V393
.R46
883
[redacted]
940
MIT LIBRARIES
3 9080 02754 1876
JAN 23 1956
#2

NAVY DEPARTMENT

THE DAVID W. TAYLOR MODEL BASIN

WASHINGTON 7, D.C.

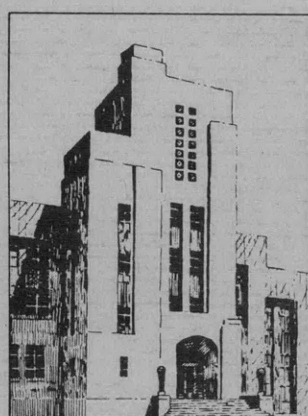
[redacted]

AN APPROXIMATION TO THE PLASTIC DEFORMATION OF A RECTANGULAR PLATE UNDER STATIC LOAD WITH DESIGN APPLICATIONS

by

J.E. Greenspon

MASS. INST. OF TECHNOLOGY
JUN 21 1976
BARKER ENGINEERING LIBRARY



June 1955 Report 940

AN APPROXIMATION TO THE PLASTIC DEFORMATION OF A RECTANGULAR PLATE UNDER STATIC LOAD WITH DESIGN APPLICATIONS

by

J.E. Greenspon

June 1955

Report 940

NS 731-037

TABLE OF CONTENTS

	Page
ABSTRACT	1
I. INTRODUCTION	1
II. METHODS OF COMPUTATION FOR PANELS WITH CLAMPED EDGES	2
A. UNIFORM STATIC LOADING	2
1. Load Resulting in a Given Set	2
2. Allowable Load and Corresponding Set According to Clarkson's Criteria	6
3. Approximation to the Failure Load of the Panel	8
4. Illustrative Problem	9
B. DYNAMIC LOADING	10
III. COMPARISON WITH EXPERIMENT, DISCUSSION, AND LIMITATIONS OF DESIGN CURVES	10
A. EXPERIMENTAL COMPARISONS	10
B. DISCUSSION AND LIMITATIONS OF THE DESIGN CURVES	12
IV. RECOMMENDATIONS	13
V. ACKNOWLEDGMENTS	13
APPENDIX - MEMBRANE THEORY	
DYNAMIC MEMBRANE THEORY	14
SPECIAL CASE OF TRANSIENT LOADING	16
UNIFORM STATIC LOAD	17
REFERENCES	19

NOTATION

a	Width of panel
b	Length of panel
C_1, C_2	Constants of integration
E	Modulus of elasticity of panel material
$F(t)$	Time-load distribution
$F(x, y, t)$	Pressure normal to panel (load per unit area)
$G(x, y)$	Space-load distribution
h	Thickness of panel
K_{mn}	Force function (having the dimensions of acceleration)
m	Factor determining the mode of deformation in the x-direction
n	Factor determining the mode of deformation in the y-direction
P_a	Allowable pressure (according to Clarkson's criterion)
P_0	Uniform normal pressure (load per unit area)
p_{mn}	Natural frequency of mode mn (radians per unit time)
q_{mn}	Time component of the deflection function w_{mn}, corresponding to mode mn
T	$\sigma_u h$
w	Lateral deflection of panel
w_m	Permanent set at center of panel
w_m/a	Permanent set ratio
ν	Poisson's ratio for panel material
ρ	Mass density of panel material
σ_s	Yield stress in pure tension
σ_u	Ultimate tensile stress of panel material
τ	Time parameter
ϕ	Function depending upon aspect ratio

ABSTRACT

Methods are given for estimating the loads and permanent sets of rectangular panels just beyond the onset of plasticity as well as for estimating very large sets and the sets corresponding to the approximate ultimate loads; the intermediate region corresponding to moderate permanent deformations is not discussed.

I. INTRODUCTION

Examination of structural failures in vessels operating in heavy seas has revealed weaknesses in the bow structure. Accordingly the Bureau of Ships requested[1] the Taylor Model Basin to obtain information on the forces acting on the bow structures of naval vessels in order to aid designers. To devise methods for the more efficient utilization of structural material in the bottom plating of ships, the following studies were planned:

a. Determine the magnitudes and character of the service loading experienced by the bottom plating of ships, especially near the bow during slamming in waves.

b. Study the literature for possible applicable methods of solving the plate design problem or for hints to methods of solution.

c. Develop new theoretical analyses as required.

d. Conduct experiments to verify existing or newly derived analytical approaches, or to provide empirical criteria when theoretical methods are unavailable or impractical.

In a number of structural problems, it is the maximum permissible amount of permanent deformation which will, or should, control the physical size and arrangement of the structure. Such designs may be lighter and more economical yet satisfy the structural requirements as well or better than a corresponding design which is based on a maximum allowable stress within the elastic limit.

In this report a theory for the design of rectangular panels which takes account of allowable permanent set is considered. Stability requirements and loads applied to the plate boundaries, which may become significant factors in the design of ship bottom plating, are not considered in this report.

In the following section, formulas and curves are given which enable the designer to estimate the permanent set that can be expected for a given uniform normal static load. Approximations are given for the cases of small permanent sets and permanent sets near failure. The intermediate region of moderately large sets is not discussed.

---

[1]References are listed on page 19.

II. METHODS OF COMPUTATION FOR PANELS WITH CLAMPED EDGES*

A. UNIFORM STATIC LOADING

1. Load Resulting in a Given Set

An approximate formula for the permanent set at the center of a panel under uniform static load, derived in the appendix, is as follows:

$$w_m = \frac{0.164}{1 + \dfrac{1}{b^2/a^2}} \frac{P_0 a^2}{\sigma_u h} \qquad [1]$$

where w_m is the set at the center of the panel,

P_0 is the uniform lateral load per unit area on the panel,

a is the width of the panel,

b is the length of the panel,

h is the thickness of the panel, and

σ_u is the ultimate stress of the panel material obtained from a standard tension test.

The following assumptions were made in the derivation of Formula [1]:

a. The tension in the plate is constant during deformation of the plate.

b. All bending stresses in the plate may be neglected.

c. Elastic deflections of the plate may be neglected.

d. The permanent set after unloading is equal to the deflection just prior to unloading.

e. The pressure on the panel is uniformly distributed.

f. The strain in the panel is large compared with the strain at the yield point.

For convenience in calculation, Formula [1] may be written as

$$\frac{w_m}{a} = \phi \frac{P_0}{\sigma_u} \frac{a}{h} \qquad [2]$$

where w_m/a is the ratio of the permanent set to the length of the shorter side,

P_0/σ_u is the ratio of the pressure to the ultimate stress,

a/h is the ratio of the length of the shorter side to the thickness,

ϕ is a parameter depending only on the aspect ratio, and

$$\phi = \frac{0.164}{1 + 1/(b^2/a^2)} \qquad [3]$$

This function ϕ is plotted in Figure 1 as a function of b/a.

*For estimations of loads corresponding to large sets, the methods, formulas, and numerical coefficients given here will be applicable to all panels which have no deflection at the edges.

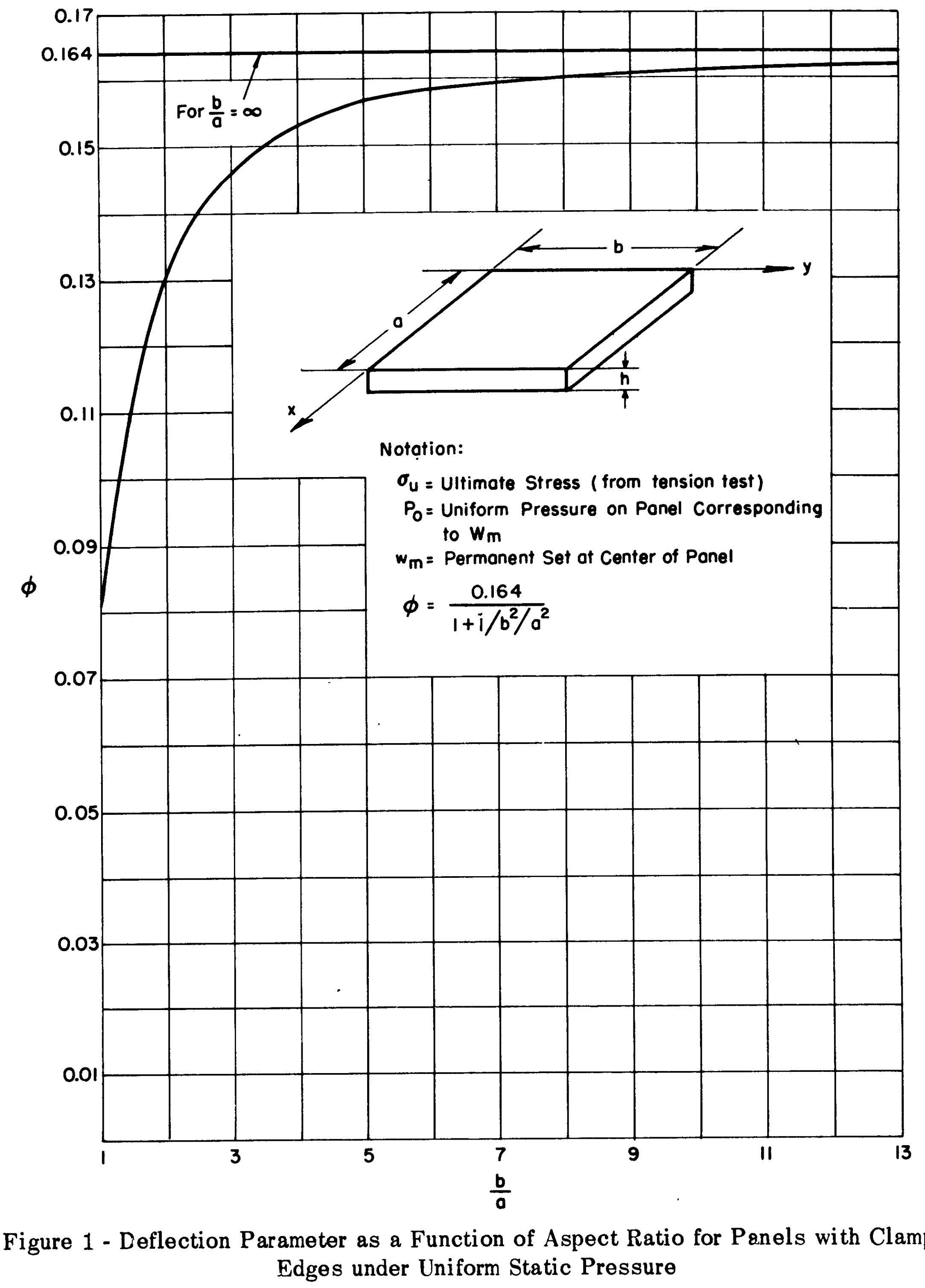


Figure 1 - Deflection Parameter as a Function of Aspect Ratio for Panels with Clamped Edges under Uniform Static Pressure

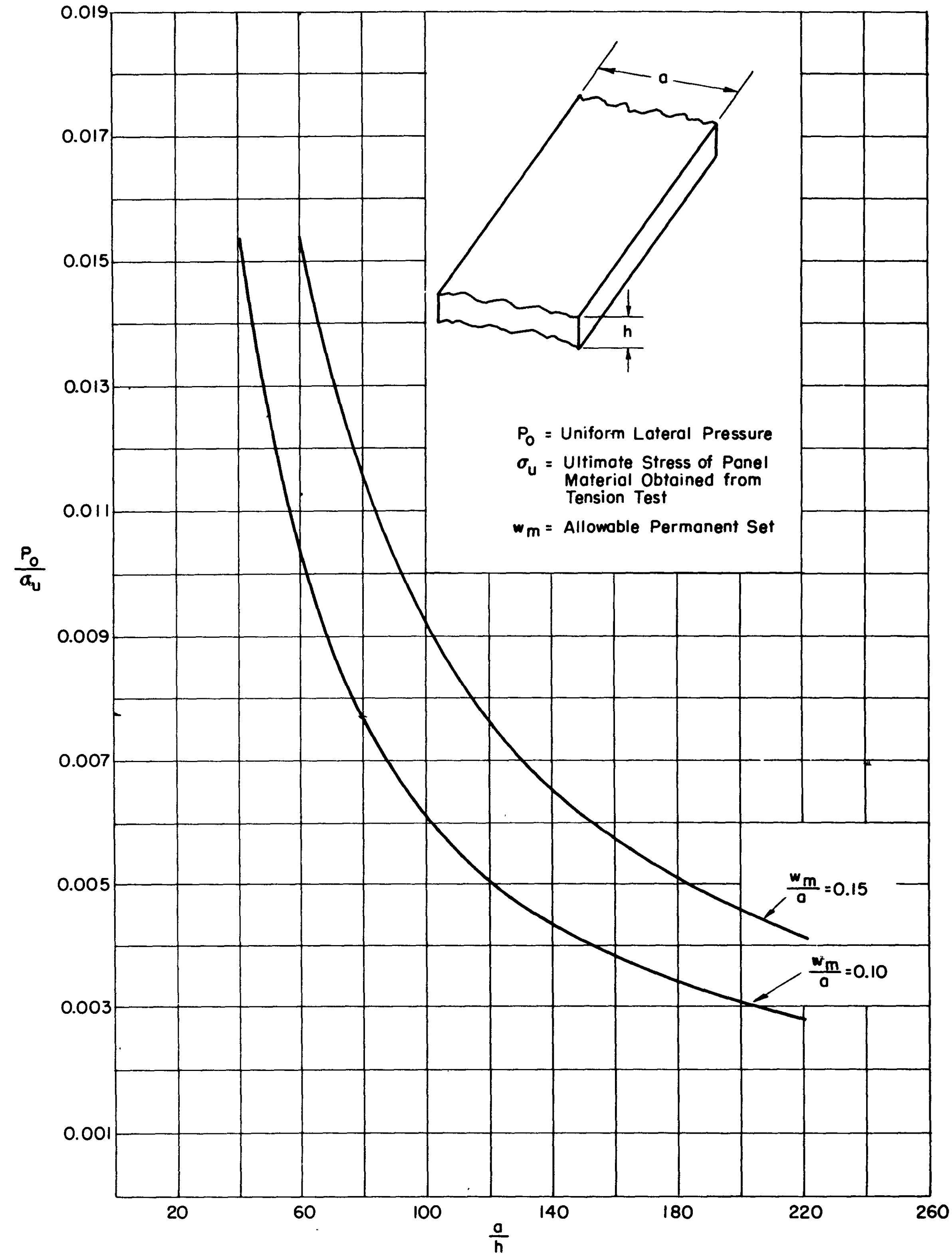


Figure 2a - $b/a = \infty$

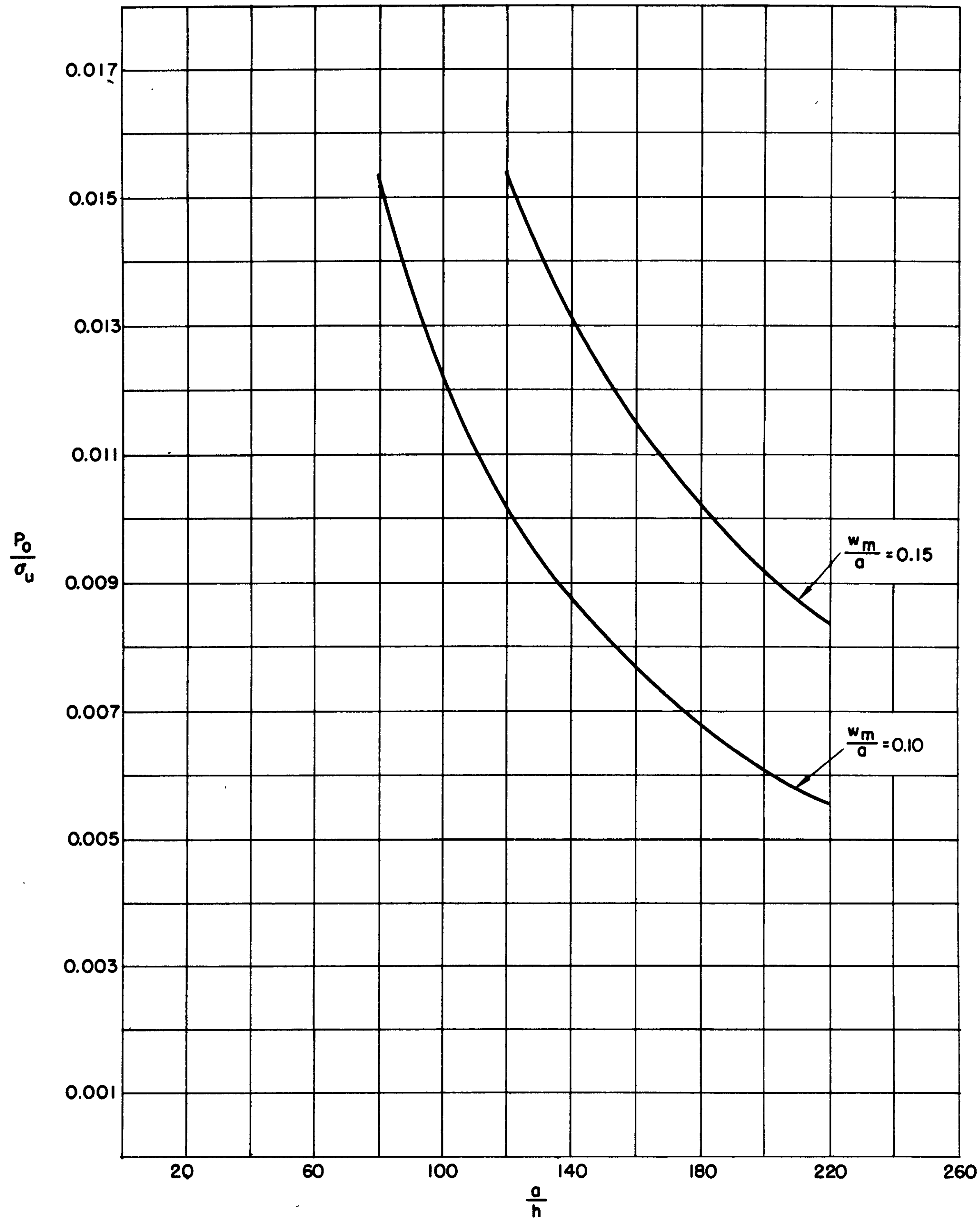


Figure 2b - $b/a = 1$

Figure 2 - Permanent-Set Curves Based on Formula [1] for Plates with Clamped Edges under Uniform Static Pressure

The ratio P_0/σ_u is plotted as a function of a/h for various values of w_m/a in Figure 2. All plates will lie between the two aspect ratios used for these curves. Thus a value of a/h may be selected for a given ratio of set to length of shorter side and a given ratio of pressure to ultimate stress.

It should be emphasized that Formula [1] and the curves in Figure 2 are based upon extensive simplifications of a very complicated problem. Since such complications as thinning of the panel in the plastic region, changes in diaphragm stresses, and changes in bending stiffness are neglected, only approximate answers can be expected from the theory. The limitations of the theory are discussed in Section III B.

It will be seen later that for pressures near the ultimate unit load, most of the deformations predicted by the theory compare favorably with the measured deformations. Consequently these curves may be expected to give the right order of magnitude for sets in which $w_m/a > 0.1$ and, with experimentally determined empirical factors discussed in Section III applied to the curves, the designer should have a fairly reliable, simple, and realistic procedure for checking the design of a panel.

2. Allowable Load and Corresponding Set According to Clarkson's Criteria

Clarkson[2] has recently written a report containing suggested design formulas for rectangular panels with fixed edge supports under uniform normal pressure. This approach differs basically from that in the preceding section of this report in that Clarkson computes an allowable load P_a corresponding to much smaller plastic deformations than are assumed in the theory of Section A 1. His criteria for the "allowable pressure" P_a are as follows:

a. For an infinitely long plate, the load-carrying capacity is either:

(1) The load when a plastic hinge has just formed along the centerline of the panel, or

(2) The load at which the membrane tension stress is equal to two-thirds of the yield stress in pure tension (σ_s).

For the case $b/a = \infty$ Clarkson gives the following approximate relationship for the allowable pressure P_a based on the criteria (1) or (2), whichever gives the lower pressure:

$$\frac{P_a E}{\sigma_s^2} = \frac{4.56}{\left(\frac{a}{h}\right)^{4/3}\left(\frac{\sigma_s}{E}\right)^{2/3}} \qquad \text{for } \frac{b}{a} = \infty. \qquad [4]$$

b. For a square plate, the load-carrying capacity is either:

(1) The load at which the pressure is 1 1/4 times the value corresponding to a completely plastic cross section at the center of the plate, or

(2) The load at which the tensile membrane stress has reached two-thirds of the yield stress in pure tension.

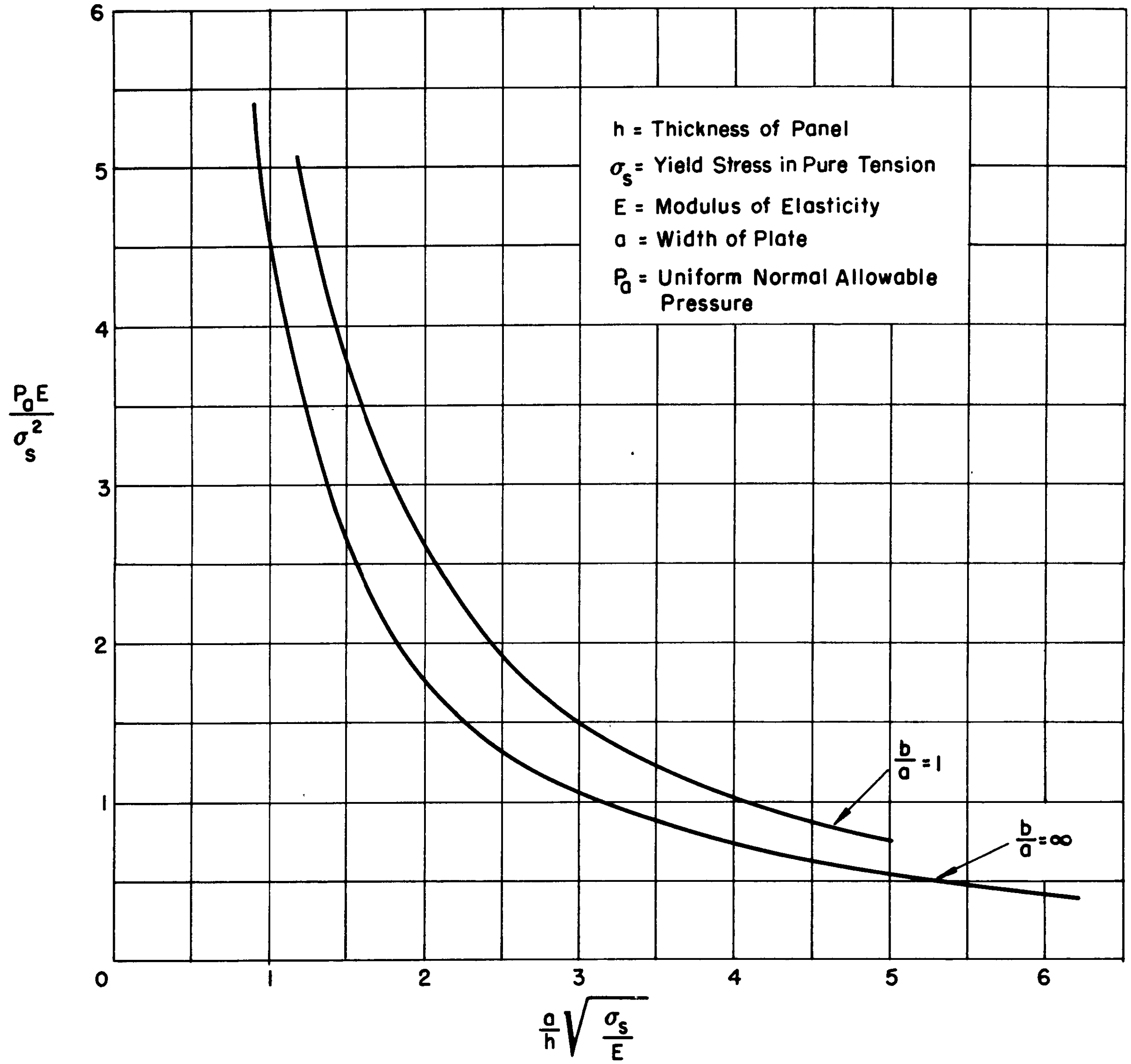


Figure 3 - Clarkson's Allowable Pressure Curves for Infinitely Long Plates and Square Plates with Clamped Edges under Uniform Static Pressure ($\nu = 0.3$)

For the case $b/a = 1$ Clarkson gives the following approximate relationship for the allowable pressure P_a based on the criteria (1) or (2), whichever gives the lower pressure:

$$\frac{P_a E}{\sigma_s^2} = \frac{6.46}{\left(\frac{a}{h}\right)^{4/3}\left(\frac{\sigma_s}{E}\right)^{2/3}} \qquad \text{for } \frac{b}{a} = 1. \qquad [5]$$

See Figure 3 for a plot of these relationships.

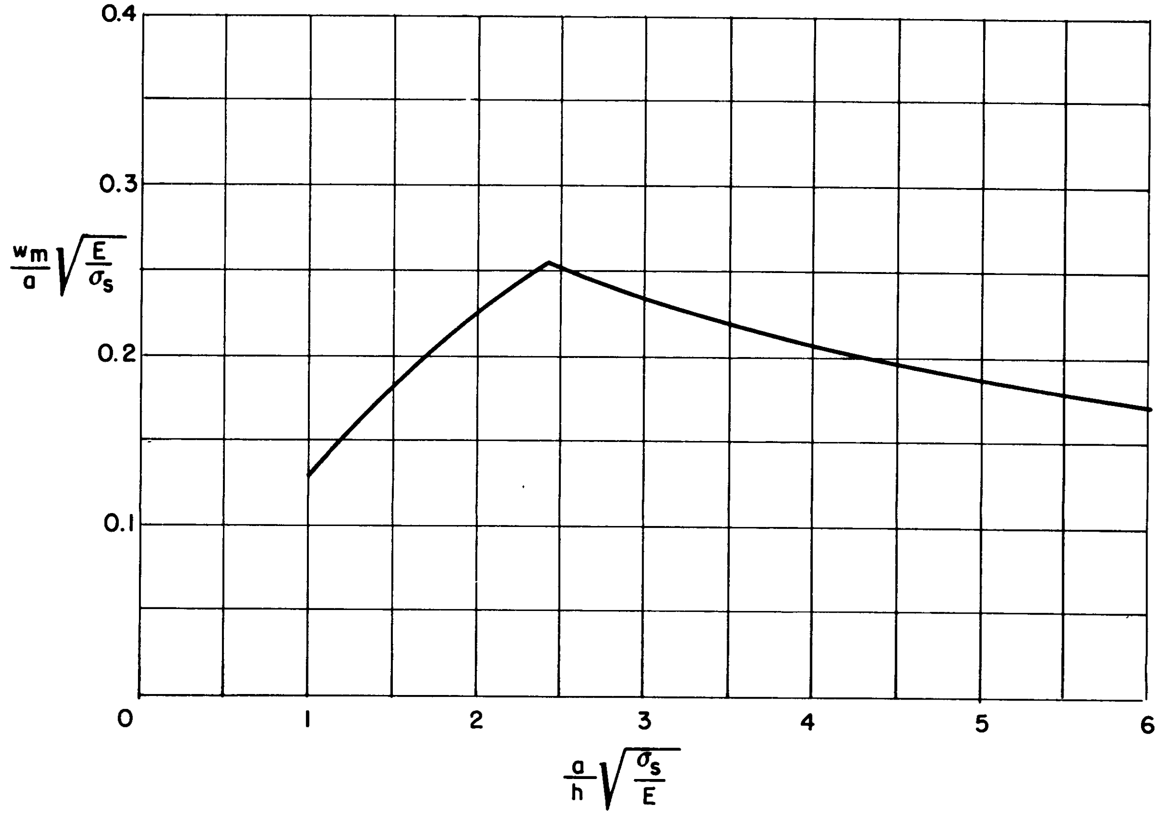


Figure 4 - Permanent Set for Infinitely Long Plates with Clamped Edges under Uniform Allowable Static Pressure Obtained from the Curve in Figure 3 ($\nu = 0.3$)

Corresponding to the allowable pressure curve for an infinitely long plate, there is a permanent-set curve which is plotted in Figure 4. Unfortunately no permanent-set curve is available for the square plate. The actual deflection curves obtained in Reference 2 show that the deflection under load is somewhat less for a square panel than it is for an infinitely long one, consequently an upper bound to the permanent set for any panel can be obtained approximately by use of Figure 4.

3. Approximation to the Failure Load of the Panel

Unfortunately an exact value for the failure load cannot be predicted with accuracy from the theories presented here. However it was found[3,4] from tests on steel diaphragms with aspect ratios of 1.55 that failure* never occurred until w_m/a reached at least 0.10. Some of the experimental data taken from References 3 and 4 are given in Table 1.

It is believed that a conservative estimate for the ultimate load may be obtained by taking $w_m/a = 0.10$ and then using Figure 2 or Formula [1] to estimate the ultimate load. It

*Failure is here defined as a rupture or fracture of the plate. The corresponding load is called the failure load or ultimate load.

TABLE 1

Experimental Data on Ultimate Loads for Clamped Edge Panels with Aspect Ratio of 1.55 under Uniform Static Pressure

The data are taken from References 3 and 4.

σ_u psi	h in.	a in.	Failure Pressure psi	$\frac{w_m}{a}$ at Failure
61,000	0.113	13.5	920	0.21
70,700	0.119	13.5	950	0.19
46,100	0.104	13.5	600	0.20
41,800	0.068	13.5	435	0.23
72,300	0.125	54	226	0.18
72,300	0.125	54	226	0.18
65,000	0.182	54	245	0.13
65,000	0.182	54	245	0.13

would be highly desirable, however, to have experimental data in order to check the validity of these estimates for a range of aspect ratios and thicknesses.

4. Illustrative Problem

To illustrate these procedures, consider the following static loading problem.

A steel panel 12 by 24 by 3/8 in. with a yield stress of 40,000 psi and an ultimate stress of 70,000 psi is subjected to a uniformly distributed pressure. Calculate

a. The allowable pressure by Clarkson's criterion (use Figure 3).

b. The approximate permanent set corresponding to Clarkson's allowable pressure (use Figure 4).

c. An estimate of the ultimate pressure (use Formula [2]).

(1) From Figure 3, for $\frac{a}{h}\sqrt{\frac{\sigma_s}{E}} = 1.2$

$\frac{P_a E}{\sigma_s^{2}}$ = 3.7 for an infinitely long plate and 5.2 for a square plate (approximately).

So P_a is less than $\frac{5.2 \times (4 \times 10^4)^2}{30 \times 10^6}$ = 280 psi approximately but greater than $\frac{3.7 \times (4 \times 10^4)^2}{30 \times 10^6}$ = 200 psi approximately. The pressure at the onset of plasticity is between 200 and 280 psi.

Which can be found by elastic
[illegible] analysis

(2) From Figure 4

$$\frac{w_m}{a}\sqrt{\frac{E}{\sigma_s}} = 0.15$$

so $w_m = 0.064$ in. Thus the permanent set at the center for onset of plasticity is less than 0.064 in.

(3) A conservative estimate of the ultimate pressure can be obtained by taking $\frac{w_m}{a} = 0.10$. Then from Formula [2], $P_0 = \frac{0.10}{0.13} \times 70{,}000 \times \frac{0.375}{12} = 1680$ approximately. Thus the ultimate pressure is at least 1680 psi for this panel.

B. DYNAMIC LOADING

In general the process of plastic deformation of a panel under dynamic loading is a very difficult theoretical problem. To obtain a complete solution to such a problem at the present time is almost an impossibility. However, if certain assumptions are made, it is believed that answers which are at least of the right order of magnitude may be obtained. The dynamic problem will not be treated in detail here; however, the dynamic membrane theory is given in the Appendix. Possibly at a later date, when more experimental evidence is available, it will be worthwhile to look further into the dynamic action of the panel in the plastic region. For design purposes, load factors that can be applied to the static deflection, could possibly be derived in much the same manner as was done for the small deflection theory in Reference 5.

III. COMPARISON WITH EXPERIMENT, DISCUSSION, AND LIMITATIONS OF DESIGN CURVES

A. EXPERIMENTAL COMPARISONS

Table 2 gives theoretical and experimental values, obtained from the original data of References 3 and 4, of P_0/σ_u for a given w_m/a and a/h with $b/a = 1.55$. Figure 5 shows curves of error factor F* as a function of the experimentally determined permanent-set ratio. A curve was drawn through the points corresponding to each plate. There is a definite pattern to these curves for permanent-set ratios greater than 0.10. The error between membrane theory and experiment for this aspect ratio (1.55) decreases as the permanent-set ratio increases.

An examination of Table 2 shows that the theoretical loads calculated from Formula [1] (which is derived in the Appendix) compare favorably with the test data for loads near the ultimate strength. Closer agreement is obtained for the larger loads (near failure) than for the smaller ones. For values of $w_m/a = 0.15$, most of the theoretical results are within 20 percent of the experimental ones.

*The error factor F is defined as the ratio of the theoretical to the experimental load for a given set.

TABLE 2

Comparison of Membrane Theory with Experiment for Uniformly Loaded Clamped Rectangular Panels with Aspect Ratio = 1.55

The values were obtained from the original data of References 3 and 4.

σ_u psi	h in.	$\frac{b}{a}$	a in.	$\frac{w_m}{a}$	$\frac{a}{h}$	$\frac{P_0}{\sigma_u}$ Experimental	$\frac{P_0}{\sigma_u}$ Formula [1]	$F = \frac{(P_0)_{Theo}}{(P_0)_{Exp}}$	Code on Figure 5
70,700	0.122	1.55	13.5	0.0644	110	0.0028	0.0051	1.82	x
				.1037		.0056	.0082	1.46	
				.1370		.0084	.0108	1.29	
				.1689		.0112	.0132	1.18	
				.1926		.0134	.0152	1.13	
46,100	0.104		13.5	.1022	130	.0043	.0068	1.58	○
				.1763		.0108	.0117	1.08	
				.2037		.0130	.0136	1.05	
41,800	0.068		13.5	.0333	199	.0012	.0015	1.25	□
				.0704		.0024	.0031	1.29	
				.1000		.0036	.0044	1.22	
				.1222		.0048	.0053	1.10	
				.1444		.0060	.0063	1.05	
				.1667		.0072	.0073	1.01	
				.2148		.0096	.0094	0.97	
				.2296		.0104	.0100	0.96	
61,000	0.119		13.5	.0533	113	.0033	.0041	1.24	△
				.0815		.0049	.0062	1.26	
				.1022		.0066	.0078	1.18	
				.1200		.0082	.0092	1.12	
				.1380		.0098	.0105	1.07	
				.1570		.0115	.0120	1.04	
				.1770		.0131	.0135	1.03	
				.1978		.0147	.0151	1.03	
72,300	0.125		54	.0426	432	.0007	.0008	1.14	●
				.0916		.0014	.0018	1.29	
				.1081		.0017	.0022	1.29	
				.1231		.0021	.0025	1.19	
				.1398		.0024	.0028	1.17	
				.1564		.0028	.0031	1.11	
				.1747		.0031	.0035	1.13	
65,000	0.182		54	.0305	297	.0008	.0009	1.13	▽
				.0676		.0015	.0020	1.33	
				.0926		.0023	.0026	1.13	
				.1111		.0031	.0032	1.03	
				0.1304		0.0038	0.0038	1.00	

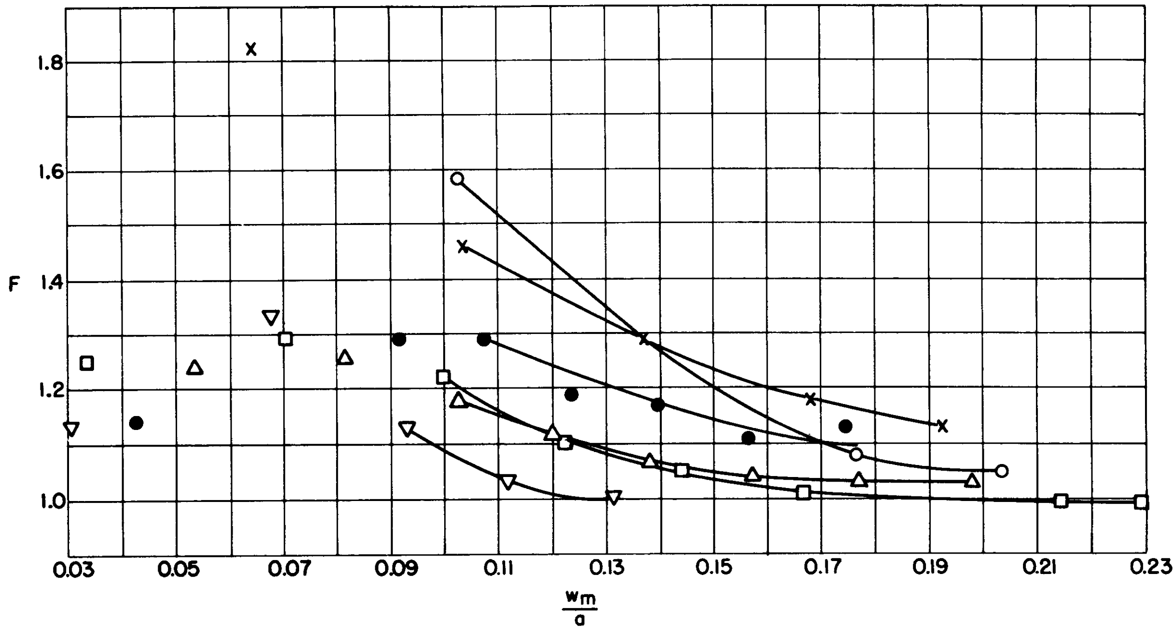


Figure 5 - Plot of Error Factor F as a Function of Permanent-Set Ratio (Aspect Ratio = 1.55)

See Table 2.

B. DISCUSSION AND LIMITATIONS OF THE DESIGN CURVES

Unfortunately no experimental verification is as yet available for Clarkson's theory so that it is difficult to say how accurate it is for predicting allowable loads for small permanent sets. On the other hand, something can be said regarding the limitations of the membrane theory. The results obtained by using Formula [1] or Figure 2, based on the theoretical-experimental comparisons for a plate with aspect ratio 1.55, should be within 20 percent of the correct value for loads near the ultimate load-carrying capacity of the panel. For w_m/a greater than 0.1, the results obtained should be of the right order of magnitude.

As these estimates of relative accuracy of predicting the permanent sets and corresponding loads are based only on tests conducted with plates of aspect ratio of 1.55, it would be of considerable value to have additional experimental data of this type for plates with larger aspect ratios and with aspect ratio of unity. It would then be practicable to apply factors based on these test results to the theoretically derived values for the purpose of design estimates.

In recent years much work has been done in the theory of plasticity, but unfortunately not much has come out in the way of design formulas and curves. The reader is referred to References 2, 6, and 7 for some of these design formulas and to References 8 and 9 for some of the more recent theoretical approaches.

IV. RECOMMENDATIONS

It is recommended that in the design of ship bottom plating, which is subject to large loads due to slamming or pounding in a seaway, the possibility of permitting a permanent plastic deformation* be considered. It is further suggested that experiments be conducted on a series of plates of large aspect ratio, and possibly also on square plates, to evaluate the utility and validity of present and future design theories, some of which are discussed in this report.

V. ACKNOWLEDGMENTS

The author wishes to express his gratitude to Mr. N.H. Jasper of the David Taylor Model Basin for his valuable criticism and suggestions. He also wishes to thank Mr. R.T. McGoldrick and Dr. E.H. Kennard for reviewing the report and making suggestions for its improvement.

---

*It is to be noted that such plates should be free of critical axial compressive loads.

APPENDIX

MEMBRANE THEORY

DYNAMIC MEMBRANE THEORY

The only stresses that are of importance in the elastic small-deflection region of plates are those corresponding to bending of the panel. As the load is increased and the plate enters the large-deflection region, the load is resisted by both bending and tension in the middle plane of the panel. As the load is further increased, the tensile stresses become predominant and ultimately the plate may be considered to act as a membrane. The following assumptions which have been mentioned before in the body of the report, are made in the theory which follows:

a. The tension in the plate is constant.

b. All bending stresses in the plate may be neglected.

c. Elastic deflections of the plate may be neglected.

d. The permanent set after unloading is equal to the deflection just prior to unloading.

e. The strain in the panel is large compared with the strain at the yield point.

The plate is therefore assumed to act as a stretched membrane. The equation of motion of a vibrating membrane is as follows:[10]

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = \frac{\rho h}{T} \frac{\partial^2 w}{\partial t^2} - \frac{F(x, y, t)}{T} \qquad [6]$$

where w is the lateral deflection,

$F(x, y, t)$ is the external lateral pressure (load per unit area), and

T is the tension per unit length.

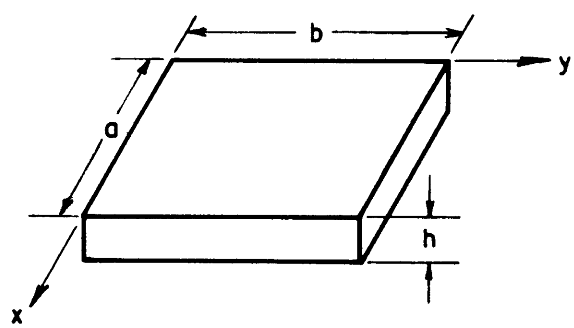


Figure 6 - Sketch of Rectangular Plate

Assuming that the boundaries of the membrane remain stationary, the boundary conditions may be written as follows:

$$w = 0 \text{ at } x = 0,\ x = a,\ y = 0,\ y = b$$

A solution of the equation of motion which satisfies these boundary conditions is

$$w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} q_{mn}(t) \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \qquad [7]$$

where $q_{mn}(t)$ is a function of time which is to be determined by substitution of this solution into the equation of motion. Performing this substitution,

$$-\sum\sum \frac{m^2\pi^2}{a^2} q_{mn}(t) \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} -$$

$$\sum\sum \frac{n^2\pi^2}{b^2} q_{mn}(t) \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \qquad [8]$$

$$= \sum\sum \frac{\rho h}{T} \ddot{q}_{mn}(t) \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} - \frac{F(x, y, t)}{T}$$

or, written more compactly,

$$\sum\sum \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \left[\frac{\rho h}{T} \ddot{q}_{mn} + \left(\frac{m^2\pi^2}{a^2} + \frac{n^2\pi^2}{b^2}\right) q_{mn}\right] = \frac{F(x, y, t)}{T} \qquad [9]$$

Now multiply both sides of the equation by $\sin \frac{k\pi x}{a} \sin \frac{l\pi y}{b}$ and integrate over the area of the panel. Due to orthogonality of the sine functions, the only terms retained are those in which $k = m$ and $l = n$, hence

$$\int_0^a \int_0^b \sin^2 \frac{m\pi x}{a} \sin^2 \frac{n\pi y}{b} \left[\frac{\rho h}{T} \ddot{q}_{mn} + \left(\frac{m^2\pi^2}{a^2} + \frac{n^2\pi^2}{b^2}\right) q_{mn}\right] dx\, dy$$

$$= \frac{1}{T} \int_0^a \int_0^b F(x, y, t) \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \, dx\, dy \qquad [10]$$

or

$$\ddot{q}_{mn} + \frac{T}{\rho h}\left(\frac{m^2\pi^2}{a^2} + \frac{n^2\pi^2}{b^2}\right) q_{mn}$$

$$= \frac{4}{\rho h a b} \int_0^a \int_0^b F(x, y, t) \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \, dx\, dy \qquad [11]$$

or

$$\ddot{q}_{mn} + p_{mn}^2 q_{mn} = \dot{K}_{mn}(t) \qquad [12]$$

where

$$p_{mn} = \sqrt{\frac{T}{\rho h}}\, \pi \left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^{1/2} \qquad [13]$$

$$K_{mn}(t) = \frac{4}{\rho h a b} \int_0^a \int_0^b F(x, y, t) \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}\, dx\, dy \qquad [14]$$

The general solution of this equation is

$$q_{mn} = C_1 \sin p_{mn} t + C_2 \cos p_{mn} t + \frac{1}{p_{mn}} \int_0^t K_{mn}(\tau) \sin p_{mn}(t-\tau)\, d\tau \qquad [15]$$

Hence the membrane deflection can be written as follows:

$$w(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left[C_1 \sin p_{mn} t + C_2 \cos p_{mn} t + \frac{1}{p_{mn}} \int_0^t K_{mn}(\tau) \sin p_{mn}(t-\tau)\, d\tau \right] \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \qquad [16]$$

SPECIAL CASE OF TRANSIENT LOADING

Assume that the load $F(x, y, t)$ can be expressed as

$$F(x, y, t) = G(x, y)\, F(t) \, \ldots\ldots \qquad [17]$$

and assume that this load is applied as a transient load. Under these assumptions the initial conditions are

$$(w)_{t=0} = (\dot{w})_{t=0} = 0 \qquad [18]$$

so that $C_1 = C_2 = 0$ and q_{mn} has the following value

$$q_{mn}(t) = \int_0^a \int_0^b G(x, y) \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}\, dx\, dy$$

$$\times \frac{4}{\rho hab p_{mn}} \int_0^t F(\tau) \sin p_{mn} (t-\tau)\, d\tau \qquad [19]$$

Thus w can be written

$$w(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left[\sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}\right]$$

$$\times \frac{4}{\rho hab} \left[\frac{\int_0^a \int_0^b G(x, y) \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}\, dx\, dy}{p_{mn}^2}\right] \left[p_{mn} \int_0^t F(\tau) \sin p_{mn} (t-\tau)\, d\tau\right] \qquad [20]$$

The last factor $\left(\text{i.e., } p_{mn} \int_0^t F(\tau) \sin p_{mn} (t-\tau)\, d\tau\right)$ is known as the response factor and has been calculated for various time functions.[11, 12] The maximum numerical value of the response factor is known as the load factor and has been used in connection with small deflections of plates; see, for example, Reference 5. For a static load the response factor has a value of unity as shown in Reference 11 so that the response for a static load is

$$w(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left[\sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}\right] \frac{4}{\rho hab} \left[\frac{\int_0^a \int_0^b G(x, y) \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}\, dx\, dy}{p_{mn}^2}\right] \qquad [21]$$

UNIFORM STATIC LOAD

Assume that the plate is subjected to a uniform static load of intensity P_0, i.e.,

$$G(x, y) = P_0$$

Then

$$w(x, y) = \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} \left[\sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}\right] \frac{16 P_0}{mn\pi^4 T\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)} \qquad [22]$$

By inspecting the series it can be seen that there are terms of order m^3 or n^3 in the denominator, which means that the first term is by far the most predominant one. As an approximation to the deflection, consider only the first term in the series. Thus

$$w(x, y) \approx \frac{16\, P_0\, b^2}{T\, \pi^4 \left(\frac{b^2}{a^2} + 1 \right)} \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} \qquad [23]$$

Now w_{max} occurs at the center of the panel, i.e., at $x = \frac{a}{2}$, $y = \frac{b}{2}$, so

$$w_{max} \approx \frac{16\, P_0\, b^2}{T\, \pi^4 \left(\frac{b^2}{a^2} + 1 \right)} \qquad [24]$$

or

$$w_{max} = \frac{0.164\, P_0\, a^2}{\sigma_u h [1 + 1/(b^2/a^2)]} \qquad [25]$$

REFERENCES

1. Bureau of Ships letter S11-3(442-440-330) of 11 Dec 1947 to David Taylor Model Basin.

2. Clarkson, J., "The Design of Uniformly Loaded Rectangular Plates," Naval Construction Research Establishment (Great Britain) Report R.247, Undex 332 (Mar 1954).

3. Day, John W., "Hydrostatic Pressure Tests on Thin Rectangular Diaphragms 13 ½ by 21 Inches," David Taylor Model Basin Report 635 (Mar 1951).

4. Day, John W., "Hydrostatic Pressure Tests on Thin Rectangular Diaphragms 84 by 54 Inches," David Taylor Model Basin Report 636 (Apr 1951).

5. Greenspon, J.E., "Stresses and Deflections in Flat Rectangular Plates under Dynamic Lateral Loads Based on Linear Theory," David Taylor Model Basin Report 774 (Apr 1955).

6. Clarkson, J., "The Deflections and Stresses of a Flat Rectangular Plate with Loading Normal to the Surface," Naval Construction Research Establishment (Great Britain) Report R.223, Undex 291 (Jul 1952).

7. Hovgaard, W., "Structural Design of Warships," U.S. Naval Institute, Annapolis, Maryland, pp. 120-126 (1940).

8. Pell, W.H. and Prager, W., "Limit Design of Plates," Brown University Report 57 (Mar 1951).

9. Gleyzal, A.N., "Plastic Deformation and Energy Absorption of a Thin Rectangular Plate under Hydrostatic Pressure," David Taylor Model Basin Report R-280 (Jan 1945).

10. Morse, P.M., "Vibration and Sound," McGraw Hill Book Company, Inc., New York (1936), Chapter V.

11. Frankland, J.M., "Effects of Impact on Simple Elastic Structures," David Taylor Model Basin Report 481 (Apr 1942).

12. Salvadori, M.G., "A Mathematical Treatment of the Generalized Hertz Impact of a Mass on a Simply Supported Beam," The Welding Journal, Vol. 26, No. 7, p. 426s (Jul 1947).

INITIAL DISTRIBUTION

Copies	
14	Chief, BuShips, Library (Code 312) 5 Tech Library 1 Nav Arch Planning Coordinator (Code 320E) 1 Appl Science (Code 370) 1 Ship Design Div (Code 410) 1 Prelim Des & Ship Protec (Code 420) 2 Hull Des (Code 440) 2 Sci-Struc & Hydro (Code 442) 1 Ship Tech Div (Code 510)
1	Chief, BuAer (TD)
1	DIR, USNRL
1	CDR, USNOL, Mech Div
1	CDR, New York Naval Shipyard, Material Lab
1	CDR, Puget Sound Naval Shipyard, Code 251
1	CDR, Mare Island Naval Shipyard
1	CDR, Boston Naval Shipyard
1	CDR, Portsmouth Naval Shipyard
1	CDR, Norfolk Naval Shipyard
1	DIR, USNEES, Annapolis, Md.
1	DIR, Langley Aero Lab, Langley Field, Va.
3	Administrator, U.S. Maritime Admin Attn: Mr. Vito L. Russo, Chief, Div of Prelim Design
1	DIR, Natl BuStand, Attn: Dr. S. Levy
1	Newport News Shipbldg & Dry Dock Co, Newport News, Va., Attn: Mr. Comstock, Asst Nav Arch
1	DIR, Appl Physics Lab, Johns Hopkins Univ, Silver Spring, Md.
10	CAPT W.N. Landers, USN (Ret), Secretary, SNAME, New York, N.Y.
1	Mr. Ralph H. Phelps, DIR, Engineering Societies Libr, New York, N.Y.
1	Dr. K.S.M. Davidson, DIR, ETT, SIT, Hoboken, N.J.
1	COMO H.A. Schade, USN (Ret), DIR, Inst of Engin Res, Univ of Calif, Berkeley, Calif.

Copies	
1	Dr. W.H. Hoppmann II, Dept of Appl Mech, Johns Hopkins Univ, Baltimore, Md.
2	Administrator, Webb Inst of Nav Arch, L.I., N.Y. Attn: Prof. Ridgely Nevitt, Stress Sec
1	Dr. Hans Bleich, School of Engin, Columbia Univ, New York, N.Y.
1	Prof. J.F. Bell, Dept of Appl Mech, Johns Hopkins Univ, Baltimore, Md.
1	Prof. W. Prager, Chairman, Graduate Div of Appl Math, Brown Univ, Providence, R.I.
1	Glenn L. Martin Co, Structural Dept, Baltimore, Md., Attn: Mr. Pusin
1	Goodyear Aircraft Corp, Structural Div, Akron, O., Attn: Mr. Jeppison
1	United Aircraft Corp, Structural Res Dept, E. Hartford, Conn.
1	Westinghouse Elec Corp, Air Arm Div, Res & Dev Br, Baltimore, Md.
1	Pratt & Whitney Aircraft Div, United Aircraft Corp, Structural Dept, E. Hartford, Conn.
1	General Electric Co., Structural Res Dept, Schenectady, N.Y.
1	Minneapolis Honeywell Regulator Corp, Brown Instr Div, Res & Dev Dept, Philadelphia, Pa.
1	Republic Aviation Corp, Stress Section, Farmingdale, L.I., N.Y.
1	Aircraft Armaments, Inc, Baltimore, Md. Attn: Mr. Kahn
1	Bendix Aviation Corp, Mech Engin Dept Towson, Md.
1	Supt, Naval Construction Research Establishment, Rosyth, Scotland, Attn: Mr. J. Clarkson
1	ALUSNA, London, England Attn: CDR L.A. Rupp
1	DIR, BSRA
3	BJSM (NS)

NAVY-DPPO PRNC WASH DC

MIT LIBRARIES DUPL
3 9080 02754 1876

JAN 23 1956