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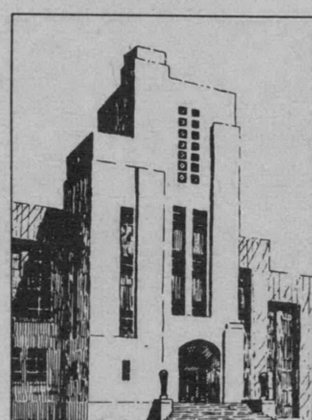
THE LAMINAR FLOW ABOUT VERY SLENDER CYLINDERS IN AXIAL MOTION, INCLUDING THE EFFECT OF PRESSURE GRADIENTS AND UNSTEADY MOTIONS

by

R.D. Cooper and M.P. Tulin



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NOTATION

(Only symbols defined directly in terms of physical quantities are listed)

p	Static pressure
r	Radial spatial variable (see Figure 1)
r_0	Radius of the cylinder
t	Time variable
u	Velocity component inside the boundary layer in the x-direction (see Figure 1)
u_1	Velocity at the outer edge of the boundary layer (potential stream velocity)
v	Velocity component inside the boundary layer in the r-direction (see Figure 1)
x	Axial spatial variable (see Figure 1)
μ	Coefficient of viscosity
ν	Kinematic viscosity, μ / ρ
ρ	Density

ABSTRACT

The effect of transverse curvature on laminar boundary layer characteristics has been investigated through the use of an approximate, linearized theory.

In the case of a cylinder in steady, axial motion with no external pressure gradient, it is shown that the transverse curvature increases the wall shear stress.

A general solution is obtained for the case of arbitrary unsteady motion of a cylinder initially at rest with no external pressure gradient. The particular case of an impulsive start is considered in detail; it is shown that for such motion, the flow field is divided into two distinct regimes, one independent of the time variable, the other independent of the axial spatial variable.

The effect of external pressure gradients in the case of steady motion is investigated, and general solutions for a large class of external pressure gradients are obtained. The particular class of Falkner-Skan type pressure gradients are treated in detail. The results of this latter investigation show that the transverse curvature increases the wall shear stress for both favorable and adverse pressure gradients. The increase for favorable gradients is less than the increase for uniform flow; the converse is true for adverse gradients.

So-called "similar" velocity profiles are shown to exist for a particular set of flows contained in the class of Falkner-Skan type external pressure gradients, and the ordinary differential equation defining them is derived.

INTRODUCTION

The prediction of boundary-layer characteristics on configurations for which the boundary-layer thickness may not be small relative to the transverse radius of curvature is of interest in several practical applications. Among these are the design of very slender, missile-like bodies and the interpretation of model tests of configurations with large transverse curvatures such as ship models with sharply turned bilges. Pertinent configurations thus may involve simple shapes, as is usually the case for missiles, or highly complex shapes, as is usually the case for ships. Furthermore, it is evident that both laminar and turbulent boundary layers are of interest.

At the present time, however, an analysis of this "transverse curvature effect" for turbulent boundary layers can be made only by resort to hypotheses for which no experimental verification yet exists.[1] In the case of laminar boundary layers, a previous investigation considered axial flow about the simple surface of a semi-infinite circular cylinder.[2] Specifically, this investigation treated the problem of incompressible, laminar boundary-layer flow on very slender cylinders in axial motion, without consideration of the effects of unsteady

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[1]References are listed on page 18.

motion or of external, axially symmetric pressure gradients. For the investigation of these latter effects, this paper applies a linearized theory for incompressible, laminar boundary layers, which has recently been discussed in connection with two-dimensional, planar flows.[3]

First, the effect of transverse curvature on the wall shear stresses of the cylinder in steady axial motion with no external pressure gradients is shown. As previously mentioned, this problem has been treated by Seban and Bond who numerically integrated the nonlinear differential equations defining the problem.[2]

The general solution of the linearized boundary layer problem foι the case of the arbitrary unsteady motion of a cylinder initially at rest (again with no external pressure gradients) is next considered, and the particular case of impulsive motion is treated in detail.

Third, the effect of external pressure gradients on the boundary-layer flow along the cylinder and with the motion steady is discussed. Solutions for a large class of external pressure gradients are obtained, and the particular class considered for two-dimensional, planar flows by Falkner and Skan[4] and by Hartree[5] is investigated in detail for the present axially symmetric condition.

Last, a class of laminar flows with transverse curvature and so-called "similar" velocity profiles is shown to exist for a particular set of external flows, and the nonlinear ordinary differential equation defining these flows is derived. The numerical solutions of this equation are not considered here.

DERIVATION OF THE APPROXIMATE EQUATION OF MOTION

The obvious difficulties involved in the exact solution of the nonlinear set of boundary-layer equations have led to the development of approximate methods of solution of boundary-layer problems. Among these approximate methods is one which involves the use of a linearized equation of motion. This linearization method has been previously used for two-dimensional, planar problems, particularly for the discussion of unsteady effects. It is regarded as especially well-suited for the initial investigation of effects such as those under consideration here since it permits meaningful results to be relatively easily obtained for a large class of problems. For the present problems, the linearized equation of motion has been slightly modified so as to lead to results in closer agreement with certain exact theory results. The derivation of the linearized equation of motion follows. It is shown that solutions of the linearized equation of motion are solutions of an appropriately linearized integral of the Prandtl boundary layer equation of motion.

The boundary -layer equations for axially symmetric flows, where the boundary-layer thickness is not necessarily small compared to the radius of the body of revolution, can be written, using the cylindrical polar coordinate system shown in Figure 1, as:[6]

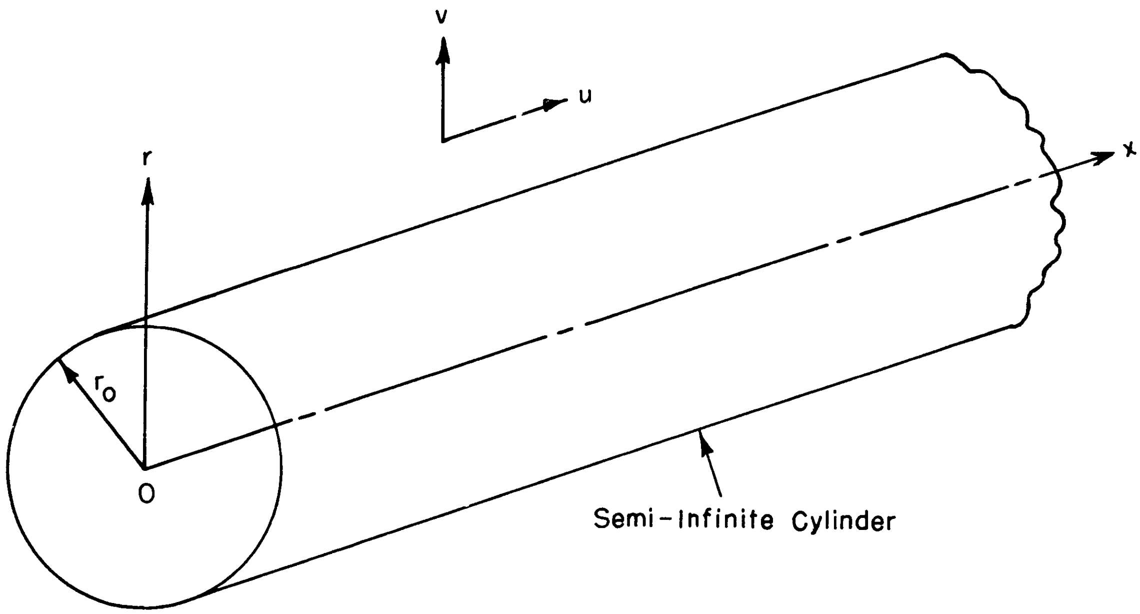

Figure 1 - The Cylindrical Polar Coordinate System for Axially Symmetric Flows

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = -\frac{1}{\rho}\frac{dp}{dx} + \frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) \qquad [1]$$

$$\frac{\partial (ru)}{\partial x} + \frac{\partial (rv)}{\partial r} = 0 \qquad [2]$$

$$-\frac{1}{\rho}\frac{dp}{dx} = \frac{du_1}{dt} = \frac{\partial u_1}{\partial t} + u_1\frac{\partial u_1}{\partial x} \qquad [3]$$

where x and r are the axial and radial spatial variables, respectively,

t is the time variable,

u and v are the velocity components in the direction of x and r, respectively,

p is the static pressure,

ρ is the density,

u_1 is the velocity at the outer edge of the boundary layer, and

ν is the kinematic viscosity.

The boundary conditions for this set of differential equations require that

$$\left.\begin{array}{l} u = v = 0 \text{ at } r = r_0 \\ \text{and} \\ u = u_1 \text{ at } r = \infty \text{ and at } x = 0 \end{array}\right\} \qquad [4]$$

where r_0 is the radius of the cylinder.

The statement that Newton's law holds "in the mean" for any transverse slice of fluid is:

$$\int_{r_0}^{\infty} \left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{dp}{dx} - \frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)\right] r\,dr = 0 \qquad [5]$$

With the aid of Equations [2], [3], and [4], this equation may be written

$$\int_{r_0}^{\infty} \left[\frac{\partial u'}{\partial t} + 2u'\frac{\partial u_1}{\partial x} + (u_1 - 2u')\frac{\partial u'}{\partial x} - \frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u'}{\partial r}\right)\right] r\,dr = 0 \qquad [6]$$

where $u' = u_1 - u$. The integrand is linearized by assuming that $u_1 - 2u' = cu_1$, where c will later be chosen so that the result for the flat plate boundary layer will give exact agreement with the Blasius result. This is a procedure similar to that employed by Lewis and Carrier in an Oseen flow approximation for the flat plate problem.[7] Indeed, the value of c found in the two cases is identical.

Finally, it is noted that solutions of the equation

$$\frac{\partial u'}{\partial t} + 2u'\frac{\partial u_1}{\partial x} + cu_1\frac{\partial u'}{\partial x} - \frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u'}{\partial r}\right) = 0 \qquad [7]$$

approximately satisfy the nonlinear equation of motion [1] in the mean.

It will be assumed that solutions of Equation [7] are meaningful approximate solutions of the boundary-layer problem defined by Equations [1], [2], and [3] when the boundary conditions [4] are satisfied. That this assumption is justified for two-dimensional, planar flows has already been shown in Reference 3.

THE SOLUTION FOR STEADY MOTION WITH NO PRESSURE GRADIENT

In the case of a cylinder in steady axial motion with no external pressure gradient, for which u_1 = const. = u_0, Equation [7] reduces to

$$cu_0\frac{\partial u'}{\partial x} - \frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u'}{\partial r}\right) = 0 \qquad [8a]$$

with the boundary conditions

$$u'(0,r) = 0, \qquad u'(x, r_0) = u_0, \qquad \lim_{r\to\infty} u'(x,r) = 0 \qquad [8b]$$

The differential equation [8a] has an interpretation that is of considerable historic interest for it may be considered as the extension to the axially symmetric case of an analysis made by Rayleigh for the corresponding two-dimensional problem (see, for instance, Reference 6, pp. 137-138). In particular, the extension of the Rayleigh analogy consists in the consideration of an infinitely long, circular cylinder initially at rest and suddenly set in motion with constant velocity u_0. The solution to this latter problem with the time variable t replaced by the modified spatial variable x/cu_0 can in accordance with the Rayleigh analysis, be considered as an approximate solution to the problem of the steady axial motion of a semi-infinite cylinder and, in fact, is an exact solution of the linearized problem defined by the system [8].

Furthermore, the differential equation [8a] is in the form of a well-known equation of physics which arises particularly in problems of diffusion and heat conduction and, consequently, is frequently referred to as the "diffusion equation." Indeed, the system [8] is easily recognized as an exact analogy to the problem of heat conduction from an infinitely long cylinder into an exterior region initially at zero temperature when the surface of the cylinder is suddenly brought to a constant temperature. This problem has been treated in detail by means of Laplace operator methods by Carslaw and Jaeger.[8] Their results, converted to the variables and parameters of the present problem, are reproduced here.

Thus, the velocity profile is given by

$$\frac{u}{u_0} = F\left(\lambda x, \frac{r}{r_0}\right) \qquad [9]$$

where

$$\lambda = \frac{\nu}{c\, u_0 r_0^2}$$

and

$$F(\alpha, \beta) = \frac{2}{\pi}\int_0^\infty e^{-\alpha\eta^2}\, \frac{J_0(\eta)\, Y_0(\beta\eta) - J_0(\beta\eta)\, Y_0(\eta)}{J_0^2(\eta) + Y_0^2(\eta)}\, \frac{d\eta}{\eta} \qquad [10]$$

In Equation [10], J_0 and Y_0 are Bessel functions of zero order of the first and second kind, respectively. If the parameter

$$\kappa = \frac{\nu x}{u_0 r_0^2}$$

is introduced, the asymptotic expansion of the velocity profile valid for small values of the parameter, i.e., for large values of r_0, is

$$\frac{u}{u_0} = 1 - \left(\frac{r_0}{r}\right)^{1/2} erfc \left\{\left(\frac{r}{r_0} - 1\right)\sqrt{\frac{c}{4\kappa}}\right\} - \frac{1}{4}\left(\frac{r_0}{r}\right)^{3/2}\left(\frac{r}{r_0} - 1\right)\left(\frac{\kappa}{c}\right)^{1/2} i\, erfc \left\{\left(\frac{r}{r_0} - 1\right)\sqrt{\frac{c}{4\kappa}}\right\}$$

$$- \frac{1}{32}\left(\frac{r_0}{r}\right)^{5/2}\left[9 - 2\frac{r}{r_0} - 7\left(\frac{r}{r_0}\right)^2\right]\frac{\kappa}{c}\, i^2 erfc\left\{\left(\frac{r}{r_0} - 1\right)\sqrt{\frac{c}{4\kappa}}\right\} - \ldots \qquad [11]$$

where $i^n erfc\{x\}$, $n = 0, 1, \ldots$, is the n^{th} repeated integral of the complementary error function.

In the heat conduction problem of Carslaw and Jaeger, the flux of heat at the surface is analogous to the wall shear stress of the present problem, that is, the wall shear stress,

$$\tau_0 = \mu\left(\frac{\partial u}{\partial r}\right)_{r = r_0},$$

where μ is the coefficient of viscosity, is given by

$$\tau_0 = \mu \frac{u_0}{r_0}\frac{4}{\pi^2}\int_0^\infty \frac{e^{-\lambda x \eta^2}}{J_0^2(\eta) + Y_0^2(\eta)}\frac{d\eta}{\eta} \qquad [12]$$

Again, when the parameter κ is small, i.e., when r_0 is large, the asymptotic solution for the wall shear stress is

$$\tau_0 = \mu\frac{u_0}{r_0}\left\{0.56419\left(\frac{\kappa}{c}\right)^{-1/2} + 0.50000 - 0.14105\left(\frac{\kappa}{c}\right)^{1/2} + 0.12500\,\frac{\kappa}{c} - 0.14693\left(\frac{\kappa}{c}\right)^{3/2} + \ldots\right\} \qquad [13]$$

(The last term of the series [13] shown here, although not given by Carslaw and Jaeger in their discussion of the heat conduction problem, is given by Jaeger and Clarke[9] in a discussion of the integral [12].) When the parameter κ is large, i.e., when r_0 is small, the asymptotic solution is

$$\tau_0 = 2\mu\frac{u_0}{r_0}\left\{\frac{1}{\log\left(4\frac{\kappa}{c}\right) - 2\gamma} - \frac{\gamma}{\left[\log\left(4\frac{\kappa}{c}\right) - 2\gamma\right]^2} - \ldots\right\} \qquad [14]$$

where $\gamma = 0.57722\ldots$ is Euler's constant.

The absence of the general term in the series solutions [11], [13], and [14] makes it difficult to discuss the convergence properties of these series, and no attempt to do so will be made here. However, it may be noted that for the range of the parameter κ considered here, values of

$$\frac{r_0\tau_0}{\mu u_0}$$

computed from [13] are in agreement with a table of values of the integral [12] published in Reference 9.

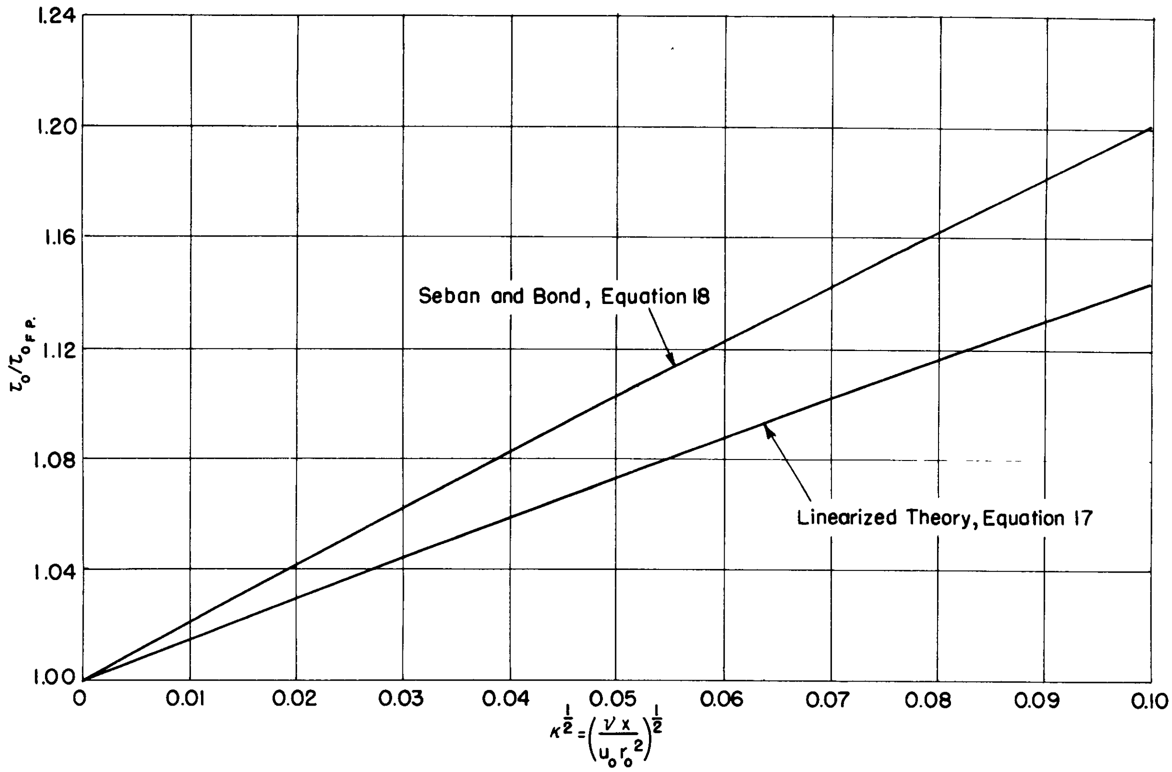

Figure 2 - The Ratio of Wall Shear Stress on a Cylinder to Wall Shear Stress on a Flat Plate in the Case of Steady, Axial Motion with no Pressure Gradient

The wall shear stress for the flat plate case can be obtained from [13] through a limiting process as r_0 approaches infinity. This yields

$$\tau_{0_{F.P.}} = 0.56419 \sqrt{c}\ \mu \sqrt{\frac{u_0^3}{\nu x}} \qquad [15]$$

The value of c is now chosen so as to give exact agreement with the Blasius solution; thus

$$c = 0.34627 \qquad [16]$$

The ratio of wall shear stress on the cylinder to that on the flat plate, for small values of κ, is then

$$\frac{\tau_0}{\tau_{0_{F.P.}}} = 1 + 1.50602\,\kappa^{1/2} - 0.72200\,\kappa + 1.08733\,\kappa^{3/2} - 2.17191\,\kappa^2 + \ldots \qquad [17]$$

It is clear from Equation [17] that the "transverse curvature effect," evaluated from the linearized Prandtl boundary-layer equations, is manifested as an increase in wall shear stress, i.e., τ_0 increases with increasing curvature. This result is in substantial agreement with that of Seban and Bond,[2] who treated the same problem by numerical integration of nonlinear differential equations. Their result is here reproduced, in the notation of the present paper, for comparison:

$$\frac{\tau_0}{\tau_{0_{\text{F.P.}}}} \doteq 1 + 2.12\,\kappa^{1/2} - 1.136\,\kappa \qquad [18]$$

Equations [17] and [18] are compared graphically in Figure 2.

THE SOLUTION FOR UNSTEADY MOTION WITH NO PRESSURE GRADIENT WITH PARTICULAR REFERENCE TO THE CASE OF AN IMPULSIVE START

In the case of a cylinder initially at rest in unsteady, axial motion with no external pressure gradient, for which $u_1 \equiv u_1\,(t)$, Equation [7] reduces to

$$\frac{\partial u'}{\partial t} + c\,u_1 \frac{\partial u'}{\partial x} - \frac{\nu}{r}\frac{\partial}{\partial r}\left(r\,\frac{\partial u'}{\partial r}\right) = 0 \qquad [19a]$$

with the boundary conditions

$$u'(0,x,r) = 0, \quad u'(t,0,r) = 0, \quad u'(t,x,r_0) = u_1, \quad \lim_{r \to \infty} u'(t,x,r) = 0 \qquad [19b]$$

where, as before, $c = 0.34627$ as defined by Equation [16].

If, for convenience, the variable transformation

$$\sigma = \frac{r}{r_0} \qquad [20]$$

is introduced, the system [19] is slightly modified to

$$\left.\begin{aligned} &\frac{\partial u'}{\partial t} + c\,u_1 \frac{\partial u'}{\partial x} - \frac{\nu'}{\sigma}\frac{\partial}{\partial \sigma}\left(\sigma\,\frac{\partial u'}{\partial \sigma}\right) = 0 \\ &u'(0,x,\sigma) = 0, \quad u'(t,0,\sigma) = 0, \quad u'(t,x,1) = u_1, \quad \lim_{\sigma \to \infty} u'(t,x,\sigma) = 0 \end{aligned}\right\} \qquad [21]$$

where

$$\nu' = \frac{\nu}{r_0^2}$$

This system is amenable to solution by operator methods; the first step is the application of the Laplace transform with respect to x, i.e.,

$$\bar{u}'(t,\alpha,\sigma) = \int_0^\infty e^{-\alpha x}\,u'(t,x,\sigma)\,dx \qquad [22]$$

This produces the system

$$\left.\begin{aligned} &\frac{\partial \bar{u}'}{\partial t} + c u_1 \alpha \bar{u}' - \frac{\nu'}{\sigma}\frac{\partial}{\partial \sigma}\left(\sigma \frac{\partial \bar{u}'}{\partial \sigma}\right) = 0 \\ &\bar{u}'(0,\alpha,\sigma) = 0, \qquad \bar{u}'(t,\alpha,1) = \frac{u_1}{\alpha}, \qquad \lim_{\sigma \to \infty} \bar{u}'(t,\alpha,\sigma) = 0 \end{aligned}\right\} \qquad [23]$$

The next step, which reduces the differential equation to the form of the diffusion equation, is the introduction of the dependent variable transformation

$$\bar{w}(t,\alpha,\sigma) = e^{c\alpha \int_0^t u_1(p)\,dp}\, \bar{u}'(t,\alpha,\sigma) \qquad [24]$$

which results in the system

$$\left.\begin{aligned} &\frac{\partial \bar{w}}{\partial t} - \frac{\nu'}{\sigma}\frac{\partial}{\partial \sigma}\left(\sigma \frac{\partial \bar{w}}{\partial \sigma}\right) = 0 \\ &\bar{w}(0,\alpha,\sigma) = 0, \quad \bar{w}(t,\alpha,1) = \frac{u_1}{\alpha} e^{c\alpha \int_0^t u_1(p)\,dp}, \quad \lim_{\sigma \to \infty} \bar{w}(t,\alpha,\sigma) = 0 \end{aligned}\right\} \qquad [25]$$

The problem defined by Equation [25] is easily treated by means of Duhamel's theorem (see, for instance, Reference 10, pp. 164-166) which yields the solution

$$\bar{w}(t,\alpha,\sigma) = \frac{\partial}{\partial t}\int_0^t \frac{u_1(\tau)}{\alpha} e^{c\alpha \int_0^\tau u_1(p)\,dp} \{1 - F[\nu'(t-\tau),\sigma]\}\, d\tau \qquad [26]$$

where the function F is as defined in Equation [10]. Since $F(0,\sigma) = 1$, this may be written

$$\bar{w}(t,\alpha,\sigma) = -\int_0^t \frac{u_1(\tau)}{\alpha} e^{c\alpha \int_0^\tau u_1(p)\,dp} \frac{\partial F[\nu'(t-\tau),\sigma]}{\partial t}\, d\tau \qquad [27]$$

from which, using the relation

$$\frac{\partial F[\nu'(t-\tau),\sigma]}{\partial t} = -\frac{\partial F[\nu'(t-\tau),\sigma]}{\partial \tau}$$

there is obtained

$$\bar{u}'(t,\alpha,\sigma) = \int_0^t \frac{u_1(\tau)}{\alpha} e^{-c\alpha \int_\tau^t u_1(p)\,dp} \frac{\partial F[\nu'(t-\tau),\sigma]}{\partial \tau}\, d\tau \qquad [28]$$

The inverse of the transform is now readily obtained as

$$u'(t,x,\sigma) = \int_0^t \left\{\begin{matrix} 0; & 0 < x < c\int_\tau^t u_1(p)dp \\ 1; & x > c\int_\tau^t u_1(p)dp \end{matrix}\right\} u_1(\tau) \frac{\partial F[\nu'(t-\tau),\sigma]}{\partial \tau}\, d\tau \qquad [29]$$

or

$$u\left(t,x,\frac{r}{r_0}\right) = u_1(t) - \int_0^t \begin{Bmatrix} 0; & 0<x<c\int_\tau^t u_1(p)dp \\ 1; & x>c\int_\tau^t u_1(p)dp \end{Bmatrix} u_1(\tau) \frac{\partial F\left[\nu'(t-\tau), \frac{r}{r_0}\right]}{\partial \tau} d\tau \qquad [30]$$

This latter equation, then, is the general solution to the linearized problem of the unsteady motion of a cylinder in axial flow in the absence of an external pressure gradient, with the sole restriction that the motion start from rest at time $t = 0$.

The case of an impulsive start, i.e., the case in which $u_1 = 0$ for $t \leq 0$ and $u_1 =$ const. $= u_0$ for $t > 0$, will now be treated in some detail as a particular case of the general solution [30]. Thus,

$$\frac{u\left(t,x,\frac{r}{r_0}\right)}{u_0} = 1 - \int_0^t \begin{Bmatrix} 0; & 0<x<c\,u_0\,(t-\tau) \\ 1; & x>c\,u_0\,(t-\tau) \end{Bmatrix} \frac{\partial F\left[\nu'(t-\tau), \frac{r}{r_0}\right]}{\partial \tau} d\tau \qquad [31]$$

Upon making the variable of integration substitution,

$$\xi = t - \tau \qquad [32]$$

Equation [31] becomes

$$\frac{u\left(t,x,\frac{r}{r_0}\right)}{u_0} = 1 + \int_0^t \begin{Bmatrix} 0; & 0<\frac{x}{c\,u_0}<\xi \\ 1; & \frac{x}{c\,u_0}>\xi \end{Bmatrix} \frac{\partial F\left(\nu'\xi, \frac{r}{r_0}\right)}{\partial \xi} d\xi \qquad [33]$$

Hence, when $x/\,c\,u_0 < t$,

$$\frac{u\left(t,x,\frac{r}{r_0}\right)}{u_0} \equiv \frac{u\left(x,\frac{r}{r_0}\right)}{u_0} = F\left(\frac{\nu' x}{c\,u_0}, \frac{r}{r_0}\right) = F\left(\lambda\, x, \frac{r}{r_0}\right) \qquad [34]$$

where, as before,

$$\lambda = \frac{\nu'}{c\,u_0} = \frac{\nu}{c\,u_0 r_0^2}$$

However, when $x/\,c\,u_0 > t$,

$$\frac{u\left(t,x,\frac{r}{r_0}\right)}{u_0} \equiv \frac{u\left(t,\frac{r}{r_0}\right)}{u_0} = F\left(\nu' t, \frac{r}{r_0}\right) \qquad [35]$$

The solution for the problem of a cylinder in linearized, axial motion experiencing an impulsive start, as given by Equations [34] and [35], is seen to be characterized by a division of the flow field into two distinct regimes, one of which is independent of the time variable t, while the other is independent of the axial spatial variable x. As shown in Figure 3, the boundary layer at a particular instant t has developed from zero thickness at the leading edge ($x = 0$) to some finite value at the point $x = c\,u_0 t$ and is constant at this value over the remaining portion of the cylinder. A comparison of Equation [34] with Equation [9] shows that the time independent regime of the cylinder in the case of an impulsive start is identical with the

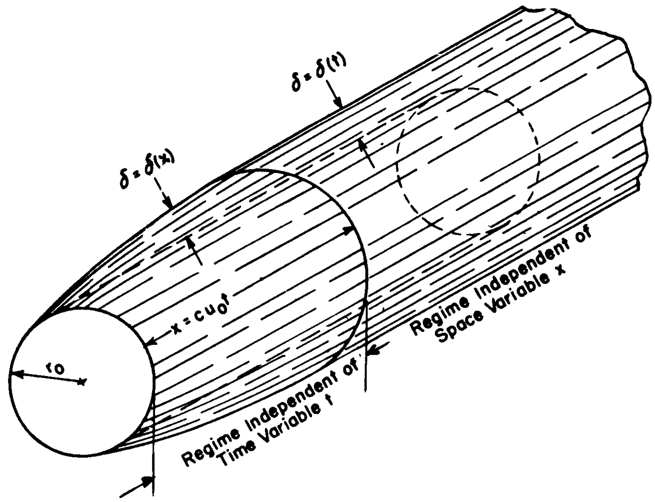

Figure 3 - The Behavior of the Boundary Layer on a Cylinder Experiencing Axially Symmetric Impulsive Motion

regime obtaining for the cylinder in steady motion. It is also of interest to note that the time dependent regime obtaining in this case, which is described by Equation [35], is identical with the case of the impulsive motion of an infinitely long cylinder previously discussed as the extension of the Rayleigh analysis to axial flow. A similar result for the case of the flat plate experiencing an impulsive start was found by Tulin.[3]

THE SOLUTION FOR STEADY MOTION WITH PRESSURE GRADIENT, WITH PARTICULAR REFERENCE TO THE FALKNER-SKAN TYPE

In the case of a cylinder in steady, axial motion subjected to an external pressure gradient, for which $u_1 = u_1(x)$, Equation [7] reduces to

$$2u'\frac{du_1}{dx} + c\,u_1\frac{\partial u'}{\partial x} - \frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u'}{\partial r}\right) = 0 \qquad [36a]$$

with the boundary conditions

$$u'(0,r) = 0, \qquad u'(x,r_0) = u_1, \qquad \lim_{r \to \infty} u'(x,r) = 0 \qquad [36b]$$

where, again, $c = 0.34627$ as defined by Equation [16].

The introduction of the dependent and independent variable transformations

$$w = u'u_1^{2/c}, \qquad X = \frac{1}{c}\int_0^x \frac{dp}{u_1(p)}, \qquad \sigma = \frac{r}{r_0} \qquad [37]$$

where it is assumed that X exists, reduces the problem to

$$\left.\begin{array}{l}\dfrac{\partial w}{\partial \mathrm{X}} - \dfrac{\nu'}{\sigma}\dfrac{\partial}{\partial \sigma}\left(\sigma \dfrac{\partial w}{\partial \sigma}\right) = 0 \\[2ex] w(0,\sigma) = 0, \quad w(\mathrm{X},1) = u_1^{\frac{2+c}{c}}, \quad \lim_{\sigma \to \infty} w(\mathrm{X},\sigma) = 0\end{array}\right\} \quad [38]$$

where

$$\nu' = \frac{\nu}{r_0^2}$$

The differential equation is now in the form of the diffusion equation, and the problem is suitable for treatment by Duhamel's theorem (loc. cit.) which produces the solution

$$w(\mathrm{X},\sigma) = \int_0^X [u_1(\xi)]^{\frac{2+c}{c}} \frac{\partial F[\nu'(\mathrm{X}-\xi),\sigma]}{\partial \xi}\, d\xi \qquad [39]$$

from which

$$\frac{u(\mathrm{X},\sigma)}{u_1(\mathrm{X})} = 1 - \int_0^X \left[\frac{u_1(\xi)}{u_1(\mathrm{X})}\right]^{\frac{2+c}{c}} \frac{\partial F[\nu'(\mathrm{X}-\xi),\sigma]}{\partial \xi}\, d\xi \qquad [40]$$

where the function F is as defined by Equation [10].

The general solution of the problem of the linearized, steady, axial motion of a cylinder, subjected to a class of arbitrary pressure gradients, is given by Equation [40]. In the absence of a pressure gradient, i.e., when $u_1(x) = \text{const.} = u_0$, [40] reduces to the solution given by Equation [9].

The so-called Falkner-Skan external pressure distribution, defined as

$$u_1(x) = bx^m \qquad [41]$$

is of particular interest, inasmuch as the exact solution of the flat-plate boundary-layer problem with such pressure distributions has been obtained by Falkner and Skan [4] and by Hartree.[5] The solution of the linearized boundary-layer equation for the steady, axial motion of a cylinder experiencing the Falkner-Skan type of external pressure gradient is contained as a special case of the general solution [40], for which case the variable X is defined as

$$\mathrm{X} = \frac{1}{c}\int_0^X \frac{dp}{bp^m} = \frac{1}{c(1-m)}\,\frac{x}{u_1(x)}, \qquad m < 1 \qquad [42]$$

The practical difficulties associated with the calculation of velocity and wall shear stress from the integral representation [40] can be circumvented by a technique which was first introduced by Goldstein [11] and used by Carslaw and Jaeger[8] in their solution of the heat conduction problem referred to previously. This technique, in essence, consists of expanding the Laplace transform of the solution in an asymptotic series and then obtaining the inversion term-by-term. The result of these operations is an asymptotic solution of the problem.

Although the velocity profiles can be obtained in this way, only the wall shear stress, which is of primary interest in the present paper, will be considered here. Thus, starting with the differential equation system [38], with $u_1(x)$ defined by [41] and X by [42], application of the Laplace transform with respect to X, i.e.,

$$\bar{w}(\alpha,\sigma) = \int_0^\infty e^{-\alpha X}\, w(X,\sigma)\, dX \qquad [43]$$

yields the ordinary differential equation system

$$\left.\begin{aligned} &\frac{d^2\bar{w}}{d\sigma^2} + \frac{1}{\sigma}\frac{d\bar{w}}{d\sigma} - \frac{\alpha}{\nu'}\bar{w} = 0 \\ &\bar{w}(\alpha,1) = \frac{A}{\alpha^{n+1}}, \qquad \lim_{\sigma\to\infty} \bar{w}(\alpha,\sigma) = 0 \end{aligned}\right\} \qquad [44]$$

where

$$A = b^{\frac{2+c}{c}}\,[bc(1-m)]^n\,\Gamma(n+1)$$

Γ is the gamma function, and

$$n = \frac{m(2+c)}{c(1-m)} \qquad [45]$$

The solution of the system [44] is

$$\bar{w}(\alpha,\sigma) = \frac{A}{\alpha^{n+1}}\,\frac{K_0\left(\sqrt{\frac{\alpha}{\nu'}}\,\sigma\right)}{K_0\left(\sqrt{\frac{\alpha}{\nu'}}\right)} \qquad [46]$$

where K_0 is the modified Bessel function of zero order of the second kind. It then follows that

$$\left[\frac{\partial\bar{w}(\alpha,\sigma)}{\partial\sigma}\right]_{\sigma=1} = -\frac{A}{\sqrt{\nu'}}\,\frac{1}{\alpha^{n+1/2}}\,\frac{K_1\left(\sqrt{\frac{\alpha}{\nu'}}\right)}{K_0\left(\sqrt{\frac{\alpha}{\nu'}}\right)} \qquad [47]$$

where K_1 is the modified Bessel function of first order of the second kind. This can now be expanded asymptotically in a form corresponding to small values of the parameter

$$\kappa = \frac{\nu x}{u_1(x)\, r_0^2}$$

(it should be noted that

$$\kappa = \frac{\nu x}{u_0 r_0^2}$$

previously defined in the case of no pressure gradient is a special case of the parameter κ here defined) to obtain

$$\left[\frac{\partial \bar{w}(\alpha,\sigma)}{\partial \sigma}\right]_{\sigma=1} = - A\left\{\frac{1}{\nu'^{1/2}\alpha^{n+1/2}} + \frac{0.50000}{\alpha^{n+1}} - \frac{0.12500\,\nu'^{1/2}}{\alpha^{n+3/2}} + \frac{0.12500\,\nu'}{\alpha^{n+2}} - \frac{0.19532\,\nu'^{3/2}}{\alpha^{n+5/2}} + \ldots\right\} \qquad [48]$$

(The details of the development of this asymptotic expansion are presented in the Appendix.)
The term-by-term inverse of this transform is

$$\left[\frac{\partial w(x,\sigma)}{\partial \sigma}\right]_{\sigma=1} = - [u_1(x)]^{\frac{2+c}{c}}\,\Gamma(n+1)\left\{\frac{1}{\Gamma\left(n+\frac{1}{2}\right)}\left[\frac{\kappa}{c(1-m)}\right]^{-1/2} + \frac{0.50000}{\Gamma(n+1)} - \frac{0.12500}{\Gamma\left(n+\frac{3}{2}\right)}\left[\frac{\kappa}{c(1-m)}\right]^{1/2} + \frac{0.12500}{\Gamma(n+2)}\,\frac{\kappa}{c(1-m)} - \frac{0.19532}{\Gamma\left(n+\frac{5}{2}\right)}\left[\frac{\kappa}{c(1-m)}\right]^{3/2} + \ldots\right\} \qquad [49]$$

from which the wall shear stress is then easily obtained as

$$\tau_0(x,\kappa,m) = - \frac{\mu}{r_0[u_1(x)]^{2/c}}\left[\frac{\partial w(x,\sigma)}{\partial \sigma}\right]_{\sigma=1} = \mu\left[\frac{c(1-m)}{\nu x}\right]^{1/2}[u_1(x)]^{3/2}\,\Gamma(n+1)\left\{\frac{1}{\Gamma\left(n+\frac{1}{2}\right)} + \frac{0.50000}{\Gamma(n+1)}\left[\frac{\kappa}{c(1-m)}\right]^{1/2} - \frac{0.12500}{\Gamma\left(n+\frac{3}{2}\right)}\,\frac{\kappa}{c(1-m)} + \frac{0.12500}{\Gamma(n+2)}\left[\frac{\kappa}{c(1-m)}\right]^{3/2} - \frac{0.19532}{\Gamma\left(n+\frac{5}{2}\right)}\left[\frac{\kappa}{c(1-m)}\right]^{2} + \ldots\right\} \qquad [50]$$

For small values of κ, [50] is the asymptotic solution of the linearized boundary-layer equation for the wall shear stress on a cylinder in steady, axial motion, experiencing an external pressure gradient of the Falkner-Skan type. For the case of zero pressure gradient, i.e., for $m = 0$, Equation [50] reduces to Equation [13]. For the case of the flat plate with a Falkner-Skan type of external pressure gradient, a limiting process as $\kappa \to 0$, and hence as $r_0 \to \infty$, reduces Equation [50] to

$$\tau_0(x,0,m) = \mu\left[\frac{c(1-m)}{\nu x}\right]^{1/2}[u_1(x)]^{3/2}\,\frac{\Gamma(n+1)}{\Gamma\left(n+\frac{1}{2}\right)} \qquad [51]$$

which contains the Blasius solution as the special case $m = 0$. In a linearized treatment of the flat plate problem with Falkner-Skan type pressure gradients, Tulin[3] obtained the same expression (with $c = 1$). It is of interest to note that, in the case of the flat plate, the present linearized theory predicts a separation velocity profile (corresponding to $\tau_0 = 0$) at $m = -0.0796$ in comparison with the value $m = -0.0904$ calculated from the exact theory by Falkner and Skan.[4]

The ratio of wall shear stresses on the cylinder to those on the flat plate, with both configurations experiencing identical external pressure gradients of the Falkner-Skan type, is then

$$\frac{\tau_0(x,\kappa,m)}{\tau_0(x,0,m)} = 1 + 0.50000\frac{\Gamma\left(n+\frac{1}{2}\right)}{\Gamma(n+1)}\left[\frac{\kappa}{c(1-m)}\right]^{1/2} - 0.12500\frac{\Gamma\left(n+\frac{1}{2}\right)}{\Gamma\left(n+\frac{3}{2}\right)}\frac{\kappa}{c(1-m)}$$
$$+ 0.12500\frac{\Gamma\left(n+\frac{1}{2}\right)}{\Gamma(n+2)}\left[\frac{\kappa}{c(1-m)}\right]^{3/2} - 0.19532\frac{\Gamma\left(n+\frac{1}{2}\right)}{\Gamma\left(n+\frac{5}{2}\right)}\left[\frac{\kappa}{c(1-m)}\right]^{2} + \ldots \qquad [52]$$

valid for small values of the parameter κ and for $m < 1$. However, the limiting process as m approaches unity yields

$$\frac{\tau_0(x,\kappa,1)}{\tau_0(x,0,1)} = 1 + 0.50000\left(\frac{\kappa}{2+c}\right)^{1/2} - 0.12500\ \frac{\kappa}{2+c} + 0.12500\left(\frac{\kappa}{2+c}\right)^{3/2}$$
$$- 0.19532\left(\frac{\kappa}{2+c}\right)^{2} + \ldots \qquad [53]$$

A significant aspect of this result is that the parameter κ is now independent of the variable x, viz.,

$$\kappa = \frac{\nu}{b r_0^2}$$

since $u_1(x) = bx$, from which it is concluded that the velocity profiles are "similar" for the case $m = 1$. Because of their existence in the linearized theory for $m = 1$, it is anticipated that "similar" velocity profiles also exist for $m = 1$ in the exact theory. An investigation confirming this supposition is presented in detail in the following section.

In Figure 4, the inverse ratio

$$\frac{\tau_0(x,0,m)}{\tau_0(x,\kappa,m)}$$

is shown as a function of the parameter κ for values of the pressure distribution index m ranging from -0.0796 to unity. It is seen that for a given pressure gradient, i.e., for a fixed m, whether favorable (m positive) or adverse (m negative), the "transverse curvature effect" increases the wall shear stress of the cylinder, i.e., τ_0 on the cylinder increases with increasing curvature. In addition, it is noted that for constant curvature, i.e., for a fixed κ, increasingly adverse pressure gradients produce increasingly greater percentage-wise increases in cylinder wall shear stress over that of the flat plate. Finally, it may be concluded that the cylinder can sustain more adverse pressure gradients without boundary layer separation than can the flat plate.

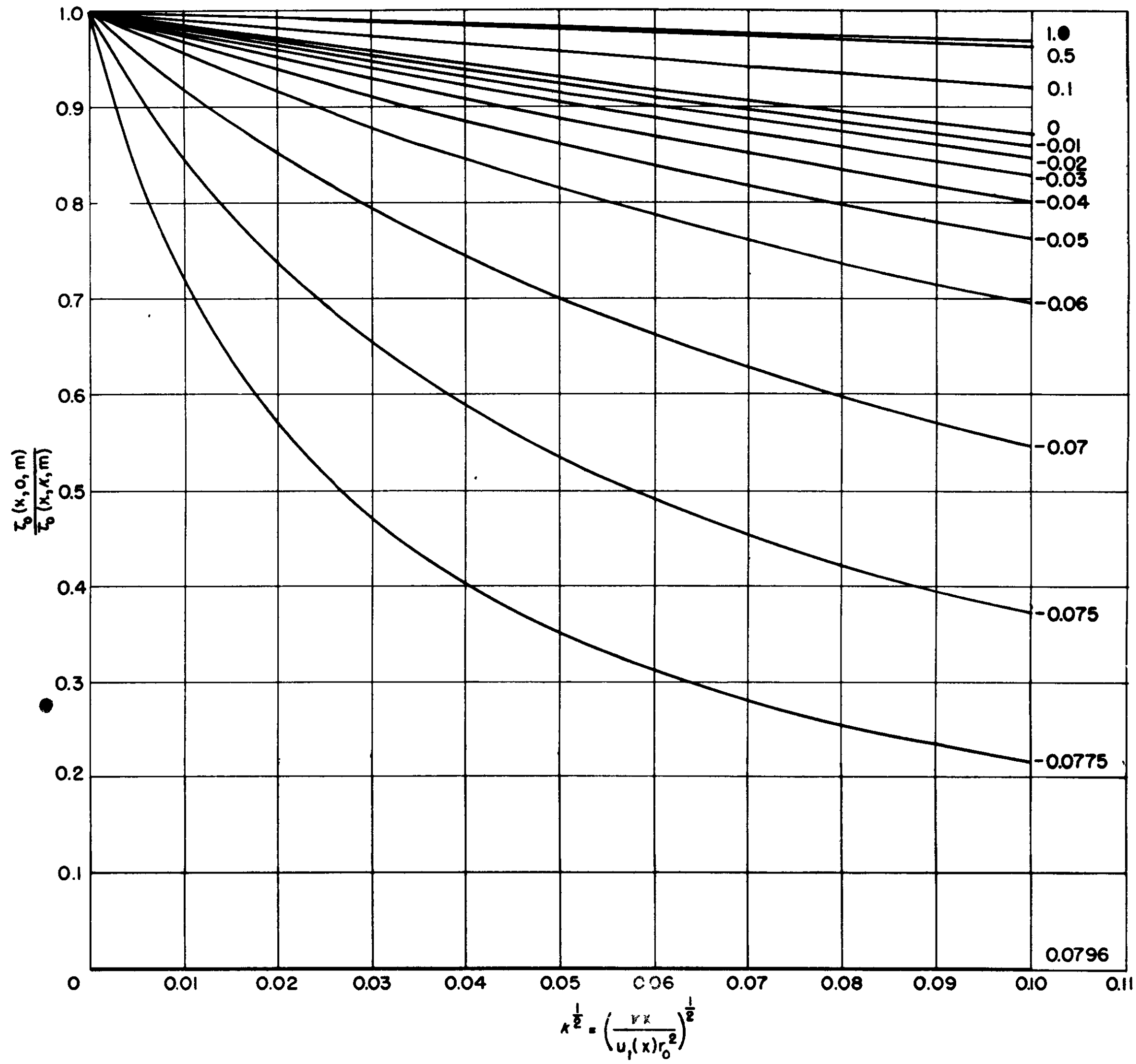

Figure 4 - The Ratio of Wall Shear Stress on a Flat Plate to Wall Shear Stress on a Cylinder in the Case of Steady, Axial Motion with External Pressure Gradient $u_1 = bx^m$

THE EXACT SOLUTIONS FOR "SIMILAR" VELOCITY PROFILES

In certain problems concerned with the motion of a fluid past an obstacle or along a surface, the equations governing the motion are the boundary-layer equation and the continuity equation, with, of course, the appropriate boundary conditions. If, by the introduction of suitable variable transformations, the problem of solving this set of partial differential equations can be reduced to the problem of solving a single ordinary differential equation, then the solution is said to be "similar" since the profiles of the velocity components at each station on the obstacle or surface have the same shape and differ only by a scalar which is a function of the distance from the origin. A number of boundary-layer problems with "similar" velocity profiles have been solved by numerical methods, and their importance lies in the fact that they represent exact solutions of the boundary-layer equations. The best known "similar" velocity profiles are those that have been found to exist for the flat plate with no pressure gradient by Prandtl[12] and with pressure gradients of the form $u_1(x) = bx^m$ by Falkner and Skan.[4] For axially symmetric flows, Pretsch[13] has found "similar" profiles for the case of the infinitely thin cylinder in the absence of pressure gradients. It will now be shown that "similar" profiles obtain on the cylinder in steady, axial motion when the external pressure gradient is defined by $u_1(x) = bx$. As anticipated from the results of the linearized theory (see the preceding section), the particular case $m = 1$ was the only one of Falkner-Skan type of pressure gradients for which "similar" profiles were found to exist.

For this case the boundary-layer and continuity equations are, respectively,

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = bx^2 + \nu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) \qquad [54]$$

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial r} = 0 \qquad [55]$$

with the boundary conditions

$$u(x,r_0) = 0, \qquad v(x,r_0) = 0, \qquad \lim_{r \to \infty} u(x,r) = bx \qquad [56]$$

The continuity equation is easily satisfied by a stream function Ψ, defined as

$$u = \frac{1}{r}\frac{\partial \Psi}{\partial r}, \qquad v = -\frac{1}{r}\frac{\partial \Psi}{\partial x} \qquad [57]$$

The introduction of the transformations

$$\zeta = \frac{br^2}{2\nu}, \qquad \Psi = \nu x \Phi(\zeta) \qquad [58]$$

then reduces the problem to the ordinary differential equation system

$$\left.\begin{aligned} &2\zeta\Phi'''(\zeta) = -\Phi(\zeta)\Phi''(\zeta) + [\Phi'(\zeta)]^2 - 1 - 2\Phi''(\zeta) \\ &\Phi(\zeta_0) = 0, \quad \Phi'(\zeta_0) = 0, \quad \lim_{\zeta \to \infty} \Phi'(\zeta) = 1 \end{aligned}\right\} \qquad [59]$$

where

$$\zeta_0 = \frac{br_0^2}{2\nu}$$

and

$$u = bx\Phi'(\zeta), \qquad v = -\frac{\nu}{r}\Phi(\zeta) \qquad [60]$$

The existence of "similar" velocity profiles in the case under consideration has thus been demonstrated. However, it may be pointed out that, because of the boundary conditions, an independent quadrature of the system [59] is required for each value of ζ_0.

Since solutions of Equation [59] are exact solutions of the boundary-layer equations for the cylinder with the particular Falkner-Skan pressure gradient $u_1(x) = bx$, such solutions would be of value in evaluating the validity of such approximate solutions as are presented in the previous portions of this paper.

SUMMARY

The effect of transverse curvature on laminar boundary-layer characteristics has been investigated through the use of an approximate, linearized theory.

In the case of a cylinder in steady, axial motion with no external pressure gradient, it is shown that the transverse curvature increases the wall shear stress.

A general solution is obtained for the case of arbitrary unsteady motion of a cylinder initially at rest with no external pressure gradient; the particular case of an impulsive start is considered in detail, and it is shown that for such motion the flow field is divided into two distinct regimes, one independent of the time variable, the other independent of the axial spatial variable.

The effect of external pressure gradients in the case of steady motion is investigated, and general solutions for a large class of external pressure gradients are obtained. The particular class of Falkner-Skan type external pressure gradients are treated in detail. The results of this latter investigation show that the transverse curvature increases the wall shear stress for both favorable and adverse pressure gradients; the increase for favorable gradients is less than the increase for uniform flow, and the converse is true for adverse gradients.

So-called "similar" velocity profiles are shown to exist for a particular set of flows contained in the class of Falkner-Skan type external pressure gradients, and the ordinary differential equation defining them is derived.

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APPENDIX

ASYMPTOTIC EXPANSION OF LAPLACE TRANSFORM

The Laplace transform

$$\left[\frac{\partial \bar{w}(\alpha,\sigma)}{\partial \sigma}\right]_{\sigma=1} = -\frac{A}{\nu'^{1/2}\,\alpha^{n+1/2}} \frac{K_1\left(\sqrt{\frac{\alpha}{\nu}}\right)}{K_0\left(\sqrt{\frac{\alpha}{\nu'}}\right)}$$

where

$$\nu' = \frac{\nu}{r_0^{\,2}}$$

is to be expanded into an asymptotic series such that its inverse will be valid for small values of the parameter

$$\kappa = \frac{\nu x}{u_1(x)\, r_0^{\,2}}$$

Since the variable α of the transform corresponds to the variable

$$X = \frac{1}{c(1-m)} \frac{x}{u_1(x)}$$

of its inverse, this requires that the expansion of the transform be valid for large values of α/ν'.

The asymptotic expansion of the modified Bessel function for large values of the argument z (see, for instance, Reference 14, p. 202) is

$$K_s \sim \left(\frac{\pi}{2z}\right)^{1/2} e^{-z}\left[1+\sum_{j=1}^{\infty}\frac{(s,j)}{(2z)^j}\right] \text{ where } (s,j) = \frac{[4s^2-1^2]\,[4s^2-3^2]\ldots[4s^2-(2j-1)^2]}{2^{2j}\cdot j!}$$

from which

$$K_1(z) \sim \left(\frac{\pi}{2z}\right)^{1/2} e^{-z}\left\{1+\frac{3}{2^2\cdot 1!}\,\frac{1}{2z}-\frac{3\cdot 5}{2^4\cdot 2!}\,\frac{1}{(2z)^2}+\frac{3\cdot 5\cdot 21}{2^6\cdot 3!}\,\frac{1}{(2z)^3}-\frac{3\cdot 5\cdot 21\cdot 45}{2^8\cdot 4!}\,\frac{1}{(2z)^4}+\ldots\right\}$$

and

$$K_0(z) \sim \left(\frac{\pi}{2z}\right)^{1/2} e^{-z}\left\{1-\frac{1}{2^2\cdot 1!}\,\frac{1}{2z}+\frac{1\cdot 9}{2^4\cdot 2!}\,\frac{1}{(2z)^2}-\frac{1\cdot 9\cdot 25}{2^6\cdot 3!}\,\frac{1}{(2z)^3}+\frac{1\cdot 9\cdot 25\cdot 49}{2^8\cdot 4!}\,\frac{1}{(2z)^4}-\ldots\right\}$$

Therefore,

$$\frac{K_1(z)}{K_0(z)} \sim \frac{1+0.37500(z)^{-1}-0.11719(z)^{-2}+0.10254(z)^{-3}-0.14420(z)^{-4}+\ldots}{1-0.12500(z)^{-1}+0.07031(z)^{-2}-0.07324(z)^{-3}+0.11215(z)^{-4}-\ldots}$$

and by simple division

$$\frac{K_1(z)}{K_0(z)} = 1+\frac{0.50000}{z}-\frac{0.12500}{z^2}+\frac{0.12500}{z^3}-\frac{0.19532}{z^4}+\ldots$$

Thus,

$$\left[\frac{\partial \bar{w}(\alpha,\sigma)}{\partial \sigma}\right]_{\sigma=1} \sim -\frac{A}{\nu'^{1/2}\alpha^{n+1/2}}\left\{1 + 0.50000\left(\frac{\nu'}{\alpha}\right)^{1/2} - 0.12500\,\frac{\nu'}{\alpha} + 0.12500\left(\frac{\nu'}{\alpha}\right)^{3/2} - 0.19532\left(\frac{\nu'}{\alpha}\right)^{2} + \ldots\right\}$$

$$\sim -A\left\{\frac{1}{\nu'^{1/2}\alpha^{n+1/2}} + \frac{0.50000}{\alpha^{n+1}} - \frac{0.12500\,\nu'^{1/2}}{\alpha^{n+3/2}} + \frac{0.12500\,\nu'}{\alpha^{n+2}} - \frac{0.19532\,\nu'^{3/2}}{\alpha^{n+5/2}} + \ldots\right\}$$

The above expression, then, is the asymptotic expansion of

$$\left[\frac{\partial \bar{w}(\alpha,\sigma)}{\partial \sigma}\right]_{\sigma=1}$$

valid for large values of α/ν'.

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The effect of external pressure gradients in the case of steady motion is investigated, and general solutions for a large class of external pressure gradients are obtained. The particular class of Falkner-Skan type pressure gradients are treated in detail. The results of this latter investigation show that the transverse curvature increases the wall shear stress for both favorable and adverse pressure gradients. The increase for favorable gradients is less than the increase for uniform flow; the converse is true for adverse gradients.

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