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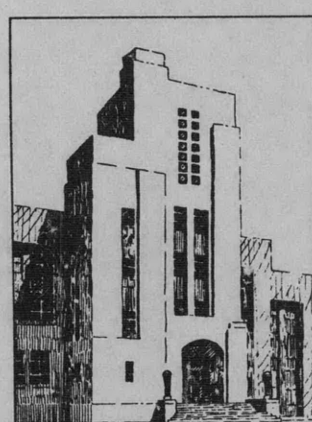
GENERALIZATION OF THE DIMENSIONLESS FREQUENCY PARAMETER IN UNSTEADY FLOWS

by

V.G. Szebehely, Dr. Eng.



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NOTATION

A	Constant
$\bar{a}(a_x, a_y, a_z)$	Acceleration vector (a_i)
$\bar{a}_c$	Convective acceleration
$\bar{a}_l$	Local acceleration
B	Constant
$b(\ )$	Arbitrary functions
C	Constant
c	Drag coefficient
F	Wetted area
$F(\ )$	Arbitrary function
$f(\ )$	Arbitrary function
$G(\ )$	Arbitrary function
$g(\ )$	Arbitrary function
$H(\ )$	Arbitrary function
K	Constant
k	Constant
k_x	Added mass coefficient in the x-direction
L	Characteristic body dimension
m	Mass
o *subscript*	Boundary or initial value (x_0, t_0)
P	Body force per unit volume
$\bar{P}(\ )$	Arbitrary vector function
p	Pressure
$R(\ )$	Arbitrary function
r	Radial coordinate
$\bar{r}(x, y, z)$	Position vector (x_i)
S	Generalized Strouhal number
$T(\ )$	Arbitrary function
t	Time
U	Free stream velocity

u_r	Radial velocity component
u_θ	Angular velocity component
$\overline{v}(u,v,w)$	Velocity vector (v_i)
$\overline{w}$	Vorticity vector
x,y,z *subscript*	Space derivatives $\left(u_x = \frac{\partial u}{\partial x}\right)$
α	Constant
ζ	Vorticity
θ	Angular coordinate
λ	Second coefficient of viscosity
μ	Ordinary coefficient of viscosity
ν	Kinematic viscosity = $\frac{\mu}{\rho}$
ρ	Density
ϕ	Potential function
ψ	Stream function
ω	Angular velocity
$\frac{\partial v_i}{\partial x_j}$	Cartesian velocity derivative tensor $(v_{i,j})$
Dot	Time derivative $\left(\frac{\partial u}{\partial t} = \dot{u}\right)$

GENERALIZATION OF THE DIMENSIONLESS FREQUENCY PARAMETER IN UNSTEADY FLOWS

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ABSTRACT

This report presents some theoretical results in connection with the unsteady flow research program at the David Taylor Model Basin. The complex and relatively unknown field of time-dependent hydrodynamic phenomena is approached from a general point of view. Only a few special flows are discussed emphasizing the diversity of unsteady flow problems. Since time effect occurs only in the acceleration term of the momentum (or Navier-Stokes) equation, an analysis of the two types of acceleration is presented in detail. A dimensionless ratio of the local and convective accelerations is introduced. It is shown that with the magnitude of this ratio, the unsteadiness of the flow can be described and characterized. Previously published results on hydrodynamic impact represent one limiting case for the above "measure of unsteadiness" ($S \to \infty$). Steady (time-independent flows) are associated with zero value of this measure ($S = 0$).

A detailed report on flows with intermediate values ($0 < S < \infty$) is under preparation. It will be published in the near future in connection with recent oscillator experiments.

A short discussion of accelerationless flows is included.

INTRODUCTION

The dimensionless parameter governing oscillation tests in hydrodynamics is the well-known Strouhal number or dimensionless frequency.[1] For nonoscillatory motions, a Fourier analysis furnishes the spectrum with which the problem can be analyzed.[2] The main problem of this report is to find a quantitative measure of unsteadiness which is applicable to any fluid motion and which therefore includes oscillation as a special case. Apart from any possible immediate application (maneuvering of bodies immersed in a fluid), the need of a generalized measure of unsteadiness seems to be justified since it forms a basis for systematic study of time-dependent fluid motions.

The acceleration vector $\bar{a}$ consists of two parts, the local $\bar{a}_l$ and the convective $\bar{a}_c$ accelerations. The local acceleration shows the time dependence of the velocity vector;

[1]References are listed on page 21.

the convective part is connected with the space dependence of velocity field. The total acceleration is often introduced as the "substantial derivative" of the velocity. This concept seems to be unnecessary since, considering the velocity a function of the time and space, the acceleration is defined as the total derivative of the velocity with respect to the time. The i^{th} component of the acceleration is

$$a_i = \frac{dv_i\,(x_1, x_2, x_3\,;\,t)}{dt}$$

and the i^{th} component of the velocity is

$$v_i = \frac{dx_i}{dt}$$

The acceleration in expanded form becomes

$$a_i = \frac{\partial v_i}{\partial t} + \frac{\partial v_i}{\partial x_j}\, v_j{}^*$$

The $\frac{\partial v_i}{\partial t}$ term is the local acceleration; the $\frac{\partial v_i}{\partial x_j} v_j$ part is called the convective part. For time-independent flows, the local acceleration is zero. For uniform (space-independent) flows, the convective part is zero. Several other expressions exist for the acceleration vector, for instance in vector form:

$$\overline{a} = \dot{\overline{v}} + \frac{1}{2}\, grad\, \overline{v}^2 - \overline{v} \times curl\, \overline{v}$$

or in scalar form:

$$a_x = \dot{u} + uu_x + vu_y + wu_z$$

$$a_y = \dot{v} + uv_x + vv_y + wv_z$$

$$a_z = \dot{w} + uw_x + vw_y + ww_z$$

The generally applicable measure of unsteadiness proposed in this report is the dimensionless ratio of the magnitudes of the local and convective accelerations:

$$S = \frac{|\dot{\overline{v}}|}{\left|\frac{1}{2}\, grad\, \overline{v}^2 - \overline{v} \times curl\, \overline{v}\right|}$$

---

*Here $\frac{\partial v_i}{\partial x_j} = v_{i,j}$ represents the Cartesian velocity derivative tensor. In computing the $v_{i,j}\, v_j$ term, the summation convention is used, i.e.,

$$v_{i,j}\, v_j = \sum_{j=1}^{3} v_{i,j}\, v_j$$

or

$$S = \frac{\sqrt{\dot{v}_i \dot{v}_i}}{\sqrt{v_{j,k} v_{j,l} v_k v_l}}$$

The quantity S will also be referred to as the generalized Strouhal number since it can be shown that it reduces to the conventional Strouhal number for linearized oscillatory flows.

In addition to an analysis of this measure of unsteadiness, two fundamental problems might be indicated at this point:

1. "Direct" problems refer to those in which the time and space dependence of the measure of unsteadiness is prescribed and inquiry is made about flows which satisfy this given unsteadiness.

2. Indirect or "inverse" problems are those in which given flows are analyzed and the generalized Strouhal numbers are computed.

ANALYSIS OF THE GENERALIZED STROUHAL NUMBER

RANGE OF UNSTEADINESS

A few definitions will be given before investigating the limiting values of the measure.

1. Steady flow is defined by the property that the velocity at every point in the flow, at any time, is independent of the time, i.e.,

$$\frac{\partial \bar{v}}{\partial t} = 0$$

2. Unsteady flow has the property that at some point(s), at certain time(s), the velocity depends on the time, i.e.,

$$\frac{\partial \bar{v}}{\partial t} \neq 0$$

3. Uniform flow is characterized by the property that the velocity at any time is the same at all points in the fluid, i.e.,

$$\frac{\partial v_i}{\partial x_j} = 0$$

4. Nonuniform flow is associated with the property that at some point(s), at certain time(s), the velocity gradient is different from zero, i.e.,

$$\frac{\partial v_i}{\partial x_j} \neq 0$$

There are four combinations of the possible space and time dependencies. It will be shown presently that the generalized Strouhal number is well defined in three cases; the fourth is of no interest.

1. For steady nonuniform flow, $S = 0$,
2. For unsteady uniform flow, $S = \infty$,
3. For unsteady nonuniform flow, $0 < S < \infty$
4. For steady uniform flow, S is not defined; this flow, however, is of no interest.

Generally the measure of unsteadiness depends on the time and on the position of the point at which the unsteadiness is computed i.e.,

$$S = S(\bar{r}, t)$$

There is no reason to expect a constant unsteadiness. Two flows might be equally unsteady at one time and at a certain point in the flow, and they might show quite different unsteadiness at different times and/or at different points. There are some important flow problems for which the measure of unsteadiness takes its limiting values. For instance, in the case of a hydrodynamic impact phenomenon, $\dot{\bar{v}}$ is very large and $\bar{v}^2 \approx 0$. This results in $S \rightarrow \infty$.*

UNIT OF UNSTEADINESS AND ACCELERATIONLESS FLOWS

An accelerationless flow possesses the property of having zero acceleration at any time at any point in the flow, i.e.,

$$\bar{a} = \bar{a}_c + \bar{a}_l = 0$$

Steady uniform flows satisfy this condition, but there are much more important solutions. From the above definition of accelerationless flow it follows that

$$|\bar{a}_c| = |\bar{a}_l|$$

and so

$$S = \frac{|\bar{a}_l|}{|\bar{a}_c|} = 1$$

An accelerationless flow, therefore, has unit unsteadiness. It might be remarked that while accelerationless flows result in $S = 1$, these are not the only unsteady flows with this property. For instance, flows with acceleration and satisfying the condition

$$\bar{a}_l = \bar{a}_c = \frac{\bar{a}}{2}$$

also have unit unsteadiness. This ambiguity can be eliminated by introducing quantities which are ratios of local and convective acceleration components and which take signs into consideration. This way, however, the simplicity and the usefulness of the measure is reduced. Investigations presented elsewhere[3] showed little advantage in introducing the quantities (elements of the Strouhal matrix) mentioned above.

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*An impact is a "very unsteady" phenomenon.

Geometrically, unit unsteadiness means that the total, the local, and the convective accelerations form an isosceles triangle (Figure 1).

This geometrical representation of unit unsteadiness is general. The $\alpha = 0$ case corresponds to accelerationless flow ($\bar{a}_c = -\bar{a}_l$); the $\alpha = \pi$ case is equivalent to $\bar{a}_c = \bar{a}_l = \frac{\bar{a}}{2}$

Despite the very special importance of accelerationless flows, very little is known about them.* For viscous fluids, the inertia forces (acceleration) are in equilibrium with the outside (or body) forces plus the pressure gradient plus the viscous forces. According to the Navier-Poisson-Duhem[7] equation:

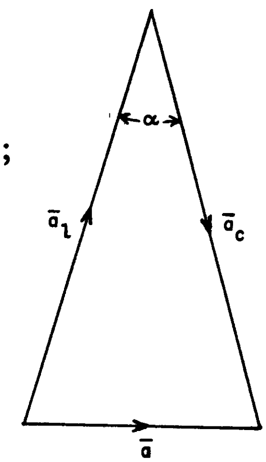

Figure 1

$$\rho a_i = -p_{,i} + P_i + (\lambda + \mu) v_{j,ji} + \mu v_{i,jj}$$

For incompressible viscous fluids

$$\rho a_i = -p_{,i} + P_i + \mu v_{i,jj}$$

In accelerationless flows, therefore, the outside forces, the pressure gradient, and the shear stresses are in equilibrium. If the fluid is ideal and the body forces are neglected, the condition for accelerationless flow is space-independent pressure.**

At this point it seems advantageous to repeat that only important accelerationless flows are under consideration here, i.e., flows for which $\bar{a}_c$ and $\bar{a}_l$ are different from zero and $\bar{a}_c = -\bar{a}_l$. The velocity of a one-dimensional accelerationless flow $[u = u(x)]$ satisfies the differential equation

$$\dot{u} + u u_x = 0$$

The momentum equation for this case is

$$-p_x + (\lambda + 2\mu) u_{xx} = 0$$

or

$$p = (\lambda + 2\mu) u_x + f(t)$$

The continuity equation for a one-dimensional flow is

$$\dot{\rho} + u \rho_x + u_x \rho = 0$$

For barotropic fluid flow, the pressure density relation is represented by the relation

$$\rho = R(p)$$

---

*A study of the literature revealed a few sentences in Neményi's paper,[4] a study by Merlin[5] using the Lagrangean method, some remarks by Hopf.,[6] etc. In what follows, only some interesting and characteristic results will be mentioned. A complete study of accelerationless flows is forthcoming.

At the time this report went to the printer, the writer's attention was called to B. Caldonazzo's paper, "On Free Motion of a Continuous Medium," Ann. Mat. pura appl., 1947, Ser. 4, Vol. 26, pp 43-55.

**The pressure is not constant in time.

Accelerationless flow of an incompressible fluid in one dimension is trivial since, for this case, the continuity equation gives $u_x = 0$ or $u = u(t)$. Now, from the equation of zero acceleration, it follows that if $u_x = 0$, $\dot{u} = 0$, then $u = constant$. The pressure will depend only on time

$$p = f(t)$$

and no effect of viscosity is observed.

For compressible nonviscous fluids

$$u_x \neq 0$$

Again the pressure depends only on the time $p = f(t)$.

Because of the $\rho = R(p)$ relation, the density depends on time alone,

$$\rho = R[f(t)] = g(t) \qquad \text{and} \qquad \rho_x = 0$$

From the continuity equation

$$u_x = \frac{-\dot{\rho}}{\rho} = \frac{-\dot{g}(t)}{g(t)} = G(t)$$

or

$$u = G(t)\, x + F(t)$$

and

$$\rho = \rho_0 e^{-\int_{t_0}^{t} G(t)\, dt}$$

The condition for zero acceleration is:

$$\dot{u} + u u_x = x(\dot{G} + G^2) + \dot{F} + GF = 0$$

Therefore, the two differential equations for $G(t)$ and $F(t)$ are:

$$\dot{G} + G^2 = 0 \qquad \text{and} \qquad \dot{F} + GF = 0$$

From these,

$$G = \frac{1}{t + t_0}$$

$$F = \frac{x_0}{t + t_0}$$

and so the velocity is

$$u = \frac{x + x_0}{t + t_0}$$

The density becomes

$$\rho = \frac{2 t_0 \rho_0}{t + t_0}$$

and the pressure is

$$p = R^{-1}(\rho)$$

Finally for compressible viscous fluids, the pressure is

$$p = (\lambda + 2\mu)\, u_x + f(t)$$

and the density

$$\rho = R\,[(\lambda + 2\mu)\, u_x + f(t)]$$

Substituting this in the continuity equation and also satisfying the condition for zero acceleration results in the velocity as a function of time and position.

The general solution of the differential equation $\dot{u} + u u_x = 0$ is obtained by the method of characteristics:

$$x = ut + f(u)$$

or

$$u = g(x - ut)$$

Choosing $f(u) = t_0 u - x_0$, the solution is $u = \dfrac{x + x_0}{t + t_0}$, as found previously.* A much more complex flow is obtained if

$$f(u) = \frac{t_0^2}{x_0}\, u^2$$

The velocity becomes multivalued, and two flows are possible:

$$u = -\frac{t x_0}{2 t_0^2} + \sqrt{\left(\frac{t x_0}{2 t_0^2}\right)^2 + \frac{x x_0}{t_0^2}} \qquad u = -\frac{t x_0}{2 t_0^2} - \sqrt{\left(\frac{t x_0}{2 t_0^2}\right)^2 + \frac{x x_0}{t_0^2}}$$

SPECIAL UNSTEADINESS DISTRIBUTIONS

Generally the measure of unsteadiness depends on the position and time, i.e.,

$$S = S(\bar{r}, t)$$

A d'Alembert type of unsteadiness is defined by

$$S(\bar{r}, t) = f(\bar{r})\, g(t)$$

For one-dimensional flow, this assumption results in the following differential equation:

---

*It is not the purpose of this report to discuss the problem of accelerationless flows. The above short analysis was presented to show that there are flows with unit unsteadiness or zero acceleration which are not trivial. In this connection, it might be mentioned that a generalization of the above ideas for three-dimensional flows, considering a velocity distribution of the type

$$v_i = a_{ij}(t)\, x_j$$

will be presented in the near future.

$$\frac{|\dot{u}|}{|u u_x|} = f(x)\, g(t)$$

For accelerationless flow, $\dot{u} + u u_x = 0$, and therefore $S = 1$. Consider now ξ and τ, new variables which are related to x and t by the following equations:

$$\xi = \int \frac{dx}{f(x)}$$

and

$$\tau = \int g(t)\, dt$$

A simple calculation shows that the velocity distribution which has d'Alembert-type unsteadiness is given as before by

$$u = G(\xi - u\tau)$$

or

$$u = G\left[\int \frac{dx}{f(x)} - u \int g(t)\, dt\right]$$

or

$$\int \frac{dx}{f(x)} = u \int g(t)\, dt + F(u)$$

Here G and F are arbitrary functions; f and g are given. Choosing, for instance, $F(u) = u$, the velocity is

$$u = \frac{\int \frac{dx}{f(x)}}{1 + \int g(t)\, dt}$$

If $g(t) = 1$, $f(x) = 1$,

$$u = \frac{x + x_0}{t + t_0}$$

as previously found. The simplest type of variable unsteadiness is

$$S(x, t) = xt$$

for which case

$$u = \frac{\ln \left(\frac{x}{x_0}\right)^2}{t^2 - t_0^2}$$

The velocity is not generally separable, but is separable only if $F(u) =$ *constant,* or if $F(u) =$ *constant* u.

For the general case $[S = S(x, t)]$ consider the differential equation:

$$\dot{u} + S(x, t)\, u u_x = 0$$

The solution can be obtained again by the method of characteristics,

$$u = H\left[x - u \int S(x, t)\, dt\right]$$

This solution, of course, reduces to the previously discussed cases if the proper assumptions are made.

EVALUATION OF THE MEASURE OF UNSTEADINESS FOR CERTAIN UNSTEADY FLOWS OF SPECIAL INTEREST

d'ALEMBERT FLOW

Separable or d'Alembert flows are defined by the following velocity distribution:

$$\bar{v}\,(\bar{r}, t) = T\,(t)\,\bar{u}\,(\bar{r})$$

The generalized Strouhal number becomes

$$S = \left| \frac{\dot{T}}{T^2}\, f\,(\bar{r}) \right|$$

where

$$f\,(\bar{r}) = \frac{|\bar{u}|}{\left| \frac{1}{2}\, grad\ \bar{u}^2 - \bar{u} \times curl\ \bar{u} \right|}$$

The unsteadiness of a separable flow is also separable. The reverse is not true, however, since d'Alembert-type unsteadiness might come from a nonseparable flow, as shown previously.

The measure of unsteadiness is time-independent if, with C = *constant,*

$$\frac{dT}{dt} = C\,T^2$$

from which

$$T = \frac{A}{1 - ACt}$$

where A is a constant of integration and, in fact, $A = T(0)$. If $C < 0$ and $A > 0$, T varies from A to *zero* as $0 \to t \to \infty$. It is an important property of the d'Alembert flows that the streamlines are pathlines and the streamline pattern is steady. The unsteadiness is constant in time for d'Alembert flows if the velocity is inversely proportional to a linear function of the time. For oscillatory d'Alembert flows, let

$$T\,(t) = A \sin \omega t + B = A\,(\sin \omega t + \alpha)$$

where A and B are positive constants and α is equal to B/A. Considering a fixed point in space, if $\alpha = 1$, the function $T(t)$ and, therefore, the velocity are zero at $t = \frac{3\pi}{2\omega} + \frac{2n\pi}{\omega}$, with $n = 0, 1, 2 \ldots$ If $\alpha > 1$, then $T\,(t) > 0$ and the velocity is unidirectional. If $0 < \alpha < 1$, the direction of the velocity alternates. For large values of α, i.e., $\alpha \gg 1$, the time dependence of the velocity is represented by a large constant plus a small oscillatory part (Figure 2).

The velocity has its maxima at

$$t_1 = \frac{\pi}{2\omega} + \frac{2n\pi}{\omega}, \qquad n = 0, 1, 2 \ldots$$

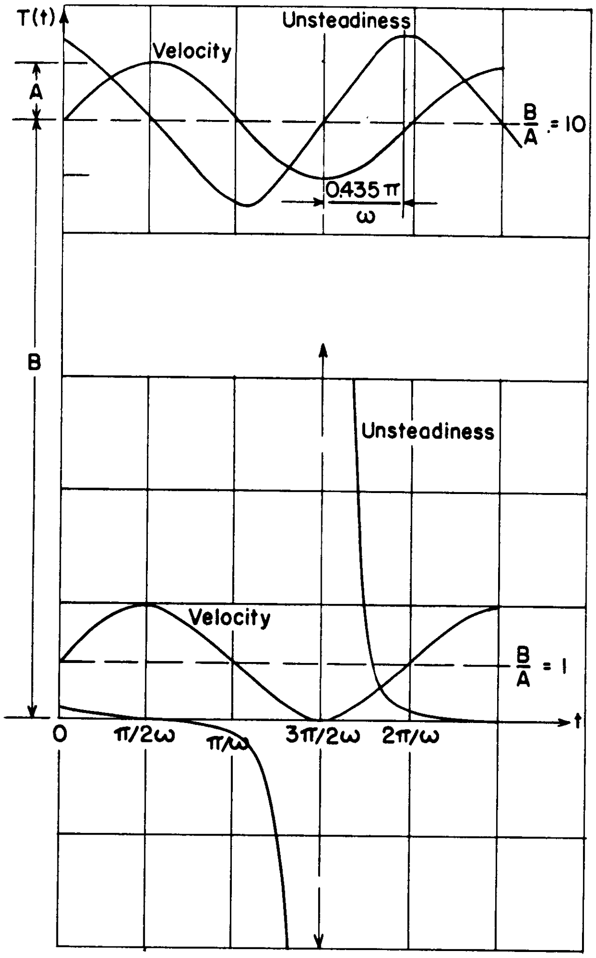

Figure 2 - Oscillatory d'Alembert Flow, $\alpha = 10$ and $\alpha = 1$

and its minima at

$$t_2 = \frac{3\pi}{2\omega} + \frac{2n\pi}{\omega}, \qquad n = 0, 1, 2 \ldots$$

The factor governing the time variation of the measure is

$$F(t) = \frac{\dot{T}}{T^2} = \frac{\omega}{A} \frac{\cos \omega t}{(\alpha + \sin \omega t)^2}$$

If $\alpha \neq 1$, the unsteadiness is zero at t_1 and t_2, i.e., when the velocity is maximum or minimum. If $\alpha = 1$, the unsteadiness is zero when the velocity is maximum and is infinite for zero velocity (= minimum velocity).

For $\alpha \gg 1$,

$$F(t) \cong \frac{\omega}{A\alpha^2} \cos \omega t$$

the maximum unsteadiness lags behind the minimum velocity by $\pi/2\omega$ and the minimum unsteadiness leads the minimum velocity by $\pi/2\omega$. This lag and lead approach zero as $\alpha \to 1$.

Figure 2 shows the lag for the case $\alpha \doteq 10$ to be $0.870 \frac{\pi}{2\omega}$ and Figure 3 shows the lag to be $0.44 \frac{\pi}{2\omega}$ for $\alpha = 2$.

In fact, the occurrence of the extreme values of unsteadiness satisfy the equation

$$\sin \omega t = \frac{\alpha}{2} - \sqrt{\left(\frac{\alpha}{2}\right)^2 + 2}$$

The maximum and minimum unsteadiness are symmetrical with respect to the minimum velocity point.

For d'Alembert flows with exponential decay, the time dependence of the velocity is given by

$$T(t) = A e^{-K^2 t}$$

and the factor governing the unsteadiness is

$$F(t) = \frac{\dot{T}}{T^2} = -\frac{K^2}{A} e^{K^2 t}$$

The unsteadiness increases exponentially in d'Alembert flows with exponential decay.

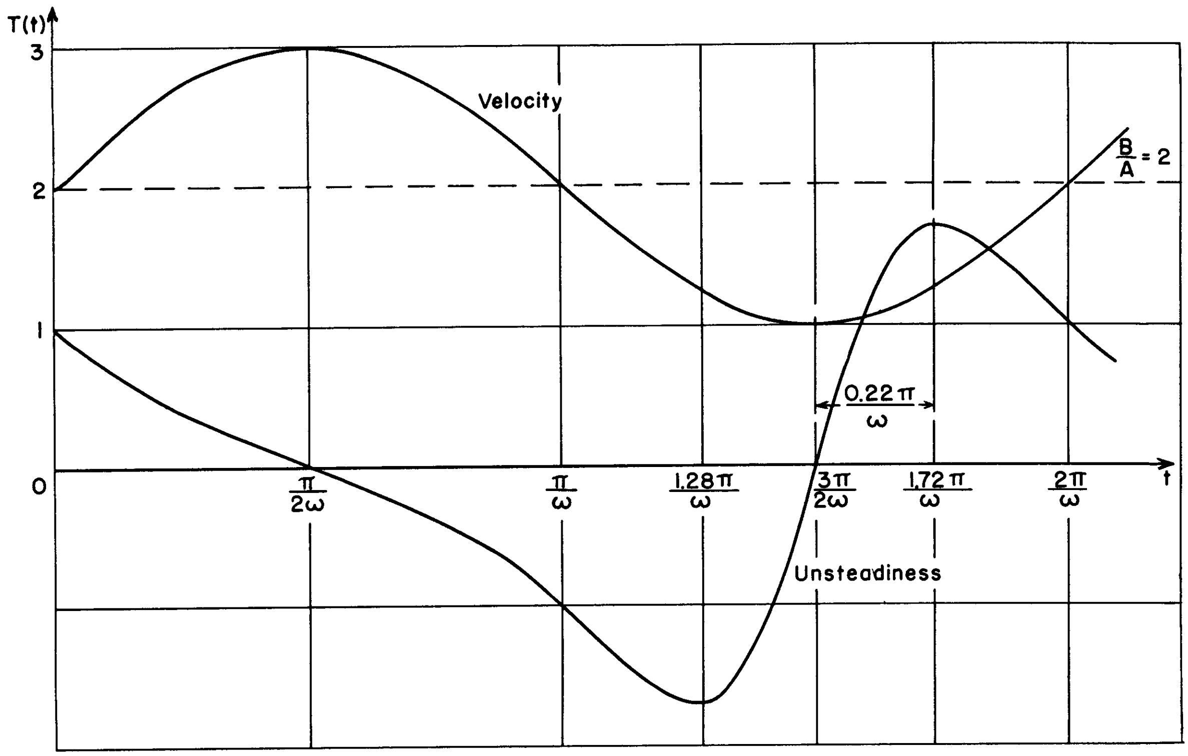

Figure 3 - Oscillatory d'Alembert Flow, $\alpha = 2$

UNSTEADY JET

An example is furnished by a special potential flow of the d'Alembert type,

$$\phi = \frac{b\,(t)}{2}\,(x^2 + y^2 - 2z^2)$$

which represents a jet impinging against a flat plate (Figure 4).

The velocity components are

$$u = b\,(t)\,x\,; \qquad v = b\,(t)\,y\,; \qquad w = -2b\,(t)\,z$$

and the acceleration components

$$a_x = \dot{b}\,x + b^2 x\,; \qquad a_y = \dot{b}\,y + b^2 y\,; \qquad a_z = -\,2\,\dot{b}\,z + 4\,b^2 z$$

The Strouhal number is

$$S = \frac{|\dot{b}|}{b^2}\sqrt{\frac{x^2 + y^2 + 4\,z^2}{x^2 + y^2 + 16\,z^2}}$$

which is constant in time, if $\dfrac{db}{dt} = C\,b^2$, where C is a dimensionless constant. The solution is again

$$b\,(t) = \frac{A}{1 - ACt}$$

where $A = b(0)$. The time-independent Strouhal number is then

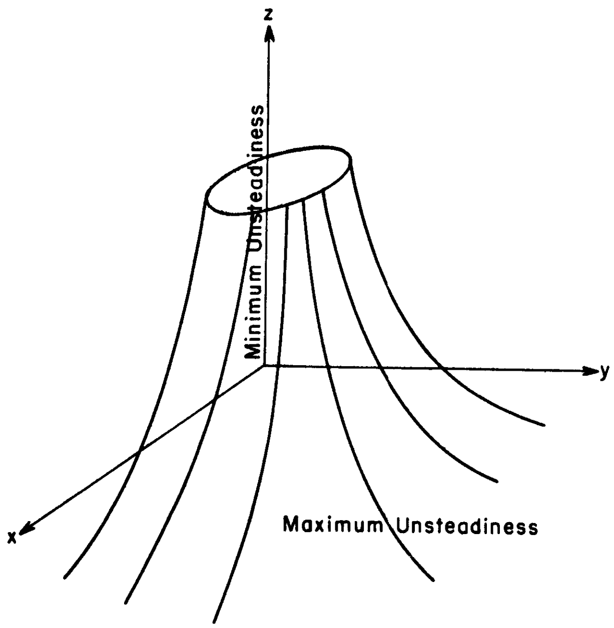

Figure 4 - Three-Dimensional Jet and Flat Plate

$$S = |C| \sqrt{\frac{x^2 + y^2 + 4z^2}{x^2 + y^2 + 16z^2}}$$

The Strouhal number is space independent on the $z = 0$ plane (where $w = 0$ and which is, therefore, the plane of impact) and on the z axis (where $w = v = 0$ and which is the center line of the jet). At any given time, the above measure has its maximum in the plane of impact ($z = 0$, $x \neq 0$, $y \neq 0$),

$$S_{max} = \frac{|\dot{b}|}{b^2}$$

and its minimum on the z axis ($x = y = 0$, $z \neq 0$),

$$S_{min} = \frac{1}{2} S_{max}$$

Returning to the case where the measure is time independent, it is seen that if $b(0) = A$ is positive, $w < 0$ for $z > 0$ and the flow approaches the plate. The variation of $b(t)$ with time is shown in Figure 5 for $C > 0$ and $C < 0$. The $C = 0$ case is equivalent to zero Strouhal number and to $b(t) = A$, i.e., to steady flow conditions. The case $C > 0$ gives increasing velocity from $t = 0$ to $t \rightarrow 1/AC$. As the time approaches $1/AC$, the velocity $\rightarrow \infty$. The case $C < 0$ represents decreasing velocity from $t = 0$ to $t \rightarrow \infty$. If the investigation is extended only to the range $0 \leqq t$ and no infinite velocity is allowed, then $C < 0$. For this case, now, if $|C_1| > |C_2|$, then the flow corresponding to C_1 will approach zero velocity quicker than the flow corresponding to C_2.

The Strouhal number is equal to $|C|$ on the (x,y) plane. In the problem under consideration, therefore, it may be concluded that large values of the Strouhal scalar are associated with large changes in velocity. As a matter of fact, a negative C is associated with a

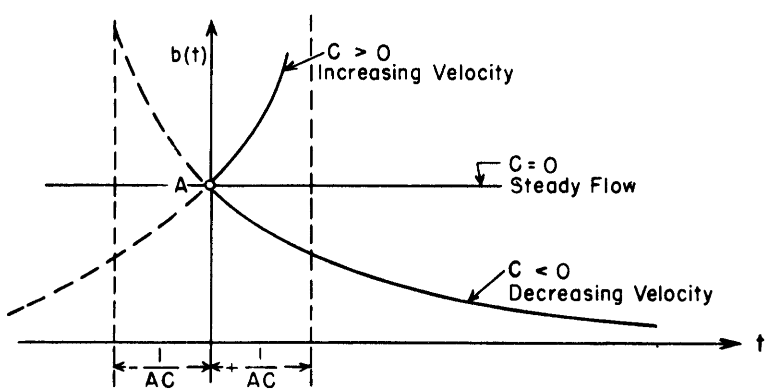

Figure 5 - Time Dependence of Jet with Constant Unsteadiness

velocity decrease and a positive C with a velocity increase, but the latter case, as is seen in Figure 5, involves infinite velocity.

DISSIPATION OF VORTICITY

It was shown by Kampé de Fériet[9] that there are only four possible two-dimensional flows of incompressible viscous fluids for which the streamlines are isocurls, i.e., the vorticity is constant along streamlines. These are flows for which the vorticity is time and space independent, parallel flows, and G.I. Taylor's two solutions, which are to be discussed in detail. Flows with constant vorticity are not considered interesting. For parallel flows, the incompressibility condition always results in zero convective acceleration. The third and fourth flows given by Taylor are those in which the streamlines are concentric circles or in which the eddies form cells. The stream function for the former case is[10]

$$\psi = \frac{A}{t}\, e^{-\frac{r^2}{4\nu t}}$$

The tangential velocity is

$$u_\theta = -\frac{\partial \psi}{\partial r} = \frac{A r}{2\nu t^2}\, e^{-\frac{r^2}{4\nu t}}$$

and the vorticity is

$$\zeta = -\frac{\partial^2 \psi}{\partial r^2} - \frac{1}{r}\frac{\partial \psi}{\partial r} = \frac{A}{\nu t^2}\left(1 - \frac{r^2}{4\nu t}\right) e^{-\frac{r^2}{4\nu t}}$$

The local and convective accelerations in the tangential and radial directions are:

$$(a_\theta)_l = \dot{u}_\theta = -\frac{A r}{2\nu t^3}\left(2 - \frac{r^2}{4\nu t}\right) e^{-\frac{r^2}{4\nu t}}$$

$$(a_\theta)_c = 0$$

$$(a_r)_l = 0$$

$$(a_r)_c = -\frac{1}{r}(u_\theta)^2 = -\frac{A^2 r}{4\nu^2 t^4}\, e^{-\frac{r^2}{2\nu t}}$$

With these values,

$$S = \left|\frac{2\nu t}{A}\left(2 - \frac{r^2}{4\nu t}\right)\right| e^{\frac{r^2}{4\nu t}}$$

This is zero on a circle of radius

$$r_1 = \sqrt{8\nu t}$$

The radius of zero unsteadiness (r_1) increases with time, and it is always larger than the radius of maximum velocity (r_2),

$$r_2 = \sqrt{2\nu t}$$

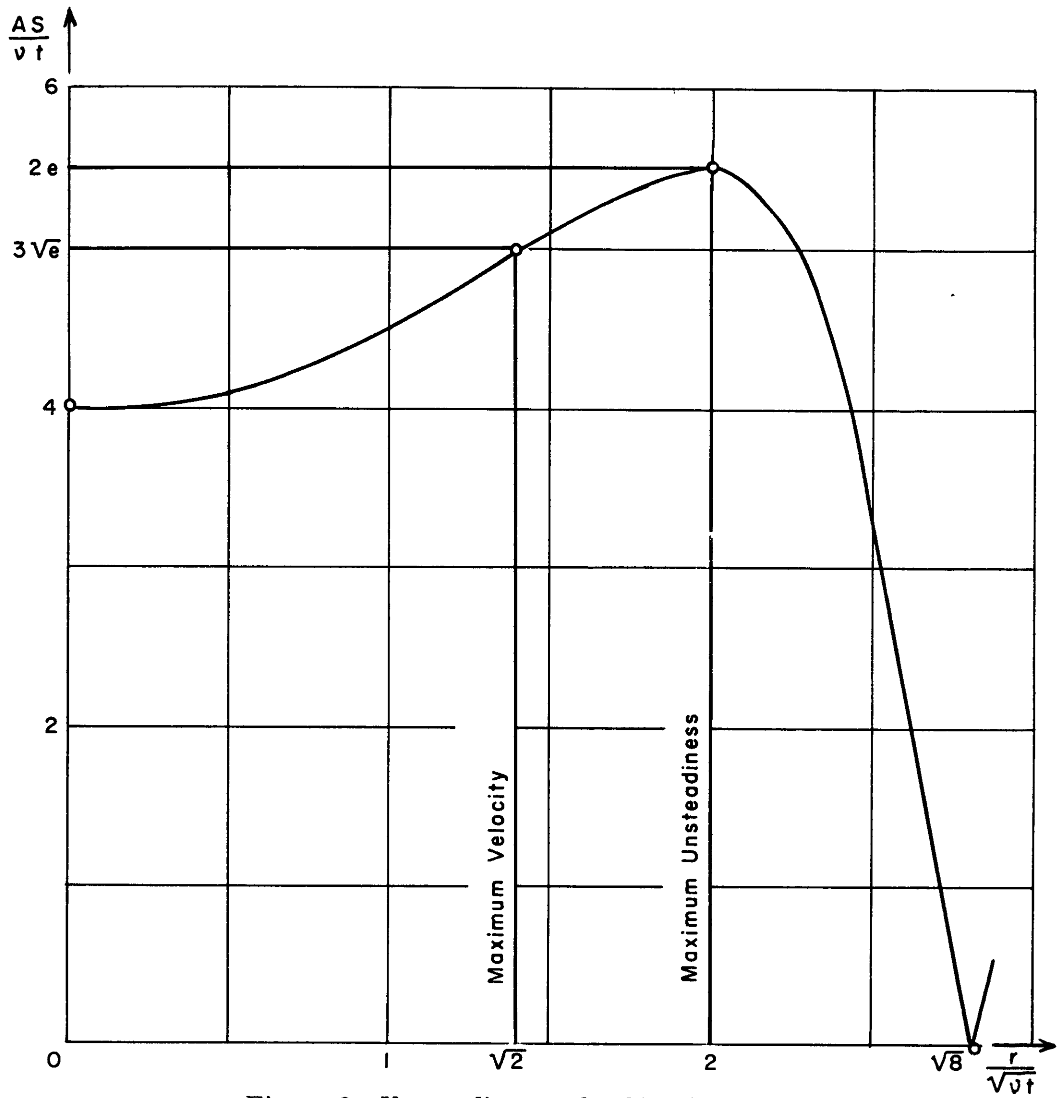

Figure 6 - Unsteadiness of a Circular Eddy

At the origin $(r = 0)$, for $A > 0$, $t > 0$,

$$S = \frac{4\,\nu t}{A}$$

i.e., the unsteadiness increases linearly with time. On the radius of maximum velocity

$$S = \frac{3\sqrt{e}\,\nu t}{A}$$

Inside the circle of zero unsteadiness, the unsteadiness is greatest at

$$r_3 = \sqrt{4\,\nu t}$$

This maximum unsteadiness is

$$S_{max} = \frac{2\,e\,\nu}{A}\,t$$

The maximum unsteadiness corresponds to zero vorticity and maximum dissipation per unit

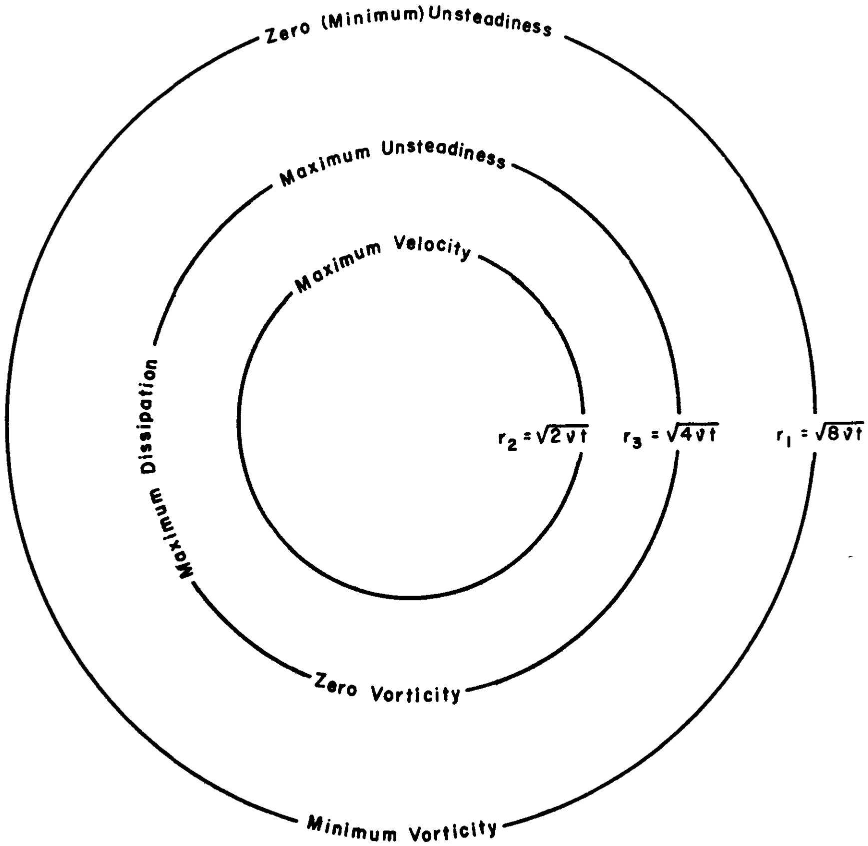

Figure 7 - Characteristics of a Circular Eddy

volume; see Figures 6 and 7.

Taylor's[11] second solution also represents an example of separable flows. This solution is more general than the previous one since both velocity components are different from zero. The stream function is

$$\psi = F(x, y)\, e^{-2\nu\alpha^2 t}$$

or for a special case

$$\psi = C \cos \alpha x \cos \alpha y\, e^{-2\nu\alpha^2 t}$$

where

$$\alpha = \frac{\pi}{d}$$

Trivial streamlines are given by $x = \pm \frac{2n+1}{2} d$ and $y = \pm \frac{2n+1}{2} d$ (Figure 8). The sign of C determines the direction of rotation. The velocity components are

$$u = -\frac{\partial \psi}{\partial y} = C\alpha \cos \alpha x \sin \alpha y\, e^{-2\nu\alpha^2 t}$$

and

$$v = \frac{\partial \psi}{\partial x} = - C\alpha \sin \alpha x \cos \alpha y \, e^{-2\nu\alpha^2 t}$$

The vorticity is

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \Delta \psi = -2\alpha^2 \psi$$

so the streamlines are isocurls.

The velocity and the vorticity approach zero as $t \to \infty$; the motion dies out quickly for large values of ν. On the other hand, the unsteadiness of the flow increases with time. The local and convective accelerations are both decreasing but the convective part dies out faster than the local component and in this way the ratio, which represents the unsteadiness, is increasing.

In fact, the Strouhal scalar is, with $C \to 0$,

$$S = \frac{2\nu e^{2\nu\alpha^2 t}}{C} \sqrt{\frac{\cos^2 \alpha x \sin^2 \alpha y + \sin^2 \alpha x \cos^2 \alpha y}{\cos^2 \alpha x \sin^2 \alpha x + \sin^2 \alpha y \cos^2 \alpha y}}$$

Now $S \to \frac{2\nu e^{2\nu\alpha^2 t}}{C}$ at the origin. For $y = \pm x$,

$$S = \frac{2\nu e^{2\nu\alpha^2 t}}{C}$$

At $x = 0$, $y = \pm \frac{d}{2}$ and at $y = 0$, $x = \pm d$, $S \to \infty$. Therefore, it is concluded that the unsteadiness is largest at points where the velocity is the largest (points A, B, C, and D).

TRKAL'S AND HAMEL'S SOLUTION

Trkal's[12] generalization of Taylor's solution is

$$\bar{v} = \bar{u}(x, y, z) \, e^{-\nu k^2 t}$$

The Navier-Poisson-Duhem equation is

$$\rho \left(\frac{\partial v_i}{\partial t} + v_{i,j} v_j \right) = -p_{,i} + (\lambda + \mu) v_{j,ji} + \mu v_{i,jj} + P_i$$

Taking the curl of this equation, and assuming incompressible fluid and conservative external force, one obtains in vector form

$$curl \, \dot{\bar{v}} - curl \, (\bar{v} \times curl \, \bar{v}) = \nu \Delta \, curl \, \bar{v}$$

Assuming a Beltrami flow ($\bar{v} \times curl \, \bar{v} = 0$), and substituting the Trkal solution, it is found that

$$-k^2 \bar{u} = \Delta \bar{u}$$

which shows the analogy between Taylor's ψ and Trkal's $\bar{u}$.

The measure of unsteadiness is time dependent and shows the same peculiar behavior as was discussed in connection with the decay of vorticity. The streamline pattern is constant in time. The unsteadiness increases as $t \to \infty$ and $|\bar{v}| \to 0$.

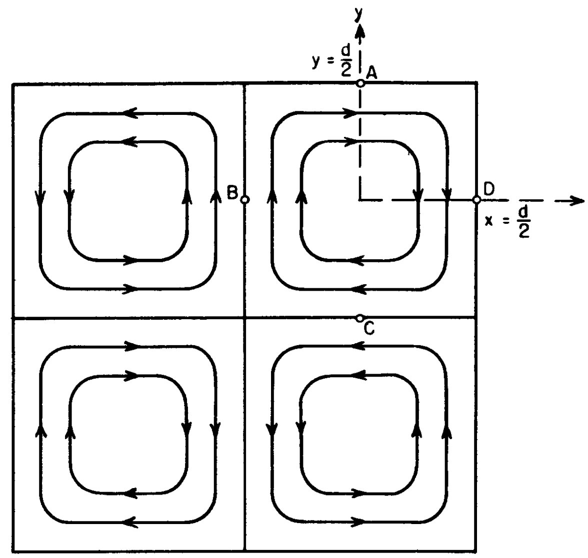

Figure 8 - Cells of Vorticity

Hamel's[13] solution gives an interesting example for two-dimensional spiral flow viscous incompressible fluid. The stream function is given by

$$\psi = C\theta + e^{nt} f_n(r)$$

where C is a constant and $f_n(r)$ is chosen to satisfy the kinematic compatibility equation.

The velocity components are

$$u_\theta = -e^{nt} f_n'(r)$$

and

$$u_r = \frac{C}{r}$$

The tangential velocity is time dependent and the radial velocity is time independent. As t becomes large, the Strouhal number S becomes time independent, differently, however, with positive or negative n. For if $n < 0$, S becomes 0 as time increases without bound, but if $n > 0$, S becomes a function of r only which is in general not zero.

If the stream function is of the form

$$\psi = A(t)\,B(r,\theta)$$

then of course $A(t)$ must satisfy the differential equation

$$\frac{d\,A(t)}{dt} = constant\ A^2(t)$$

(as was shown before) in order to obtain time-independent unsteadiness.

PSEUDO FLOWS

A pseudo plane flow of the first kind is defined by the velocity distribution[14]

$$u = f(x, y, z; t), \qquad v = g(x, y, z; t), \qquad w = 0$$

i.e., the trajectories are in planes parallel with the (x,y) plane. The local acceleration components are given by $\dot{f}$, $\dot{g}$, and 0. The convective acceleration has the components

$$ff_x + gf_y\,,\ fg_x + gg_y \quad \text{and} \quad 0.$$

Therefore, the acceleration takes place in planes parallel to the *(x,y)* plane.

The Strouhal number is:

$$S = \sqrt{\frac{\dot{f}^2 + \dot{g}^2}{(f_x f + f_y g)^2 + (g_x f + g_y g)^2}}$$

A very special case of the above is the general Poisseuille flow $[u = f(y, z; t),$ $v = 0, w = 0]$ which is a rotational, non-Beltrami flow, with zero convective and one-dimensional local acceleration. The unsteadiness is infinite. For the "slow motion" (Stokes) unsteady problems, and for impact problems where the convective acceleration is neglected, the measure becomes infinite. For a pseudo plane flow of the second kind, the motion is identical in planes parallel to the *(x,y)* plane and it is defined by $u = f(x, y; t)$, $v = g(x, y; t)$, $w = h(x, y; t)$. The Strouhal number is again generally not zero and finite.

Rotational symmetric flows show the same behavior with respect to the measure as the pseudo plane flows. A pseudo rotational symmetric flow of the first kind is given by: $v_r = f(r, \theta, z; t)$, $v_\theta = 0$ and $v_z = h(r, \theta, z; t)$. The pseudo rotational symmetric flow of the second kind $[v_r = f(r, z; t)$, $v_\theta = g(r, z; t)$ and $v_z = h(r, z; t)]$ shows perfect analogy with the corresponding planar case and the measure is, in both cases, generally not zero and finite. For the notation, see Figure 9.

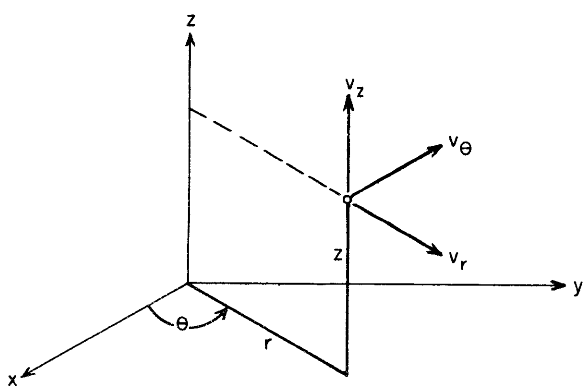

Figure 9 - Coordinate System for Rotational Symmetric Flows

LINEARIZED FLOW

If a small perturbation velocity $\bar{v}$ is superimposed on a constant flow velocity $\bar{U}$, the actual velocity becomes

$$\bar{v}' = \bar{v} + \bar{U}, \quad \text{or} \quad v_i' = v_i + U_i$$

where $|\bar{v}|^2 \ll |\bar{U}|^2$.

The actual acceleration is

$$a_i' = \frac{\partial v_i'}{\partial t} + v_{i,j}' v_j'$$

The linearized acceleration becomes

$$a_i = \dot{v}_i + v_{i,j} U_j$$

The Strouhal number is

$$S = \frac{\sqrt{\dot{v}_i \dot{v}_i}}{\sqrt{v_{j,k} v_{j,l} U_k U_l}}$$

If the constant flow is along the x-axis, then

$$U_1 = U; \qquad U_2 = U_3 = 0$$

and

$$S = \frac{\sqrt{\dot{v}_i \dot{v}_i}}{U \sqrt{v_{j,1} v_{j,1}}}$$

where

$$v_{j,1} = \frac{\partial v_j}{\partial x}$$

This measure of unsteadiness is generally time and space dependent.

If we further assume a special perturbation where

$$u = v = 0 \qquad w = A x e^{i \omega t}$$

the measure of unsteadiness becomes

$$S = \frac{x \omega}{U}$$

Substituting a characteristic length *(L)* for *x*, the conventional Strouhal number or dimensionless frequency ratio is obtained,

$$S = \frac{L \omega}{U}$$

The following facts should be emphasized:

1. The acceleration was linearized,
2. the perturbation was not of the d'Alembert type, and
3. the measure of unsteadiness depends on *x*.

INERTIA METHOD FOR SHIP RESISTANCE MEASUREMENT

The inertia method of measuring ship resistance can be connected with the analysis of the following differential equation:

$$(1 + k_x) m \frac{du}{dt} = - c \frac{\rho}{2} u^2 F$$

where m is the mass of the ship and u is the instantaneous velocity of the body in the direction of motion.

The actual experiment suggested by W. Froude consists of a deceleration test during which the velocity decreases from $u_0 = u(0)$ to $u = 0$. The above equation assumes that the drag coefficient c is independent of the acceleration, i.e., the instantaneous velocity alone gives the drag which is independent of higher order derivatives of the velocity and so $c = c(u)$. This is, of course, not the case since from dropping experiments Lunnon[15,16] has shown that the drag computed from the instantaneous velocity is less than the actual drag in accelerated motion. There is also little reason to believe that the added mass is independent of the acceleration in a viscous fluid.[17-19] Assuming furthermore $c(u) = c_0$ = *constant*, the solution is:

$$u = \frac{u_0}{1 + B u_0 t}$$

where

$$B = \frac{\frac{\rho}{2} F c_0}{(1 + k_x) m}$$

A comparison with the discussion on pages 12-13 shows that u_0 corresponds to A, and B to $-C$. Since B is essentially positive, $C < 0$, which means that the velocity is decreasing. Assuming that a d'Alembert flow can represent the flow occurring in connection with the inertia method, the conclusion is made that the assumptions made in the inertia method are equivalent to assuming a time-independent unsteadiness.

The purpose of the above analysis was to investigate the inertia method from the point of view of the measure of unsteadiness and not to discuss the method generally. There are, of course, several very important details in connection with the method which were not mentioned at all.

RECOMMENDATIONS FOR FUTURE RESEARCH

A thorough study of accelerationless flows is strongly indicated, since for these flows the unsteadiness is constant, i.e., space as well as time independent. The investigation should be extended to two- and three-dimensional velocity distributions. Further examples should be given for three-dimensional incompressible unsteady flows. At the present time it seems that an analysis of compressible fluids is in some respects simpler than that of incompressible fluids. The theoretical investigation presented in this paper should be connected more strongly with the motion of submerged bodies. In this respect, a rather encouraging fact is that the generalized Strouhal number reduces to the conventional dimensionless frequency parameter if certain linearizations are performed.

CONCLUSIONS

The unsteadiness of a nonuniform flow can be measured by comparing the local and convective parts of the acceleration. The measure is generally space and time dependent. Separable flows with hyperbolic decay of the velocity show constant unsteadiness in time, increasing unsteadiness for exponential decay of the velocity, and oscillatory unsteadiness for sinusoidal velocity variation. Linearized flow shows time-independent unsteadiness for harmonic oscillation and gives the conventional dimensionless frequency ratio as the measure of unsteadiness.

Three examples are discussed in detail: an unsteady jet, the decay of vorticity, and the inertia method of measuring ship resistance. An unsteady jet striking a plate has maximum unsteadiness on the plate and minimum unsteadiness on the axis of the jet. The two types of vorticity dissipation studied are fundamentally different. A circular eddy is represented by a nonseparable velocity distribution and a set of "rectangular" vortices by a d'Alembert flow. The velocity dies out and the unsteadiness increases with time in both cases. The assumptions made in the inertia method of measuring ship resistance are equivalent with assuming time-independent unsteadiness. The conventional dimensionless frequency ratio is obtained by linearizing the generalized measure of unsteadiness.

ACKNOWLEDGMENTS

This report presents the results of research on one phase of unsteady flow problems. When first outlined, this research program met several objections. In this most critical state, the author received great encouragement from Dr. L. Landweber, whose pure scientific attitude made this research possible.

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The idea to establish a measure of unsteadiness was suggested by Dr. Truesdell's unpublished paper entitled "Two Measures of Vorticity."

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