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THE DETERMINATION OF THE LOCAL SKIN FRICTION AND THE THICKNESS OF TURBULENT BOUNDARY LAYERS FROM THE VELOCITY SIMILARITY LAWS

by

Paul S. Granville

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NOTATION

A	Slope of logarithmic velocity law
B_1, B_3	Intercepts of logarithmic velocity law
D_1, D_2	Velocity profile constants
d	Width of pitot tube
$\tilde{d}$	Deviation of effective center of pitot tube from geometric center
J_1	Transitional sublayer factor
k, k_1, k_2	Linear parameters defining roughness
k^*	Roughness Reynolds number
R_x	Reynolds number based on length
R	Remaining terms of a series
U	Velocity at outer edge of boundary layer
u	Tangential velocity in shear flows
u_τ	Shear velocity
x	Distance along boundary layer
y	Normal distance from wall
$\tilde{y}$	Normal distance from wall to geometric center of pitot tube
y^*	Nondimensional y
y^*_L	Nondimensional thickness of laminar sublayer
δ	Thickness of boundary layer
μ	Viscosity of fluid
ν	Kinematic viscosity
ρ	Density of fluid
σ	Local skin friction parameter
τ_w	Local skin friction or shearing stress at wall

ABSTRACT

The applicability of pipe calibrations of Preston tubes to boundary layers is discussed on the basis of the logarithmic similarity law and the displacement effect of the tubes. A new logarithmic formula for the boundary-layer thickness of flat plates which is especially useful for high Reynolds numbers is also derived from the similarity laws.

INTRODUCTION

Two of the many applications of the similarity laws of the velocity profile of turbulent boundary layers are: (1) the experimental determination of local skin friction; and (2) the prediction of the boundary-layer thickness of flat plates.

The determination of the local skin friction of turbulent boundary layers on bodies moving in fluids is of fundamental importance in experimental fluid mechanics. The inherent difficulties of the direct measurement of local skin friction by means of small movable areas have led to the indirect methods of velocity measurements within the boundary layer. Here the similarity laws of the velocity profile provide the linkage between local skin friction and velocity.

The oldest method is that of Stanton, who used a half pitot tube on the wall to measure the velocity within the laminar sublayer. A more recent and less difficult method is that of Preston, who used a round pitot tube on the wall to measure the velocity outside of the laminar sublayer. However, questions have arisen concerning the calibration and the range of use of Preston tubes. The validity of the calibration of Preston tubes in fully developed pipe flow for use in boundary layers has been challenged by investigators of the National Physical Laboratory in England. Another question concerns the applicability of laboratory calibrations at low Reynolds numbers to tests at high Reynolds numbers such as on ships and aircraft. A third question involves the suitability of the Preston tubes for measurements on rough surfaces. This report will attempt to resolve these questions by means of a relation obtained from the logarithmic velocity law by explicitly considering the displacement effect of the tube.

In boundary-layer studies on full-scale vessels, comparison is often made with flat-plate prediction. The existing 1/5-power law of Von Kármán for boundary-layer thickness is, however, not adequate for conditions at high Reynolds numbers. To overcome this deficiency, a new logarithmic law for boundary-layer thickness is developed from the similarity laws which behaves somewhat differently than the 1/5-power law. The relative insensitivity of boundary-layer thickness, as indicated by the new relation, to changes in speed at high Reynolds numbers is in agreement with measurements on full-scale vessels.

SIMILARITY LAWS

INNER AND OUTER LAWS

For turbulent shear flows, such as fully developed viscous flow in pipes or boundary-layer flow on flat plates, the two laws which provide similarity in the mean-velocity profile by means of the local skin friction are:[1]

1. The inner law or law of the wall which applies to the flow immediately adjacent to the solid wall; and

2. The outer law or velocity-defect law which applies to the outer region of the shear flow.

The overlapping of the ranges of the two laws leads to a logarithmic functional form for both laws in the common region.

The inner law for the general case of rough surfaces is given nondimensionally as

$$\frac{u}{u_\tau} = f\left(y^*, k^*, \frac{k}{k_1}, \frac{k_1}{k_2}, \ldots\right) \qquad [1]$$

where u is the mean velocity of the turbulent flow parallel to the wall

$$y^* = \frac{u_\tau y}{\nu} \qquad [2]$$

y is the normal distance away from the wall,

$u_\tau = \sqrt{\frac{\tau_w}{\rho}}$ is the shear velocity,

τ_w is the shearing stress or local skin friction,

ρ is the density of the fluid,

$\nu = \frac{\mu}{\rho}$ is the kinematic viscosity of the fluid,

μ is the coefficient of viscosity of the fluid,

$$k^* = \frac{u_\tau k}{\nu} \qquad [3]$$

$k, k_1, k_2, \ldots$ are linear parameters defining the roughness.

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[1] References are listed on page 15.

In the case of smooth surfaces, k, k_1, k_2, ... are zero, and Equation [1] reduces to

$$\frac{u}{u_\tau} = f(y^*) \tag{4}$$

Although Equation [1] or [4] as functional relations holds both for pipe flows and for boundary-layer flows on flat plates, numerical differences have been found to exist between them.

An important characteristic of the inner law is that it has been empirically found to hold, even in pressure gradients.

The outer law may be stated as

$$\frac{U - u}{u_\tau} = f\left(\frac{y}{\delta}\right) \tag{5}$$

where U is the maximum velocity in the pipe flow or the velocity just outside the boundary layer, and δ is the thickness of the shear flow or of the boundary layer.

Significantly, the outer law has been found to be independent of Reynolds number or of roughness.

Where the inner and outer laws overlap, logarithmic forms necessarily ensue or

$$\frac{u}{u_\tau} = A \ln y^* + B_1 \tag{6}$$

for the inner law and

$$\frac{U - u}{u_\tau} = - A \ln \frac{y}{\delta} + B_3 \tag{7}$$

for the outer law.

For a particular geometric configuration, A and B_3 are always constants whereas B_1 is a constant only for smooth surfaces and a function of $k^*, \frac{k}{k_1}, \frac{k_1}{k_2}, \ldots$ for rough surfaces. The factors A, B_1, and B_3 have been found to differ numerically for fully developed pipe and boundary-layer flows.

SUBLAYERS

According to the behavior of the velocity profile, various sublayers may be distinguished in turbulent shear layers.

The laminar sublayer is the very thin layer of flow in contact with the wall wherein the flow, owing to the presence of the wall, is considered to be effectively laminar. The velocity profile is simply stated as

$$\frac{u}{u_\tau} = y^* \qquad [8]$$

The thickness of the laminar sublayer diminishes with increasing roughness and finally vanishes in the case of the fully rough regime.

An intermediate layer between the laminar sublayer and the main turbulent flow of the shear layer is the transitional sublayer where both the laminar and turbulent shearing stresses are considered significant. The velocity profile for smooth surfaces and for rough surfaces in the general rough regime may be stated as

$$\frac{u}{u_\tau} = A \ln (y^* - J_1) + B_1 \qquad [9]$$

where

$$J_1 = B_1 - A + A \ln A \qquad [10]$$

Two sublayers may be distinguished in the turbulent flow proper. The inner turbulent sublayer is the region where the logarithmic law, Equation [6] or [7], prevails, and the outer turbulent sublayer is the region where the outer law holds but not the inner law.

DETERMINATION OF LOCAL SKIN FRICTION FROM SIMILARITY LAWS

GENERAL

The inner law, by linking the velocity distribution to the local skin friction or shearing stress, provides a convenient means for measuring skin friction with a pitot tube. Once the velocity law is known, the principal difficulty in determining the local skin friction is due to the finite size of the tube, which causes the velocity reading to correspond to some point other than the geometric enter of the tube. This displacement of the effective center of the tube from the geometric center can be taken into consideration, however, by proper calibration.

If $\widetilde{y}$ is the distance from the wall to the geometric center of the pitot tube and $\widetilde{d}$ is the deviation of the effective center of the velocity reading from the geometric center, the inner law for smooth surfaces may be stated from Equation [4] as

$$\frac{u}{u_\tau} = f\left[\frac{u_\tau \widetilde{y}}{\nu}\left(1 + \frac{\widetilde{d}}{\widetilde{y}}\right)\right] \qquad [11]$$

or

$$\frac{u}{u_\tau} = f\left(\frac{u_\tau \widetilde{y}}{\nu}, \frac{\widetilde{d}}{\widetilde{y}}\right) \qquad [12]$$

With $q = \frac{1}{2} \rho u^2$ as the dynamic pressure and $u_\tau = \sqrt{\frac{\tau_w}{\rho}}$, the inner law may be written also as

$$\frac{q\tilde{y}^2}{\rho\nu^2} = \frac{\tau_w \tilde{y}^2}{\rho\nu^2} f \left[\frac{\tau_w \tilde{y}^2}{\rho\nu^2} \left(1 + \frac{\tilde{d}}{\tilde{y}}\right)^2 \right] \qquad [13]$$

or

$$\frac{q\tilde{y}^2}{\rho\nu^2} = f \left(\frac{\tau_w \tilde{y}^2}{\rho\nu^2}, \frac{\tilde{d}}{\tilde{y}} \right) \qquad [14]$$

Since the displacement effect $\tilde{d}$ is a function of the size of the tube d and the velocity profile which is governed by u_τ, $\tilde{y}$, and ν

$$\frac{\tilde{d}}{\tilde{y}} = f \left(\frac{d}{\tilde{y}}, \frac{u_\tau \tilde{y}}{\nu} \right) \qquad [15]$$

or

$$\frac{\tilde{d}}{\tilde{y}} = f \left(\frac{d}{\tilde{y}}, \frac{\tau_w \tilde{y}^2}{\rho\nu^2} \right) \qquad [16]$$

Then, in general,

$$\frac{q\tilde{y}^2}{\rho\nu^2} = f \left(\frac{\tau_w \tilde{y}^2}{\rho\nu^2}, \frac{d}{\tilde{y}} \right) \qquad [17]$$

For tubes resting on the surface $\tilde{y} = \frac{d}{2}$

$$\frac{qd^2}{4\rho\nu^2} = \frac{\tau_w d^2}{4\rho\nu^2} f \left[\frac{\tau_w d^2}{4\rho\nu^2} \left(1 + 2\frac{\tilde{d}}{d}\right)^2 \right] \qquad [18]$$

$$\frac{\tilde{d}}{d} = f \left(\frac{\tau_w d^2}{4\rho\nu^2} \right) \qquad [19]$$

and

$$\frac{qd^2}{4\rho\nu^2} = f \left(\frac{\tau_w d^2}{4\rho\nu^2} \right) \qquad [20]$$

STANTON TUBES

In some of the earliest experimental studies of turbulent boundary layers, Stanton[2] found that it required an especially narrow kind of pitot tube to probe the very thin laminar sublayer. This was accomplished by using a knife edge and the wall itself to form the boundaries of the pitot tube. This half pitot tube, known henceforth as the Stanton tube, required calibration in known laminar flows to allow for the displacement effect due to the finite size of the tube.

There followed further studies of the capabilities of the Stanton tube by a number of investigators. Fage and Falkner[3] used a modified form of the Stanton tube for measurements of local skin friction on an airfoil. G.I. Taylor[4] studied the displacement effect of very small tubes. Hool[5] developed a simple tube by soldering razor blades to the wall. Recent investigators[6] have used a flattened pitot tube resting on the wall for supersonic flows.

Although not originally thought as such, the Stanton tube represents the earliest application of the inner law to measurements of local skin friction. From Equation [8] the relation for the Stanton tube in the laminar sublayer becomes

$$\frac{qd^2}{4\rho\nu^2} = \frac{1}{2}\left(\frac{\tau_w d^2}{4\rho\nu^2}\right)^2 \left(1 + 2\frac{\tilde{d}}{d}\right)^2 \qquad [21]$$

The marked variation of $\frac{\tilde{d}}{d}$ with $\frac{\tau_w d^2}{4\rho\nu^2}$ is shown in Figure 1 for various types of Stanton tubes. The values of $\frac{\tilde{d}}{d}$ become quite large for low values of $\frac{\tau_w d^2}{4\rho\nu^2}$.

The magnitude of the laminar sublayer is given by y_L^* which is approximately

$$y_L^* = B_1 + A \ln A \qquad [22]$$

Also by definition

$$\left(\frac{\tau_w d^2}{4\rho\nu^2}\right)_L = y_L^{*\,2} \qquad [23]$$

For flat plates $y_L^* = 7.8$ and consequently $\left(\frac{\tau_w d^2}{4\rho\nu^2}\right)_L = 60.8$.

PRESTON TUBES

Inasmuch as the Stanton tube, by virtue of its small size, represents a difficult technique for measuring local skin friction, Preston[7] conceived the idea of using ordinary pitot tubes on the wall, of such size as to satisfy the inner law by lying within the inner turbulent

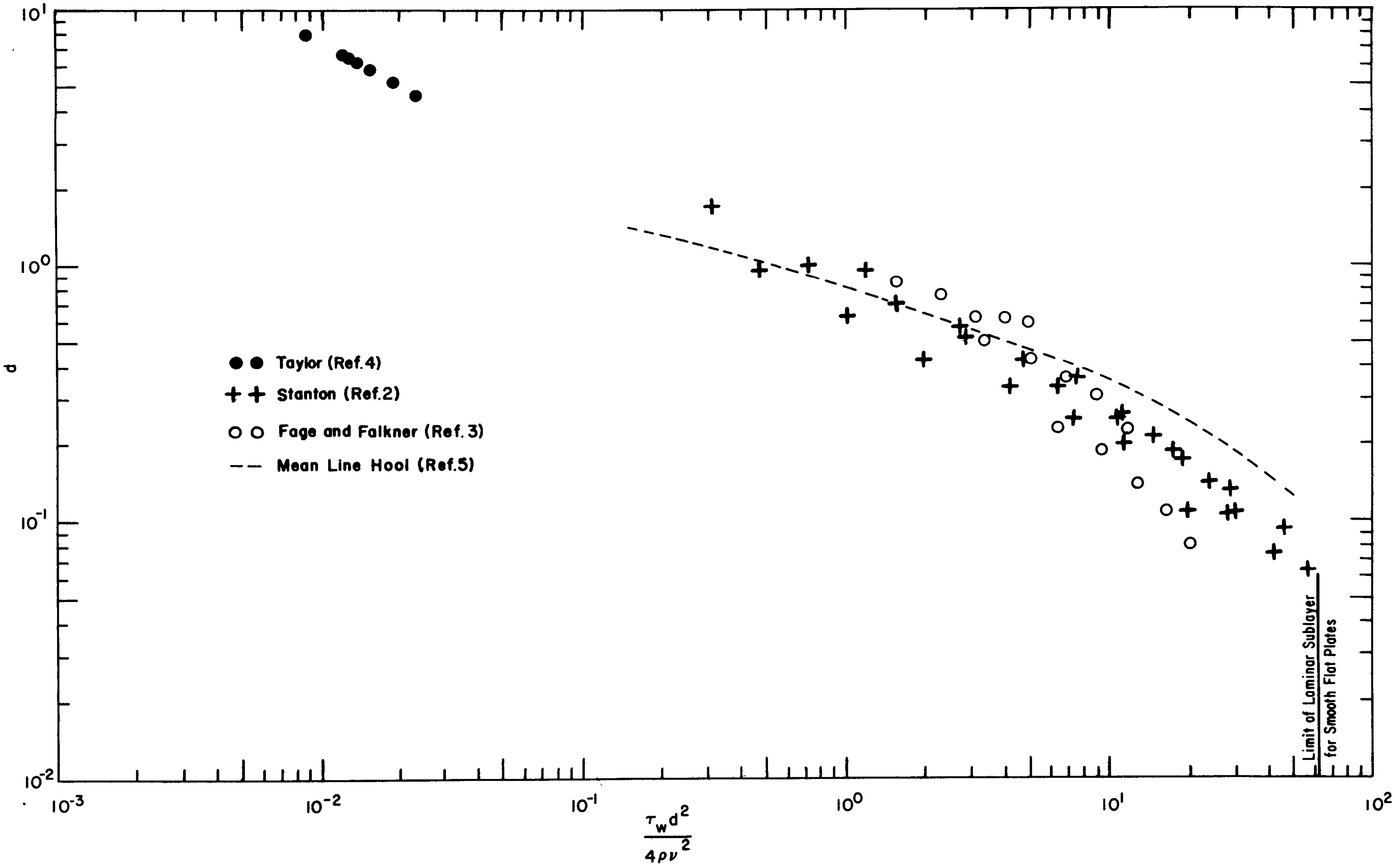

Figure 1 – Displacement of Effective Center for Stanton Tubes

sublayer. In the analysis of the relationship between the dynamic pressure read by the pitot tube and the local skin friction, the displacement effect of the tube was implicitly incorporated into the dimensionless analysis to give $\frac{qd^2}{4\rho\nu^2}$ as a function of $\frac{\tau_w d^2}{4\rho\nu^2}$ as stated in Equation [20]. To standardize the procedure, Preston decided on round tubes with square-cut ends and with the inner diameter six-tenths of the outer diameter. By considering the inner law to be universally valid, Preston conveniently calibrated the tubes in fully developed pipe flow so that the local skin friction could be obtained from the pressure gradient along the pipe. A power law was fitted to the calibration in the inner turbulent sublayer

$$\log_{10} \frac{\tau_w d^2}{4\rho\nu^2} = -1.396 + \frac{7}{8} \log_{10} \frac{qd^2}{4\rho\nu^2} \qquad [24]$$

A controversy soon developed on whether the calibration of Preston tubes in pipes held precisely for boundary layers. Relf and other aerodynamicists[8] of the National Physical Laboratory obtained a somewhat different calibration on a flat plate, using values of local skin friction deduced from the total drag coefficient and Reynolds number. Dutton[9] then substantiated Preston on the basis of measurements of momentum thickness on a flat plate. The National Physical Laboratory[10] riposted with more reliable flat-plate measurements which agreed with Relf, using more direct measurements of local skin friction by means of a Stanton tube. This calibration was stated as

$$\log_{10} \frac{\tau_w d^2}{4\rho\nu^2} = -1.350 + \frac{7}{8} \log_{10} \frac{qd^2}{4\rho\nu^2} \qquad [25]$$

Further support for the National Physical Laboratory came from the National Advisory Committee for Aeronautics,[11] wherein the calibration of the Preston tube for flat plates was obtained from direct readings of local skin friction from a floating-element dynamometer. This calibration, which is close to that of the National Physical Laboratory, is

$$\log_{10} \frac{\tau_w d^2}{4\rho\nu^2} = -1.366 + 0.877 \log_{10} \frac{qd^2}{4\rho\nu^2} \qquad [26]$$

For completeness of record, it should be noted that an earlier independent investigation by Hsu[12] confirmed Preston's pipe calibration for flat plates on the basis of values of local skin friction calculated from values of the measured momentum thickness.

The matter of the calibration of Preston tubes may be better understood if the displacement effect is separately considered in the analysis. For the logarithmic velocity law, Equation [6], the relationship in pitot-tube parameters as in Equation [13] becomes

$$\sqrt{\frac{qd^2}{4\rho\nu^2}} = \frac{1}{\sqrt{2}}\sqrt{\frac{\tau_w d^2}{4\rho\nu^2}}\left[A \ln\sqrt{\frac{\tau_w d^2}{4\rho\nu^2}} + B_1 + A \ln\left(1 + 2\frac{\tilde{d}}{d}\right)\right] \qquad [27]$$

Any differences in the calibration of Preston tubes for pipe and boundary-layer flows must lie then in the values of A, B_1, and $\frac{\tilde{d}}{d}$. Numerous measurements in pipes and on flat plates have substantiated differences in A and B_1.[1] Despite their scatter, the data of Figure 2 furthermore show no appreciable differences in the values of $\frac{\tilde{d}}{d}$ for pipe and boundary-layer flows. On the basis of the differences in A and B_1 there ought to be then a different calibration of Preston tubes for pipe and boundary-layer flows.

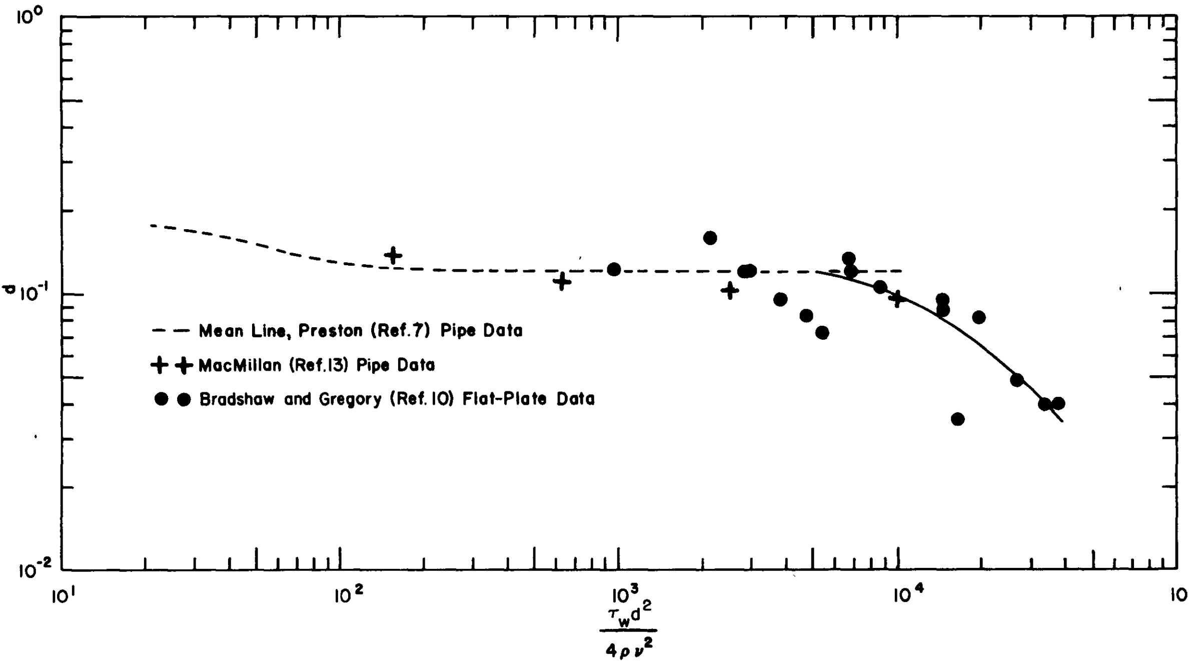

Figure 2 – Displacement of Effective Center for Preston Tubes

Since the calibrations, Equations [24], [25], and [26], stated in terms of power laws are associated with the power law relations of the inner law which are known to vary with Reynolds number, it is more valid to use the logarithmic law representation of Equation [27] together with the mean of the measured values of $\frac{\tilde{d}}{d}$ of Figure 2. This was done in Figure 3, using the flat-plate values of $A = 2.6$ and $B_1 = 4$ from Landweber.[14] The agreement with flat-plate calibrations, Equations [25] and [26], is excellent. To cover the lower range of the curve for the

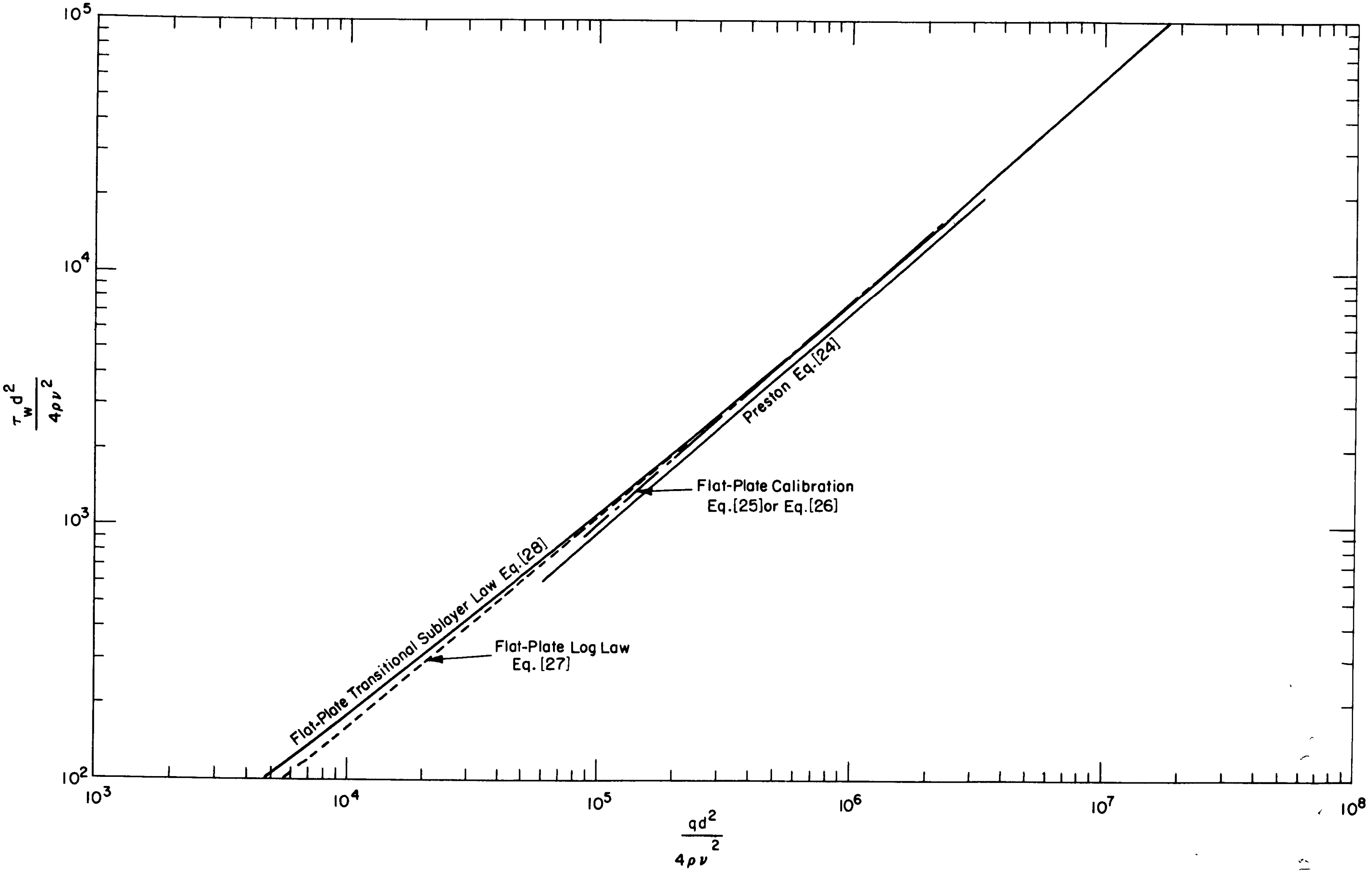

Figure 3 – Calibration of Preston Tubes

transitional sublayer, Equation [9] was used in the form

$$\sqrt{\frac{qd^2}{4\rho\nu^2}} = \frac{1}{\sqrt{2}}\sqrt{\frac{\tau_w d^2}{4\rho\nu^2}}\left\{A \ln\left[\sqrt{\frac{\tau_w d^2}{4\rho\nu^2}}\left(1 + 2\frac{\tilde{d}}{d}\right) - J_1\right] + B_1\right\} \qquad [28]$$

Since B_1 varies with k^* for rough surfaces, it is obvious from Equation [27] that a different calibration would be necessary for each value of k^*. Furthermore, it will be difficult to correctly position the Preston tube on the wall in the case of irregular roughnesses. In general, it seems that the Preston tube is not suited for the rough surfaces found in practice.

SURVEY OF INNER LAW

To determine the local skin friction of rough surfaces, recourse can be made to surveying the velocity profile of the inner law with a rake of pitot tubes or a traversing pitot tube. Kempf and Karhan[15] measured the inner law on a full-scale vessel at different times during a voyage to ascertain the increase in frictional resistance due to progressive fouling of the hull. A similar procedure was done more recently by Cutland.[16]

From the inner law, Equation [6],

$$u = \sqrt{\frac{\tau_w}{\rho}}\left(A \ln\sqrt{\frac{\tau_w}{\rho}} + B_1\right) + A\sqrt{\frac{\tau_w}{\rho}} \ln y \qquad [29]$$

Hence in a plot of velocity u against the logarithm of distance y, the slope of the straight line can be used to obtain the local skin friction τ_w. If the pitot tube is relatively large, it may be necessary to correct for the displacement effect to obtain the correct value of y.

BOUNDARY-LAYER THICKNESS OF FLAT PLATES

USE OF SIMILARITY LAWS TO DEFINE THICKNESS

The gradual merging of the boundary-layer flow with the external flow has made the exact determination of boundary-layer thickness quite uncertain. This is an important matter in the plotting of the velocity-defect law which involves the ratio y/δ. In the case of flat plates with smooth surfaces, Landweber[14] presented a consistent procedure based on the similarity laws as follows:

Equating the velocity u of the logarithmic velocity laws, Equations [6] and [7], produces

$$\frac{U}{u} = \sigma = A \ln \frac{u_\tau \delta}{\nu} + B_1 + B_3 \qquad [30]$$

or

$$\frac{y}{\delta} = e^{-\frac{B_3}{A}} \, y^* \, e^{\frac{\sigma - B_1}{A}} \qquad [31]$$

For smooth surfaces where B_1 is constant, the velocity defect $\frac{U - u}{u_\tau}$ may be plotted against $y^* e^{\frac{\sigma}{A}}$. For rough surfaces, however, where B_1 is a function of roughness, the velocity defect must be plotted against $y^* e^{\frac{\sigma - B_1}{A}}$.

Landweber and Siao[17] have more recently introduced for smooth surfaces a somewhat different interpretation of the previous procedure. They define the velocity-defect law in terms of a convenient natural length instead of boundary-layer thickness. An earlier method of Rotta[18] also uses the similarity laws to define the boundary-layer thickness in terms of the displacement thickness.

BOUNDARY-LAYER THICKNESSES OF FLAT PLATES IN ZERO PRESSURE GRADIENT

Although a prediction of the thickness is a fundamental requirement in any studies of boundary layers, it is still surprising to find quoted in the literature Von Kärmän's 1/5-power law for smooth flat plates with turbulent boundary layers.[19]

$$\frac{\delta}{x} = \frac{0.37}{R_x^{1/5}} \qquad [32]$$

where x is the distance from the leading edge and $R_x = \frac{Ux}{\nu}$. Since this law is derived from 1/7-power velocity profiles it has application to a limited range of Reynolds numbers, and since it is also based on pipe data it has only an approximate validity for flat plates.

More recently, Allan and Cutland[20] presented the following empirical fit to the measured boundary-layer thickness on a towed plank

$$\delta^2 + 4.0\,\delta = 0.065\,x \qquad [33]$$

where δ and x are measured in feet. This is similar to an older expression of Baker[21] obtaine from ship measurements*

$$\delta^2 + 1.5\,\delta = 0.02\,x \qquad [34]$$

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*Telfer, in his discussions of References 20 and 21, fitted the data with expressions of form $\frac{\delta}{x} = \text{const.} + \frac{\text{const.}}{R_x^{1/3}}$.

The similarity laws, however, provide a ready means for obtaining the values of boundary-layer thickness for flat plates over any range of Reynolds numbers. From Equation [30]

$$\frac{u_\tau \delta}{\nu} = e^{\frac{\sigma - B_1 - B_3}{A}} = \left(\frac{u_\tau}{U}\right)\left(\frac{U\delta}{\nu}\right) = \frac{1}{\sigma}\left(\frac{U\delta}{\nu}\right) \qquad [35]$$

or

$$\ln\frac{U\delta}{\nu} = \frac{\sigma}{A} + \ln\sigma - \frac{(B_1 + B_3)}{A} \qquad [36]$$

Also

$$\frac{\delta}{x} = \frac{1}{R_x}\left(\frac{U\delta}{\nu}\right) \qquad [37]$$

Since, in the case of smooth plates, R_x is uniquely related to σ then $\frac{\delta}{x}$ is a function of R_x. With Landweber's[14] tabulated values of $R_x - C$ and the value of $C = -0.096 \times 10^5$ from Reference 22, the following table results:

TABLE 1

Calculated Values of Boundary-Layer Thickness From Similarity Laws

σ	$(U\delta)/\nu$	R_x	δ/x
19.64	3.686×10^3	1.141×10^5	3.230×10^{-2}
20	4.308	1.447	2.977
21	6.640	2.466	2.693
22	1.021×10^4	4.212	2.424
24	2.400	1.168×10^6	2.055
26	5.601	3.114	1.799
28	1.300×10^5	8.10	1.605
30	3.000	2.07×10^7	1.449
32	6.893	5.219	1.321
34	1.578×10^6	1.301×10^8	1.213
36	3.600	3.209	1.122
38	8.186	7.852	1.043
40	1.852×10^7	1.906×10^9	0.972

An expression giving a close fit to the tabulated values of boundary-layer thicknesses may be derived from the similarity laws as follows:

From Equation [80] of Reference 1

$$R_x = \left(\frac{u_\tau \delta}{\nu}\right) D_1 \sigma^2 \left[1 - \left(\frac{D_2}{D_1} + 2A\right)\frac{1}{\sigma} + 2A\left(A + \frac{D_2}{D_1}\right)\frac{1}{\sigma^2} + \ldots\right] \qquad [38]$$

where D_1 and D_2 are constants, and then from Equation [35]

$$\frac{x}{\delta} = D_1\sigma - \left(\frac{D_2}{D_1} + 2A\right) + R_1 \qquad [39]$$

where R_1 represents the remaining terms of the series. If R_1 is linearized with respect to σ or

$$R_1 = \text{constant}\ \sigma + \text{constant} \qquad [40]$$

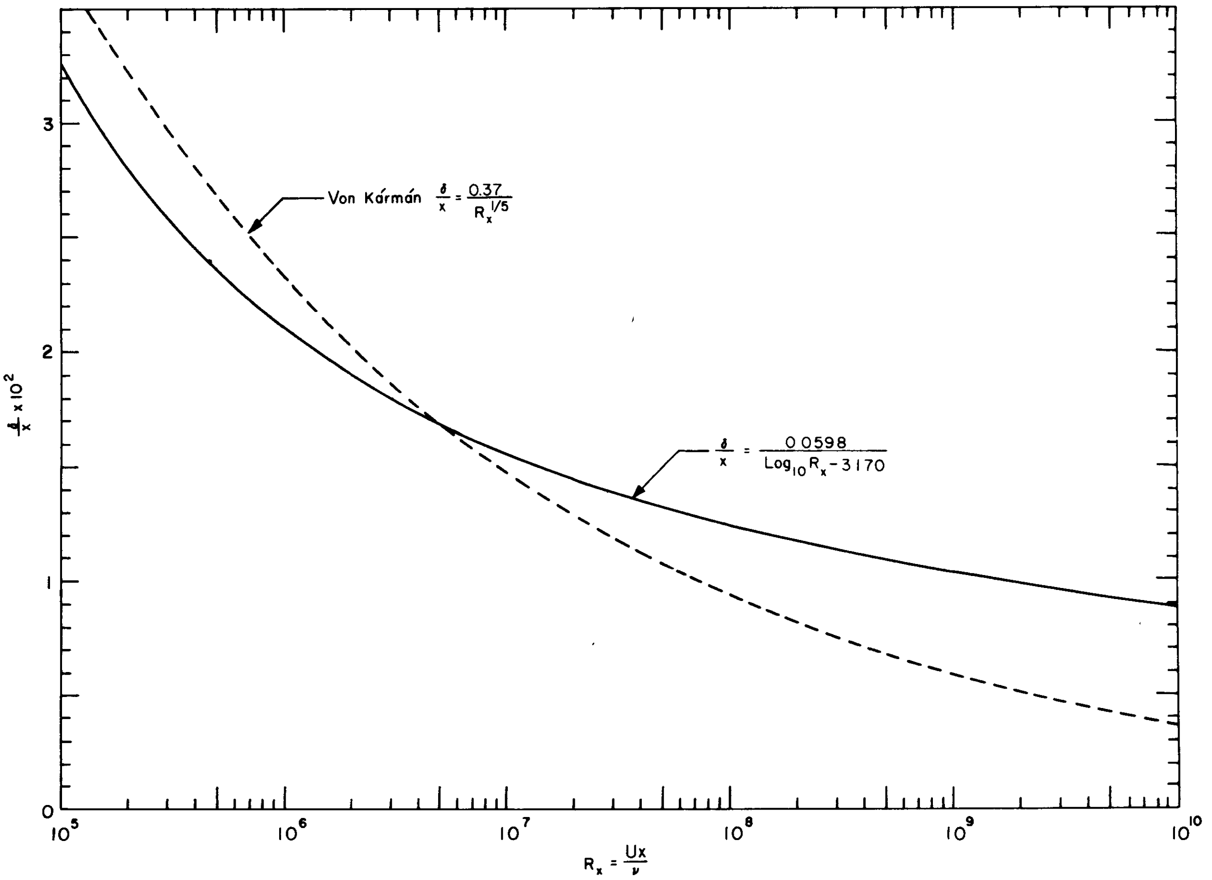

Figure 4 – Boundary-Layer Thickness of Flat Plates

then

$$\frac{x}{\delta} = \text{constant } \sigma + \text{constant} \tag{41}$$

Now from Equation [38]

$$\ln R_x = \frac{\sigma}{A} - \frac{B_1}{A} - \frac{B_3}{A} + \ln D_1 + R_2 \tag{42}$$

where R_2 represents the remaining terms of the series. If R_2 is also linearized with respect to σ, there results

$$\sigma = \text{constant } \ln R_x + \text{constant} \tag{43}$$

which, when substituted into Equation [41], gives finally

$$\frac{x}{\delta} = \text{constant } \ln R_x + \text{constant} \tag{44}$$

From the plot of the tabulated values of $\frac{x}{\delta}$, a very close fit results which is

$$\frac{x}{\delta} = 16.72 \log_{10} R_x - 53.0 \qquad [45]$$

or

$$\frac{\delta}{x} = \frac{0.0598}{\log_{10} R_x - 3.170} \qquad [46]$$

Figure 4 shows a comparison between the 1/5-power law and the new relation, Equation [46]. The greatest differences lie at the low and the high values of Reynolds numbers. The shallowness of the new curve at high Reynolds numbers, which indicates inappreciable changes in boundary-layer thickness with speed, agrees with observations of tests of full-scale vessels.[21]

ACKNOWLEDGMENTS

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