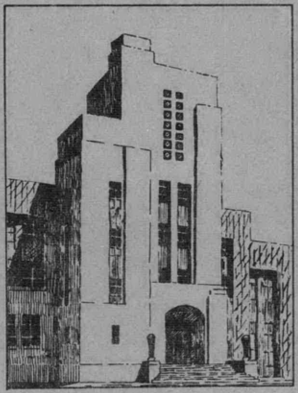
THE DAVID W. TAYLOR MODEL BASIN

UNITED STATES NAVY

A CARRIER-TYPE STRAIN INDICATOR

BY GEORGE W. COOK



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PERSONNEL

The design of the circuits described in this report, as well as the analytical discussion and assembly of material in the report, is the work of George W. Cook of the Electronics Section of the David Taylor Model Basin. The preliminary and final models of the completed strain indicators were constructed by other members of this section.

PREFACE

This report is not intended to serve as a technical service manual, nor will it be found entirely suitable for use as an operating instruction pamphlet. The subject matter is confined to a generalized discussion of the fundamental principles on which the design of the TMB Type 1A Strain Indicator is based and the operation of that strain indicator in combination with strain gages of the wire-resistance type.

It is considered that the circuit employed is not necessarily the ultimate in circuit design; however, the four models of strain indicators developed so far have several features which are not usually found in equipment of the carrier type. As a result of these departures from conventionality in circuit design, it is now possible to make measurements which could not be made conveniently before these instruments were available.

Considerable interest in the TMB Type 1A Strain Indicator has been shown by other agencies, both Governmental and private. It is hoped that this description of the present state of development of the instrument may assist others in their quest for more detailed information concerning the characteristics and behavior of engineering structures.

Except for certain aspects of the circuit deemed to be of particular interest, no attempt is made to justify or explain the use of each component part. A complete schematic diagram of the instrument, with all electrical values, is included for use by anyone interested in technical circuit details. It is assumed that readers who have an interest in these circuit details will find more elaborate explanations unnecessary.

A CARRIER-TYPE STRAIN INDICATOR

ABSTRACT

An electronic instrument is described which is suitable for measuring and recording static and dynamic strains by means of strain gages of the wire-resistance type and suitable recording apparatus. The circuit is discussed, with particular emphasis on the fundamental principles of design. Certain novel features in the design of the instrument are brought out by mathematical treatment. Schematic wiring diagrams complete with electrical values are included, together with photographs of the completed instrument.

INTRODUCTION

An electronic instrument has been developed at the David Taylor Model Basin for measuring strain in materials and structures. This instrument is known as the TMB Type 1A Strain Indicator; see Figure 1.

This strain indicator depends for its performance upon a particular electronic circuit, also developed at the Taylor Model Basin. Although the basic circuit was devised specifically for use in this instrument, the circuit has been employed successfully in the solution of many other measurement problems.

The Type 1A Strain Indicator was designed primarily for use with the 120-ohm Type A SR-4 strain gage. The present report is concerned chiefly, however, with a description of the electronic circuit as it is employed in the strain indicator. Therefore, very few of the gage problems and applications are discussed.

GENERAL CONSIDERATIONS

Many different types of strain-measuring apparatus have been developed in the past decade. The most important single contribution is undoubtedly the SR-4 wire-resistance strain gage, known also as the metalectric strain gage. This gage is manufactured in a variety of shapes and sizes, each for certain types of strain measurements. It is widely used in the measurement of strain in structural members and has opened a vast new field of possibilities for the critical examination of physical properties of materials and the behavior of structures under service conditions. Constructional details of this gage are shown diagrammatically in Figure 2.

As loading is applied to the structure, the resultant dimensional changes in the member are accompanied by corresponding changes in the resistance of the wire in the gage. If the gage is part of a suitable electrical

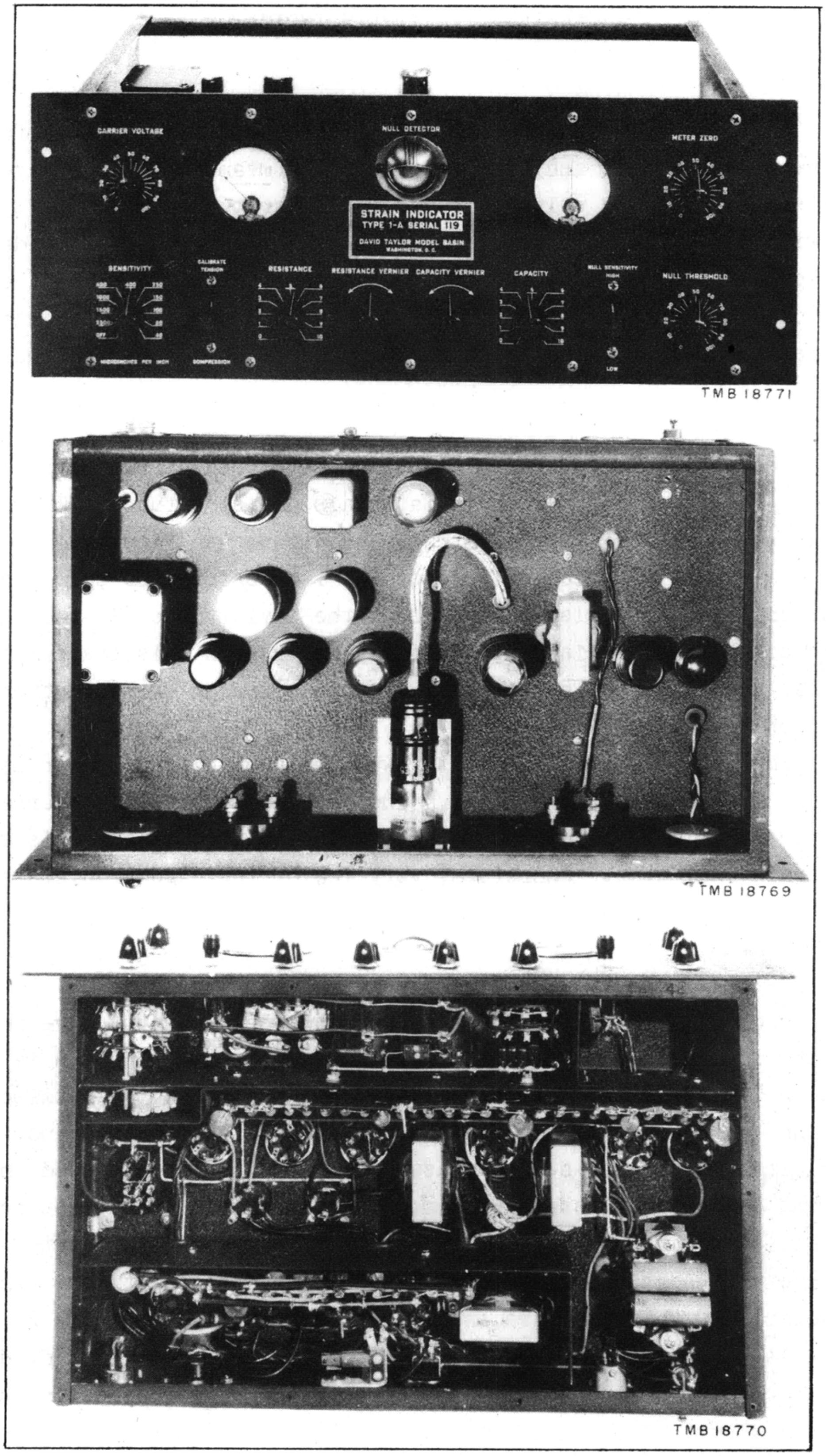

Figure 1 - Front, Top, and Bottom Views of the Type 1A Strain Indicator

circuit, these changes in length and resistance of the gage may be resolved in terms of strain in the material under observation, at the point where the gage is mounted.

An important part of the investigation in this new field involves the precise determination of the behavior of structural members under various conditions of loading. This branch of the field, considered in a broad sense, may well be expanded to include the use of a structural member as the compliant element in a more or less complicated mechanical gage system in which the SR-4 strain gage may be effectively used as the transducing agent. An example of this is a coupling rod fitted with one or more gages and used as a tension and compression dynamometer. When employed with proper electronic instrumentation, such combinations are virtually limitless in their uses and applications. It is becoming apparent that the measurement possibilities depend almost entirely on the ingenuity and imagination of the designers of mechanical parts, gages, and electronic instruments.

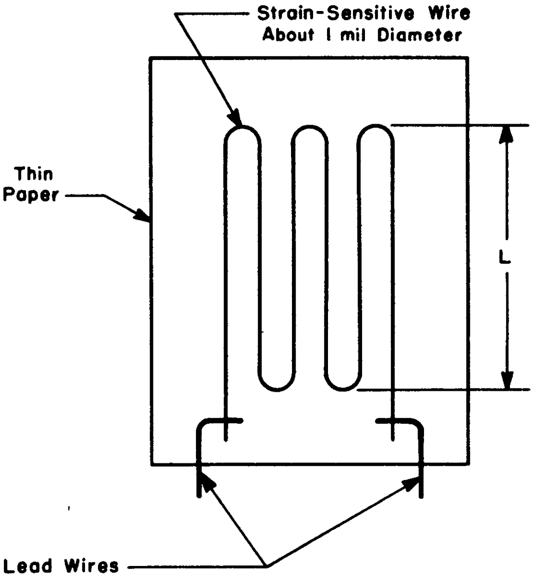

Figure 2 - Diagram of the SR-4 Strain Gage

This gage consists of a grid of fine wire laid on a small sheet of thin paper and held in place by a suitable cement. This gage is easily put in place by cementing the paper backing to the surface of the material on which strain is to be measured. The gage operates on the principle that a change in the length of the loops of fine wire produces a proportional change in its electrical resistance by varying both its length and its area.

The SR-4 strain gage depicted in Figure 2, notwithstanding its many advantages and possibilities, is like most other devices in that it possesses some undesirable characteristics. While it faithfully indicates the magnitude of strain due to stress in a specimen by a corresponding change in resistance, it also indicates strain sometimes wanted but more often not, when the surface on which it is mounted undergoes a dimensional change from any other cause, such as normal expansion or contraction due to changes in temperature of the specimen.

Spurious indications of this nature can be largely offset by the simple expedient of using the gage in a Wheatstone bridge circuit. A second gage, exactly like the first, is placed electrically in the circuit so that the active gage and the second gage are adjacent arms in the bridge. The second or compensating gage must be mounted on the specimen or on a similar specimen which is exposed to the same temperature effects. In this way, the compensating gage is subjected to the same spurious changes as the active gage, but not to the load strain being measured.

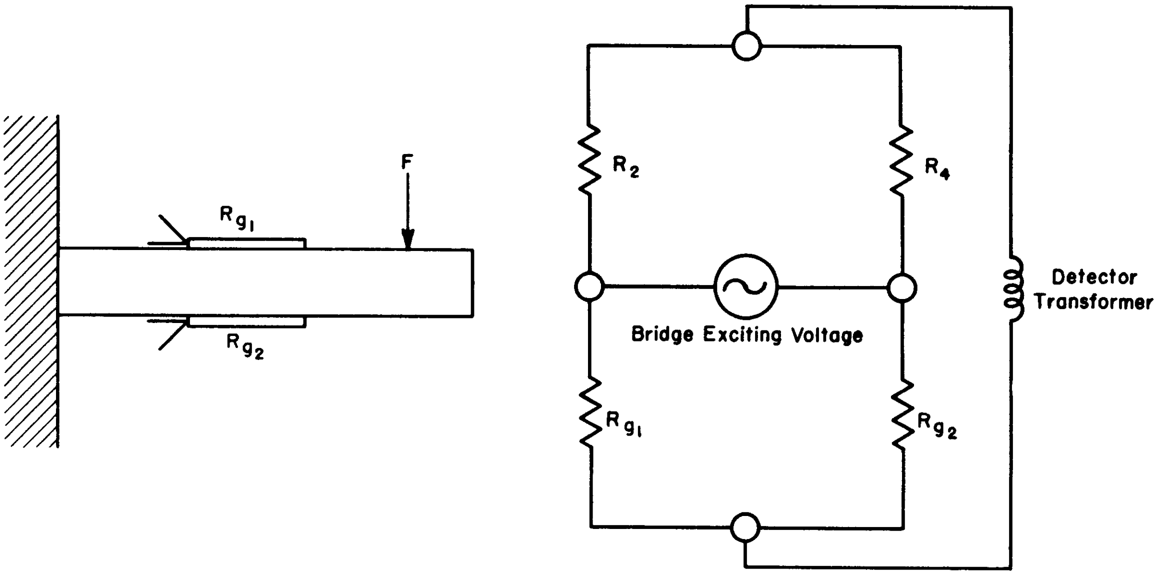

Figure 3 - Strain-Gage Connection for Measurement of Bending

Gages on opposite sides of the specimen are connected in adjacent arms of the bridge to measure bending. This connection also provides temperature compensation.

In special cases where strain due to bending in a plate or bar is to be measured, the compensating gage may take an active part in the measurement by being mounted directly opposite the primary gage on the reverse surface of the material; see Figure 3. For the measurement of bending moments, the resistance changes in this case are additive for the two gages. All longitudinal strain indications, as well as temperature expansion or contraction effects, are canceled.

From the standpoint of practicing accurate strain-measurement technique, it is highly desirable to measure the strain produced by bending whenever possible. In some types of strain measurements it is expedient to mount gages in multiple series-parallel arrangements, i.e., with groups of four gages connected to serve as one gage. Many complex strain-measurement problems have been solved in this way.

THE TYPE 1A STRAIN INDICATOR

The strain indicator consists of 15 essential units, as indicated in the block diagram, Figure 4.

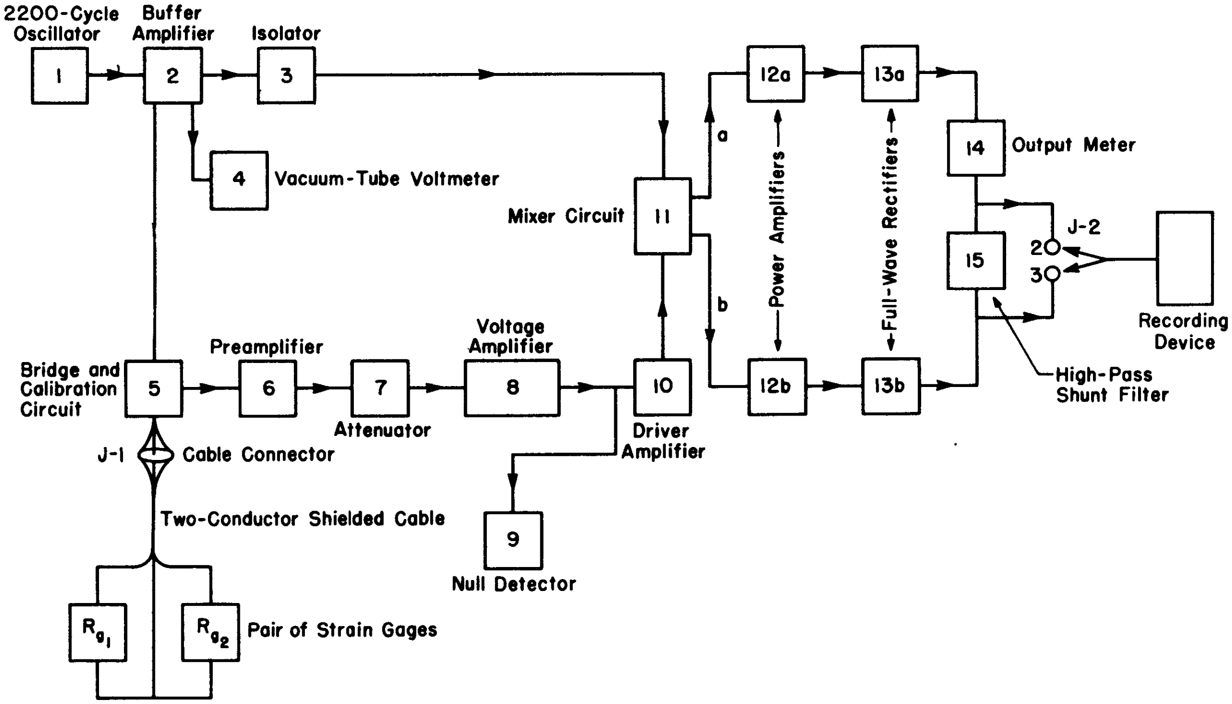

Figure 4 - Block Diagram of Type 1A Strain Indicator

Unit 1 is a sinusoidal oscillator whose frequency is 2200 cycles per second. A buffer amplifier, Unit 2, which is driven by the oscillator, furnishes power to excite the bridge, Unit 5. It also delivers a voltage through the isolator, Unit 3, to the mixer circuit, Unit 11. The output or signal voltage from the bridge is passed by the preamplifier, Unit 6, to the attenuator, Unit 7. The output voltage of the attenuator is further amplified by Unit 8, which is a fixed-gain voltage amplifier. The amplified signal voltage from Unit 8 is impressed on the null detector, Unit 9, and also passes through the driver, Unit 10, to the mixer circuit, Unit 11. The composite output voltage from the mixer circuit drives two independent power amplifiers, 12a and 12b. These in turn operate into full-wave rectifiers, 13a and 13b respectively. Unit 14 is a zero-center milliammeter whose full-scale current is 5 mils either side of center. The recording device is connected to the output terminals, 2 and 3. These terminals are shunted with a simple high-pass filter, Unit 15. Unit 4 comprises a vacuum-tube voltmeter which serves to indicate the magnitude of the voltage supplied to the bridge and mixer circuit. Two strain gages, R_{g1} and R_{g2}, are connected by cable to the bridge circuit. These gages are electrically an integral part of the bridge and calibrating circuit, as explained later.

A separate power-supply unit was designed for use with this instrument. The power furnished by this unit is sufficient for the operation of either one or two strain indicators. The power unit is designed to plug into a standard 115-volt 60-cycle single-phase power line. The power consumed with one strain indicator connected is 135 watts. When two indicators are used, the power consumed is 175 watts.

TABLE 1

Range and Sensitivity of Strain Indicator

Full-Scale Indications	
Strain microinches per inch	Approximate Stress in Steel pounds per square inch
40	1,200
60	1,800
100	3,000
150	4,500
250	7,500
400	12,000
600	18,000
1,000	30,000
1,500	45,000
2,500	75,000

The range and sensitivity scale of the Type 1A Strain Indicator are shown in Table 1. Good resolution may usually be obtained from indications as small as 1/5 of full scale.

The built-in calibration method has an accuracy of 0.25 per cent or better when the gage factor* is equal to 2 and the gage resistance is 120 ohms. Some users of this instrument report verified accuracy within 0.2 per cent in records obtained in field measurements.

This strain indicator is designed primarily to be used with multichannel electromagnetic recording oscillographs, which are equipped with bifilar suspension or other types of galvanometers. Linear output current of 5 milliamperes in either polarity is available for driving a 10- to 20-ohm load. Other types of recording or indicating devices may be used if their driving-power requirements are in this range. Higher currents may be consumed if some nonlinearity can be accepted; for example, about 1.5 to 2 per cent deviation at 8 milliamperes.

The usable frequency range of the instrument is from zero to about 200 cycles per second. With some correction, however, higher frequencies can be used. This frequency range is influenced by the resistance of the recording galvanometer; see Figure 5.

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* The "gage factor" is defined as the ratio of unit change in gage resistance to unit change in gage length; i.e., gage factor is equal to $\Delta R/R$ divided by $\Delta L/L$.

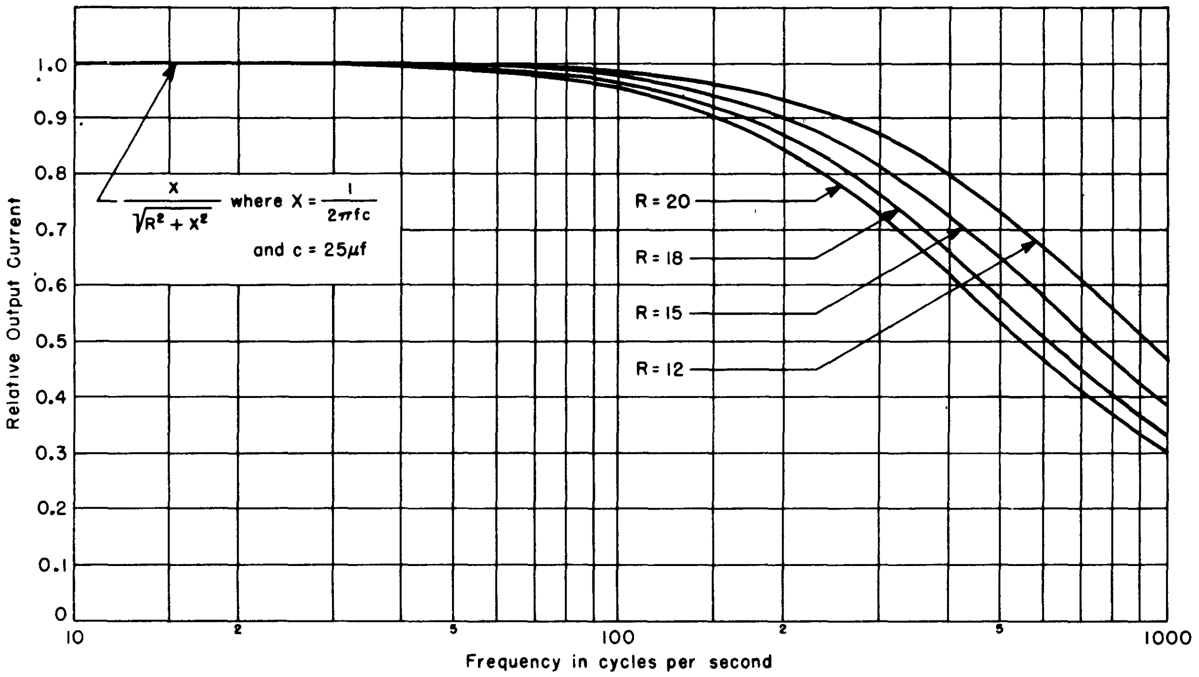


Figure 5 - Output Current As a Function of Frequency

If an alternating strain of constant amplitude is measured, the output current, as a function of the frequency of alternation, is shown in this figure. Below 10 cycles per second, the response is essentially uniform up to and including zero frequency. Response curves near and including zero frequency are not conveniently shown on a logarithmic frequency scale. The response curve is influenced by the resistance R of the recording galvanometer. Curves for four values of R are shown. Curves for other values of R may be computed and plotted as indicated in the figure.

BRIDGE CIRCUIT

The bridge circuit employed in this instrument is a fairly complex elaboration of the usual Wheatstone bridge circuit. Since mathematical analysis of the behavior of this bridge circuit is complicated, it will be sufficient to state, in the words of the immortal Oliver Heaviside, "It works!" This is an alternating-current bridge which is excited by the output voltage of the buffer amplifier.

It might reasonably be asked why a direct-current supply is not used for bridge excitation. The Type 1A Strain Indicator is designed to measure strains which may vary in a frequency range extending from zero to 200 cycles per second. The sensitivity of the instrument is such that good resolution of strains as low as 8 microinches per inch may be obtained. The signal voltage from the bridge, when a strain of this magnitude is introduced, is about 5 microvolts. Now, if a direct current were used to excite the bridge, a constant strain, i.e., a strain of zero frequency, would produce a d-c voltage output from the bridge. Uncontrollable variations in circuit parameters preclude the use of any known d-c amplifying system at signal levels of this order. The use of a carrier system is, then, virtually essential to the extension of the low-frequency range of the instrument to zero.

Provision has been made for balancing the bridge for both resistance and reactance. The output voltage from such a bridge circuit is, of course, equal to zero while the bridge is in this balanced condition. The null detector serves to determine when this condition is established. A change in resistance in any arm of the bridge disturbs this balanced condition, and an output voltage develops at the output terminals of the bridge. This voltage is proportional to the degree of unbalance in the bridge.

It is important to note that the bridge circuit has no sense of direction; i.e., a resistance increment introduced in an arm of the bridge produces the same voltage output whether the increment represents a resistance increase or decrease. A sense of direction in the bridge could be obtained if the bridge were initially unbalanced. Of course, the degree of unbalance would have to be greater than the peak amplitude of the modulation produced by the strain being measured.

There are several valid reasons for not using the unbalanced bridge; one of them is a matter of convenience to the operator. It can easily be seen that a new unbalance would have to be established for each setting of the attenuator; furthermore, the operator could not conveniently keep a constant check on the condition of balance of the bridge. Probably the most important reason for using the balanced bridge will appear when the output stage is discussed. For the moment let it be pointed out that it is desirable to obtain from the bridge only the side-band components of any modulation produced in the bridge.

Resistance balance in the bridge is accomplished in the manner shown in Figure 6. The lower arms of the bridge, indicated as R_{g_1} and R_{g_2}, are wire-resistance strain gages, 120-ohm, Type A. The upper arms R_2 and R_4 are precision resistors of 120 ohms each.

The bridge is balanced for stray capacity effects by the method shown in Figure 7. A precise degree of balance may be obtained by alternate manipulation of the resistance and capacitance compensating controls.

CALIBRATION METHOD

Of the several methods in general use for calibrating strain-measuring apparatus, the one which probably is most widely used consists of direct measurements of

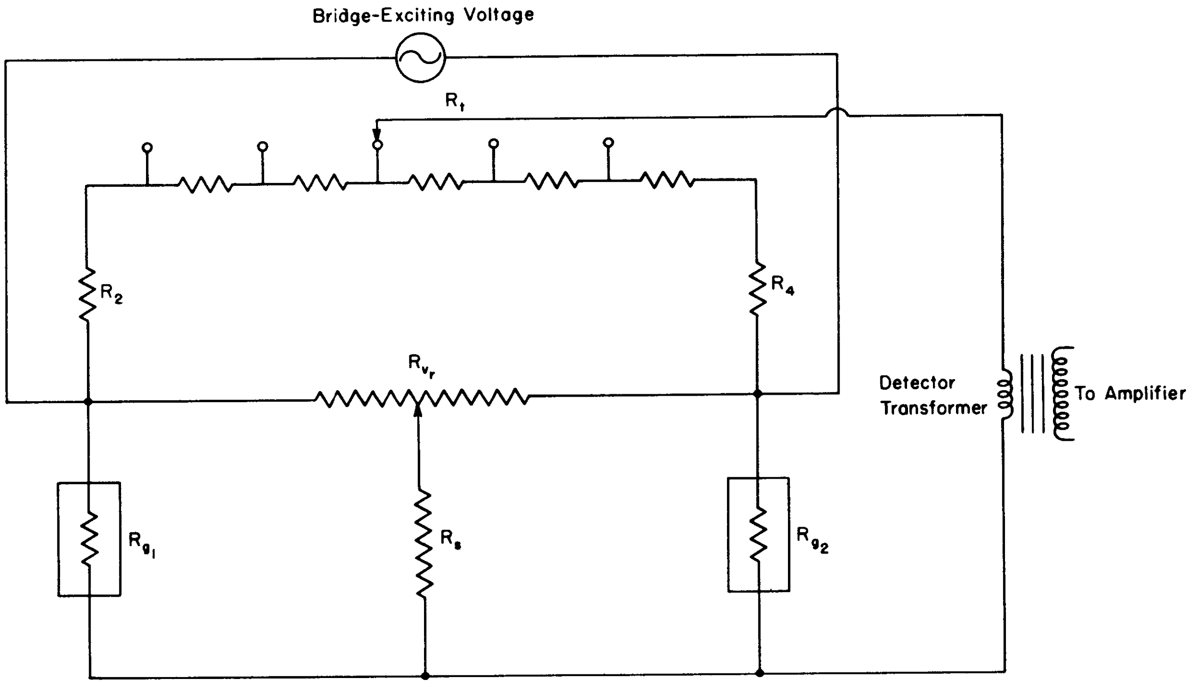


Figure 6 - Wheatstone Bridge with Resistance Balance

A resistance balance can be accomplished in this simplified bridge circuit by adjustment of the rough control R_t and the vernier potentiometer R_{v_r}. The effectiveness of moving the vernier control from one extreme position to the other must of course be equal to or exceed the effectiveness of moving the rough control one step. A 10 per cent overlap is provided in this instrument.

a. the voltage gain of the amplifying apparatus,
b. the exact resistance of the gage, and
c. the magnitude of the exciting current in the gage.

With the results of these three measurements at hand, the strain may readily be computed. If particular care is exercised in making these measurements, the only limitation on the accuracy of the calibration is the accuracy of the measuring devices used in these operations. The quantities involved in the computations must often be carried to several decimal places. Neglecting the time consumed in making these three measurements and in computing the results, it will be noted that there is a chance for human error in each of four operations. Furthermore, several accessory pieces of apparatus are usually required to make the three essential measurements.

Another method which is widely used consists in operating on, or actually loading, the member on which a strain measurement is to be made, in such a manner as to produce predictable results. It is the object of such a

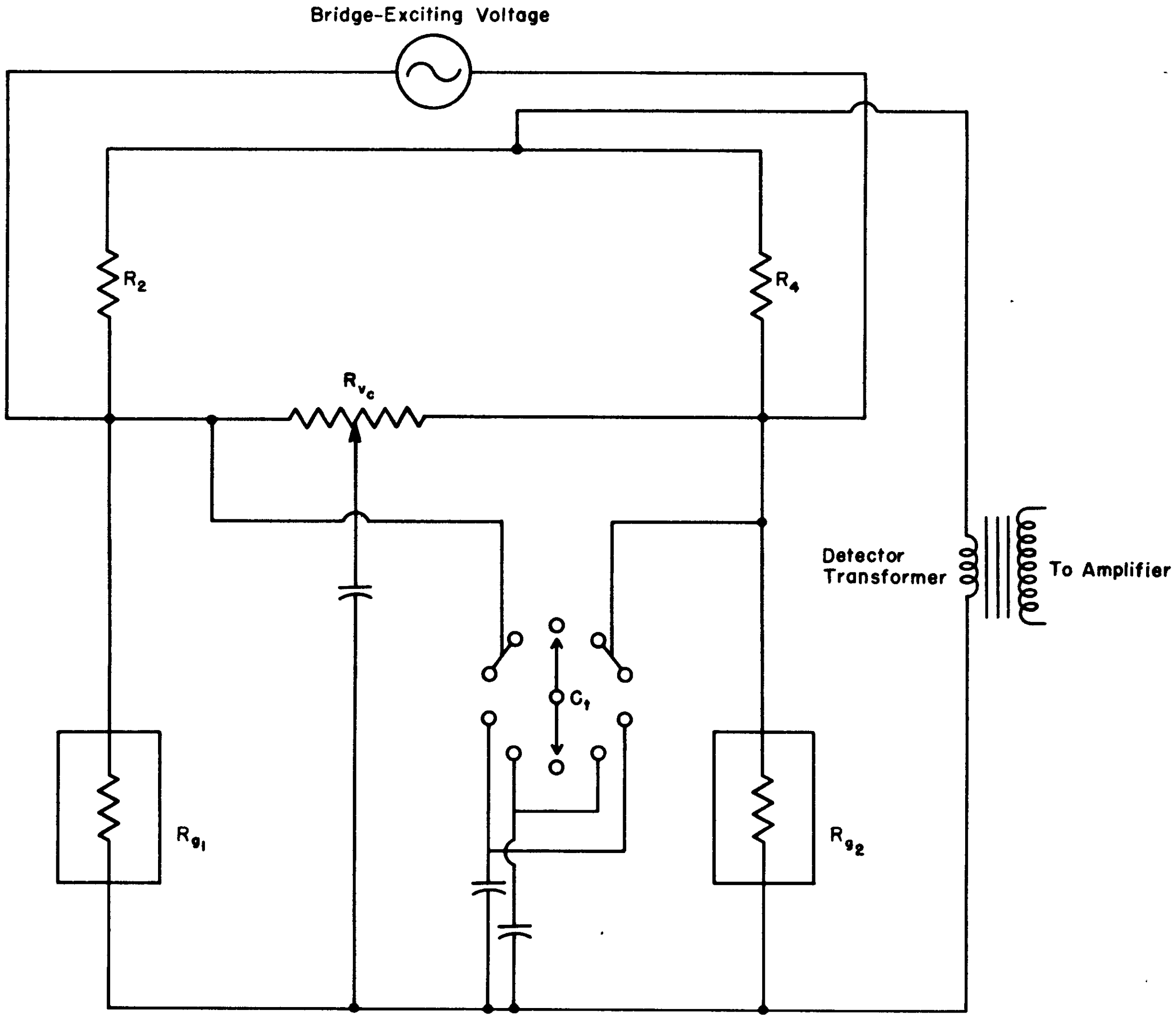


Figure 7 - Wheatstone Bridge with Reactance Balance

Assuming that resistance balance has been established, a capacitance or reactance balance can be accomplished by adjustment of the rough control C_t and the vernier control R_{v_c}. The effectiveness of moving the vernier control from one extreme position to the other overlaps the effectiveness of one step of the rough control by about 10 per cent.

calibration method to produce a deflection or indication at the output of the strain-measuring apparatus with which the effect of applying an unknown load to the member may be directly compared.

There is a valid objection, however, to this method of calibration; it is not always possible to load a structural member in a predictable manner. The use of strain-measuring apparatus depending on this type of calibration, consequently, is limited to test conditions where structural members can be thus loaded.

A method of calibration which is much more likely to produce satisfactory results consists in introducing a known resistance change in the gage circuit. This change can be introduced either by shunting the gage with another resistor whose resistance is accurately known, or by introducing a known resistance in series with the gage.

Calibrating by shunting either presumes a prior knowledge of the exact resistance of the gage and the connecting wires or cable, or requires the measurement of this total resistance at the time of calibration. It is seldom convenient to install a switching arrangement at the gage location, and for this reason most shunt-type calibration methods include the resistance of the cable with that of the gage. A somewhat cumbersome computation is always involved in this method, together with the bother of determining the gage-plus-cable resistance. Incidentally, this resistance value is seldom found to be a round number which is easy to handle in computation formulas.

The series calibrating method, on the other hand, neatly avoids the difficulties just mentioned. The absolute value of the gage-plus-cable resistance has little effect on the accuracy of the calibration, because the resistance change introduced in the arm of the bridge by the calibration operation is independent of the arm resistance. There are, however, also possibilities for error in this method. For example, some strain-measuring apparatus is provided with a means for opening the gage circuit and inserting a known resistance. This scheme is quite usable if the resistance change ΔR thus introduced in the circuit is fairly large; but when the calibrating resistance to be inserted is of the order of a few thousandths of an ohm, the resistance of the switch contacts employed in the operation may have an effect which is comparable to that of the calibrating resistor being inserted. Needless to say, the result of calibration under these conditions is indeterminate.

The calibrating method which is incorporated in the Type 1A Strain Indicator is of the series type, but the switch-contact-resistance problem is solved by the method shown in Figure 8. A 2-ohm precision resistor is permanently connected in series with each of the lower or gage arms of the bridge. A resistance change may be produced conveniently in either of these arms by shunting one or the other of these 2-ohm resistors with a calibrating resistor. The calibrating resistor has an electrical resistance which is greater than that of the 2-ohm resistor, and the switch contact resistance is now in series with this relatively large resistor. For this reason any possible error due to switch contact resistance is reduced to one part in many thousands.

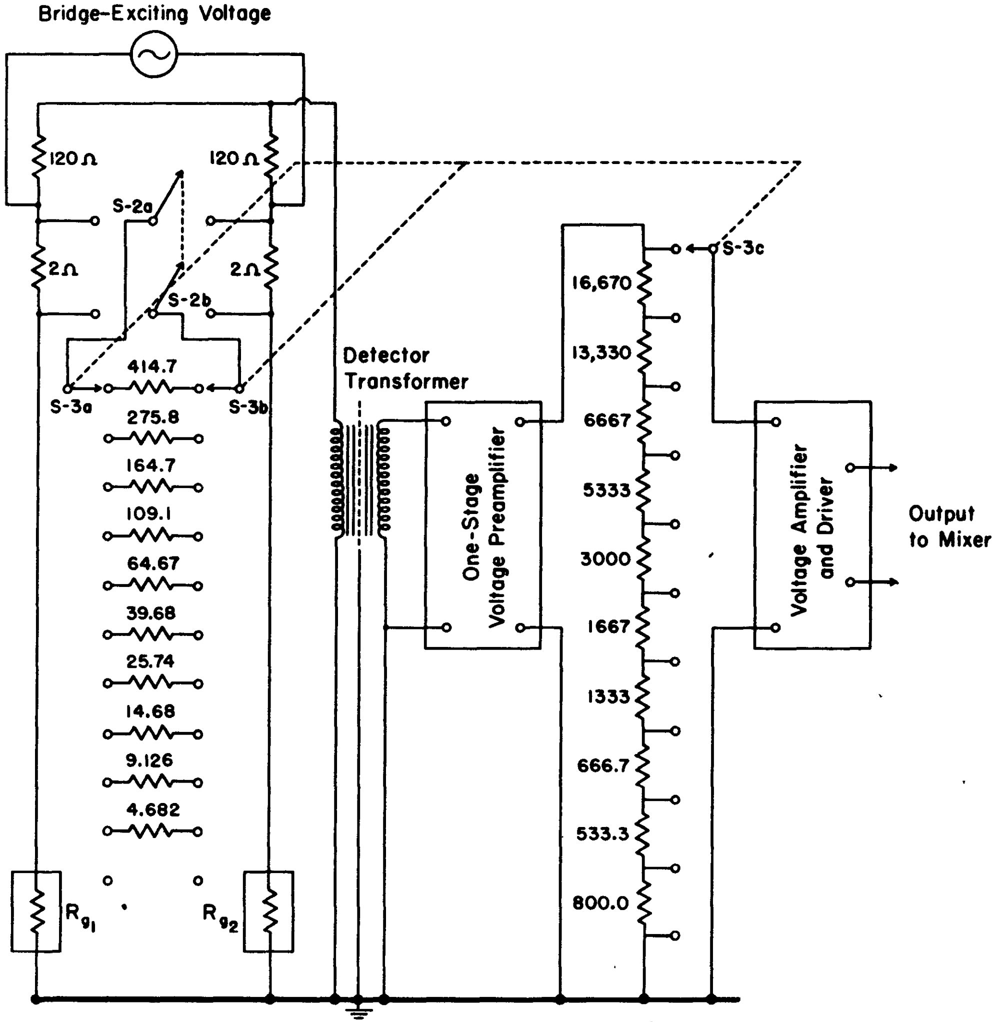


Figure 8 - Calibration Method

The desired calibrating resistor is selected by the switches S-3a and S-3b. Manipulation of the gaged switches S-2a and S-2b places the selected resistor in parallel with the 2-ohm resistor on either side of the bridge. This method of calibration has many distinct advantages over other methods, as explained in the text.

The calibration switch used for this purpose is a three-position spring-return lever-type switch, standing normally in the central position. When desired, it may be actuated by the operator. When it is thrown in one direction, a decrease in resistance is introduced in series with one lower arm of the bridge. When it is thrown in the other direction, a resistance decrease is introduced in series with the other lower arm. The resistance decrease is identical for both conditions since the same calibrating resistor is used. The resultant deflection or indication on the recording device caused by these accurately known resistance changes may be directly compared with the deflection obtained by a strain in the specimen under observation.

The bidirectional operation of this calibration method provides calibration indications which correspond to strains in either tension or compression. It should be noted that calibration is accomplished in one simple operation, that no accessory apparatus is involved, and that no tedious measurements or computations are necessary.

The signal or unbalance voltage from the bridge is stepped up and amplified by a transformer of high turn ratio and preamplifier stage respectively. This amplified voltage is in turn impressed on a special attenuator. At this point there may be some curiosity concerning the odd-valued resistances which comprise this attenuator. These resistance values are directly related step by step to the series of calibrating resistors in the bridge circuit. For example, if the uppermost calibrating resistor, 414.7 ohms, is selected, and the calibrating switch is operated, an unbalance voltage is developed across the bridge which has the same magnitude as that which would be produced if one gage were subjected to a compressive strain of 40 microinches per inch. This is true because the shunting of a 2-ohm resistor with one of 414.7 ohms results in a resistance change of 9.6 milliohms, and this change is the same as the ΔR obtained by compressing a 120-ohm Type A gage 40 microinches per inch, assuming the gage factor is equal to 2. The design of the complete amplifying system is such that when this calibrating resistor is used, full-scale deflection will be indicated on the recording device if the attenuator is rotated to the maximum-output position.

Now let the next calibrating resistor, 275.8 ohms, be selected. Operation of the calibration switch will produce a resistance unbalance which is equivalent to a strain in one gage of 60 microinches per inch. The unbalance voltage from the bridge will, of course, be much larger. Unless the voltage gain in the amplifying system is reduced, the recording device will obviously be driven off scale. The resistance values in the attenuator are apportioned so as to permit uniform deflection on the recording device when the calibrating switch is actuated, provided that the switches which select the calibrating resistors and the attenuator switch are kept in step. This condition is assured by the simple expedient of ganging the switches; i.e., they are all mounted on the same control shaft and rotate simultaneously.

The use of this scheme makes it feasible to designate the position of the attenuator in microinches per inch for full-scale deflection rather than in attenuation units, in voltage gain factors, or in abstract figures representing its rotary position. The designation of the attenuator position in units of strain presumes the use of 120-ohm Type A gages with gage factors exactly equal to 2. When it is necessary to use gages with gage factors other than 2, a multiplying factor must be applied to indications in order to obtain exactness in test results.

The record is easily corrected in either of two ways: the output indications produced by the measured strain may be multiplied by the quantity $\frac{2}{\text{Actual gage factor}}$, or the indication produced by actuation of the calibrating switch may be multiplied by the quantity $\frac{\text{Actual gage factor}}{2}$. Where multiple series-parallel gage combinations are employed and the gage factors are mixed, remedial measures are left to the ingenuity of the user.

OUTPUT CIRCUIT

Two amplifier stages of conventional design are interposed between the attenuator rotor and the step-down transformer which drives the mixer circuit. A sacrifice in voltage gain in the transformer is accepted for the purpose of reducing the impedance of the mixer circuit to a low level. As shown in Figure 9, a portion of the carrier voltage, which excites the bridge, is imposed in parallel on the control grids of two power-amplifier tubes. The

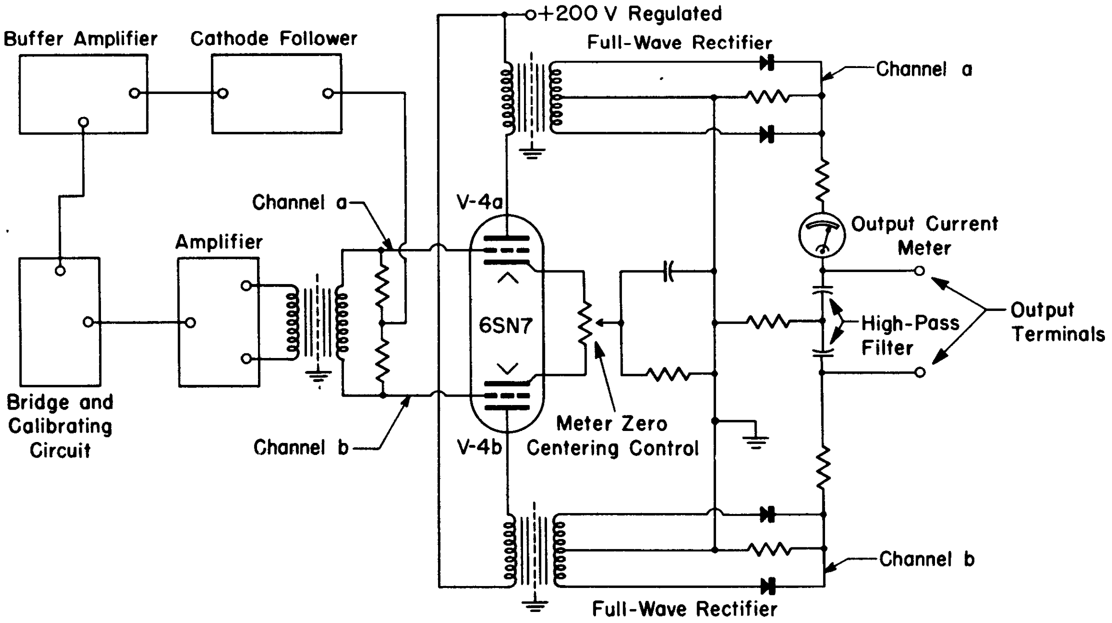


Figure 9 - Skeleton Circuit Diagram of Mixer Circuit and Output Stage

output voltage of the signal amplifier drives the same two grids differentially. If we may borrow terminology from the field of communications, this state of affairs might be called an "injected carrier" system; but in a communications system the composite output voltages of the power amplifiers would be transmitted in combination to a demodulator.

In this method of demodulation, however, the power amplifiers are split and operate independently. Each one drives its own full-wave rectifier circuit. The output voltage of each of these rectifiers consists of a series of adjoining half-cycles, the familiar pattern obtained when a full-wave rectifier operates on a sinusoidal wave. The rate of repetition is twice the carrier frequency or 4400 half-cycles per second.

The two rectifier output circuits are so arranged that they dissipate a fairly large current in their respective equal load resistors. This condition is, of course, essential to the successful driving of a current-operated recording device. When the bridge is balanced, and no voltage is delivered to the mixer from the signal amplifier, equal voltages are impressed on these two load resistors. The recording device, which is connected to these two equipotential points, records zero current transfer between the points.

When the bridge is unbalanced, for example when the calibrating switch is thrown in one direction, the output voltage of the signal amplifier combines with the carrier voltage at the mixer. The reaction is such that the effective driving voltage at the grid of one power-amplifier tube is increased and that at the other grid is decreased. The resulting currents in the two rectifiers and their load resistors are no longer equal.

In the ensuing struggle to obey Kirchhoff's laws governing the behavior of electric currents in a network, the difference-current flows through the recording device. This current is linearly related to the degree of unbalance in the bridge. If the calibrating switch is thrown in the other direction, the reaction in the mixer is reversed, and the current in the recording device is likewise reversed.

The instrument thus has recovered a sense of direction and not only indicates accurately the magnitude of a resistance change in an arm of the bridge, but also resolves the change into either an increase or a decrease in resistance. This resolution may in turn be defined as strain in either tension or compression, as the case may be.

The simple high-pass filter which shunts the recording device serves to bypass a large portion of any ripple which may develop. The zero-centered milliammeter which is in series with the recording device provides a convenient means for reading the magnitude of a static strain and also furnishes a continual indication of the current in the recording device.

As previously stated, the signal amplifier is a conventional resistance-capacitance coupled amplifier. As such, it issues spurious voltages to the mixer circuit which did not originate in the bridge. Some of this disturbance is introduced into the amplifier circuits as a result of the use of alternating current for filament heating. This effect is particularly noticeable in the early stages of the amplifier. Thermal agitation, shot and flicker effects, and everyday tube microphonics each contribute a generous share toward a conglomerate amplifier output voltage.

In general, these disturbing voltages have no appreciable frequency counterpart in the carrier voltage which is injected at the mixer. It is interesting to note that unless the frequency of a disturbing voltage approaches that of the carrier, it induces nearly equal current increases in the rectifiers and their load resistors. The recording device is scarcely affected by such currents because little difference-current is developed.

This desirable property of the Type 1A Strain Indicator makes it possible to obtain good clean records even though the input signal from the bridge is no more than a few microvolts. Also, the use of a bridge-exciting voltage far below that which is ordinarily used is feasible. With this reduction in bridge voltage, heating and distortion of the specimen by electrical-power dissipation in the gages becomes a negligible factor. The importance of this point can be appreciated when one considers the difficulties encountered in the determination of very small strains in a specimen which is continually undergoing a process of dimensional creepage.

The full-wave rectifiers employed in the output stage are copper-oxide contact rectifiers. Selenium rectifiers have also been successfully used. It is recognized that, in general, the nonlinear characteristics of rectifiers of this type impair the accuracy of records obtained with their use. It may be pointed out that in the Type 1A Strain Indicator the output circuit is differentially driven. If reasonable care is used in pairing off these rectifiers, the nonlinear characteristics are canceled, or at least the undesirable effects are greatly reduced and are no longer a considerable factor.

A study of the behavior of the mixer circuit and output stage when the resistance in one or more arms of the bridge circuit is varied in a sinusoidal manner is of particular interest. This condition is often encountered when strain is measured in a structural member that is subjected to vibratory loading.

Two strain gages may be mounted in contraposition on opposite faces of a bar, as shown in Figure 3. Let this bar be set in sustained vibration so that the free end of the bar oscillates in a simple harmonic mode. It is assumed that the bridge circuit was first balanced for both resistance and reactance. Under these conditions the bridge is unbalanced first in one direction and then in the other. The excursions from balance are equal, sinusoidal in nature, and symmetrical about the initial balance point. The wave form of the resultant output voltage from the bridge is shown in Figure 10. The amplified replica of this voltage is impressed on the mixer circuit.

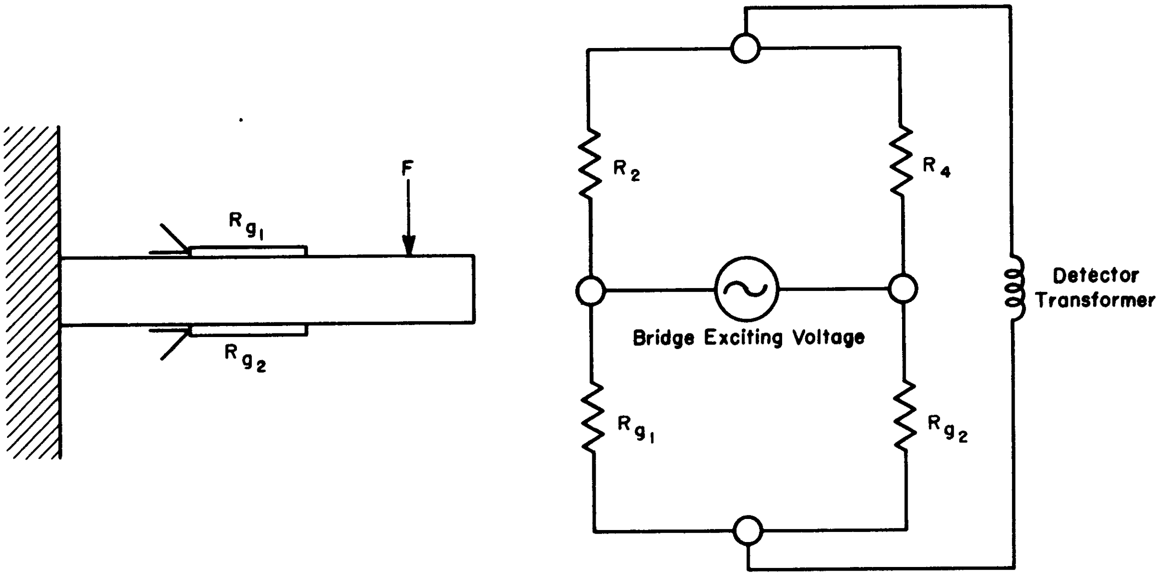


Figure 3 - Strain Gage Connection for Measurement of Bending

It can be shown that the voltage e which is impressed on the mixer circuit by the amplifier is

$$e = E_s k A \epsilon \qquad [1]$$

where $E_s = E \cos \omega_e t$, the bridge exciting voltage,

k is the gage factor,

ϵ is the instantaneous value of the strain in the bar,

A is the voltage gain of the amplifying system,

E is the maximum amplitude of the bridge exciting voltage, and

ω_e is the angular frequency of the bridge exciting voltage.

Since the bar is oscillating in a sinusoidal manner

$$\epsilon = \epsilon_m \cos \omega_m t \qquad [2]$$

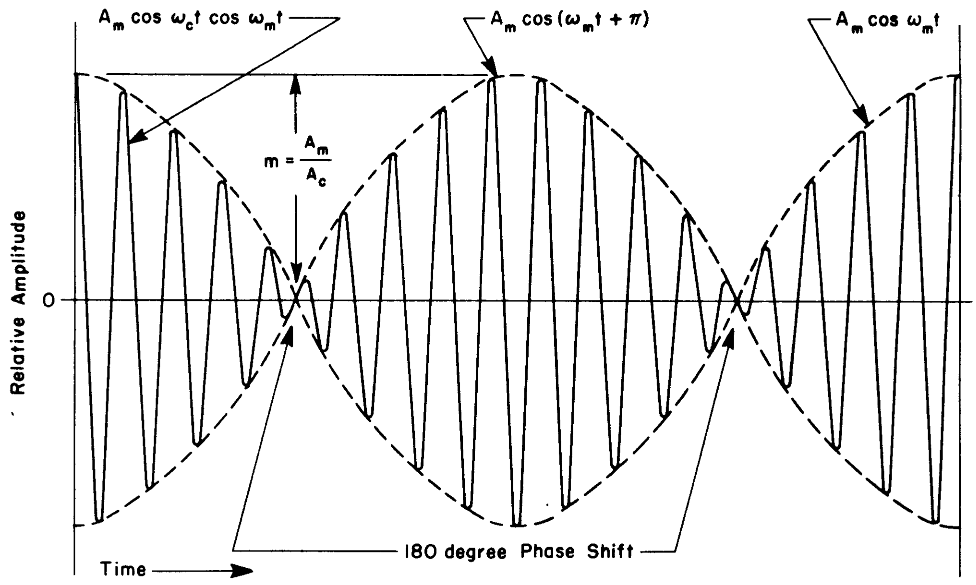


Figure 10 - Wave Form at Output Terminals of Bridge Circuit When Vibratory Strain Is Measured

where ϵ_m is the maximum value of the strain in the bar, and
ω_m is the angular frequency of the bar vibration.

Since k, A, E, and ϵ_m are all constants, let their product be equal to an amplitude factor A_m so that

$$A_m = EkA\epsilon_m$$

Then

$$e = A_m \cos \omega_c t \cos \omega_m t \qquad [3]$$

By trigonometric manipulation, Equation [3] may be rewritten

$$e = \frac{A_m}{2}\left[\cos(\omega_c + \omega_m)t + \cos(\omega_c - \omega_m)t\right] \qquad [4]$$

This expression shows the complete absence of voltages of angular frequencies ω_c and ω_m. The only components present in the wave are two side-band frequenquencies of equal amplitude, symmetrically spaced on a linear frequency spectrum either side of a mid-angular-frequency ω_c.

Injection at this point of another voltage of amplitude A_c and angular frequency ω_c produces a new total voltage e_a. The expression for this voltage may be written

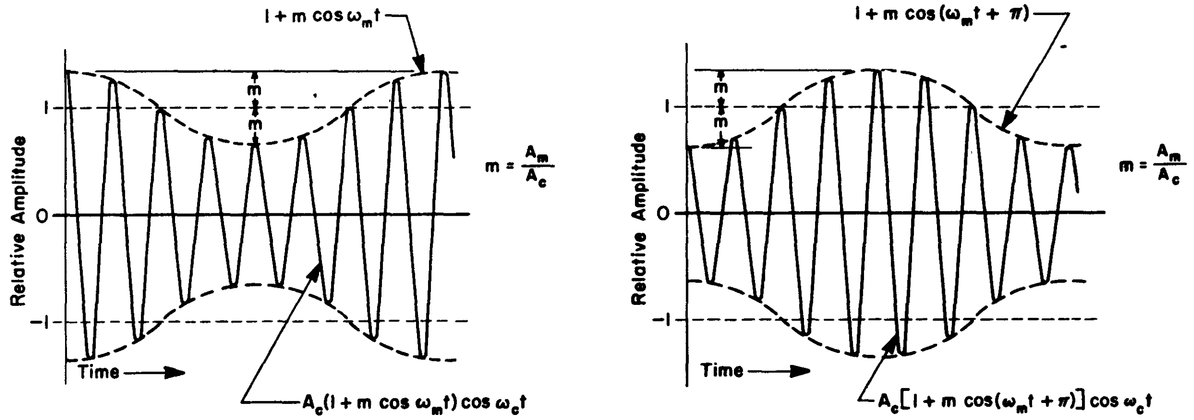


Figure 11a - Wave Form of Effective Driving Voltage of Channel a

Figure 11b - Wave Form of Effective Driving Voltage of Channel b

Figure 11 - Wave Forms of Driving Voltages for Channels a and b

$$e_a = A_c \cos \omega_c t + \frac{A_m}{2} \cos(\omega_c + \omega_m)t + \frac{A_m}{2} \cos(\omega_c - \omega_m)t \qquad [5]$$

Let

$$\frac{A_m}{A_c} = m \text{ so that } A_m = A_c m$$

where m is the degree of modulation or the modulation factor. Then Equation [5] may be rewritten

$$e_a = A_c\left[\cos \omega_c t + \frac{m}{2} \cos(\omega_c + \omega_m)t + \frac{m}{2} \cos(\omega_c - \omega_m)t\right] \qquad [6]$$

By trigonometric rearrangement and simplification

$$e_a = A_c(1 + m \cos \omega_m t) \cos \omega_c t \qquad [7]$$

This equation is the familiar expression for an amplitude-modulated carrier wave which is free from harmonic distortion.

The effective driving voltage e_a for channel a is impressed on the grid of the power amplifier tube V-4a. A similar voltage e_b is impressed on the grid of the power amplifier tube V-4b in channel b. However, the voltage e_b has one point of difference: The modulation envelope is shifted π radians or 180 degrees with respect to the modulation envelope of the voltage e_a. A direct comparison of the wave forms of these two voltages may be made by referring to Figure 11.

As previously stated, the channels a and b operate independently and feed into separate full-wave rectifiers and load resistors. The wave forms of the voltages impressed on these load resistors are shown in Figure 12. The output voltage for channel a may be expressed as

$$e_{0_a} = A_a\left[\frac{2}{\pi} - \frac{4}{\pi}\sum\frac{\cos\beta\omega_c t\cos^2\beta\frac{\pi}{2}}{\beta^2 - 1}\right]\left[1 + m\cos\omega_m t\right] \qquad [8]$$

where β is the order of even harmonics and is successively equal to 2, 4, 6, 8, ···, (1).* Equation [8] may be expanded thus:

$$e_{0_a} = \frac{2A_a}{\pi}\left[1 - \frac{2\cos 2\omega_c t}{1\times 3} - \frac{2\cos 4\omega_c t}{3\times 5} - \frac{2\cos 6\omega_c t}{5\times 7}\cdots\right](1 + m\cos\omega_m t) \qquad [9]$$

The output voltage for channel b may be expressed in a similar manner as

$$e_{0_b} = A_b\left[\frac{2}{\pi} - \frac{4}{\pi}\sum\frac{\cos\beta\omega_c t\cos^2\beta\frac{\pi}{2}}{\beta^2 - 1}\right]\left[1 + m\cos(\omega_m t + \pi)\right] \qquad [10]$$

Equation [10] may be expanded thus:

$$e_{0_b} = \frac{2A_b}{\pi}\left[1 - \frac{2\cos 2\omega_c t}{1\times 3} - \frac{2\cos 4\omega_c t}{3\times 5} - \frac{2\cos 6\omega_c t}{5\times 7}\cdots\right]\left[1 + m\cos(\omega_m t + \pi)\right] \qquad [11]$$

The amplitude coefficients A_a and A_b may or may not be equal, depending on the position of the meter zero-centering control. It will be noted that the wave forms of these two voltages differ again only in the phase relationship of the modulation envelopes. This relationship is readily apparent in Figure 12. The voltage e_0 which is impressed on the recording device is the difference between these two voltages; i.e.,

$$e_0 = e_{0_a} - e_{0_b}$$

In the special case where $A_a = A_b = A_0$

$$e_0 = \left[\frac{2}{\pi} - \frac{4}{\pi}\sum\frac{\cos\beta\omega_c t\cos^2\beta\frac{\pi}{2}}{\beta^2 - 1}\right]A_0 m\left[\cos\omega_m t - \cos(\omega_m t + \pi)\right] \qquad [12]$$

---

* Numbers in parentheses indicate references on page 28 of this report.

$$e_0 = 2mA_0\left[\frac{2}{\pi} - \frac{4}{\pi}\sum \frac{\cos\beta\omega_c t\cos^2\beta\frac{\pi}{2}}{\beta^2 - 1}\right]\cos\omega_m t \qquad [13]$$

The wave form expressed in this equation is shown in Figure 13. Equation [13] may be expanded thus:

$$e_0 = \frac{4mA_0}{\pi}\left[1 - \frac{2\cos 2\omega_c t}{1\times 3} - \frac{2\cos 4\omega_c t}{3\times 5} - \frac{2\cos 6\omega_c t}{5\times 7}\cdots\right]\cos\omega_m t \qquad [14]$$

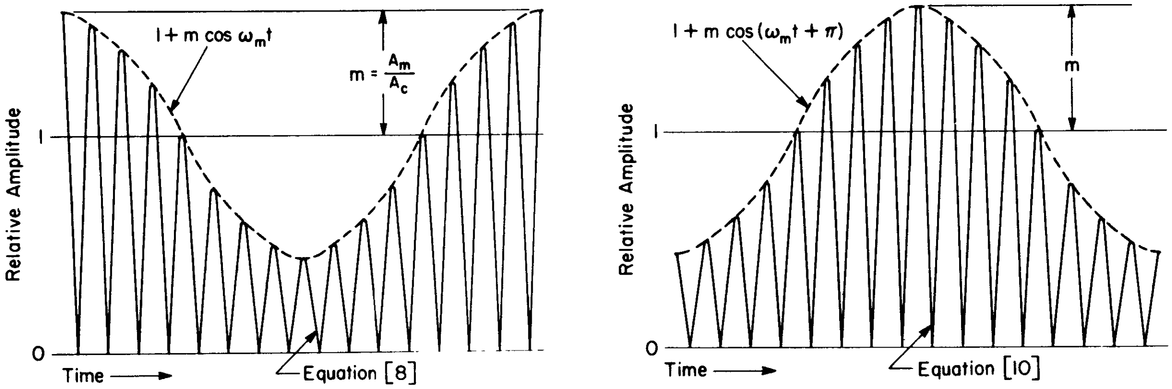


Figure 12a - Wave Form of Output Voltage of Channel a

Figure 12b - Wave Form of Output Voltage of Channel b

Figure 12 - Wave Forms Developed by Dual Demodulator

The indulgence in mathematical exercises, Equations [1] through [14], has for its purpose the focusing of attention on several salient features in this carrier system. It is observed that modulation of the carrier is accomplished by the simple addition of three voltages: The carrier component and two symmetrical side-band components. A voltage of the modulating frequency is not used in the process, nor are conventional tuned circuits or other nonlinear elements employed. Theoretically, the use of this method for modulation results in a perfect modulating operation.

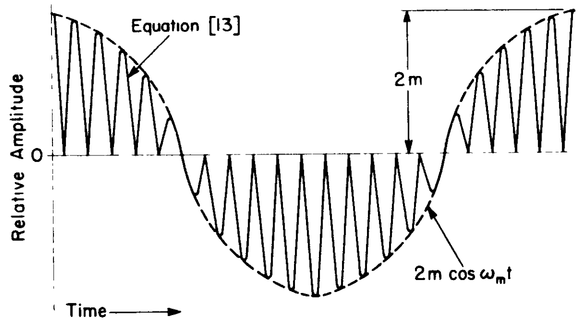


Figure 13 - Wave Form of Equation [13] As Observed at the Output Terminals before Installation of the High-Pass Filter

The dual demodulation of the carrier in the twin-output stage provides for direct cancellation of even-harmonic distortion of the component of angular modulating frequency ω_m. The net result of demodulation by this method is the accomplishment of a distortionless operation, insofar as this frequency is concerned. The truth of this last statement is, of course, dependent on exact symmetry in the electrical characteristics of the component parts which comprise the mixer and output circuits.

The suppression of one side band is known to introduce strong even-harmonic components in a modulated carrier wave. During the development of this carrier system, it was experimentally proved that either the upper or the lower side band can be completely suppressed without noticeable distortion in the output indications. This characteristic is a strong point in favor of this system of demodulation. It is easy to see that almost all of the amplitude distortion arising in the amplifying system is canceled by the differential action of the output stage in the process of demodulating the carrier.

Amplitude distortion as a function of frequency is purposely introduced at the output terminals by the insertion of the high-pass filter in shunt with the recording device; see Figure 5 for the response curve. This filter has no noticeable effect on the dual demodulators but serves only to reduce the response of the recording device at the ripple frequency.

NULL DETECTOR

A very simple experiment establishes the necessity for a null detector as an integral part of this instrument. Assume that the bridge circuit is initially balanced for both resistance and reactance and that the output current meter reads zero. Let a reactance unbalance be introduced in the bridge circuit by rotation of one of the capacitance-balancing controls. The output meter will no longer read zero. The resistance-balancing controls may be adjusted so that the output meter again reads zero without disturbing the capacitance-balancing controls. Now the bridge circuit is unbalanced for both resistance and reactance. Further capacitance unbalance and resistance readjustment can be made until the amplifying system is overloaded, but this condition is not indicated by the output meter.

Needless to say, the instrument is not usable for strain measurements while this condition exists. Attempts to restore the initial balanced condition in the bridge circuit are almost certain to fail unless some indicator other than the output meter is available, because the possible combinations of resistance and reactance unbalance that will induce zero current in the output meter are almost limitless.

The null detector includes a high-gain pentode voltage amplifier which is used in a special reflex circuit. The a-c components appearing at the grid of the mixer-driver tube are impressed by a cathode-follower isolator on the grid of the pentode amplifier tube V-7 as shown in Figure 14. The amplified counterphase voltage at the plate of the pentode is impressed on a half-wave rectifier diode. The d-c component of this rectified voltage is passed through a simple resistance-capacitance filter network and is imposed as a biasing voltage on the grid of the same pentode amplifier tube. This biasing voltage changes the d-c plate current in the tube.

An electron ray indicator tube, which is commonly known as a "magic eye" tube, is employed as the visual indicator. The cathode of the indicator tube is fixed in potential with respect to ground by a bleeder network. This

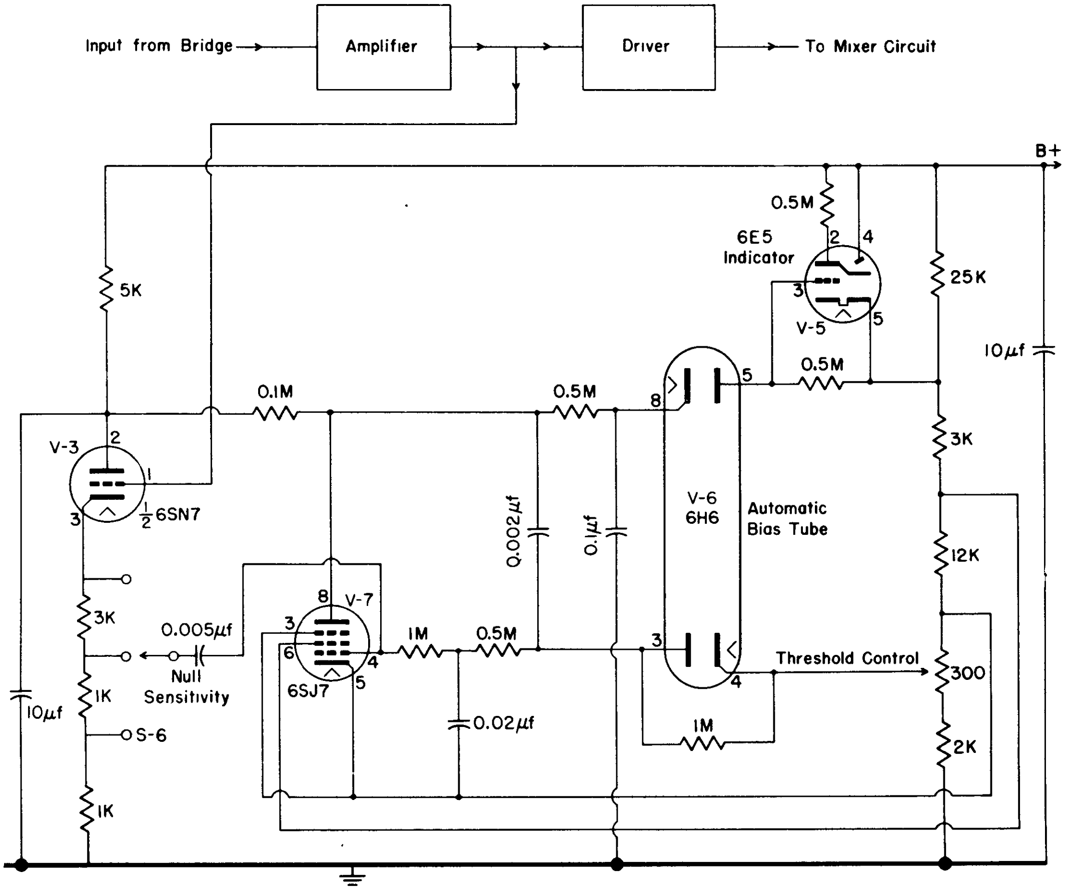


Figure 14 - Null Detector Schematic Circuit

network is apportioned so that this potential is slightly positive with respect to the potential at the plate of the pentode when no signal is impressed on its grid. Application of signal voltage to the grid of the pentode induces a decrease in plate current, as previously mentioned, causing the plate to swing positive with respect to its previous potential position. The initial grid bias on the indicator tube is thereby removed, whereupon the eye on the indicator tube opens. Destructive grid current in the indicator tube is prevented by nonconduction in the connecting diode between the pentode plate and the grid of the indicator tube. When the signal voltage is removed or reduced to zero, by balancing the bridge circuit, for example, the eye closes.

When the bridge is greatly unbalanced, the attenuator in the amplifier may be rotated to the least sensitive position, the sensitivity switch S-6 in the null detector may be moved to "low," and the null threshold control adjusted so as to compel the original potential relation between the pentode plate and the cathode of the indicator tube, with consequent closure of the eye. By suitable adjustments of the bridge-balancing controls and backing off of the null threshold control, improvement in bridge balance is readily made. The attenuator may be rotated for greater sensitivity as this improvement progresses, until full sensitivity is used. Final adjustments are made with the null sensitivity control in the "high" position. The complete circuits, of which Figures 8, 9, and 14 represent parts, are shown in Figures 17 and 18, at the end of the report.

The voltage gain of the reflex amplifier in the null detector together with the voltage gain of the signal amplifier provide an overall voltage gain of about 10 million times. A d-c voltage of 2.5 volts is required to actuate the eye of the indicator tube from "open" to "close." Output voltages from the bridge of 0.05 microvolt are therefore discernible, and bridge balance may easily be accomplished to a high degree of accuracy. Figures 15a and 15b are graphical presentations of the relationship between bridge voltage output and null detector indications.

OSCILLATOR CIRCUIT

After careful consideration of the respective merits of the many types of oscillator circuits which have been developed, the "bootstrap" type of oscillator circuit was chosen for use in the Type 1A Strain Indicator. Published literature (2) dealing with the basic design principles affecting the performance of this oscillator circuit is available, and further technical discussion seems unnecessary.

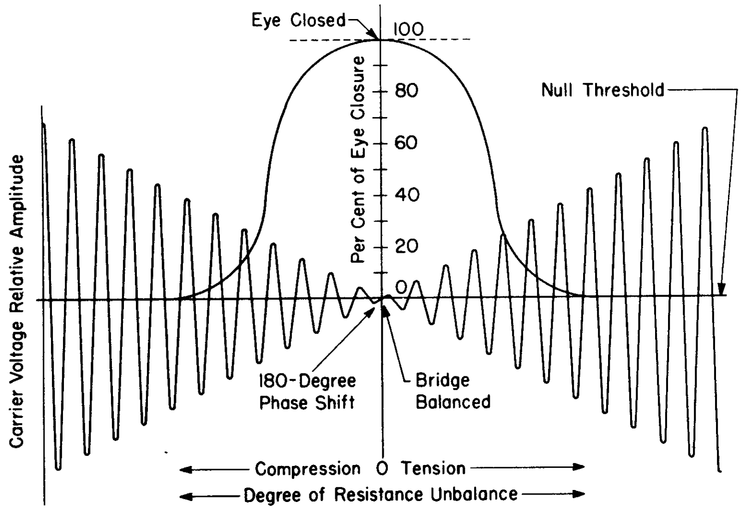


Figure 15a - Null Detector Indication As a Function of Bridge Resistance Unbalance When Stray Capacity Is Balanced

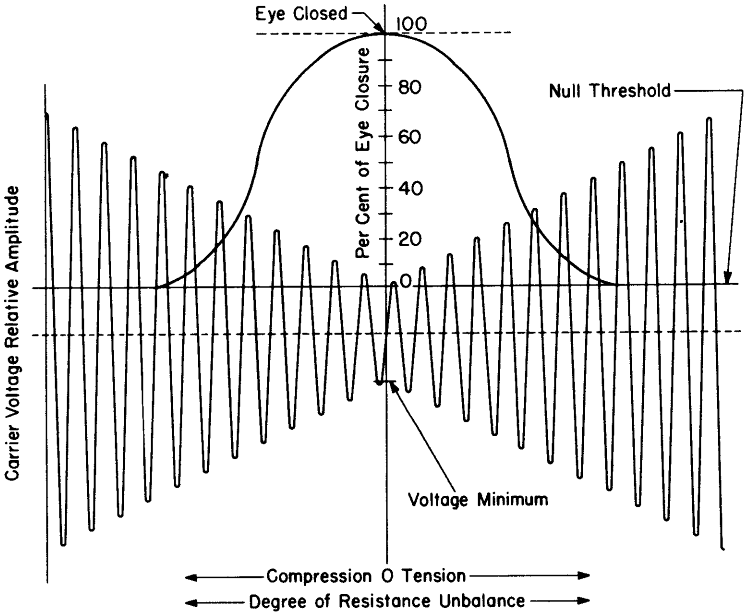


Figure 15b - Null Detector Indication As a Function of Bridge Resistance Unbalance When Stray Capacity Is Not Balanced

Figure 15 - Null Detector Indication as a Function of Bridge Resistance Unbalance

The inherent simplicity in operating principle exhibited by this oscillator circuit was an important factor influencing the choice of this particular circuit. It possesses good frequency stability characteristics and, with only one major modification, was immediately applicable for use in this instrument. The modification just mentioned consists of resistance isolation of the "tank" coil-and-capacitor combination.

It is imperative that the bridge-exciting voltage have good wave form; i.e., it should be essentially free from harmonic distortion because of the well-known fact that a practical a-c bridge circuit cannot be perfectly balanced at fundamental and harmonic frequencies simultaneously. Virtual elimination of harmonic frequency components in the output voltage of the oscillator could be accomplished by imposing a sharply tuned filter between the oscillator and the load; but a serious difficulty immediately arises. If the characteristics of the filter are sharp enough effectively to reduce harmonic terms in its output voltage to negligible quantities, rigid control must be imposed on the frequency of the oscillator; otherwise continual amplitude variations will occur at the output of the filter as the oscillator frequency wanders. Rigid control of the frequency of oscillation is not practical in small instruments of this kind, but the problem was solved by resistance isolation of the "tank" coil-and-capacitor, as previously mentioned. The final circuit, with modifications, is shown in Figure 16. The "tank" serves as a

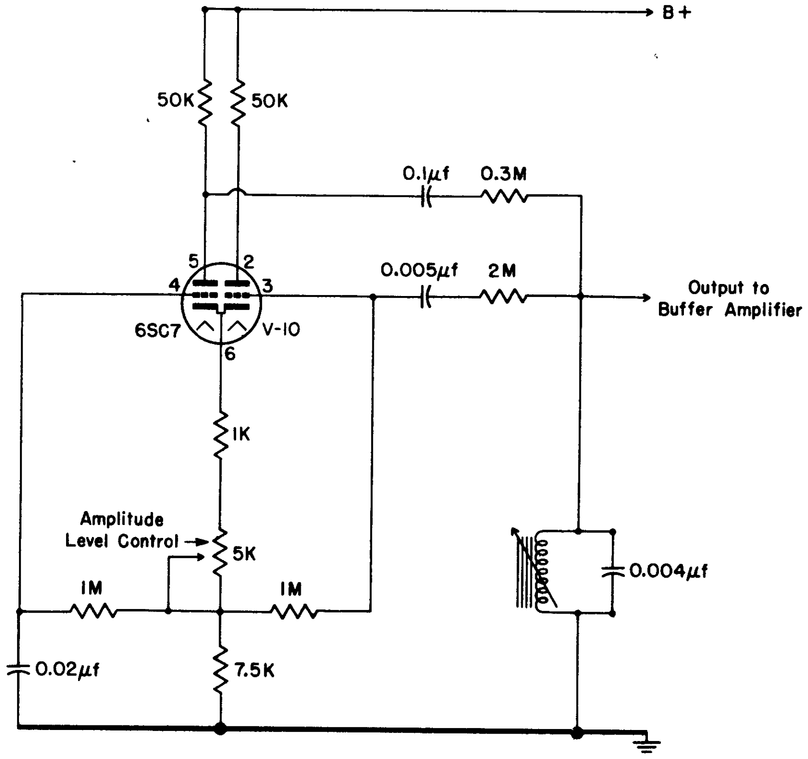


Figure 16 - Oscillator Schematic Circuit

filter, but it simultaneously serves as the frequency-determining element in the oscillator circuit. Since the same element is employed for both purposes, the natural frequency of the filter and the natural frequency of the oscillator necessarily coincide.

Some test conditions require two or more strain indicators to be used on one test specimen with the gages and connecting leads proximate to each other. If each strain indicator operates with its own oscillator, the frequency of which is nearly the same as that of its neighbor's oscillator, spurious modulation caused by "beating" is produced at the respective outputs, proportional to the degree of coupling between the gages and leads.

This undesirable modulation vanishes when all strain indicators involved are excited by a common oscillator. For connection to a common oscillator a switch jack J-4, Figure 17, is provided on the back plate of each instrument. Leads from an accessory oscillator may be plugged into each instrument. When the plug is inserted in the jack, the internal oscillator is disconnected automatically, and connection is made directly to the buffer amplifier.

CONCLUSION

In the last few years many definite improvements have been made in methods and apparatus for measuring and recording strain. These improvements have done much to widen the scope of scientific investigations of the physical properties and behavior of materials and structures. In this march of progress the Type 1A Strain Indicator is serving to assist in the program of removing the guesswork from engineering practice.

Four models of this instrument have been designed and built, all of them based on the design principles discussed in this report. The Type 2A Strain Indicator is a 2-channel unit which was built particularly for use in flight testing in aircraft. The Type 3A Strain Indicator is a single-channel unit with a special output stage designed for electrostatic cathode-ray tube deflection. This model is used for indicating strain in various parts of large engines. With auxiliary equipment, a pattern of strain as a function of piston position is shown on a cathode-ray oscilloscope. The Type 5A Strain Indicator is somewhat similar to the Type 1A, but there are several points of difference between the two. The carrier frequency in the Type 5A is 5000 cycles per second, whereas it is 2200 in the Type 1A; also in the Type 5A the bridge circuit is completely external, a shunt type of calibrating method is used, and the output stage includes dual demodulation filters and dual cathode-follower drivers for the recording device.

An impressive array of difficult strain measurements have been made with these instruments, many of them under adverse field-test conditions. These measurements include the determination and recording of strains in turret structures and gun foundations during training and elevating operations; the recording of strains in model-scale landing craft during beaching operations; in conjunction with special gages, the recording of the pressure in pounds per square inch exerted on the human body by safety harness in plane crashes, in this case deriving important information concerning the nature and magnitude of the forces involved; in conjunction with special dynamometers, the measurement and recording of the towing capacity of marine tugs; in conjunction with special accelerometers, the determination and recording of recoil acceleration in naval gun mechanisms; in conjunction with special blast gages, the recording of the blast pressure about aerial rockets during launching; in conjunction with special resistance types of pressure gages, the measurement and recording of pressures developed in hydraulic recoil chambers on new large-caliber naval guns; and the determination and recording of combined steady and oscillating torque in drive shafts. These and many other measurements of a more or less routine nature have been made with a fairly high degree of accuracy in test results.

REFERENCES

(1) "Über die Gleichrichtung von Strömen" (The Rectification of Currents), by August Hund, Elektrotechnik und Maschinenbau, Vol. 40, No. 4, January 1922, p. 38.

(2) "Cathode-Coupled Oscillators," by F. Butler, Wireless Engineer, Vol. 21, No. 254, November 1944, p. 521.

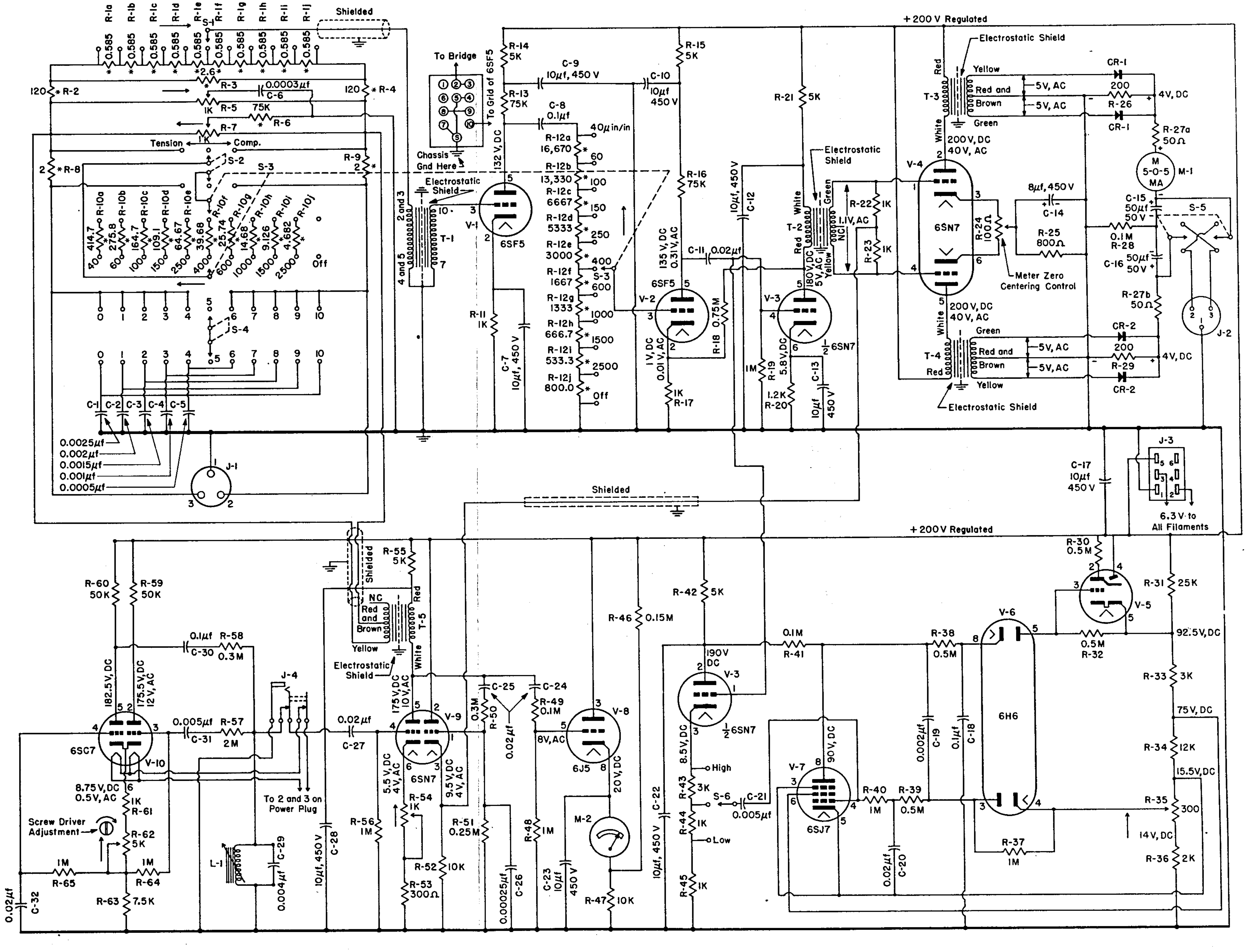
Parts List

Part	Value	Description
R-1a		0.585 ohms, Precision, Shallcross Type BX172 ±1/4%
R-1b		
R-1c		
R-1d		
R-1e		
R-1f		
R-1g		
R-1h		
R-1i		
R-1j		
R-2	120 ohms	Precision, Shallcross Type BX172 ±1/4%
R-3	2.6 ohms	
R-4	120 ohms	
R-5	1000 ohms, Helipot	
R-6	75 K Precision, Shallcross Type BX172	
R-7	1000 ohms, Helipot	
R-8	2 ohms	Precision, Shallcross Type BX172 ±1/4%
R-9	2 ohms	
R-10a	414.7 ohms	
R-10b	275.8 ohms	
R-10c	164.7 ohms	
R-10d	109.1 ohms	
R-10e	64.67 ohms	
R-10f	39.68 ohms	
R-10g	25.74 ohms	
R-10h	14.68 ohms	
R-10i	9.126 ohms	
R-10j	4.682 ohms	
R-11	1000 ohms, IRC, BT-1	
R-12a	16,670 ohms	Precision, Shallcross Type BX172 ±1/4%
R-12b	13,330 ohms	
R-12c	6667 ohms	
R-12d	5333 ohms	
R-12e	3000 ohms	
R-12f	1667 ohms	
R-12g	1333 ohms	
R-12h	666.7 ohms	
R-12i	533.3 ohms	
R-12j	800.0 ohms	
R-13	75 K	IRC, BT-1
R-14	5 K	
R-15	5 K	
R-16	75 K	
R-17	1 K	
R-18	0.75 M	
R-19	1 M	
R-20	1.2 K	
R-21	5 K	
R-22	1 K, IRC, BT-1, matched values	
R-23		
R-24	100 ohms, potentiometer	
R-25	800 ohms, IRC, BT-1	
R-26	200 ohms	IRC, BT-1, matched values
R-29	200 ohms	
R-27a	50 ohms	
R-27b	50 ohms	

Part	Value	Description
R-28	0.1 M	IRC, BT-1
R-30	0.5 M	
R-31	25 K	
R-32	0.5 M	
R-33	3 K	
R-34	12 K	
R-35	300 ohms, potentiometer	
R-36	2 K	IRC, BT-1
R-37	1 M	
R-38	0.5 M	
R-39	0.5 M	
R-40	1 M	
R-41	0.1 M	
R-42	5 K	
R-43	3 K	IRC, BT-1
R-44	1 K	
R-45	1 K	
R-46	0.15 M	
R-47	10 K	
R-48	1 M	
R-49	0.1 M	
R-50	0.3 M	
R-51	0.25 M	
R-52	10 K	
R-53	300 ohms	
R-54	1000 ohms, potentiometer	
R-55	5 K	IRC, BT-1
R-56	1 M	
R-57	2 M	
R-58	0.3 M	
R-59	50 K	
R-60	50 K	
R-61	1 K	
R-62	5 K, potentiometer	
R-63	7.5 K, IRC, BT-1	
R-64	1 M, IRC, BT-1/2	
R-65		
C-1	0.0025 μf	500 v, silvered mica
C-2	0.002 μf	
C-3	0.0015 μf	
C-4	0.001 μf	
C-5	0.0005 μf	
C-6	0.0003 μf	
C-7	10 μf, 450 v, electrolytic	Cornell-Dubilier
C-8	0.1 μf, 600 v, paper	
C-9	10 μf, 450 v, electrolytic	
C-10	10 μf, 450 v, electrolytic	
C-11	0.02 μf, 600 v, paper	
C-12	10 μf, 450 v, electrolytic	
C-13	10 μf, 450 v, electrolytic	
C-14	8 μf, 450 v	electrolytic, Blue Beaver
C-15	50 μf, 50 v	
C-16	50 μf, 50 v	
C-17	10 μf, 450 v, electrolytic	
C-18	0.1 μf	600 v, paper
C-19	0.002 μf	
C-20	0.02 μf	

Part	Description
C-21	0.005 μf, 500 v, silvered mica
C-22	10 μf, 450 v, electrolytic
C-23	
C-24	0.02 μf, 600 v, paper
C-25	
C-26	0.00025 μf, 500 v, silvered mica
C-27	0.02 μf, 600 v, paper
C-28	10 μf, 450 v, electrolytic
C-29	0.004 μf, 500 v, silvered mica
C-30	0.1 μf, 400 v, paper
C-31	0.005 μf, 500 v, mica
C-32	0.02 f, 600 v, paper
T-1	Kenyon bridge transformer
T-2	Audio transformers, 8000 to 500 ohms, Audio Development Co., A-5311
T-3	
T-4	
T-5	
L-1	Choke coil, U.T. Co. VI-C12
S-1	Switch, Centralab K-121, one B section
S-2	Switch, Centralab 1455
S-3	Switch, Centralab K-121, one B section, two J sections
S-4	Switch, Centralab K-121, two J sections
S-5	Switch, DPDT toggle switch, H and H
S-6	Switch, Centralab 1454
J-1	Jack, 3-pin male, Cannon XK-3-14
J-2	Jack, 3-pin male, Cannon X-3-14
J-3	Jack, 6-pin male, Jones P-306-RP
J-4	Jack, circuit breaking, Mallory 705
CR-1	Rectifier, copper oxide, Connant Type M
CR-2	Rectifier, copper oxide, Connant Type M
M-1	Milliameter, 5-0-5 ma, Weston Model 506
M-2	Milliameter, 0-1 ma, Weston Model 506
V-1	6SF5
V-2	6SF5
V-3	6SN7
V-4	6SN7
V-5	6E5
V-6	6H6
V-7	6SJ7
V-8	6J5
V-9	6SN7
V-10	6SC7

The unit of resistance is ohms.
K corresponds to a multiplying factor of 10^3.
M corresponds to a multiplying factor of 10^6.



* Indicates Precision Resistors
→ Indicates Clockwise Motion
NC Indicates No Connection

Figure 17 - Schematic Circuit Diagram of Type 1A Strain Indicator

Parts List

R-1	0.1 M, 1 w	
R-2	0.1 M, 1 w	
R-3	0.5 M, 1/2 w	
R-4	15 K (approx.), 1 w	
R-5	1 M, 1/2 w	
R-6	50 K, 1 w	
R-7	50 K, 1 w	
R-8	6 K, 10 w	
R-9	0.1 M, 1 w	
R-10	0.1 M, 1 w	
R-11	0.5 M, 1/2 w	
R-12	15 K (approx.), 1 w	
R-13	1 M, 1/2 w	
R-14	50 K, 1 w	
R-15	50 K, 1 w	
R-16	6 K, 10 w	
C-1	4 μf, 600 v,	Dykanol Cornell-Dubilier TLA 6040
C-2	4 μf, 600 v,	
C-3	4 μf, 600 v,	
C-4	4 μf, 600 v,	
C-5	0.5 μf, 600 v,	Dykanol Cornell-Dubilier DYR 6050
C-6	0.5 μf, 600 v,	
C-7	20 μf, 450 v,	electrolytic, Cornell-Dubilier BR 2045
C-8	20 μf, 450 v,	
L-1	8 h, 150 ma, Thordarson T-13C30	
L-2		
L-3		
T-1	Thordarson power transformer T-13R16	
T-2	Thordarson filament transformer T-19F81	
T-3	Thordarson filament transformer T-19F98	
T-4	Thordarson filament transformer T-19F81	
T-5	Thordarson filament transformer T-19F98	
J-1	Chassis receptacle, male, Hubbell 6808	
J-2	6-pin female receptacle, Jones S-306-FP	
J-3	6-pin female receptacle, Jones S-306-FP	
S-1	SPST toggle switch, H and H	
F-1	5-amp fuse, Littlefuse	

V-1	5Z4
V-2	5Z4
V-3	6B4-G
V-4	6SJ7
V-5	VR-105/30
V-6	6B4-G
V-7	6SJ7
V-8	VR-105/30

Pilot Light 6.3 v, 150 ma
General Electric Mazda 47

The unit of resistance is ohms.
K corresponds to a multiplying factor of 10^3.
M corresponds to a multiplying factor of 10^6.

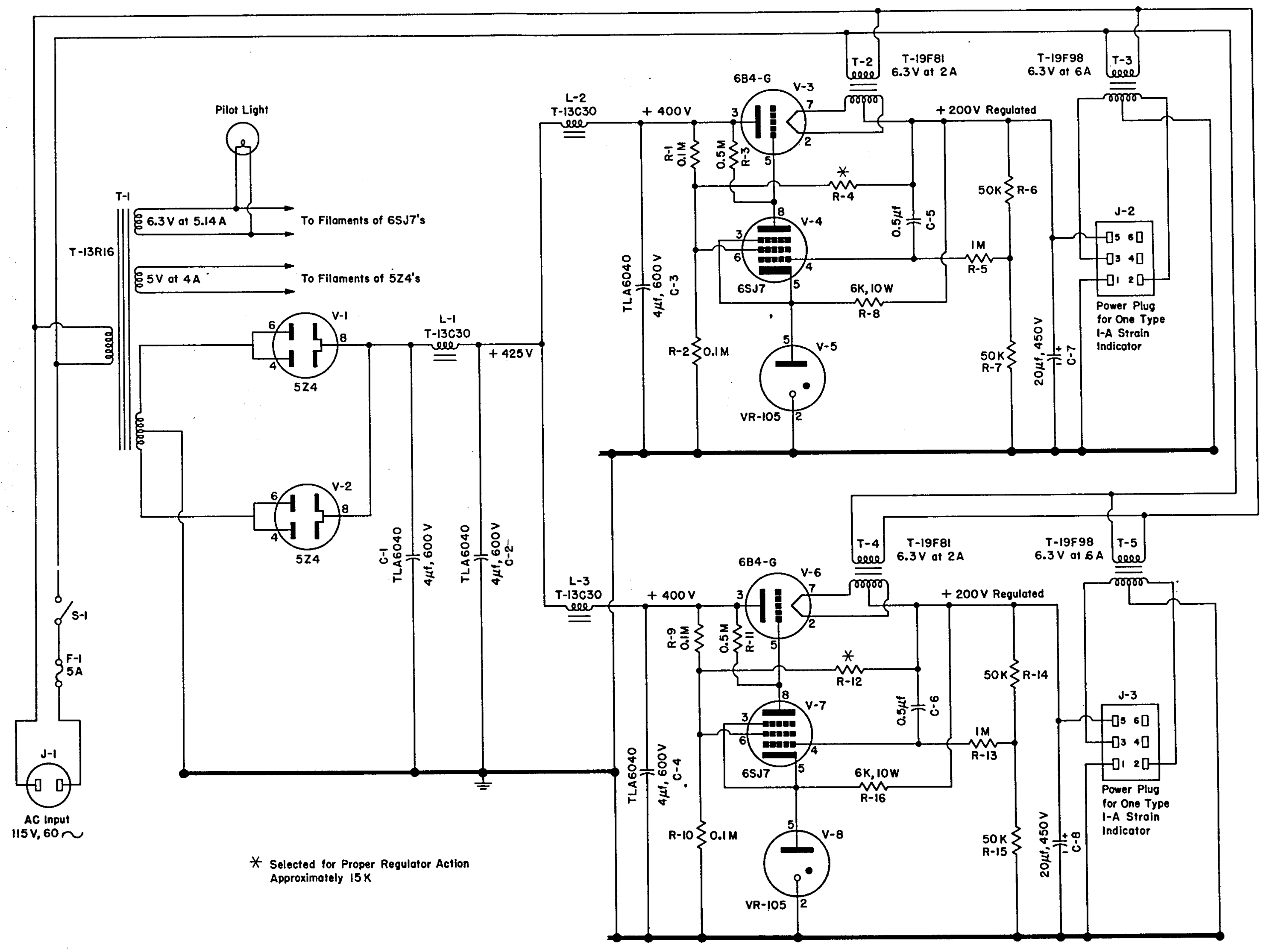


Figure 18 - Schematic Circuit Diagram of Power Supply Unit