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STRUCTURAL SIMILITUDE FOR IMPACT PHENOMENA

(From C and R Bulletin No. 13-A)

by

CDR S. R. Heller, Jr., USN

STRUCTURAL MECHANICS LABORATORY

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PREFACE

In June 1939 the Bureau of Construction and Repair (now part of the Bureau of Ships) published Bulletin No. 13-A – CONFIDENTIAL – by R.D. Conrad (then LIEUTENANT (CC), USN; now deceased) which reported model investigations of armored structures. Much of the material included in the Bulletin was unclassified, but a great deal was classified and, accordingly, was protected.

During the two decades that have elapsed, the scientific literature has been deprived of Conrad's work on similitude of structural models subjected to impact. It, therefore, seemed wise to separate the similitude portion of the original Bulletin from that portion devoted exclusively to model tests of armored structures and to the development of construction details.

The author desires to state unequivocally that this report, essentially the unclassified portion of Bulletin No. 13-A, should properly have Conrad's byline. The words are those of the author; the ideas are Conrad's.

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ABSTRACT

The conditions for geometric and dynamic similitude of structural models subjected to impact are developed. Various factors considered to have secondary influence on dynamic similitude are discussed, and their scale effects are explored. Although complete dynamic similitude of such a complex problem is probably not attainable, the essential requirements of similitude can be met to the end that model investigations can be justified by results obtained—not merely by economic considerations.

INTRODUCTION

The development that follows was spurred by the renascence of capital ship design in 1935 after a hiatus of about 20 years. In the interval new problems had arisen to plague the ship designer. Greater thicknesses of horizontal armor were required to meet the plunging fire at anticipated increased battle ranges and to offset aerial bombing attacks. Torpedo improvement required a corresponding improvement in underwater protection. Turrets and, therefore, barbettes had become larger. Weight was severely limited by the several treaties governing naval construction. What might have been of even greater significance than any or all of the technical problems was the financial stringency associated with the Great Depression.

To meet these new problems, past practice in armoring was critically re-examined. The resistance of plates to penetration was well known from vast quantities of proving ground experience. The prediction of projectile conditions for penetration is, however, a separate ballistic problem. Thus, it became apparent that the design of armored structures to withstand impacts incapable of penetration was still obscure. Not only was there a lack of a reasonable theoretical approach, but there was also a dearth of actual experience.

In the past it had apparently been considered essential to make full-scale projectile-impact tests of armored structures in order to predict performance. Consequently, few such tests had ever been made. The time and money required to carry out the carefully controlled experiments needed to furnish design information were practically prohibitive. Accordingly, the possibilities of the model approach were explored. From these studies it was concluded that, despite uncertainties associated with reduced scales, much might be learned from small-scale model tests. In order to design these model tests properly the conditions of similitude for impact phenomena were developed.

DIMENSIONAL ANALYSIS OF PROJECTILE-IMPACT PHENOMENA

The phenomena which occur when a projectile strikes an armored structure may conveniently be classed as ballistic and structural. The action of the *projectile* after impact and the *local* damage to the *armor* are *ballistic* effects. The momentum acquired by the armor plate causes it to move and damage the supporting structure. Although some consideration of the ballistic effects is necessary to study the effects on the structure, the motion of the armor and the resulting *structural* effects are of primary concern.

As has already been mentioned in the INTRODUCTION, the resistance of an armor plate to penetration can be predicted very accurately from proving-ground data, but little is known about the mechanics of impact. In addition, the momentum acquired by the armor, which is the principal determinant of the degree to which armor plates are dislodged and the supporting structure is damaged, is greatest for nonpenetrative impacts in which the projectile rebounds.

The complete dimensional analysis of the problem involves a number of factors which appear to be of secondary importance—or which cannot be controlled in a model test. What is needed is an approach similar to that used in a model towing test for resistance and propulsion information. Obviously, both the Reynolds and Froude numbers for the prototype cannot conveniently be maintained for the model. Similarly, for impact testing of models representing armored structures the factors of secondary importance must be temporarily ignored, and corrections added later. Consequently, a simplified treatment will be given first, and then the omitted quantities will be discussed later in an effort to improve the admittedly incomplete results first obtained.

Although the dimensional method is useful in providing some insight to the details of complex phenomena, it must be borne in mind that facts cannot be established by dimensional reasoning alone but can be obtained from experiments conforming to the principles of similitude obtained by dimensional analysis.

PRIMARY REQUIREMENTS

The elementary case is shown schematically in Figure 1. The first step in dimensional analysis is the listing of quantities which, if changed, would change the phenomenon—in this case, the damage to the supporting structure. This list should include:

E the modulus of elasticity of the material

G the shear modulus of the material

I the moment of inertia of the armor with respect to the axis about which it tends to rotate when struck

l a characteristic linear dimension defining the size of the system

M mass (effective) of the armor

m mass of projectile

v striking velocity of projectile

θ the angle of attack measured from the normal to the armor

ν the Poisson's ratio of the material

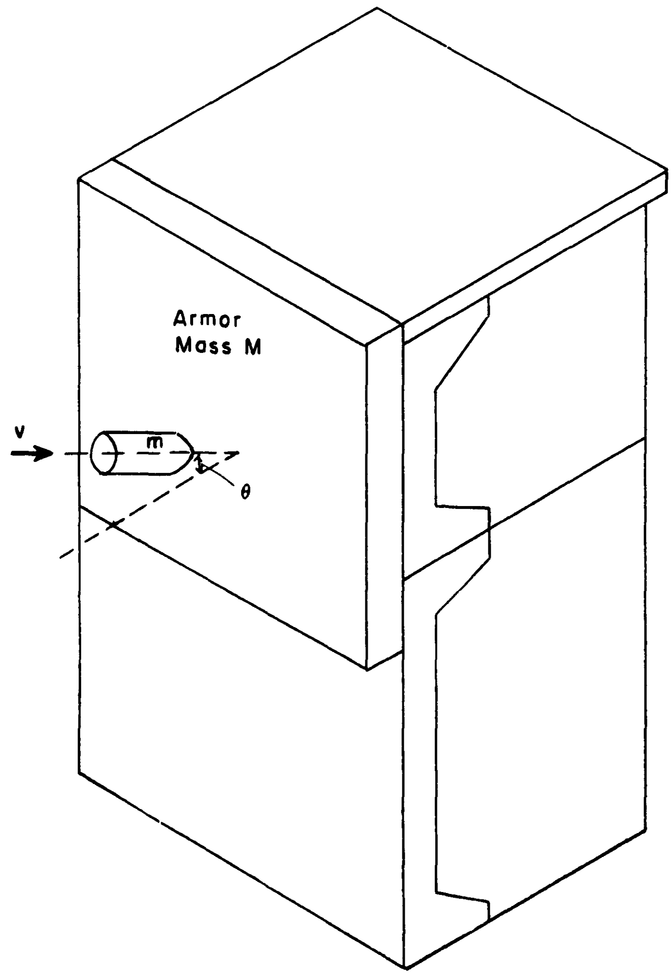

Figure 1 - Schematic Representation of Armored Structure Subjected to Impact

ρ the mass density of the material

σ the stress at some particular point in the supporting structure at maximum deformation

σ_0 the initial stress at that point

σ_y the yield stress of the material

If the mass system of dimensions (M,L,T) is used, not more than ten independent dimensionless combinations of these 13 quantities can be formed to define the dimensionally homogeneous equation relating the quantities.* Since the systematic control of as many as ten combinations of quantities is a rather arduous task, a longer look at the quantities and the effects of preliminary assumptions seems warranted prior to deriving the dimensionless combinations.

*See any standard text on dimensional analysis; e.g., Bridgman, P.W., "Dimensional Analysis," Yale University Press, New Haven (1922).

First, complete geometrical similarity of the two systems, model and prototype, is a normal practice. Thus a single linear dimension is sufficient to define shape, and the ratio of any two corresponding lengths defines the scale factor λ.

Second, other things being equal, the same material could be used in both model and prototype. This would eliminate E, G, ν, ρ, and σ_y since they would be the same. Poisson's ratio ν is dimensionless and, hence, would contribute nothing to the analysis. It is easily demonstrated that the combination involving ρ is simply $\rho l^3/M$, and this surely has the same value in geometrically similar models of the same density. Moreover, the quantities E, G, σ, σ_0, and σ_y all have the same dimensions ($ML^{-1}T^{-2}$). Hence, all ratios formed from them are dimensionless. It will therefore be necessary to derive only a single combination corresponding to them; the remainder can be written by inspection.

Third, the angles of attack must obviously be the same for model and prototype; for unless they are there could be no hope of obtaining comparable results. θ is itself dimensionless and can, therefore, be omitted.

Fourth, the moment of inertia I of any object is completely determined by its size, shape, mass, and distribution of density. Since a model of the same material as and geometrically similar to the prototype specifies these characteristics, I is superfluous and can be omitted.

Finally, either m or M may be omitted, for with the same material and geometrical similarity only one mass need be specified, just as only one length is required.

The list of quantities now reduces to:

Quantity	Dimensions
M	M
v	LT^{-1}
l	L
E, G, σ, σ_0, σ_y	$ML^{-1}T^{-2}$

These eight variables will yield not more than five independent combinations. These are derived by choosing three of the quantities and combining them successively with those remaining.

It seems logical to expect damage to be reproduced if stresses are reproduced. Thus it appears that σ should be one of the initial quantities since it will appear in all dimensionless combinations, and further transformation will be unnecessary to obtain the relations necessary to assure equal stress. For equally obvious reasons M and l are chosen for the other two initial quantities.

The following dimensionless combinations are easily obtained:

$$\pi_v = v\sqrt{\frac{M}{\sigma l^3}} \qquad \pi_{\sigma_0} = \frac{\sigma_0}{\sigma}$$

$$\pi_{\sigma_y} = \frac{\sigma_y}{\sigma} \qquad \pi_E = \frac{\sigma}{E} \qquad [1]$$

$$\pi_G = \frac{\sigma}{G}$$

It must now be recalled that model and prototype are to be made of the same material and will be geometrically similar. Under these conditions M/l^3 is a constant. Thus the first of Equations [1] reduces to $v/\sqrt{\sigma}$.

The conditions for dynamic similitude corresponding to Equations [1] become, using subscripts to refer to the model:

$$\frac{\sigma_1}{\sigma} = \left(\frac{v_1}{v}\right)^2 \qquad \frac{\sigma_1}{\sigma} = \frac{\sigma_{0_1}}{\sigma_0}$$

$$\frac{\sigma_1}{\sigma} = \frac{\sigma_{y_1}}{\sigma_y} \qquad \frac{\sigma_1}{\sigma} = \frac{E_1}{E} \qquad [2]$$

$$\frac{\sigma_1}{\sigma} = \frac{G_1}{G}$$

The first of Equations [2] requires *equal striking velocities for equal stresses at corresponding points.* The last two of Equations [2] require the same elastic properties for both model and prototype. Since these are about the same for most steels, there is some latitude in the choice of model material.

Superficially the third of Equations [2] requires equal yield points for the two materials. Actually it requires identical stress-strain curves. Similitude of elastic effects is obtained if elastic properties, E and G, are the same for both materials; but similitude of plastic effects requires the remainder of the stress-strain curve to be the same. This will not generally be possible, so some scale effect is expected.*

The second of Equations [2] requires equal initial stresses at corresponding points in the two structures. Gravity will be discussed later, but there are other sources of initial stress to be considered. Local concentrations caused by riveting and welding and the general bending of the ship girder must be reproduced somehow in the model.

***Note: At the time of the original writing this was true, but modern heat-treating techniques can produce identical stress-strain curves for the same material regardless of thickness.**

ADDITIONAL CONSIDERATIONS

It is easy to imagine other quantities which could influence dynamic similitude. Some of these are:

- c the rate of propagation of stress waves in the material
- e the energy absorbed per unit volume
- F the force on any area at any instant during impact
- g the acceleration of gravity
- H the energy dissipated in heat
- mv the momentum
- t the time required for any event to occur
- γ the rate of straining of the material

Very likely some of the foregoing are redundant, and perhaps others are not susceptible to dimensional analysis. It does appear, however, that the total list does include all quantities likely to affect results. Each will be examined separately and in those combinations that appear to be significant.

Stress Waves

The velocity of a stress wave c will be the same in model and prototype, since it depends only on the density and elasticity of the material—which has already been chosen as the same. The corresponding dimensionless combination is v/c. This will be the same in model and prototype for equal velocities of impact.

Stress waves are reflected at the boundaries of a member and partly reflected and transmitted at all joints and discontinuities of section. These reflections may add to or cancel each other depending on whether the boundaries are fixed or free.[1] The length of a stress wave depends on the duration of impact, and the number of waves which any element of a structure can contain depends on the length of the element in the direction of propagation of the wave. Further discussion of the effect of stress waves will be postponed until time is discussed.

Energy Absorbed

The energy absorbed per unit volume of material e has the dimensions $ML^{-1}T^{-2}$, the same as stress and elasticity. Thus $\pi_e = e/\sigma$, which shows that, for equal stresses, e will be the same for both model and prototype. This is also shown by considering $\pi_v = v\sqrt{M/\sigma l^3}$. If the velocity V imparted to the model is the same fraction of v as in full scale, V may be

[1]References are listed on page 12.

substituted for v in π_v. Subsequent squaring yields $MV^2/\sigma l^3$ which, for equal stresses, shows that the kinetic energy per unit volume has the same value in both systems if the striking velocities are the same and $M \propto l^3$ —conditions already prescribed.

Force

The dimensionless combination involving force is

$$\pi_F = \frac{F}{\sigma l^2} \qquad [3]$$

Equal velocities, determined by π_v, ensure equal stresses. Therefore, the only contribution from π_F is that $F \propto l^2$, which is the requirement for equal stresses in any similar structures regardless of the type of loading. In this particular problem the forces cannot be controlled directly, but, since holding the striking velocities constant gives equal stresses, the forces are necessarily in the ratio l^2. This dimensionless combination is included merely as an illustration of the redundant information occasionally furnished by dimensional analysis.

Gravity

The dimensionless combination involving gravity is

$$\pi_g = \frac{Mg}{\sigma l^2} \qquad [4]$$

For equal stresses, this requires that $M \propto l^2$. For materials of equal density, however, $M \propto l^3$ and the model will be too light.

A solution to the dilenma is, of course, to find a material for the model the ratio of whose density to the prototype material density is l/l_m. Then the condition that $M \propto l^2$ corresponding to π_g would be satisfied. Substitution of this into π_v gives the corresponding velocity requirement

$$v \propto \sqrt{l} \qquad [5]$$

Correct gravity (deadweight) stresses would be given by the increased density, and the reduced velocity would produce the proper dynamic stresses. This "ideal" solution is virtually impossible, since no known material exists which has the same elastic properties as steel and a much greater density. This dilemma is tantamount to the problem in hydromechanics of maintaining the Reynolds number vL/ν constant at the same time as the Froude number $v/\sqrt{gL}$ is held constant.

Fortunately, the problem rarely exists. In most dynamic problems the gravity stresses in full scale are negligibly small in comparison with the dynamic stresses, and, hence, the condition that $M \propto l^2$ can be disregarded. Occasionally, however, gravity stresses will be sufficiently large to influence the effects of impact loading as, for example, the study of impact on turrets. In such cases an artifice must be employed to increase dead weight without changing the mass of the model and without inhibiting motion. Tie rods and stiff springs can be so used, but neither is entirely satisfactory.

Heat

The nature of heat effects is obscure. There have been occasional observations of the fusing of projectile to plate. Frequently, when an analysis of test data has been attempted, the major amount of energy available from the projectile (or explosive) has been unaccounted for and assumed to have been dissipated as heat. The effects of rate of heat transfer, temperature, specific heat, and thermal conductivity can be included in the dimensional analysis, but, since these factors cannot be controlled in the model and some doubt exists as to their interpretation, the results have been omitted.

Moreover, it is not likely that heat is an important factor in influencing damage, even in the armor itself. Proving-ground experiments reveal that undeformed projectiles, regardless of the severity of impact, show no effects of heating although there is the same opportunity for "heat damage" as in the plate. Apparently projectile penetration and shock transmission occur so much more rapidly than appreciable conduction of heat into undamaged material that the material ahead of the projectile is not heated sufficiently to alter its properties.

Ultimately, of course, all energy in any mechanical system degenerates into heat. The primary effects in this case, as in ship resistance, are not believed to be influenced by such a transformation. The kinetic energy of ship motion is apparent in the creation of a wave system and frictional wake, but the degeneration of this energy into heat occurs so gradually that it is not considered in the analytical description of primary phenomena.

Momentum

Proving-ground experience shows that the resistance of armor to penetration is best related by an "energy coefficient." It is very likely, however, that damage to armor supports and dislocation of armor itself are as much affected by momentum of a projectile as by its kinetic energy. Thus it appears that momentum should also be considered.

The dimensionless combination corresponding to momentum is

$$\pi_{mv} = \frac{mv}{\sqrt{M\sigma l^3}} \qquad [6]$$

Dimensionally this is equivalent to

$$v\sqrt{\frac{M}{\sigma l^3}}$$

which is identically π_v as defined in the first of Equations [1]. Nothing new has been found by including momentum in the analysis. Nor was it expected since both m and v had been included separately. It may be noted also that, for conditions of equal stress and density, $M \propto l^3$, and Equation [6] reduces to

$$\pi_{mv} = \frac{mv}{l^3} \qquad [6a]$$

which shows that the momentum per unit volume is also constant.

Time

The dimensionless combination resulting from the inclusion of time is

$$\pi_t = \frac{\sigma l t^2}{M} \qquad [7]$$

For the case of the same material, model and prototype, $M \propto l^3$, and Equation [7] reduces to

$$\pi_t = \sigma\left(\frac{t}{l}\right)^2 \qquad [7a]$$

For equal stresses this requires that $t \propto l$. It has been demonstrated[2] from the equations of motion that time in similar systems varies as the scale if impact velocity is constant.

With the same impact velocities in model and prototype, the velocities of all corresponding parts of the two structures are probably sensibly the same. Furthermore, if equal stresses produce geometrically similar deformations (as should be the case for materials with the same stress-strain curve), the distances through which corresponding parts move vary as the scale. Thus the condition that time must vary as the scale seems to be fulfilled automatically.

That time intervals of corresponding events vary as the scale has already been demonstrated for elastic impacts by Donnell[3] through an analysis of stress waves. Donnell derived an approximate formula which, in the nomenclature used herein, is

$$t = \frac{l}{c}\left[\pi\sqrt{\frac{m}{M} + \frac{1}{2}} - \frac{1}{2}\right] \qquad [8]$$

If the scale is changed and the materials remain the same, the only term on the right-hand side of Equation [8] that is changed is l. Thus it can be seen that, in similar systems, the duration of impact varies directly as the scale. Since the intensities of reflected and transmitted waves in a structure depend upon the rigidity of connections and the fixity of supports for which exact similitude in a small model is well-nigh impossible, the propagation of stress waves is probably not very well reproduced. This, in turn, will influence the duration of impact. Similarly, partial penetration may not be very well reproduced with consequent dissimilarity in duration of impact.

Rate of Strain

The rate of straining of the material γ is associated with force and time. Since time varies as the scale for equal stresses, strain rate should vary inversely as the scale. That this is true can be seen from the dimensionless combination involving γ:

$$\pi_\gamma = \gamma\sqrt{\frac{M}{\sigma l}} \qquad [9]$$

For the same material, model and prototype, $M \propto l^3$, and Equation [9] reduces to:

$$\pi_\gamma = \frac{\gamma l}{\sqrt{\sigma}} \qquad [9a]$$

Therefore, for equal stresses, $\gamma \propto 1/l$. This relation is somewhat similar to that involving time. Neither time nor strain rate can be controlled, but theoretically both will be correctly proportional if impact velocities are the same and penetration is similar.

Some experimental work indicates that, as strain rate is increased, the yield point is raised and the stress-strain curve beyond the yield point is also raised. The primary requirements of dynamic similitude indicate that the stress-strain curves of both model and prototype material must be identical. Thus the material itself constitutes a possible source of scale effect.

Since the material itself introduces possible error, the similarity of results is questionable. Three possibilities exist:

1. The yield point is not reached either in model or prototype. Since response is elastic, there will be no damage.

2. The yield point is not exceeded in model but is in prototype. The model then does not predict full-scale damage.

3. The yield point is exceeded in both model and prototype.

The last case is fortunately the typical one. There are here two opposing effects: The model material will stretch more because of the higher load rate, but the duration of loading is less. If the load is applied long enough to cause fracture in both structures, it is probable that the elongation in the model will be greater. If no fracture occurs, the relative damage is still questionable.

SUMMARY

Dynamic similitude of impact requires then:

1. Geometrical similitude

2. Identical material (identical density and stress-strain curves)

3. Equal striking velocities

4. Nonpenetrative impacts

5. Gravity and other initial stresses either negligible or suitably reproduced.

The *initial* forces must be in the ratio of the square of the scale, and contact stresses must be equal since these are fixed only by the mass, scale, and velocity. The total amount of energy per unit volume will be the same in the two structures. A decrease in damage at contact, however, will apparently increase damage elsewhere. The differences in energy distribution are inextricably associated with the dynamic properties of material about which too little is known to predict with certainty. It appears safe to assume, however, that the proportions of projectile energy absorbed by armor and its supports are the same in both cases.

The duration of the impact depends on the degree of penetration and on the propagation and reflection of stress waves. The latter depend only on the material and the scale. The propagation velocity is merely $\sqrt{E/\rho}$; whereas the vibrations or waves of displacement depend upon mass distribution and rigidity of connections rather than on material properties. There is no difficulty in reproducing masses, but some trouble is anticipated in obtaining the proper rigidity of model connections. Fortunately, stress-wave velocity involves the square root of material properties; and displacement-wave velocity, the square root of rigidity. Therefore, unless the model is a very poor replica of the prototype, the speeds of action should be sensibly the same and similitude of time ($t \propto l$) should exist. Unfortunately, the degree of penetration cannot be controlled. Even full-scale penetrations are erratic. Hence the distortion of the similitude of time must be accepted. The safest viewpoint is the conservative one: Penetrations in the model are correspondingly less than in full scale, and full-scale damage may be underestimated.

As previously discussed, the temperature rise in advance of the projectile is believed to be too small to make the heat generated in deforming a factor of any importance. It is considered that *differences* in damage caused by heat will be negligible.

The shorter time intervals on the model necessarily require higher strain rates. These, in turn, raise the yield point of the material and the remainder of the stress-strain curve beyond the yield point an undetermined amount. Thus the initial stress wave of yield-point intensity, to which all reflected and transmitted intensities are proportional, is somewhat higher in the model than in the prototype. Model material will consequently withstand somewhat higher stress than the prototype without yielding. This all combines to predict that full-scale damage will extend somewhat deeper into the structure than will model damage.

At points where damage occurs, similitude of effects depends on the dynamic properties of the material in plastic flow. The load on the model is applied at a higher rate but for a shorter time. Within this time, the material reaches its yield point later but probably flows more for a given load. In view of the conflicting tendencies it must be conservatively assumed that the model damage is correspondingly *less* than the full-scale damage.

Finally, since the propagation of stress depends on the rigidity of structural joints, special care must be taken with the model joints lest model damage be minimized.

It may be concluded, therefore, that, for the conditions of similitude specified, the damage in full scale will:

1. Occur at the points indicated by the model,
2. Penetrate somewhat deeper into the structure, and
3. Not be less than the corresponding damage at any point on the model.

The dynamic properties of the material appear to be the only inherent source of scale effect. Pending amplifying information, it is best to make a generous allowance for more than corresponding damage in full scale.

Although the problem, as revealed by this study, is so complex that complete dynamic similitude is probably not attainable, it is considered that the requirements of similitude can be met to the extent that model investigations will be justified by the results obtained.

REFERENCES

1. Timoshenko, S., "Theory of Elasticity," McGraw-Hill Book Co., New York (1934), Chapter 12.

2. Lamb, E.H., "Notes on the Dynamics of Impact Tests," Proceedings Institute of Civil Engineers (1920-1).

3. Donnell, L.H., "Longitudinal Wave Transmission and Impact," Transaction American Society Mechanical Engineers, Applied Mechanics Division, Vol. 55, No. 22, pp. 153-167 (1930).

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