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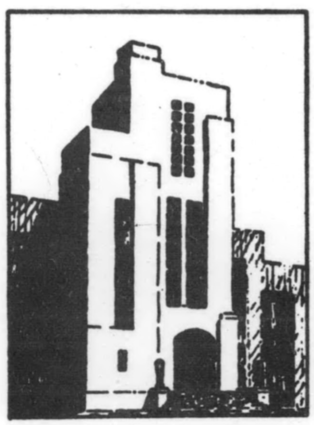

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The Theoretical Dynamic Longitudinal Stability of a Constant-Lift Hydrofoil System

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ABSTRACT

A qualitative treatment is given for an idealized constant-lift system of hydrofoils that do not pierce the surface. It is shown that as a prerequisite for stability the center of lift on the hydrofoil must move aft as the surface is approached. At submergences deeper than the order of one chord, where the movement of the center of lift can be neglected, the system cannot be stable.

INTRODUCTION

The use of hydrofoils for sustentation of water-borne craft has been advocated for many years and many principles of operation have been proposed, leading to a variety of distinctive hydrofoil systems. One of the systems proposed recently may be described as the "constant-lift" system. The essential feature of this system is that some mechanism is employed that tends to keep the hydrodynamic lift developed by the hydrofoils at a constant value in spite of variation in angle of attack. This paper* presents a brief analysis of the stability characteristics such a system would have inherently (without manual or automatic control), if the constant-lift mechanism were ideal. No attempt is made to discuss the response of such a system in a seaway.

*This report is the result of a study made under Research task NR 060-171, financed by the Office of Naval Research.

EQUATIONS OF MOTION

Following Reference 1*, and using the nomenclature of Reference 2, the dimensional equations of motion of a hydrofoil system may be written

$$\left.\begin{aligned} m\frac{dw}{dt} - mU\frac{d\theta}{dt} &= w\frac{\partial Z}{\partial w} + z_o\frac{\partial Z}{\partial z_o} + \theta\frac{\partial Z}{\partial \theta} + \frac{d\theta}{dt}\frac{\partial Z}{\partial q} + Z(t) \\ mk_y{}^2\frac{d^2\theta}{dt^2} &= w\frac{\partial M}{\partial w} + z_o\frac{\partial M}{\partial z_o} + \theta\frac{\partial M}{\partial \theta} + \frac{d\theta}{dt}\frac{\partial M}{\partial q} + M(t) \end{aligned}\right\} \quad [1]$$

where Z(t) and M(t) are disturbance terms.

EVALUATION OF DERIVATIVES

Were it not for the constant-lift mechanism the partial derivatives appearing in Equation [1] could be evaluated by methods given in Reference 1. In evaluating these derivatives with constant-lift it will be assumed that the mechanism is ideal; that is, that any change in angle of attack, α, of each hydrofoil instantly effects a rotation of the hydrofoil such that α is returned to its initial value. This assumption presents the constant-lift system in its best light because it neglects important lags that would be present in any actual system due to the virtual and mass inertias of the hydrofoil and unsteady-lift effects. For simplicity, the treatment will be limited to hydrofoils that do not pierce the surface.

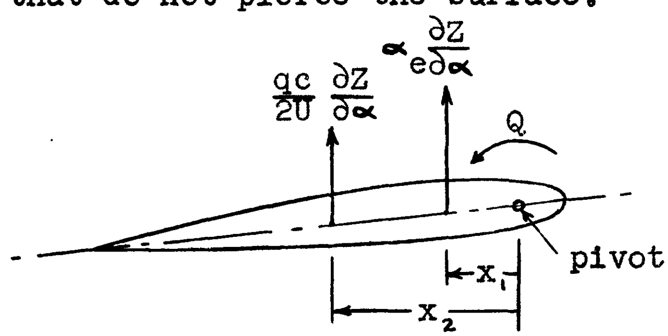

Figure 1 - Forces on Pitching Hydrofoil

* References are listed on page 8.

In Figure 1 assume that the constant-lift mechanism exerts a constant torque Q on the hydrofoil. With the hydrofoil initially at the equilibrium angle of attack α_e (assumed small), a force is developed near the quarter chord such that

$$x_1\alpha_e \frac{\partial Z}{\partial \alpha} = Q \qquad [2]$$

where x_1 is the distance from the pivot point to the quarter chord (positive with the pivot behind the quarter chord). For a small incremental increase in angle of attack $\Delta\alpha$

$$-x_1\Delta\alpha\frac{\partial Z}{\partial \alpha} = \Delta Q_1 \qquad [3]$$

There is no restraint to oppose ΔQ_1, hence the hydrofoil will rotate until $\Delta\alpha$ is reduced to zero. The imposition of the constant torque Q is, therefore, the necessary and sufficient condition for constant lift with changing angle of attack. For such a system

$$\frac{\partial Z}{\partial w} = \frac{\partial M}{\partial w} = 0 \qquad [4]$$

Emphasis can be concentrated on the basic contribution of the constant-lift action to the stability by eliminating shallow hydrofoil submergences (less than the order of one chord) from consideration. This restriction will eliminate the near-surface effects so the derivatives $\partial Z/\partial z_0$ and $\partial M/\partial z_0$ will be insignificant. It follows that the derivatives $\partial Z/\partial\theta$ and $\partial M/\partial\theta$ can then also be ignored. It will be assumed, therefore, that

$$\frac{\partial Z}{\partial z_0} = \frac{\partial M}{\partial z_0} = \frac{\partial Z}{\partial \theta} = \frac{\partial M}{\partial \theta} = 0 \qquad [5]$$

Normally the principal contribution to the remaining derivatives, $\partial Z/\partial q$ and $\partial M/\partial q$, results from the change in the angle of attack at the hydrodynamic center of the hydrofoil brought about by the pitching motion, and this change is nullified by the constant-lift mechanism. A secondary contribution to these derivatives results, however, from an effective change in camber for the pitching hydrofoil. Referring to Figure 1, the changed effective camber associated with a pitching hydrofoil produces an incremental Z force

$$\Delta Z = \frac{qc}{2U}\frac{\partial Z}{\partial \alpha} \qquad [6]$$

Equation [6] is obtained by eliminating the angle-of-attack contribution from Equation [19] of Reference 1. This force has a point of application near the half chord; the associated incremental moment about the hydrofoil pivot is

$$\Delta Q_2 = -x_2 \frac{qc}{2U} \frac{\partial Z}{\partial \alpha}$$

or

$$\Delta Q_2 = -(x_1 - 0.25c) \frac{qc}{2U} \frac{\partial Z}{\partial \alpha} \qquad [7]$$

The hydrofoil will rotate to nullify ΔQ_2 by decreasing the angle of attack by an amount such that

$$\Delta \alpha = - \frac{x_1 - 0.25c}{x_1} \frac{qc}{2U} \qquad [8]$$

This rotation will cause a reduction in the incremental Z force, from that given by Equation [6], to a final value of

$$\Delta Z = \frac{0.25c}{x_1} \frac{qc}{2U} \frac{\partial Z}{\partial \alpha} \qquad [9]$$

Thus

$$\frac{\partial Z}{\partial q} = \frac{0.125c^2}{x_1 U} \frac{\partial Z}{\partial \alpha} \qquad [10]$$

Unless the midchord of the hydrofoil is located directly below the center of gravity, the incremental Z force given by equation [9] will produce a pitching moment about the center of gravity. In general, therefore, the derivative $\partial M/\partial q$ will have a value other than zero, and the equations of motion for the constant-lift system will be

$$\left.\begin{aligned} m \frac{dw}{dt} - mU \frac{d\theta}{dt} &= \frac{d\theta}{dt} \frac{\partial Z}{\partial q} + Z(t) \\ mk_y^2 \frac{d^2\theta}{dt^2} &= \frac{d\theta}{dt} \frac{\partial M}{\partial q} + M(t) \end{aligned}\right\} \qquad [11]$$

STABILITY OF THE MOTION

Upon making use of the symbol p to represent differentiation with respect to time, the characteristic equation corresponding to Equations [11] can be written as the determinant

$$\begin{vmatrix} mp & -(mU + \frac{\partial Z}{\partial q})p \\ 0 & mk_y{}^2p^2 - \frac{\partial M}{\partial q}\,p \end{vmatrix} = 0 \qquad [12]$$

which expands to

$$p^2(mk_y{}^2p - \frac{\partial M}{\partial q})m = 0 \qquad [13]$$

The characteristic equation has two zero roots and one root with the value

$$p = \frac{1}{mk_y{}^2}\frac{\partial M}{\partial q} \qquad [14]$$

Consequently two modes of the motion can never be other than neutrally stable. The remaining mode will be unstable if $\partial M/\partial q$ is positive. Thus at best the system will be neutrally stable with respect to depth of submergence and pitch angle when the hydrofoils are deeply submerged.

STABILITY AT SHALLOW SUBMERGENCE

The earlier-imposed condition that the submergence be greater than one chord will now be considered. It is well known that as the hydrofoil approaches the surface a loss of lift occurs at a given angle of attack and the slope of the lift curve is also reduced (see Reference 1). If, at shallow submergence, the lift due to angle of attack continued to be developed at the quarter chord, however, the constant-lift mechanism would simply produce a new equilibrium angle of attack and the conclusions regarding stability would be the same as for deep submergence.

Experimental work done by Ausman at low Froude numbers indicates that the center of lift moves aft at shallow submergence (Reference 3). Kotchin, and Keldysch and Lavrentiev give theoretical developments that indicate a forward movement of the lift center as the surface is approached under high-speed operation (see References 4 and 5). The apparent disagreement as to the direction of movement needs to be resolved, but one may examine the near-surface stability in the light of the movement.

With the torque Q adjusted for constant lift at some operating depth near the surface, the derivatives $\partial Z/\partial w$ and $\partial M/\partial w$ will be zero. The derivative $\partial Z/\partial z_o$ has a value other than zero only if the lift center changes position with change in operating depth; the sign of the derivative depends on the direction of movement. Values of derivatives $\partial M/\partial z_o$, $\partial Z/\partial \theta$ and $\partial M/\partial \theta$ depend on the direction of the moment and the geometry of the system. Practical limitations on the geometry are such that $\partial M/\partial \theta$ will have the same sign as $\partial Z/\partial z_o$.

The equations of motion are

$$\left.\begin{aligned} m\frac{dw}{dt} - mU\frac{d\theta}{dt} &= z_o\frac{\partial Z}{\partial z_o} + \theta\frac{\partial Z}{\partial \theta} + \frac{d\theta}{dt}\frac{\partial Z}{\partial q} + Z(t) \\ mk_y^2\frac{d^2\theta}{\partial t^2} &= z_o\frac{\partial M}{\partial z_o} + \theta\frac{\partial M}{\partial \theta} + \frac{d\theta}{dt}\frac{\partial M}{\partial q} + M(t) \\ \frac{dz_o}{dt} &= w - \theta U \end{aligned}\right\} \qquad [15]$$

The corresponding characteristic equation is

$$m^2k_y^2p^4 - m\frac{\partial M}{\partial q}p^3 - m\left(k_y^2\frac{\partial Z}{\partial z_o} + \frac{\partial M}{\partial \theta}\right)p^2$$

$$-\left(\frac{\partial M}{\partial z_o}\frac{\partial Z}{\partial q} - \frac{\partial M}{\partial q}\frac{\partial Z}{\partial z_o}\right)p - \left(\frac{\partial M}{\partial z_o}\frac{\partial Z}{\partial \theta} - \frac{\partial M}{\partial \theta}\frac{\partial Z}{\partial z_o}\right) = 0 \qquad [16]$$

A necessary requirement for stability is that all of the coefficients of Equation [16] have the same sign. Therefore, $\partial Z/\partial z_o$ and $\partial M/\partial \theta$ must be negative as a condition for stability. This condition will be met only if the center of lift moves aft as the surface is approached. If the results given in References 4 and 5 are valid, therefore, the constant-lift system will be unstable at high speeds. In spite of a favorable movement of the center of pressure at low speed, stability of the system is not assured, however, because the criterion for no variation in sign of the coefficients of Equation [16] is a necessary but not a sufficient condition for stability. Routh's discriminant (see Reference 6) would have to be evaluated in any given case in order to have assurance of stability.

CONCLUDING REMARKS

This study indicates that the constant-lift system will not be stable unless it operates with the hydrofoils near the surface. For stability the further requirement that the center of lift on the hydrofoil must move aft as the hydrofoil approaches the surface is essential. Without this favorable travel of the center of pressure the constant-lift system will not be stable under any circumstance.

These conclusions were reached under the assumption that the constant-lift mechanism functions perfectly. Any actual embodiment of the system will introduce resonances, lags and dead zones caused by inertia, solid friction, etc. As in any other dynamic system, such problems will have a deleterious effect on the system stability.

REFERENCES

1. Imlay, Frederick H., "Theoretical Motions of Hydrofoil Systems", NACA Report No. 918, May 9, 1947.

2. "Nomenclature for Treating the Motion of a Submerged Body Through a Fluid", SNAME Technical and Research Bulletin No. 1-5, April 1950.

3. Ausman, John Stanley, "Pressure Limitation on the Upper Surface of a Hydrofoil", University of California, Graduate Division, 5 March 1954.

4. Kotchin, N. E., "On the Wave-Making Resistance and Lift of Bodies Submerged in Water", Transactions of the Conference on the Theory of Wave Resistance, U.S.S.R., Moscow, 1937; English translation by A.I.(T) Air Ministry, R.T.P. No. 666, March 1938; SNAME Technical and Research Bulletin No. 1-8, August 1951.

5. Keldysch, M. V., and Lavrentiev, M. A., "On the Motion of an Aerofoil Under the Surface of a Heavy Fluid, i.e., a Liquid", paper to ZAHI, Moscow, 1935; English translation by Science Translation Service, STS-75, November 1949.

6. Routh, E. J., "Advanced Rigid Dynamics", Vol. II, Macmillian & Co. (London), 1905, pp. 223-230.

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2. Hydrofoils - Stability
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II. Office of Naval Research. NR 060-171

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