#2

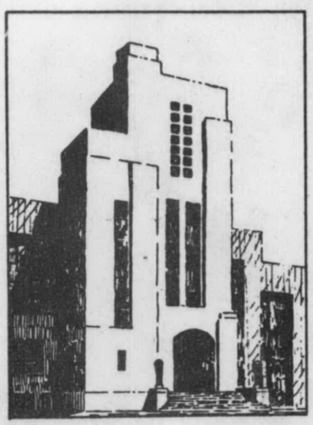
# NAVY DEPARTMENT
## THE DAVID W. TAYLOR MODEL BASIN
WASHINGTON 7, D.C.

SLAMMING DUE TO PURE PITCHING
MOTION

By

M. Alison Todd





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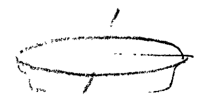
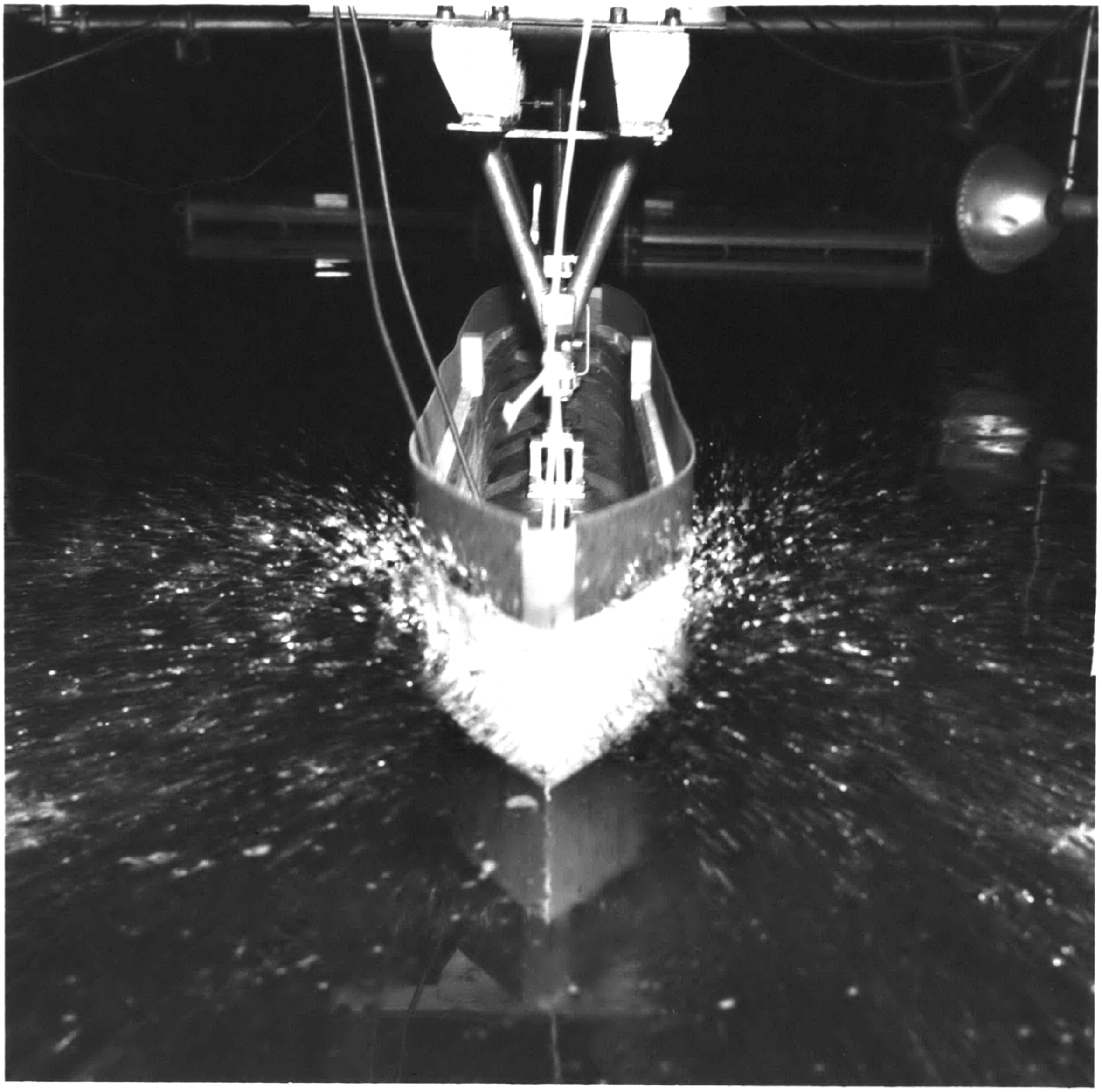
# NOTATION

| | | |
|---|---|---|
| $A_i\ (\theta)$ | = | Immersed cross-sectional area of $i^{th}$ transverse section |
| $\bar{A}\ (\theta)$ | = | Waterplane area |
| $B$ | = | Total beam at waterline |
| $F_W$ | = | Force due to weight of model |
| $F_B$ | = | Force on model due to buoyancy of displaced water |
| $H$ | = | Draft at pivot point |
| $I$ | = | Moment of inertia of model about axis  |
| $I_a(\theta)$ | = | Added moment of inertia of model |
| $M_a(\theta)$ | = | Statical moment due to added mass of model |
| $M_B(\theta)$ | = | Moment due to buoyancy of displaced water |
| $M_W$ | = | Moment due to weight of model |
| $S$ | = | Keel wetted length |
| $U$ | = | Forward speed of advance |
| $V_i$ | = | Vertical velocity of ith transverse section |
| $X,Y,Z$ | = | Coordinate System translating with the model |
| $Z_i$ | = | Draft of $i^{th}$ transverse section |
| $b_i$ | = | Half beam of ith section |
| $c_i$ | = | Modified half beam (considering piled up water) |
| $g$ | = | Acceleration due to gravity |
| $\bar{i},\bar{j},\bar{k}$ | = | Unit vectors in x,y,z directions |
| $\bar{i}',\bar{j}',\bar{k}'$, | = | Unit vectors in X,Y,Z directions |
| $m$ | = | Mass of model |
| $M_a$ | = | Added mass of model |
| $p$ | = | Instantaneous pressure at a point on the hull |
| $p_o$ | = | $\frac{\rho V^2}{2}$ |

| | | |
|---|---|---|
| $q$ | = | $\sqrt{\Phi} = \overset{d\theta/dt}{\dot{\theta}}$ = Angular velocity in xz plane |
| $t$ | = | Time |
| $u(c)$ | = | $dZ(c)/dc$ = Slope of modified transverse section |
| $u, w$ | = | Velocity components of an element in xz directions |
| $x, y, z$ | = | Coordinate system translating and rotating with model |
| $\Delta x_i$ | = | Longitudinal distance between stations $i-\frac{1}{2}$, and $i+\frac{1}{2}$ |
| $z_i(y)$ | = | Equation of ith transverse section |
| $z_i(c)$ | = | Equation of modified transverse section |
| $\gamma$ | = | Constant |
| $\delta$ | = | $V^2/\dot{V}c$ |
| $\eta$ | = | $y/c$ = Dimensionless beam coordinate |
| $\theta$ | = | Pitch angle |
| $\theta_o$ | = | Initial pitch angle |
| $\mathcal{K}_i(\theta)$ | = | Inertia coefficient of ith section |
| $\xi$ | = | $\sqrt{1-\eta^2}$ |
| $\rho$ | = | Density of water |
| $\phi$ | = | $(\dot{\theta})^2$ $\left(\frac{d\theta}{dt}\right)^2$ WHAT IS $\Phi$ |

Superscript dot denotes differentiation with respect to time



Slamming of the M.S.San Francisco Model.

# SLAMMING DUE TO PURE PITCHING MOTION

By

M. Alison Todd

## Abstract

In this report a method is developed for predicting maximum slamming forces experienced by a ship's hull, when its impact with the free water surface is due to pure pitching motion.

The results of the theoretical investigation are compared with those obtained from a series of experiments performed upon an eight foot model of the M.S. SAN FRANCISCO, in which the model is given an initial angular displacement about a fixed axis of rotation about 30 percent of the length from aft, and allowed to pitch in her natural period. The results are given in the form of theoretical and experimental acceleration traces. The pressure distribution over some of the forward sections is also obtained theoretically, and the total moment due to slamming is estimated both from this and from the differential equation of slamming. In the case of pure pitching motion about this axis, the general location of the region of high pressure at the time of slamming is found to be about 20 percent aft of the forward perpendicular.

The effect of slamming on the pitching motion of the ship is shown to be very slight. The effect on slamming of finite speed of advance, for the case of an artificially pitched model moving through calm water, is also very small.

## Introduction

Dr. Szebehely(1) has investigated the hydrodynamic principles involved in impact problems of ship's hulls with the free water surface, with special application to the phenomenon of slamming. Slamming may be defined as a sudden change in the vertical or angular acceleration of a ship, which results in the elastic vibration of the hull. The problem has been divided into three phases. The first was the establishment of the basic principles(1) The predictions of slamming forces based on the application of these principles to a V-shaped wedge were checked by dropping a model of the M.S. SAN FRANCISCO onto the water surface. The present report constitutes a certain refinement of this case, since actual ship lines were used in the calculations and the draft of each section was varied with time to correspond to the pitching motion.

The last phase, on which considerable work has already been done, is concerned with the slamming forces experienced by a ship towed freely through regular and random waves.

The problem of slamming due to pure pitching motion is considered first for the more simple case of no forward speed, in which case the differential equation can be handled directly by numerical methods; and secondly for finite speed of advance, in which case even a first integral of the equation cannot be obtained immediately, and a method of successive approximations must be used.

In the latter case attempts were made to circumvent this complication, and by means of certain assumptions to produce a simple method of solution at least in the immediate region of slamming. The details of this will be presented later.

In obtaining the calculated acceleration curve certain factors such as viscosity were neglected, but attempts were made to include all factors which would influence the results appreciably. The correction for the finite aspect ratio in the calculation of the added moment of inertia had a maximum effect of about 4 percent, and so was included.

The slamming forces experienced by a ship in pitching motion are dependent to a large degree upon the shape of the hull. A fine ship, for example, will experience much less violent slamming forces than a ship with fuller lines. For this reason it was necessary to perform the calculations illustrating the method for a particular ship (the M.S. SAN FRANCISCO), and, in the course of the calculations, to fit the pertinent parts of her lines with analytical curves, in actual fact with high degree polynomials (see fig. 3). This would have to be done for any other ship for which this method of predicting slamming forces were to be used. The M.S. SAN FRANCISCO was chosen for this work because a great deal of model and full-scale trial data are available for that ship.

## The Differential Equation of Slamming

In the general case the moment equation may be written:*

$$-\left[I + I_a(\theta)\right]\frac{d\theta}{dt} + \frac{U}{g}\left[\theta M_a(\theta) + \theta_0 M_a(\theta_0)\right] = \int_{t=0}^{t}\left[M_w + M_B(\theta)\right]dt \qquad (1)$$

?

where

| | | |
|---|---|---|
| $I$ | = Moment of inertia of model | $= \sum_i (\Delta m)_i x_i^2$ |
| $I_a(\theta)$ | = Added moment of inertia of model | $= \sum_i (\Delta m_a)_i x_i^2$ |
| $M_w$ | = moment due to weight of model | $= g\sum_i (\Delta m)_i x_i$ |
| $M_a(\theta)$ | = statical moment due to added mass | $= g\sum (\Delta m_a)_i x_i$ |
| $M_B(\theta)$ | = moment of buoyancy of displaced water | $= \sum (\Delta F_B)_i x_i$ |
| $U$ | = forward speed of advance | |
| $\theta$ | = angle of pitch | |
| $g$ | = acceleration due to gravity | |
| $\theta_0$ | = initial pitch angle (at time t = o) | |

* The derivation of this equation is given in the appendix.

In the derivation of this equation and in the calculation of the coefficients of equation (1) an element theory is used.

We define $(\Delta m)_i$ = mass of ith element
$(\Delta m_a)_i$ = added mass of ith element normal to the keel
$(\Delta F_B)_i$ = force on ith element due to buoyancy
$x_i$ = longitudinal coordinate of ith element

The origin is taken at the pivot point; moments are taken positive clockwise; $x_i$ is measured positive forward and forces are positive downward. The pitch angle θ is measured positively upward from the water surface. See fig. 1.

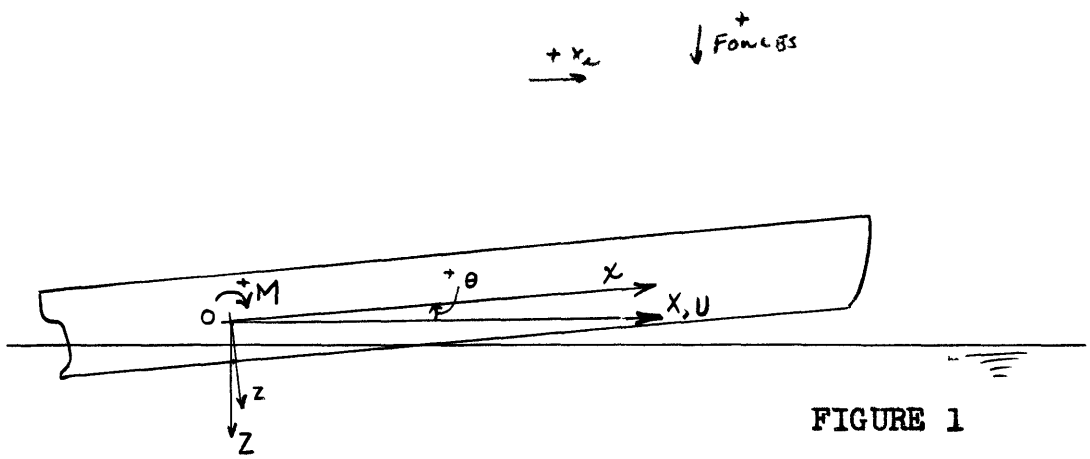



FIGURE 1

In the case of zero speed of advance this equation reduces to

$$\left[I + I_a(\theta)\right]\frac{d\theta}{dt} = \int_{t=0}^{t}\left[M_w + M_B(\theta)\right]dt \qquad (2)$$

The solution of equations (1) and (2) necessitates the evaluation of the functions $I_a(\theta)$, $M_a(\theta)$, $M_B(\theta)$ and the methods of achieving this constitute the next part of this paper.

Calculation of the Statical Moment (or First Moment) Due to Added Mass $M_a(\theta)$

The statical moment is obtained from considerations of added mass, on which subject the literature is quite considerable. The method used here is that first suggested by Professor Lewis (2), in which the element of added mass for a ship's section may be obtained from that of an elliptical section. The method involves multiplying the expression for the added mass of the elliptical

CROSS SECTION AREA

COEF = $\frac{AREA}{BN}$ ??

NO VALUES GIVEN IN GRAPH.

PILED UP WATER?

where? ⇉

?

$C_N$ IS EFFECTIVE BEAM - DEFINITION ??
(IT IS TO ACCOUNT FOR PILED UP WATER BUT STILL WHAT IS $C_N$)

COULD THIS BE TURNED AROUND

section of the same beam to draft ratio (B/H) as that of the ship by a correction factor, which is dependent on the shape of the section. According to this we have the expression for the added mass.

$$(\Delta m_a)_i = K_i \frac{\rho}{2} b_i^2 \pi \Delta x_i \qquad \text{-----------} \quad (3)$$

where $b_i$ = half beam at waterline
$\rho$ = density of fluid
$K_i$ = inertia coefficient
$\Delta x_i = \dfrac{x_{i+1}+x_i}{2} - \dfrac{x_i + x_{i-1}}{2} = \dfrac{x_{i+1}}{2} - \dfrac{x_{i-1}}{2}$

Professor Lewis has obtained the values of $K_i$ for a series of sections similar to those of a ship, and Dr. Todd(3), by plotting these against the B/H ratio and the cross-sectional area coefficient has eliminated the necessity of fitting the lines of a particular ship to the standard series of sections considered by Professor Lewis. These curves are reproduced in fig. (2).

One further correction to this expression for the added mass is necessary for work in the field of impact. The effect of piled up water must be taken into account by the introduction of an effective beam measurement. Dr. Szebehely has given a method for doing this in his consideration of a wedge hitting the water surface(1).

The effective beam $c_i$ is obtained from the analytical curves fitted to the transverse section and replaces $b_i$ in formula (3). It is immediately evident from physical considerations that both $c_i$ and $K_i$ are functions of the pitch angle θ as well as of the shape of the ship's sections.

To find the modified beam $c_i(\theta)$ suppose the ith semi-transverse section to be represented by the equation:*

$$z_i(y) = a_0 + a_1 y_i + a_2 y_i^2 + \cdots + a_n y_i^n \qquad \text{------} \quad (4)$$

when $y=0$ $z=0$ . $a_0=0$

where $y_i$ = coordinate along beam
$z_i$ = coordinate perpendicular to x-y plane
$a_0, a_1, \ldots a_n$ are constants. The origin of the z and y coordinates is the keel at the ith section.

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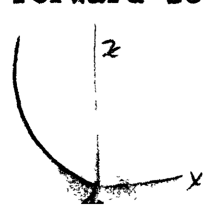
* The equations for the forward sections are given in the appendix.



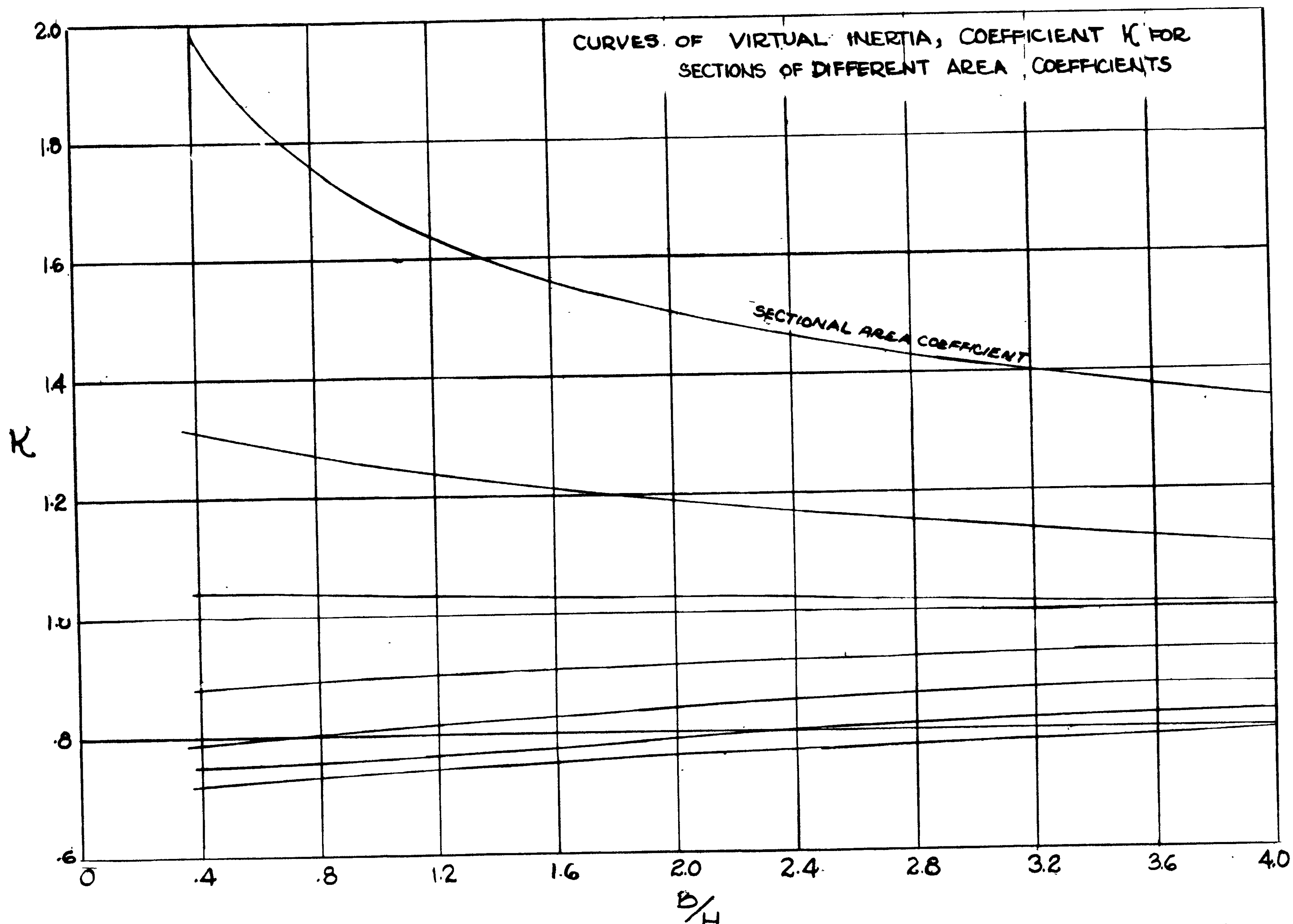

CURVES OF VIRTUAL INERTIA, COEFFICIENT K FOR
SECTIONS OF DIFFERENT AREA COEFFICIENTS
SECTIONAL AREA COEFFICIENT
K
2.0
1.8
1.6
1.4
1.2
1.0
.8
.6
0
.4
.8
1.2
1.6
2.0
2.4
2.8
3.2
3.6
4.0
B/H

FIG (2)

$a_0 = 0$
$a_1 = ?$

IS THERE ANY DIFF
BETWEEN C AND $C_N$

CHECK →

ADDED MASS IS A FUNCTION
OF ACCELERATION

The constant and the linear term can be found by inspection. Plotting the differences between the ship line and the tangent, represented by the first two terms, on a logarithmic scale, gives the power of the next term as the slope of this difference line. The coefficient of this term can be found by comparison of one point. Higher powers with small coefficients may be necessary in order to fit the section further from the keel. From this equation for Z(y) we obtained a modified cross-section Z(c) taking account of piled-up water.

From reference (1)

$$u(c) = \frac{dz(c)}{dc} = \frac{2}{\pi}a_1 + a_2 c + \alpha_2 c^2 + \alpha_3 c^3 + \text{------} + \alpha_n c^n \qquad \text{———} (5)$$

whence

$$z(c) = \frac{2}{\pi}a_1 c + \frac{a_2}{2}c^2 + \frac{\alpha_2}{3}c^3 + \text{------} + \frac{\alpha_n}{n+1}c^{n+1} \qquad \text{———} (6$$

where

$$\alpha_n = a_{n+1} \cdot \begin{cases} \dfrac{2}{\pi}\,\dfrac{2.4.6\cdots\cdots n}{1.3.5\cdots\cdots(n-1)} & \text{for n even} \\ \dfrac{1.3.5.7\cdots\cdots n}{2.4.6\cdots\cdots(n-1)} & \text{for n odd} \end{cases}$$

Fig. (3), given as an example, represents one transverse section, the mathematical approximation, and the modified form $z(C)$. From such figures c may be obtained as a function of x and Z (or θ).

The expression for the added mass is:

$$m_a(\theta) = \sum_i K_i(\theta)\frac{\rho}{2}\pi\, c_i^2(\theta)\,\Delta x_i \qquad \text{————} (7)$$

and for the statical moment due to added mass:

$$M_a(\theta) = g\sum_i K_i(\theta)\frac{\rho}{2}\pi\, x_i\, c_i^2(\theta)\,\Delta x_i \qquad \text{————} (8)$$

where $x_i$ is measured from the pivot point.

A plot of $M_a(\theta)$ for the M.S. SAN FRANCISCO (Draft H = 1.09", aft pivot point) is given in figure (4).

Calculation of the Added Moment of Inertia, $I_a(\theta)$

The added moment of inertia or second moment is also a function of the pitch angle since any change in this angle will result in a change in the mass and distribution of the displaced

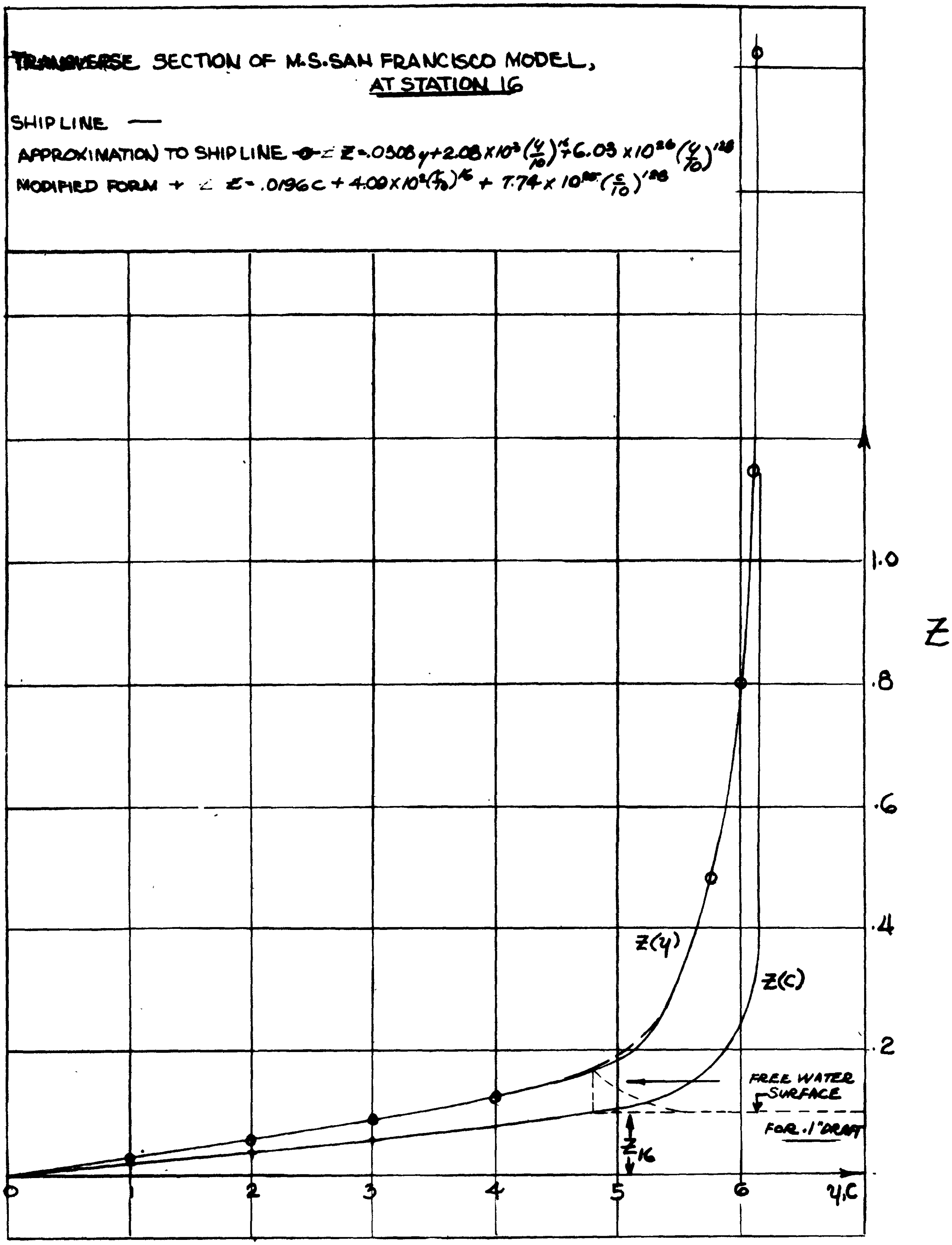

TRANSVERSE SECTION OF M.S. SAN FRANCISCO MODEL,
AT STATION 16
SHIPLINE —
APPROXIMATION TO SHIPLINE $Z = .0308y + 2.08 \times 10^{3}(\frac{y}{10})^{16} + 6.03 \times 10^{26}(\frac{y}{10})^{128}$
MODIFIED FORM + $Z = .0196c + 4.00 \times 10^{2}(\frac{c}{10})^{16} + 7.74 \times 10^{25}(\frac{c}{10})^{128}$
Z(y)
Z(c)
FREE WATER SURFACE
FOR .1" DRAFT
$Z_{16}$
Z
1.0
.8
.6
.4
.2
0
1
2
3
4
5
6
y,c

FIG.(3)

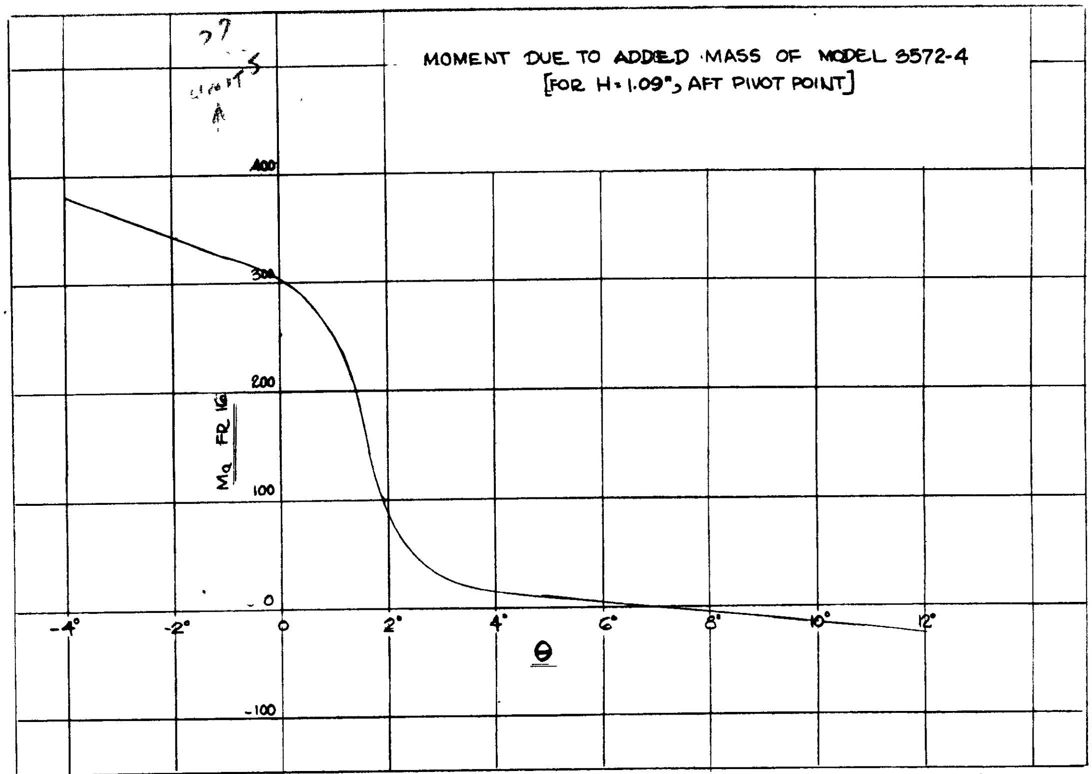

MOMENT DUE TO ADDED MASS OF MODEL 3572-4
[FOR H=1.09", AFT PIVOT POINT]
Ma FR LB
400
300
200
100
0
-100
-4°
-2°
0
2°
4°
6°
8°
10°
12°
θ

FIG. (4)

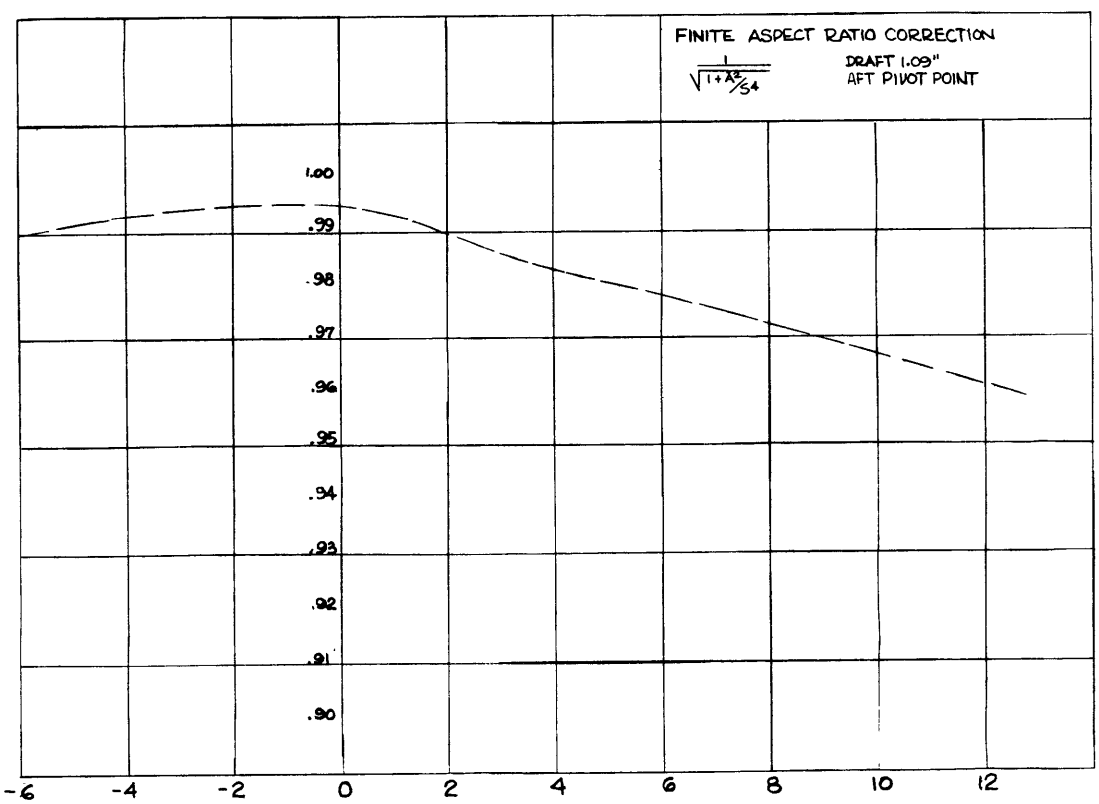

FINITE ASPECT RATIO CORRECTION
$\frac{1}{\sqrt{1+A^2/S4}}$
DRAFT 1.09"
AFT PIVOT POINT
1.00
.99
.98
.97
.96
.95
.94
.93
.92
.91
.90
-6
-4
-2
0
2
4
6
8
10
12

FIG. (5)

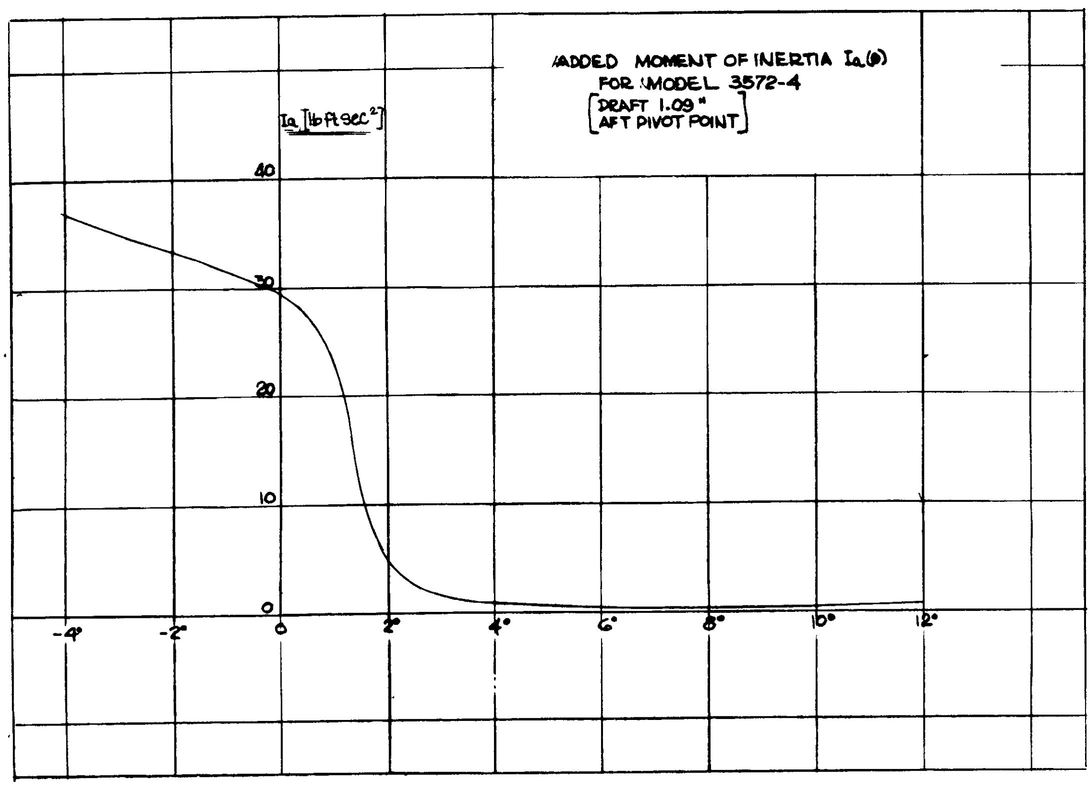

ADDED MOMENT OF INERTIA $I_a(\theta)$
FOR MODEL 3572-4
[DRAFT 1.09"
AFT PIVOT POINT]
$I_a$ [lb ft sec$^2$]
40
30
20
10
0
-4°
-2°
0
2°
4°
6°
8°
10°
12°

FIG.(6)

THIS MAY NOT
BE ONLY
A FUNCTION OF PITCH
ANGLE BUT OF THE
PITCHING FREQ AND WAVE
PARAMETERS

fluid. It can be found immediately from the first moment with an additional correction for the finite aspect ratio of the ship.[1]

$$I_a(\theta) = \frac{1}{\sqrt{\frac{\bar{A}^2(\theta)}{S^4(\theta)} + 1}} \sum_i x_i^2 \, (\Delta m_a)_i \qquad (9)$$

where $\sqrt{\bar{A}^2(\theta)/S^4(\theta)+1}$ is the above-mentioned correction, Fig. (5)

$\bar{A}(\theta)$ = waterline area

$S(\theta)$ = wetted keel length

These last two functions were obtained from trigonometry and graphical integration. The added moment of inertia is analytically very complicated and for subsequent work it was used only numerically, (Fig. (6)).

Calculation of Moment Due to Buoyancy $M_B(\theta)$

Like the added moment of inertia, the moment due to buoyancy is dependent upon the mass and distribution of the displaced fluid and thus is a function of the pitch angle $\theta$.

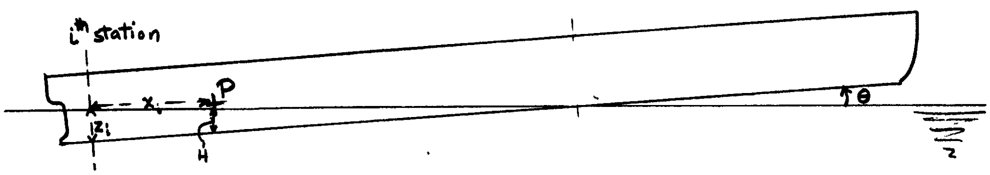



Fig. 7

If P is the pivot point, and R the point at which (for some $\theta$) the keel leaves the water, then we introduce the following notation:

$H$ = draft at pivot point (constant if $\theta$ is small)
$Z_i(\theta)$ = depth of immersion at $i^{th}$ station

$A_i(Z_i)$ = cross-sectional area immersed.

Then $$M_B(\theta) = \sum_i \left(\Delta M_B(\theta)\right)_i \qquad (10)$$

where $$\left(\Delta M_B(\theta)\right)_i = \rho g x_i \left[A_i(Z_i)\right] \Delta x_i \qquad (11.)$$

$\gamma$ $(A_i \, \Delta x_i)$

$\underbrace{\text{Vol} \times \gamma}_{\text{WEIGHT}}$ $\times x$ = MOM

GOOD POINT

$$-I\frac{d^2\theta}{dt^2} - \frac{dI_a}{dt}\dot{\theta} - \frac{d^2\theta}{dt^2} I_a = M_w + M_a(\theta)$$

CHECK THIS

$$\frac{I\dot{\Phi}}{2} = I\ddot{\theta}$$

$$\frac{1}{2}\frac{d\dot{\Phi}}{dt} = \frac{d^2\theta}{dt^2}$$

All these calculations were performed for a fixed draft H, considerably less than that of the load-water line, since it was at this light draft that the experimental results showed slamming. It is evident that these three functions are, however, dependent upon this factor, and the moment due to buoyancy was calculated for a second value of H. This showed that for small θ, (i.e. in the region of slamming) the moment due to buoyancy is extremely sensitive to draft - an increase of 45 percent in draft gives a factor of 4 in $M_B(\theta)$ in this range. [see Fig. (8)]. These functions therefore have always to be computed for a particular ship and a particular draft.

Solution of the Differential Equation for Zero Speed of Advance

We return now to equation (2) for the case of zero speed of advance:

$$-[I + I_a(\theta)]\dot{\theta} = \int_{t=0}^{t} [M_w + M_B(\theta)]\,dt \qquad \phi = 2\frac{d\theta}{dt}\cdot\frac{d^2\theta}{dt^2}$$

or, after differentiation

$$-I\alpha -$$

$$-I\ddot{\theta} - \frac{d}{dt}[I_a(\theta)\dot{\theta}] = M_w + M_B(\theta) \qquad (12)$$

Make the substitution $(\dot{\theta})^2 = \phi(\theta)$
Then equation (12) becomes

$$\left[\frac{d\theta}{dt}\right]^2 = \phi \qquad \frac{d\theta}{dt} = \phi^{1/2}$$

$$2\frac{d\theta}{dt}\cdot\frac{d^2\theta}{dt^2} = \frac{1}{2}\phi^{-1/2}\frac{d\phi}{dt}$$

$$-\frac{1}{2\dot{\theta}}[I_a(\theta) + I]\dot{\phi} - I_a'(\theta)\phi = M_w + M_B(\theta) \qquad (13)$$

where the prime denotes differentiation with respect to θ. The solution of this equation for $\phi$ is:

$$\left(\frac{d\theta}{dt}\right)^2 = \phi = \frac{-2\int_{\theta_0}^{\theta}[M_w + M_B(\theta)][I + I_a(\theta)]\,d\theta}{[I + I_a(\theta)]^2} \qquad (14)$$

$$\frac{d^2\theta}{dt^2} = \frac{1}{2}\frac{1}{\frac{d\theta}{dt}}\frac{d\phi}{dt} \qquad \frac{d\phi}{dt} = 2\frac{d\theta}{dt}\cdot\frac{d^2\theta}{dt^2}$$

Using the previously calculated functions $I_a(\theta)$, $M_B(\theta)$ and the values $M_W$ = 109.1 ft. lb. and I = 27.4 ft. lb. sec$^2$, equation (14) may be solved numerically for $\dot{\theta}$ as a function of θ, and thus, by integrating, for θ in terms of t. This curve of angular velocity versus time is given in figure (9). Graphical differentiation was then used to produce the final acceleration-time curve, figure (10). It is interesting to note that, while the impact

$\frac{.6}{.134} = 4.48 \text{ rad/sec}^2$

$\frac{1}{2}(.134 \text{ sec} \times .6 \text{ rad/sec})$

$= .0803$ RAD

$= 4.6$ DEGREES

$r = 18.481$
$1.363$
$\underline{35.812}$
$55.656$

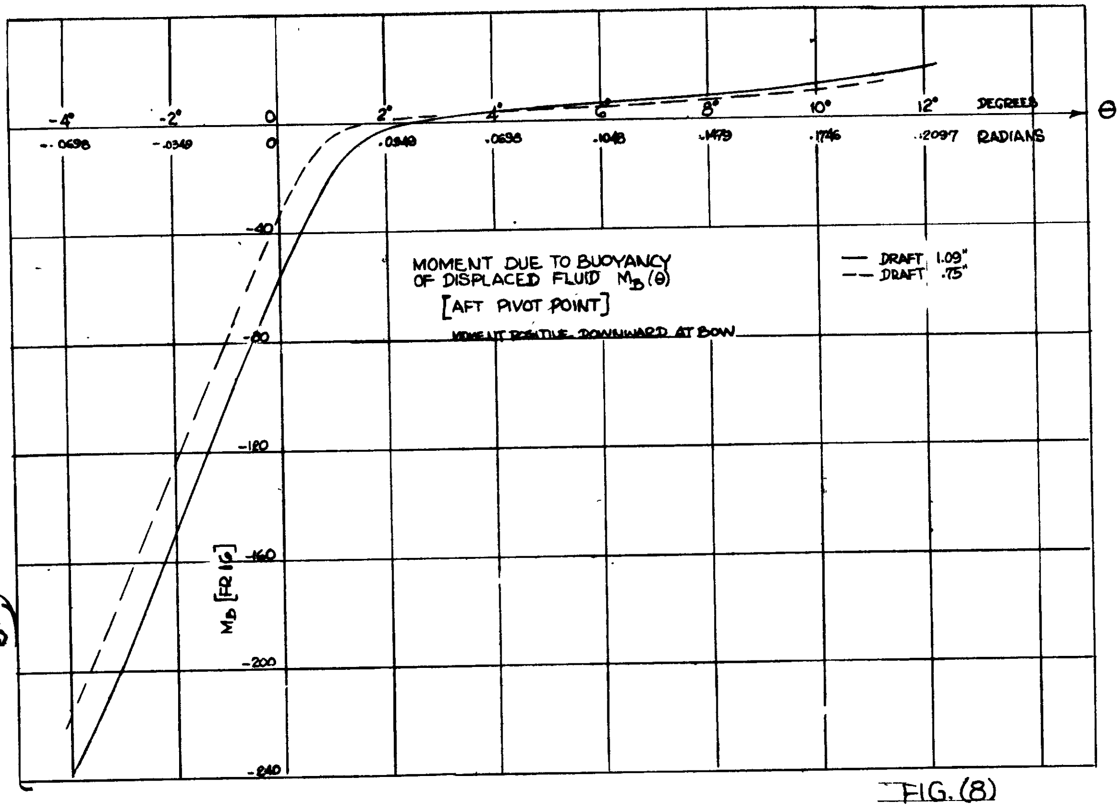
$r = 4.63$ ft

$12.2°$
$\underline{-4.6°}$
$7.6$

$\therefore a = 4.48 \text{ rad/sec}^2 \times 4.63$ ft

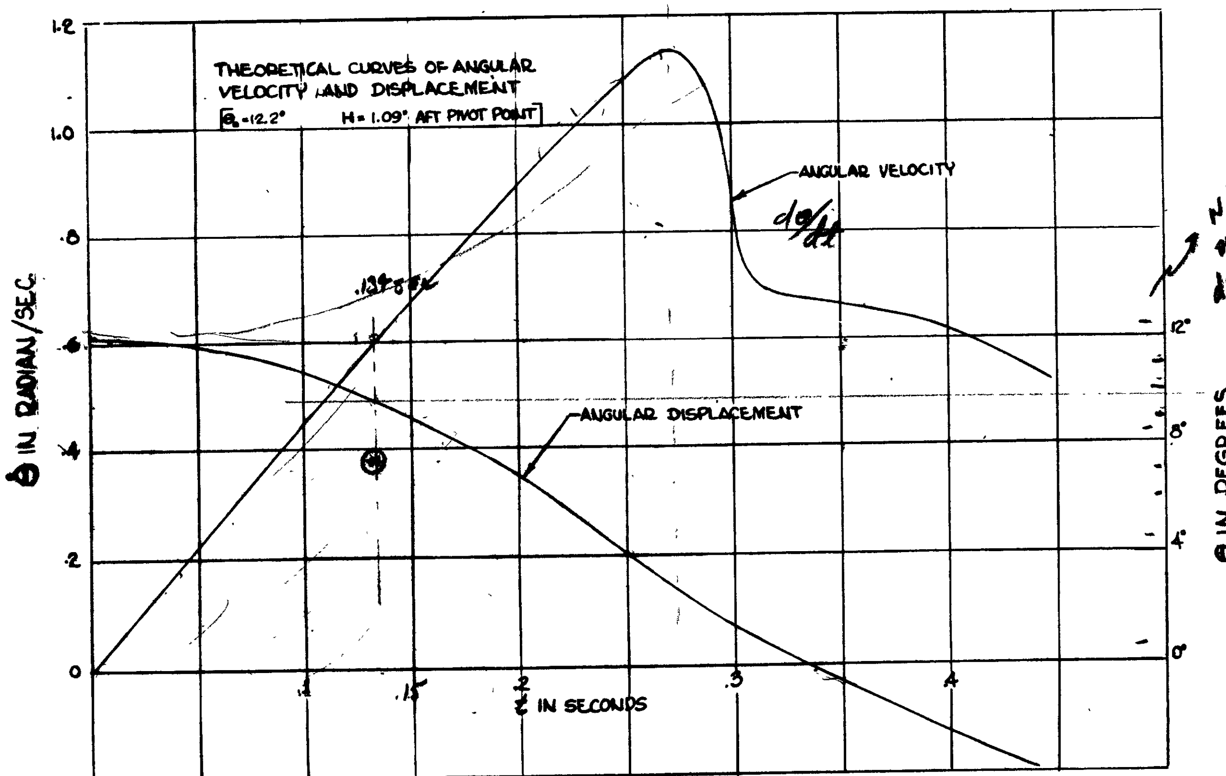
$a = 20.8 \text{ ft/sec}^2$

$a = .646$ g's ← THIS AGREES WITH FIG 10




Units?? ft # (I presume)

FIG. (8)




NEG on $d\theta/dt$ NEG

FIG. (9)

How do you get negative displacement when $d\theta/dt$ is never negative ← depends on where you start

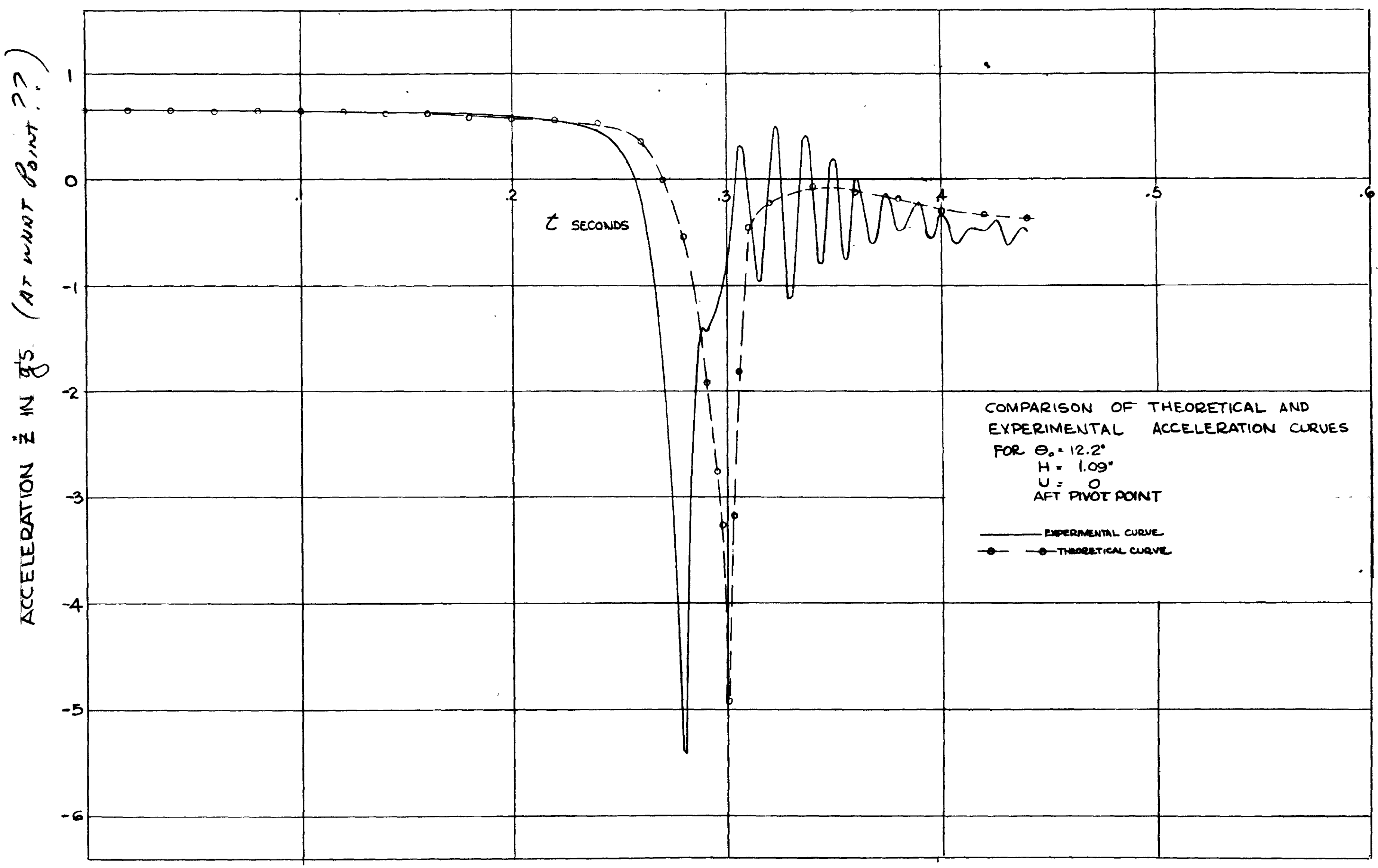

ACCELERATION $\ddot{z}$ IN g's (AT WHAT POINT ??)
1
0
-1
-2
-3
-4
-5
-6
.1
.2
.3
.4
.5
.6
t SECONDS
COMPARISON OF THEORETICAL AND EXPERIMENTAL ACCELERATION CURVES
FOR $\theta_0$ = 12.2°
H = 1.09"
U = 0
AFT PIVOT POINT
EXPERIMENTAL CURVE
THEORETICAL CURVE

FIG.(10)

GOOD POINT

POSITION OF THESE WEIGHTS IMPORTANT

WHAT WOULD HAPPEN IF BOTH WEIGHTS WERE SHIFTED TOWARD THE CENTER.

shows up quite clearly in a sudden decrease in velocity, its effect on the displacement curve obtained by the integration is practically unnoticeable. From this we may conclude that, despite the large magnitude of the deceleration experienced in slamming, its duration is so short that its effect on the actual motion of the ship is almost negligible.

Figure (10) shows also the experimental curve of acceleration for identical conditions of draft and loading. In general, the agreement is quite good. There is some time lag and difference in magnitude of the maximum deceleration. The latter could be explained by the inherent inaccuracy of graphical differentiation. Both discrepancies could be explained by a hypothesis that the draft of the model was not measured accurately enough . This is quite possible since this measurement presents considerable practical difficulty. Approximations in the basic equation such as the use of an element theory, and in the computation of coefficients, may also contribute appreciably to the discrepancies.

The oscillations which appear after the impact in the experimental curve, represent the resultant elastic vibration of the hull. No attempt was made to take such into account in the equations of the theoretical work. One further investigation was carried out for the case of zero speed of advance. The effect of a change of loading of the ship was studied by calculating the acceleration curve for a changed position of a 25 lb. weight located in the stern. The moment due to weight and the moment of inertia are both changed by this shift while the added moment of inertia and the moment due to buoyancy are unaltered. As a result the initial acceleration is altered giving rise to a further shift in the time of maximum deceleration and to a change in the maximum attained velocity which influences the magnitude of the slamming force. In the case considered, the distance between the weight and the pivot point was decreased by about 27 percent, increasing $M_W$ by 17 percent and decreasing I by 11 percent. These changes resulted in an increase of the initial acceleration from .66g to .83g. The maximum deceleration was experienced 0.01 second earlier, and its magnitude was increased by about 20 percent.

<u>Solution of the Differential Equation for Finite Speed of Speed of Advance</u>

In this case we have the equation

$$-\left[I + I_a(\theta)\right]\frac{d\theta}{dt} + \frac{U}{g}\left[\theta M_a(\theta) - \theta_0 M_a(\theta_0)\right] = \int_{t=0}^{t}\left[M_w + M_B(\theta)\right]dt \quad ----(1$$

or differentiating

$$-[I+I_a(\theta)]\ddot{\theta} - \dot{\theta}\,\dot{I}_a(\theta) + \frac{U}{g}\left[\dot{\theta}\,M_a(\theta) + \theta\,\dot{M}_a(\theta)\right] = M_W + M_B(\theta) \quad --- (15)$$

Substituting $\phi = (\dot{\theta})^2$ as before we obtain

$$-\frac{1}{2}\frac{d\phi}{d\theta} - \frac{\phi\, dI_a(\theta)/d\theta}{I+I_a(\theta)} + \frac{U\sqrt{\phi}}{g}\,\frac{\left[M_a(\theta) + \theta\,\frac{dM_a(\theta)}{d\theta}\right]}{I+I_a(\theta)} = \frac{M_W + M_B(\theta)}{I+I_a(\theta)} \quad --- (16)$$

This equation may be solved by means of successive approximations. It may be put into the form

$$\sqrt{\phi_1(\theta)} = \frac{-\int_{\theta_0}^{\theta} \frac{M_B + M_W}{\sqrt{\phi}} + \frac{U}{g}\left[\theta\,M_a(\theta) - \theta_0 M_a(\theta_0)\right]}{I + I_a(\theta)} \quad ----(17)$$

$\frac{d\theta}{dt}$ ANGULAR VELOCITY

where the first approximate form for the velocity, $\sqrt{\phi(\theta)}$ was taken as that for zero speed of advance and $\sqrt{\phi_1(\theta)}$ is the second approximation. After five successive approximations the difference in slope of the velocity time curves did not exceed the limit of the accuracy of the slope drawing and of the initial curve. It may be concluded that the finite speed of advance has very little direct influence on the magnitude of slamming in the particular case of a ship which is pitched artifically while moving through calm water. This conclusion is substantiated by the experimental results (see page 23).

An attempt was made to find a simpler method of solution for equation (16), at least in the region of slamming. As can be seen from the curves of $I_a(\theta)$ and $M_B(\theta)$ the ratio $\frac{M_W + M_B(\theta)}{I + I_a(\theta)}$ is very small in this region and in order to simplify the equation it was assumed that this term could be neglected giving the homogeneous equation:

$$\frac{d\phi}{d\theta} + \frac{2\phi\, dI_a(\theta)/d\theta}{I+I_a(\theta)} = 2\sqrt{\phi}\,\frac{U}{g}\,\frac{\left[M_a(\theta) + \theta\,\frac{dM_a(\theta)}{d\theta}\right]}{I+I_a(\theta)} \quad -----(18)$$

which is of the form

$$\phi' + f(\theta)\,\phi = g(\theta)\,\phi^{1/2}$$

where f and g are the functions given in equation (18). Substituting $q = \phi^{1/2}$ we obtain the solution for $q$:

$$q = e^{-\int \frac{f(\theta)}{2} d\theta} \left[ \gamma + \int \frac{g(\theta)}{2} e^{\int \frac{f(\theta)}{2} d\theta} d\theta \right] + \text{CONSTANT} \qquad (19)$$

where
$$\int \frac{f(\theta)}{2} d\theta = \int \frac{\frac{d}{d\theta}\left(I_a(\theta)\right)}{I + I_a(\theta)} d\theta = \ln\left(I + I_a(\theta)\right)$$

Therefore

$$q = \dot{\theta} = \frac{1}{I + I_a(\theta)} \left[ \gamma + \frac{U}{g}\left(\theta M_a(\theta) - \theta_0 M_a(\theta_0)\right) \right] \qquad (20)$$

The constant $\gamma$ was found from initial conditions. These were taken at time t = .24 sec., just before the deceleration began. Using the values of $\theta$, $I_a(\theta)$ and $M_a(\theta)$ at this time, $\gamma = -30.43$. The resulting velocity curve is shown on figure (11), curve (2). Putting then U = 0 in equation (20) and neglecting the $M_W + M_B(\theta)$ term,

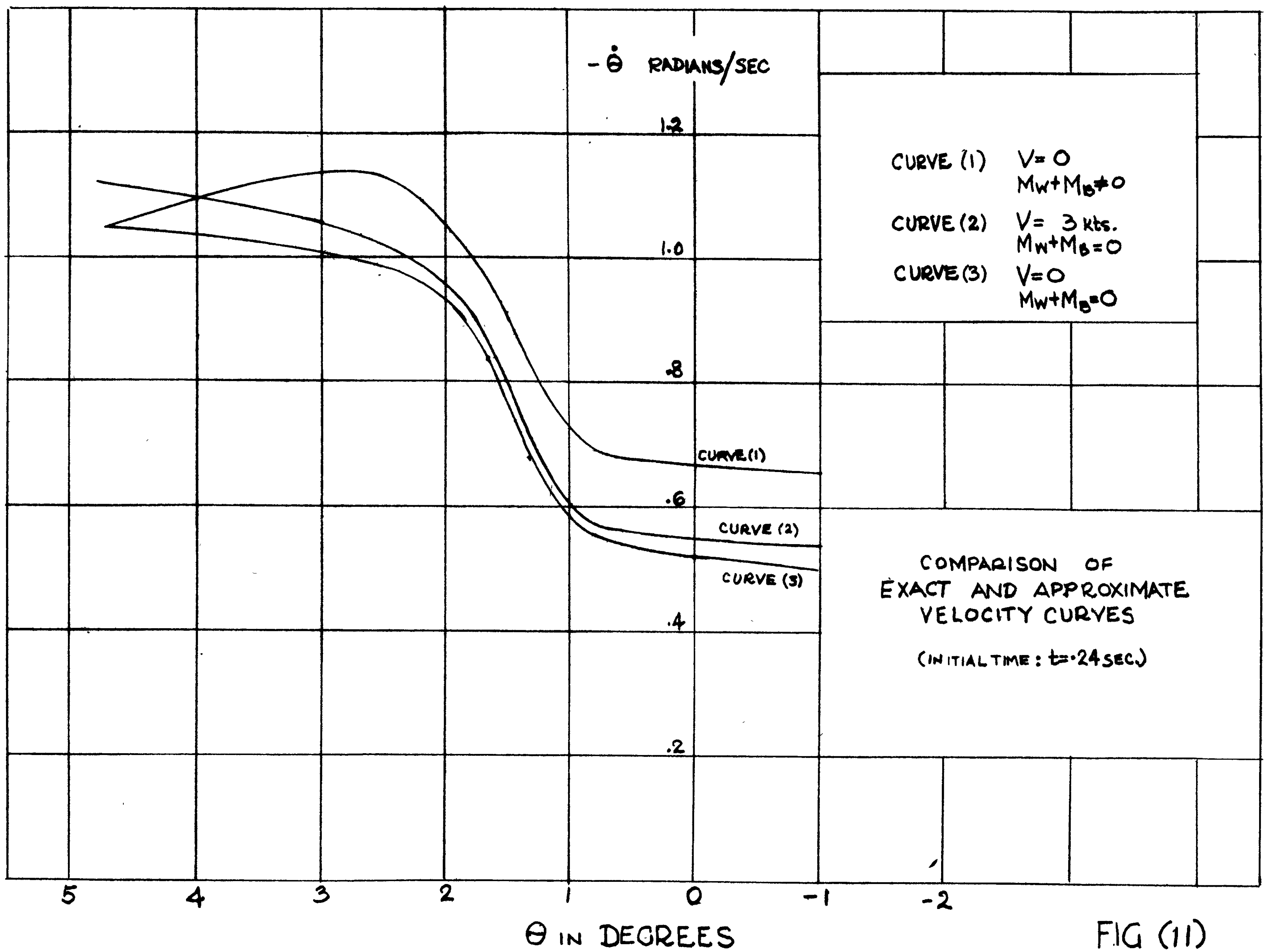
$$\dot{\theta} = \frac{K}{I + I_a(\theta)} \quad \text{where K is some constant.}$$

Again from initial conditions at the same time we find that K =-29.36.

The resulting velocity curve is again shown on figure (11), curve (3). The exact solution for zero speed of advance is represented by curve (1). From this figure it appears that the angular velocity is scarcely influenced by the speed of advance, but that the neglected moment term is important. The former fact is emphasized by the successive approximation solution, which is also rendered necessary by the invalidity of the above method of simplification.

## Theoretical Pressure Distribution

So far we have investigated the maximum decelerations of a ship during impact. These in themselves, however, are not the cause of the damage to the hull which is the main concern of the practical naval architect and shipbuilder. For purpose of structural design it is the pressure distribution on the hull which is of greater interest and importance.



- θ̇ RADIANS/SEC
1.2
1.0
.8
.6
.4
.2
5
4
3
2
1
0
-1
-2
θ IN DEGREES
CURVE (1) V= O Mw+MB≠O
CURVE (2) V= 3 kts. Mw+MB=O
CURVE (3) V=O Mw+MB=O
CURVE (1)
CURVE (2)
CURVE (3)
COMPARISON OF EXACT AND APPROXIMATE VELOCITY CURVES
(INITIAL TIME: t=·24 SEC.)


FIG (11)

The pressure will vary across a certain transverse section and from section to section at any given time. At any section the pressure distribution varies with time. Thus the pressure at any point is a function of three variables, the position given by x and y and the time t.

Reference (1) equation (73) gives the equation for the pressure at any point:

$$p = \frac{\rho}{2} V^2 \left[ \frac{2}{u(c)\sqrt{1 - y^2/c^2}} + \frac{2\dot{V} c}{V^2} \sqrt{1 - y^2/c^2} - \frac{y^2/c^2}{1 - y^2/c^2} \right] \quad \text{---(21}$$

where V = instantaneous velocity of transverse section under consideration
$\rho$ = density of water
y = beam coordinate of point
$\dot{V}$ = instantaneous acceleration
$u(c) = \frac{dZ(c)}{dc}$ [see added mass investigation] PAGE 6
c = modified beam at water level for piled up water effect.

Introducing $\eta = y/c$, $\delta = \frac{V^2}{\dot{V} c}$ and $p_o = \frac{\rho V^2}{2}$ equation (21) becomes

$$p = p_o \left[ \frac{2}{u(c)\sqrt{1 - \eta^2}} + \frac{2}{\delta}\sqrt{1 - \eta^2} - \frac{\eta^2}{1 - \eta^2} \right] \quad \text{-------(22}$$

At the keel

$$p_{keel} = 2 p_o \left[ \frac{1}{u(c)} + \frac{1}{\delta} \right]$$

At any instant, from previous results, we know the angular velocity and acceleration, and therefore V and $\dot{V}$ may be found for any section.

The formula (21) is valid for a wedge of small deadrise angle. By using the u(c) calculated for the particular sections, it holds for these more complicated shapes, but again only for small deadrise angle. Thus the pressure distribution on any section can be calculated only for a very short time after that section touches the water surface, that is where the slope of Z(c) is small. If it is applied to sections having a deeper draft, negative pressure distributions result.

Figure (12) shows the pressure distribution on section 17 at the instant of slamming ($\theta = 1.42^{\circ}$) and at times very shortly after.

Figure (13) shows the pressure distribution on sections 16 and 17 at the instant of slamming.

The y coordinate of that point on any section, at which the pressure is a maximum at a certain time, may very easily be calculated.

For maximum pressure

$$\frac{\partial p}{\partial \eta} = 0$$

where's 23?

which, from equation (23), gives

$$\frac{\xi}{u(c)} - \frac{\xi^3}{\delta} = 1 \qquad (24)$$

WHERE DID $\frac{dy}{du}$ go. $u_c$ IS A FUNCTION OF c

where

$$\xi = \sqrt{1 - \eta^2}$$

Since in this equation $u(c)$ is small and $\xi < 1$, we take as a first approximation $\xi = u(c)$. Substituting in equation (24) we get

$$1 - \frac{u^3(c)}{\delta} \cong 1$$

The accuracy of this approximation, therefore, depends on the magnitude of $\frac{u^3(c)}{\delta}$. The maximum value of this ratio, for conditions giving positive pressure distributions, was at station 17 and $\theta = 1.0^{\circ}$, in which case $u(c) = 0.0911$ and $\delta = -0.985$ and $\frac{u^3(c)}{\delta} = -0.00076$. Thus the maximum error introduced by taking the position of the pressure maximum at $\xi = u(c)$ or $y = c\sqrt{1-u^2(c)}$ is 0.076 percent.

Assuming this solution for y we obtain a simple formula for the maximum pressure

$$p_{max} \cong p_0 \left[1 + \frac{1}{u^2(c)}\right]$$

An attempt was made to estimate the moment due to slamming from these calculated pressure distributions. The highly localised pressure at station 17 at the instant of slamming would contribute

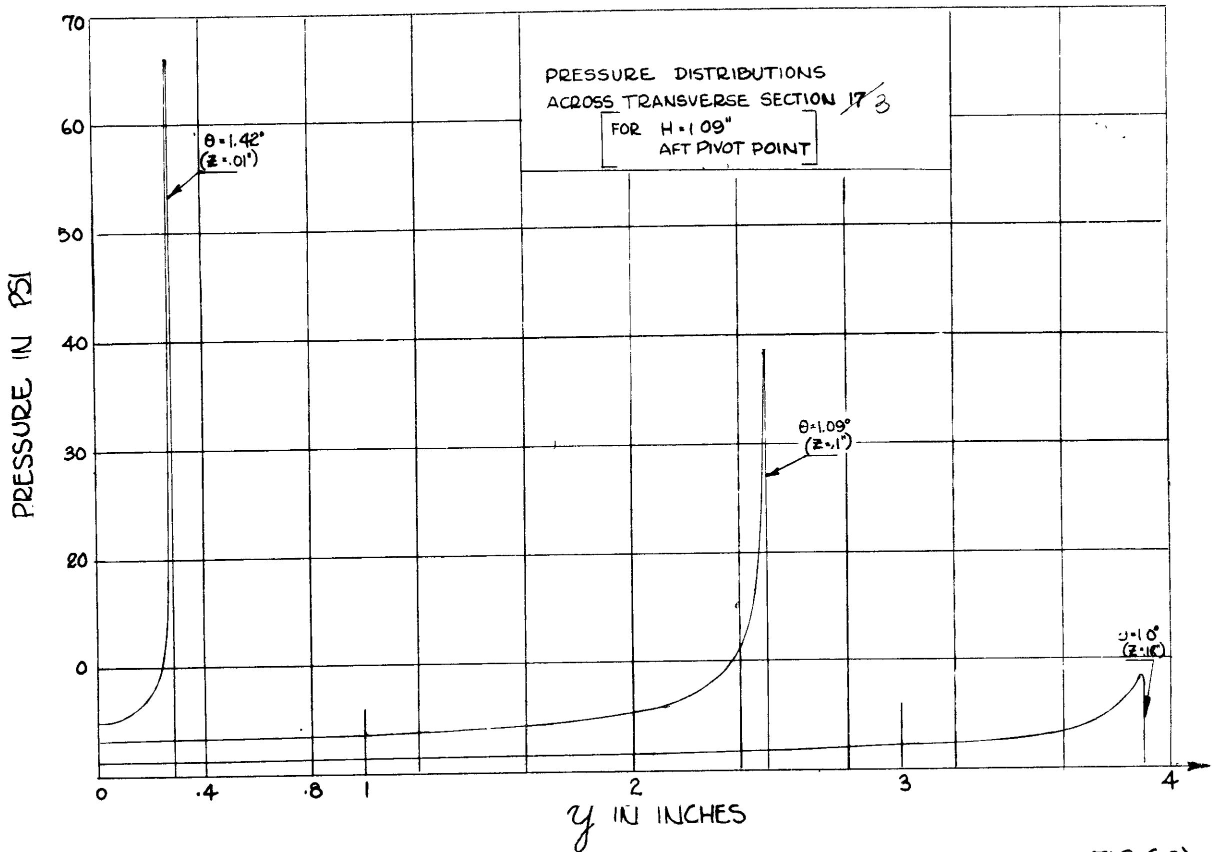

PRESSURE DISTRIBUTIONS
ACROSS TRANSVERSE SECTION ~~17~~ 3
FOR H = 1.09"
AFT PIVOT POINT
θ = 1.42°
(Z = .01")
θ = 1.09°
(Z = .1")
θ = 1.0°
(Z = .15")
PRESSURE IN PSI
y IN INCHES
70
60
50
40
30
20
0
0
.4
.8
1
2
3
4

FIG. (12)

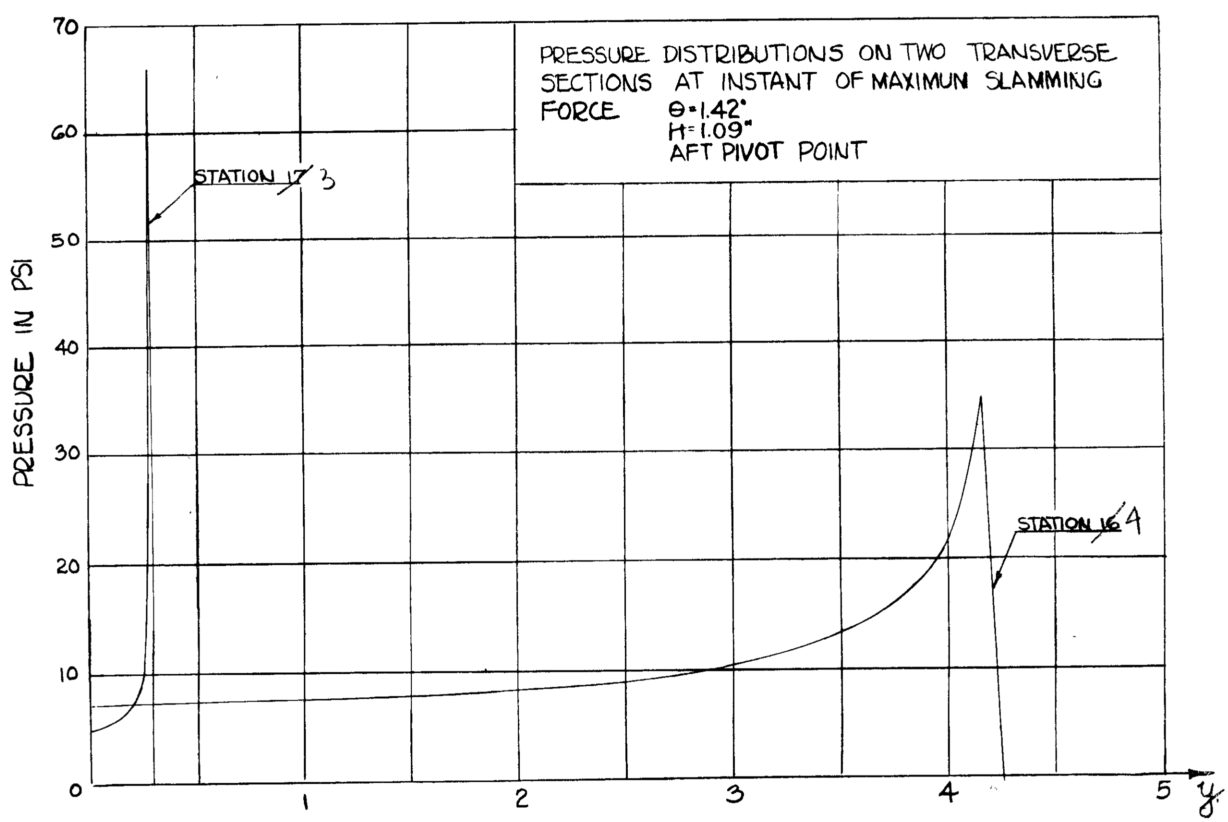

PRESSURE DISTRIBUTIONS ON TWO TRANSVERSE
SECTIONS AT INSTANT OF MAXIMUN SLAMMING
FORCE θ = 1.42°
H = 1.09"
AFT PIVOT POINT
STATION ~~17~~ 3
STATION ~~16~~ 4
PRESSURE IN PSI
y
70
60
50
40
30
20
10
0
1
2
3
4
5

FIG. (13)

practically nothing to the total force. However, the pressure distribution at station 16 would give rise to a considerable force. Suppose we assume a linear distribution of force per unit ship length, over the two sections 15 to 17 with the maximum at station 16. Then the total moment is 1500 ft. lbs.

This agrees in order of magnitude with the 1000 ft. lbs. for this moment obtained from equation (12): → 0 Fwd speed

Moment due to slamming = $\frac{d}{dt}(I_a(\theta)\dot{\theta}) = -M_W - M_g(\theta) - I\ddot{\theta}$.
For the full-scale ship, whose length is approximately 400 feet, this would correspond to a slamming moment of three million foot tons. This agrees in order of magnitude with the measured bending moment for this ship.

This investigation gives the general location of the region of high pressure at the instant of slamming. However, it cannot be assumed that the maximum contribution to the force is actually given by station 16. It is impossible to interpolate the pressure distributions between stations 15, 16 and 17. In order to make reasonable predictions as to the damage caused by slamming, it would be necessary to obtain a pressure map for this region, by calculating the pressure distributions at many intermediate sections.

## Experimental Methods and Results

The model of the M.S. SAN FRANCISCO (TMB Model 3572-A) used in the slamming tests had a B.P. length of 7.8 feet, a beam of 12.9 inches and had a freeboard of 9.6 inches. An artificial freeboard was added to avoid shipping water into the accelerometer. For most of the tests the model was pivoted at a point 2.3 feet forward from the aft perpendicular [see figure (14)]. Weights were placed in the stern, and about half-way forward of the midship section, to balance the model in a horizontal position when resting freely in the water. A 5g accelerometer was placed 10½ inches aft of the forward perpendicular. This was connected to an oscillograph and the results were obtained in the form of acceleration traces. The model was pitched by the simple method of giving it an initial angular displacement, (by pulling up the bow by means of a rope) and releasing it to oscillate with its own natural frequency.

The ship was towed at several drafts and at several forward speeds. Runs were made in which the model was pivoted at the midship section, at a forward point and at the above-mentioned aft point. Appreciable changes in acceleration were recorded only in the last case. For a light draft (7/8") and a 9° initial angle slight slamming (-.6g) was noted even when the model was pivoted about the midship section. For any larger draft (3½", 6"), and for all drafts using the forward pivot point no slamming was experienced.

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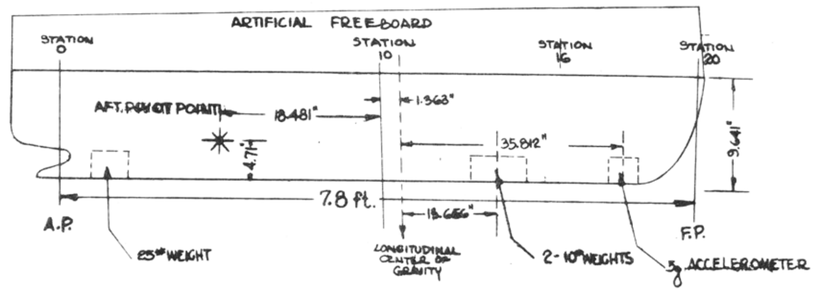
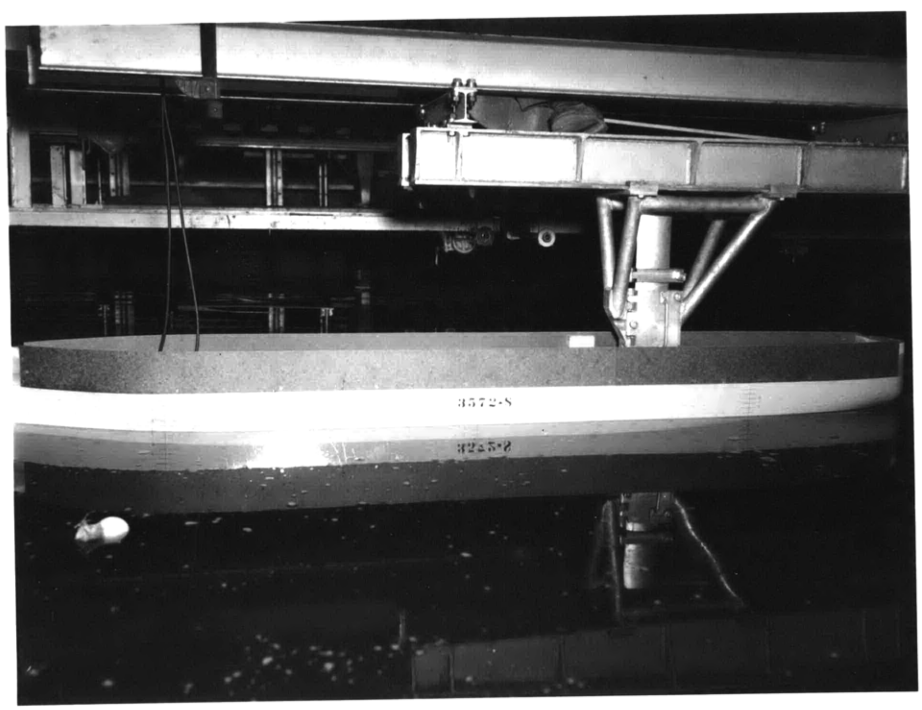



DIAGRAM OF MODEL 3572-4
M.S. SAN FRANCISCO

FIG. (14)



Experimental Arrangement.

The experiments performed with the model pivoted at the after end yield greater slamming. With an initial draft of 3 inches and initial angle $6\frac{1}{2}^{\circ}$ the model experiences slight slamming (less than 1g deceleration); for greater draft and this angle, none. With a draft of 1.09" the slamming varied with the initial angle: for 12.2° the maximum deceleration was 4.2g; for 9°, 3.1g; and for 6°, 1.9g. All these values are for zero speed of advance. If the draft is further reduced to .75" the magnitudes are again increased to the order of 5 or 6g, and a similar variation with inital angle is noticeable. These summarized results are given in Table 1.

TABLE 1

| Pivot Point | Initial Angle | Draft H | Slamming |
|---|---|---|---|
| Forward | — | — | None |
| Midship | 7° - 9° | 7/8" | Slight (-.2g to -.6g) |
| | " | $3\frac{1}{2}$ to 6" | None |
| Aft | 8° - 11° | .75 | -5.4g to -6g |
| | 5.7° - 12.2° | 1.09" | -1.9g to -4.3g |
| | " | 3" - 6" | None |

At least in this artificial case, with no waves present, slamming seems to be hardly sensitive to forward speed. There is a slight trend in some cases towards an increase in the maximum deceleration at 1 knot, and a falling off in the 2 and 3 knot runs, but the spread is slight and scarcely exceeds the possible error caused by the difficulty in measuring the initial angle. No conclusion should be drawn, therefore, except that of the insensitivity of slamming to forward speed in this particular experimental set-up. It must be emphasized that when the model is towed in waves the forward speed will partially determine the period of encounter, changing the effective wave-length and thus the maximum pitch angle, to which, as we have seen, slamming is sensitive. In addition, in the case of waves the forward velocity will have a component perpendicular to the water surface.

Figure (15) shows the effect of draft for zero speed of advance. The larger angles in cases (1) and (2) would tend to increase slamming but this effect is more than compensated by the heavier draft. Comparison of cases (1) with (2) and (3) with (4) shows the effect of angle for fixed draft.

In all these curves, the maximum deceleration due to impact is given by the first peak. The high frequency oscillations following this are caused by the elastic vibration of the hull.

As will be seen in fig. (15) the experimental traces of acceleration, as a result of friction generated in the bearings of the experimental set up, start at zero at time

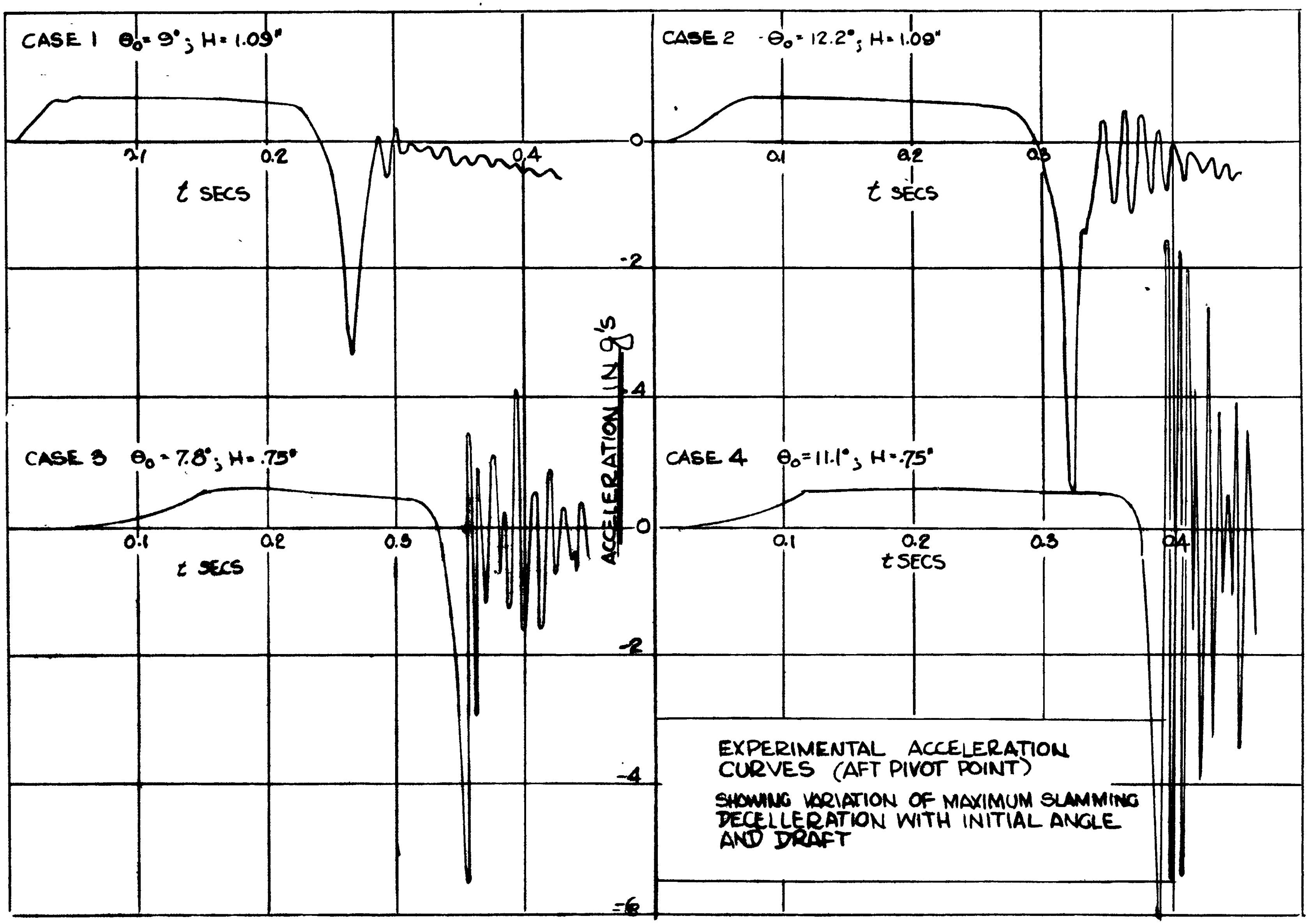

CASE 1 θ₀ = 9°; H = 1.09"
CASE 2 θ₀ = 12.2°; H = 1.09"
CASE 3 θ₀ = 7.8°; H = .75"
CASE 4 θ₀ = 11.1°; H = .75"
t SECS
ACCELERATION IN g's
0.1
0.2
0.3
0.4
0
-2
-4
-6
EXPERIMENTAL ACCELERATION CURVES (AFT PIVOT POINT)
SHOWING VARIATION OF MAXIMUM SLAMMING DECELLERATION WITH INITIAL ANGLE AND DRAFT

FIG. (15)

zero and increase to a constant value. The curve used for comparison with the theoretical results was corrected for this effect by assuming the initial acceleration to be that later attained and adjusting the starting time to leave the total area under the curve the same. This resulted in a shift of .008 second in the time scale.

The accelerations obtained from all these runs were integrated graphically to give velocity and displacement curves. A representative case is given in figure (16).

It is interesting to note that the extremely sharp peak in the acceleration curve causes a sudden drop in the velocity curve but does not bring it to zero. Thus the vertical motion of the model does not cease completely due to the impact. After the second integration the resulting displacement shows scarcely any effect of the peak. Thus we conclude that, since the impact has such a short duration, the ship is not halted in its downward travel, and that in fact, the impact has very little effect on the ship's motion.

## Conclusions

From this work we can conclude that given the initial position of a ship in calm water it is possible to solve numerically the diferential equations of motion and to predict whether or not the ship will slam, the approximate time and magnitude of slamming and the pressure distribution over that part of the hull which suffers impact at the instant of maximum deceleration.

It has been pointed out that slamming is very sensitive to change in draft, less sensitive to change in initial angle and, when towed in calm water and artifically pitched about a fixed pivot point, practically independent of forward speed.

There is a great deal of further numerical work which should be done in order to produce a practically useful pressure map. The whole problem of predicting slamming forces for a model towed freely through waves has yet to be solved.

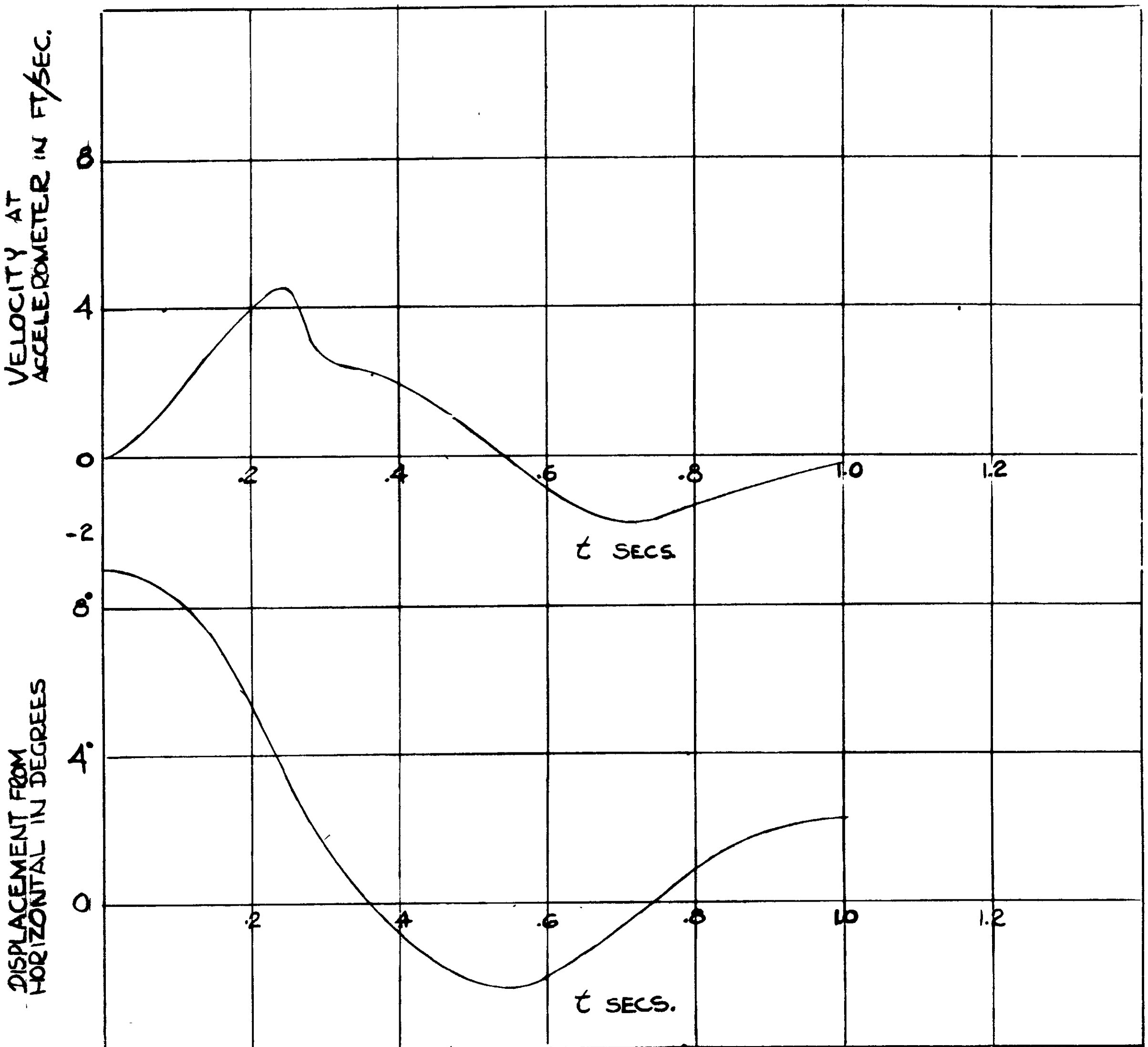
## Summary of Equations for Numerical Calculations

1. Calculation of elements of added mass

$$(\Delta m_a)_i = K_i \frac{\rho}{2} c_i^2 \pi \Delta x_i$$

Equation of $i^{th}$ transverse section:

$$z_i(y) = a_0 + a_1 y + a_2 y^2 + \cdots + a_n y^n$$




EXPERIMENTAL VELOCITY AND DISPLACEMENT CURVES FOR CASE 1 OF FIG. 12

$\theta_0 = 9°$
H = 1.09"
AFT PIVOT POINT

FIG. (16)

Equation of modified section taking into account the piled-up water effect:

$$z_i(c) = \frac{2}{\pi} a_1 c + \frac{a_2}{2} c^2 + \frac{\alpha_2}{3} c^3 + \text{------} + \frac{\alpha_n}{n+1} c^{n+1}$$

where

$$\alpha_n = a_{n+1} \begin{cases} \dfrac{2.\ 2.4.6 \cdots n}{\pi.\ 1.3.5 \cdots (n-1)} & \text{for n even} \\ \dfrac{1.3.5 \cdots n}{2.4.6 \cdots (n-1)} & \text{for n odd} \end{cases}$$

This equation gives $c(\theta)$ for any section, since $z$ is a function of $\theta$. $K_i$ **is obtained from fig. (1) and depends on the cross-sectional area and B/H ratio of the immersed section.**

2. Calculation of Statical Moment Due to Added Mass

$$M_a(\theta) = g \sum_i x_i (\Delta m_a)_i$$

3. Calculation of Added Moment of Inertia

$$I_a(\theta) = \frac{1}{\sqrt{\bar{A}^2(\theta)/S^4(\theta) + 1}} \sum_i x_i^2 (\Delta m_a)_i$$

where $\bar{A}(\theta)$ = waterline area
$S(\theta)$ = wetted keel length

4. Calculation of Moment due to Buoyancy

$$M_B(\theta) = \sum \rho g x_i A_i(\theta) \Delta x_i$$

where $A_i(\theta)$ = immersed cross-sectional area.

5. Solution of Differential Equation for Zero Speed of Advance

This solution is given by

$$\phi = \frac{-2\int_{\theta_0}^{\theta} (M_w + M_B(\theta))(I + I_a(\theta))\, d\theta}{(I + I_a(\theta))^2}$$

where $\phi = (\dot{\theta})^2$

This gives a numerical solution for $\dot{\theta}$ as a function of $\theta$:

$$\frac{d\theta}{dt} = \sqrt{\phi}$$

Then $t = \displaystyle\int_{\theta_0}^{\theta} \frac{d\theta}{\sqrt{\phi}}$

or by numerical integration $\theta = \theta(t)$.

From this we obtain immediately $\dot{\theta}$ as a function of t, and by graphical differentiation $\ddot{\theta}$ as a function of t.

6. Solution of differential equation for finite speed of advance by successive approximations

Assuming the solution $\varpi(\theta)$ from previous case as a first approximation, the next approximation becomes

$$\sqrt{\phi_1(\theta)} = \frac{-\int_{\theta_0}^{\theta} \frac{M_N + M_a(\theta)}{\sqrt{\phi(\theta)}}\, d\theta + \frac{U}{g}\left[\theta M_a(\theta) - \theta_0 M_a(\theta_0)\right]}{I + I_a(\theta)}$$

7. Calculation of pressure at any point across one transverse section

$$p = \frac{\rho V^2}{2}\left[\frac{2}{u(c)\sqrt{1 - y^2/c^2}} + \frac{2\dot{V}c}{V^2}\sqrt{1 - y^2/c^2} - \frac{y^2/c^2}{1 - y^2/c^2}\right]$$

or

$$p = p_0\left[\frac{2}{u(c)\sqrt{1-\eta^2}} + \frac{2}{\delta}\sqrt{1-\eta^2} - \frac{\eta^2}{1-\eta^2}\right]$$

where

$$\eta = y/c \quad , \quad p_0 = \rho V^2/2 \quad , \quad \delta = V^2/\dot{V}c$$

$$u(c) = \frac{dZ(c)}{dc} = \frac{2}{\pi}a_1 + a_2 c + \alpha_2 c^2 + \cdots + \alpha_n c^n$$

$$p_{keel} = 2 p_0\left[\frac{1}{u(c)} + \frac{1}{\delta}\right]$$

On any transverse section the point at which the pressure is a maximum is given by

$$y = c\sqrt{1 - u^2(c)}$$

and this maximum pressure is

$$p_{max} \cong p_0 \left(1 + \frac{1}{u^2(c)}\right)$$

## ACKNOWLEDGEMENTS

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Appreciation should also be expressed to Dr. L. Landweber for his thorough review of the work and helpful criticism.

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## APPENDIX

Equations of Forward Transverse Sections of the

M.S.San Francisco Model

Station 6

$$z(y) = .025y + 32(y/10)^{8} + 480(y/10)^{16}$$

Station 7

$$z(y) = .0154y + 1.4 \times 10^{4}\,(y/10)^{22}$$

Station 8 - 14

$$z(y) = .0154y + 6.23 \times 10^{5}\,(y/10)^{32} + 6.31 \times 10^{10}\,(y/10)^{64}$$

Station 15

$$z(y) = .0154y + 64.1\,(y/10)^{12} - 6 \times 10^{4}(y/10)^{28} + 1.4 \times 10^{10}(y/10)^{56}$$

Station 16

$$z(y) = .0308y + 2.08 \times 10^{3}(y/10)^{16} + 6.03 \times 10^{26}(y/10)^{128}$$

Station 17

$$z(y) = .06y + 26.85\,(y/10)^{6} + 4.55 \times 10^{2}\,(y/10)^{12} - 3.52 \times 10^{12}(y/10)^{48}$$

Station 18

$$z(y) = .182y + 30\,(y/10)^{4} + 2.58 \times 10^{12}(y/10)^{32}$$

Station 19

$$z(y) = .66 + .237y + 92.2\,(y/10)^{3} + 4.57 \times 10^{10}(y/10)^{16}$$

( These equations hold for the eight foot model. z and y are measured in inches.)

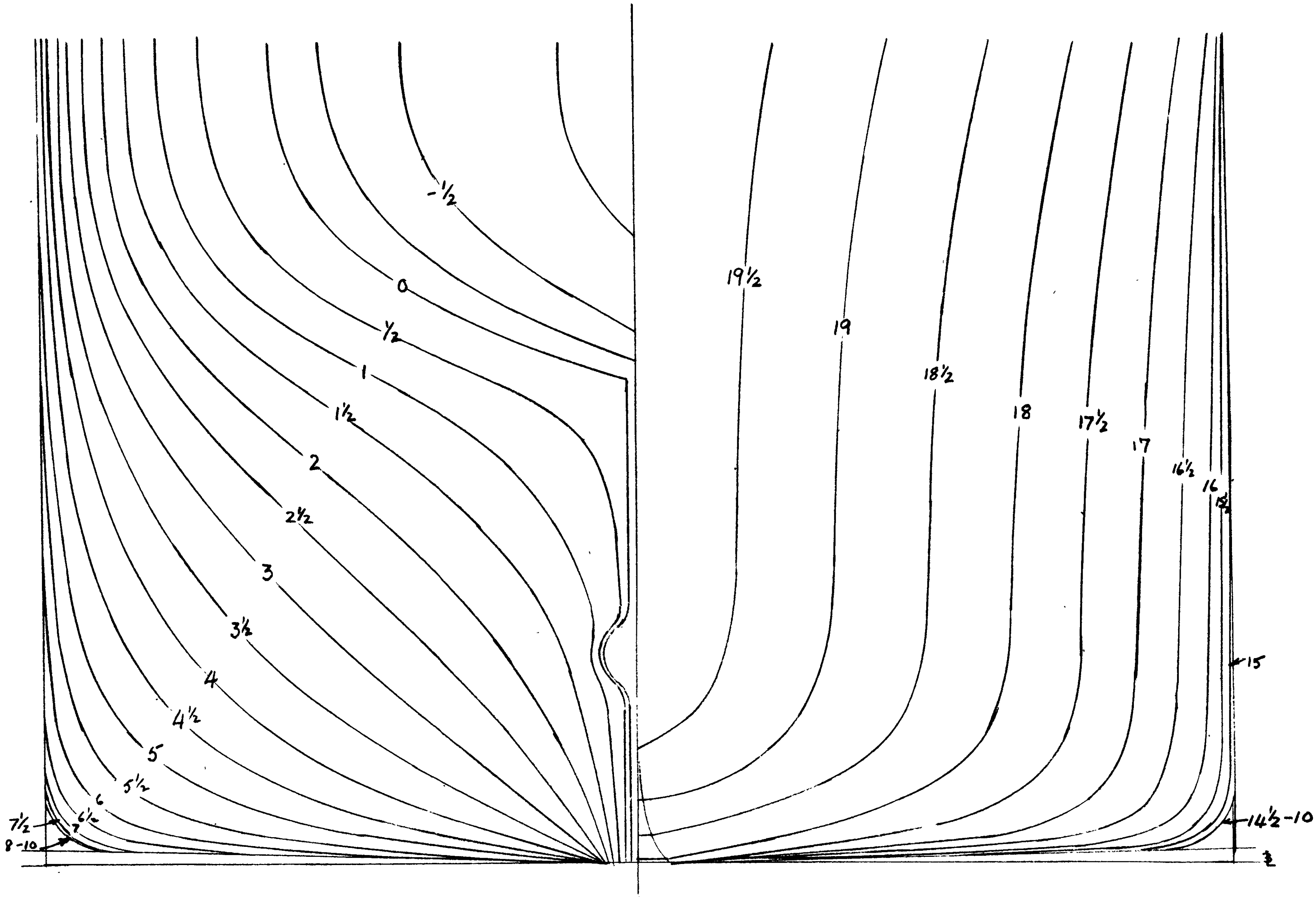
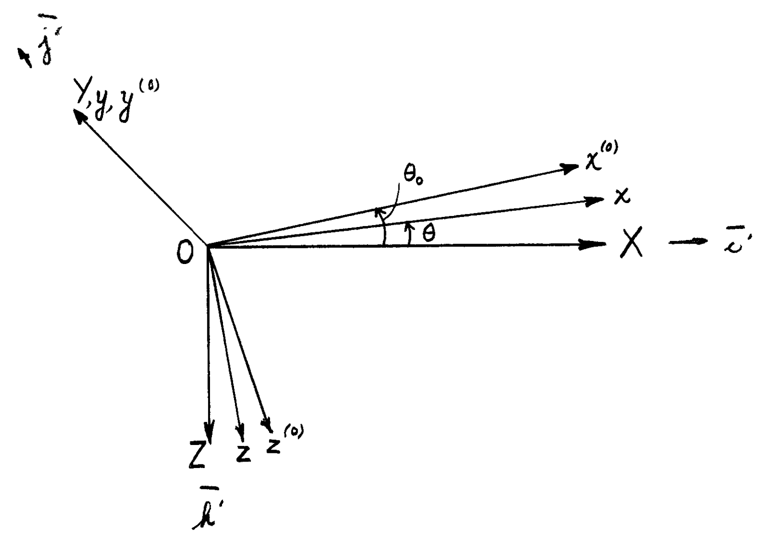



Figure 17
Transverse Sections of M.S.San Francisco.

## Derivation of the general differential equation of slamming

We define the following coordinate systems




The two coordinate systems (X,Y,Z) and (x,y,z) have their origin at the pivot point O. The (X,Y,Z) system (unit vectors $\bar{i}', \bar{j}', \bar{k}'$) translates with the origin only; $\bar{i}'$ is always parallel to the water surface. The (x,y,z) system (unit vectors $\bar{i}, \bar{j}, \bar{k}$) translates with the origin and rotates with the body in the x,z plane. Thus, x is always along the ship. The superscipt zero again denotes conditions at time zero.

The forward velocity U is always parallel to the X axis and can be written as the vector $U\bar{i}'$. We assume that an element theory can be used and that the mass of each element of the model is concentrated at the centroid of that element. In the absence of a heavy superstructure, as in this case, the approximation is reasonable.

The position vector of the centroid of an element is $\bar{i}x + \bar{k}z$. (The subscript i denoting the ith element will be ommitted for convenience).

The absolute velocity of this centroid is given by

$$u\bar{i} + w\bar{k}$$

where u and w are the components of the velocity along the x and z directions, respectively.

$$u\bar{i} + w\bar{k} = U\bar{i}' + q\bar{j} \times (\bar{i}x + \bar{k}z)$$

$$= U\bar{i}' + q(-\bar{k}x + \bar{i}z) \qquad (A.1)$$

where $q\bar{j}$ is the angular velocity vector about O. From (A.1) we obtain

$$u = U + qz, \; w = U\theta - qx, \; \dot{w} = U\dot{\theta} - \ddot{\theta}x \qquad (A.2)$$

We assume that the added mass of an element may be represented by its component normal to the model keel, $\Delta m_a$, and that the momentum of the fluid due to the motion of the element is

$$m_a \, w\bar{k}$$

If the mass of the element is denoted by $\Delta m$ the equation of motion for that element is:

$$\frac{d}{dt}\left[\Delta m(u\bar{i} + w\bar{k}) + \Delta m_a w\bar{k}\right] = [\Delta F_W + \Delta F_B]\bar{k}' \qquad (A.3)$$

where $\Delta F_W$ = force due to weight of element

$\Delta F_B$ = force due to buoyancy, acting on element.

Integrating (A.3) we obtain

$$\bar{i}u\Delta m + \bar{k}w(\Delta m + \Delta m_a) - \bar{i}^{(o)}u^{(o)}\Delta m - \bar{k}^{(o)}w^{(o)}(\Delta m + \Delta m_a{}^{(o)})$$

$$= \bar{k}'\int_0^t (\Delta F_W + \Delta F_B)dt \qquad (A.4)$$

If we take the normal components of all terms in (A.4) at time t, and consider the angles involved small, we have:

$$w(\Delta m + \Delta m_a) - (\theta - \theta)u^{(o)}\Delta m - w^{(o)}(\Delta m + \Delta m_a{}^{(o)})$$

$$= \int_0^t (\Delta F_W + \Delta F_B)dt$$

or using (A.2)

$$(U\theta - gx)(\Delta m + \Delta m_a) - (\theta - \theta_o)U\Delta m - U\theta_o(\Delta m + \Delta m_a{}^{(o)})$$

$$= \int_0^t (\Delta F_W - \Delta F_B)dt$$

or

$$u\theta\Delta m_a - U\theta_o\Delta m_a^{(o)} - qx(\Delta m + \Delta m_a) = \int_o^t (\Delta F_W - \Delta F_B)dt \qquad (A.5)$$

Multiplying equation (A.5) by x, summing over all the elements and introducing the previous notation results at once in the equation

$$-\dot{\theta}[I + I_a] + \frac{u}{g}[\theta M_a(\theta) - \theta_o M_a(\theta_o)] = \int_o^t (M_W + M_B)dt$$

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SLAMMING DUE TO PURE PITCHING MOTION, by M. Alison Todd. February 1954. iv, 36 p. incl. table, figs., refs. (Preliminary copy) UNCLASSIFIED

In this report a method is developed for predicting maximum slamming forces experienced by a ship's hull, when its impact with the free water surface is due to pure pitching motion.
The results of the theoretical investigation are compared with those obtained from a series of experiments performed upon an eight foot model of the M.S. SAN FRANCISCO, in which the model is given an initial angular displacement about a fixed axis of rotation about 30 percent of the length from aft, and allowed to pitch in her natural period. The results are given in the form of theoretical and experimental acceleration traces. The pressure distribution over some of the forward sections is also obtained

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3. Pitching
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The effect of slamming on the pitching motion of the ship is shown to be very slight. The effect on slamming of finite speed of advance, for the case of an artificially pitched model moving through calm water, is also very small.

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