K_{R_2}} = M_0 x_2 + M_1 x_2 + M_2 x_2 + M_3 x_3 + \frac{1}{2} P_0 \ [x_0^2 - (a + b)^2]$$
$$- \frac{1}{2} R_1 (x_1^2 - b^2) - \frac{1}{2} R_2 x_2^2 - \frac{1}{2} R_3 x_3^2 \qquad [15]$$

At $x = x_3$, we obtain by combining Equations [7] and [11]

$$\frac{BR_3}{K_{L_3}} = \frac{1}{2} M_0 x_3^2 + \frac{1}{6} P_0 x_3^2 (2x_0 + a + b + c) - \frac{1}{6} R_1 x_3^2 (2x_1 + b + c)$$
$$- \frac{1}{6} R_2 x_3^2 (2x_2 + c) - \frac{1}{3} R_3 x_3^3 + \frac{1}{2} M_1 x_3^2 + \frac{1}{2} M_2 x_3^2 + \frac{1}{2} M_3 x_3^2 \qquad [16]$$

and similarly,

$$-\frac{BM_3}{K_{R_3}} = M_0 x_3 + M_1 x_3 + M_2 x_3 + M_3 x_3 + \frac{1}{2} P_0 [x_0^2 - (a + b + c)^2] - \frac{1}{2} R_1 [x_1^2 - (b + c)^2] - \frac{1}{2} R_2 (x_2^2 - c^2) - \frac{1}{2} R_3 x_3^2 \qquad [17]$$

After terms are collected, Equations [12] through [17] may, for the sake of convenience, be written in the following form:

$$R_1\left(\frac{B}{K_{L_1}} + \frac{x_1^3}{3}\right) + \frac{R_2 x_2^2}{2}\left(\frac{2x_2}{3} + b\right) + \frac{R_3 x_3^2}{2}\left(\frac{2x_3}{3} + b + c\right) - \frac{M_1 x_1^2}{2} - M_2 x_2\left(\frac{x_2}{2} + b\right) - M_3 x_3\left(\frac{x_3}{2} + b + c\right) = \frac{M_0 x_1^2}{2} + \frac{P_0 x_1^2}{6}(2x_0 + a)$$

$$R_1\left(\frac{x_1^2}{2}\right) + \frac{R_2 x_2^2}{2} + \frac{R_3 x_3^2}{2} - M_1\left(x_1 + \frac{B}{K_{R_1}}\right) - M_2 x_2 - M_3 x_3 = M_0 x_1 + \frac{P_0}{2}(x_0^2 - a^2)$$

$$\frac{R_1 x_2^2}{6}(2x_1 + b) + R_2\left(\frac{B}{K_{L_2}} + \frac{x_2^3}{3}\right) + \frac{R_3 x_3^2}{2}\left(\frac{2x_3}{3} + c\right) - \frac{M_1 x_2^2}{2} - \frac{M_2 x_2^2}{2} - M_3 x_3\left(\frac{x_3}{2} + c\right) = \frac{M_0 x_2^2}{2} + \frac{P_0 x_2^2}{6}(2x_0 + a + b)$$

$$\frac{R_1}{2}(x_1^2 - b^2) + \frac{R_2 x_2^2}{2} + \frac{R_3 x_3^2}{2} - M_1 x_2 - M_2\left(x_2 + \frac{B}{K_{R_2}}\right) - M_3 x_3 = M_0 x_2 + \frac{P_0}{2}[x_0^2 - (a + b)^2]$$

$$\frac{R_1 x_3^2}{6}(2x_1 + b + c) + \frac{R_2 x_3^2}{6}(2x_2 + c) + R_3\left(\frac{B}{K_{L_3}} + \frac{x_3^3}{3}\right) - \frac{M_1 x_3^2}{2} - \frac{M_2 x_3^2}{2} - \frac{M_3 x_3^2}{2} = \frac{M_0 x_3^2}{2} + \frac{P_0 x_3^2}{6}(2x_0 + a + b + c)$$

$$\frac{R_1}{2}[x_1^2 - (b + c)^2] + \frac{R_2}{2}(x_2^2 - c^2) + \frac{R_3 x_3^2}{2} - M_1 x_3 - M_2 x_3 - M_3\left(x_3 + \frac{B}{K_{R_3}}\right) = M_0 x_3 + \frac{P_0}{2}[x_0^2 - (a + b + c)^2]$$

The unknown reactions R_i and M_i, found by solving this set of simultaneous equations, are then used to determine the deflections δ_0 and θ_0 at $x = x_0$. For example,

$$B\delta_0 = B(y_0 + y_0') = \frac{1}{2} M_0 x_0^2 + \frac{1}{3} P_0 x_0^3 - \frac{1}{2} R_1 x_1^2 \left(\frac{2}{3} x_1 + a\right)$$
$$- \frac{1}{2} R_2 x_2^2 \left(\frac{2}{3} x_2 + a + b\right) - \frac{1}{2} R_3 x_3^2 \left(\frac{2}{3} x_3 + a + b + c\right)$$
$$+ M_1 x_1 \left(\frac{1}{2} x_1 + a\right) + M_2 x_2 \left(\frac{1}{2} x_2 + a + b\right) \qquad [18]$$
$$+ M_3 x_3 \left(\frac{1}{2} x_3 + a + b + c\right)$$

and similarly,

$$B\theta_0 = B(\gamma_0 + \gamma_0') = M_0 x_0 + M_1 x_1 + M_2 x_2 + M_3 x_3 + \frac{1}{2} P_0 x_0^2$$
$$- \frac{1}{2} R_1 x_1^2 - \frac{1}{2} R_2 x_2^2 - \frac{1}{2} R_3 x_3^2 \qquad [19]$$

To obtain $\delta_0^P = \delta^P$, let $M_0 = 0$, $P_0 = 1$ in Equation [18]. To obtain $\theta_0^P = \theta^P$, let $M_0 = 0$, $P_0 = 1$ in Equation [19]. To obtain $\delta_0^M = \delta^M$, let $M_0 = 1$, $P_0 = 0$ in Equation [18]. To obtain $\theta_0^M = \theta^M$, let $M_0 = 1$, $P_0 = 0$ in Equation [19]. Conservation of energy requires that $\delta^M = \theta^P$.

PROCEDURE FOR DETERMINING INFLUENCE COEFFICIENTS FOR A SPECIFIC PROPELLER-SHAFT SYSTEM FOR USE WITH FORMULA [3]

The method for determining influence coefficients just presented is applied here to the propeller-shaft system shown in Figure 2. In the present application only δ^P will be determined for use with Formula [3]. The frequency obtained will be the lowest-mode second-order forward whirling frequency, $G = 0$, which would be an underestimate, that is, an estimate on the side of safety, of the first-order forward whirling frequency.

1. Evaluate the following elements:

$$a_{11} = +\left(\frac{B}{K_L} + \frac{x_1^3}{3}\right); \qquad a_{12} = +\frac{x_2^2}{2}\left(\frac{2}{3} x_2 + b\right); \qquad a_{13} = -\frac{1}{2} x_1^2$$

$$a_{21} = +\frac{1}{2} x_1^2; \qquad a_{22} = +\frac{1}{2} x_2^2; \qquad a_{23} = -\left(x_1 + \frac{B}{K_R}\right)$$

$$a_{31} = +\frac{x_2^2}{6}(2x_1 + b); \qquad a_{32} = +\frac{1}{3} x_2^3; \qquad a_{33} = -\frac{1}{2} x_2^2$$

$$c_1 = +\frac{x_1^2}{6}(2x_0 + a); \qquad c_2 = +\frac{1}{2}(x_0^2 - a^2); \qquad c_3 = +\frac{x_2^2}{6}(2x_0 + a + b)$$

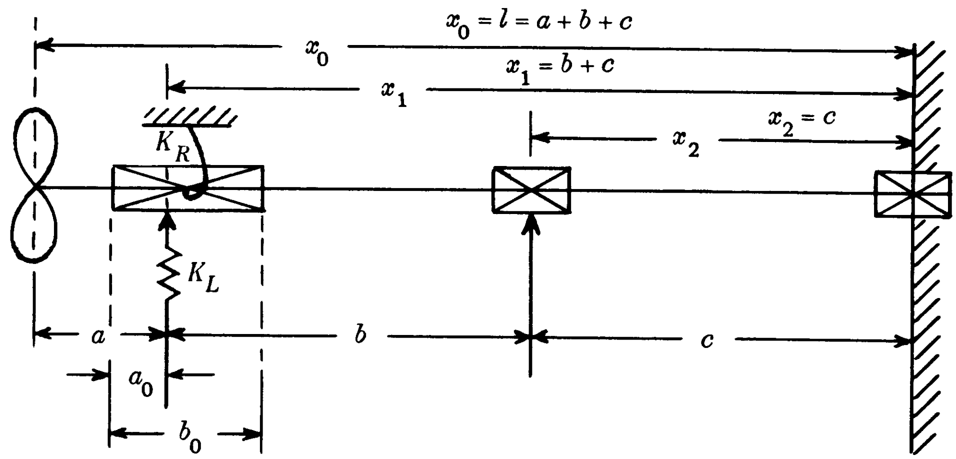


Figure 2 - Schematic Sketch of a Propeller-Shaft System in Which Flexibilities in the After Bearing Only Are Considered

$a_0 = \frac{b_0}{2}$ **for self-aligning, articulated bearings, such as types A and C, as shown in Figure 4.**

$a_0 = \frac{b_0}{3}$ **for standard stave bearings, such as type B, as shown in Figure 4.**

2. Evaluate R_1, R_2, and M_1 from the following determinants:*

$$R_1 D = \begin{vmatrix} c_1 & a_{12} & a_{13} \\ c_2 & a_{22} & a_{23} \\ c_3 & a_{32} & a_{33} \end{vmatrix}, \quad R_2 D = \begin{vmatrix} a_{11} & c_1 & a_{13} \\ a_{21} & c_2 & a_{23} \\ a_{31} & c_3 & a_{33} \end{vmatrix},$$

$$M_1 D = \begin{vmatrix} a_{11} & a_{12} & c_1 \\ a_{21} & a_{22} & c_2 \\ a_{31} & a_{32} & c_3 \end{vmatrix}, \text{ where } D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

3. Evaluate

$$\delta^P = \frac{1}{B}\left[\frac{1}{3} x_0^3 - \frac{1}{2} R_1 x_1^2 \left(\frac{2}{3} x_1 + a\right) - \frac{1}{2} R_2 x_2^2 \left(\frac{2}{3} x_2 + a + b\right) + M_1 x_1 \left(\frac{x_1}{2} + a\right)\right]$$

Substitution of this influence coefficient into Formula [3] gives the frequency. This was the procedure used in calculating the frequencies listed in Table 1 and at the bottom of Table 2.

***In this case the reactions are due only to P_0 since the constants are evaluated for M_0 taken equal to zero. For the middle bearing $K_{L_2} = \infty$, $K_{R_2} = 0$.**

INFLUENCE COEFFICIENTS FOR SEVERAL IDEALIZED PROPELLER-SHAFT SYSTEMS

The method of deriving influence coefficients given on page 3 was also applied to several simple, equivalent, propeller-shaft systems. Sketches of these systems and the corresponding formulas for their influence coefficients are given in Figure 3. Also given, for completeness only, is a pinned-pinned arrangement which was derived by a variation of the method. The flexibility of the aftermost shaft support is one of the most important factors which affect the natural whirling frequencies. The Appendix contains illustrations of several types of bearing supports and indicates methods which may be used in determining the effective stiffnesses for each type of support.

COMPARISON OF COMPUTED AND EXPERIMENTALLY DETERMINED NATURAL WHIRLING FREQUENCIES

Table 1 lists frequencies obtained experimentally, by means of an electrical analog[1] and the UNIVAC computer,[1] and by Formula [3]. The propeller-shaft arrangement utilized in evaluating δ^P for use in Formula [3] is shown in Figure 2. The constants used in the computation of frequencies by Formula [3] are listed in Table 2.

TABLE 1

Computed and Experimental First-Mode Natural Whirling Frequencies of Five Ships

Shafting of the CVA 59 and the DL-1 was made of Alloy 4 steel.

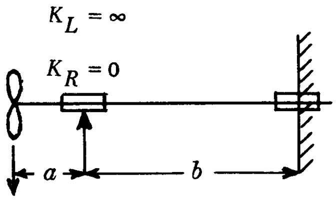
Ship	Frequencies in cpm			
	Nonrotating Shaft in Air, $h = 0$			Formula [3] ($G = 0$ in Air)†
	Experimental*	Electrical Analog	UNIVAC Computer	
CVA 59**	325	260	275	262
BB 61††	460	480		378
SSK 241	790[4]	645		641
SSK 243	630[4]	500	539	555
DL-1	447		344	410

*Horizontal vibration in air, excited by means of a vibration generator.

†The equivalent shaft system used is shown in Figure 2 (includes m_{es}).

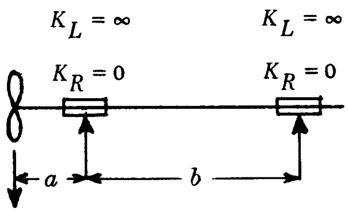
**Inboard propeller shaft.

††Outboard propeller shaft.



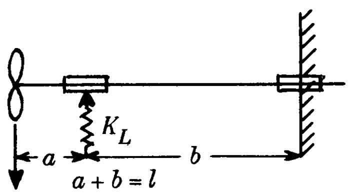
$$\delta^P = \frac{a^2}{EI}\left(\frac{a}{3} + \frac{b}{4}\right), \quad \theta^M = \frac{1}{EI}\left(a + \frac{b}{4}\right)$$

$$\theta^P = \delta^M = \frac{a}{2EI}\left(a + \frac{b}{2}\right)$$



$$\delta^P = \frac{a^2}{3EI}(a + b), \quad \theta^M = \frac{1}{EI}\left(a + \frac{b}{3}\right)$$

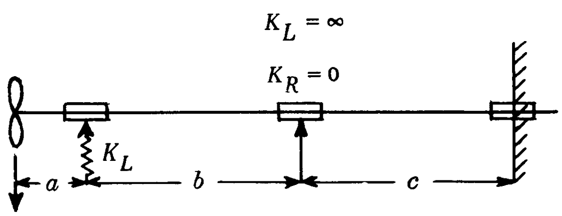
$$\theta^P = \delta^M = \frac{a}{6EI}(3a + 2b)$$



$$\delta^P = \frac{1}{EI}\left[\frac{l^3}{3} - b^4\left(\frac{3}{\frac{3EI}{K_L} + b^3}\right)\left(\frac{a}{2} + \frac{b}{3}\right)^2\right]$$

$$\theta^M = \frac{1}{EI}\left[l - \left(\frac{3}{\frac{3EI}{K_L} + b^3}\right)\left(\frac{b^4}{4}\right)\right]$$

$$\delta^M = \theta^P = \frac{1}{EI}\left[\frac{l^2}{2} - \frac{b^4}{2}\left(\frac{3}{\frac{3EI}{K_L} + b^3}\right)\left(\frac{a}{2} + \frac{b}{3}\right)\right]$$



$$a + b + c = l \qquad b + c = k$$

$$\theta^M = \frac{1}{EI}\left[l - \left(\frac{1}{\frac{b}{9} + \frac{c}{12} + \frac{EI}{3K_L b^2}}\right)\left(\frac{k^2}{12} + \frac{EI\,c}{4K_L b^2}\right)\right]$$

$$\delta^P = \frac{1}{EI}\left\{\frac{l^3}{3} - \frac{1}{\frac{b}{9} + \frac{c}{12} + \frac{EI}{3K_L b^2}}\left[\frac{k^2}{2}\left(\frac{a^2}{6} + \frac{2ab}{9} + \frac{ac}{6} + \frac{2b^2}{27} + \frac{7bc}{54} + \frac{c^2}{18}\right) + \frac{EIc\,(2l + a + b)^2}{36K_L b^2}\right]\right\}$$

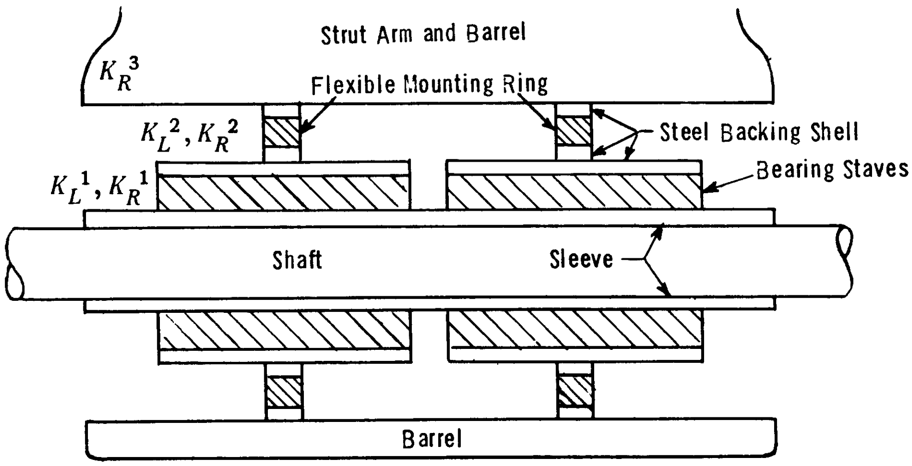
$$\theta^P = \delta^M = \frac{1}{EI}\left\{\frac{l^2}{2} - \frac{1}{\frac{b}{9} + \frac{c}{12} + \frac{EI}{3K_L b^2}}\left[\frac{k^2}{2}\left(\frac{a}{6} + \frac{b}{9} + \frac{c}{12}\right) + \frac{EIc\,(2l + a + b)}{12\,K_L b^2}\right]\right\}$$

Figure 3 - Schematic Sketches of Several Idealized Propeller-Shaft Systems and Their Corresponding Influence Coefficient Formulas

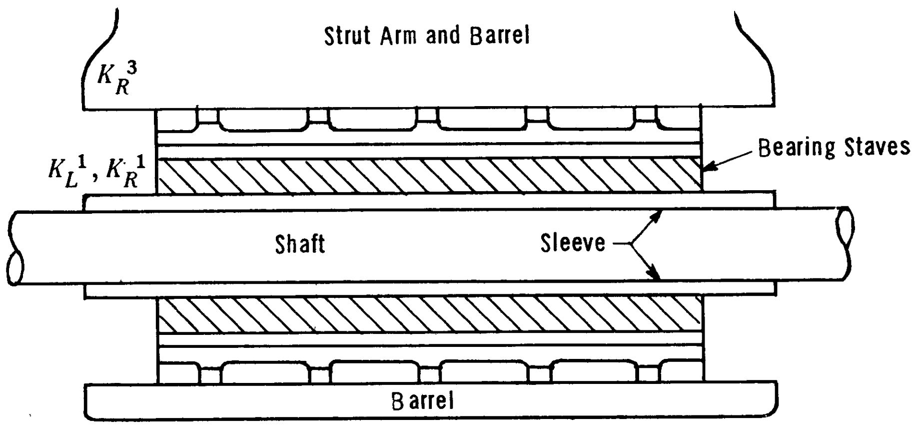
TABLE 2

Constants Used in Computation of Natural Whirling Frequencies of Several Propeller-Shaft Systems

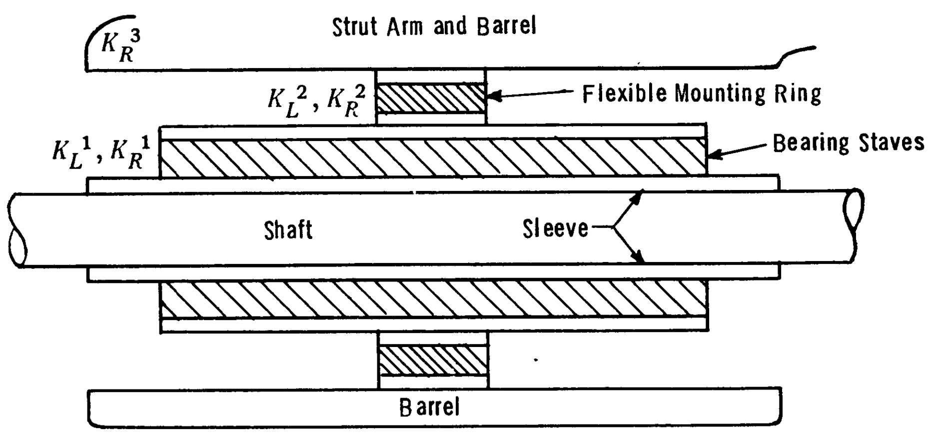
Item	Definition of Terms	Units	CVA 59	BB 61	SSK 241	SSK 243	DL-1
			Inboard Shaft Self-Aligning Rubber Bearing	Outboard Shaft Wooden Bearing	Wooden Bearing	Self-Aligning Rubber Bearing	Rubber Bearing
Type of Bearing	See Figure 4		A	B	B	C	B
$B = EI$	Bending Rigidity	lb in.^{2}	249×10^9	426×10^9	8.73×10^9	8.73×10^9	133×10^9
m_p	Mass of Propeller	lb sec^2/in.	181.35	105.6	7.876	7.876	106.2
a	See Figure 2	in.	96	97	28.75	35.75	61
b	See Figure 2	in.	595	562	249.5	242.5	529.6
$c = x_2$	See Figure 2	in.	720	413	209.5	209.5	576
$b+c=x_1$	See Figure 2	in.	1315	975	459	452	1105.6
$l = x_0$	See Figure 2	in.	1411	1072	487.75	487.75	1166.6
μ	See Notation	lb sec^2/in^2	0.154	0.197	0.0358	0.0358	0.116
m_s	$m_s = \mu l$	lb sec^2/in.	217.3	211.2	17.45	17.45	135.326
K_L	See Figures 2,4	lb/in.	1.195×10^6	0.921×10^6	0.141×10^6	0.073×10^6	0.858×10^6
K_R	See Figures 2,4	lb in./radian	1.3066×10^9	1.18×10^9	12.252×10^6	1.053×10^6	0.592×10^9
a_0	See Figure 2	in.	60	58	14	19 13/16	32
b_0	See Figure 2	in.	120	174	42	39 5/8	96
δ^P	See Notation	in/lb	5.541×10^{-6}	4.574×10^{-6}	16.122×10^{-6}	30.818×10^{-6}	4.482×10^{-6}
m_{es}	See Notation	lb sec^2/in.	$0.27\, m_s$	$0.16\, m_s$	$0.338\, m_s$	$0.10\, m_s$	$0.11\, m_s$
Ω_N	See Notation	radians/sec	27.43	39.6	67.1	58.1	42.9
f_N	See Notation	cycles/min	262	378	641	555	410



Type A - CVA 59



Type B - BB 61, SSK 241, DL-1



Type C - SSK 243

Figure 4 - Schematic Sketches of Main Bearing Assemblies

See Notation for the definition of the symbols denoting stiffnesses.

APPENDIX

DETERMINATION OF EFFECTIVE STIFFNESS OF BEARING SUPPORTS

Figure 4 shows three types of main strut bearing assemblies for which the resultant linear and rotational stiffnesses have been computed. These stiffnesses are listed in Table 2; the methods used for their computation are given in Reference 2.

Illustrated in Figure 4 are the main strut bearing assemblies of the USS FORRESTAL (CVA 59), USS IOWA (BB 61), USS BASHAW (SSK 241), USS BREAM (SSK 243), and the USS NORFOLK (DL-1). The stiffnesses indicated in the sketches are those which must be considered when computing the resultant stiffnesses K_R and K_L. For bearings Types A and C

$$\frac{1}{K_L} = \frac{1}{K_L^{1}} + \frac{1}{K_L^{2}}\ ; \qquad \frac{1}{K_R} = \frac{1}{K_R^{1}} + \frac{1}{K_R^{2}} + \frac{1}{K_R^{3}}$$

and for Type B bearings

$$K_L = K_L^{1}\ ; \qquad \frac{1}{K_R} = \frac{1}{K_R^{1}} + \frac{1}{K_R^{3}}$$

Approximate formulas for component stiffnesses are given in Reference 2. Table 2 lists the bearing materials used on each ship, the length of the bearing, and the resultant stiffnesses.

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4. "Experimental Investigation of Natural Frequencies and Modes of Vibration of Propeller Shafts of USS BASHAW (SSK 241) and USS BREAM (SSK 243)," San Francisco Naval Shipyard Report 17-53 (12 Jun 1953).

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