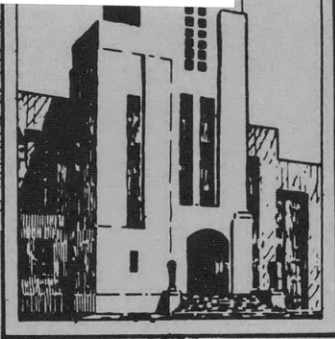


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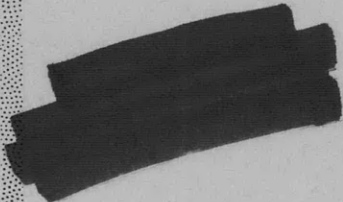


DEPARTMENT OF THE NAVY  
DAVID TAYLOR MODEL BASIN

HYDROMECHANICS

ELASTIC GENERAL INSTABILITY OF RING-STIFFENED CYLINDERS  
WITH INTERMEDIATE HEAVY FRAMES UNDER  
EXTERNAL HYDROSTATIC PRESSURE

AERODYNAMICS



by

William F. Blumenberg

and

Thomas E. Reynolds

STRUCTURAL  
MECHANICS



STRUCTURAL MECHANICS LABORATORY

RESEARCH AND DEVELOPMENT REPORT

APPLIED  
MATHEMATICS

December 1961

Report 1588



DEPARTMENT OF THE NAVY  
DAVID TAYLOR MODEL BASIN  
WASHINGTON 7, D. C.

IN REPLY REFER TO  
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16 Jan 1962

From: Commanding Officer and Director, David Taylor Model Basin  
To: Chief, Bureau of Ships (442) (in duplicate)

Subj: Investigation of elastic general instability of ring-stiffened cylinders with intermediate heavy frames; forwarding of report on

Encl: (1) DATMOBAS Report 1588 entitled, "Elastic General Instability of Ring-Stiffened Cylinders with Intermediate Heavy Frames under External Hydrostatic Pressure"  
2 copies

1. The advantages offered by the use of intermediate heavy frames in submarine pressure hulls have prompted the David Taylor Model Basin to include under its general program of submarine frame strength research an investigation of the elastic general instability of ring-stiffened cylinders having intermediate heavy stiffeners. The objective of this study is to develop through analysis and experiment a reliable theory which would be helpful in establishing criteria for the efficient design of heavy frames. Enclosure (1) presents the first results of both the experimental and theoretical phases of this investigation.

2. Results of the experimental phase concern nondestructive tests of a machined ring-stiffened cylinder whereby general instability strength is determined as a function of heavy frame size. A short empirical formula is developed which shows excellent agreement with the experimental results. However, the range of data is regarded as too limited for an adequate evaluation of the formula. The analytical results consist of an examination of existing theory and a description of certain modifications which have been developed at the Model Basin to



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obtain greater generality. This modified solution is found to agree better with experiment but it is indicated that some deficiencies still remain.



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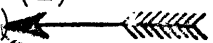
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**ELASTIC GENERAL INSTABILITY OF RING-STIFFENED CYLINDERS  
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**December 1961**

**Report 1588  
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## ABSTRACT

A machined ring-stiffened cylinder was subjected to hydrostatic pressure to investigate the effect of various sizes of central heavy frames on its elastic overall-buckling strength. The cylinder was tested initially with a large central frame, which was systematically reduced by machining after each successive non-destructive test until it was the size of a typical frame.

The tests demonstrated that a heavy frame can be an effective substitute for an internal bulkhead in increasing the general-instability strength of a cylindrical pressure hull.

A short empirical solution based on the Lévy ring formula has been developed which yields results in close agreement with the experimental findings.

Kendrick's Part IV theory showed generally good agreement with experiment for the geometry investigated only for cases in which the heavy frame was somewhat less than fully effective. A modification of this theory is presented which agrees better with experiment.

## INTRODUCTION

The efficiency of intermediate heavy frames in increasing the elastic general-instability strength of hydrostatically loaded ring-stiffened cylinders is being investigated at the David Taylor Model Basin as a part of the submarine structural research program. This information is important for the efficient structural design of long compartments of deep-diving submarines. No proven design methods are presently available for determining the properties and locations of such heavy frames.

For the experimental phase of this project, a ring-stiffened cylinder was designed with a central heavy frame, which was expected to divide the cylinder so that each half would act independently. As the size of the heavy frame was reduced in stages by machining and the cylinder was tested nondestructively, an experimental curve of collapse pressure versus the size of the heavy frame was obtained. Based on these results, a short empirical formula for predicting collapse pressures was developed.

The analytical phase of the problem began with an examination of a theoretical solution obtained by Kendrick.<sup>1</sup> It was decided that the accuracy of this solution could be improved through modifications. Accordingly, a new solution was developed, and both were programmed for the IBM 7090 computer.

In this report, both the experimental and theoretical phases of the investigation are described, and the results from theory and experiment are compared.

---

<sup>1</sup>References are listed on page 18.

## EXPERIMENT

### DESCRIPTION OF MODEL

The model was accurately machined to the dimensions shown in Figure 1 from a steel tube whose high yield strength (100,000 psi) was sufficient to prevent inelastic action prior to the onset of buckling. The central frame was undercut as shown so that subsequent reductions in its width would not significantly change the unsupported length between it and the two end rings.

The sequence of reductions of the heavy frame followed in the testing program is shown schematically in Figure 2 and the dimensions of each case are given in Table 1. After the first test in which the dimensions of the heavy frame were as shown for Case 1, the frame was machined equally on both sides to the dimensions given for Case 2. This procedure was repeated until Case 8 was tested, after which the height of the heavy frame was reduced, in stages, to typical size (Case 14).

The sequence was arranged so that removal of a unit of frame material would result in a large reduction in torsional rigidity for Cases 1 through 8, and a large reduction in bending rigidity for Cases 9 through 14. In this way, it was hoped that the effects of these two stiffness properties could be examined separately.

The parameter chosen as a realistic measure of bending rigidity is the moment of inertia of the frame-shell section, which includes the stiffener plus one typical bay length of shell. A schematic diagram of a frame-shell section is shown in Figure 3, and the parameters for each case are given in Table 2. The moment is taken about the longitudinal centroidal axis.

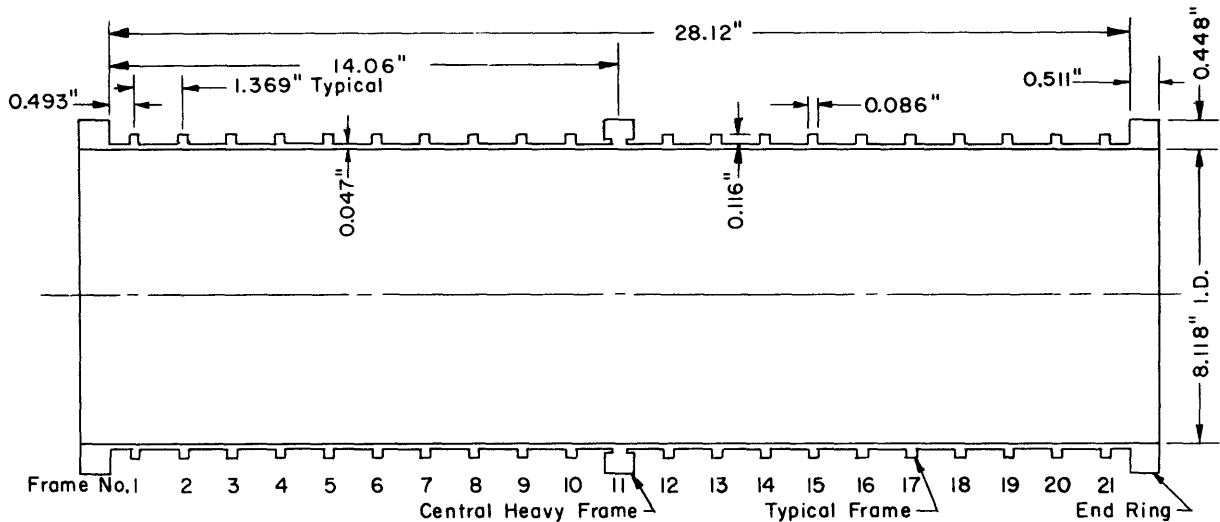


Figure 1 – Schematic Diagram of Axial Section of Ring-Stiffened Cylinder with Central Heavy Frame

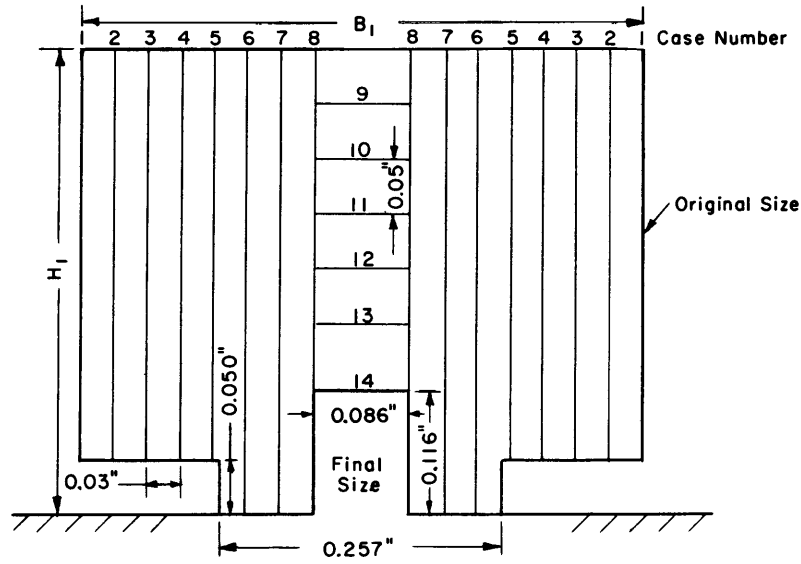


Figure 2 – Sequence of Reductions of Heavy Frame

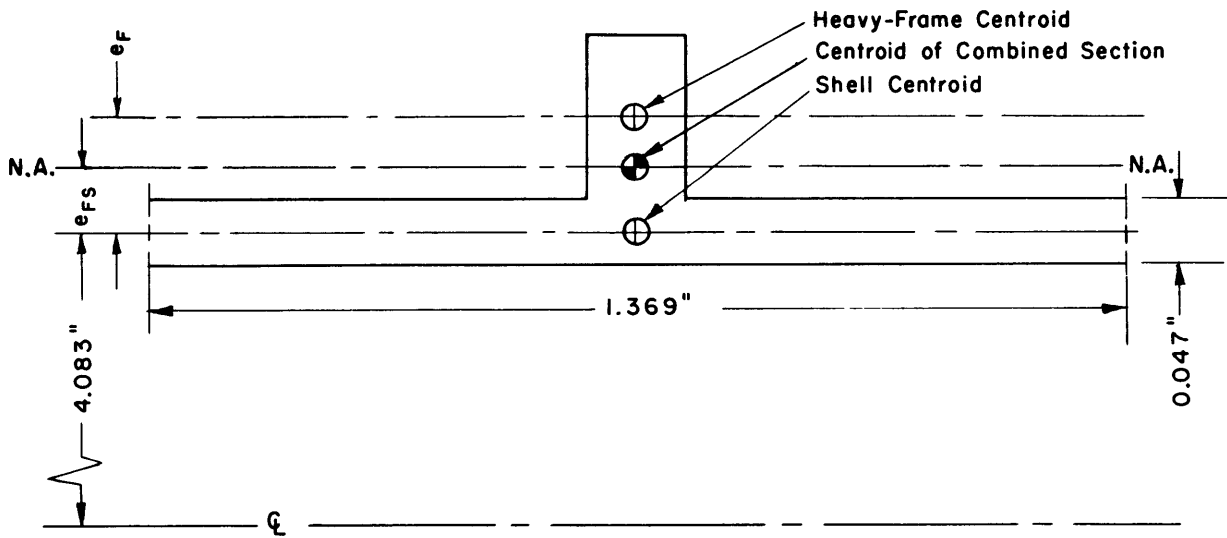


Figure 3 – Schematic Diagram of Frame-Shell Section

TABLE 1

Heavy Frame Parameters

Case	Width $B$ in.	Height $H$ in.
1	0.511	0.401
2	0.451	0.401
3	0.391	0.401
4	0.331	0.401
5	0.271	0.401
6	0.211	0.401
7	0.151	0.401
8	0.086	0.401
9	0.086	0.351
10	0.086	0.301
11	0.086	0.251
12	0.086	0.201
13	0.086	0.151
14	0.086	0.116

TABLE 2

Parameters of Frame-Shell Section

Case	$e_F^*$ in.	$e_{FS}^{**}$ in.	$I_{FS}^\dagger$ in. <sup>4</sup>	$\frac{I_{FS}^{\dagger\dagger}}{I_{fs}}$
1	0.236	0.177	0.004940	62.5
2	0.234	0.170	0.004614	58.4
3	0.232	0.162	0.004263	53.9
4	0.229	0.153	0.003878	49.1
5	0.225	0.141	0.003451	43.7
6	0.224	0.127	0.002950	37.3
7	0.224	0.109	0.002367	30.0
8	0.224	0.078	0.001588	20.1
9	0.199	0.064	0.001135	14.4
10	0.174	0.050	0.000767	9.7
11	0.149	0.037	0.000484	6.1
12	0.124	0.026	0.000280	3.5
13	0.099	0.017	0.000143	1.8
14	0.082	0.011	0.000079	1.0

\* $e_F$  is distance from centroid of shell to centroid of frame.

\*\* $e_{FS}$  is distance from centroid of shell to centroid of frame-shell section.

† $I_{FS}$  is moment of inertia of heavy-frame-shell section about its centroid.

†† $I_{fs}$  is moment of inertia of typical-frame-shell section about its centroid.

## INSTRUMENTATION AND TEST PROCEDURE

The model was instrumented with electrical-resistance strain gages oriented circumferentially on the exterior surface of the frames. Gages were located every 15 deg on Frames 6, 11, and 16 to determine the circumferential buckling pattern. The longitudinal deflection pattern was determined by gages located on every frame along one generator. Figure 4 shows the cylinder instrumented for the first test. The critical pressures were obtained by applying the Southwell method<sup>2\*</sup> to the strain-pressure plots of the gages on Frames 6 and 16 for the  $n = 4$  buckling mode and on Frame 11 for the  $n = 3$  buckling mode.

In addition, acceleration pickups were attached to the surface of the shell to determine the critical buckling pressures by a nondestructive vibration method.\*\* Tests were carried

\*The accuracy of the Southwell method as applied to stiffened shells has been shown from earlier tests<sup>2</sup> to be within 3 percent of the actual collapse pressures.

\*\*The vibration method is described briefly in References 3 and 4.



out with internal as well as external pressure applied to the cylinder. This method permits determination of the pressures associated with the noncritical as well as with the critical modes.

The end-closure arrangement used on the model for the tests is the same as that used for Case 1 in Reference 5, where it was found that experimental pressures obtained with these closure plates agreed closely with the theory of Reference 6 (second solution).

## TEST RESULTS AND DISCUSSION

In all the static tests the characteristic lobar strain patterns appeared at pressures well below buckling and grew rapidly as pressure was increased. These strains provided the necessary data for the application of the Southwell method to determine elastic buckling pressures and also indicated the associated buckling modes.

The experimental pressures obtained by the Southwell and the vibration methods for each case are listed in Table 3 and are represented graphically in Figure 5, where the abscissa scale represents the ratio of the moment of inertia of the heavy-frame-shell section ( $I_{FS}$ ) to that of the typical-frame-shell section ( $I_{fs}$ ).

The curves of Figure 5 show the transition in the critical pressure and buckling mode as successive decreases in the heavy frame effectively lengthen the cylinder. Figure 6 shows the variation of the critical pressure and mode with overall length of a uniformly stiffened cylinder having the same typical geometry as the model tested. The theory of Reference 6 (second solution) was used to calculate the curves. For the heaviest central frame investigated (Case 1), the critical pressure of 412 psi obtained by the Southwell method is in close agreement with the pressure of 400 psi ( $n = 4$ ) obtained from Figure 6 for a uniformly framed cylinder of one-half the total model length. The experimental buckling mode was  $n = 4$  for each half of the cylinder, and the patterns were staggered with respect to each other as shown in Figure 7.

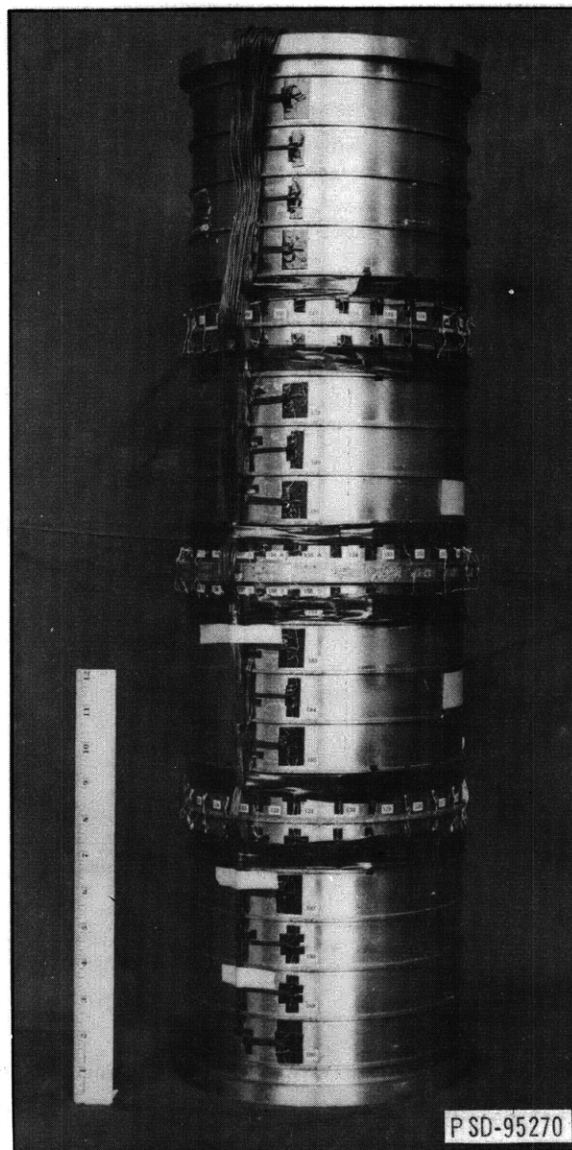


Figure 4 - Cylinder Instrumented for First Test

**TABLE 3**  
**Experimental Pressures from Southwell and**  
**Vibration Methods**

Case	Experimental Pressure, psi			
	Southwell Method		Vibration Method	
	$n^* = 3$	$n = 4$	$n = 3$	$n = 4$
1		412		438
2		409		
3		404		
4		399		
5		399		
6		406	492	422
7		396	458	422
8	380		401	419
9	344		365	413
10	312		334	423
11	275		302	
12	249		276	
13	224		247	402
14	210		237	406

\* $n$  is the number of circumferential buckling lobes.

There was little reduction in the critical pressure as the heavy frame was reduced in size until a point between Cases 7 and 8 was reached. Thus this experimental evidence indicates that a very rigid internal bulkhead can be replaced by a frame of far less weight with no apparent loss in strength. Beginning with Case 8 the deformation pattern changed to the  $n = 3$  mode, and the lobes extended over the full length of the cylinder; see Figure 7. Figure 8 shows the transition from the  $n = 4$  to the  $n = 3$  mode in the deformation pattern observed at Frames 6 and 16. Also shown is the deformation pattern of Frame 11 which had three lobes for all cases tested. The transition from  $n = 4$  to  $n = 3$  occurred at a Southwell pressure of 394 psi which agrees extremely well with the pressure of 388 psi determined from Figure 6.

Further reduction in the size of the heavy frame was accompanied by a large nonlinear reduction in the critical pressure of the cylinder. For the limiting case of a typical frame at the center of the model (Case 14), the Southwell experimental pressure of 210 psi ( $n = 3$ ) was exactly the same as that predicted from Figure 6.

As previously discussed, reductions in the torsional rigidity of the heavy frame were greatest in progressing from Case 1 to Case 8. Yet Figure 5 shows that these reductions had little effect on the general-instability strength of the cylinder even though it was in this range that the antisymmetric longitudinal pattern required twisting of the heavy frame (Figure 7). The significant losses in strength appeared in the range from Case 8 to Case 14 where reductions in the bending rigidity were greatest and where the longitudinal pattern was symmetric with no twisting of the heavy frame.

It would thus appear that torsional rigidity of external frames, at least, is of relatively little importance and bending rigidity is of prime importance in determining the ability of intermediate heavy frames to increase the general-instability strength of a ring-stiffened cylinder.

### EMPIRICAL HEAVY-FRAME FORMULA

A convenient empirical method, based on the Lévy ring formula,<sup>7</sup> has been developed for determining the elastic overall-buckling pressure of a uniformly ring-stiffened cylinder with intermediate heavy frames. The ring formula of Reference 7 has been adapted to the

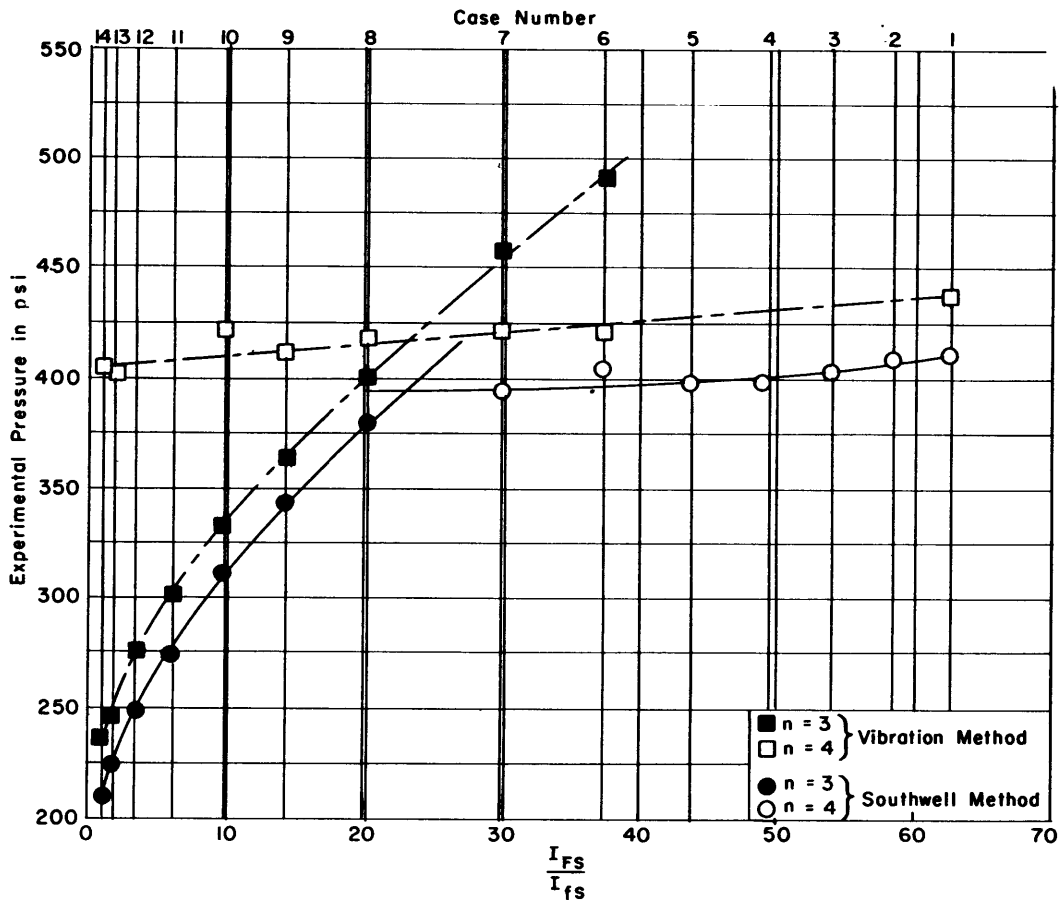


Figure 5 - Experimental Pressures versus Heavy Frame Parameter

present problem by redefining the geometric parameters as follows:

$$I_{FS} = \frac{p_{cr} R_C^3 L_e}{(m^2 - 1) E} \quad [1]$$

where it is assumed that

$$L_e \approx L_f + (L_F - L_f) \left[ \frac{p_{cr} - p_B}{p_x - p_B} \right]$$

for the range where  $p_x \geq p_{cr} \geq p_B$ . Here

$I_{FS}$  is the moment of inertia of the heavy-frame-shell section,

$p_{cr}$  is the critical pressure of the cylinder,

$E$  is the modulus of elasticity of the cylinder material,

$R_C$  is the radius from the axis of the cylinder to the centroid of the heavy-frame-shell section,

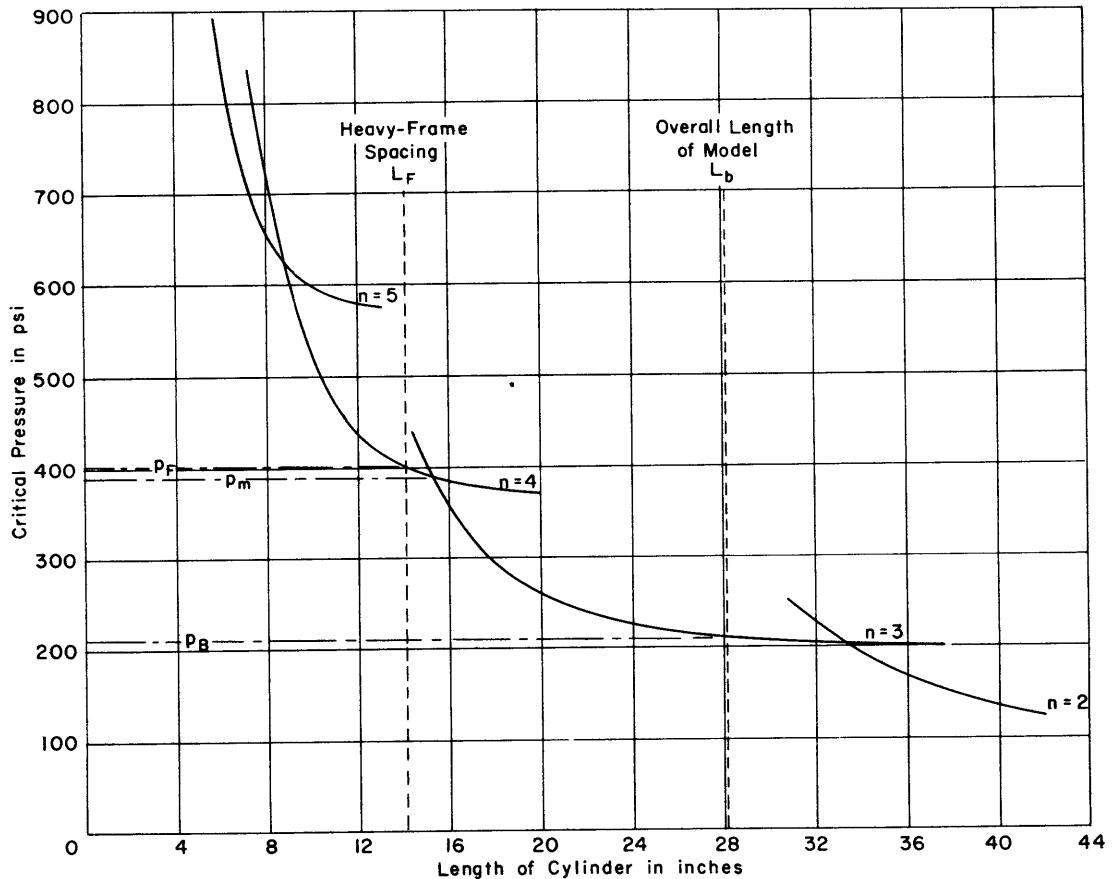


Figure 6 – Kendrick Part III (Second Solution) versus Length of Uniformly Stiffened Cylinder

$L_e$  is the effective length of that portion of the cylinder which loads the heavy frame,  $p_B$  is the critical pressure of the cylinder with the heavy frames replaced with typical frames (Figure 6),

$m$  is the critical buckling mode determined for  $p_B$ ,

$L_f$  is the typical frame spacing,

$L_F$  is the heavy-frame spacing,

$p_x$  is equal to  $p_F$  or  $p_m$ , whichever is the lower pressure,

$p_F$  is the critical pressure of the uniformly stiffened cylinder of length  $L_F$  (Figure 6),\* and

$p_m$  is the critical pressure at which the critical mode changes from  $m$  to  $m + 1$  as the length of the uniformly stiffened cylinder is reduced (Figure 6).

Thus the size of the heavy frame in the range  $p_x \geq p_{cr} \geq p_B$  is dependent upon two limiting conditions. For the lower pressure limit  $p_B$ , the heavy frame is equal in size to a typical frame and the load acting on it is the pressure over one typical frame spacing of shell. As

\*This is the maximum pressure obtainable for a stiffened cylinder with intermediate heavy frames.

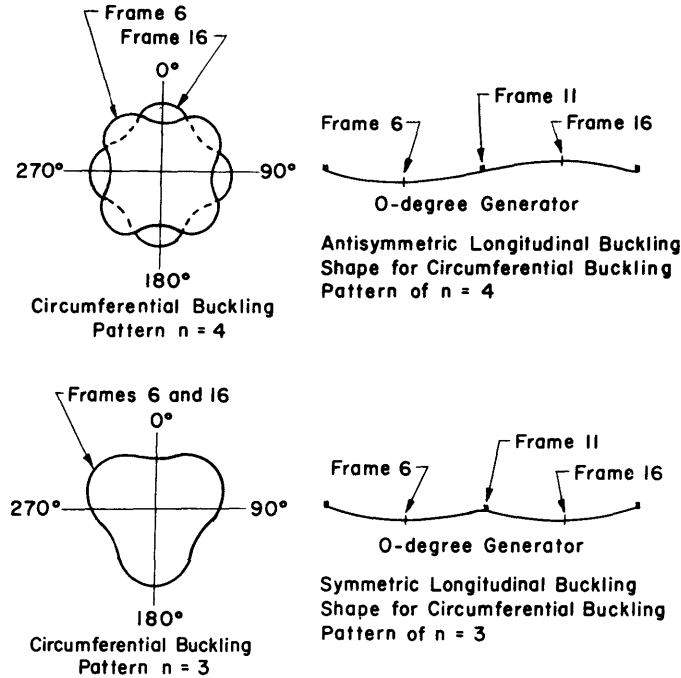


Figure 7 – Experimental Buckling Configurations

the heavy frame is made larger, it assumes increasingly more of the total load. At the upper pressure limit  $p_x$ , the maximum pressure for which there exists an overall symmetrical buckling shape (Figure 7), the heavy frame is loaded by the pressure acting on one heavy-frame spacing of cylinder.

The size of the heavy frame in the pressure range  $p_F \geq p_{cr} \geq p_m$  can be calculated from the formula

$$I_{FS} = I_m + (I'_{FS} - I_m) \left[ \frac{p_{cr} - p_m}{p_F - p_m} \right] \quad [2]$$

where  $I'_{FS} = \frac{p_F R_C^3 L_F}{3E}$  and  $I_m$  is the moment of inertia of the heavy-frame-shell section determined for  $p_m$  from Equation [1]. Any further increase in the strength of the heavy frame will not increase the critical pressure because the failure will occur between the heavy stiffeners.

In Figure 9, the experimental results are compared with empirical curves based on Equations [1] and [2] and determined by two different elastic overall-instability solutions. The solid curve was obtained by determining the values of  $p_B$ ,  $p_F$ , and  $p_m$  from Figure 6.\*

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\*An alternative method for determining the values of  $p_B$ ,  $p_F$ , and  $p_m$  with reasonable accuracy is by using the graphical solution of Reference 8.

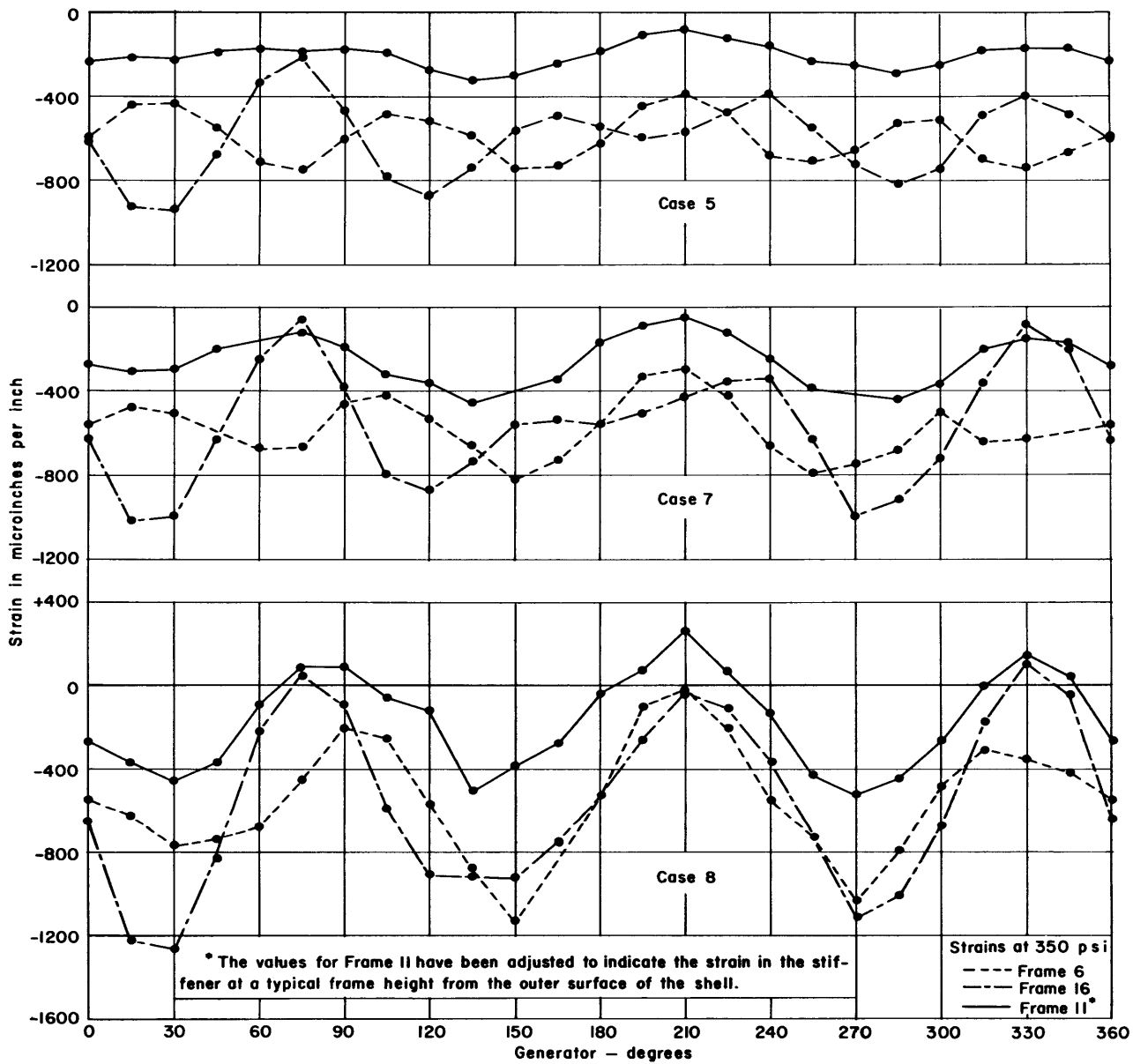


Figure 8 - Observed Circumferential Buckling Patterns

The agreement between the experimental and the calculated results is very good for all sizes of the heavy frame. The broken-line curve in Figure 9 was obtained by determining the values of  $p_B$ ,  $p_F$ , and  $p_m$  from curves similar to those in Figure 6 but computed from Bryant's equation\*<sup>9</sup> for elastic general-instability strength. The agreement between the broken-line curve and the experimental results is also good, but somewhat unconservative, for all sizes of the heavy frame. The results thus calculated would be more conservative if the effective moment of inertia ( $I_e$ ) found from Equation [1] of Reference 8 is used in the second term of Bryant's equation. Closer agreement with experimental results is also obtained if the values of  $p_B$  and  $p_F$  are computed from Bryant's equation and  $p_m$  is approximated by the equation

$$p_m \approx \frac{m(m+2)E I_{fs}}{L_f R_C^3}$$

In this way, the need for drawing curves similar to those of Figure 6 is eliminated.

## THEORY

### RESUMÉ OF ANALYTICAL WORK

Kendrick's (Part IV) analysis<sup>1</sup> of the buckling of a ring-stiffened cylinder with intermediate heavy stiffeners is the only theoretical treatment of this problem known to the authors.

---

\*Bryant's equation is as follows:

$$p = \frac{Eh}{R} \frac{\lambda^4}{\left(n^2 + \frac{\lambda^2}{2} - 1\right)(n^2 + \lambda^2)^2} + \frac{EI(n^2 - 1)}{L_f R_C^3}$$

where  $h$  is the shell thickness,

$R$  is the mean radius of the shell,

$n$  is the number of circumferential lobes,

$\lambda = \pi R/L_b$ , and

$L_b$  is the overall length of the cylinder.

The moment of inertia  $I$  is taken about the centroid of a section comprising one small frame plus a portion of shell of length  $L_f$ . It can be written

$$I = \frac{A_f e_f^2}{1 + \frac{A_f}{L_f h}} + I_f + L_f \frac{h^3}{12}$$

where  $A_f$  is the area of a small frame,

$I_f$  is the moment of inertia of the frame, and

$e_f$  is the distance from the midsurface of the shell to the centroid of the frame.

Bryant's equation is usually written with  $R$  instead of  $R_C$  in the second term. Use of  $R_C$  leads to slightly more conservative results.

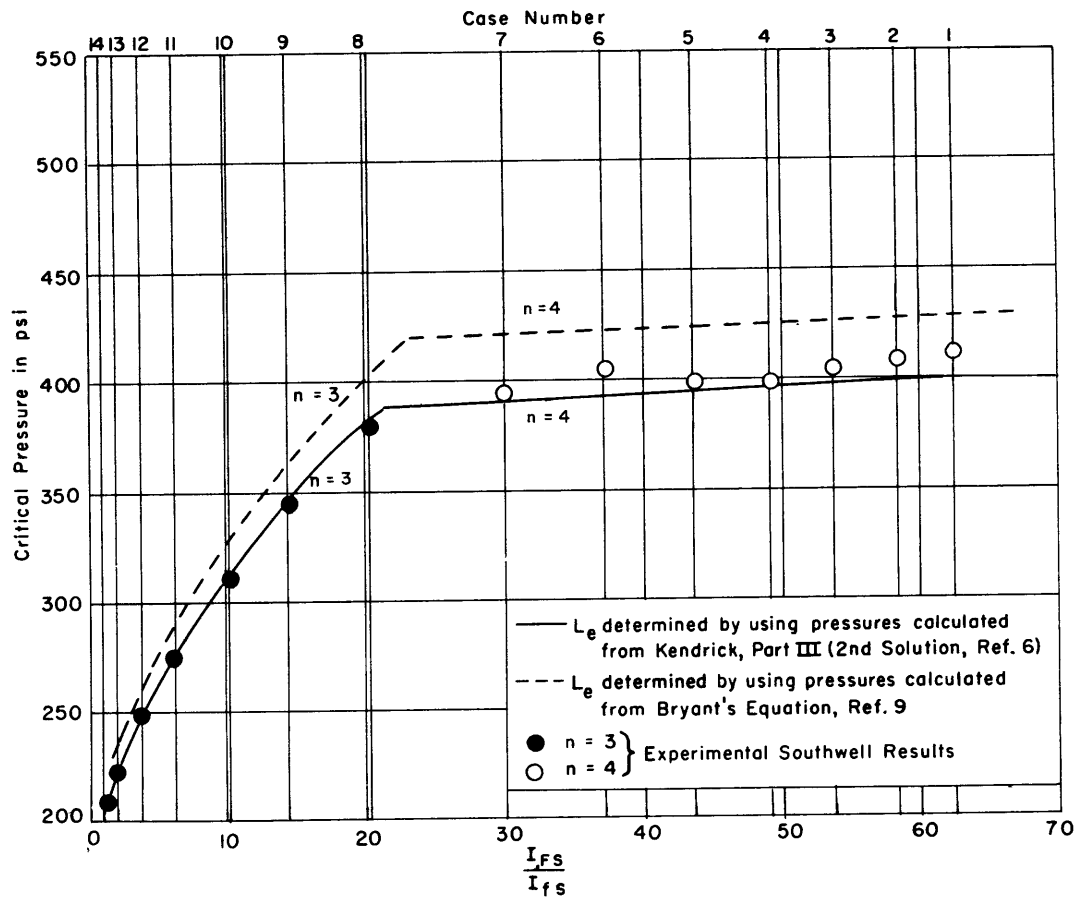


Figure 9 – Empirical Heavy Frame Formula Compared with Experimental Swallow Critical Pressures

The analysis considers a cylinder of finite length (shown in Figure 10) stiffened by sets of equally spaced light frames located between equally spaced heavy frames and terminated by rigid bulkheads. The approach is the same as that used by Kendrick in previous analyses.<sup>6</sup> The total potential of the system is obtained, a set of buckling displacements is assumed, and the buckling equations are then derived from the condition of minimum potential energy. The assumed buckling displacements used by Kendrick for the present problem are:

$$u = A_1 u_1(x) \cos n\theta$$

$$v = \left[ B_1 v_1(x) + B_2 \left| \sin \frac{\pi x}{L_f} \right| + B_3 \left| \sin \frac{\pi x}{L_F} \right| \right] \sin n\theta \quad [3]$$

$$w = \left[ C_1 w_1(x) + C_2 \left| \sin \frac{\pi x}{L_f} \right| + C_3 \left| \sin \frac{\pi x}{L_F} \right| \right] \cos n\theta$$



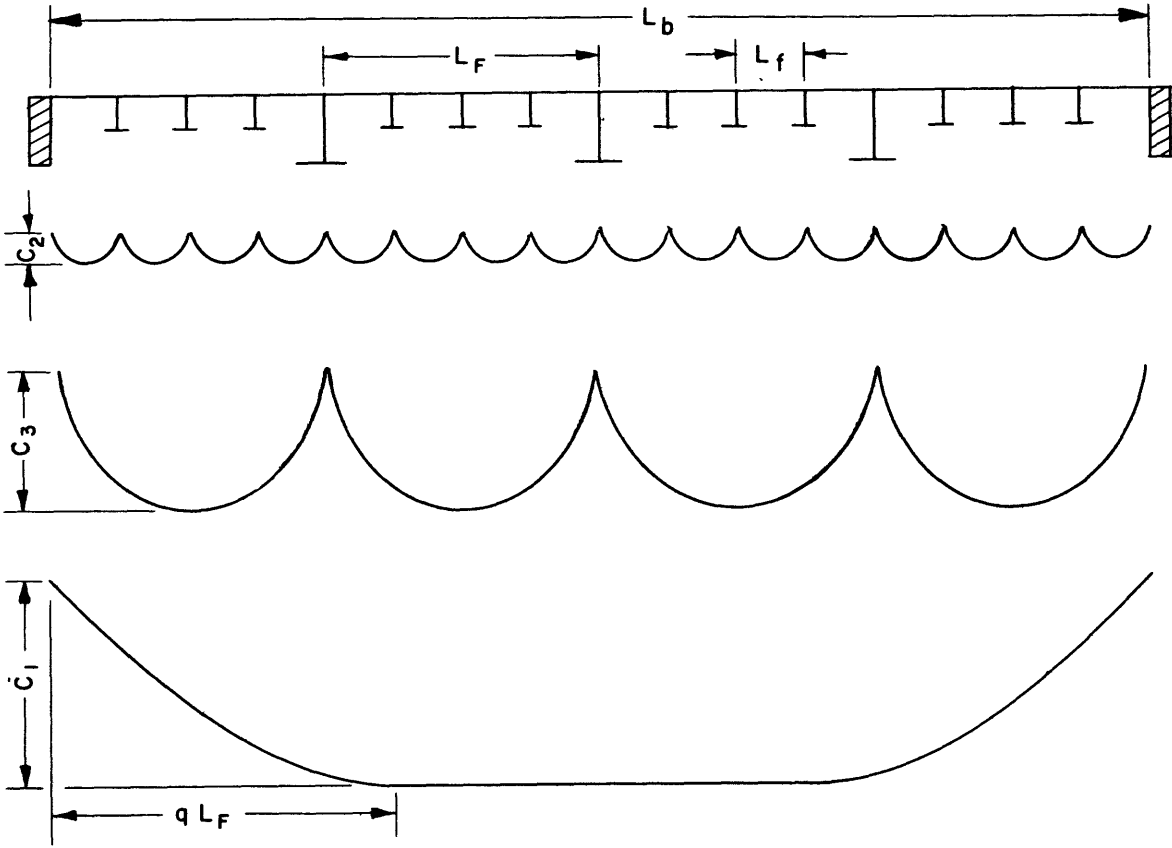


Figure 10 – Longitudinal Configuration of Radial Buckling Displacement (Kendrick Part IV)

where

$$\left. \begin{aligned} u_1(x) &= \cos \frac{\pi x}{2qL_f} \\ v_1(x) = w_1(x) &= \sin \frac{\pi x}{2qL_f} \end{aligned} \right\} 0 \leq x \leq qL_f \quad [4]$$

$$\left. \begin{aligned} u_1(x) &= 0 \\ v_1(x) = w_1(x) &= 1 \end{aligned} \right\} qL_f \leq x \leq L_b - qL_f \quad [5]$$

$$\left. \begin{aligned} u_1(x) &= \cos \left[ \pi \frac{(x - L_b + 2qL_f)}{2qL_f} \right] \\ v_1(x) = w_1(x) &= \sin \left[ \pi \frac{(x - L_b + 2qL_f)}{2qL_f} \right] \end{aligned} \right\} L_b - qL_f \leq x \leq L_b \quad [6]$$

and where  $x$  and  $\theta$  are the axial and circumferential coordinates,  
 $u$ ,  $v$ , and  $w$  are the axial, tangential, and radial displacements,  
 $A_1$ ,  $B_1$ , etc. are arbitrary coefficients,  
 $L_b$  is the bulkhead spacing, and  
 $q$  is an arbitrary number which permits the formation of a flat central portion of variable length as shown in Figure 10.

When  $q$  is given the value  $L_b/2L_f$ , the shape of the radial deflection between bulkheads is a half sine wave. Since there are seven arbitrary coefficients in the buckling deformations [3], the system possesses, in a sense, seven degrees of freedom which allow independent deformations between the light frames, the heavy frames, and the bulkheads (also shown in Figure 10). It should be pointed out that the use of this deflection pattern results in neglect of the torsional rigidity of the heavy frames, an assumption that is not unreasonable in view of the experimental observations.

After some deliberation, it was concluded that Kendrick's theory, while basically sound, might be improved on two counts. First, the generality could be expanded by permitting one additional degree of freedom. This was done by adding to the axial displacement  $u(x, \theta)$  a second component varying periodically between adjacent heavy frames, i.e.,

$$u = \left[ A_1 u_1(x) + \frac{L_F}{\pi} A_3 \frac{d}{dx} \left| \sin \frac{\pi x}{L_F} \right| \right] \cos n\theta \quad [7]^*$$

The contribution of this additional degree of freedom can, perhaps, be better understood by considering the case in which the heavy frames are infinitely rigid. The overall-displacement components between bulkheads would then be expected to vanish (i.e.,  $A_1 = B_1 = C_1 = 0$ ), and the problem should reduce to that of a uniformly stiffened cylinder of length  $L_F$ . For the solution of that problem Kendrick<sup>6</sup> found that five degrees of freedom were needed. In his Part IV analysis only four would remain, but with the addition of the axial component appearing in Equation [7] the buckling configuration becomes identical with the first solution of Reference 6.

The second shortcoming in the theory of Reference 1 concerns the circumferential stresses associated with the initial or prebuckling deformation. In previous analyses of uniformly stiffened cylinders, Kendrick<sup>6</sup> has shown that it is sufficiently accurate to approximate this deformation by a uniform contraction of the cylinder. The circumferential stress  $\sigma_\theta$

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\*While this function violates continuity of the shell at the heavy frames, it is, nevertheless, consistent with the discontinuities in slope arising from the configurations assumed for the other two displacements  $v$  and  $w$ . It would be logical to add a ninth degree of freedom to allow periodic axial deflections between light frames. However, since the theory of Reference 6, which excludes this component, has shown excellent agreement with experiment,<sup>4,5</sup> its absence is evidently not a serious defect. It would conceivably become so only in cases where the buckling is of the interframe (von Mises) type.

is thus constant everywhere. In his heavy-frame analysis, Kendrick continues the use of this approximation and assumes that the circumferential stress is given by

$$\sigma_{\theta} = - \frac{p R L_F}{A_F + N_1 A_f + h L_F} \quad [8]$$

where  $A_F$  is the area of a heavy frame,

$A_f$  is the area of a light frame,

$N_1$  is the number of light frames between heavy frames,

$h$  is the shell thickness,

$L_f$  is the small-frame spacing, and

$L_F$  is the heavy-frame spacing equal to  $(N_1 + 1)L_f$ .

So long as  $A_f$  and  $A_F$  are approximately equal, the above approximation for  $\sigma_{\theta}$  is reasonable. However, if  $A_F$  becomes very large, the stresses in the small frames and in the shell will be greatly underestimated. Thus, as  $A_F$  increases, the buckling pressure also increases without limit. This is contrary to the experimental results (Figure 5) which show that beyond a certain size further increases in the heavy frame had practically no effect on the collapse pressure. Consequently, it would appear that the use of Equation [8] can lead to unconservative results.

The second modification, therefore, was to eliminate this difficulty by replacing  $A_F$  by  $A_f$  in the equation for  $\sigma_{\theta}$ . Now

$$\sigma_{\theta} = - \frac{p R L_f}{A_f + h L_f} \quad [9]$$

This equation is identical with that used in Kendrick's Part III analysis<sup>6</sup> and so should be reasonably accurate when the heavy frames are of sufficient strength to act as bulkheads.

Both Kendrick's Part IV analysis and the TMB solution incorporating the two modifications introduced by Equations [7] and [9] were programmed for the IBM 7090 computer, and calculations were carried out for comparison with each other and with experiment.

## RESULTS AND DISCUSSION

In Figure 11 the elastic general-instability pressures computed from both Kendrick's Part IV solution and the TMB solution are plotted against the ratio  $I_{FS}/I_{fs}$  together with the experimental results. In all cases  $q$  was given the value  $L_b/2L_f$  (the computer program will accept any value for  $q$ ) to restrict the overall buckling shape to a half sine wave.\* The

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\*In general, the buckling pressure will be a minimum for some particular value of  $q$  which need not be  $L_b/2L_f$ . The restriction on  $q$  was made only to simplify the results of this initial investigation. Despite this restriction the results of the TMB solution were always conservative. The effect of varying  $q$  will be considered in a subsequent report.

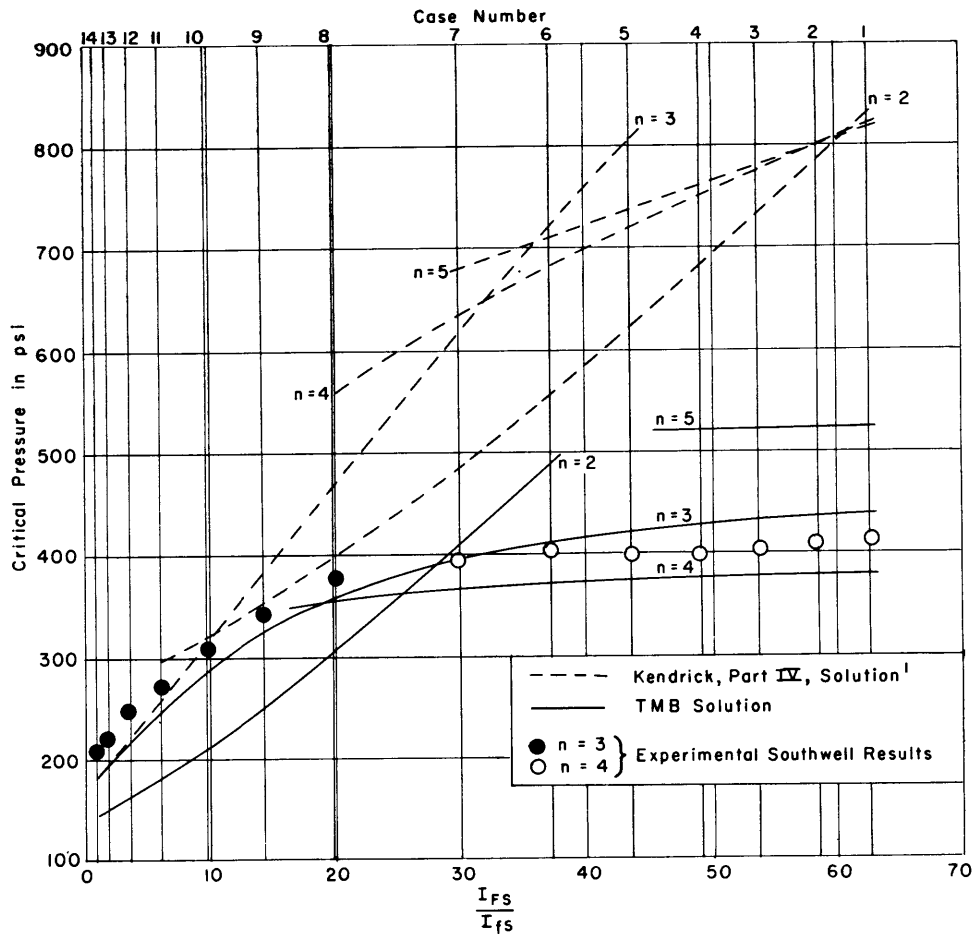


Figure 11 – Theoretical Heavy Frame Solutions Compared with Experimental Southwell Critical Pressures

broken-line curves of Figure 11 show that Kendrick's Part IV solution agrees well with experiment only in the region where the third mode is critical. When  $I_{FS}/I_{fs}$  exceeds 10, the second mode, according to Kendrick's solution, becomes critical, a transition that was never observed experimentally. As the heavy frame continues to increase in size, the Kendrick predicted pressures rise rapidly with no apparent limit, so that the solution is highly unconservative in the upper range of  $I_{FS}/I_{fs}$ .

The solid curves show that the TMB solution is conservative for all cases tested and that the agreement with experiment is very good for the modes  $n = 3$  and  $n = 4$ . In particular, this solution, like the experimental pressures, approaches an upper limit. However, the unrealistically low pressures which this solution gives for the second mode in the lower range of  $I_{FS}/I_{fs}$  cannot be disregarded. There the solution not only fails to predict correctly the critical mode but grossly underestimates the effectiveness of the heavy frame.

Evidently both solutions have a shortcoming which, for the cylinder tested, results in the appearance of the second mode at pressures far below those for the other modes in many of the cases. The difficulty probably arises from the fact that in both solutions the shell is assumed hinged at the heavy frame. As the frame is reduced in size, the weakening effect

of this hinge becomes more important until the typical size is reached, at which point both solutions are, in effect, for a cylinder of length  $L_F$  with one end simply supported and the other end in effect free. Although, for the cylinder tested, theoretically the second mode appears to be seriously influenced, it must be expected that, in general, other modes may be similarly affected. At present, no simple means has been found for eliminating this shortcoming.

### SUMMARY OF RESULTS

To summarize the results of this study Table 4 presents the experimental buckling pressures as determined by the Southwell Method together with the pressures given by the empirical and analytical solutions, for all cases tested. Results of the empirical solution correspond to the curves of Figure 9 and are obtained using pressures calculated from, in one case, Kendrick's Part III, second solution,<sup>6</sup> and in the other, Bryant's equation.<sup>9</sup> Of all four methods, it appears that the empirical solution employing Kendrick's analysis agrees best with experiment.

TABLE 4  
Summary of Results

Case	$\frac{l_{FS}}{l_s}$	Critical Pressure, psi				
		Experimental (Southwell Method)	Empirical Solution*		Theoretical Solutions	
			A	B	Kendrick Part IV	TMB
14	1.0	210 (3)**	210 (3)	224 (3)	186 (3)	143 (2)
13	1.8	224 (3)	223 (3)	234 (3)	196 (3)	148 (2)
12	3.5	249 (3)	248 (3)	253 (3)	222 (3)	161 (2)
11	6.1	275 (3)	277 (3)	280 (3)	262 (3)	181 (2)
10	9.7	312 (3)	312 (3)	312 (3)	318 (3)	211 (2)
9	14.4	344 (3)	342 (3)	337 (3)	354 (2)	253 (2)
8	20.1	380 (3)	385 (3)	378 (3)	397 (2)	305 (2)
7	30.0	396 (4)	396 (4)	391 (4)	484 (2)	367 (4)
6	37.3	406 (4)	399 (4)	401 (4)	556 (2)	371 (4)
5	43.7	399 (4)	403 (4)	409 (4)	621 (2)	374 (4)
4	49.1	399 (4)	406 (4)	417 (4)	679 (2)	376 (4)
3	53.9	404 (4)	409 (4)	423 (4)	734 (2)	378 (4)
2	58.4	409 (4)	412 (4)	429 (4)	784 (2)	379 (4)
1	62.5	412 (4)	414 (4)	434 (4)	819 (5)	379 (4)

\* A – Calculations based on Kendrick, Part III, second solution.<sup>6</sup>  
 B – Calculations based on Bryant's Equation.<sup>9</sup>  
 \*\* Numbers in parentheses indicate the number of circumferential lobes.

## CONCLUSIONS

1. Experiments indicate that an intermediate heavy frame can serve as an adequate substitute for an internal bulkhead to increase the general-instability strength of a ring-stiffened cylinder.
2. In determining the proportions of an adequate heavy stiffener, bending rigidity is shown by these tests to be of much greater importance than torsional rigidity, at least for external frames.
3. The excellent agreement with experiment shown by the empirical heavy frame formula for the geometry tested is promising, but further experimental evaluation is needed.
4. Modifications in Kendrick's heavy-frame theory have resulted in a substantial improvement, although some deficiencies still remain. Additional tests must be conducted before the theory can be properly evaluated.

## RECOMMENDATIONS FOR FUTURE WORK

The accuracy of the empirical and theoretical heavy-frame solutions should be investigated through destructive tests of inexpensive machined cylinders of small diameter and through non-destructive tests of larger cylinders with interchangeable rings simulating heavy frames.

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