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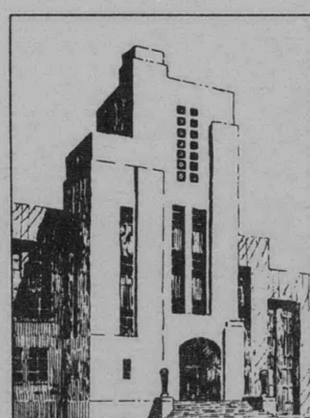
WALL CORRECTIONS FOR FLOW ABOUT TWO- AND THREE-DIMENSIONAL SYMMETRICAL BODIES IN RECTANGULAR CHANNELS OF INFINITE AND FINITE LENGTHS

by

Avis Borden, Ph. D.



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NOTATION

b	tank length
$2c$	source-sink separation
d	tank depth
e	eccentricity of an ellipse or spheroid
K	special value of k
k	index of summation
k_1	added mass coefficient
L	over-all length of hydrodynamic body
l	half-length of hydrodynamic body
m	index of summation
n	index of summation
p	pressure coefficient from whole array of bodies
p^0	pressure coefficient of the original body
p'	$p - p^0$ pressure coefficient contributed by image bodies alone
Q	source strength
$R(x,y,z)$	velocity induced by the original doublet of unit strength
$r(x,y,z)$	velocity induced by the original point source of unit strength and the 2K images above and below
$S(x,y,z)$	velocity induced by a line doublet of unit strength extending from $-(K + \frac{1}{2})d$ to $(K + \frac{1}{2})d$
$s(x,y,z)$	velocity induced by a line source of unit strength extending from $-(K + \frac{1}{2})d$ to $(K + \frac{1}{2})d$
t	half-width of hydrodynamic body at its maximum section
u	x-component of perturbation velocity induced by whole array of bodies
u^0	x-component of perturbation velocity of the original body
u'	$u - u^0$
V	volume of body
v^0	y-component of perturbation velocity of the original body
w^0	z-component of perturbation velocity of the original body
x	longitudinal coordinate
y	horizontal transverse coordinate
z	vertical transverse coordinate

α a particular value of x

β a particular value of y

λ shape parameter

μ doublet strength

ξ integration parameter

σ velocity induced by the whole array of unit doublets

σ' velocity induced by the whole array of unit doublets excluding the original doublet

τ velocity induced by whole array of unit sources

τ' velocity induced by the whole array of unit sources excluding the original source

Subscripts:

2 two-dimensional flows

3 three-dimensional flows

ABSTRACT

Methods for obtaining the wall interference in the flow about two- and three-dimensional bodies at any point in a rectangular channel of infinite and finite lengths have been developed. Exact expressions have been obtained for the Rankine oval and the circular cylinder, and reasonably accurate approximations have been derived for the Rankine ovoid and the sphere. Methods for extending the results to bodies of other shapes are discussed.

INTRODUCTION

When a body is placed along the centerline of a rectangular channel which is infinitely long, an infinite two-dimensional array of images is formed by the multiple reflections of the body in the four walls of the channel. The flow in a channel is then equivalent to the flow past such an array of bodies. Velocity and pressure measurements made at any point in the channel are affected not only by the original body but by the whole field of images. If the channel is finite in length, the flow is further restricted by the end walls which, by the multiple reflections of the two-dimensional array of images, form an infinite three-dimensional array.

The present study was undertaken to determine the magnitude of the wall interference for models of different sizes and shapes in a rectangular electrolytic tank which is equivalent to a rectangular channel of finite length. Reflections in the tank bottom and free water surface are identical with mirror reflections in solid walls. Sources and doublets are reflected with a change of sign in the end walls.

An optimum model size for an electrolytic tank or wind tunnel would be one for which the wall correction falls within the experimental error of the measurement. As it often happens that this optimum size is too small to produce the required experimental accuracy in the measurements, methods are presented for finding the wall correction at any point in the channel for both two- and three-dimensional symmetric bodies.

Many papers have been written on wall interference, particularly for wind tunnels. Lock[1] has calculated the wind tunnel interference at the position of the body for both two- and three-dimensional symmetrical bodies which are small compared with the width of the tunnel. His results will be extended to include larger bodies and to find the pressure at any point in a channel of infinite or finite length. By the present method, expressions may be found for the wall interference of bodies which may be represented by a source-sink or doublet distribution along the axis of the symmetry. It will also be shown how the results may be extended to bodies of other shapes by the use of Lock's shape parameter.

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[1]References are listed on page 31.

SOURCE-SINK AND DOUBLET REPRESENTATION OF HYDRODYNAMIC BODIES

Since the flow about many hydrodynamic bodies may be represented as the flow past a distribution of sources and sinks or doublets, preliminary remarks will be made about the flow past simple sources and doublets. Then the flow about any hydrodynamic body may be expressed as a linear combination of expressions for the flow about the component singularities.

If u, v, and w are the dimensionless components of the perturbation velocity produced by a hydrodynamic body in a uniform flow of unit velocity in the negative x-direction, the dimensionless pressure or the pressure coefficient at any point in the field is

$$p = 2u - (u^2 + v^2 + w^2) \qquad [1]$$

In two-dimensional flow, there is no z-component of velocity.

In the outer regions of the field, the perturbation velocity components are small compared with unity, and squares of these quantities may be neglected. In determining the wall interference in a channel or an electrolytic tank, the contribution of the image bodies to the pressure is given approximately by

$$p' = 2u' \qquad [2]$$

where the primes refer to the sum of the effects due to all the bodies of the image space. Hence, only the x-component of the perturbation velocity is needed to find the magnitude of the wall interference.

For future reference, the x-component of the perturbation velocity will be listed for various types of singularities. The superscript 0 refers to flow unrestricted by channel walls. In two-dimensional flows, the perturbation velocities for a line source of strength Q_2 and line doublet of strength μ_2, oriented parallel to the flow velocity, are respectively

$$u_2^0\,(x, y) = \frac{Q_2\, x}{x^2 + y^2} \qquad [3]$$

$$u_2^0\,(x, y) = \frac{\mu_2\,(x^2 - y^2)}{(x^2 + y^2)^2} \qquad [4]$$

when the singularities are situated at the origin. In three-dimensional flows, the perturbation velocities for a point source of strength Q_3 and point doublet of strength μ_3, situated at the origin, are respectively

$$u_3^0\,(x, y, z) = \frac{Q_3\, x}{(x^2 + y^2 + z^2)^{3/2}} \qquad [5]$$

$$u_3^0(x, y, z) = \mu_3 \left[\frac{3x^2}{(x^2 + y^2 + z^2)^{5/2}} - \frac{1}{(x^2 + y^2 + z^2)^{3/2}} \right] \qquad [6]$$

LINE SOURCES AND SINKS IN A RECTANGULAR CHANNEL

If a line source is placed at $x = c$, $y = 0$, midway between two parallel walls which are unit distance apart, image sources are produced at $y = n$ where n takes on all integral values from $-\infty$ to $+\infty$; see Figure 1. The x-component of the velocity induced by this row of sources, including the original source, is

$$u_2(x - c, y) = Q_2 \tau_2(x - c, y) = Q_2 \sum_{n=-\infty}^{\infty} \frac{x - c}{(x - c)^2 + (y - n)^2} \qquad [7]$$

The sum, denoted by $\tau_2(x - c, y)$, has the value[2]

$$\tau_2(x - c, y) = \pi \frac{\sinh 2\pi(x - c)}{\cosh 2\pi(x - c) - \cos 2\pi y} \qquad [8]$$

Values of $\tau_2(x - c, y)$ for different values of $x - c$ and y are tabulated in Table 1, page 18. The velocity induced by the image sources alone is

$$u_2'(x - c, y) = Q_2 \tau_2'(x - c, y) = u_2(x - c, y) - u_2^0(x - c, y) \qquad [9]$$

Values of $\tau_2'(x - c, y)$ are also tabulated in Table 1 as well as $u_2^0(x - c, y)/Q_2$ for a single line source in an infinite fluid.

If in addition to a source at $x = c$, $y = 0$, a sink is placed at $x = -c$, $y = 0$, the x-component of the velocity induced by the resultant source-sink pair and all the images is

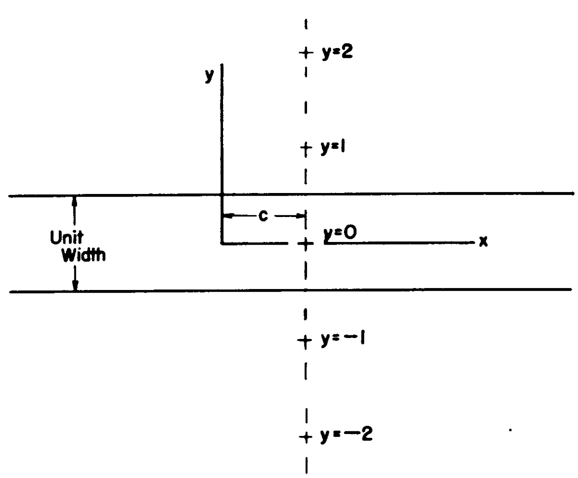

Figure 1 - Images Formed by a Line Source on the Centerline of an Infinitely Long Channel

$$u_2(x, y) = Q_2[\tau_2(x - c, y) - \tau_2(x + c, y)] \qquad [10]$$

In unrestricted flow the stagnation streamline past a line source-sink pair generates a Rankine oval. Although the longitudinal walls distort the shape somewhat, Equation [10] may be considered as a good approximation for the flow induced by a Rankine oval in an infinitely long channel. To find the velocity induced by the images alone, it is necessary to subtract the velocity induced by the original source-sink pair

$$u_2^0(x,y) = Q_2\left[\frac{x - c}{(x - c)^2 + y^2} - \frac{x + c}{(x + c)^2 + y^2}\right] \qquad [11]$$

The velocity induced by the images alone is

$$u_2'(x, y) = Q_2[\tau_2'(x - c, y) - \tau_2'(x + c, y)] = u_2(x, y) - u_2^0(x, y) \qquad [12]$$

Values of $u_2(x, y)/Q_2$, $u_2^0(x, y)/Q_2$ and $u_2'(x, y)/Q_2$ for Rankine ovals with $c = 0.10$, 0.20, and 0.30 are tabulated in Tables 2 through 4. The quantities in these tables were obtained by combining appropriate values of $\tau_2(x - c, y)$, $u_2^0(x - c, y)/Q_2$, and

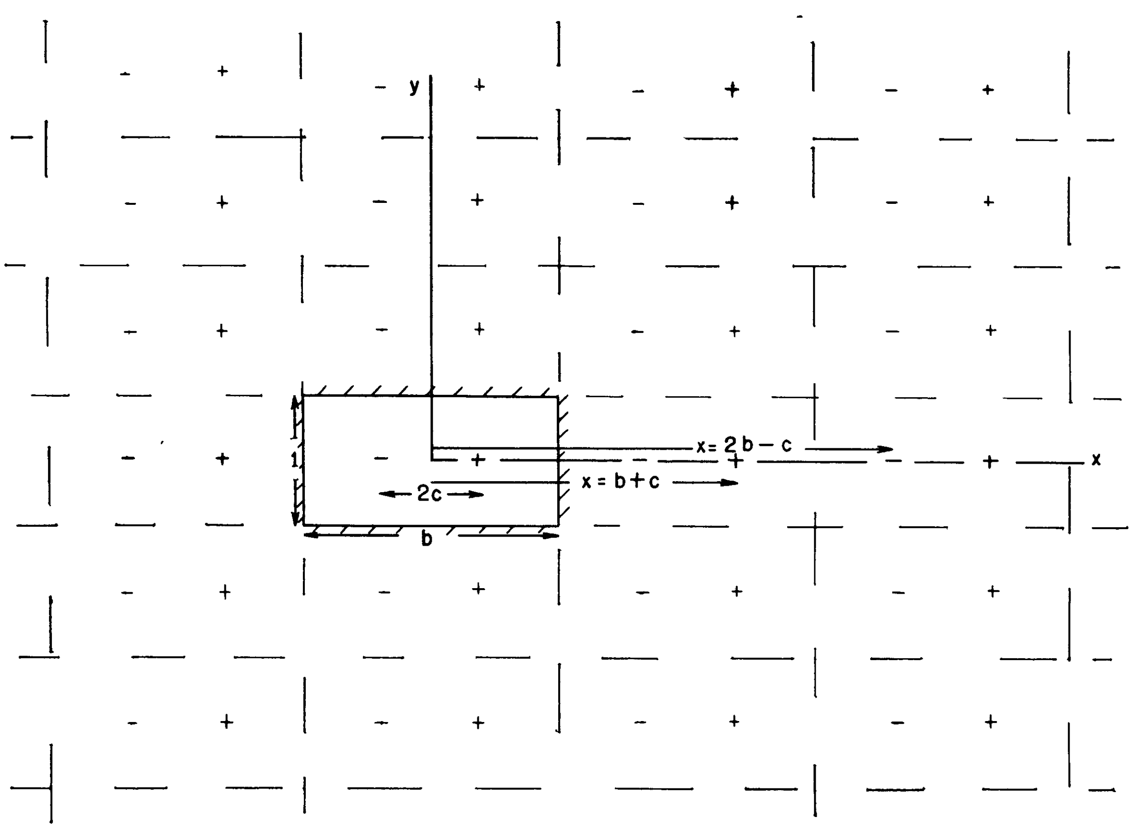

Figure 2 - Images Formed by a Line Source-Sink Pair on the Centerline of a Channel of Unit Width and Finite Length

$\tau_2'(x - c, y)$ from Table 1. Constants for obtaining Rankine ovals of different sizes and shapes may be obtained from the curves and formulas of the Appendix.

If a Rankine oval is placed at the center of a rectangular electrolytic tank of unit width and finite length b, images are formed in the ends of the tank as well as in the sides; see Figure 2. Since the end walls are conducting surfaces, sources are reflected as alternate sinks and sources at the image points. For the source-sink pair, it can be shown that sources are induced at $x = c + mb$ and sinks at $x = -c + mb$, where m takes on all integral values from $-\infty$ to ∞. The velocity induced by the original body and all the images becomes

$$u_2(x, y) = \pi Q_2 \sum_{m=-\infty}^{\infty} \left[\frac{\sinh 2\pi(x - c - mb)}{\cosh 2\pi(x - c - mb) - \cos 2\pi y} - \frac{\sinh 2\pi(x + c - mb)}{\cosh 2\pi(x + c - mb) - \cos 2\pi y} \right] \quad [13]$$

Since both terms of Equation [13] converge to π rapidly as the argument $x \pm c - mb$ increases, the corrections for the end walls may be found by summing only a few terms which are already tabulated in Tables 2 through 4. For example, to find the perturbation velocity u_2 at $x = 0.2$, $y = 0.125$, in a tank of length $b = 0.8$, it is necessary to add values of u_2 in the $y = 0.125$ column for the following values of x: 0.2, − 0.6, 1.0, − 1.4, 1.8, etc. The perturbation velocity from the images alone is found by subtracting the value of u_2^0 at $x = 0.2$, $y = 0.125$.

CIRCULAR CYLINDER IN A RECTANGULAR CHANNEL

When the source and sink which generate the Rankine oval are brought infinitesimally close together, they form a doublet of strength μ_2 and the stagnation streamline generates a circle. It may be shown that in a flow of unit velocity, the doublet strength is

$$\mu_2 = t^2 \quad [14]$$

where t is the radius of the circle. The x-component of the perturbation velocity of the unrestricted flow past a circular cylinder is given by Equation [4].

If a circular cylinder is placed at the center of an infinitely long channel of unit width, image cylinders or doublets are produced at every $y = n$. As a first approximation, the velocity may be assumed to be induced by an infinite row of doublets

$$u_2(x, y) = \mu_2 \sigma_2(x, y) = \mu_2 \sum_{n=-\infty}^{\infty} \frac{x^2 - (y - n)^2}{[x^2 + (y - n)^2]^2} \quad [15]$$

The sum $\sigma_2(x, y)$ is the negative x-derivative of the sum given in Equation [7]. Since the value of the sum in Equation [7] is given in Equation [8], $\sigma_2(x, y)$ will be given by the negative x-derivative of Equation [8]. Then

$$\sigma_2(x, y) = -2\pi^2 \frac{\cosh 2\pi x \cos 2\pi y - 1}{[\cosh 2\pi x - \cos 2\pi y]^2} \qquad [16]$$

Values of $\sigma_2(x, y)$, $u_2^0(x, y)/\mu_2$, and $\sigma_2'(x, y)$ are tabulated in Table 5. Values of $u_2^0(x, y)$ are obtained from Equation [4] and

$$u_2'(x, y) = \mu_2 \sigma_2'(x, y) = \mu_2 \sigma_2(x, y) - u_2^0(x, y) \qquad [17]$$

Values of $u_2(x, y)$ and $u_2'(x, y)$ for tanks of finite lengths may be found by adding the effect of images in the ends of the tank as already described for the case of the Rankine oval.

POINT SOURCES AND SINKS IN A RECTANGULAR CHANNEL

The same general procedure as that followed for two-dimensional bodies will be followed for three-dimensional bodies in a rectangular channel or tank. Here, however, image bodies are produced above and below the channel by the multiple reflections in the upper and lower surfaces.

Before considering particular three-dimensional bodies in a channel, the effects of a single pair of surfaces on a point source will be considered. If the source is at $x = \alpha$, $y = \beta$, $z = 0$, halfway between parallel horizontal walls with spacing d, a column of image sources is induced above and below the original source at the points $z = kd$, where k takes on all integral values from $-\infty$ to ∞. From Equation [5], the x-component of the perturbation velocity becomes

$$u_3(x-\alpha, y-\beta, z) = Q_3 \sum_{k=-\infty}^{\infty} \frac{x-\alpha}{[(x-\alpha)^2 + (y-\beta)^2 + (z-kd)^2]^{3/2}} \qquad [18]$$

The sum in Equation [18] may be evaluated by summing over the $2K + 1$ sources nearest to the plane of the original body and integrating over the more distant images. Usually sufficient accuracy is obtained by choosing $K = 2$ or even 1. Then

$$\begin{aligned} u_3(x-\alpha, y-\beta, z) &= Q_3 \sum_{k=-K}^{K} \frac{x-\alpha}{[(x-\alpha)^2 + (y-\beta)^2 + (z-kd)^2]^{3/2}} \\ &+ \frac{Q_3}{d} \int_{-\infty}^{-(K+\frac{1}{2})d} \frac{(x-\alpha)\, d\xi}{[(x-\alpha)^2 + (y-\beta)^2 + (z-\xi)^2]^{3/2}} \qquad [19] \\ &+ \frac{Q_3}{d} \int_{(K+\frac{1}{2})d}^{\infty} \frac{(x-\alpha)\, d\xi}{[(x-\alpha)^2 + (y-\beta)^2 + (z-\xi)^2]^{3/2}} \end{aligned}$$

This equation may be integrated and expressed as a linear combination of three terms

$$u_3(x-\alpha,\ y-\beta,\ z) = u_2(x-\alpha,\ y-\beta) + Q_3[r(x-\alpha,\ y-\beta,\ z) - s(x-\alpha,\ y-\beta,\ z)] \qquad [20]$$

where (1)

$$u_2(x-\alpha,\ y-\beta) = \frac{2Q_3}{d}\,\frac{x-\alpha}{(x-\alpha)^2+(y-\beta)^2} \qquad [21]$$

is the velocity induced by a two-dimensional source of strength

$$Q_2 = \frac{2Q_3}{d} \qquad [22]$$

(2)

$$r(x-\alpha,\ y-\beta,\ z) = \sum_{k=-K}^{K} \frac{x-\alpha}{[(x-\alpha)^2+(y-\beta)^2+(z-kd)^2]^{3/2}} \qquad [23]$$

is the velocity induced by the original point source of unit strength and the $2K$ images above and below, and

(3)

$$s(x-\alpha,\ y-\beta,\ z) = \frac{x-\alpha}{d[(x-\alpha)^2+(y-\beta)^2]}\left\{\frac{(K+\tfrac{1}{2})\,d-z}{\sqrt{(x-\alpha)^2+(y-\beta)^2+[(K+\tfrac{1}{2})\,d-z]^2}} + \frac{(K+\tfrac{1}{2})\,d+z}{\sqrt{(x-\alpha)^2+(y-\beta)^2+[(K+\tfrac{1}{2})\,d+z]^2}}\right\} \qquad [24]$$

is the velocity induced by a line source extending from $z = -(K+\tfrac{1}{2})\,d$ to $z = (K+\tfrac{1}{2})\,d$.

Next consider a point source at $x = c$, $y = 0$, $z = 0$, at mid-height and on the centerline of a channel of unit width, of height d, but of infinite length. Now columns of image sources are induced at every $y = n$ and the perturbation velocity becomes

$$u_3(x-c,\ y,\ z) = Q_3\tau_3(x-c,\ y,\ z) = \sum_{n=-\infty}^{\infty}[u_2(x-c,\ y-n) + Q_3\, r(x-c,\ y-n,\ z) - Q_3\, s(x-c,\ y-n,\ z)] \qquad [25]$$

and

$$\tau_3(x-c,\ y,\ z) = \frac{2}{d}\tau_2(x-c,\ y) + \sum_{n=-\infty}^{\infty}[r(x-c,\ y-n,\ z) - s(x-c,\ y-n,\ z)] \qquad [26]$$

Values of $\tau_2(x-c,\ y)$ are already tabulated in Table 1. Values of $r(x-c,\ y,\ 0) - s(x-c,\ y,\ 0)$ which show how greatly the flow about a row of sources differs from that about a line source are tabulated in Tables 6 and 7 for two channel configurations $d = 1.0$ and 0.5 and for $z = 0$. In these calculations $K = 2$ was used for depth 0.5. The use of larger values of K did not change the values appreciably. The total correction at some value of x at a distance $y = 0.125$ off the centerline is found in channels of unit height by adding the corrections at $y = 0.125$ and -0.875. At a height $d = 0.5$, the correction at $y = -0.875$ is negligible.

Tables 8 and 9 give values of $\tau_3(x - c, y, 0)$ for infinitely long channels of unit width and heights $d = 1.0$ and 0.5, and $u_3^0(x - c, y, 0)$ from Equation [5] for a single point source in unrestricted flow. Values of $\tau_3'(x - c, y, 0)$ which gives the wall interference were obtained from the relation

$$u_3'(x - c, y, z) = Q_3 \dot{\tau}_3'(x - c, y, z) = Q_3 \; \tau_3(x - c, y, z) - u_3^0(x - c, y, z) \quad [27]$$

In analogy with the two-dimensional case, if a sink of strength $-Q_3$ is placed at $x = -c$ downstream from a source of equal strength at $x = c$, the stagnation streamline generates a Rankine ovoid. If this source and sink are on the centerline of an infinitely long channel of unit width, image sources are produced at $x = c$, $y = n$, $z = kd$, and image sinks at $x = -c$, $y = n$, $z = kd$. The x-component of the perturbation velocity becomes

$$u_3(x, y, z) = Q_3 [\tau_3(x - c, y, z) - \tau_3(x + c, y, z)] \quad [28]$$

Using appropriate values from Tables 8 and 9, $u_3(x, y, 0)/Q_3$ and $u_3^0(x, y, 0)/Q_3$ are tabulated in Tables 10 through 15 for various Rankine ovoids in infinitely long channels in which $d = 1.0$ and 0.5. The wall interference alone, $u_3'(x, y, 0)/Q_3$ in these tables, was found from the relation

$$u_3'(x, y, z) = Q_3 \; [\tau_3'(x - c, y, z) - \tau_3'(x + c, y, z)] = u_3(x, y, z) - u_3^0(x, y, z) \quad [29]$$

The effect of the end walls in an electrolytic tank of finite length may be obtained by the method described for the case of the Rankine oval.

SPHERE IN A RECTANGULAR CHANNEL

In the limiting condition when the source and sink are brought infinitesimally close together, the source-sink pair becomes a doublet. In terms of the sphere radius t, the doublet strength in a free stream of unit velocity is

$$\mu_3 = \tfrac{1}{2} t^3 \quad [30]$$

The x-component of the perturbation velocity is given in Equation [6].

A sphere midway between a pair of parallel horizontal walls of infinite extent and a distance d apart induces an infinite column of image spheres with spacing d above and below the original one. The velocity induced at any point by a sphere at $x = \alpha$, $y = \beta$, $z = 0$, is given approximately by

$$u_3(x-\alpha,\ y-\beta,\ z) = \mu_3 \sum_{k=-\infty}^{\infty} \left\{ \frac{3(x-\alpha)^2}{[(x-\alpha)^2+(y-\beta)^2+(z-kd)^2]^{5/2}} - \frac{1}{[(x-\alpha)^2+(y-\beta)^2+(z-kd)^2]^{3/2}} \right\} \quad [31]$$

As in the case of the Rankine ovoid, Equation [31] will be evaluated by summing over near images and integrating over more distant images. This is equivalent to considering the column of doublets as an infinite line doublet except over the central portion where discrete doublets are substituted for a portion of the line. Then Equation [31] reduces to a linear combination of three terms

$$u_3(x-\alpha,\ y-\beta,\ z) = u_2(x-\alpha,\ y-\beta) + \mu_3[R(x-\alpha,\ y-\beta,\ z) - S(x-\alpha,\ y-\beta,\ z)] \quad [32]$$

where (1)

$$u_2(x-\alpha,\ y-\beta) = \frac{2\mu_3}{d}\,\frac{(x-\alpha)^2-(y-\beta)^2}{[(x-\alpha)^2+(y-\beta)^2]^2} \quad [33]$$

is the velocity induced by an infinite line doublet of strength

$$\mu_2 = \frac{2\mu_3}{d}$$

(2)

$$R(x-\alpha,\ y-\beta,\ z) = \sum_{k=-K}^{K} \left\{ \frac{3(x-\alpha)^2}{[(x-\alpha)^2+(y-\beta)^2+(z-kd)^2]^{5/2}} - \frac{1}{[(x-\alpha)^2+(y-\beta)^2+(z-kd)^2]^{3/2}} \right\} \quad [34]$$

is the velocity induced by the original doublet of unit strength and the 2K images above and below, and

(3)

$$S(x-\alpha,\ y-\beta,\ z) = \frac{2(x-\alpha)^2-(y-\beta)^2}{d[(x-\alpha)^2+(y-\beta)^2]^2} \left\{ \frac{(K+\tfrac{1}{2})d-z}{\sqrt{(x-\alpha)^2+(y-\beta)^2+[(K+\tfrac{1}{2})d-z]^2}} + \frac{(K+\tfrac{1}{2})d+z}{\sqrt{(x-\alpha)^2+(y-\beta)^2+[(K+\tfrac{1}{2})d+z]^2}} \right\} - \frac{(x-\alpha)^2}{d[(x-\alpha)^2+(y-\beta)^2]^2} \times$$

$$\left\{ \frac{[(K+\tfrac{1}{2})d-z]^3}{\left[(x-\alpha)^2+(y-\beta)^2+[(K+\tfrac{1}{2})d-z]^2\right]^{3/2}} + \frac{[(K+\tfrac{1}{2})d+z]^3}{\left[(x-\alpha)^2+(y-\beta)^2+[(K+\tfrac{1}{2})d+z]^2\right]^{3/2}} \right\} \quad [35]$$

is the velocity induced by a finite line doublet extending from $-(K + \frac{1}{2})\,d$ to $(K + \frac{1}{2})\,d$.

Next consider a point doublet at $x = 0$, $y = 0$, $z = 0$, at mid-depth on the centerline of a channel of unit width and infinite length. Since there are columns of doublets induced at every $y = n$

$$u_3(x, y, z) = \mu_3\, \sigma_3(x, y, z)$$
$$= \sum_{n=-\infty}^{\infty} \left\{ u_2(x, y - n) + \mu_3 [R(x, y - n, z) - S(x, y - n, z)] \right\} \qquad [36]$$

or

$$\sigma_3(x, y, z) = \frac{2}{d}\sigma_2(x, y) + \sum_{n=-\infty}^{\infty} [R(x, y - n, z) - S(x, y - n, z)] \qquad [37]$$

Computations of $R(x, y, 0) - S(x, y, 0)$, which show how greatly the flow about a single column of discrete spheres of unit spacing differs from the flow about a line doublet, are tabulated in Table 16. Table 17 gives values of $\sigma_3(x, y, 0)$ in an infinitely long channel of unit width and unit height and values of $u_3^0(x, y, 0)/Q_3$ for the unrestricted flow about a doublet from Equation [6]. Values for the image spheres alone $\sigma_3'(x, y, z)$ were obtained from the relation

$$u_3'(x, y, z) = \mu_3\, \sigma_3'(x, y, z) = u_3(x, y, z) - u_3^0(x, y, z) \qquad [38]$$

The effect of the end walls in an electrolytic tank of finite length may be computed by the method already described for the Rankine oval.

EXTENSION OF RESULTS TO BODIES OF OTHER SHAPES

In his paper on wind-tunnel interference, Lock[1] has derived expressions for the wall interference for symmetrical bodies on the centerline of an infinitely long rectangular channel by replacing the bodies of the image space by point doublets of appropriate strengths. Although Lock computed the wall correction only at the center of the body, his results may readily be extended to obtain the wall correction at any point in the channel by means of relations already obtained in former sections of this report:

$$u_2'(x, y) = \mu_2\, \sigma_2'(x, y) \qquad [17]$$

$$u_3'(x, y, z) = \mu_3\, \sigma_3'(x, y, z) \qquad [38]$$

Lock writes the dipole moment of the body in terms of its maximum half-width t and a shape parameter λ.

$$\mu_2 = \lambda t^2 \qquad [39]$$

$$\mu_3 = \tfrac{1}{2} \lambda t^3 \qquad [40]$$

It is evident that $\lambda = 1$ for a circle or sphere and is greater than 1 for elongated bodies. The formulas in Table 18 give the functional form of λ for some of the common symmetrical bodies.[1] Lock's graphs of λ as a function of the length-width ratio are plotted in Figure 3.

If the velocity distribution of an arbitrary body in a free stream of unit velocity is known, expressions for the dipole moments may be calculated from the following integrals:[3]

$$\mu_2 = \frac{1}{\pi} \int_0^L U_s \frac{\partial s}{\partial x} y \, dx \qquad [41]$$

$$\mu_3 = \frac{1}{4} \int_0^L U_s \frac{\partial s}{\partial x} y^2 \, dx \qquad [42]$$

where U_s is the velocity along the body surface in a free stream of unit velocity and L is the over-all body length.

The wall correction obtained by representing the bodies of the image space by point doublets gives a reasonably accurate estimate of the wall correction, provided the over-all length of the body is small compared with the channel width. A better approximation may be obtained by representing the image bodies by source-sink pairs with finite separation $2c$. Then the wall interference, obtained from Equations [12] and [29] by substituting $\mu/2c$ for the source strength, is given by the relations:

$$u_2'(x, y) = \frac{\mu_2}{2c} [\tau_2'(x - c, y) - \tau_2'(x + c, y)] = \mu_2 \, \sigma_2'(x, y : c) \qquad [43]$$

$$u_3'(x, y, z) = \frac{\mu_3}{2c} [\tau_3'(x - c, y, z) - \tau_3'(x + c, y, z)] = \mu_3 \, \sigma_3'(x, y, z : c) \qquad [44]$$

As c approaches zero, $\sigma_2'(x, y; c)$ approaches $\sigma_2'(x, y)$ and $\sigma_3'(x, y, z; c)$ approaches $\sigma_3'(x, y, z)$. Graphs have been prepared in Figures 4 through 7 showing the variation of $\sigma_2'(x, y; c)$ and $\sigma_3'(x, y, z; c)$ with c for various channel configurations. In addition to the data already presented in the earlier sections of the report, the wall interference along the centerline of a channel of height-width ratio 0.8 is presented in Figure 7.

As the values of $\sigma_2'(x, y; c)$ and $\sigma_3'(x, y, z; c)$ change slowly with c, it is not necessary to determine c exactly. The value of c for the equivalent Rankine oval or ovoid with the same maximum half-width and shape parameter gives a reasonably accurate value of c for any other arbitrary body. Thus c may be determined by the use of the graphs of Figures 3, 8, and 9. The length-width ratio of a Rankine oval or ovoid having the same shape parameter as the original body may be found on Figure 3. Then with the aid of the graphs or by formula 5 or 6 of Table 18, the value of c may be determined for that oval or ovoid with the same maximum half-width as the original body.

TABLE 18

λ for Some Common Symmetrical Bodies

1. Circle	$\lambda = 1$
2. Sphere	$\lambda = 1$
3. Ellipse	$\lambda = \frac{1}{2}\left(1 + \frac{l}{t}\right)$
4. Spheroid	$\lambda = \frac{4}{3}\left(\frac{l}{t}\right)^3 \frac{e^3}{ln\,\frac{1+e}{1-e} - \frac{2e}{1-e^2}}$
5. Rankine Oval	$\lambda = \frac{l^2 - c^2}{t^2}$
6. Rankine Ovoid	$\lambda = \sqrt{\frac{c^2}{t^2} + 1}$

TABLE 19

Comparison of the Ratio $\frac{c}{t}$ of a Spheroid with that of the Equivalent Rankine Ovoid

l/t	Spheroid		Rankine Ovoid
	λ	c/t	c/t
2	1.613	1.155	1.266
3	2.244	1.885	2.009
4	2.884	2.582	2.705
5	3.530	3.266	3.386
7	4.833	4.619	4.728
9	6.147	5.963	6.065

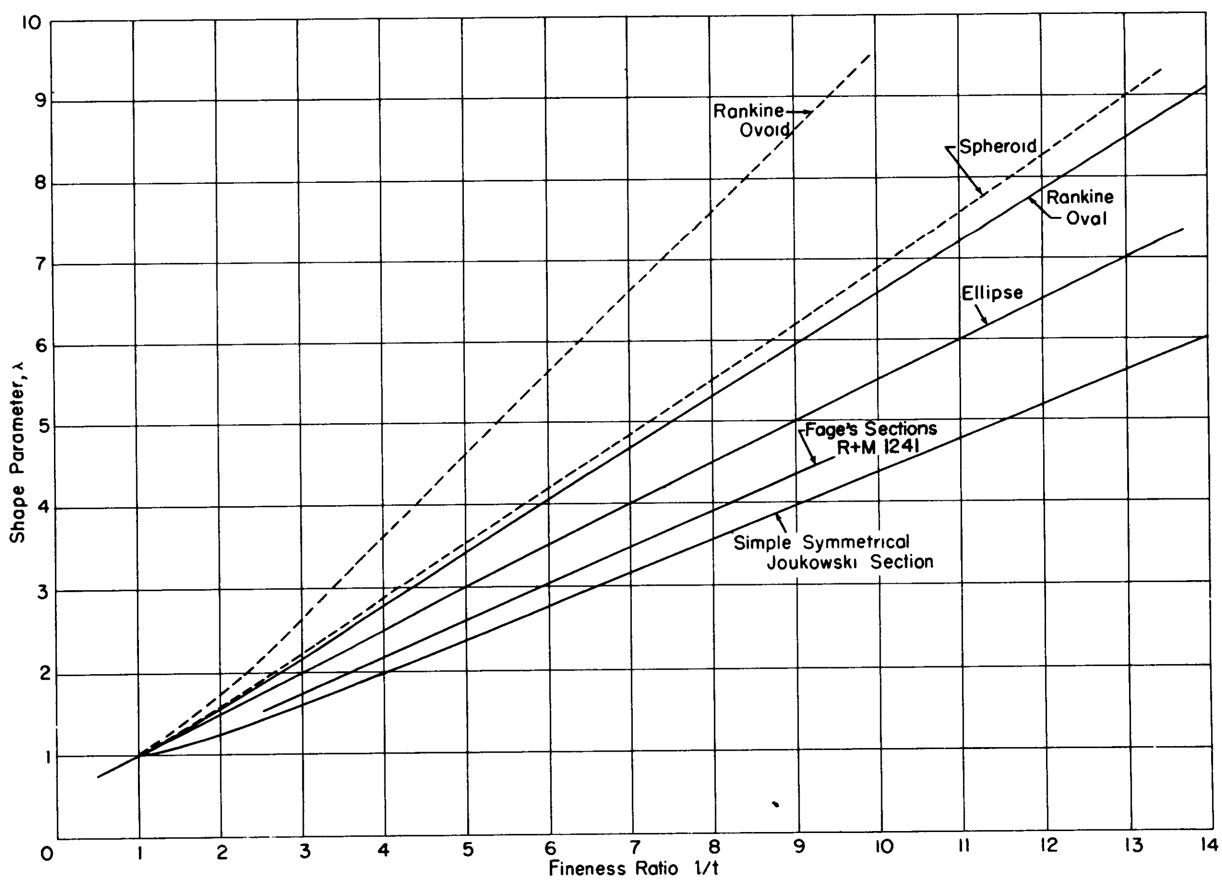

Figure 3 - Lock's Shape Parameter as a Function of Fineness Ratio for Bodies of Different Shapes.[3]

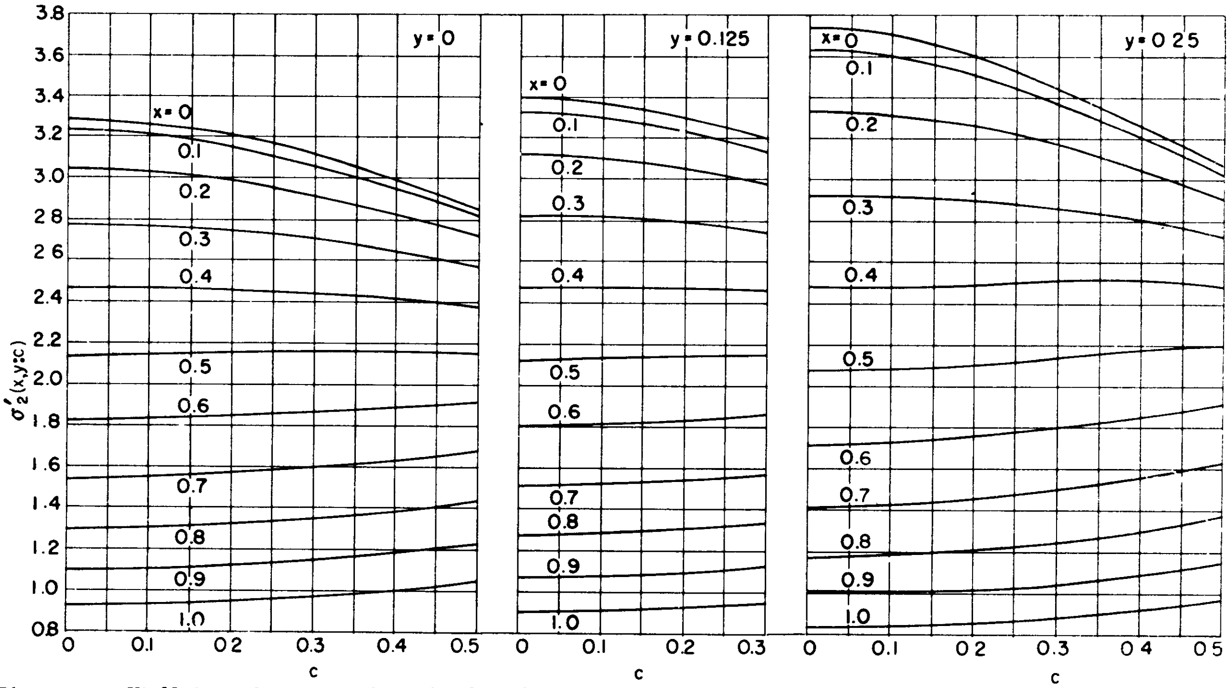

Figure 4 - Wall Interference of a Dipole of Unit Strength Consisting of a Line Source-Sink Pair of Separation $2c$ on the Centerline of an Infinitely Long Channel of Unit Width

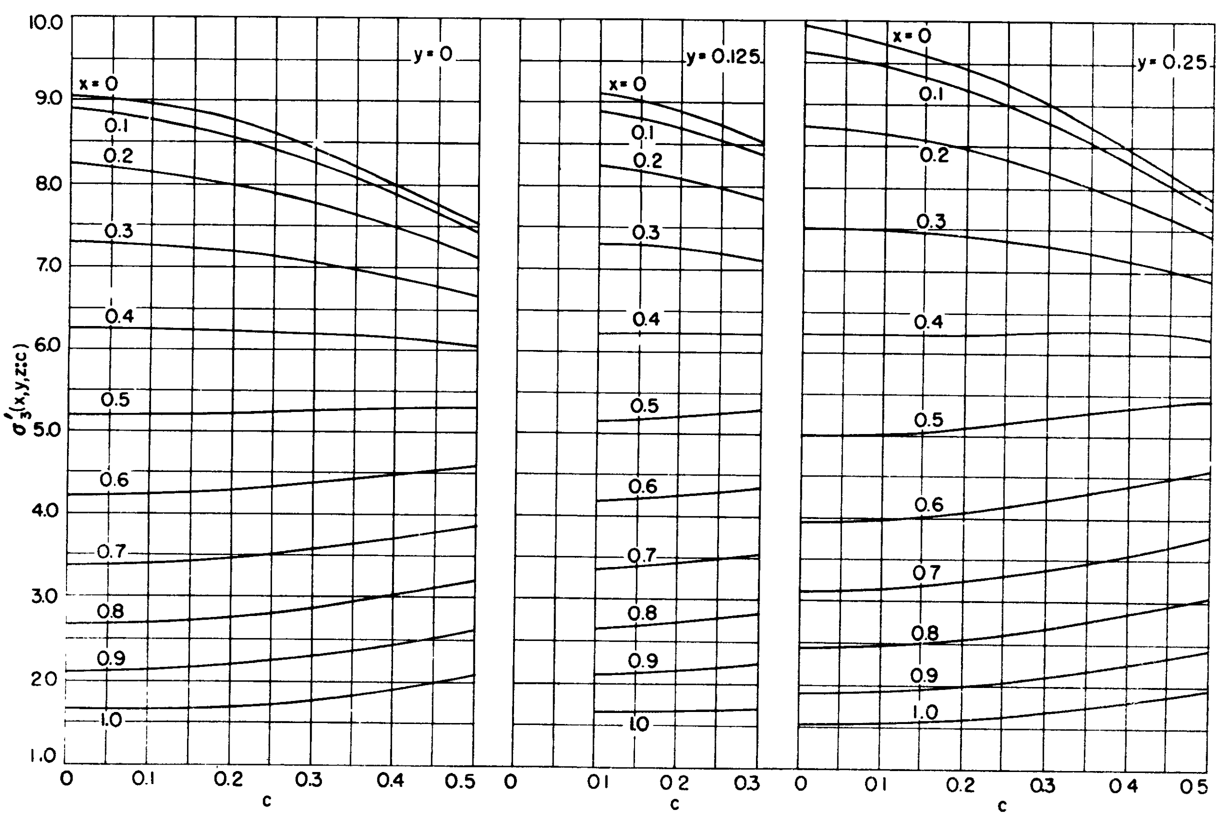

Figure 5 - Wall Interference of a Dipole of Unit Strength Consisting of a Point Source-Sink Pair of Separation $2c$ at the Centerline of an Infinitely Long Channel of Height-Width Ratio $d = 1.0$

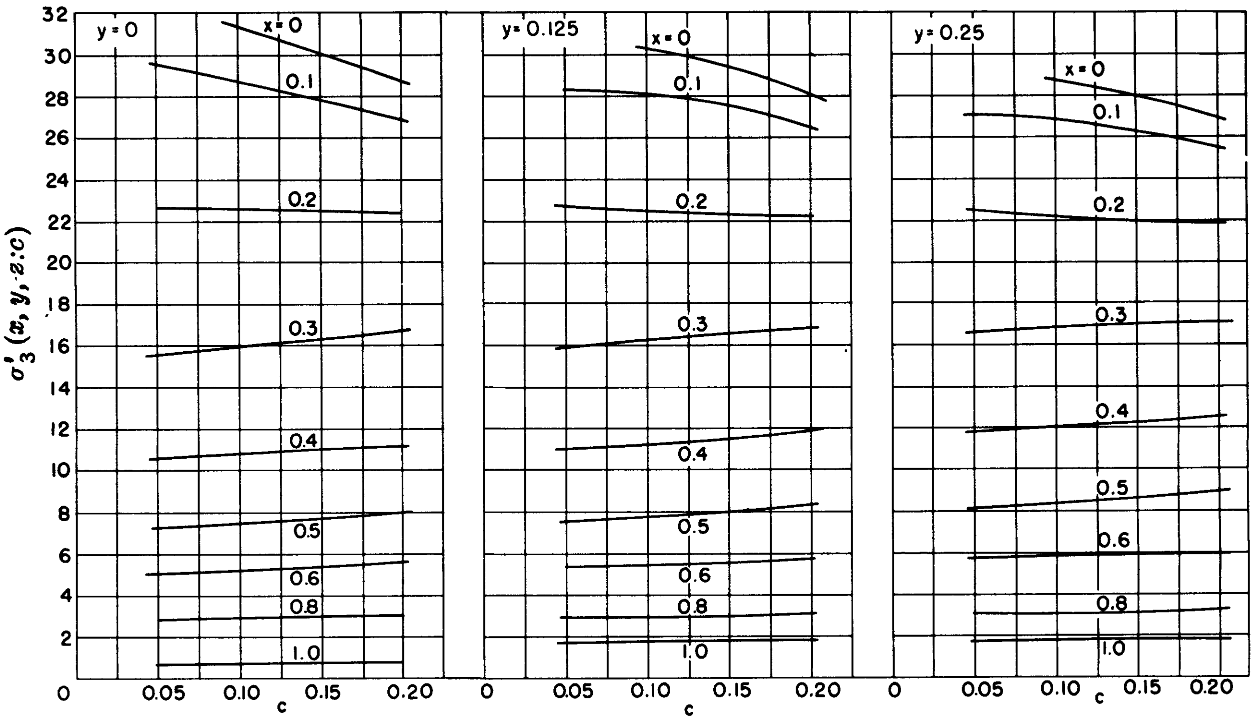

Figure 6 - Wall Interference of a Dipole of Unit Strength Consisting of a Point Source-Sink Pair of Separation $2c$ at the Centerline of an Infinitely Long Channel of Height-Width Ratio $d = 0.5$

The accuracy of this approximation was tested for spheroids of different length-width ratios. Since the source distribution of a spheroid varies linearly between the two foci, the centroids of the total positive and negative charges are at two thirds of the focal distances. The shape parameter may be calculated by Formula 4 of Table 18 or more readily by means of the following formula for μ_3 [4]*

$$\mu_3 = \frac{1 + k_1}{4\pi} V \qquad [45]$$

where V is the volume and k_1 the added mass coefficient of the body. Then for the spheroid

$$\lambda = \frac{2}{3}(1 + k_1)\frac{l}{t} \qquad [46]$$

Values of k_1 for spheroids of different length-width ratios are tabulated in Reference 4. Then c/t for the Rankine ovoid of the same shape parameter may be computed by the use of Formula 6 of Table 18. The error of the approximation may be found by comparing c/t for the spheroid and Rankine ovoid in Table 19. Since c is made dimensionless by dividing by the channel width, a 5 to 10 percent error in c/t results in an error of no more than 1 or 2 percent in c.

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*This formula for obtaining μ_3 may be used for any body for which the added mass coefficient k_1 is known. For two-dimensional bodies the corresponding formula is $\mu_2 = (1 + k_1)(\text{sectional area})/2\pi$

If the source distribution of a body is known or if it is determined by a numerical integration of the integrand of Equation [41] or [42], the wall correction may be determined with as much accuracy as desired. If the source distribution is broken down into a discrete number of sources and sinks at different points along the axis, the wall correction at any point may be found by summing over the wall corrections for the individual sources and sinks.

ACKNOWLEDGMENT

Many of the calculations in this report were made and checked by Mrs. Rose M. Sayre. The author is grateful to Mr. M.P. Tulin for helpful discussions which led to the general relations for the shape parameters of arbitrary bodies.

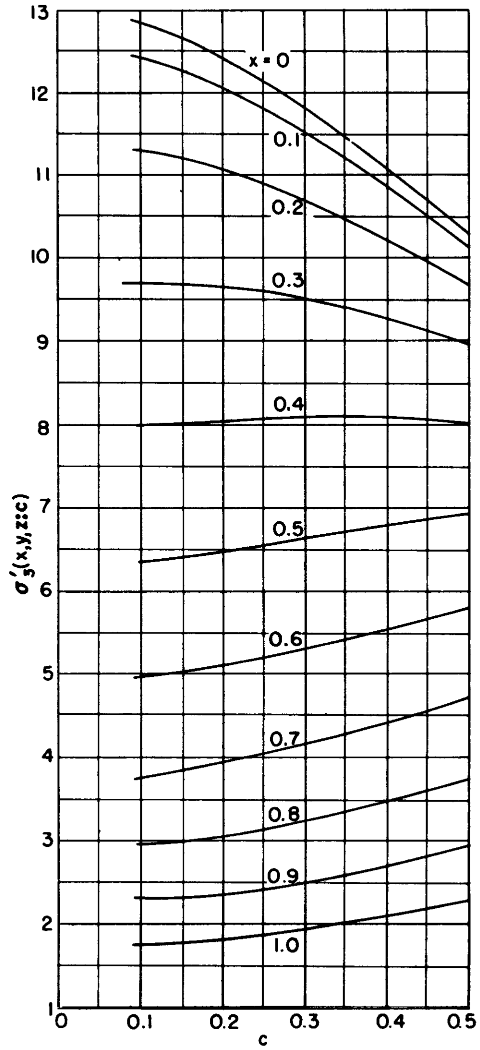

Figure 7 - Wall Interference of a Dipole of Unit Strength Consisting of a Point Source-Sink Pair of Separation $2c$ at the Centerline of an Infinitely Long Channel of Height-Width Ratio $d = 0.8$

APPENDIX

PARAMETERS FOR RANKINE OVALS AND OVOIDS

In order to facilitate the choice of source-sink separation and source strength for generating Rankine ovals and ovoids of different shapes and sizes, relations have been obtained between the pertinent parameters and are plotted in dimensionless form in Figures 8 and 9.

From standard textbooks in hydrodynamics, the following relations are obtained for the Rankine oval in a free stream of unit velocity[4]

$$\frac{Q_2}{c} = \frac{1}{2}\left(\frac{l^2}{c^2} - 1\right)$$

$$\frac{2\frac{t}{c}}{\frac{t^2}{c^2} - 1} = \tan\left(\frac{c}{Q_2} \cdot \frac{t}{c}\right) \qquad [47]$$

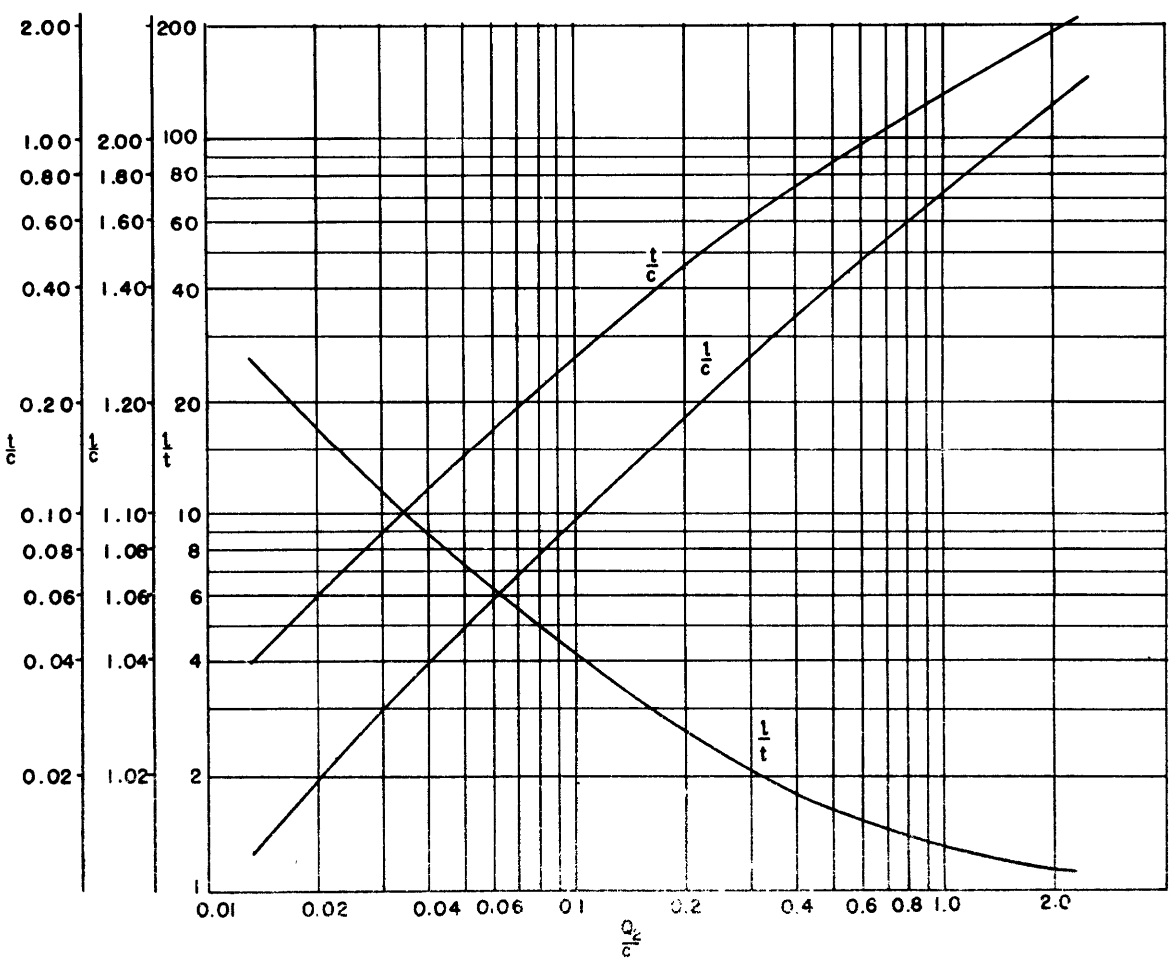

Figure 8 - Relations Between Parameters for a Rankine Oval

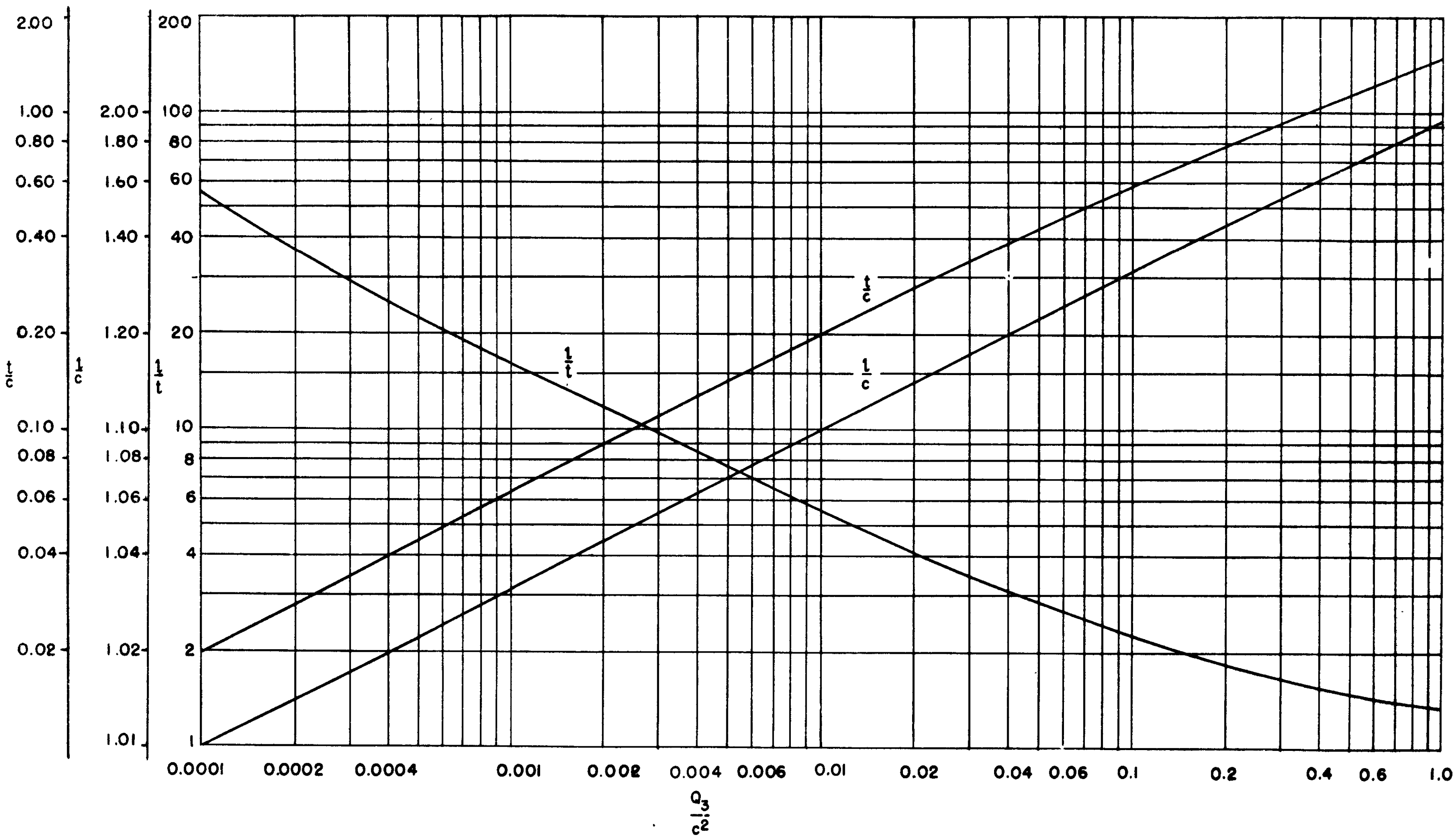

Figure 9 - Relations between Parameters for a Rankine Ovoid

In Figure 8, t/c, l/c, and l/t are plotted as functions of Q_2/c.

The corresponding relations for the Rankine ovoid are

$$\frac{Q_3}{c^2} = \frac{c}{4l}\left(\frac{l^2}{c^2} - 1\right)^2 = \frac{t^2}{4c^2}\sqrt{1 + \frac{t^2}{c^2}} \qquad [48]$$

In Figure 9, t/c, l/c, and l/t are plotted as functions of Q_3/c^2.

TABLE 1

Wall Interference for a Line Source at $x = c$, $y = 0$, on the Centerline of an Infinitely Long Channel of Unit Width

	$y = 0$			$y = 0.125$			$y = 0.25$			$y = 0.375$			$y = 0.50$		
$x - c$	$\tau_2(x-c, y)$	u_2^0/Q_2	$\tau_2'(x-c, y)$	$\tau_2(x-c, y)$	u_2^0/Q_2	$\tau_2'(x-c, y)$	$\tau_2(x-c, y)$	u_2^0/Q_2	$\tau_2'(x-c, y)$	$\tau_2(x-c, y)$	u_2^0/Q_2	$\tau_2'(x-c, y)$	$\tau_2(x-c, y)$	u_2^0/Q_2	$\tau_2(x-c,y)$
0	∞	∞	0	0	0	0	0	0	0	0	0	0	0	0	0
0.1	10.326	10.000	0.326	4.239	3.902	0.337	1.750	1.379	0.371	1.102	0.664	0.438	0.956	0.385	0.571
0.2	5.641	5.000	0.641	4.255	3.596	0.659	2.671	1.951	0.720	1.946	1.107	0.839	1.750	0.690	1.060
0.3	4.266	3.333	0.933	3.797	2.840	0.957	3.000	1.967	1.033	2.483	1.301	1.182	2.313	0.882	1.431
0.4	3.695	2.500	1.195	3.499	2.278	1.221	3.101	1.798	1.303	2.784	1.331	1.453	2.671	0.976	1.695
0.5	3.426	2.000	1.426	3.333	1.882	1.451	3.130	1.600	1.530	2.950	1.280	1.670	2.881	1.000	1.881
0.6	3.290	1.667	1.623	3.244	1.597	1.647	3.138	1.420	1.718	3.039	1.199	1.840	3.000	0.984	2.016
0.7	3.220	1.429	1.791	3.196	1.384	1.812	3.141	1.267	1.874	3.087	1.110	1.977	3.065	0.946	2.119
0.8	3.183	1.250	1.933	3.171	1.220	1.951	3.141	1.139	2.002	3.112	1.025	2.087	3.101	0.899	2.202
0.9	3.164	1.111	2.054	3.157	1.090	2.067	3.142	1.032	2.110	3.126	0.947	2.179	3.120	0.849	2.271
1.0	3.153	1.000	2.153	3.150	0.985	2.165	3.142	0.941	2.201	3.133	0.877	2.256	3.130	0.800	2.330
1.1	3.148	0.909	2.239	3.146	0.898	2.248	3.142	0.864	2.278	3.137	0.814	2.323	3.135	0.753	2.382
1.2	3.145	0.833	2.312	3.144	0.824	2.320	3.142	0.799	2.342	3.139	0.759	2.380	3.138	0.710	2.428
1.3	3.144	0.769	2.375	3.143	0.762	2.381	3.142	0.742	2.400	3.140	0.710	2.430	3.140	0.670	2.470
1.4	3.143	0.714	2.429	3.142	0.709	2.433	3.142	0.692	2.450	3.141	0.667	2.474	3.141	0.633	2.508
1.5	3.142	0.667	2.475	3.142	0.662	2.480	3.142	0.649	2.493	3.141	0.628	2.513	3.141	0.600	2.541
1.6	3.142	0.625	2.517	3.142	0.621	2.521	3.142	0.610	2.532	3.141	0.593	2.548	3.141	0.569	2.572
1.7	3.142	0.588	2.554	3.142	0.585	2.563	3.142	0.576	2.566	3.141	0.561	2.580	3.141	0.541	2.600
1.8	3.142	0.556	2.586	3.142	0.553	2.589	3.142	0.545	2.597	3.142	0.532	2.610	3.142	0.516	2.626

TABLE 2

Wall Interference for a Rankine Oval ($c = 0.1$) on the Centerline of an Infinitely Long Channel of Unit Width

x	$y = 0$			$y = 0.125$			$y = 0.25$			$y = 0.375$			$y = 0.5$		
	u_2/Q_2	u_2^0/Q_2	u_2'/Q_2	u_2/Q_2	u_2^0/Q_2	u_2'/Q_2	u_2/Q_2	u_2^0/Q_2	u_2'/Q_2	u_2/Q_2	u_2^0/Q_2	u_2'/Q_2	u_2/Q_2	u_2^0/Q_2	u_2'/Q_2
0	−20.652	−20.000	−0.652	−8.478	−7.804	−0.674	−3.500	−2.758	−0.742	−2.204	−1.328	−0.876	−1.912	−0.770	−1.142
0.1	∞	∞	−0.641	−4.255	−3.596	−0.659	−2.671	−1.951	−0.720	−1.946	−1.107	−0.839	−1.750	−0.690	−1.060
0.2	+ 6.060	+ 6.667	−0.607	+0.442	+ 1.062	−0.620	−1.250	−0.588	−0.662	−1.381	−0.637	−0.744	−1.357	−0.497	−0.860
0.3	1.946	2.500	−0.554	0.756	1.318	−0.562	−0.430	+0.153	−0.583	−0.838	−0.224	−0.614	−0.921	−0.286	−0.635
0.4	0.840	1.333	−0.493	0.464	0.958	−0.494	−0.130	0.367	−0.497	−0.467	+0.021	−0.488	−0.568	−0.118	−0.450
0.5	0.405	0.833	−0.428	0.255	0.681	−0.426	−0.037	0.378	−0.415	−0.255	0.132	−0.387	−0.329	−0.008	−0.321
0.6	0.206	0.571	−0.365	0.137	0.498	−0.361	−0.011	0.333	−0.344	−0.137	0.170	−0.307	−0.184	+0.054	−0.238
0.7	0.107	0.417	−0.310	0.073	0.377	−0.304	−0.003	0.281	−0.284	−0.073	0.174	−0.247	−0.101	0.085	−0.186
0.8	0.056	0.318	−0.262	0.039	0.294	−0.255	−0.001	0.235	−0.236	−0.039	0.163	−0.202	−0.055	0.097	−0.152
0.9	0.030	0.250	−0.220	0.021	0.235	−0.214	−0.001	0.198	−0.199	−0.021	0.148	−0.169	−0.029	0.099	−0.128
1.0	0.016	0.202	−0.186	0.011	0.192	−0.181	0	0.168	−0.168	−0.011	0.133	−0.144	−0.015	0.096	−0.111
1.1	0.008	0.167	−0.159	0.006	0.161	−0.155	0	0.142	−0.142	−0.006	0.118	−0.124	−0.008	0.090	−0.098
1.2	0.004	0.140	−0.136	0.003	0.136	−0.133	0	0.122	−0.122	−0.003	0.104	−0.107	−0.005	0.083	−0.088
1.3	0.002	0.119	−0.117	0.002	0.115	−0.113	0	0.107	−0.107	−0.002	0.092	−0.094	−0.003	0.077	−0.080
1.4	0.002	0.102	−0.100	0.001	0.100	−0.099	0	0.093	−0.093	−0.001	0.082	−0.083	−0.001	0.070	−0.071
1.5	0.001	0.089	−0.088	0	0.088	−0.088	0	0.082	−0.082	0	0.074	−0.074	0	0.064	−0.064
1.6	0	0.079	−0.079	0	0.077	−0.077	0	0.073	−0.073	0	0.067	−0.067	0	0.059	−0.059

TABLE 3

Wall Interference for a Rankine Oval (c = 0.2) on the Centerline of an Infinitely Long Channel of Unit Width

	$y=0$			$y=0.125$			$y=0.25$			$y=0.375$			$y=0.5$		
x	u_2/Q_2	u_2^0/Q_2	u_2'/Q_2	u_2/Q_2	u_2^0/Q_2	u_2'/Q_2	u_2/Q_2	u_2^0/Q_2	u_2'/Q_2	u_2/Q_2	u_2^0/Q_2	u_2'/Q_2	u_2/Q_2	u_2^0/Q_2	u_2'/Q_2
0	−11.282	−10.000	−1.282	−8.510	−7.192	−1.318	−5.342	−3.902	−1.440	−3.892	−2.214	−1.678	−3.500	−1.380	−2.120
0.1	−14.592	−13.333	−1.259	−8.036	−6.742	−1.294	−4.750	−3.346	−1.404	−3.585	−1.965	−1.620	−3.269	−1.267	−2.002
0.2	∞	∞	−1.195	−3.499	−2.278	−1.221	−3.101	−1.798	−1.303	−2.784	−1.331	−1.453	−2.671	−0.976	−1.695
0.3	+ 6.900	+ 8.000	−1.100	+0.906	+2.020	−1.114	−1.380	−0.221	−1.159	−1.848	−0.616	−1.232	−1.925	−0.615	−1.310
0.4	2.351	+ 3.333	−0.982	1.011	1.999	−0.988	−0.467	+0.531	−0.998	−1.093	−0.092	−1.001	−1.250	−0.294	−0.956
0.5	1.046	1.904	−0.858	0.601	1.456	−0.855	−0.141	0.700	−0.841	−0.604	+0.191	−0.795	−0.752	−0.064	−0.688
0.6	0.512	1.250	−0.738	0.328	1.058	−0.730	−0.040	0.659	−0.699	−0.328	0.306	−0.634	−0.430	+0.077	−0.507
0.7	0.262	0.889	−0.627	0.176	0.792	−0.616	−0.012	0.568	−0.580	−0.176	0.333	−0.509	−0.239	0.151	−0.390
0.8	0.137	0.667	−0.530	0.094	0.612	−0.518	−0.004	0.479	−0.483	−0.094	0.322	−0.416	−0.130	0.184	−0.314
0.9	0.072	0.520	−0.448	0.050	0.486	−0.436	−0.001	0.403	−0.404	−0.050	0.296	−0.346	−0.070	0.193	−0.263
1.0	0.038	0.417	−0.379	0.027	0.396	−0.369	−0.001	0.340	−0.341	−0.027	0.266	−0.293	−0.037	0.189	−0.226
1.1	0.020	0.342	−0.322	0.014	0.328	−0.314	0	0.290	−0.290	−0.014	0.237	−0.251	−0.020	0.179	−0.199
1.2	0.010	0.286	−0.276	0.008	0.276	−0.268	0	0.249	−0.249	−0.008	0.210	−0.218	−0.011	0.167	−0.178
1.3	0.006	0.242	−0.236	0.004	0.236	−0.232	0	0.215	−0.215	−0.004	0.186	−0.190	−0.006	0.153	−0.159
1.4	0.003	0.208	−0.205	0.002	0.203	−0.201	0	0.189	−0.189	−0.002	0.166	−0.168	−0.003	0.141	−0.144
1.5	0.002	0.181	−0.179	0.001	0.177	−0.176	0	0.166	−0.166	−0.001	0.149	−0.150	−0.001	0.129	−0.130
1.6	0.001	0.158	−0.159	0	0.156	−0.156	0	0.147	−0.147	−0.001	0.135	−0.136	−0.001	0.117	−0.118

TABLE 4

Wall Interference for a Rankine Oval (c = 0.3) on the Centerline of an Infinitely Long Channel of Unit Width

	$y = 0$			$y = 0.125$			$y = 0.25$			$y = 0.375$			$y = 0.5$		
x	u_2/Q_2	u_2^0/Q_2	u_2'/Q_2	u_2/Q_2	u_2^0/Q_2	u_2'/Q_2	u_2/Q_2	u_2^0/Q_2	u_2'/Q_2	u_2/Q_2	u_2^0/Q_2	u_2'/Q_2	u_2/Q_2	u_2^0/Q_2	u_2'/Q_2
0	− 8.532	− 6.666	−1.866	−7.594	−5.680	−1.914	−6.000	−3.934	−2.066	−4.966	−2.602	−2.364	−4.626	−1.764	−2.862
0.1	− 9.336	− 7.500	−1.836	−7.754	−5.874	−1.880	−5.772	−3.749	−2.023	−4.730	−2.438	−2.292	−4.421	−1.666	−2.755
0.2	−13.752	−12.000	−1.752	−7.572	−5.784	−1.788	−4.880	−2.979	−1.901	−4.052	−1.944	−2.108	−3.837	−1.385	−2.452
0.3	∞	∞	−1.623	−3.244	−1.597	−1.647	−3.138	−1.420	−1.718	−3.039	−1.199	−1.840	−3.000	−0.984	−2.016
0.4	+ 7.106	+ 8.571	−1.465	+1.043	+2.518	−1.475	−1.391	+0.112	−1.503	−1.985	−0.446	−1.539	−2.109	−0.561	−1.548
0.5	2.458	3.750	−1.292	1.084	2.376	−1.292	−0.470	0.812	−1.282	−1.166	+0.082	−1.248	−1.351	−0.209	−1.142
0.6	1.102	2.222	−1.120	0.640	1.750	−1.110	−0.142	0.935	−1.077	−0.643	0.354	−0.997	−0.807	+0.033	−0.840
0.7	0.542	1.500	−0.958	0.349	1.293	−0.944	−0.041	0.857	−0.898	−0.349	0.454	−0.803	−0.459	0.176	−0.635
0.8	0.278	1.091	−0.813	0.187	0.984	−0.797	−0.012	0.736	−0.748	−0.187	0.466	−0.653	−0.254	0.247	−0.501
0.9	0.145	0.834	−0.689	0.100	0.773	−0.673	−0.004	0.621	−0.625	−0.100	0.440	−0.540	−0.138	0.274	−0.412
1.0	0.076	0.660	−0.584	0.053	0.622	−0.569	−0.001	0.525	−0.526	−0.053	0.400	−0.453	−0.075	0.276	−0.351
1.1	0.040	0.536	−0.496	0.029	0.511	−0.482	−0.001	0.447	−0.448	−0.029	0.358	−0.387	−0.040	0.266	−0.306
1.2	0.022	0.444	−0.422	0.015	0.428	−0.413	0	0.383	−0.383	−0.015	0.319	−0.334	−0.021	0.249	−0.270
1.3	0.011	0.375	−0.364	0.008	0.364	−0.356		0.331	−0.331	−0.008	0.284	−0.292	−0.011	0.231	−0.242
1.4	0.006	0.321	−0.315	0.004	0.313	−0.309		0.288	−0.288	−0.004	0.253	−0.257	−0.006	0.212	−0.218
1.5	0.003	0.277	−0.274	0.002	0.271	−0.269		0.254	−0.254	−0.003	0.227	−0.230	−0.004	0.194	−0.198
1.6	0.002			0.001						−0.002			−0.002		

TABLE 5

Wall Interference for a Circular Cylinder on the Centerline of an Infinitely Long Channel of Unit Width

	$y = 0$			$y = 0.125$			$y = 0.25$			$y = 0.375$			$y = 0.5$		
x	σ_2	u_2^0/μ_2	σ_2'	σ_2	u_2^0/μ_2	σ_2'	σ_2	u_2^0/μ_2	σ_2'	σ_2	u_2^0/μ_2	σ_2'	σ_2	u_2^0/μ_2	σ_2'
0	∞	∞	−3.290	−67.392	−64.000	−3.392	−19.739	−16.000	−3.739	−11.563	−7.111	−4.452	−9.870	−4.000	−5.870
0.1	96.761	100	−3.239	−11.884	− 8.566	−3.318	−13.617	− 9.988	−3.629	−10.006	−5.757	−4.249	−8.956	−3.550	−5.406
0.2	21.954	25.000	−3.046	+ 4.763	+ 7.878	−3.115	− 5.473	− 2.142	−3.331	− 6.809	−3.084	−3.725	−6.809	−2.497	−4.312
0.3	8.332	11.111	−2.779	3.850	6.667	−2.817	− 1.739	+ 1.183	−2.922	− 4.028	−0.952	−3.076	−4.518	−1.384	−3.134
0.4	3.785	6.25	−2.465	2.209	4.681	−2.472	− 0.511	1.970	−2.481	− 2.223	+0.214	−2.437	−2.736	−0.535	−2.201
	1.864	4.000	−2.136	1.199	3.322	−2.123	− 0.147	1.920	−2.067	− 1.200	0.717	−1.917	−1.568	0	−1.568
0.6	0.954	2.778	−1.824	0.642	2.441	−1.799	− 0.042	1.667	−1.709	− 0.643	0.875	−1.518	−0.870	+0.296	−1.166
	0.498	2.041	−1.543	0.343	1.856	−1.513	− 0.012	1.401	−1.413	− 0.341	0.879	−1.220	−0.474	0.438	−0.912
0.8	0.263	1.563	−1.300	0.183	1.453	−1.270	− 0.003	1.170	−1.173	− 0.186	0.819	−1.005	−0.256	0.492	−0.748
	0.139	1.235	−1.096	0.098	1.165	−1.067	− 0.001	0.995	−0.996	− 0.098	0.741	−0.839	−0.137	0.498	−0.635
1.0	0.074	1.000	−0.926	0.052	0.954	−0.902	0	0.830	−0.830	− 0.052	0.661	−0.713	−0.074	0.480	−0.554
	0.039	0.827	−0.788	0.028	0.795	−0.767	0	0.715	−0.715	− 0.028	0.586	−0.614	−0.039	0.450	−0.489
1.2	0.021	0.694	−0.673	0.015	0.672	−0.657	0	0.610	−0.610	− 0.015	0.520	−0.535	−0.021	0.417	−0.438
	0.011	0.592	−0.581	0.008	0.576	−0.568	0	0.530	−0.530	− 0.008	0.462	−0.470	−0.012	0.383	−0.395
1.4	0.006	0.510	−0.504	0.004	0.498	−0.494				− 0.004	0.412	−0.416	−0.006	0.350	−0.356
	0.002	0.391	−0.389	0.002	0.435	−0.433				− 0.002	0.369	−0.371	−0.004	0.320	−0.324
1.6	0.001	0.309	0.308	0.001	0.384	−0.383				− 0.001	0.332	−0.333	−0.002	0.293	−0.295
	0	0.250	−0.250	0						0					
1.8															
2.0															

TABLE 6

Corrections to Be Added to τ_2 for a Line Source to Convert to τ_3 for a Column of Discrete Point Sources with Spacing d = 1.0

$x - c$	$r(x-c, y, 0) - s(x-c, y, 0)$						
	$y = 0$	$y = 0.125$	$y = 0.25$	$y = 0.5$	$y = 0.75$	$y = 0.875$	$y = 1.0$
0	∞	0	0	0	0	0	0
0.1	80.238	16.804	2.576	0.164	0.018	0.007	0.003
0.2	15.459	8.500	2.612	0.250	0.031	0.013	0.006
0.3	5.092	3.693	1.702	0.248	0.035	0.014	0.006
0.4	2.048	1.662	0.960	0.198	0.035	0.015	0.007
0.5	0.907	0.780	0.514	0.139	0.029	0.013	0.007
0.6	0.425	0.378	0.271	0.090	0.022	0.011	0.006
0.7	0.207	0.188	0.143	0.056	0.016	0.009	0.006
0.8	0.104	0.095	0.076	0.035	0.012	0.007	0.005
0.9	0.053	0.050	0.041	0.022	0.009	0.006	0.004
1.0	0.032	0.030	0.023	0.014	0.007	0.005	0.004
1.1	0.017	0.016	0.014	0.009	0.006	0.005	0.004
1.2	0.011	0.010	0.009	0.007	0.005	0.004	0.004
1.3	0.008	0.007	0.006	0.005	0.005	0.004	0.004
1.4	0.006	0.006	0.006	0.005	0.004	0.004	0.004
1.5	0.005	0.005	0.005	0.005	0.004	0.004	0.003
1.6	0.005	0.005	0.005	0.004	0.004	0.004	0.003
1.7	0.004	0.004	0.004	0.004	0.004	0.003	0.003
1.8	0.004	0.004	0.004	0.004	0.004	0.003	0.003

TABLE 7

Corrections to Be Added to τ_2 for a Line Source to Convert to τ_3 for a Column of Discrete Point Sources with Spacing d = 0.5

$x - c$	$r(x-c, y, 0) - s(x-c, y, 0)$			
	$y = 0$	$y = 0.125$	$y = 0.25$	$y = 0.50$
0	-	0	0	0
0.1	61.834	10.470	1.001	0.021
0.2	8.191	3.849	0.790	0.028
0.3	1.701	1.089	0.361	0.026
0.4	0.414	0.306	0.138	0.020
0.5	0.116	0.094	0.055	0.017
0.6	0.043	0.039	0.028	0.015
0.7	0.024	0.023	0.020	0.014
0.8	0.018	0.018	0.017	0.013
0.9	0.017	0.016	0.015	0.013
1.0	0.014	0.015	0.014	0.012
1.1	0.013	0.013	0.013	0.011
1.2	0.013	0.012	0.011	0.010

Wall Interference for a Point Source at $x = c$, $y = 0$, $z = 0$, on the Centerline of an Infinitely Long Channel of Height-Width Ratio $d = 1.0$

	$y = 0$			$y = 0.125$			$y = 0.25$			$y = 0.50$		
$x-c$	$T_3(x-c, y, 0)$	u_3^0/Q_3	$T_3'(x-c, y, 0)$	$T_3(x-c, y, 0)$	u_3^0/Q_3	$T_3'(x-c, y, 0)$	$T_3(x-c, y, 0)$	u_3^0/Q_3	$T_3'(x-c, y, 0)$	$T_3(x-c, y, 0)$	u_3^0/Q_3	$T_3'(x-c, y, 0)$
0	∞	∞	0	0	0	0	0	0	0	0	0	0
0.1	100.896	100.000	0.896	25.289	24.378	0.911	6.094	5.123	0.971	2.240	0.754	1.486
0.2	26.753	25.000	1.753	17.023	15.245	1.778	7.985	6.095	1.890	4.000	1.281	2.719
0.3	13.636	11.111	2.525	11.301	8.739	2.762	7.737	5.038	2.699	5.122	1.513	3.609
0.4	9.452	6.250	3.202	8.675	5.435	3.240	7.197	3.811	3.386	5.738	1.524	4.214
0.5	7.773	4.000	3.773	7.459	3.652	3.807	6.803	2.862	3.941	6.040	1.414	4.626
0.6	7.017	2.778	4.239	6.877	2.606	4.271	6.569	2.185	4.384	6.180	1.259	4.921
0.7	6.659	2.041	4.618	6.589	1.947	4.642	6.441	1.705	4.736	6.242	1.100	5.142
0.8	6.480	1.563	4.917	6.444	1.507	4.937	6.370	1.359	5.011	6.272	0.953	5.319
0.9	6.389	1.235	5.154	6.370	1.200	5.270	6.334	1.104	5.230	6.284	0.825	5.459
1.0	6.346	1.000	5.346	6.335	0.977	5.358	6.314	0.913	5.401	6.288	0.716	5.572
1.1	6.321	0.826	5.495	6.313	0.811	5.502	6.304	0.766	5.538	6.288	0.624	5.664
1.2	6.309	0.694	5.615	6.302	0.683	5.619	6.298	0.652	5.646	6.290	0.546	5.744
1.3	6.304	0.592	5.712	6.300	0.583	5.717	6.297	0.561	5.736	6.290	0.481	5.809
1.4	6.299	0.510	5.789	6.298	0.504	5.794	6.295	0.487	5.808	6.291	0.426	5.865
1.5	6.296,	0.444	5.852	6.296	0.440	5.856	6.294	0.427	5.867	6.291	0.379	5.912
1.6	6.295	0.391	5.904	6.295	0.387	5.908	6.293	0.377	5.916	6.292	0.340	5.952
1.7	6.294	0.346	5.948	6.294	0.343	5.951	6.292	0.335	5.957	6.292	0.306	5.986
1.8	6.294	0.309	5.985	6.294	0.306	5.988	6.292	0.300	5.992	6.292	0.276	6.016

TABLE 9

Wall Interference for a Point Source at $x = c$, $y = 0$, $z = 0$ at the Centerline of an Infinitely Long Channel of Height-Width Ratio $d = 0.5$

	$y = 0$			$y = 0.125$			$y = 0.25$			$y = 0.50$		
$x - c$	$\tau_3(x-c, y, 0)$	u_3^0/Q_3	$\tau_3'(x-c, y, 0)$	$\tau_3(x-c, y, 0)$	u_3^0/Q_3	$\tau_3'(x-c, y, 0)$	$\tau_3(x-c, y, 0)$	u_3^0/Q_3	$\tau_3'(x-c, y, 0)$	$\tau_3(x-c, y, 0)$	u_3^0/Q_3	$\tau_3'(x-c, y, 0)$
0	∞	∞	0	0	0	0	0	0	0	0	0	0
0.1	103.138	100.000	3.138	27.426	24.378	3.048	8.001	5.123	2.878	3.865	0.754	3.111
0.2	30.755	25.000	5.755	20.869	15.245	5.624	11.474	6.095	5.379	7.056	1.281	5.775
0.3	18.765	11.111	7.654	16.277	8.739	7.542	12.361	5.038	7.323	9.303	1.513	7.790
0.4	15.194	6.250	8.944	14.302	5.435	8.867	12.542	3.811	8.731	10.724	1.524	9.200
0.5	13.820	4.000	9.820	13.426	3.652	9.774	12.575	2.862	9.713	11.557	1.414	10.143
0.6	13.203	2.778	10.425	13.015	2.606	10.409	12.580	2.185	10.395	12.029	1.259	10.770
0.7	12.904	2.041	10.863	12.807	1.947	10.860	12.584	1.705	10.879	12.287	1.100	11.187
0.8	12.750	1.563	11.187	12.702	1.507	11.195	12.584	1.359	11.225	12.430	0.953	11.477
0.9	12.673	1.235	11.438	12.644	1.200	11.444	12.583	1.104	11.479	12.505	0.825	11.680
1.0	12.626	1.000	11.626	12.615	0.977	11.638	12.582	0.913	11.669	12.543	0.716	11.827
1.1	12.605	0.826	11.779	12.597	0.811	11.786	12.581	0.766	11.815	12.561	0.624	11.937
1.2	12.593	0.694	11.901	12.588	0.683	11.905	12.580	0.652	11.922	12.572	0.546	12.026

TABLE 10

Wall Interference for a Rankine Ovoid ($c = 0.1$) at the Centerline of an Infinitely Long Channel of Height-Width Ratio $d = 1.0$

	$y = 0$			$y = 0.125$			$y = 0.25$			$y = 0.5$		
x	u_3/Q_3	u_3^0/Q_3	u_3'/Q_3	u_3/Q_3	u_3^0/Q_3	u_3'/Q_3	u_3/Q_3	u_3^0/Q_3	u_3'/Q_3	u_3/Q_3	u_3^0/Q_3	u_3'/Q_3
0	−201.792	−200.000	−1.792	−50.578	−48.756	−1.822	−12.188	−10.246	−1.942	−4.480	−1.508	−2.972
0.1	∞	∞	−1.753	−17.023	−15.245	−1.778	− 7.985	− 6.095	−1.890	−4.000	−1.281	−2.719
0.2	+ 87.260	+ 88.889	−1.629	+13.988	+15.639	−1.651	− 1.643	+ 0.085	−1.728	−2.882	−0.759	−2.123
0.3	17.301	18.750	−1.449	8.348	9.810	−1.462	+ 0.788	2.284	−1.496	−1.738	−0.243	−1.495
0.4	5.863	7.111	−1.248	3.842	5.087	−1.245	0.934	2.176	−1.242	−0.918	+0.099	−1.017
0.5	2.435	3.472	−1.037	1.798	2.829	−1.031	0.628	1.626	−0.998	−0.442	0.265	−0.707
0.6	1.114	1.959	−0.845	0.870	1.705	−0.835	0.362	1.157	−0.795	−0.202	0.314	−0.516
0.7	0.537	1.215	−0.678	0.433	1.099	−0.666	0.199	0.826	−0.627	−0.092	0.306	−0.398
0.8	0.270	0.806	−0.536	0.219	0.747	−0.528	0.107	0.601	−0.494	−0.042	0.275	−0.317
0.9	0.134	0.563	−0.429	0.109	0.530	−0.421	0.056	0.446	−0.390	−0.016	0.237	−0.253
1.0	0.068	0.409	−0.341	0.057	0.389	−0.332	0.030	0.338	−0.308	−0.004	0.201	−0.205
1.1	0.037	0.306	−0.269	0.033	0.294	−0.261	0.016	0.261	−0.245	−0.002	0.170	−0.172
1.2	0.017	0.234	−0.217	0.013	0.228	−0.215	0.007	0.205	−0.198	−0.002	0.143	−0.145
1.3	0.010	0.184	−0.174	0.004	0.179	−0.175	0.003	0.165	−0.162	−0.001	0.120	−0.121
1.4	0.008	0.148	−0.140	0.004	0.143	−0.139	0.003	0.134	−0.131	−0.001	0.102	−0.103
1.5	0.004	0.119	−0.115	0.003	0.117	−0.114	0.002	0.110	−0.108	−0.001	0.086	−0.087
1.6	0.002	0.098	−0.096	0.002	0.097	−0.095	0.002	0.092	−0.090	−0.001	0.073	−0.074
	0.001	0.082	−0.081	0.001	0.081	−0.080	0.002	0.077	−0.075	0	0.064	−0.064

TABLE 11

Wall Interference for a Rankine Ovoid ($c = 0.2$) at the Centerline of an Infinitely Long Channel of Height-Width Ratio $d = 1.0$

	$y = 0$			$y = 0.125$			$y = 0.25$			$y = 0.5$		
x	u_3/Q_3	u_3^0/Q_3	u_3'/Q_3	u_3/Q_3	u_3^0/Q_3	u_3'/Q_3	u_3/Q_3	u_3^0/Q_3	u_3'/Q_3	u_3/Q_3	u_3^0/Q_3	u_3'/Q_3
0	− 53.506	− 50.000	−3.506	−34.046	−30.490	−3.556	−15.970	−12.190	−3.780	−8.000	−2.562	−5.438
0.1	−114.532	−111.111	−3.421	−36.590	−33.117	−3.473	−13.831	−10.161	−3.670	−7.362	−2.267	−5.095
0.2	∞	∞	−3.202	− 8.675	− 5.435	−3.240	− 7.197	− 3.811	−3.386	−5.738	−1.524	−4.214
0.3	+ 93.123	+ 96.000	−2.877	+17.830	+20.726	−2.896	− 0.709	+ 2.261	−2.970	−3.800	−0.660	−3.140
0.4	19.736	22.222	−2.486	10.146	12.639	−2.493	+ 1.416	3.910	−2.494	−2.180	+0.022	−2.202
0.5	6.977	9.070	−2.093	4.712	6.792	−2.080	1.296	3.333	−2.037	−1.120	0.413	−1.533
0.6	2.972	4.687	−1.715	2.231	3.928	−1.697	0.827	2.452	−1.625	−0.534	0.571	−1.105
0.7	1.384	2.765	−1.381	1.089	2.452	−1.363	0.469	1.758	−1.289	−0.244	0.589	−0.833
0.8	0.671	1.778	−1.107	0.542	1.629	−1.087	0.255	1.272	−1.017	−0.108	0.543	−0.651
0.9	0.338	1.215	−0.877	0.276	1.136	−0.860	0.137	0.939	−0.802	−0.046	0.476	−0.522
1.0	0.171	0.869	−0.698	0.142	0.824	−0.682	0.072	0.707	−0.635	−0.018	0.407	−0.425
1.1	0.085	0.643	−0.558	0.070	0.617	−0.547	0.037	0.543	−0.506	−0.006	0.344	−0.350
1.2	0.047	0.490	−0.443	0.037	0.473	−0.436	0.019	0.426	−0.407	−0.003	0.290	−0.293
1.3	0.025	0.382	−0.357	0.017	0.371	−0.354	0.010	0.339	−0.329	−0.003	0.245	−0.248
1.4	0.014	0.303	−0.289	0.007	0.296	−0.289	0.005	0.275	−0.270	−0.002	0.206	−0.208
1.5	0.010	0.246	−0.236	0.006	0.240	−0.234	0.005	0.226	−0.221	−0.002	0.175	−0.177
1.6	0.005	0.201	−0.196	0.004	0.198	−0.194	0.003	0.187	−0.184	−0.001	0.150	−0.151

TABLE 12

Wall Interference for a Rankine Ovoid (c = 0.3) at the Centerline of an Infinitely Long Channel of Height-Width Ratio d = 1.0

	$y = 0$			$y = 0.125$			$y = 0.25$			$y = 0.5$		
x	u_3/Q_3	u_3^0/Q_3	u_3'/Q_3	u_3/Q_3	u_3^0/Q_3	u_3'/Q_3	u_3/Q_3	u_3^0/Q_3	u_3'/Q_3	u_3/Q_3	u_3^0/Q_3	u_3'/Q_3
0	− 27.272	− 22.222	−5.050	−22.602	−17.478	−5.124	−15.474	−10.076	−5.398	−10.244	−3.026	−7.218
0.1	− 36.205	− 31.250	−4.955	−25.698	−20.680	−5.018	−15.182	− 9.906	−5.276	− 9.738	−2.805	−6.933
0.2	−108.669	−104.000	−4.669	−32.748	−28.030	−4.718	−12.897	− 7.985	−4.912	− 8.280	−2.168	−6.112
0.3	∞	∞	−4.239	− 6.877	− 2.606	−4.271	− 6.569	− 2.185	−4.384	− 6.180	−1.259	−4.921
0.4	+ 94.237	+ 97.959	−3.722	+18.700	+22.431	−3.731	− 0.347	+ 3.418	−3.765	− 4.002	−0.346	−3.656
0.5	20.273	23.437	−3.164	10.579	13.738	−3.159	+ 1.615	4.736	−3.121	− 2.272	+0.328	−2.600
0.6	7.247	9.876	−2.629	4.931	7.539	−2.608	1.403	3.934	−2.531	− 1.162	0.688	−1.850
0.7	3.106	5.250	−2.144	2.340	4.458	−2.118	0.883	2.898	−2.015	− 0.550	0.808	−1.358
0.8	1.452	3.174	−1.722	1.146	2.841	−1.695	0.499	2.096	−1.597	− 0.248	0.790	−1.038
0.9	0.708	2.084	−1.376	0.575	1.923	−1.348	0.271	1.533	−1.262	− 0.110	0.713	−0.823
1.0	0.355	1.449	−1.094	0.289	1.364	−1.075	0.144	1.144	−1.000	− 0.048	0.619	−0.667
1.1	0.181	1.053	−0.872	0.146	1.003	−0.857	0.075	0.872	−0.797	− 0.019	0.527	−0.546
1.2	0.093	0.791	−0.698	0.074	0.760	−0.686	0.040	0.677	−0.637	− 0.007	0.446	−0.453
1.3	0.051	0.609	−0.558	0.040	0.590	−0.550	0.021	0.536	−0.515	− 0.004	0.376	−0.380
1.4	0.027	0.480	−0.453	0.019	0.468	−0.449	0.012	0.431	−0.419	− 0.004	0.318	−0.322
1.5	0.015	0.385	−0.370	0.008	0.377	−0.369	0.006	0.352	−0.346	− 0.002	0.270	−0.272

TABLE 13

Wall Interference for a Rankine Ovoid (c = 0.05) at the Centerline of an Infinitely Long Channel of Height-Width Ratio d = 0.5

	$y = 0$			$y = 0.125$			$y = 0.25$			$y = 0.5$		
x	u_3/Q_3	u_3^0/Q_3	u_3'/Q_3	u_3/Q_3	u_3^0/Q_3	u_3'/Q_3	u_3/Q_3	u_3^0/Q_3	u_3'/Q_3	u_3/Q_3	u_3^0/Q_3	u_3'/Q_3
0.05	∞	∞	−3.138	−27.426	−24.378	−3.048	−8.001	−5.123	−2.878	−3.865	−0.754	−3.111
0.15	72.383	75	−2.617	+ 6.557	+ 9.133	−2.576	−3.473	−0.972	−2.501	−3.191	−0.527	−2.664
0.25	11.990	13.889	−1.899	4.592	6.506	−1.914	−0.887	+1.057	−1.944	−2.247	−0.232	−2.015
0.35	3.571	4.861	−1.290	1.975	3.304	−1.329	−0.181	1.227	−1.408	−1.421	−0.011	−1.410
0.45	1.374	2.250	−0.876	0.876	1.783	−0.907	−0.033	0.949	−0.982	−0.833	+0.110	−0.943
0.55	0.617	1.222	−0.605	0.411	1.046	−0.635	−0.005	0.677	−0.682	−0.472	0.155	−0.627
0.65	0.299	0.737	−0.438	0.208	0.659	−0.451	−0.004	0.480	−0.484	−0.258	0.159	−0.417
0.75	0.154	0.478	−0.324	0.105	0.440	−0.335	0	0.346	−0.346	−0.143	0.147	−0.290
0.85	0.077	0.328	−0.251	0.058	0.307	−0.249	+0.001	0.255	−0.254	−0.075	0.128	−0.203
0.95	0.047	0.235	−0.188	0.029	0.223	−0.194	+0.001	0.191	−0.190	−0.038	0.109	−0.147
1.05	0.021	0.174	−0.153	0.018	0.166	−0.148	+0.001	0.147	−0.146	−0.018	0.092	−0.110
1.15	0.012	0.132	−0.120	0.009	0.128	−0.119	+0.001	0.114	−0.113	−0.011	0.078	−0.089

TABLE 14

Wall Interference for a Rankine Ovoid (c = 0.10) at the Centerline of an Infinitely Long Channel of Height-Width Ratio d = 0.5

	$y = 0$			$y = 0.125$			$y = 0.25$			$y = 0.5$		
x	u_3/Q_3	u_3^0/Q_3	u_3'/Q_3	u_3/Q_3	u_3^0/Q_3	u_3'/Q_3	u_3/Q_3	u_3^0/Q_3	u_3'/Q_3	u_3/Q_3	u_3^0/Q_3	u_3'/Q_3
0	−206.276	−200	−6.276	−54.852	−48.756	−6.096	−16.002	−10.246	−5.756	−7.730	−1.508	−6.222
0.1	∞	∞	−5.755	−20.869	−15.245	−5.624	−11.474	− 6.095	−5.379	−7.056	−1.281	−5.775
0.2	+ 84.373	+ 88.889	−4.516	+11.149	+15.639	−4.490	− 4.360	+ 0.085	−4.445	−5.438	−0.759	−4.679
0.3	15.561	18.750	−3.189	6.567	9.810	−3.243	− 1.068	2.284	−3.352	−3.668	−0.243	−3.425
0.4	4.945	7.111	−2.166	2.851	5.087	−2.236	− 0.214	2.176	−2.390	−2.254	+0.099	−2.353
0.5	1.991	3.472	−1.481	1.287	2.829	−1.542	− 0.038	1.626	−1.664	−1.305	0.265	−1.570
0.6	0.916	1.959	−1.043	0.619	1.705	−1.086	− 0.009	1.157	−1.166	−0.730	0.314	−1.044
0.7	0.453	1.215	−0.762	0.313	1.099	−0.786	− 0.004	0.826	−0.830	−0.401	0.306	−0.707
0.8	0.231	0.806	−0.575	0.163	0.747	−0.584	+ 0.001	0.601	−0.600	−0.218	0.275	−0.493
0.9	0.124	0.563	−0.439	0.087	0.530	−0.443	0.002	0.446	−0.444	−0.113	0.237	−0.350
1.0	0.068	0.409	−0.341	0.047	0.389	−0.342	0.002	0.338	−0.336	−0.056	0.201	−0.257
	0.033	0.306	−0.273	0.027	0.294	−0.267	0.002	0.261	−0.259	−0.029	0.170	−0.199

TABLE 15

Wall Interference for a Rankine Ovoid (c = 0.20) at the Centerline of an Infinitely Long Channel of Height-Width Ratio d = 0.5

	$y = 0$			$y = 0.125$			$y = 0.25$			$y = 0.5$		
x	u_3/Q_3	u_3^0/Q_3	u_3'/Q_3	u_3/Q_3	u_3^0/Q_3	u_3'/Q_3	u_3/Q_3	u_3^0/Q_3	u_3'/Q_3	u_3/Q_3	u_3^0/Q_3	u_3'/Q_3
0	− 61.510	− 50.000	−11.510	−41.738	−30.490	−11.248	−22.948	−12.190	−10.758	−14.112	−2.562	−11.550
0.1	−121.903	−111.111	−10.792	−43.703	−33.117	−10.586	−20.362	−10.161	−10.201	−13.168	−2.267	−10.901
0.2	∞	∞	− 8.944	−14.302	− 5.435	− 8.867	−12.542	− 3.811	− 8.731	−10.724	−1.524	− 9.200
0.3	+ 89.318	+ 96.000	− 6.682	+14.000	+20.726	− 6.726	− 4.574	+ 2.261	− 6.835	− 7.692	−0.660	− 7.032
0.4	17.552	22.222	− 4.670	7.854	12.639	− 4.785	− 1.106	3.910	− 5.016	− 4.973	+0.022	− 4.995
0.5	5.861	9.070	− 3.209	3.470	6.792	− 3.322	− 0.223	3.333	− 3.556	− 2.984	0.413	− 3.397
0.6	2.444	4.687	− 2.243	1.600	3.928	− 2.328	− 0.042	2.452	− 2.494	− 1.706	0.571	− 2.277
0.7	1.147	2.765	− 1.618	0.782	2.452	− 1.670	− 0.008	1.758	− 1.766	− 0.948	0.589	− 1.537
0.8	0.577	1.778	− 1.201	0.400	1.629	− 1.229	− 0.002	1.272	− 1.274	− 0.514	0.543	− 1.057
0.9	0.299	1.215	− 0.916	0.210	1.136	− 0.926	+ 0.003	0.939	− 0.936	− 0.274	0.476	− 0.750
1.0	0.157	0.869	− 0.712	0.114	0.824	− 0.710	+ 0.004	0.707	− 0.703	− 0.142	0.407	− 0.549

TABLE 16

Corrections to be Added to $\frac{2}{d}\sigma_2$ for a Line Doublet to Convert to σ_3 for a Column of Discrete Doublets with Unit Spacing

	$R(x, y, 0) - S(x, y, 0)$					
x	$y = 0$	$y = 0.25$	$y = 0.50$	$y = 0.75$	$y = 1.00$	$y = 1.25$
0	–	−34.262	−1.842	−0.214	−0.043	−0.016
0.1	1797.65	−12.236	−1.357	−0.183	−0.035	−0.016
0.2	72.904	+ 7.527	−0.393	−0.109	−0.031	−0.014
0.3	50.124	8.936	+0.418	−0.027	−0.021	−0.013
0.4	17.432	5.817	0.579	+0.027	−0.011	−0.009
0.5	7.080	3.260	0.543	0.054	−0.003	−0.007
0.6	3.130	1.739	0.405	0.065	+0.002	−0.005
0.7	1.456	0.912	0.270	0.050	0.004	−0.003
0.8	0.701	0.477	0.169	0.042	0.005	−0.001
0.9	0.347	0.278	0.104	0.029	0.005	0
1.0	0.178	0.135	0.064	0.021	0.006	+0.001
1.1	0.094	0.075	0.039	0.016	0.006	0.002
1.2	0.054	0.046	0.026	0.013	0.006	0.002
1.3	0.034	0.028	0.019	0.011	0.006	0.003
1.4	0.022	0.021	0.015	0.009	0.006	0.003
1.5	0.018	0.017	0.013	0.009	0.006	0.003
1.6	0.015	0.014	0.011	0.008	0.005	0.004
1.7	0.013	0.012	0.010	0.007	0.005	0.004
1.8	0.011	0.010	0.008	0.006	0.005	0.004

TABLE 17

Wall Interference for a Sphere at the Centerline of an Infinitely Long Channel of Height-Width Ratio d = 1.0

	$y = 0$			$y = 0.25$			$y = 0.5$		
x	σ_3	u_3^0/μ_3	σ_3'	σ_3	u_3^0/μ_3	σ_3'	σ_3	u_3^0/μ_3	σ_3'
0	∞	∞	−9.04	−73.970	−64.000	−9.970	−21.582	−8.000	−13.583
0.1	1991.10	2000	−8.90	−39.669	−30.030	−9.639	−19.269	−6.671	−12.598
0.2	116.75	125	−8.25	− 3.542	+ 5.203	−8.745	−14.011	−3.753	−10.258
0.3	66.766	74.074	−7.308	+ 5.418	12.938	−7.520	− 8.618	−1.038	− 7.580
0.4	24.980	31.250	−6.270	4.813	11.027	−6.214	− 4.893	+0.651	− 5.544
0.5	10.802	16.000	−5.198	3.013	8.014	−5.001	− 2.593	1.415	− 4.008
0.6	5.042	9.259	−4.217	1.715	5.667	−3.952	− 1.335	1.617	− 2.952
0.7	2.460	5.831	−3.371	0.935	4.044	−3.109	− 0.678	1.550	− 2.228
0.8	1.237	3.906	−2.669	0.512	2.943	−2.431	− 0.343	1.379	− 1.722
0.9	0.635	2.744	−2.109	0.305	2.190	−1.885	− 0.170	1.184	− 1.354
1.0	0.338	2.000	−1.662	0.157	1.665	−1.508	− 0.084	1.002	− 1.086
1.1	0.184	1.503	−1.319	0.093	1.291	−1.198	− 0.039	0.843	− 0.882
1.2	0.108	1.157	−1.049	0.061	1.018	−0.957	− 0.016	0.708	− 0.724
1.3	0.068	0.910	−0.842	0.042	0.816	−0.774	− 0.005	0.597	− 0.602
1.4	0.046	0.729	−0.683	0.033	0.663	−0.630	+ 0.003	0.505	− 0.502
1.5	0.034	0.593	−0.559	0.029	0.546	−0.517	0.005	0.430	− 0.425
1.6	0.027	0.488	−0.461	0.026	0.454	−0.428	0.007	0.368	− 0.361
1.7	0.023	0.407	−0.384	0.023	0.382	−0.359			
1.8	0.021	0.343	−0.322	0.020	0.324	−0.304			

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