HYDROSTATIC TESTS OF INELASTIC AND ELASTIC STABILITY
OF RING-STIFFENED CYLINDRICAL SHELLS
MACHINED FROM STRAIN-HARDENING STEEL

by

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DEPARTMENT OF THE NAVY
DAVID TAYLOR MODEL BASIN

HYDROMECHANICS

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STRUCTURAL MECHANICS

APPLIED MATHEMATICS

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RESEARCH AND DEVELOPMENT REPORT

December 1961

Report 1501
From: Commanding Officer and Director, David Taylor Model Basin  
To: Chief, Bureau of Ships (442) (in duplicate)  

Subj: Experimental investigation of inelastic and elastic stability of ring-stiffened cylindrical shells machined from strain-hardening steel; forwarding of report on  

Encl: (1) DATMOBAS Report 1501 entitled "Hydrostatic Tests of Inelastic and Elastic Stability of Ring-Stiffened Cylindrical Shells Machined from Strain-Hardening Steel"  

1. The results of an experimental investigation of ten ring-stiffened cylindrical shells machined from strain-hardening steel having a range of "thinness ratio" $\lambda$ from 0.85 to 1.67 are presented in enclosure (1).  

2. Agreement within 4 percent was obtained between the experimental collapse pressure and the theoretical pressures computed from Reynolds' inelastic lobar buckling theory for those cylinders with thinness ratios of 0.85 to 1.10 and from Reynolds' elastic lobar buckling theory for those cylinders with thinness ratios greater than 1.40. However, agreement was only fair between experimental pressures and the pressures computed from Reynolds' inelastic theory for those cylinders with thinness ratios between 1.10 and 1.40; these cylinders were still in the inelastic range but tending toward the elastic range.

"E. E. JOHNSON"

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ABSTRACT

Ten ring-stiffened cylindrical shells machined from strain-hardening steels with yield strengths of 70,000 psi and 100,000 psi were tested to collapse under external hydrostatic pressure. The cylinders were designed to cover a range of "thinness ratio" $\lambda$ from 0.85 to 1.67. Four of the cylinders collapsed in a combined asymmetric (lobar) and axisymmetric mode; the other six cylinders failed only in the lobar mode.

Agreement within 4 percent was obtained between the experimental collapse pressures and the theoretical pressures computed from Reynolds' inelastic lobar buckling theory for those cylinders with thinness ratios of 0.85 to 1.10 and from Reynolds' elastic lobar buckling theory for those cylinders with thinness ratios greater than 1.40. However, agreement was only fair between experimental pressures and the pressures computed from Reynolds' inelastic theory for those cylinders with thinness ratios between 1.10 and 1.40; these cylinders were still in the inelastic range but tending toward the elastic range.

INTRODUCTION

As part of a test program designed to provide information on the resistance of ring-stiffened cylindrical shells to explosive loading, ten machined externally-framed cylinders were tested to collapse under hydrostatic pressure. The cylinders were designed with geometries in the lobar buckling range and were machined from strain-hardening steel. The static tests were conducted primarily to provide data on static strength for use in evaluating the subsequent results of dynamic tests of similar cylinders. In addition, the static results also afforded an excellent opportunity to compare experimental collapse pressures with the pressures calculated from theories recently developed by Reynolds for inelastic and elastic lobar buckling of stiffened cylinders under hydrostatic pressure.

This report deals only with the static phase of the test program. The design, instrumentation, and testing of the cylinders that were collapsed under hydrostatic pressure are described; experimental and theoretical strains are compared; and the experimental collapse pressures are discussed and compared with theoretical pressures.
BACKGROUND

Two basic inelastic modes of shell failure can occur when ring-stiffened cylinders fabricated from strain-hardening material are subjected to external hydrostatic pressure. These are: axisymmetric buckling, a phenomenon in which circumferential corrugations develop in the shell, and asymmetric or lobar buckling in which alternate inward and outward lobes appear around the circumference of the shell. Recently at the David Taylor Model Basin, analyses \(^1,2\) based on stability considerations were developed for determining the buckling pressures of a stiffened cylinder collapsing in either of these modes. The buckling pressures were found to be a function of the cylinder geometry and the secant and tangent moduli of the shell material as determined from a uniaxial stress-strain curve.

Prior to these recent developments, \(^\ast\) collapse of the axisymmetric type was usually treated theoretically as failure due to yielding precipitated by high stress rather than as a buckling phenomenon. These past analyses were based on the assumption of an idealized material which, at a certain stress level depending upon the failure criterion used, underwent an abrupt transition to the perfectly plastic state. Analyses of this type are applicable for stiffened cylinders fabricated from material with an ideally plastic (plateau-type) stress-strain curve. For a structure made of a strain-hardening material, these past analyses have been proven inadequate since the determination of collapse pressure was predicated on the state-of-equilibrium stresses and was not based on stability considerations. Investigations of inelastic lobar buckling have been rather limited in the past and empirical in nature.

Another possible mode of failure which could be analyzed on the basis of stability considerations is that termed "elastic lobar buckling."

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\(^1\)References are listed on page 15.

\(^\ast\)A recent Japanese paper dealing with the axisymmetric collapse problem has come to the attention of the Model Basin. It will soon be available as TMB Translation T-298 (Reference 3).
The theoretical buckling pressures for this case are independent of yield strength and secant and tangent moduli of the material and can be determined from the cylinder geometry, Poisson's ratio, and Young's modulus of elasticity for the shell material. In 1929, Von Mises developed a rigorous solution for the elastic buckling of a thin-walled cylinder of finite length under hydrostatic pressure, assuming simple support conditions at the ends. He further assumed that the prebuckling deformations of the stiffened cylinder could be approximated by the more simple ones corresponding to an unstiffened cylinder of infinite length. In 1932, Von Sanden and Tölke extended this work; they also assumed simple support conditions at the frames but considered the exact prebuckling deformations of Von Sanden and Günther for a stiffened cylinder in an approximate manner to make the mathematics tractable. Recently at the Model Basin, Reynolds approached the problem more rigorously than these past investigators by considering more realistic boundary conditions at the frames and describing the prebuckling deformations in the form of a Fourier series so as to permit a better approximation to the actual state.

Since the cylinders presented in this report were designed with geometries in the lobar buckling range and were machined from strain-hardening steel, the test results are used to evaluate the theories that have recently been developed by Reynolds for inelastic and elastic lobar buckling.

DESCRIPTION OF MODELS

The models were machined from AISI-4140 seamless steel tubing, heat treated to a yield strength of approximately 70,000 psi for two models and 100,000 psi for the other eight models. Stress-strain curves obtained from uniaxial compression specimens cut from the heat-treated tubing indicated that the material was of the strain-hardening type. Typical stress-strain curves of the materials with yield strengths of about 70,000 psi and about 100,000 psi are plotted in Figure 1; the curves shown represent specimens of the material used for Models 12 and 55.
The geometry and material yield strength (based on a 0.2 percent offset) for each of the models are summarized in Table 1. All models were designed with an end arrangement as shown in the schematic diagram of Table 1 in order to preclude premature failure near the ends. The models were made pressure-tight by integral heavy rings at both ends, of which one was attached to a closure bulkhead and the other was bolted to the inside surface of the pressure tank head.

Figure 1 - Stress-Strain Curves of Materials Used in Models
TABLE 1

Geometrical and Material Properties of Models

A nominal value of $E = 30.0 \times 10^6$ psi was assumed as Young's modulus of elasticity for the models.

<table>
<thead>
<tr>
<th>Model</th>
<th>Radius $R$ (in.)</th>
<th>Shell Thickness $h$ (in.)</th>
<th>Frame Spacing $L_f$ (in.)</th>
<th>Frame Width $b$ (in.)</th>
<th>Frame Depth $F$ (in.)</th>
<th>Number of Typical Frame Spacings</th>
<th>Yield Strength $\sigma_y$ (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>4.0813</td>
<td>0.0446</td>
<td>1.140</td>
<td>0.079</td>
<td>0.314</td>
<td>8</td>
<td>68,100</td>
</tr>
<tr>
<td>13</td>
<td>4.0788</td>
<td>0.0396</td>
<td>0.912</td>
<td>0.078</td>
<td>0.311</td>
<td>13</td>
<td>96,500</td>
</tr>
<tr>
<td>22</td>
<td>4.0768</td>
<td>0.0356</td>
<td>1.140</td>
<td>0.068</td>
<td>0.272</td>
<td>10</td>
<td>70,500</td>
</tr>
<tr>
<td>23</td>
<td>4.0745</td>
<td>0.0310</td>
<td>1.140</td>
<td>0.061</td>
<td>0.242</td>
<td>10</td>
<td>97,500</td>
</tr>
<tr>
<td>54</td>
<td>4.0704</td>
<td>0.0229</td>
<td>0.684</td>
<td>0.057</td>
<td>0.228</td>
<td>15</td>
<td>104,400</td>
</tr>
<tr>
<td>55</td>
<td>4.0727</td>
<td>0.0274</td>
<td>0.912</td>
<td>0.059</td>
<td>0.237</td>
<td>11</td>
<td>103,000</td>
</tr>
<tr>
<td>56</td>
<td>4.0758</td>
<td>0.0337</td>
<td>1.368</td>
<td>0.063</td>
<td>0.251</td>
<td>6</td>
<td>103,700</td>
</tr>
<tr>
<td>62</td>
<td>4.0827</td>
<td>0.0475</td>
<td>1.368</td>
<td>0.078</td>
<td>0.311</td>
<td>7</td>
<td>104,000</td>
</tr>
<tr>
<td>65</td>
<td>4.0827</td>
<td>0.0475</td>
<td>0.912</td>
<td>0.078</td>
<td>0.311</td>
<td>11</td>
<td>104,000</td>
</tr>
<tr>
<td>73</td>
<td>4.0727</td>
<td>0.0274</td>
<td>1.368</td>
<td>0.078</td>
<td>0.312</td>
<td>7</td>
<td>103,200</td>
</tr>
</tbody>
</table>

INSTRUMENTATION AND TEST PROCEDURE

All models were instrumented with SR-4 electrical-resistance strain gages to study their elastic and inelastic behavior during pressure loading and to facilitate interpretation of the results. The shells were instrumented in order to obtain data for comparison with the elastic strains computed from the Von Sanden and Günther theory, which theory constitutes the basis for designing ring-stiffened cylindrical shells subjected to hydrostatic pressure. The rectangular frames were not instrumented since the width of the frames was too narrow for strain gages.
The models were tested to failure in two or more pressure runs. Preliminary runs were made to obtain sufficient data on the elastic behavior of each model for comparison with the elastic strains computed from the Von Sanden and Günther theory. A period of 5 min. was allowed to elapse before strains were recorded to permit stabilization of stresses in the models. All models were tested to collapse in the Model Basin 10-in. diameter hydrostatic pressure tank.

TEST RESULTS

Six of the ten models (Models 22, 23, 55, 56, 62, and 73) failed in the shell by lobar buckling. The other four models (Models 12, 13, 54, and 65) failed in a combined mode of lobar buckling and axisymmetric collapse.

TABLE 2
Experimental Collapse Pressure, Mode of Failure, and Thinness
Ratio for Each Model

<table>
<thead>
<tr>
<th>Model</th>
<th>Experimental Collapse Pressure psi</th>
<th>Mode of Failure</th>
<th>Thinness Ratio λ</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>975</td>
<td>BY*</td>
<td>0.855</td>
</tr>
<tr>
<td>13</td>
<td>1160</td>
<td>BY</td>
<td>0.986</td>
</tr>
<tr>
<td>22</td>
<td>735</td>
<td>B</td>
<td>1.035</td>
</tr>
<tr>
<td>23</td>
<td>705</td>
<td>B</td>
<td>1.354</td>
</tr>
<tr>
<td>54</td>
<td>695</td>
<td>BY</td>
<td>1.340</td>
</tr>
<tr>
<td>55</td>
<td>730</td>
<td>B</td>
<td>1.358</td>
</tr>
<tr>
<td>56</td>
<td>725</td>
<td>B</td>
<td>1.443</td>
</tr>
<tr>
<td>62</td>
<td>1335</td>
<td>B</td>
<td>1.111</td>
</tr>
<tr>
<td>65</td>
<td>1695</td>
<td>BY</td>
<td>0.893</td>
</tr>
<tr>
<td>73</td>
<td>475</td>
<td>B</td>
<td>1.671</td>
</tr>
</tbody>
</table>

* B denotes lobar buckling; BY denotes combination of lobar buckling and axisymmetric collapse.
Figure 2a - Model 12 ($\lambda = 0.855$)

Figure 2b - Model 13 ($\lambda = 0.986$)

Figure 2c - Model 22 ($\lambda = 1.035$)

Figure 2d - Model 23 ($\lambda = 1.354$)

Figure 2 - Models after Collapse Showing Modes of Failure
Figure 2e - Model 54 ($\lambda = 1.340$)

Figure 2f - Model 55 ($\lambda = 1.358$)

Figure 2g - Model 56 ($\lambda = 1.443$)

Figure 2h - Model 62 ($\lambda = 1.111$)
The experimental collapse pressures, modes of failure, and corresponding values of "thinness ratio" $\lambda^*$ for each model are summarized in Table 2.

It is interesting to note that three of the four models which failed in the combined lobar buckling and axisymmetric mode had the lowest values of the thinness ratio $\lambda$. The mode of collapse of each model is shown in Figure 2.

A strain-sensitivity factor was determined for each strain gage. This factor is the slope of the linear portion of the pressure-strain curve and is measured in microinches per inch per psi of pressure. Pertinent strain data from gages located in the center region of each model are summarized in Table 3 together with strain-sensitivity factors computed from the theory of Von Sanden and Günther. The corresponding experimental and theoretical strains compare favorably.

\[
\lambda^* = \frac{4}{\sqrt{\left( \frac{h/2R}{L/2R} \right)^3 \cdot \sqrt{\frac{\sigma_y}{E}}}}
\]

where $L = L_f - b$
### TABLE 3

Comparison of Measured and Theoretical Strain-Sensitivity Factors

<table>
<thead>
<tr>
<th>Model</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experimental</td>
<td>Theoretical</td>
<td>Experimental</td>
<td>Theoretical</td>
</tr>
<tr>
<td>12</td>
<td>-1.47(5)</td>
<td>-1.56</td>
<td>-2.18(12)</td>
<td>-2.17</td>
</tr>
<tr>
<td>13</td>
<td>-1.53(3)</td>
<td>-1.63</td>
<td>-1.99(16)</td>
<td>-2.11</td>
</tr>
<tr>
<td>22</td>
<td><strong>-1.90</strong></td>
<td>-2.90</td>
<td>-3.46</td>
<td>-3.46</td>
</tr>
<tr>
<td>23</td>
<td>-2.16(5)</td>
<td>-2.18</td>
<td>-3.36(18)</td>
<td>-3.45</td>
</tr>
<tr>
<td>54</td>
<td>-2.29(6)</td>
<td>-2.55</td>
<td>-3.23(21)</td>
<td>-3.45</td>
</tr>
<tr>
<td>55</td>
<td>-2.10(6)</td>
<td>-2.36</td>
<td>-3.28(18)</td>
<td>-3.38</td>
</tr>
<tr>
<td>56</td>
<td>-1.78(5)</td>
<td>-2.03</td>
<td>-3.20(20)</td>
<td>-3.38</td>
</tr>
<tr>
<td>62</td>
<td>-1.36(6)</td>
<td>-1.52</td>
<td>-2.17(19)</td>
<td>-2.24</td>
</tr>
<tr>
<td>65</td>
<td>-1.40(3)</td>
<td>-1.50</td>
<td>-1.75(9)</td>
<td>-1.77</td>
</tr>
<tr>
<td>73</td>
<td>-1.62(9)</td>
<td>-1.75</td>
<td>-4.10(21)</td>
<td>-4.28</td>
</tr>
</tbody>
</table>

*Numbers in parentheses denote number of strain gages used in averaging process to determine experimental value.

**Strain data not available.
### Table 4

Theoretical and Experimental Collapse Pressures

<table>
<thead>
<tr>
<th>Model</th>
<th>Thinness Ratio $\lambda$</th>
<th>Experimental Collapse Pressure</th>
<th>Axisymmetric Collapse Pressures</th>
<th>Inelastic Lobar Buckling $^1$ Pressures</th>
<th>Elastic Lobar Buckling Pressures</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Von Sanden</td>
<td>Hencky-</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>and Günther Formula $^2$</td>
<td>Von Mises at Outer Fiber $^3$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Hinge $^8$</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>0.855</td>
<td>975(BY) $^6$</td>
<td>788</td>
<td>859</td>
<td>946</td>
</tr>
<tr>
<td>65</td>
<td>0.893</td>
<td>1695(BY)</td>
<td>1451</td>
<td>1543</td>
<td>1703</td>
</tr>
<tr>
<td>13</td>
<td>0.986</td>
<td>1160(BY)</td>
<td>1104</td>
<td>1149</td>
<td>1301</td>
</tr>
<tr>
<td>22</td>
<td>1.035</td>
<td>735(B)</td>
<td>622</td>
<td>686</td>
<td>747</td>
</tr>
<tr>
<td>62</td>
<td>1.111</td>
<td>1335(B)</td>
<td>1208</td>
<td>1348</td>
<td>1446</td>
</tr>
<tr>
<td>54</td>
<td>1.340</td>
<td>695(BY)</td>
<td>712</td>
<td>719</td>
<td>834</td>
</tr>
<tr>
<td>23</td>
<td>1.354</td>
<td>705(B)</td>
<td>732</td>
<td>816</td>
<td>878</td>
</tr>
<tr>
<td>55</td>
<td>1.358</td>
<td>730(B)</td>
<td>732</td>
<td>787</td>
<td>877</td>
</tr>
<tr>
<td>56</td>
<td>1.443</td>
<td>725(B)</td>
<td>820</td>
<td>931</td>
<td>977</td>
</tr>
<tr>
<td>73</td>
<td>1.671</td>
<td>475(B)</td>
<td>644</td>
<td>731</td>
<td>770</td>
</tr>
</tbody>
</table>

$^a$ (B) denotes lobar buckling failure; (BY) denotes combination of lobar buckling and axisymmetric mode of failure.

$^{10}$ The theory of Reference 2 indicates that the values in brackets are elastic buckling pressures. In the elastic range, the theory of Reference 2 degenerates to a solution similar to the Von Sanden and Tölke theory.
DISCUSSION AND INTERPRETATION OF RESULTS

Theoretical collapse pressures for various modes of failure for each of the ten models are given in Table 4 in order of increasing thinness ratio $\lambda$ together with the corresponding experimental collapse pressures and observed modes of failure. The parameters given in Table 1 were used in calculating the collapse pressures in Table 4. The secant and tangent moduli determined from uniaxial stress-strain curves of the material for each of the models were used to calculate the inelastic axisymmetric buckling and asymmetric (lobar) buckling pressures from the theories of References 1 and 2, respectively.

In Table 4, the first three theoretical pressures are for axisymmetric collapse of stiffened cylinders fabricated from an ideally plastic material. Since the geometry and material properties of the models tested were not in this range, these theoretical pressures are given in Table 4 merely for comparison with the theoretical pressures that are associated with the lobar buckling mode of failure. The pressures computed from Formula [92a] are those at which the outer-fiber circumferential stress midway between adjacent frames reaches the strength of the shell material. The Hencky-Von Mises pressure is the pressure at which yielding begins on the outside surface of the shell. It is the pressure at which the biaxial stress, computed by applying the Hencky-Von Mises criterion to the Von Sanden-Gunther stresses on the outer fiber of the shell at midbay, reaches the yield strength of the shell material. The "plastic hinge" pressures were computed from a theory, based on the Hencky-Von Mises criterion, which allows for the plastic reserve strength after yielding begins. This plastic hinge analysis assumes that the shell material possesses a stress-strain relationship in compression which consists of a straight line through the origin for the elastic range and a horizontal line for the plastic range, that is, the material is elastic, perfectly plastic. The yield strength necessary for computing these three pressures can only be arbitrarily determined for materials exhibiting a continuous stress-strain curve. Standard procedure at the Model Basin has been to use the 0.2-percent-offset method for determining the yield strength of such materials.
The pressures mentioned so far apply to cylinders which fail due to axisymmetric yielding. It has been found from measurements prior to collapse that this type of failure is usually accompanied by large nonlinear strains in the shell of the cylinder. In contrast, for these ten models the nonlinear strains measured prior to collapse were relatively small, and the failures occurred rather abruptly. This is indicative of a buckling phenomenon. Therefore, the three theoretical collapse pressures discussed so far do not apply as such to the models presented in this report.

The axisymmetric buckling \(^1\) pressures and asymmetric (lobar) buckling \(^2\) pressures given in Table 4 were computed from analyses which are representative of inelastic modes of failure encountered by ring-stiffened cylinders fabricated from strain-hardening materials. It can be seen that the pressures computed from Reference 1 are higher than those computed from Reference 2. This is not at all surprising since the geometries of the models were in the lobar buckling range \((\lambda > 0.8)\). However, excellent agreement was observed between the experimental collapse pressures of Models 12 and 65, the models with the lowest \(\lambda\) values, and the corresponding theoretical collapse pressures computed from Reference 1.

The elastic lobar buckling pressures given in Table 4 as computed from the theories of Von Sanden and Tölke \(^5\) and Von Mises \(^9\) are lower than the corresponding pressures computed from the theory of Reynolds, \(^7\) which is based on more realistic assumptions as mentioned earlier. Both the Von Mises \(^4\) and Von Sanden and Tölke \(^5\) elastic buckling theories are based on the assumption of simple support at the shell edges whereas Reynolds' theory \(^7\) considers the torsional restraint afforded the shell by the reinforcing rings. Further, Reynolds assumed a more representative prebuckling state of stress in the shell, reflecting the reinforcing action of the rings, which leads to a higher critical buckling pressure.

Therefore, to compare the experimental collapse pressures with theory in a logical way, attention must be focused on the inelastic and elastic lobar buckling pressures computed from References 2 and 7, respectively. Table 4 shows that, for stiffened cylinders with \(\lambda\) values from 0.85 to 1.10, excellent agreement was realized between the experimental collapse pressures and the corresponding theoretical pressures computed from Reynolds' inelastic lobar buckling theory. \(^2\) Likewise, for stiffened cylinders with
\( \lambda \) values greater than 1.40, excellent agreement was realized between the experimental collapse pressures and the corresponding theoretical pressures computed from Reynolds' elastic lobar buckling theory.\(^7\) However, only fair agreement with the inelastic theory of Reference 2 was observed for those cases with \( \lambda \) values between 1.10 and 1.40, which are still in the realm of inelastic buckling but which tend toward the elastic range. The fact that this agreement is not as good as for cases where \( \lambda < 1.10 \) is due primarily to the simplified form of the elastic buckling terms used in developing the inelastic buckling theory of Reference 2. In the limit, the inelastic theory of Reference 2 predicts an elastic buckling pressure which represents an upper bound of the inelastic influences on the buckling process. However, this upper bound or elastic buckling pressure of Reference 2 is much lower than the corresponding elastic pressure computed from Reference 7. Therefore, if more rigorous elastic buckling terms, such as those used in Reference 7, could be incorporated in the inelastic theory of Reference 2, better agreement could be expected between this latter theory and stiffened cylinders with \( \lambda \) values between 1.10 and 1.40; however, this would lead to a very cumbersome and complicated buckling theory.

CONCLUSIONS

1. The prebuckling strains measured on the stiffened cylinder models agreed very well with strains computed from the axisymmetric elastic theory of Von Sanden and Günther.\(^6\)

2. For the stiffened cylinders with \( \lambda \) values ranging from 0.85 to 1.10, excellent agreement was realized between the experimental collapse pressures and the corresponding pressures computed from Reynolds' inelastic lobar buckling theory.\(^2\)

3. For the stiffened cylinders with \( \lambda \) values greater than 1.40, excellent agreement was realized between the experimental collapse pressures and the corresponding pressures computed from Reynolds' elastic lobar buckling theory.

4. For the stiffened cylinders with \( \lambda \) values between 1.10 and 1.40, only fair agreement was realized between the experimental collapse pressures and those calculated from the theory of Reference 2. The disagreement appears to be due to the simplified form of the elastic buckling influences used in developing the theory of Reference 2.
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