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SERVICE STRESSES AND MOTIONS OF THE ESSO ASHEVILLE, A T-2 TANKER, INCLUDING A STATISTICAL ANALYSIS OF EXPERIMENTAL DATA

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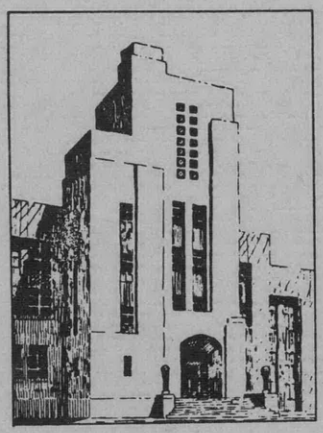

RESEARCH AND DEVELOPMENT REPORT

September 1955

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STATISTICAL TERMS AND DEFINITIONS

It should be noted that in the present work only estimates of the statistical parameters can be given.

variate	Property under study, for example, magnitude of stress variation, height of waves.
$P(x)$	Probability of the event x with respect to the random experiment ϵ. The probability P means that in a long series of repetitions of ϵ it is practically certain that the relative frequency of occurrence of x is equal to P where $0 < P < 1$. A value of $P = 1$ indicates that the event will occur with every experiment ϵ, $P = 0$ indicates that the event will never occur.
$p(x)$	$= \frac{d}{dx} P(x)$
population	Aggregate of all the members of the distribution subject to a statistical study. The members may be individual measurements or the results of individual experiments.
sample	Number of N members drawn (usually at random) from the population. Here N is the "sample size."
c.d.f.	Abbreviation of the term "cumulative distribution function" $P(x)$ which denotes the probability that the variate x will take on a value less than or equal to x, i.e., $P(x_a) = P\,[x \leqq x_a]$.
d.f.	Abbreviation of the term "distribution function" $p(x)$ which is the derivative of $P(x)$ with respect to x, i.e., $p(x) = \frac{d}{dx} P(x)$ and $\int_{-\infty}^{+\infty} p(x)\, dx = 1$
mean value of x	$= \frac{\sum_{i=1}^{N} x_i}{N}$ where x_i is the individual member of the population of sample and N is the number of members of the population or sample.
$\bar{x}$	Mean value of a sample of values of x.
ξ	Mean value of the population.
variance of x	$= \frac{\sum_{i=1}^{N} (x_i - \bar{x})^2}{N - 1}$ The variance is a measure of the dispersion of the values of x. The greater the variance the greater the dispersion.
s^2	Variance of a sample.
σ^2	Variance of the population.

standard deviation	Square root of the variance.
quantile or fractile	Corresponding to a given value of the c.d.f. $P(x)$, there is a value of the variate below which a fraction $P(x)$ of all the members of the population lie. The value of this variate is the quantile or fractile corresponding to $P(x)$.
confidence bands	Interval within which the "true" distribution will fall with a certain probability.
random	A method of drawing a sample when each member of the population has an equal chance of selection.

ABSTRACT

Stresses amidships as well as pitching and heaving accelerations were measured on the T-2 tanker ESSO ASHEVILLE during the vessel's normal commercial operations over the period 24 August 1952 to 3 April 1954. The data were obtained automatically, using a sampling procedure, and are analyzed by statistical methods.

Analysis indicates that the dynamic stress variations associated with the ship's motion in waves will rarely exceed 12,000 psi peak-to-peak. Also, the stress variations due to changes in temperature distribution occasionally are of the same order of magnitude as those due to changes in dead-load distribution. The maximum measured stress variation (free of stress concentration) corresponding to changes in temperature was about 11,000 psi.

The occurrence of wave-induced service stresses and of the ship motions experienced by the ESSO ASHEVILLE may be approximated by a logarithmically normal distribution. Expected extreme wave-induced stresses are predicted by use of the methods of statistics.

INTRODUCTION

Service stresses and motions were measured on a T-2 tanker, the ESSO ASHEVILLE,* during the period from 24 August 1952 to 3 April 1954 when the vessel was engaged in coastal traffic along the Atlantic seaboard, Central America, and the Gulf coast; see Table 1. The vessel is a single-deck, longitudinally framed tanker with a length between perpendiculars of 503 ft, a molded beam of 68 ft, a molded depth of 39 ft 3 in., a displacement of 21,800 tons, and a midship section modulus of 47,700 in.^{2}-ft.

The general purpose of these tests was to determine the magnitudes and frequency distributions of the dynamic stresses (due to wave action) and the rigid-body motions of the ship under actual service conditions over an extended period of time. It was also possible to measure the deck stresses amidships during variations in static load distribution as well as those stresses incident to a change in the temperature distribution throughout the hull girder. Inasmuch as there have been numerous structural failures of this type of vessel, these data may be of value in assessing the relative significance of these different service stresses in contributing to such failures. The dynamic stresses and motions will be considered first; then the static load and temperature stresses will be discussed. An outline of the statistical methods utilized in this report is given in Appendix 1.

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*This report supersedes an interim report, TMB Report 875.

TABLE 1

Voyages of the ESSO ASHEVILLE during the Period August 1952 to April 1954

Dates	Ports	Condition
30 Jul to 1 Aug 1952	Portland to Baltimore	Ballast
24 Aug to 28 Aug	Baltimore to Aruba	Ballast
30 Aug to 11 Sep	Aruba to W.C. Central America	Loaded
12 Sep to 18 Sep	W.C. Central America to C. Christi	Ballast
20 Sep to 24 Sep	Corpus Christi to Portland	Loaded
25 Sep to 30 Sep	Portland to Aruba	Ballast
1 Oct to 5 Oct	Aruba to Buenavent	Loaded
9 Oct to 14 Oct	Buenavent to Baytown	Ballast
16 Oct to 20 Oct	Baytown to Wilmington	Loaded
22 Oct to 26 Oct	Wilmington to Baytown	Ballast
26 Oct to 3 Nov	Baytown to L. Charles to Portland	Loaded
4 Nov to 10 Nov	Portland to Baytown	Ballast
12 Nov to 21 Nov	Baytown to Montreal	Loaded
25 Nov to 2 Dec	Montreal to Baton Rouge	Ballast
8 Dec to 12 Dec	Baton Rouge to Philadelphia	Loaded
14 Dec to 19 Dec	Philadelphia to Baytown	Ballast
20 Dec to 25 Dec	Baytown to Providence	Loaded
27 Dec to 31 Dec	Providence to Baton Rouge	Ballast
3 Jan to 7 Jan 1953	Baton Rouge to Philadelphia	Loaded
9 Jan to 15 Jan	Philadelphia to Baytown	Ballast
16 Jan to 21 Jan	Baytown to N. Haven	Loaded
22 Jan to 29 Jan	N. Haven to Baytown	Ballast
30 Jan to 2 Feb	Baytown to Jacksonville	Loaded
3 Feb to 6 Feb	Jacksonville to Baytown	Ballast
7 Feb to 12 Feb	Baytown to Norfolk	Loaded
13 Feb to 17 Feb	Norfolk to Baton Rouge	Ballast
19 Feb to 24 Feb	Baton Rouge to Boston	Loaded
26 Feb to 3 Mar	Boston to Baytown	Ballast
5 Mar to 11 Mar	Baytown to Boston	Loaded
12 Mar to 18 Mar	Boston to Galveston	Ballast
22 Mar to 27 Mar	Baytown to N. Haven	Loaded
29 Mar to 4 Apr	N. Haven to Baytown	Ballast
5 Apr to 10 Apr	Baytown to New York	Loaded
13 Apr to 18 Apr	New York to Freeport	Ballast
20 Apr to 23 Apr	Freeport to Pt. Everglades	Loaded
24 Apr to 26 Apr	Pt. Everglades to Baton Rouge	Ballast
29 Apr to 3 May	Baton Rouge to Providence	Loaded
4 May to 10 May	Providence to Baytown	Ballast
11 May to 16 May	Baytown to Philadelphia	Loaded
17 May to 22 May	Philadelphia to Galveston	Ballast
25 May to 30 May	Baytown to New York	Loaded
2 Jun to 7 Jun	New York to Baytown	Ballast
9 Jun to 13 Jun	Baytown to Norfolk	Loaded
14 Jun to 17 Jun	Norfolk to Pt. Isabel	Ballast
20 Jun to 25 Jun	Pt. Isabel to Norfolk	Loaded
26 Jun to 30 Jun	Norfolk to Baytown	Ballast
2 Jul to 6 Jul	Baytown to Baltimore	Loaded
8 Jul to 11 Jul	Baltimore to Baton Rouge	Ballast
14 Jul to 18 Jul	Baton Rouge to New York	Loaded
20 Jul to 22 Jul 1953	New York to Jacksonville	Ballast

TABLE 1 (continued)

Dates	Ports	Condition
7 Aug to 10 Aug 1953	Jacksonville to Baytown	Ballast
12 Aug to 17 Aug	Baytown to Boston	Loaded
19 Aug to 25 Aug	Boston to Baytown	Ballast
26 Aug to 31 Aug	Baytown to Portland	Loaded
2 Sep to 7 Sep	Portland to Aruba	Ballast
9 Sep to 13 Sep	Aruba to Buenavent	Loaded
15 Sep to 20 Sep	Buenavent to Baton Rouge	Ballast
22 Sep to 26 Sep	Baton Rouge to Tiverton	Loaded
28 Sep to 3 Oct	Tiverton to Baton Rouge	Ballast
5 Oct to 10 Oct	Baton Rouge to Boston	Loaded
12 Oct to 17 Oct	Boston to Curacao	Ballast
18 Oct to 18 Oct	Curacao to Punta Cardon	Loaded
20 Oct to 22 Oct	Punta Cardon to Santiago de Cuba	Loaded
24 Oct to 27 Oct	Santiago to Galveston	Ballast
1 Nov to 5 Nov	Baytown to Morehead City	Loaded
8 Nov to 12 Nov	Morehead City to Corpus Christi	Ballast
13 Nov to 18 Nov	Corpus Christi to Norfolk	Loaded
19 Nov to 24 Nov	Norfolk to Houston	Ballast
25 Nov to 1 Dec	Houston to Boston	Loaded
3 Dec to 8 Dec	Boston to Aruba	Ballast
9 Dec to 12 Dec	Aruba to Buenavent	Loaded
14 Dec to 19 Dec	Buenavent to Baytown	Ballast
21 Dec to 26 Dec	Baytown to Boston	Loaded
27 Dec to 2 Jan 1954	Boston to Baytown	Ballast
4 Jan to 9 Jan	Baytown to Norfolk	Loaded
10 Jan to 14 Jan	Norfolk to Baytown	Ballast
15 Jan to 21 Jan	Baytown to Boston	Loaded
23 Jan to 23 Jan	Boston to Portland	Loaded
24 Jan to 30 Jan	Portland to Baton Rouge	Ballast
1 Feb to 5 Feb	Baton Rouge to Philadelphia	Loaded
7 Feb to 12 Feb	Philadelphia to Galveston	Ballast
15 Feb to 18 Feb	Baytown to Charleston	Loaded
20 Feb to 20 Feb	Charleston to Morehead City	Loaded
22 Feb to 26 Feb	Morehead City to Baytown	Ballast
27 Feb to 4 Mar	Baytown to New York	Loaded
5 Mar to 10 Mar	New York to Aruba	Ballast
12 Mar to 13 Mar	Aruba to Santiago	Loaded
15 Mar to 17 Mar	Santiago to Aruba	Ballast
18 Mar to 26 Mar	Aruba to Recife	Loaded
27 Mar to 30 Mar	Recife to Rio	Loaded
31 Mar to 6 Apr	Rio to Belem	Loaded
9 Apr to 13 Apr	Belem to Amuay	Ballast
14 Apr to 22 Apr	Amuay to Montreal	Loaded
25 Apr to 3 May	Montreal to Galveston	Ballast

The sea trials discussed here were conducted under the joint sponsorship of the S-10 panel of the Society of Naval Architects and Marine Engineers and the U.S. Navy with the cooperation of the Esso Shipping Company.

INSTRUMENTATION

The stresses in the fore-and-aft direction were determined at the port and starboard edges of the main deck at Frame 57, at a point approximately 7 in. inboard from the gunwale; see Figure 1. These measurements were made by means of SR-4 strain gages connected electrically so as to give an output signal proportional to the stress in the fore-and-aft direction; see Appendix 2. The measured stresses included the stress components due to static and dynamic load and those due to temperature variations. The strain gages were compensated so as to exclude strains unaccompanied by stress. A linear accelerometer measuring the heaving acceleration was installed at Frame 57 on the centerline;* a 10-in. electromechanical strain gage was installed at the same location. The rigid-body pitching acceleration of the ship was measured by means of two linear accelerometers spaced 12 ft 5 in. apart on the bridge deck. These accelerometers were connected so as to act electrically in opposition, thus giving an output proportional to pitching acceleration. All accelerometers and the 10-in. strain gage were built by the Schaevitz Engineering Company and utilized differential transformers as the sensing elements.

The outputs of these gages were recorded automatically, at hourly intervals, for 2 min. The recorder used was the TMB automatic ship's motion recorder which is described in Reference 1.** The automatic timing motor on this recorder occasionally failed to operate properly, resulting in the loss of sampled data for some periods of time. The heaving accelerometer and 10-in. strain gage were damaged occasionally by salt-water corrosion and were repaired when practicable; the SR-4 gages and pitching accelerometers operated satisfactorily throughout the tests.

On 11 April 1953 a Sperry roll and pitch recorder was installed on the ship; it was arranged so as to record simultaneously with the TMB recorder. The Sperry recorder was inoperative after 9 November 1953.

See Figure 1 for photographs of the test installation.

DYNAMIC STRESSES AND MOTIONS INDUCED BY WAVES

Inasmuch as many thousands of variations of stress and motion were measured, it will be advantageous to utilize some of the statistical methods which have been developed for the purpose of handling large masses of numerical data. It may be helpful to make a few remarks about these methods.

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*This location is close to the fore-and-aft position of the ship's center of gravity.

**References are listed on page 44.

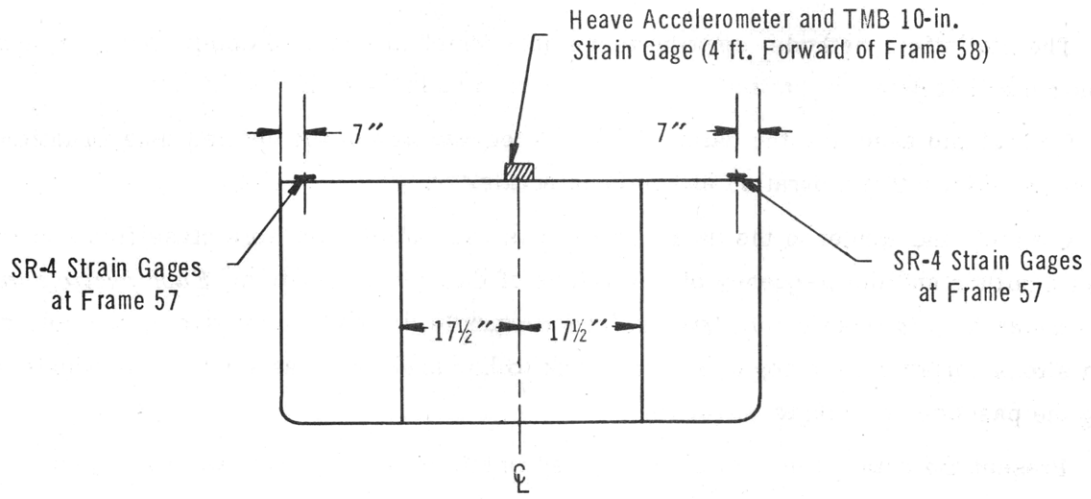

Figure 1a - Section at Frame 57, Looking Aft, Showing Location of Gages

The pitch accelerometers were located on the bridge deck.

Figure 1b - Location of SR-4 Strain Gages and Cable Runways, Looking Aft

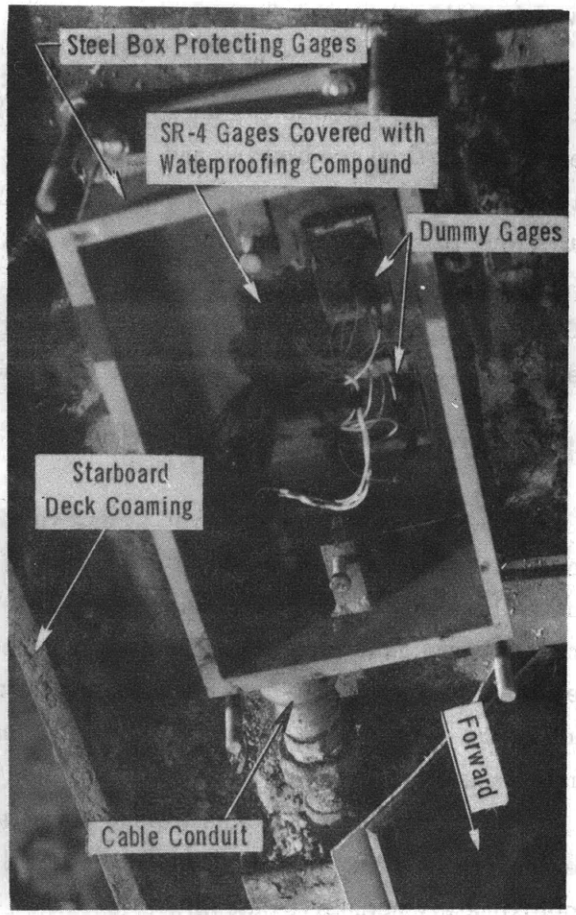

Figure 1c - Closeup of SR-4 Strain Gage Installation on Starboard Side of Main Deck

Figure 1 - Test Installation

The photographs were taken at about 1500 on 1 August 1953 at Jacksonville, Florida.

The statistical methods comprise techniques which are used to obtain, analyze, and present numerical data. In particular, they have been used here to:

1. Collect and assemble the data. Many hundred *samples* of the desired time variations were *recorded* for a 2-min duration at hourly intervals.

2. Classify and condense the data. The stress and motion data were classified according to the magnitude and the frequency of occurrence of the cyclic variation. *Each cyclic variation is understood to mean a complete cycle*; for example, in the case of stress, it would denote a stress variation from hog to sag and back to hog again such as would be encountered during the passage of a single wave.

3. Present the data. The data are presented in tabular form as well as in the form of "distribution patterns." These distribution patterns indicate the relative probability of occurrence of a quantity as a function of the magnitude of that quantity. This type of presentation is intended to bring out the more important useful items of information that may be derived from the measured data.

4. Analyze the data. With the experimental data presented in a concise manner, an analytic expression is fitted to the data, and tests of significance are applied to determine whether certain statistical predictions and conclusions can be reached with a fair degree of confidence on the basis of the data. For example, it will be found that, for the ESSO ASHEVILLE, the variations in stress and motion can be reasonably well represented by a logarithmically normal distribution. It should be emphasized that the specific numerical results given here on dynamic stresses and motions are valid for the conditions encountered by the vessel from 1 September 1953 to 3 April 1954 and may differ appreciably for other ships or other service conditions.

DISTRIBUTION PATTERNS OF STRESSES

Table 2 gives the observed stress variations at the port and starboard sides of the main deck, classified according to the magnitude of the stress variation (peak-to-peak) for both load and ballast conditions of the ship. The stress variations are tabulated separately for the port and starboard gage locations. Table 3 indicates the number of stress variations (in a 30-day period at sea) which have magnitudes greater than the indicated values, as obtained by summation of the data in Table 2. Tables 2 and 3 furnish the data in a form suitable for plotting histograms.*

Considerable effort was expended in fitting an analytic distribution function to the experimental distribution; it was found that the logarithm of the stress variation was distributed approximately normally, that is,

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*A histogram is a bar graph indicating the frequency of occurrence of a measurement as a function of the magnitude of the measurement.

$$p = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(\log h - u)^2}{2\sigma^2}} \quad \text{and} \quad \int_{-\infty}^{+\infty} p\, d\,(\log h) = 1 \qquad [1]$$

where p is the probability density for $\log h$, $p\,d\,(\log h)$ being proportional to the frequency of occurrence of values of $\log h$ in the interval $d\,(\log h)$,

h is the independent variable (the stress variation),

u is the mean value of $\log h$, and

σ is the standard deviation of $\log h$.

Thus the frequency distribution of the wave-induced stresses may be described in terms of the quantities σ and u. If the logarithmically normal distribution is a good approximation to the actual distribution of the stress variations, then the statistics and tables that have been developed for the normal distribution pattern may be applied to the collection, presentation, analysis, and prediction of these sea-induced stress variations.

Statistics is a specialized field and it is not practical to develop or explain here, in detail, the methods utilized. Appendix 1 discusses some of the more important concepts, and the reader may, in addition, consult References 2 through 8 for further study.

The stress and ship-motion data given in this report furnish what is known as a "truncated" or "censored" distribution* because only values larger than a given lower limit (500 psi in our case) were tabulated. Fisher[2] has devised a method for determining, from the measured truncated data, the parameters σ and u which define the normal distribution. If the analytically expressed distribution thus determined fits the measured data reasonably well, it is probable that the data (logarithm of the stress) follow a normal distribution pattern. Application of the Fisher method to the data in Table 3 gives the distribution patterns shown in Figure 2.** Inasmuch as inspection of Table 3 indicated no large differences between stress variations under load and ballast conditions, the average of these data was also used in these computations.

It was observed that on each day there was a predominant period of stress variations. A histogram showing the distribution of the predominant periods is shown in Figure 3.

The plotted points in Figure 2 were obtained by use of the theoretically determined degree of truncation (that is, percent of the distribution falling below the point of truncation), together with the data of Table 2, and thus their scatter from the straight lines should give an unbiased measure of the applicability of the assumed normal distribution within the range (above 500 psi) throughout which the stress measurements were analyzed. Let us examine

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*Truncated at the lower end of the distribution.

**The coordinate paper in Figure 2 is specially devised so that a logarithmically normal distribution will plot as a straight line.

TABLE 2

Basic Data on Wave-Induced Stresses for the Period 1 September 1953 to 3 April 1954

Type of Measurement	Actual or Expanded Counts	Load or Ballast	Days at Sea Subject to Study	Days at Sea during Which Suitable Data were Measured	Percent of Time at Sea Subject to Sampling	Minutes of Recording Time	Number of Stress Cycles Falling between Stated Limits† (for a 30-Day Period at Sea)												
							For a Sample of 2 min Taken Every Hour							For a Sample of 2 min Taken Every Fourth Hour††						
							Expansion* Factor K_1	2,000-4,000 psi	4,000-6,000 psi	6,000-8,000 psi	8,000-10,000 psi	10,000-12,000 psi	Over 12,000 psi	Expansion* Factor K_2	500-1,000 psi	1,000-2,000 psi	2,000-3,000 psi	3,000-4,000 psi	4,000-5,000 psi	5,000-6,000 psi
Stress	A	L	75	59	79	2,832	-	1,208	221	31	3	0	0							
Stbd	E	L					15.25	18,422	3,370	473	46	0	0							
	A	L	68	40.9	60	2,136								-	1,226	758	206	95	43	4
	E	L												80.89	99,171	61,315	16,663	7,685	3,478	324
Stress	A	B	69	65	94	3,120		1,690	142	14	0	0	0							
Stbd	E	B					13.85	23,406	1,967	194	0	0	0							
	A	B	61.9	61.9	100	3,304								-	1,409	1,336	427	115	17	5
	E	B												52.3	73,691	69,873	22,332	6,015	889	262
Stress	A	L	75	65	87	3,120		1,767	343	55	8	4	0							
Port	E	L					13.85	24,473	4,751	762	111	55	0							
	A	L	68	34.8	51	1,888								-	987	706	222	77	0	1
	E	L												91.53	90,340	64,620	20,320	7,048	0	92
Stress	A	B	69	58	84	2,784		1,442	280	38	2	0	0							
Port	E	B					15.52	22,380	4,345	590	31	0	0							
	A	B	61.9	35.2	57	1,922								-	785	685	280	125	48	18
	E	B												89.90	70,752	61,582	25,172	11,238	4,315	1,618

*The factors K_1 and K_2 are the factors by which the cycles measured on the oscillogram samples are multiplied in order to estimate the number of stress cycles that would have been obtained if continuous measurements had been taken over a period of 30 days at sea.

†Peak-to-peak variation.

††Every fourth sample recorded was utilized in this analysis.

TABLE 3

Distribution of Wave-Induced Stress Variations for a Period of 30 Days at Sea between 1 September 1953 and 3 April 1954

The average period of the measured stress variation was about 9.2 sec. The most frequently observed period was in the range 5 to 7 sec; see Figure 3.

Stress psi	Estimated Number of Cycles of Magnitude† Greater than the Indicated Stress (For a period of 30 days at sea)								
	Average No. Cycles P and S Load and Ballast	Ship Loaded				Ship in Ballast			
		Starboard Gage		Port Gage		Starboard Gage		Port Gage	
	**	*	**	*	**	*	**	*	**
500	181,128		188,723		188,007		174,072		173,710
1,000	97,640		89,552		97,667		100,381		102,958
2,000	33,292	22,311	28,237	30,152	33,047	25,567	30,508	27,346	41,376
3,000	12,170		11,574		12,727		8,176		16,204
4,000	4,174	3,889	3,889	5,679	5,679	2,161	2,161	4,966	4,966
5,000	1,139		843		1,020		456		2,239
6,000	566	519	519	928	928	194	194	621	621
8,000	61	46	46	166	166	0	0	31	31
10,000	13	0	0	55	55	0	0	0	0
12,000	0	0	0	0	0	0	0	0	0

*This estimate is based on oscillograms taken for a duration of 2 min at intervals of 1 hr.

**This estimate is similar to the preceding one except that, for stress values less than 4,000 psi, sample oscillograms of 2-min duration, taken at intervals of 4 hr, were utilized in Figure 2.

†Peak-to-peak variation.

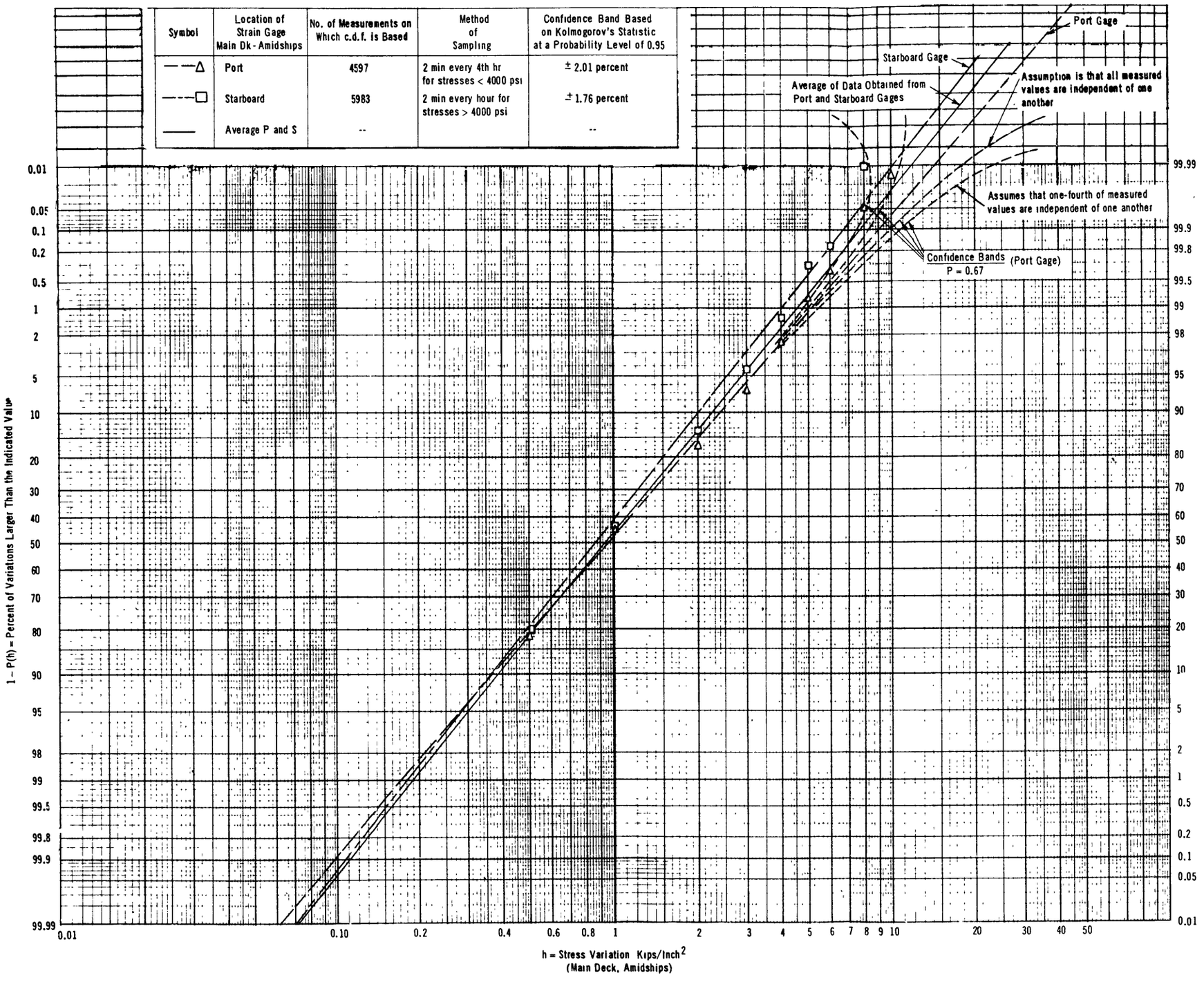

Figure 2 - Statistical Cumulative Distribution of Hull Girder Stress Amidships

Approximately 220,000 stress variations are expected to occur for each 30-day period at sea.

The straight lines, representing log-normal distributions, have been directly computed from the measured truncated data (18.4 percent truncation). The only assumption utilized is that the distribution is logarithmically normal. For each gage the average of the load and ballast conditions has been used for the plots. Confidence bands plotted here are computed on the basis of the assumption that the quantiles of the logarithm of the stress are normally distributed (see Reference 5) and that the measured data are random and independent samples.

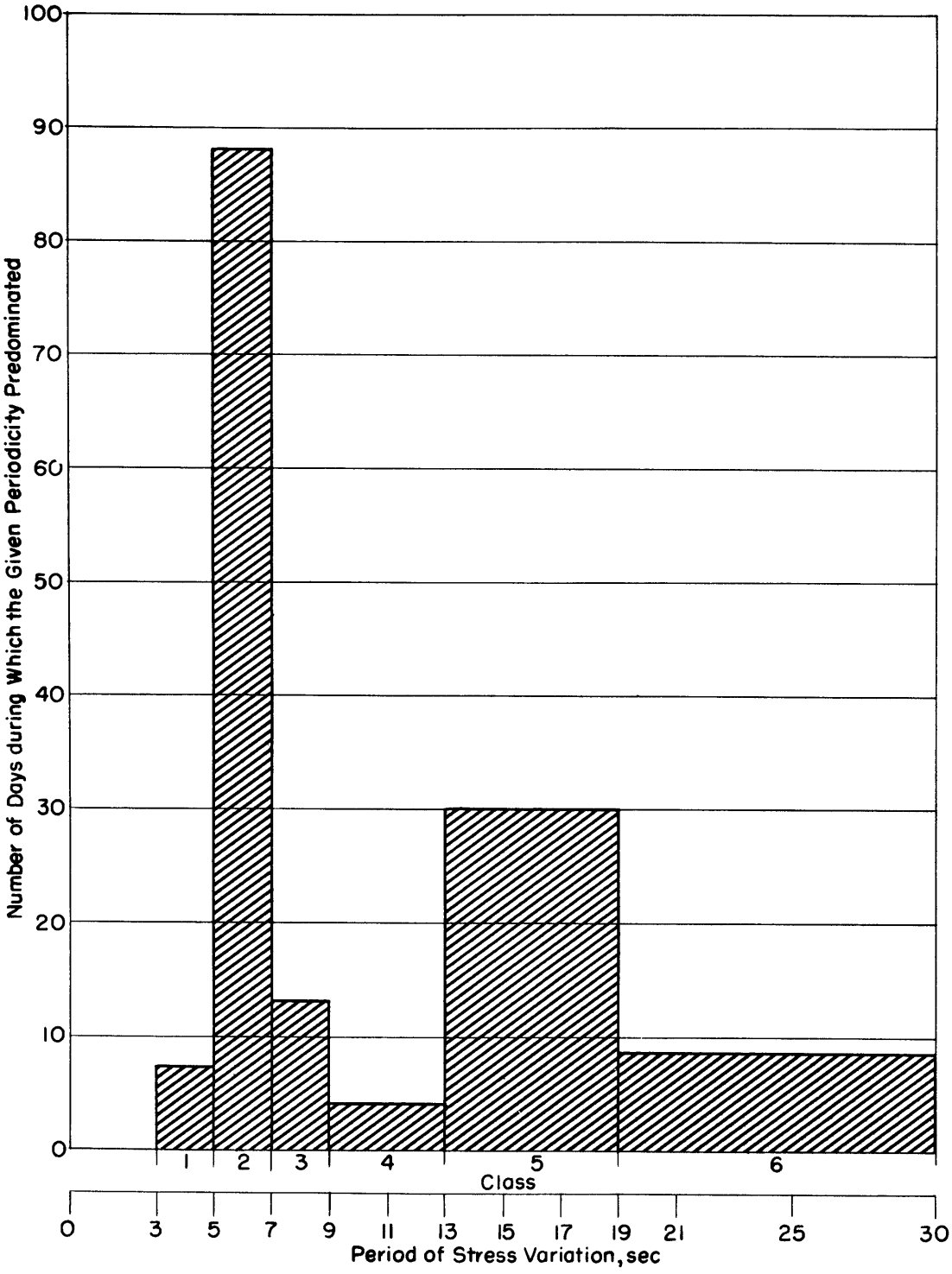

Figure 3 - Frequency of Occurrence of Stress Variations of Various Periods

The estimated natural period of pitching is 6 sec. The average period of predominant stress variation is 7.2 sec.

the hypothesis that, for this vessel and service, the logarithm of the sea-induced stresses is normally distributed; this distribution will be called, for short, the log-normal distribution.

There are a number of methods available for quantitatively testing the validity of this hypothesis, but unfortunately they assume that the test data consist of random independent measurements taken from the population. Consecutive cycles of variations in stress and motion are *not* independent. Autocorrelations made for these functions have indicated that observations separated by about six cycles are "reasonably" independent of each other in time. It will not be possible to give accurate probabilities for the hypothesis considered; however, if it is assumed (falsely) that all data are random and independent, then a better fit will be required between the assumed distribution (log-normal c.d.f.) and the experimental data than should really be necessary. Otherwise stated, *if the hypothesis is probably true for independent observations, it is certainly even more valid for the actual observations.*

One particular test of significance that was used is the chi-square test which compares the observed grouped distribution with the theoretical distribution.* It is of the go, no-go type, and one poor experimental grouped value may cause a rejection of the hypothesis. If all observations are assumed to be independent, this test rejects the hypothesis. If only every sixth observation is considered to be an independent observation, then this test indicates that the deviations of the experimental data from the log-normal distribution are well within the limits of expected chance variations.

Other more informative tests of significance are available, which establish bounds about the theoretical distribution within which the experimental data are expected to fall for given probability levels. Such bounds are known as confidence limits. One of the simplest of such methods utilizes Kolmogorov's statistic[4] which permits one to assert, with a probability P, that the true continuous probability distribution function is contained within the confidence band everywhere. Kolmogorov's statistic was used to fit confidence bands to the motion data plotted in Figures 4 and 5.

A more accurate (and tedious) way of fitting confidence bands utilizes a theorem of statistics[5] which states that, if many samples of the same size are taken at random from any one-dimensional distribution, then any given quantile of the samples will be asymptotically normally distributed as the size of the sample becomes larger and larger. The latter method has been used to fit confidence limits to the distribution of stress variations measured on the port side amidships, as shown in Figure 2 and Table 4. It is evident from an inspection of Figure 2 that the experimental data are approximated by the log-normal distribution with a high degree of probability, especially when the accuracy with which the stress data were obtained is considered. To avoid confusion, confidence bands have not been fitted to those straight lines in Figure 2 which represent the c.d.f. of the average and of the starboard

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*See Chapter 23 of Reference 3.

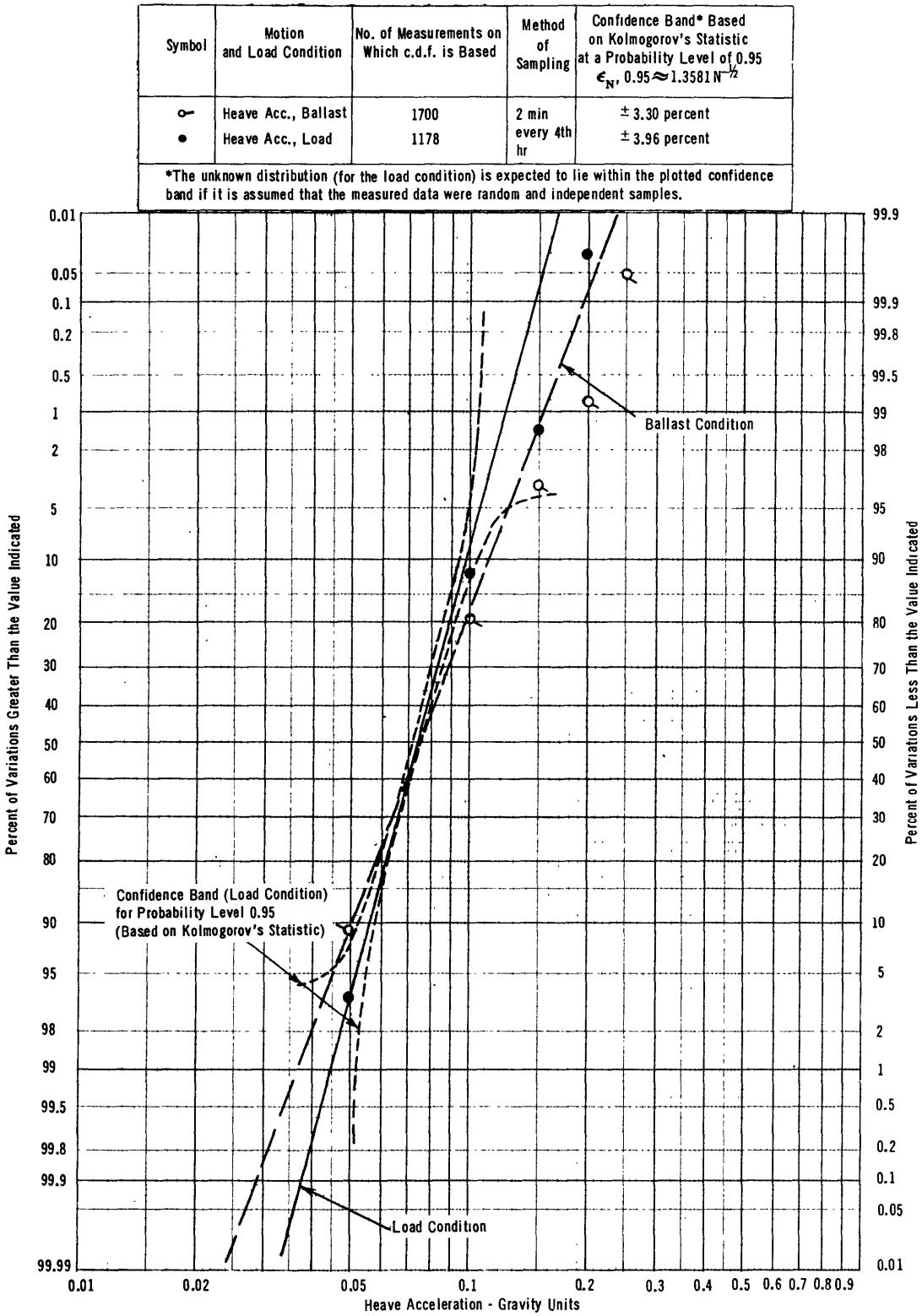

Figure 4 - Statistical Cumulative Distribution of Heave Acceleration at Center of Gravity of Ship

These distributions are based on the data given in Tables 5 and 6.

Symbol	Motion and Load Condition	No. of Measurements on Which c.d.f. is Based	Method of Sampling	Confidence Band* Based on Kolmogorov's Statistic at a Probability Level of 0.95 ϵ_N, $0.95 \approx 1.3581\, N^{-1/2}$
o-	Pitch Acc., Ballast	1575	2 min every 4th hr	± 3.42 percent
●	Pitch Acc., Load	1578		± 3.42 percent

*The unknown distribution (for the load condition) is expected to lie within the plotted confidence band if it is assumed that the measured data were random and independent samples.

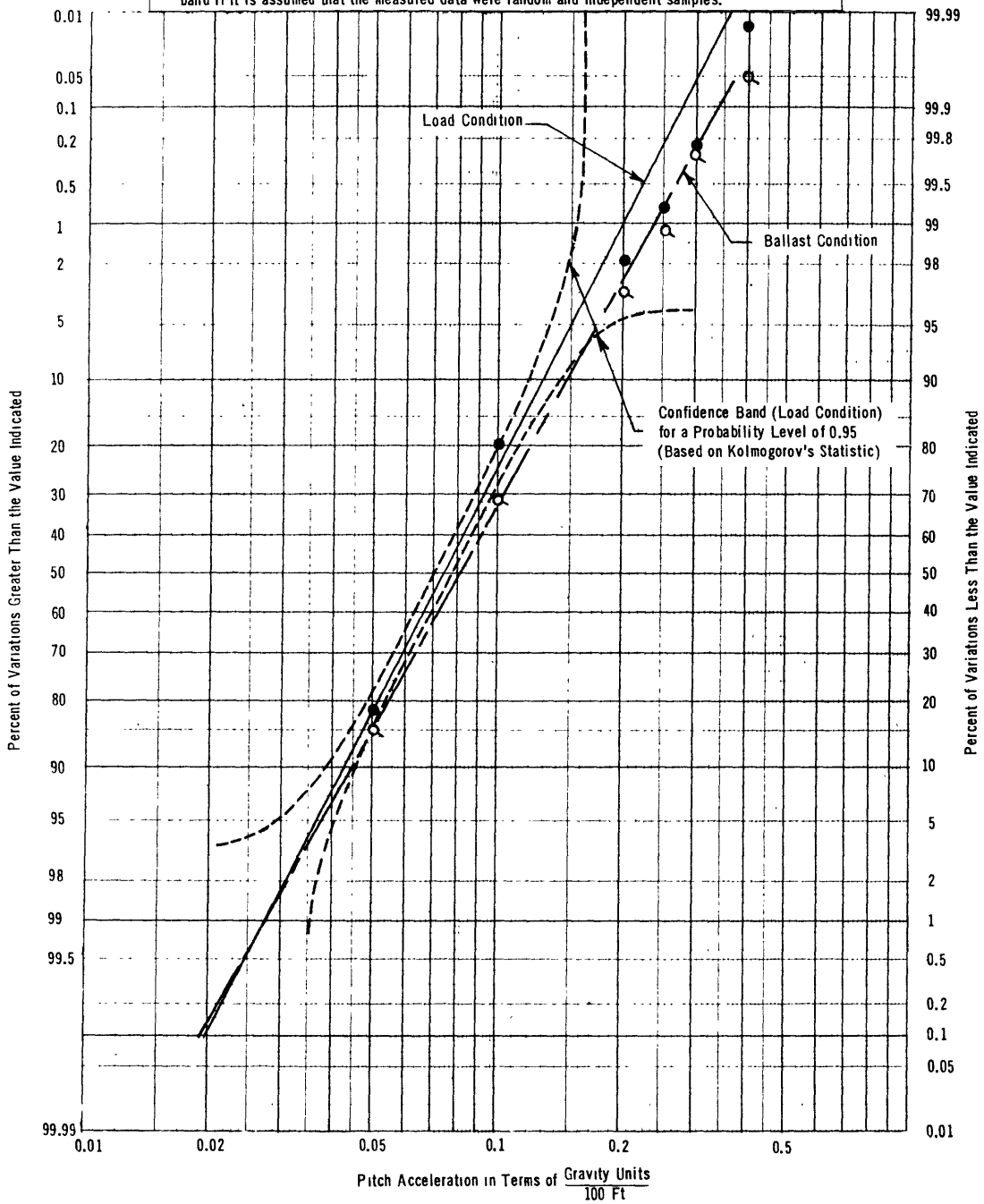

Figure 5 - Statistical Cumulative Distribution of Pitch Acceleration

These distributions are based on the data given in Tables 5 and 6.

TABLE 4

Computation of Confidence Limits for Distribution of Stress Variations, Port Side, Shown in Figure 2

Stress psi x	Log_{10} Stress x_L*	$(\bar{x}_L - x_L)$ γ	γ^2	$f(x_L)$**	$q = (1-p)\times 10^2$	p^3	$\frac{1}{f}\sqrt{\frac{pq}{N}} = S_x$	$x_L + S_x$ = Log x_{upper}	$x_L - S_x$ = Log x_{lower}	Confidence Limits $P = 0.67$	
										x_{upper} psi	x_{lower} psi
20,000	4.3010	−1.3255	1.757	1.980×10^{-4}	0.0011	0.99999	0.247 (0.494)†	4.5480 (4.7950)	4.0540 (3.8070)	35,300 (62,400)	11,300 (6,400)
16,000	4.2041	−1.2286	1.5095	6.798×10^{-4}	0.005	0.99995	0.153 (0.306)	4.3571 (4.5101)	4.0511 (3.8981)	22,800 (32,400)	11,100 (7,900)
14,000	4.1461	−1.1706	1.3703	1.361×10^{-3}	0.009	0.99991	0.103 (0.206)	4.2491 (4.3521)	4.0431 (3.9401)	17,900 (22,500)	10,900 (8,700)
10,000	4.0000	−1.0245	1.050	6.718×10^{-3}	0.050	0.9995	0.0491 (0.0982)	4.0491 (4.0982)	3.9509 (3.9018)	11,300 (12,700)	8,900 (8,000)
8,000	3.9031	−0.9276	0.8604	1.729	0.150	0.9985	0.0335 (0.0670)	3.9366 (3.9701)	3.8696 (3.8361)	8,600 (9,300)	7,400 (6,900)
4,000	3.6021	−0.6266	0.3926	0.1781	2.3	0.977	0.0126 (0.0252)	3.6147 (3.6273)	3.5895 (3.5769)	4,100 (4,200)	3,900 (3,800)
1,000	3.0000	−0.0245	0.00060	1.256	47.0	0.53	0.0060 (0.0120)	3.0060 (3.0120)	2.940 (2.9880)	1,010 (1,040)	990 (970)

*The quantiles of x_L are distributed normally with the mean value x_L and standard deviation $\frac{1}{f(x_L)}\sqrt{\frac{pq}{N}}$ where p is the theoretical value of the cumulative distribution function corresponding to x_L and $f(x_L)$ is the value of the logarithmic distribution function corresponding to x_L. $N = 4{,}597$ observations.

**The distribution function of x_L (log of stress on port side of deck) has a mean value $\bar{x}_L = 2.9755$ and a standard deviation $\sigma_L = 0.3167$, that is $f = \frac{1}{\sigma_L\sqrt{2\pi}} \exp\left[-\frac{(\bar{x}_L - x_L)^2}{2\sigma_L^2}\right]$ or $\log_{10} f = 0.1004 - 2.1650\,(\bar{x}_L - x_L)^2$.

†The figures in parentheses are computed on the basis of the assumption that every fourth observation was an independent random observation, $N = \frac{1}{4}(4{,}597)$.

stresses. However, visual inspection indicates that here, too, there is good agreement between the theoretical and experimental distribution.

Finally, the hypothesis may be tested by inquiring into the relationship that exists between the underlying cumulative distribution of *all stress variations* and the cumulative "extreme value" distribution of the stress variations.

It has been shown in the literature[5] that, if a distribution is normal, the largest values in repeated large samples from this distribution have a cumulative distribution $F(x)$ of their own[6-8] which approaches, as the sample size becomes larger and larger, the form

$$F(x) = \exp\left[-e^{-(x-u_1)\alpha}\right] \qquad [2]$$

In the present case x is a possible value of the extreme of the logarithm of the stress and u and α are constants for a particular distribution. Special coordinate paper has been

devised (see Figure 6) on which such a distribution (c.d.f.) will plot as a straight line. Gumbel[7] has devised a method for estimating the parameters u and α of the distribution of extremes on the basis of measurements of extremes; see Appendix 1 for a short discussion of this method. The method is applied here to the extreme sea-induced stress variations measured in 2-min samples taken at hourly intervals during 19 trips, each averaging 121 hours underway.

The resulting extreme value distributions for the port and starboard strain gages are plotted in Figures 6a and 6b. If all values of x are different, $F(x)$ for each observed point equals $i/(n+1)$ where i is the order number of the observed value when they are arranged in order of increasing magnitude and n is the total number of observed extremes. For the treatment of equal values of x see Appendix 1. The plotted points represent the individual measured extremes whereas the straight line in Figures 6a and 6b represents the best fit to the measured data (utilizing the method of Gumbel[7]) under the assumption that the underlying distribution* is of the log-normal type.

On this assumption, $F(x)$ represents the fraction of all trips which, on the average, are expected to give an extreme value of the logarithm of the stress less than x. On the bottom scale values of $1/(1-F)$, denoted by T, are shown; T is the number of trips the ship must make, on the average, in order to exceed the limit during at least one of the 2-min sampling periods. The dashed confidence limits drawn in Figure 6 denote the limits within which 67 percent of the experimental data would be expected to fall if measurements of extremes were conducted indefinitely and if the assumed type of distribution were valid. Inasmuch as the scatter lies well within these limits, it is concluded that, for the service conditions encountered, the extreme value distribution (from hourly 2-min samples) of the sea-induced hull-girder stresses is approximately of the form given by Equation [2] and that the c.d.f. of both the underlying and extreme value distributions are compatible.

It is concluded, on the basis of these several tests of significance, that the basic hypothesis to be tested is "probably" true, that is, the logarithm of the sea-induced stresses are, at least approximately, normally distributed.

The more important information, and some of the conclusions based thereon, obtained for the sea-induced stress variations and considered applicable for the conditions under which the tests were conducted, are as follows:

1. The underlying or basic distribution is very nearly a logarithmically normal distribution.

2. The extreme value distribution is approximately of the type given by Equation [2], at least over the range tested.

3. Only a small difference was evident between the basic distribution patterns of the stress variations measured by the port and starboard gages.

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*Figures 6c and 6d represent estimated extreme value distributions arrived at by direct application of the Gumbel method to the measured extreme values of the stress variations under the assumption that the basic distribution is unknown; see Appendix 1 for a discussion of this assumption.

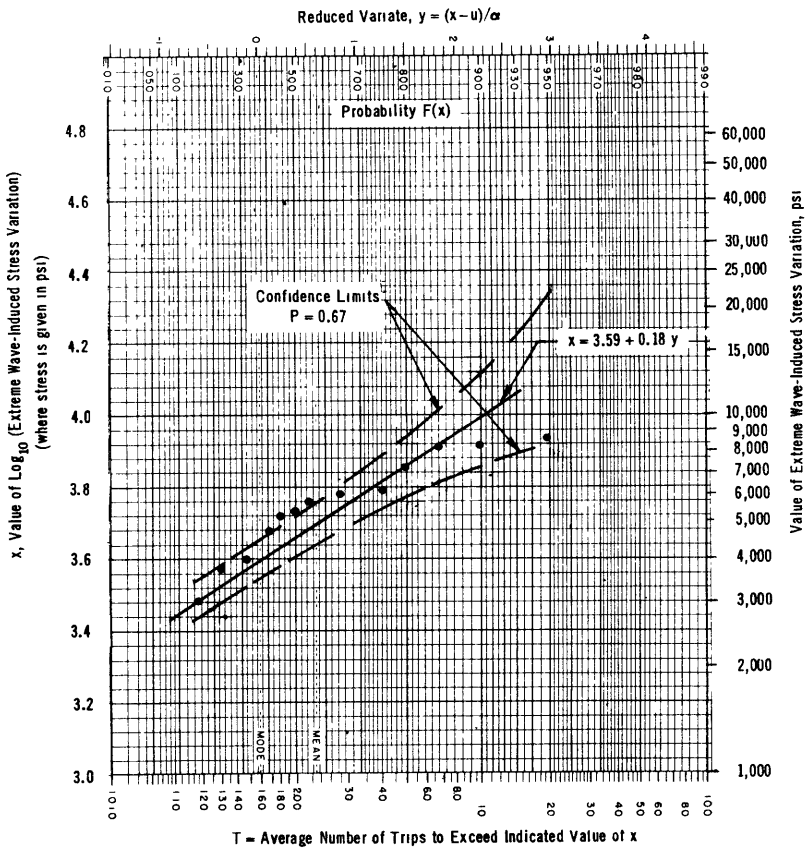


Figure 6a - Starboard Side, Amidships, Main Deck
(Based on 19 trips averaging 121 hours at sea each)

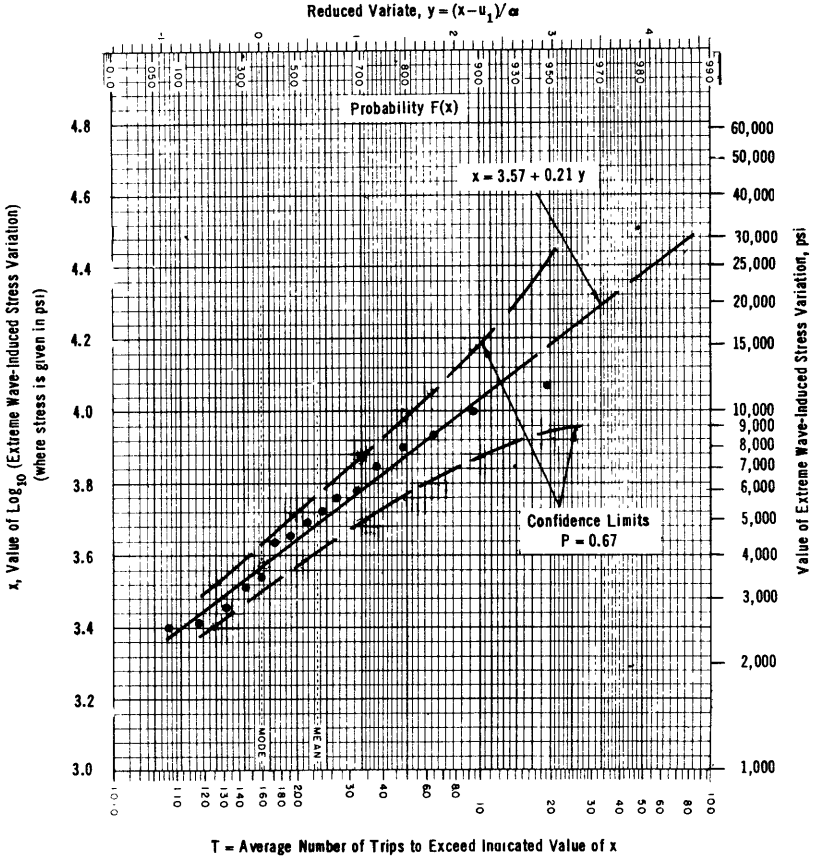


Figure 6b - Port Side, Amidships, Main Deck
(Based on 18 trips averaging 121 hours at sea each)

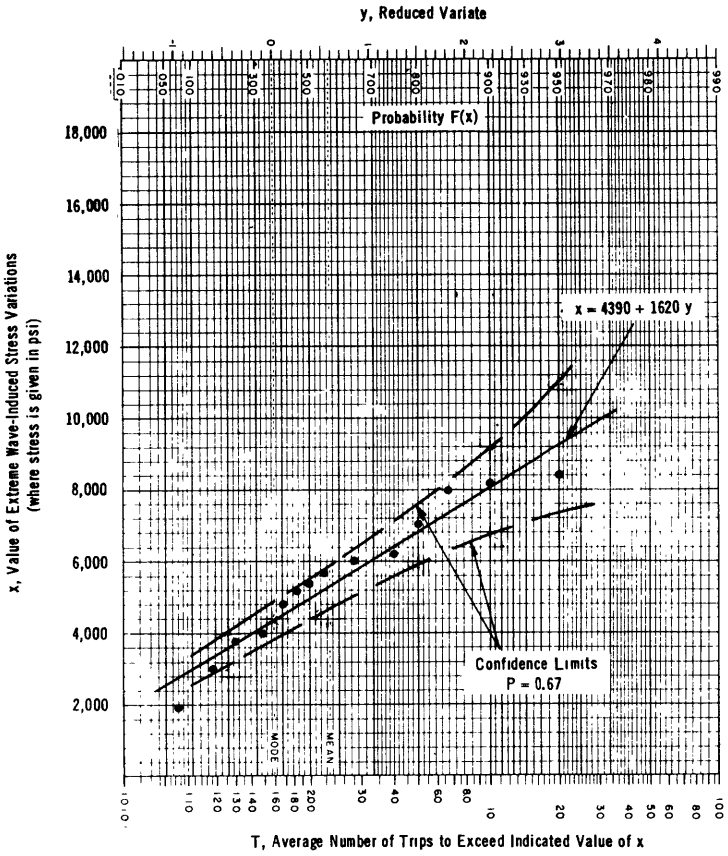


Figure 6c - Starboard Side, Amidships, Main Deck

The same observed values of the extreme stress variations are used here as for Figure 6a.

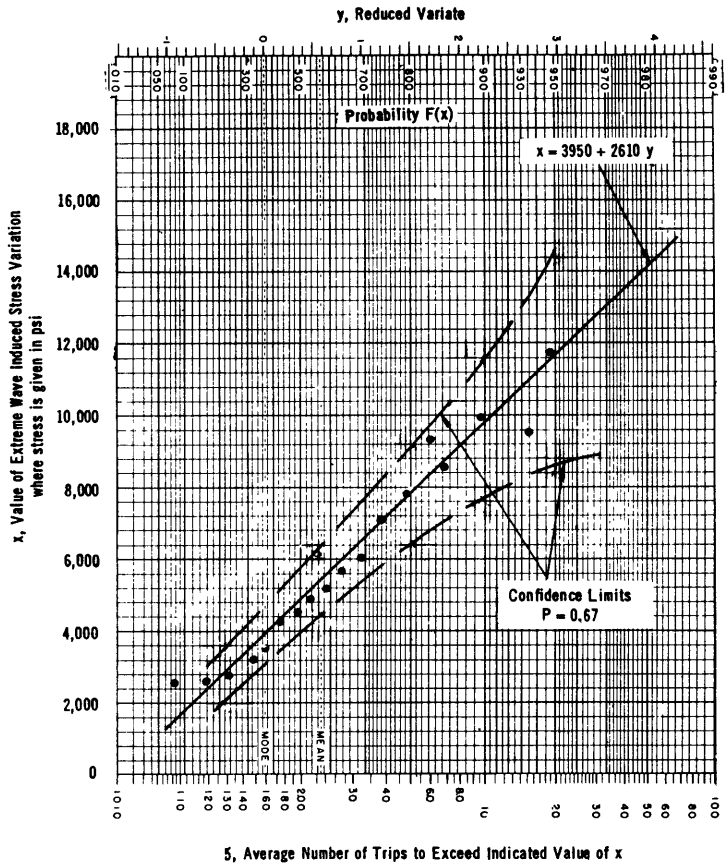


Figure 6d - Port Side, Amidships, Main Deck

The same observed values of the extreme stress variations are used here as for Figure 6b.

Figure 6 - Distribution of Probable Extreme Stress Variation

See Appendix 1 for an explanation of this distribution and a definition of terms.

TABLE 5

Basic Ship Motion Data for the Period 1 September 1953 to 3 April 1954

Quantity Measured	Actual or Expanded Counts	Load or Ballast	Days at Sea Subject to Study	Days at Sea during Which Suitable Data Were Measured	Percent of Time at Sea Subject to Sampling	Minutes of Recording Time	Number of Cycles Falling between Stated Limits* (Peak-to-Peak) for a Period of 30 Days at Sea											
							For Sample of 2 min every Hour					For Sample of 2 min every Fourth Hour						
							Expansion** Factor	H** 0.1 - 0.2 / P** 0.05 - 0.10	H 0.2 - 0.4 / P 0.10 - 0.25	H 0.4 - 0.6 / P 0.25 - 0.50	H 0.6 - 0.8 / P 0.50 - 1.00	Expansion** Factor	H 0.05 - 0.10 / P 0.05 - 0.10	H 0.10 - 0.15 / P 0.10 - 0.20	H 0.15 - 0.20 / P 0.20 - 0.30	H 0.20 - 0.25 / P 0.30 - 0.40	H 0.25 - 0.30 / P 0.40 - 0.50	H 0.30 - 0.35 / P 0.50 - 1.00
Heave Accel. (Amidships)	A	L	75	59	79	2,832		339	14	0	0							
	E	L					15.25	5,170	214	0	0							
	A	L	68	58.2	86	3,030							1,031	130	14	3	0	0
	E	L										57.03	58,798	7,414	798	171	0	0
Heave Accel. (Amidships)	A	B	71	51	72	2,448		1,080	58	0	0							
	E	B					17.65	19,062	1,024	0	0							
	A	B	61.9	51.7	84	2,792						-	1,341	291	55	12	1	0
	E	B										61.89	82,994	18,010	3,404	743	62	0
Pitch Accel.	A	L	75	74	99	3,552		1,819	809	41	0							
	E	L					12.16	22,119	9,837	499	0							
	A	L	68	68	100	3,518						-	1,208	333	33	4	1/4	0
	E	L										49.12	59,337	16,570	1,621	200	13	0
Pitch Accel.	A	B	69	53	77	2,544	-	1,410	1,099	56	0							
	E	B					16.98	23,941	18,660	951	0							
	A	B	61.9	51.9	84	3,196						-	949	565	56	4	1	0
	E	B										54.08	60,337	30,550	3,028	216	54	0

*H denotes Heave Acceleration, its intensity is specified in terms of units of gravity (G's).
P denotes Pitch Acceleration, its intensity is specified in terms of an equivalent linear acceleration in units of gravity. This value is equal to $\left(\frac{\text{angular acceleration}}{\text{in rad/sec}^2}\right) \times (100 \text{ ft}) \div 32.2 \frac{\text{ft}}{\text{sec}} = \frac{\text{gravity units}}{100 \text{ ft length}}$.

**The expansion factor has been defined in Table 2.

TABLE 6

Distribution of Heave and Pitch Accelerations for a 30-Day Period at Sea

The average estimated period of the measured motions is 9.2 sec (from stress oscillograms).

Pitch		Heave	Estimated Number of Cycles of Magnitude† Greater than the Indicated Value (For a period of 30 days at sea during the rough weather season)									
			Heave Acceleration					Pitch Acceleration				
Radians Sec^2	G's 100 ft	Units of Gravity	Average L & B	Loaded		Ballast		Average L & B	Loaded		Ballast	
			**	*	**	*	**	**	*	**	*	**
0.016	0.05	0.05	86,328		67,224		105,432	85,964	32,455	77,741	43,552	94,185
0.032	0.10	0.10	15,432	5,384	8,426	20,086	22,438	26,126	10,336	18,404	19,611	33,848
0.064	0.20	0.15	2,720		1,012		4,428	2,566		1,834		3,298
0.081	0.25	0.20	619	214	214	1,024	1,024		499		951	
0.097	0.30	0.25	31		0		62	242		213		270
0.129	0.40	0.30	0		0		0	34		13		54
0.161	0.50	0.35	0		0		0	0	0	0	0	0
0.193	0.60	0.40	0	0	0	0	0					
0.322	1.00	0.60	0	0	0	0	0					

*This estimate is based on oscillograms taken for a duration of 2 min at intervals of 1 hr.
**This estimate is based on oscillograms taken for a duration of 2 min at intervals of 4 hr (used for plotting).
†Peak-to-peak variation.

4. There was a significant difference between the extreme value distributions of the port and starboard stress variations, the port extreme stresses being more severe.

5. The basic distribution patterns are useful as a concise specification of the service stresses the hull girder structure experiences. However, the scatter of the measured values from the theoretical distribution may be expected to be much greater at the higher stresses, which are experienced but rarely, than for the more frequently experienced stress variations. An estimate of the most probable extreme stress expected during a given service period, for example, over the service life of the ship, can best be obtained from an extreme value distribution. The extreme value distribution may be used to specify the expected upper limit of the service stresses which are given by the basic distribution.

6. Wave-induced stress variations of magnitude greater than 12,000 psi (peak-to-peak) occurred infrequently.

DISTRIBUTION PATTERNS OF SHIP MOTIONS

Table 5 gives the variations of pitching and heaving accelerations, classified according to magnitude and frequency of occurrence, for both the load and ballast conditions of the ship. The tabulation in Table 6 is obtained by summation of the data in Table 5. Table 6 indicates the number of variations that would be expected during a 30-day period at sea under the conditions encountered by the ESSO ASHEVILLE from 1 September 1953 to 3 April 1954. The same method that was applied to determine the basic distribution of the stress data was also applied to the ship-motion data. The results are shown in Figures 4 and 5. The classification used for the motion data (acceleration) was coarser than that used for the stress data, thus the fit of the experimental data would not be expected to be good. Kolmogorov's statistic was the only measure of significance that was applied in this case. It is apparent from an inspection of Figures 4 and 5 that the log-normal distribution (c.d.f.) fits the data fairly well.

The extreme value distribution was not determined for ship motions; however, since the basic distribution is of the log-normal type, the extreme value distribution for the logarithm of the heaving and pitching acceleration should follow the same type of distribution as was shown to be applicable in the case of sea-induced stress variations; see Figure 6.

INDIVIDUAL MAXIMUM VALUES

The absolute maximum values of sea-induced stresses and motions measured during the entire period 24 August 1952 to 3 April 1954 are given in Table 7. Data from the ship's log and Weather Bureau are given in Table 8 for the last two conditions. The calculated dead-load still-water bending moments corresponding to the ship's loading on 24 September 1953 are given in Figure 7. The oscillogram of 31 December 1952 indicated that the starboard stress variations were consistently greater than those on the port side; the reverse was true for the oscillograms taken on 24 September 1953; see Figure 8. The Weather Bureau and the

TABLE 7

Maximum Values of Sea-Induced Stresses and Motions

Quantity	Peak-to-Peak Variations	Loaded or Ballast Condition	Date Measured	Ports
Heave Acceleration	0.34 g	B	0700 30 Mar 1953	New Haven-Baytown
Pitch Acceleration	0.142 rad/sec^2 0.44 g/100 ft	B	2000 24 Feb 1954	Morehead City-Baytown
Stress, Main Dk ⌧ Starboard	9,400 psi	B	0800 31 Dec 1952	Providence-Baton Rouge
Stress, Main Dk ⌧ Port	11,700 psi	L	1900 24 Sep 1953	Baton Rouge-Tiverton

ship's log data indicate that on 31 December 1952 the wind and sea came from the port side whereas on 24 September 1953 the wind, sea, and swell came from the starboard direction. These observations appeared to be part of a general pattern which always held true when definite data were available as to the heading of the ship relative to the sea; that is, observation indicated that the deck stresses were greater on the side of the vessel which was opposite to the side from which the waves came; or stated otherwise, the stresses are greater on the leeward side. A reasonable explanation of this behavior is obtained by consideration of the combined action of longitudinal and transverse bending moments induced by the sea, together with the change of distance of the relative position of the neutral axis incident to the rolling motion of the ship.

Typical severe conditions encountered throughout the test period were as follows:

1. 0600 on 23 December to 2400 on 24 December 1952 - loaded condition, enroute from Baytown to Providence. Pitching motion was predominant. The following maxima were recorded at 1500 on 23 December:

Heaving acceleration	0.27 g
Pitching acceleration	0.10 rad/sec^2
Stress, starboard gage	8000 psi
Stress, port gage	7200 psi

All values are peak-to-peak variations. Note that the stress variations were approximately equal on the port and starboard sides.

2. 0100 on 24 September to 0900 on 25 September 1953 - loaded condition, enroute from Baton Rouge to Tiverton; see Figure 8. During this period the most severe stress variations encountered throughout the tests were measured. Pitching motion was predominant. The period of stress variation was about 6 sec. At 0945 on 24 September the conditions were

Text continued on page 25.

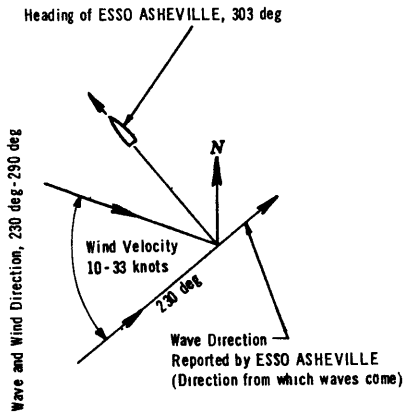
TABLE 8

Log and Weather Bureau Data for Two Dates on Which the ESSO ASHEVILLE Experienced Large Wave-Induced Stresses

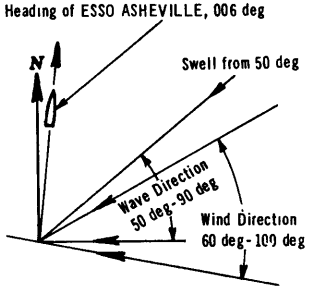
The Ship's log and the Weather Bureau data are taken from C.R. Nevitt's memorandum of January 1954 to members of the SNAME research panel S-10.

The corresponding oscillograms are shown in Figure 8.

	Ship →	ESSO ASHEVILLE	R.C. TUTTLE	BRINTON LYKES	VINLAND	REDSTONE	SABINE SUN	KEY TANKER	TILLAMOOK	CROWN TRADER	ESSO ASHEVILLE
From ESSO ASHEVILLE Log	Load Condition	Ballast									Load
	Enroute from	Providence									Baton Rouge
	Enroute to	Baton Rouge									Tiverton
	Record Date	31 Dec 52	31 Dec 52	31 Dec 52	31 Dec 52	31 Dec 52	24 Sep 53	24 Sep 53	24 Sep 53	24 Sep 53	24 Sep 53
	Time on Record	0800									0945 1900
	Ship's Position	87°46'W-28°24'N									79.7°W-27.7°N ?
	Ship's Heading	303°									006° ?
	Wind Direction	270°									100° ?
	Wind Force-Beaufort	6									4-5 ?
	Sea	Rough									Beam Sea-Heavy Swell
	Sea Direction	-									Swell from 50°
	Weather	Scattered clouds									Scattered Clouds
	Shaft rpm	80 (Reduced)									88.7 ?
	Ship's Speed-knots	13.7									15.4 ?
	Draft Fwd-feet	10.0									29.63 29.63
	Draft Aft-feet	24.0									30.46 30.46
	Cargo or Ballast Distribution	Tanks 3, 6, 8 Port Center & Stbd Full									53 tons Tank 5 C.L. 22 tons Tank 6 Wings All other tanks 98% full
From Test Oscillograms	Measured Max. Stress Variation, Port	3,200 psi									10,800 psi 11,700 psi
	Measured Max. Stress Variation, Stbd	9,400 psi									7,100 psi 8,400 psi
From Photostat of Log of Ship's Weather Observations Furnished by Weather Bureau	Greenwich Time	1200	1200	1200	1200	1200	1600	1800	1200	1800	1800
	Ship's Position	88.1°W-28.4°N	89.5°W-26.3°N	90.4°W-28.0°N	87.5°W-26.3°N	86.6°W-27.5°N	80.4°W-29.5°N	79.4°W-26.5°N	79.8°W-27.2°N	79.6°W-28.8°N	79.7°W-28.7°N
	Wave Direction	230°	290°	270°	270°	290°	90°	90°	60°	90°	50°
	Wave Period - sec	7-9	7 - 9	13 - 15	7 - 9	7-9	11 - 13	5 - 7	5 - 7	9 - 11	?
	Wave Height - feet	8	8	11	9 1/2	14	12.6	5	6 1/2	8	8
	Wind, Direction and Speed	230°, 33 knots	290°, 30 knots	290°, 30 knots	270°, 10 knots	290°, 30 knots	90°, 20 knots	100°, 15 knots	60°, 27 knots	90°, 15.knots	80°, 21 knots
	Temperature - Air, Water	67°, 70°	68°, 72°	63°, 73°	72°, 76°	66°, 72°	86°, 82°	83°, 84°	82°, 87°	84°, 84°	84°, 85°
	Weather	Clouds, clearing	Clouds forming	Clouds forming	Clouds, clearing	Clouds forming	Clouds forming	Clouds forming	Squalls	Clouds, clearing	Clouds, clearing
Nominal Calculated Stress (S)		24,500 psi									



Heading of Ship, Sea, and Wind on 31 December 1952



Heading of Ship, Sea, and Wind on 24 September 1953

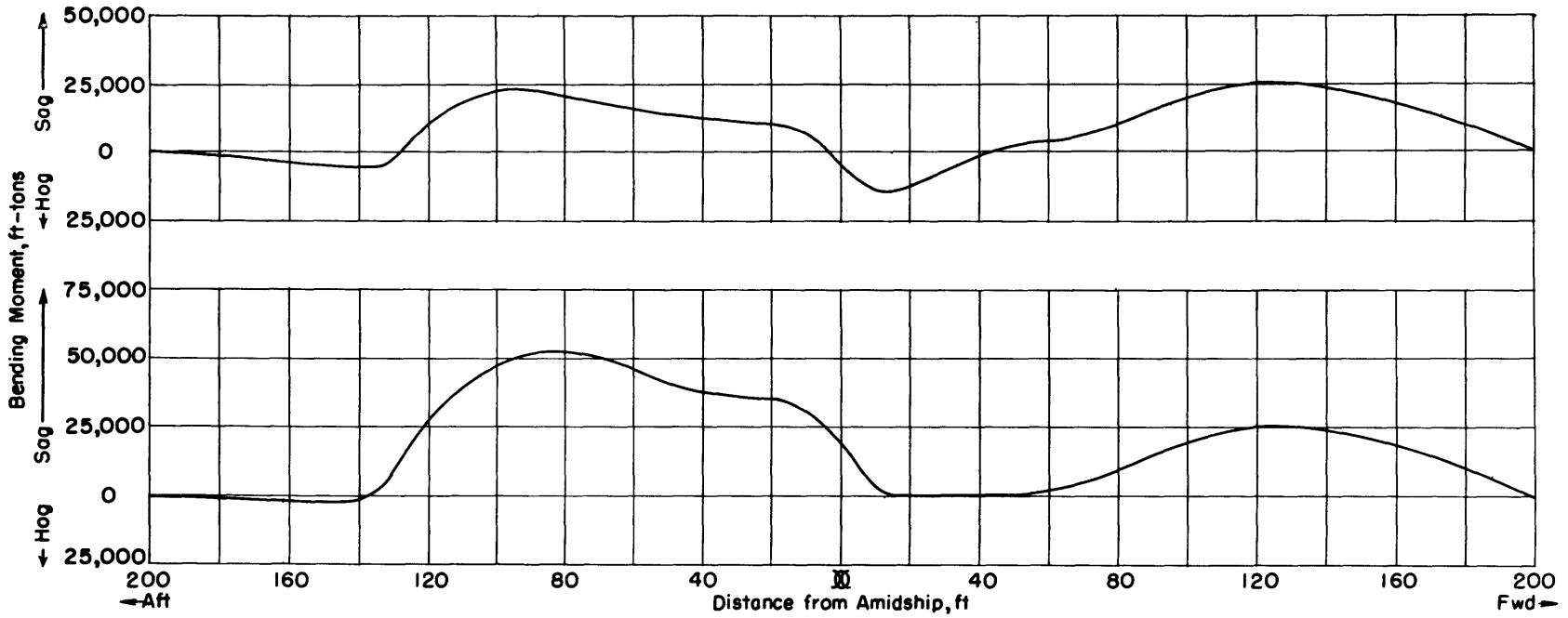


Figure 7 - Still-Water Bending-Moment Curves Corresponding to Loading That Existed during Most Severe Measured Sea-Induced Stresses

This curve was furnished by the Shipbuilding Division, Bethlehem Steel Company

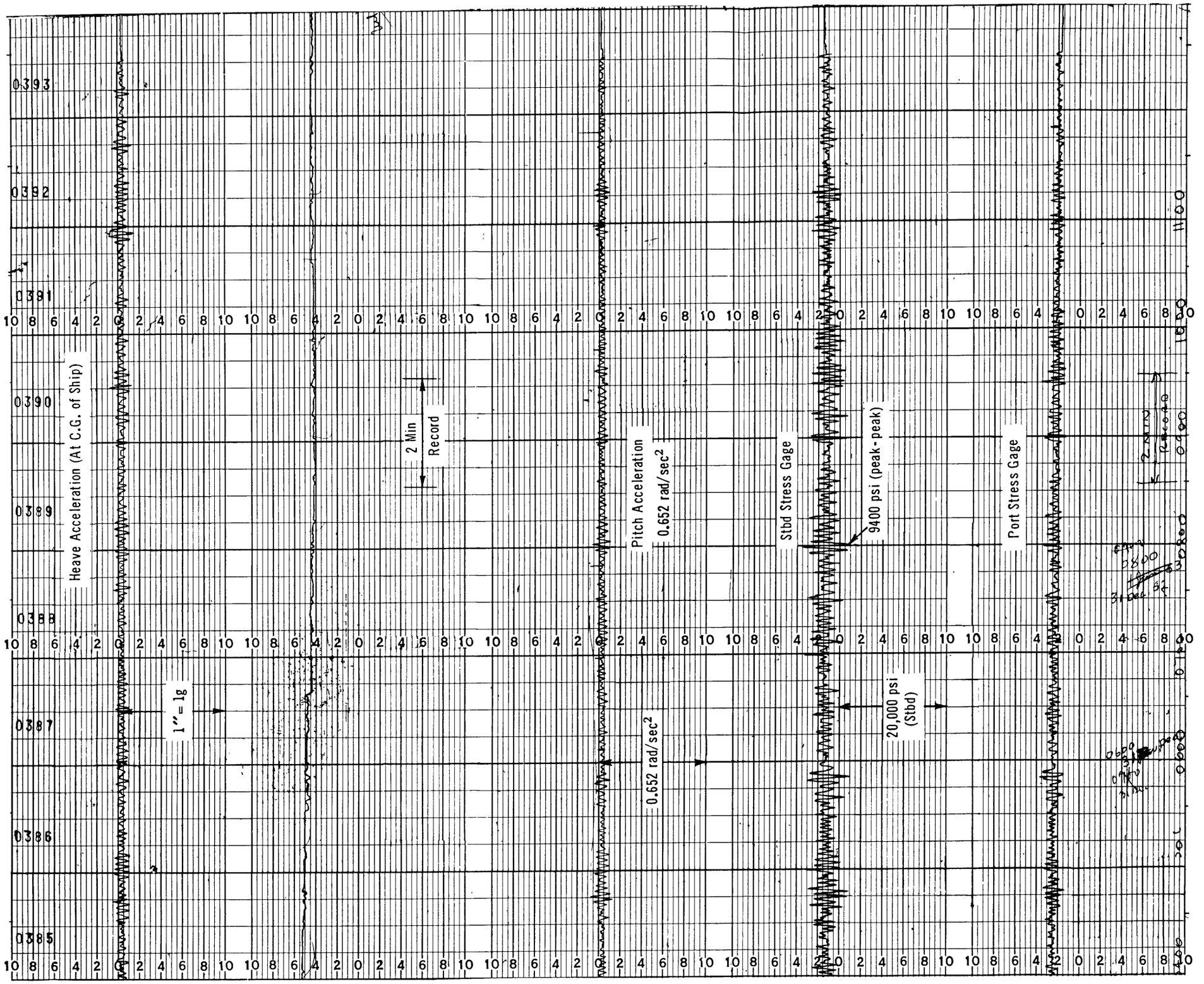


Figure 8a - SS ESSO ASHEVILLE enroute Providence to Baton Rouge 31 Dec 1952 (In ballast condition)

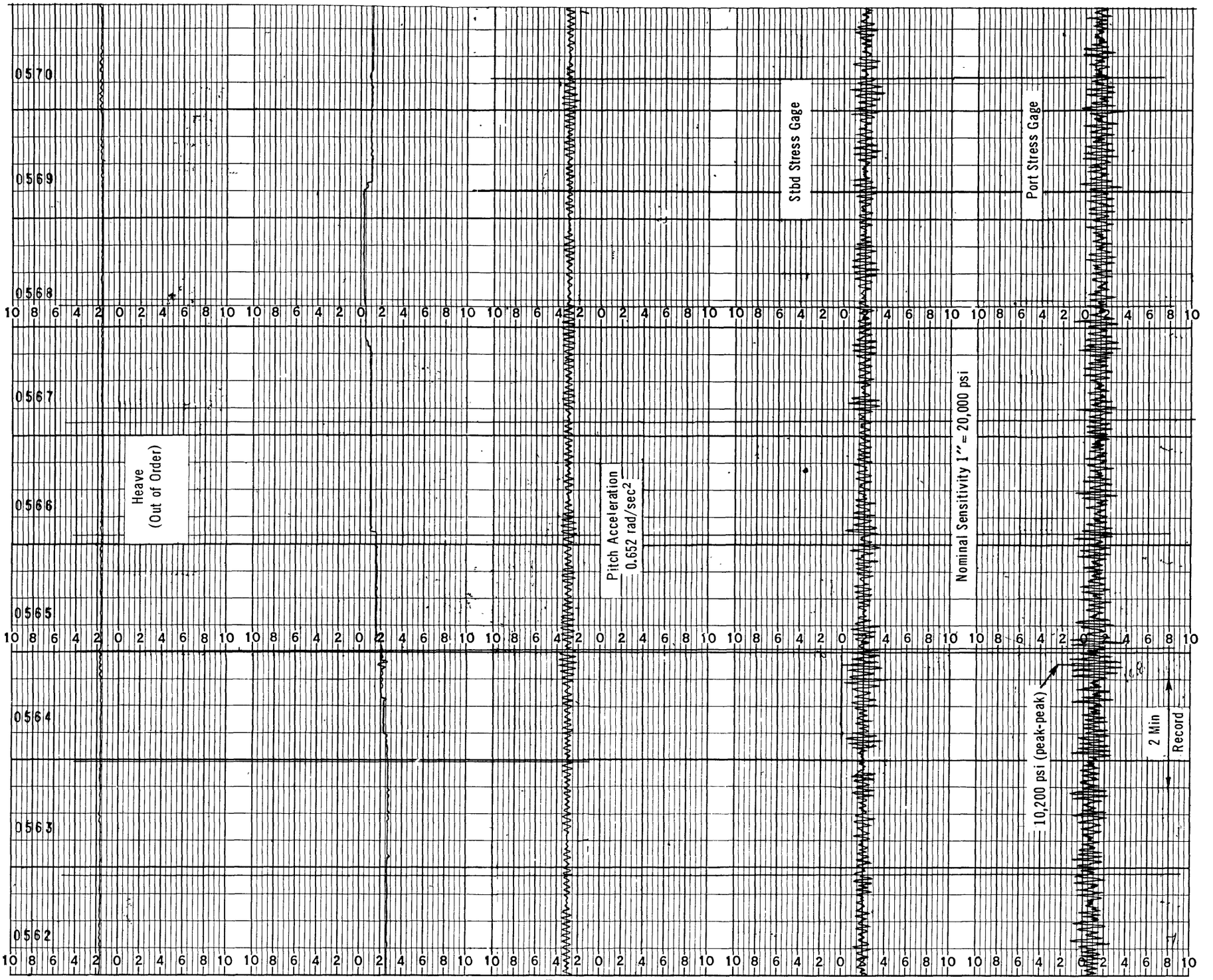


Figure 8b - SS ESSO ASHEVILLE enroute Providence to Baton Rouge 24 Sep 1953 (In loaded condition)
Course 6 degrees Wind 100 degrees Force 4-5 Heavy Beam Sea Heavy Swell from 50 degrees

Figure 8 - Sample Oscillograms Showing Large Wave-Induced Stress Variation

course 6 deg, wind direction 100 deg, wind force 5, heavy beam sea, heavy swell from 50 deg. The following maximum peak-to-peak variations were recorded simultaneously at about 1900 on 24 September:

Heaving acceleration	Instrument out of order
Pitching acceleration	0.42 g/100-ft length or 0.135 rad/sec^2
Stress, starboard gage	8,400 psi
Stress, port gage	11,700 psi

Presumably the conditions at 1900 were similar to those at 0945.

STRESS VARIATIONS INCIDENT TO CARGO LOADING

Table 9 is a tabulation of changes in the stress in the main deck, amidships, measured incident to changes in cargo loading. It should be remembered, however, that a portion of these stress changes may well be due to changes in the distribution of temperature throughout the hull girder. The temperature-induced stress variations have been discussed separately in Reference 9 but will also be commented on in the following section.

TABLE 9

Stress Variations Recorded during Changes in Cargo Loading

Date				Operation		Stress Variation,* psi	
Time	Day	Month	Year	Loading, Unloading	Ports	Starboard	Port
0130	30	Sep	1952	During Loading	Aruba	− 8,300	− 7,800
1700	30	Sep	1952				
				Loading Completed	Aruba	+11,700	+10,200
0830	1	Oct	1952	Aground - Unloaded	Aruba	−14,000	−10,200
1200	7	Oct	1952		Buenaventura		
0530	23	Apr	1953	During Discharge of Cargo	Pt. Everglades	−11,200	−16,400
0430	24	Apr	1953				
	4	Oct	1953	Loading	Baton Rouge	− 9,300	−12,200
	10-11	Oct	1953	Unloading	Boston	+11,900	+14,800
0600	12	Nov	1953	Unloading Ballast	Corpus Christi	+ 9,200	+ 8,100
2000	12	Nov	1953	Empty	Corpus Christi		
1300	13	Nov	1953	Loading	Corpus Christi	−10,600	− 9,600
2000	8	Jan	1954	Unloading	Norfolk	+10,800	+ 8,700
2200	9	Jan	1954				
0900	21	Feb	1954	Unloading	Morehead City	− 4,000	− 3,800
1400	21	Feb	1954				
0100	22	Feb	1954	Unloading	Morehead City	+ 9,600	+10,600
0900	22	Feb	1954			− 6,800	− 8,000

*Negative sign indicates increasing compression, positive sign indicates increasing tension. There is a remote possibility that the signs of stress variations measured up to July 1953 should be reversed.

STRESS VARIATIONS INDUCED BY CHANGES IN THE TEMPERATURE DISTRIBUTION THROUGHOUT THE HULL GIRDER

Inspection of the oscillograms which show a signal output proportional to fore-and-aft stress in the main deck amidships has, on many occasions, indicated a definite pattern of stress variation with a periodicity of about one day. A stress variation in the direction of increasing compression occurs when going from night to day (during normal atmospheric conditions); the maximum compressive (sag) stress is reached in the afternoon. The stress then changes to the tensile (hog) direction, reaching a maximum value sometime early in the morning. This phenomenon is best observed with the ship underway, or at other times when load changes are nil. The stress variation is generally not the same on the port and the starboard side of the deck; thus it is implied that the temperature changes are not symmetrical with respect to the fore-and-aft centerline plane of the ship. These slowly varying stresses are quite appreciable; the maximum measured variation was about 10,900 psi measured 18 April 1953; see Figure 9.

The following simplified explanation is advanced for the foregoing phenomenon; for a more detailed explanation and a method for computing temperature-induced stresses, see TMB Report 937.[9] The stress variations are believed to be caused by nonuniform changes in temperature in the ship girder. At night the temperature of the deck is relatively cool, probably approximately the same as that of the air. As the sun rises above the horizon, the portion of the ship above the waterline is heated and that portion of the ship below the waterline remains substantially at constant temperature (probably nearly equal to that of the water). The heated part of the upper portion of the hull wants to expand, but the submerged cooler portion resists the expansion, thus inducing compressive stresses in the deck. The reverse conditions occur as the deck cools during the night. Inasmuch as the sun does not shine on all parts of the exposed portions of the ship with equal intensity, combined horizontal and vertical bending and possibly torsion may be expected.

Very little quantitative information is available[10, 11] which would indicate the actual stress variations and temperature variations throughout the ship that might be expected in service. The ESSO ASHEVILLE tests are, to the author's knowledge, the first occasion where service stresses induced by temperature changes have been observed over a fairly long period of time (about 1½ yr).

In Table 10 are tabulated values of diurnal stress variations incident to variations in temperature gradient measured while the ship was enroute between ports except for one set of records taken in port at Jacksonville during an overhaul. A positive sign indicates increasing tension, a negative sign denotes decreasing tension.

It may be inferred, from the data given in Table 10, that the temperature-induced stress variations are of the same order of magnitude as those due to changes in the cargo load.

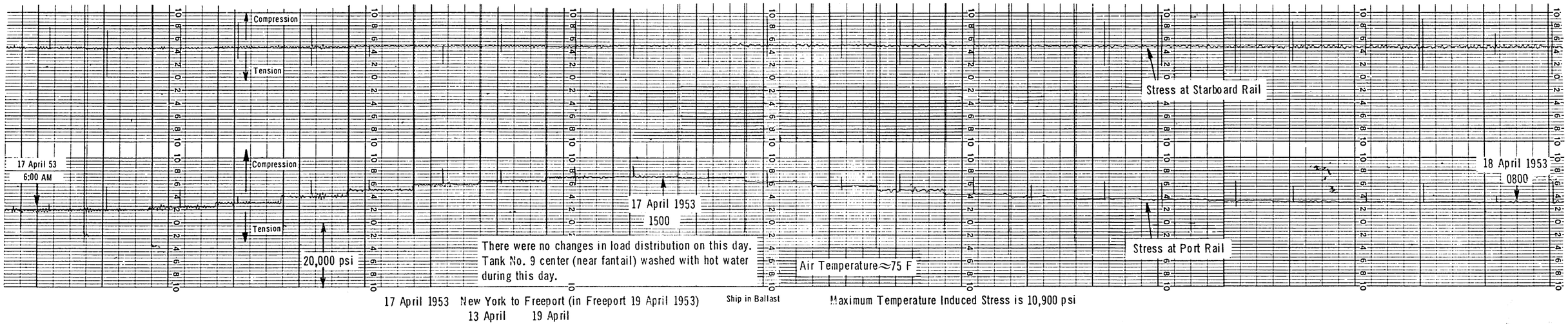


Figure 9 - Sample Oscillogram Showing Temperature-Induced Stress Variation on 17 April 1953

TABLE 10

Diurnal Stress Variations

Date	Time	Enroute between Ports	Diurnal Stress Variation	
			Starboard	Port
2 Oct 1952	0130	Aruba to Buenaventura (loaded)		
2 Oct 1952	1330	Aruba to Buenaventura (loaded)	−1000	−9600
3 Oct 1952	0330	Aruba to Buenaventura (loaded)	0	+9600
3 Oct 1952	1430	Aruba to Buenaventura (loaded)	0	−7400
28 Dec 1952	0200	Providence to Baton Rouge (ballast)		
28 Dec 1952	1000	Providence to Baton Rouge (ballast)	−3500	−3000
29 Dec 1952	0200	Providence to Baton Rouge (ballast)	+3200	+1700
17 Apr 1953	0330	Baytown to New Haven (loaded)		
17 Apr 1953	1530	Baytown to New Haven (loaded)	−1000	−10,900
18 Apr 1953	0430	Baytown to New Haven (loaded)	+1000	+8600
2 Aug 1953	0600	Jacksonville,* during overhaul		
2 Aug 1953	Afternoon	Jacksonville, during overhaul	−3000	−6500
3 Aug 1953	0600	Jacksonville, during overhaul	+3000	+7400
3 Aug 1953	Afternoon	Jacksonville, during overhaul	−2000	−7400
4 Aug 1953	0600	Jacksonville, during overhaul	+2000	+7400
4 Aug 1953	Afternoon	Jacksonville, during overhaul	−2000	−7000
5 Aug 1953	0600	Jacksonville, during overhaul	-	+7800
16 Aug 1953	0620	Baytown to Boston (loaded)		
16 Aug 1953	1300	Baytown to Boston (loaded)	−1500	−5000
17 Aug 1953	0620	Baytown to Boston (loaded)	+5500	+5500
2 Oct 1953	0500	Tiverton to Baton Rouge (ballast)		
2 Oct 1953	1400-1700	Tiverton to Baton Rouge (ballast)	−3500	−6100
3 Oct 1953	0500	Tiverton to Baton Rouge (ballast)	+3500	+7000

*It is interesting to note that the diurnal stress variations very nearly repeated themselves during the period 2-4 August 1953 while the vessel was in a shipyard at Jacksonville, Florida for an overhaul. Presumably the vessel's position alongside the dock was not changed during this 3-day period.

DISCUSSION

An attempt to apply the methods of statistics to the collection, presentation, and analysis of the wave-induced stress variations and motions measured on the ESSO ASHEVILLE has been described. The measures of statistical significance (confidence limits) that have been applied indicate that, for this ship and service, the wave-induced stresses and motions may be represented by a logarithmically normal distribution. The type of data which was used to determine the basic distributions did not consist of random independent samples. Therefore the measures of significance, which assume that the samples are random and independent, are not strictly applicable. If confidence limits are derived under the assumption that the measured data are random and independent, then these limits will be narrower or more stringent than they should be; thus the fit of the assumed log-normal distribution is better than would be indicated by the confidence limits, which were based on the assumption of random independent measurements.

In the case of the extreme-value distributions, plotted in Figure 6, the experimental sampled extreme stresses were independent and the confidence limits as plotted should be valid. It is important to note that the extreme value distribution, Figure 6, and the basic distribution, Figure 2, are derived by two independent methods and that the patterns evidenced by these two distributions complement each other; that is, if the basic distribution is log-normal (a straight line in Figure 2), then the extreme value distribution for the logarithm must be of the type $F(y) = \exp[\epsilon^{-y}]$, which is represented by a straight line in Figures 6a and 6b.

The existence of a predictable distribution pattern would greatly simplify the collection and analysis of service data, as well as furnish a concise and graphic manner of specification of service conditions for given ship types in a given service. The limited amount of data given here do not, by themselves, establish that such a pattern exists for other ships and other service conditions. The data do establish, with a good degree of reliability or confidence, the distribution pattern for the ESSO ASHEVILLE type of tanker, for the conditions actually encountered. Examination of References 12 and 13 indicates that this pattern may be applicable to other ships also. Predictions of the largest stress and motion to be expected over a long period of time are basically less reliable than a prediction of those values which occur more frequently. However, the reliability of the estimated value also is predictable, and furthermore, such predictions for, say, two different designs, are comparable to the extent that it can be stated that one design gives a higher stress or motion than the other design. In addition, the relative magnitudes to be expected can be specified.

The parameters computed for the log-normal distribution pattern fitted to the experimental data are not appreciably affected by variations or scatter of those values or events which occur relatively infrequently. For this reason, *provided the type of basic distribution pattern has been established*, it is possible to obtain a reasonably good approximation to the distribution pattern with a limited amount of experimental data, because it is not necessary to continue measurements until the more extreme and infrequently occurring members of the population

have been sampled. Inasmuch as ship motions and hull-girder stresses are of rather small magnitudes during the greater portion of time, it is necessary to measure the smaller magnitudes in order to have a sufficiently representative sample for the statistical analysis. Too large a degree of truncation of the data would make it difficult to establish the distribution patterns with acceptable accuracy.

In naval architecture very large values (extremes) of stress and motion, although they occur very seldom, may be of considerable importance. The reliability of predicting the magnitude of these extreme, but rare, values is of necessity very poor; nevertheless it would be desirable to have experimental data in this range if such predictions are to be made. In any case, if statistical predictions are to be made, it is necessary that measurements of the lower magnitudes, which occur relatively more often than the larger values, be available. However, if rare extremes are not of particular interest, the reliability of theoretical predictions based on experimental data which do not include these rare extremes will not be affected appreciably by lack of observations of these extreme values.

SUMMARY AND CONCLUSIONS

The following statements apply to the ESSO ASHEVILLE for the test conditions considered.

1. The dynamic stress variations in the hull girder, free of stress concentration, incident to the motion of the ship in a seaway will rarely exceed 12,000 psi peak-to-peak. The greatest dynamic stresses measured during the period 24 August 1952 to 3 April 1954 were 11,700 psi at the port edge, main deck, amidships and 8,400 psi at the starboard edge, main deck, amidships. The largest number of stress variations were of a magnitude less than 2000 psi peak-to-peak. The periods of the variations in stress, heave, and pitch are most often nearly the same. The predominant periods of the wave-induced stress variations were in the ranges 5 to 7 sec and 13 to 19 sec.

2. It was noted that, over-all, more severe wave-induced stress variations were measured on the port side than on the starboard side amidships. However, there were long periods of time, of the order of hours, when the reverse was true; and there were also periods during which these stress variations were nearly of the same magnitude on both sides of the main deck. The difference between these stresses are believed to be due to transverse bending and to the motion of the ship incident to an oblique course of the vessel relative to the wave crests. Observations, as well as a rough analysis, indicate that the larger stresses occur on the leeward side, although this should not be interpreted to mean that this is always so inasmuch as occasional exceptions are probable.

3. The largest hull motions measured during the period 24 August 1952 to 3 April 1954 were:

Heaving accelerations, amidships near the center of gravity of the ship, 0.34 gravity units.

Pitching acceleration, 0.142 rad/sec^2 or 0.44 gravity units per 100-ft length.

The pitching acceleration makes a major contribution to the linear acceleration experienced at points far from the center of gravity of the ship. More severe pitching and heaving motions were experienced in the ballasted condition than in the loaded condition.

4. The maximum stress variation measured amidships on the main deck during the loading and unloading of cargo or ballast was 16,400 psi. This value includes stress variations due to temperature changes because the latter could not be separated from those due to changes in cargo loading.

5. Diurnal stress variations of appreciable magnitude were induced in the hull girder, very probably as a consequence of changes in the temperature distribution throughout the ship. These temperature-induced stresses attain values of the same order of magnitude as those due to changes in the cargo load. The largest measured diurnal stress variation (presumably due to temperature changes) was 10,900 psi.

6. The wave-induced hull-girder stress variations and the ship motions experienced by the ESSO ASHEVILLE for the test conditions considered may be approximated by a logarithmically normal distribution, that is, the logarithms of the stress and of the motion appear to be normally distributed; see Figures 2, 4, and 5.

7. The extreme value distribution of the hull-girder stresses measured on the ESSO ASHEVILLE is of the type illustrated by Figure 6.

8. The service conditions under which this tanker operated, as specified by wave-induced hull bending moments (stresses) and motions, may be concisely represented by simple graphs such as Figures 2 and 6.

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Mr. Wigle performed much of the detail analysis of the recorded data. Mrs. A.W. Mathewson checked the mathematical work. The advice and opinions of statisticians from various government and private activities have been freely solicited and used. In particular, the advice of Mr. I.R. Savage and Dr. J. Lieblein of the National Bureau of Standards and of Dr. Suzuki of the Taylor Model Basin was most helpful. The still-water bending moment curves, Figure 8, were obtained through the courtesy of Mr. J.H. McDonald of the Shipbuilding Division, Bethlehem Steel Company. Professor C. Ridgely-Nevitt of the Webb Institute of Naval Architecture secured the ship's log and Weather Bureau data given in Table 8. The author wishes to express his appreciation to Dr. E.H. Kennard of the Taylor Model Basin for the critical and constructive review he has given this paper as well as for the lively, illuminating discussions on statistics with him.

APPENDIX 1

STATISTICAL DISTRIBUTION PATTERNS

BASIC DISTRIBUTIONS

It will be necessary to define a few terms that are widely used in statistics. For more detail on the methods, however, the reader is referred to References 3 to 5.

The *cumulative distribution function* $P(x_a)$ denotes the probability that the variate x (for example, stress variation) will take on a value less than or equal to x_a, i.e.,

$$P(x_a) = P\{x \leqq x_a\}$$

The probability P may take on values between 0 and 1. $P(x_a) = 0$ signifies that there are no variates equal to or less than x_a, and $P(x_a) = 1$ signifies that all values of the variate are equal to or less than x_a. $P(x_a) = 0.6$ means that 60 percent of all members of the population* have values less than x_a. The probability that x takes on values less than x_b and larger than x_a is $P(x_b) - P(x_a)$. The derivative of the cumulative distribution function

$$\frac{dP(x)}{dx} = p(x)$$

is called the *distribution function* of x.

The best known and most widely used distribution function is the "normal distribution." It is theoretically possible, but not often practicable, to convert distributions which are not "normal" distributions into a normal distribution by a proper choice of a change of variables. Special coordinate paper can and has been devised so that a particular type of cumulative distribution function will plot as a straight line on such a chart. Figures 2, 4, and 5 are examples of such charts devised for the logarithmically normal distributions. In order to make published statistical tables and charts generally applicable to all normal distributions, the data are usually "normalized" by expressing the variate in terms of the "standard deviation."

The probability $P(x)$ takes on definite numerical values for a given population. In many practical cases, as in the subject of this report, an estimate of $P(x)$ is desired on the basis of samples of the members of the population. Statistics provides the tools for making such estimates. A measure is often required as to the reliability of the estimate or of the significance of the assumptions that may have been made regarding the distribution of the population. The methods of statistics provide such "tests of significance" which enable one to estimate the probable variation of the estimated parameters from the true value. The normal and log-normal distributions are the ones considered in this report.

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*The population consists of all the individual members under consideration, each individual of which is specified by a magnitude of the variate.

The *normal distribution* is defined as follows:

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(x-\bar{x})^2}{2\sigma^2}}, \qquad -\infty < x < +\infty \qquad [3]$$

$$P(x) = \int_{-\infty}^{x} p(x)\, dx. \qquad [4]$$

where x is the variate (stress variation, motion),

$\bar{x}$ is the mean value of x,

σ is the standard deviation of x, and

σ^2 is the variance of x.

Thus the two parameters $\bar{x}$ and σ define the normal distribution completely.

The *logarithmically normal distribution* is a normal distribution of the logarithm of x, thus

$$p(\log x) = \frac{1}{\sigma\sqrt{2\pi}}\, \epsilon^{-\frac{(\log x - u)^2}{2\sigma^2}} \qquad [5]$$

where u is the mean value of $\log x$ and σ is the standard deviation of $\log x$.

DISTRIBUTION OF EXTREMES

The distributions of stress variations, as shown in Figure 2, are called the underlying distributions (also known as initial or basic distribution). They represent the distribution of *all* the wave-induced stresses. Often, however, the engineers might require, among other things, an estimate of the extreme wave-induced stress that that structure would be subjected to during a given service life. The following considerations are then of interest.

If the underlying distribution is known, it is possible[5] to determine therefrom the distribution of the largest* values occurring in repeated large samples of equal size taken from the underlying distribution. This distribution is known as the "extreme value distribution for samples of the given size." Such a distribution can also be determined directly (assuming that it is predictable) from observations of extremes alone, where these extremes are considered as members of a population of extremes.

It has been shown in the literature[5] that, if the underlying distribution is of a certain type, including the normal distribution, the extreme value distribution approaches, as the sample size becomes larger and larger, the limiting distribution of the population of extremes defined by $F(x)$ where

*This statement can also be made for the distribution of the smallest value.

$$F(x) = \exp\left[-e^{-(x-u_1)\alpha}\right] \text{ or } \quad F(y) = \exp\left[-e^{-y}\right] \qquad [6]$$

where y $= (x-u_1)\alpha$ and is called the reduced variate,

x is the magnitude of the actual variate (extreme value),

u_1 and α are constants for a particular distribution, and

$F(x)$ is the fraction of the population of extremes with values less than x.

Here u_1 is the most probable value or mode of the distribution of extremes.

The process of fitting this analytical form of an extreme value distribution to the observed data requires an estimate of the population parameters u_1 and α. Gumbel[7] has devised a method for estimating these parameters for small sample sizes which utilizes a variation of the method of least squares to fit the function $F(x)$ to the observed cumulative frequency distribution. For further discussion of Gumbel's method see References 6, 7, and 8. The procedure suggested by Gumbel is as follows:

To fit an extreme value distribution $F(x)$ to experimental data,

let N be the number of observed extreme values,

x the variate (extreme value),

$\bar{x}$ the mean value of the N observed extremes, $s^2 = \sum \frac{(x-\bar{x})^2}{N-1}$,

s the standard deviation of the observed extremes,

σ_N the expected standard deviation of the reduced extremes y for a given N, as obtained from a standard table reproduced as Table 11, and

$\bar{y}_N$ the expected mean of the reduced variate taken from Table 11.

Then

$$\left(\frac{1}{\alpha}\right) = \frac{s}{\sigma_N} \text{ and } \quad u_1 = \bar{x} - \frac{\bar{y}_N}{\alpha} \qquad [7]$$

Procedure: (See Table 12 and Figure 6 for an illustrative example)

1. (a) Tabulate the different observed values x, ordered according to magnitude, and note the frequency n with which each magnitude was observed (columns 1, 2). If the last magnitude occurs n_N times, write this magnitude twice, with respective frequencies $n_N - 1$ and 1.

(b) Number also the actual observations in the order of increasing (or equal) magnitude; call this number m. As the rank m_g of each distinct magnitude take

$$m_g = \sqrt{m\,(m+n-1)}$$

where m has its lowest value for the n equal observations. Thus, if $n = 1$, $m_g = m$. (Column 3)

(c) The plotting position $F(x)$ for observed values of x on the extreme probability chart, such as is shown in Figure 6, is determined by dividing the rank of the observation by $(N + 1)$, see column 4 of Table 12. Plot the points $[F(x), x]$ on the chart.

2. Calculate $\bar{x}$ and s.

3. For the given sample size N, obtain the corresponding values of $\bar{y}_N$ and σ_N from Table 11.

4. From Equations [7] determine $\left(\frac{1}{\alpha}\right)$ and u_1.

5. The equation of the straight line on the extreme probability chart (Figure 6) which best fits the given experimental data is

$$x = u_1 + \left(\frac{1}{\alpha}\right) y \qquad [8]$$

6. The theoretical distribution $F(x)$ is then given by Equation [6].

If the assumed form $F(y) = \exp[-e^{-y}]$ does represent the extreme value distribution for the population of extremes under consideration, then the observed values of extremes plotted on extreme probability paper should be scattered closely about the straight line represented by Equation [8].

In order to evaluate the degree of scatter that may be expected with the hypothesis that the assumed form of the distribution of extremes holds, use is made of a theorem in statistics which states that the sample quantiles* (the plotted points) are asymptotically normal to the corresponding quantile of the population (the values on the straight line).

Otherwise stated: if a sample of N extremes is chosen at random and if these extremes are numbered in the order of increasing magnitude, then the value of any particular extreme, say the rth, cannot be predicted in advance of the drawing of the sample; there exists for it a certain probability curve. If N is large, and if neither r/N nor $(N-r)/N$ is too small, then the probability curve for the rth extreme is approximately normal in form. If N is increased and if r is also increased so as to keep r/N as nearly constant as possible (as for a fixed quantile of the sample), then the approximation to normality increases, becoming perfect in the limit as $N \to \infty$; the width of the probability spread simultaneously decreases, becoming zero in the limit.

If a curve is plotted on each side of the straight line (on Figure 6) at a distance of one standard deviation of the distribution of the corresponding quantiles, then two-thirds of the observed extremes should lie between these limits if a great number (theoretically an infinite number) of samples from the distributions of extremes were taken. This confidence or control curve is therefore labeled with the corresponding probability level, i.e., $P = 0.67$.

In many cases a visual inspection of the scatter is sufficient to indicate that the assumed distribution of extremes is an acceptable representation of the actual distribution.

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*A quantile denotes the variate corresponding to a given fraction, i.e., the 0.40 quantile is that value of the variate below which 40 percent of all values lie.

TABLE 11

Expected Means and Standard Deviations of Reduced Extremes for Use in Fitting a Theoretical Extreme Value Distribution Pattern to Observed Extreme Values

This table was compiled by Dr. E. J. Gumbel for Environmental Protection Section, Research and Development Branch, Office of the Quartermaster General, Washington, D.C.

Sample Size N	Reduced Mean $\bar{y}_N$	Standard Deviation σ_N	Sample Size N	Reduced Mean $\bar{y}_N$	Standard Deviation σ_N	Sample Size N	Reduced Mean $\bar{y}_N$	Standard Deviation σ_N
15	0.5128	1.0206	50	0.5485	1.1607	85	0.5578	1.1973
			51	0.5489	1.1623	86	0.5580	1.1980
			52	0.5493	1.1638	87	0.5581	1.1987
			53	0.5497	1.1658	88	0.5583	1.1994
			54	0.5501	1.1667	89	0.5585	1.2001
20	0.5236	1.0628	55	0.5504	1.1681	90	0.5586	1.2007
21	0.5252	1.0696	56	0.5508	1.1696	91	0.5587	1.2013
22	0.5268	1.0754	57	0.5511	1.1708	92	0.5589	1.2020
23	0.5283	1.0811	58	0.5515	1.1721	93	0.5591	1.2026
24	0.5296	1.0864	59	0.5518	1.1734	94	0.5592	1.2032
25	0.5309	1.0915	60	0.5521	1.1747	95	0.5593	1.2038
26	0.5320	1.0961	61	0.5524	1.1759	96	0.5595	1.2044
27	0.5332	1.1004	62	0.5527	1.1770	97	0.5596	1.2049
28	0.5343	1.1047	63	0.5530	1.1782	98	0.5598	1.2055
29	0.5353	1.1086	64	0.5533	1.1793	99	0.5599	1.2060
30	0.5362	1.1124	65	0.5535	1.1803	100	0.5600	1.20649
31	0.5371	1.1159	66	0.5538	1.1814			
32	0.5380	1.1193	67	0.5540	1.1824	150	0.5646	1.22534
33	0.5388	1.1226	68	0.5543	1.1834			
34	0.5396	1.1255	69	0.5545	1.1844	200	0.5672	1.23598
35	0.5403	1.1285	70	0.5548	1.1854	250	0.5688	1.24292
36	0.5410	1.1313	71	0.5550	1.1863			
37	0.5418	1.1339	72	0.5552	1.1873	300	0.5699	1.24786
38	0.5424	1.1363	73	0.5555	1.1881			
39	0.5430	1.1388	74	0.5557	1.1890	400	0.5714	1.25450
40	0.5436	1.1413	75	0.5559	1.1898	500	0.5724	1.25880
41	0.5442	1.1436	76	0.5561	1.1906			
42	0.5448	1.1458	77	0.5563	1.1915	750	0.5738	1.26506
43	0.5453	1.1480	78	0.5565	1.1923			
44	0.5458	1.1499	79	0.5567	1.1930	1000	0.5745	1.26851
45	0.5463	1.1519	80	0.5569	1.1938	Inf.	0.5772	1.28255
46	0.5468	1.1538	81	0.5570	1.1945			
47	0.5473	1.1557	82	0.5572	1.1953			
48	0.5477	1.1574	83	0.5574	1.1960			
49	0.5481	1.1590	84	0.5576	1.1967			

TABLE 12

Computations for Fitting a Theoretical Extreme Value Distribution to Extreme Wave-Induced Stress Variations Measured on Port Side, Main Deck

This distribution is based on 18 trips averaging 121 hours at sea each. The data cover the period 2 September 1953 to 13 March 1954. Both load and ballast conditions are included in the data. See Table 1 for routes traveled during this period.

STEP 1. Complete the Tabulation below.

Observed Extreme Stress psi h	$\text{Log}_{10}\, h$ x	No. of Observations n	Rank m_g	Plotting Position $F(x) = \frac{m_g}{N+1}$	$\lvert x-\bar{x}\rvert$	$(x-\bar{x})^2 \times 10^{-2}$	Hours at Sea
2,500	3.3979	2	1.41	0.074	0.2754	7.585	109 142
2,600	3.4150	1	3	0.158	0.2583	6.672	108
2,800	3.4472	2	4.5	0.237	0.2261	5.112	118 103
3,200	3.5051	1	6	0.316	0.1682	2.829	125
3,500	3.5441	1	7	0.368	0.1292	1.669	120
4,300	3.6355	1	8	0.421	0.0378	0.144	132
4,500	3.6532	1	9	0.474	0.0201	0.040	145
4,900	3.6902	1	10	0.526	0.0169	0.029	111
5,200	3.7160	1	11	0.579	0.0427	0.182	127
5,700	3.7559	1	12	0.632	0.0826	0.682	110
6,000	3.7782	1	13	0.684	0.1049	1.100	131
7,100	3.8513	1	14	0.737	0.1780	3.168	144
7,800	3.8921	1	15	0.789	0.2188	4.787	122
8,500	3.9294	1	16	0.842	0.2561	6.559	120
9,900	3.9956	1	17	0.894	0.3223	10.388	99
11,700	4.0682	1	18	0.947	0.3943	15.547	116
						$\Sigma = 79.190$	

STEP 2. Mean ($\log_{10}$ Extreme Stress) $= \bar{x} = 3.6733$, $N = 18$

Standard Deviation $s = \left[\frac{(x-\bar{x})^2}{N-1}\right]^{1/2} = 0.216$

STEP 3. For $N = 18$, Table 11 gives $\bar{y}_N \approx 0.519$, $\sigma_N = 1.046$

STEP 4. $\frac{1}{\alpha} = \frac{s}{\sigma_N} = 0.207 \qquad u_1 = \bar{x} - \frac{y_N}{\alpha} = 3.566$

STEP 5. $x = u_1 + \frac{1}{\alpha}\, y = 3.566 + 0.207\, y$ where x = value of $\log_{10}$ (Extreme Stress Variation)

This straight line is plotted in Figure 6b.

In a variety of applications of the distribution of extreme values to problems in which the underlying basic distribution was unknown, it has been found that the formulation [2] often describes the distribution of extremes of physical phenomena under study, such as floods and gust velocities. Thus, if all knowledge of the underlying distribution of the stress variations is disregarded, we may fit the distribution of extreme values [2] to the observed extreme stress variations directly (not to the extremes of the logarithm of the stress variations) by utilizing the method of Gumbel just described. The resulting distributions are shown in Figures 6c and 6d.

Since the fit of the experimental data to the assumed type of distribution is good, one could conclude that the extreme values of the stress variations are approximated by the formulation, Equation [2]. Furthermore, since it does not appear reasonable that both the logarithm of the stress variation (Figures 6a, 6b) and the stress variation itself (Figures 6c, 6d) should follow the extreme value distribution given by Formula [2] for all values of the stress variation, it may be concluded that the underlying distribution is not exactly log-normal, although it may be very nearly so over the range of observed values.

Let an estimate be made for the stress on the port side, main deck of the ESSO ASHEVILLE for a service period of 10 trips* of 101 hr (at sea) each. Figure 6b indicates that, if 2-min measurements were made at hourly intervals, the expected maximum measured stress variation would be 11,000 psi.

It will be of interest to make an estimate of the largest stress to be expected in a given period of time at sea, on the basis of the underlying log-normal distribution shown in Figure 2. For this purpose a formula will be developed which will give the fraction f of all samples of size N (belonging to a given underlying distribution) which will have at least one (extreme) value of $h > h_{m1}$. The formula is

$$f = 1 - e^{-N[1 - P(h_{m1})]} \qquad [9]$$

This formula may be developed as follows (as was pointed out by Dr. Lieblein of the National Bureau of Standards):

Let $P(h_{m1})$ = fraction of underlying (basic) distribution of h with $h < h_{m1}$.

Take a sample of N h's in such a way that each h is chosen at random from the basic distribution, and repeat such sampling many times. Then the probability of choosing a value less than h_{m1} when one single value of h is chosen from one sample of size N, is $[P(h_{m1})]$ and consequently the probability that some members of the sample of size N exceed h_{m1} is $1 - [P(h_{m1})]^N$.

Therefore the fraction f of all (a very large number) samples of size N which will have one or more values of h greater than h_{m1} is

$$f = 1 - [P(h_{m1})]^N$$

*Environmental conditions are assumed to be those for which the experimental data were obtained.

This formula is exact, but for large N it is inconvenient. Now for large N it is practically certain that in any given sample, h_m has such a value that $P(h_m)$ is close to 1. For values of h_{m1} such that $1 - P(h_{m1})$ is small, a more convenient approximate formula can be found. We have

$$\log (1 - f) = N \log P(h_{m1}) = N \log \{1 - [1 - P(h_{m1})]\}$$

$$\log (1 - f) \approx - N [1 - P(h_{m1})]$$

by use of the series, $\log (1 - x) = - x - x^2/2 \cdots\cdots$ Thus f is given approximately by the expression shown in Equation [9]. This result is independent of the form of P. As a first example of the application of Formula [9] let it be required to estimate the extreme stress variation that would be exceeded by one-half of all ships ($f = 1/2$) if 10,400 random stress measurements ($N = 10{,}400$) were made on each of many ships. Assume that the underlying distribution applicable to these ships is that given for the port side of the main deck amidships in Figure 2.

$$f = 1/2, \qquad N = 10{,}400$$

From Formula [9]

$$f = \frac{1}{2} = 1 - e^{-10{,}400\,[1 - P(h_{m1})]}$$

$$-10{,}400\,[1 - P(h_{m1})] = - 0.6931$$

or

$$1 - P(h_{m1}) = 6.66 \times 10^{-5}$$

The corresponding value of the stress variation h_{m1} is read from Figure 2 as 16,000 psi (from hog to sag). Similarly an estimate of the stress variation which will be exceeded in 20 winter seasons (26,700,000 stress variations) in one-half of all ships ($f = 1/2$) is about 50,000 psi (from hog to sag).

Not much confidence can be placed on the accuracy of this last prediction because the measurements on which the prediction is based did not include magnitudes of the order of the estimate, i.e., the estimates are not expected to be reliable if the estimated extreme value lies outside the range in which experimental data are available.

A better method for making an estimate of the probable extreme stress is by use of the experimentally obtained distribution of extreme values; this being the most direct method.

APPENDIX 2

INSTRUMENTATION

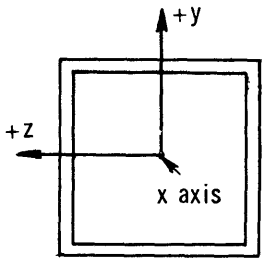
It may be well to look into the validity of the strain gage measurements. Strains cause the electrical resistance of the gage to vary; therefore anything that causes the resistance to vary will be reflected as an apparent strain. Dummy gages can be effective only if they are at substantially the same temperature as those gages which are directly cemented to the deck; see Figure 10 and the discussion of the strain gage circuity given later.

Before the replacement of the strain gages at Jacksonville in August 1953, tests were made in which the steel deck around the gages was heated by means of blow torches. During the heating and cooling of the deck, temperatures of the steel at the location of active and dummy gages were measured by means of a thermocouple. The temperature of the steel at the gage locations reached 160 F rising 22 F in 20 min; the temperature lag between active and dummy gages was small, at all times less than 5 F. Temperature equalization and compensation is therefore considered satisfactory.

The change in leakage resistance between the strain gage and the steel hull is another possible source of error. A slow drift in the strain gage signal may occur due to a change in leakage resistance to ground as well as due to aging of vacuum tubes and capacitors. The leakage resistance to ground was checked during every visit to the ship, and the maximum change in strain gage signal due to this cause was about 3 μ in/in. (90 psi) for the period of 13 December 1952 to February 1954 (corresponding change in leakage resistance was from 9.3 megohms to 20 megohms). Most of the time since August 1953 the leakage resistance has been of the order of 1000 megohms. Therefore the drift in the strain gage signal due to this cause is practically nil. The drift due to the aging of circuit components is more important, but this type of drift is very slow and would not affect the signal appreciably during a 24-hr period, so that it may be neglected as far as effect on the analysis of thermal stress is concerned.

STRAIN GAGE CIRCUITY

The relationship between stresses, strains, and temperature changes, as given in article 65 of Timoshenko's "Theory of Elasticity" are given below as Equation [10]



$$e_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)] + \alpha T,$$

$$e_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)] + \alpha T \qquad [10]$$

$$e_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)] + \alpha T$$

These relationships specify the actual strains corresponding to any actual change in stress and the corresponding change in temperature. It is immaterial whether the stresses are due to temperature changes alone, or to external loading, or to a combination of both—these relationships are valid in either case.

For the stresses in the plating of a ship girder it may be assumed that the stress σ_z normal to the thickness of the plating is zero.

Utilizing the expressions for e_x and e_z, Equations [10], the stress σ_z may be eliminated between them, and the following expression for σ_x results:

$$\sigma_x = \frac{E}{1-\nu^2}\left[e_x + \nu e_z - (1+\nu)\alpha T\right] \qquad [11]$$

where σ_x is the change in normal stress in the longitudinal direction,

e_x, e_z are the total changes in strains in longitudinal and transverse directions, respectively,

ν is Poisson's ratio,

α is the temperature coefficient of expansion, and

T is the change in temperature at the point of measurement.

If there are no variations in external loads, then the stress σ_x and the strains e_x, e_z will be due only to the changes in temperature throughout the structure, including the temperature change T at the point x, y, z at which σ_x and e_x, e_z are determined.

To determine σ_x it is then only necessary to measure the quantities e_x, e_z, and T, assuming that E, ν, and α are known constants, and substitute these values in Equation [11].

Another, more convenient method is to arrange an electrical circuit which performs the operation indicated by the right-hand side of Equation [11], thus yielding a signal which is proportional to the actual stress variation in the structure. This is the procedure which was adopted for the stress measurements on the T-2 tanker ESSO ASHEVILLE. The circuity is shown in Figure 10, and its operation is as follows. R_1, R_2, R_3, and R_4 are 60-ohm SR-4 strain gages mounted on the deck structure. B_1 and B_2 are 120-ohm SR-4 strain gages mounted on steel bars which are free to expand and which are at the same temperature as is the deck. Bars B are physically located next to resistors R_1, R_2, R_3, and R_4. These gages are all situated close together at the location at which the stress is measured.

The resistance change of an SR-4 strain gage is proportional to the strain of the metal to which it is attached and also proportional to the resistance of the gage.

Consider Equation [11] term by term and consider the component of the circuit which provides an electrical output proportional to that term.

1. A signal proportional to $+e_x$ is provided by 180 ohms of strain gage wire ($R_1+R_2+R_4$) physically oriented in the longitudinal x direction. Elementary electrical theory shows that the voltage changes across R_1, R_2, and R_4 are additive at the output terminals.

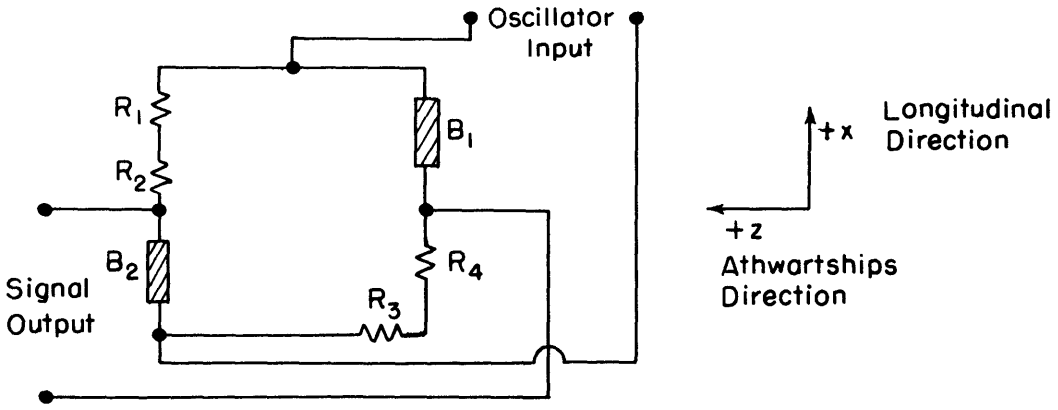


Figure 10 - Circuit for Measuring σ_x

2. A signal proportional to $+\nu e_z$ is provided by ν times 180 ohms of strain gage wire (R_3) physically oriented in the athwartships z direction. Taking $\nu = 1/3$, it is evident that $1/3 \times 180 = 60$ ohms are required for R_3. The signal output of gage R_3 is electrically additive to that of gages R_1, R_2, and R_4.

3. A signal proportional to $-(1 + \nu)\alpha T = -4/3\alpha T$ is provided by $4/3 \times 180 = 240$ ohms of strain gage wiring attached to steel bars which are free to elongate an amount αT per unit of length where αT is the temperature induced "stress-less" strain. The electrical output of gages B_1 and B_2 is electrically subtracted from the output of R_1, R_2, R_3, and R_4.

It should be noted that any change of resistance of the strain gage wire itself, as a consequence of temperature changes, is automatically compensated by having the same amount of strain gage wire, 120 ohms in this case, in each arm of the bridge.

It has been shown, therefore, that the electrical output of the strain gage bridge is proportional to the stress σ_x at the surface of the structure (i.e., at the location of the gage).

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