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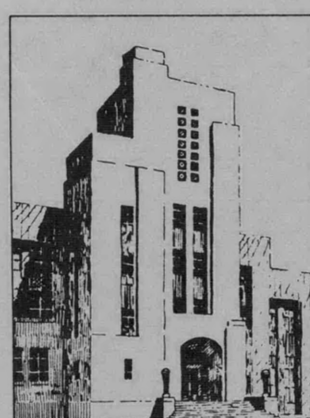
STRESSES AND DEFLECTIONS IN FLAT RECTANGULAR PLATES UNDER DYNAMIC LATERAL LOADS BASED ON LINEAR THEORY

by

Joshua E. Greenspon



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NOTATION

A	Maximum deflection parameter
A_b	Cross-sectional area of beam
A_p	Area of plate
a	Width of plate
B	Maximum stress parameter
b	Length of plate
b/a	Aspect ratio
c	Distance of a given fiber from the neutral axis of a beam
$D = \dfrac{Eh^3}{12(1-\nu^2)}$	Plate modulus
dA	Differential element of area
E	Modulus of elasticity of plate material
F	Natural frequency in cps
$f(t)$	Time load distribution (known as shape of pulse)
G	Space load distribution factor such that $P = GF(t)$
h	Thickness of plate
I	Moment of inertia of beam with respect to neutral axis
K	Frequency number for plate (involves mode and aspect ratio)
L	Load factor (maximum value of response factor R_m)
M_n, M_{ns}	Moment resultants (have dimensions of moment per unit length)
N_n, N_{ns}, Q_n	Force resultants (have dimensions of force per unit length)
n	Distance in direction normal to boundary of a flat plate of arbitrary shape (has dimensions of length) (n lies in plane of plate)
P	Lateral load acting on plate (a function of space and time) (in pressure units)
P_0	Maximum value (in time) of uniformly distributed load
p_r	Circular frequency of rth mode of vibration
q_m	A function of time such that $w = w_m q_m$ satisfies the plate equation $D\nabla^4 w + \rho h \dfrac{\partial^2 w}{\partial t^2} = P(x, y, t)$
$R_m(t)$	Response factor
s	Distance in direction of boundary of a flate plate of arbitrary shape (has dimensions of length)

T	Natural period of vibration
t	Time variable
t_0, t_1	Times of application of pulses
V_n	Shearing force resultant (has dimensions of load per unit length)
w	Lateral deflection
w_r	Deflection function in rth mode of vibration
X_i, Y_j	Normal mode functions for the modes of vibration of a beam
α_i, α_j	Factors defining modes of vibration of a beam
β_i, β_j	Frequency numbers for the modes of vibration of a beam
γ	Static deflection parameter as defined in Table 1
δ	Static stress parameter as defined in Table 1
ν	Poisson's ratio
ρ	Mass per unit volume of plate material
σ_n	Normal stress
σ_x	Stress in beam
τ_{ns}, τ_{nz}	Shear stresses
∇^4	Differential operator $\left(\frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 dy^2} + \frac{\partial^4}{\partial y^4}\right)$ in rectangular coordinates
$\frac{\partial w}{\partial n}$	Slope of plate boundary

ABSTRACT

Approximate formulas are given for the determination of natural frequencies and maximum deflections and stresses in rectangular plates subjected to normal transient loading as well as to static loading. The stresses and deflections resulting from dynamic loads are obtained by the application of a "load factor" to the values computed for a static load. The appendixes give the small-deflection theory of forced vibration of plates with approximate solutions for plates with edges which are clamped, simply supported, or free. It is assumed that the plate deflection may be expressed as a series, each term having the form of the normal-mode function of a uniform beam.

INTRODUCTION

The David Taylor Model Basin is studying the effects of wave forces on the bows of ships[1] to gain information on which to base improved design criteria. This leads to questions concerning the effect of these dynamic forces on the bottom plating of ships. The published technical literature lacks specific procedures and formulas which would be of direct utility to the designer in computing the deflections and stresses in plates under impact loading. Application of available material to the design of plate panels subjected to impact loads requires extensive calculation before one can obtain numerical answers.

It is the purpose of this report to present approximate methods and formulas for the calculation of natural frequencies, deflections, and stresses in plates in a form suitable for direct application by the designer. For those who are interested in the theory and in the derivation of the approximate methods, there is an appendix containing the small deflection theory of forced vibration of plates together with an approximate solution of the problem. This solution is obtained by approximating the deflection of the plate in terms of the normal modes of lateral vibration of a simply supported, clamped, or free uniform beam. For design purposes, stresses and deflections are obtained by the application of "load factors"* to the magnitudes computed under the assumptions of the static application of the maximum load. Numerical examples illustrate the use of the methods. The approximate solutions given here are compared with known exact solutions in order to give some idea as to the degree of approximation. The effects of virtual mass and damping are neglected.

[1]References are listed on page 21.

*The load factor is defined as the maximum numerical value of the response factor. The response factor is that factor by which the static response in each mode must be multiplied to give the dynamic response.

PROPOSED DESIGN PROCEDURE FOR SIMPLY SUPPORTED OR CLAMPED RECTANGULAR PLATES UNDER UNIFORM IMPULSIVE PRESSURE

The recommended procedure for obtaining natural frequency, maximum deflection, and maximum stress in a plate subjected to uniform impact loading is as follows:

1. The *natural frequency* of the plate in Figure 1 is given by the general formula

$$F = \frac{K}{2\pi a^2}\sqrt{\frac{D}{\rho h}}$$

where K is the frequency number (given in Table 1 for several modes of simply supported and clamped rectangular plates) which is a function of aspect ratio and mode,

ρ is the mass per unit volume of the plate material, and

$$D = \frac{Eh^3}{12(1-\nu^2)}$$

where E is the modulus of elasticity and ν is Poisson's ratio for the plate material.

For a steel plate with $E = 30 \times 10^6$ psi, $\nu = 0.30$,

$$\rho = \frac{0.284 \text{ lb-sec}^2}{386 \text{ in.}^4}, \quad F = 9730\,\frac{Kh}{a^2} \text{ cps}$$

where h and a are measured in inches.

For an aluminum plate with $E = 10 \times 10^6$ psi, $\nu = 0.33$,

$$\rho = \frac{0.098 \text{ lb-sec}^2}{386 \text{ in.}^4}, \quad F = 9570\,\frac{Kh}{a^2} \text{ cps}$$

where h and a are measured in inches.

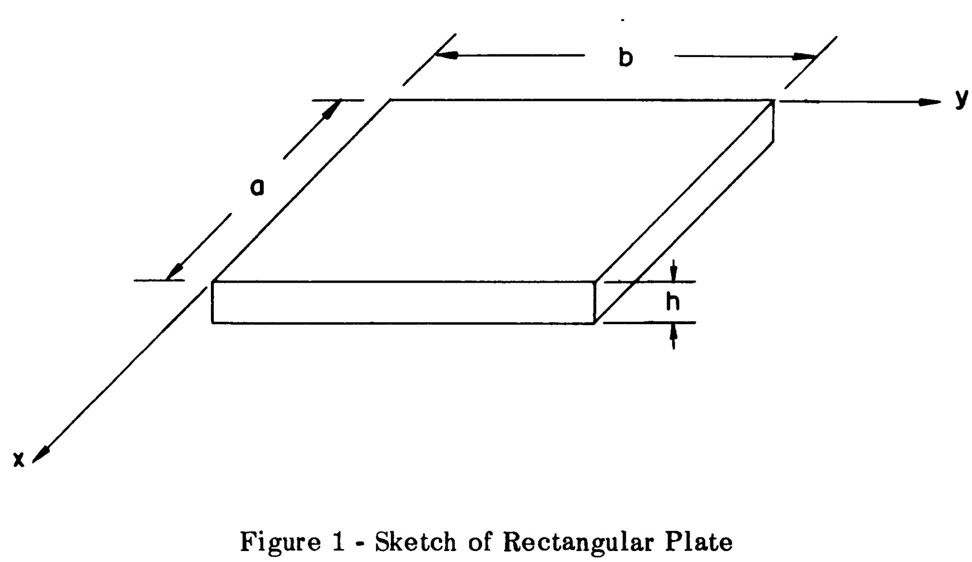

Figure 1 - Sketch of Rectangular Plate

2. The *maximum deflection* w_{max} in either a fixed or simply supported plate occurs at the center. An approximate upper limit for its value is computed* as follows, assuming that the maximum contributions of the first four terms are additive:

$$w_{max} \approx \frac{P_0 a^4}{E h^3} [A_1 L_1 + A_2 L_2 + A_3 L_3 + A_4 L_4]$$

where A_1, A_2, A_3, and A_4 are tabulated in Table 1, and L_1, L_2, L_3, and L_4 are load factors for modes corresponding to A_1, A_2, A_3, A_4 to be read off one of the curves given in Figure 2.

Column 15 of Table 1 gives an approximate value of the maximum deflection for static load application.

3. The *maximum tensile* or *compressive stress* occurs at the surfaces of a plate. Under uniformly distributed loading; it is located at the center of the plate in a simply supported plate and at the middle of the long side in a fixed plate. In both cases, the maximum tensile or compressive stress is in the direction of the short side. In a simply supported plate, there is a compressive stress on the top surface (where the load is applied) at the center, but in a clamped plate, there is a tensile stress on the top surface at the edge. An upper limit for the maximum stress is computed as follows, assuming that the maximum contributions of the first four terms are additive:

$$\sigma_{max} \approx \frac{6}{h^2} P_0 a^2 [B_1 L_1 + B_2 L_2 + B_3 L_3 + B_4 L_4]$$

where B_1, B_2, B_3, and B_4 are tabulated in Table 1, and L_1, L_2, L_3, and L_4 are dynamic load factors to be read off one of the curves given in Figure 2.

Column 16 of Table 1 gives an approximate value of the maximum stress for static load application.

The plates considered in Table 1 are assumed to be fixed on all edges or simply supported on all edges. Tables 3 and 4 contain calculations for plates with other boundary conditions. The use of the first 14 columns of Table 1 is explained in the procedure. Columns 15 and 16 give the theoretical static response based on Equations [17] and [21] of Appendix 1.

In the expressions for w_{max} and σ_{max} in the proposed design procedure, it was assumed that the vibratory modes were all in phase; therefore the results obtained by using these expressions should always give maximum deflections or stresses higher than the values obtained by computing the response as a function of time. For example, if the load factor is assumed to be unity (which is the case for static loading), then the deflection or stress would be ob-

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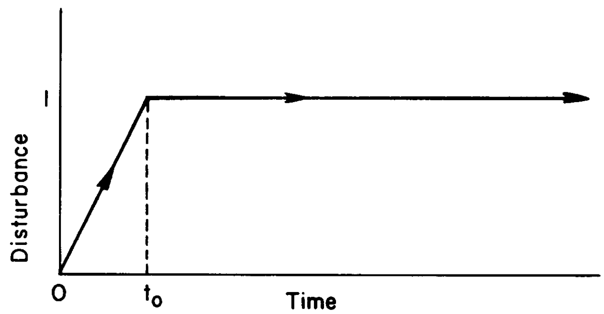
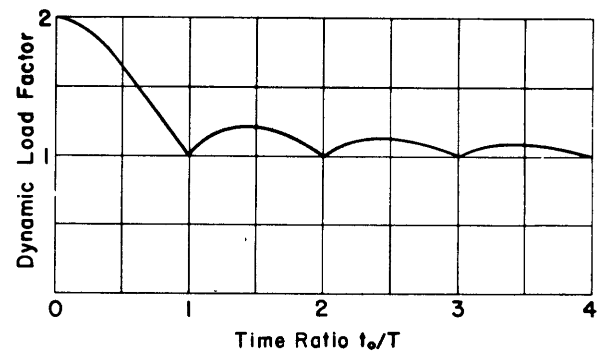
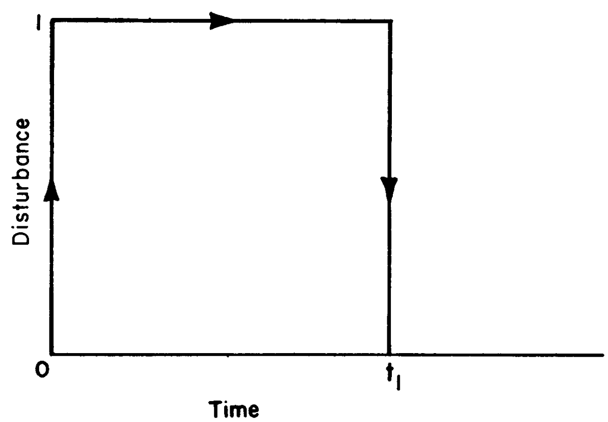
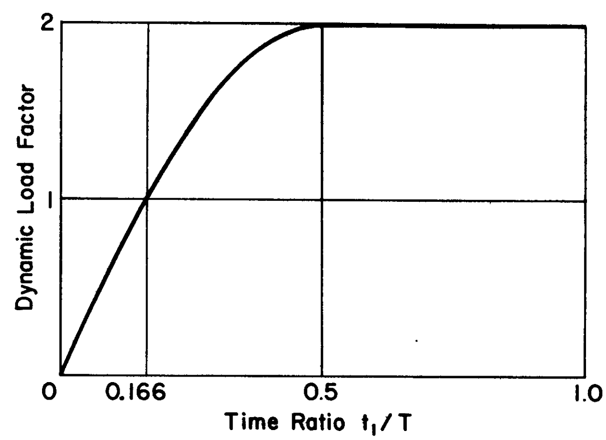
*Warburton[2] has a procedure for calculating the natural frequencies of rectangular plates also based on the assumption that the modes can be represented by beam functions. He uses the Rayleigh method for his computations and gives an account of the limitations of assuming that the modes are composed of a single product of beam functions. However, he does not consider the calculation of deflections and stresses.



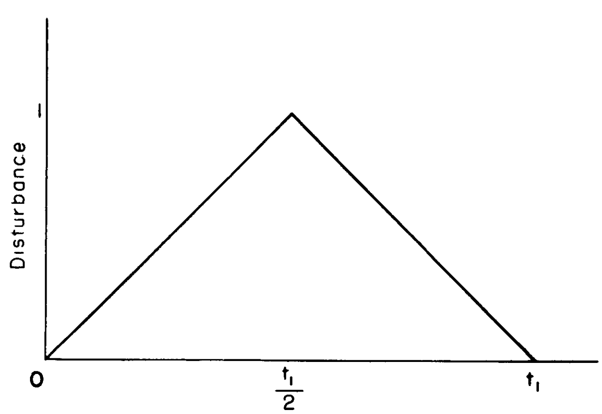


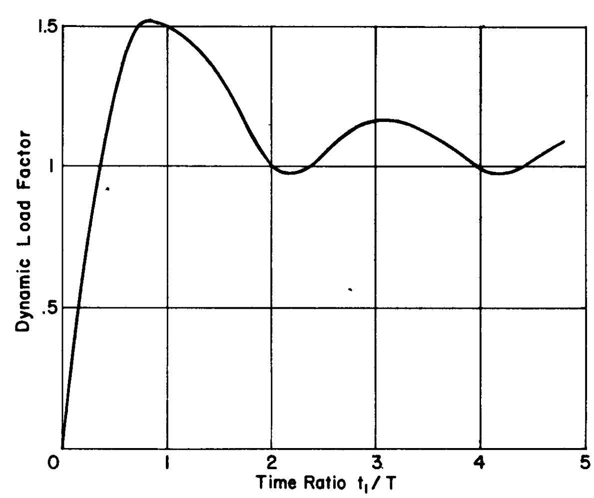
Load Applied Gradually and Maintained Indefinitely



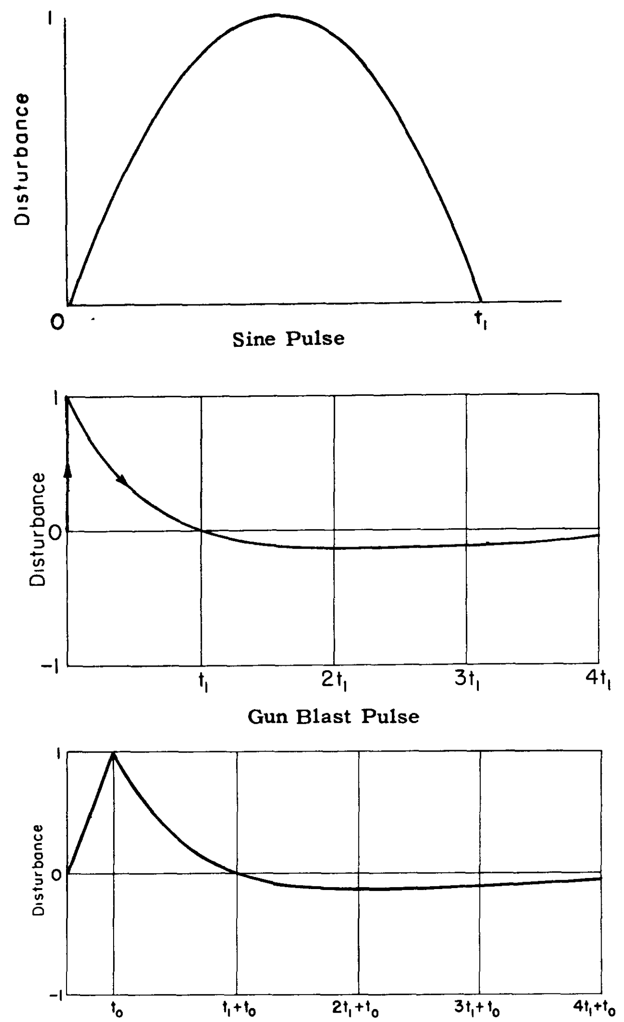


Rectangular Pulse





Triangular Pulse



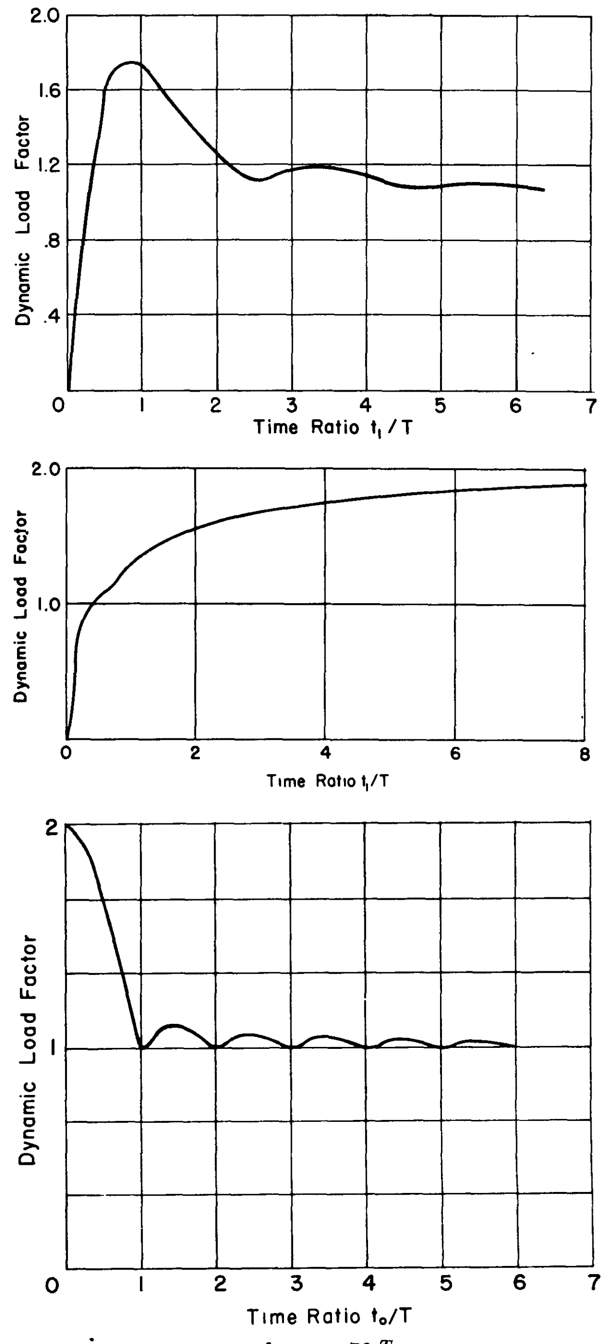


Figure 2 - Pulse Shapes and Corresponding Load Factor Curves

The procedure for obtaining the load factor for any mode of vibration is as follows:

1. Compute the natural frequency for that mode in cps; see p 3.
2. Invert this to obtain the period, T, for that mode.
3. For a given shape and duration of pulse, calculate t_1/T or t_0/T (where t_0 or t_1 is the time duration as shown in the pulse curves) and use the appropriate curve to determine the load factor for the mode of period T

TABLE 1

Frequencies, Deflections, and Stresses in Simply Supported and Clamped Rectangular Plates under Uniform Impulsive Load ($\nu = 0.3$)

(Note that only the symmetrical modes are excited in this case).

$\frac{b}{a}$	First Symmetrical Mode $w = X_1Y_1$			Second Symmetrical Mode $w = X_1Y_3$			Third Symmetrical Mode $w = X_1Y_5$			Fourth Symmetrical Mode $w = X_3Y_1$			First Antisym Mode $w=X_1Y_2$	Second Antisym Mode $w=X_2Y_2$	Static Values: Deflection w, $w=\gamma\frac{P_0a^4}{Eh^3}$, $\gamma = A_1 - A_2 + A_3 - A_4$	Static Values: Stress σ, $\sigma = \delta\frac{6P_0a^2}{h^2}$, $\delta = B_1 - B_2{}^* + B_3 \pm B_4$
	K	A_1	B_1	K	A_2	B_2	K	A_3	B_3	K	A_4	B_4	K	K	γ	δ
SIMPLY SUPPORTED PLATE																
1.0	19.70	0.0454	0.0533	99.0	0.0006	0.0020	256.6	0.00005	0.0004	99.0	0.00060	0.0051	49.2	79.0	0.0443	0.0466
1.2	16.70	.0632	.0691	71.6	.0011	.0030	181.2	.00011	.0006	95.5	.00064	.0053	37.2	66.8	.0616	.0614
1.4	14.86	.0796	.0830	55.2	.0019	.0041	135.8	.00019	.0008	93.8	.00067	.0056	30.0	59.5	.0772	.0741
1.6	13.70	.0939	.0948	44.6	.0030	.0055	106.3	.00032	.0011	92.5	.00068	.0056	25.3	54.8	.0905	.0848
1.8	12.90	.1061	.1047	37.4	.0042	.0070	86.0	.00048	.0014	91.8	.00069	.0057	22.0	51.5	.1017	.0934
2.0	12.30	.1163	.1130	32.0	.0057	.0086	71.6	.00069	.0018	91.3	.00070	.0058	19.7	49.2	.1106	.1004
2.2	11.90	.1248	.1198	28.2	.0074	.0104	60.9	.00096	.0022	90.6	.00071	.0058	18.0	47.5	.1177	.1058
2.4	11.58	.1319	.1254	25.3	.0092	.0122	52.7	.00127	.0027	90.3	.00072	.0059	16.7	46.2	.1233	.1100
2.6	11.30	.1379	.1301	23.0	.0111	.0140	46.4	.00165	.0031	90.1	.00072	.0059	15.7	45.3	.1277	.1133
2.8	11.10	.1429	.1341	21.2	.0131	.0159	41.4	.00207	.0037	90.0	.00072	.0059	14.9	44.5	.1312	.1160
3.0	10.95	.1472	.1374	19.8	.0151	.0177	36.2	.00255	.0042	89.7	.00073	.0060	14.2	43.8	.1339	.1179
4.0	10.46	.1609	.1482	15.4	.0247	.0261	25.3	.00554	.0074	89.4	.00074	.0060	12.4	41.9	.1410	.1235
5.0	10.25	.1680	.1536	13.4	.0326	.0327	19.7	.00908	.0107	89.0	.00074	.0061	11.4	41.0	.1437	.1255
6.0	10.12	.1720	.1567	12.3	.0386	.0375	16.8	.01258	.0137	88.8	.00074	.0061	11.0	40.5	.1452	.1268
7.0	10.01	.1750	.1588	11.7	.0431	.0410	14.9	.01601	.0167	88.7	.00074	.0061	10.7	40.2	.1472	.1284
8.0	10.00	.1760	.1601	11.3	.0465	.0438	13.6	.01905	.0192	88.7	.00074	.0061	10.5	40.0	.1478	.1294
9.0	9.99	.1773	.1609	11.0	.0491	.0459	12.9	.02124	.0210	88.7	.00074	.0061	10.4	39.9	.1483	.1299
10.0	9.95	.1780	.1610	10.7	.0509	.0472	12.3	.02326	.0226	88.7	.00074	.0061	10.3	39.8	.1496	.1303
CLAMPED PLATE																
1.0	36.00	0.0146	0.0376	132.5	0.0004	0.0011	310.0	0.00005	0.00012	132.5	0.00042	.0066	73.8	109.0	.0138	.0432
1.2	30.80	.0199	.0515	96.1	.0008	.0021	219.0	.00010	.00026	128.7	.00045	.0070	56.0	92.5	.0188	.0567
1.4	28.00	.0242	.0624	74.5	.0013	.0034	164.0	.00017	.00045	126.5	.00046	.0073	45.5	83.3	.0226	.0668
1.6	26.35	.0273	.0705	60.6	.0020	.0052	129.0	.00028	.00073	125.1	.00047	.0074	39.0	77.6	.0251	.0734
1.8	25.35	.0296	.0764	51.5	.0028	.0072	105.0	.00043	.00111	124.2	.00048	.0075	34.8	73.8	.0268	.0778
2.0	24.60	.0313	.0807	45.0	.0036	.0094	87.5	.00061	.00158	123.5	.00048	.0076	32.8	71.5	.0278	.0805
2.2	24.15	.0325	.0839	40.3	.0045	.0117	75.0	.00084	.00215	123.0	.00049	.0076	30.0	69.5	.0284	.0820
2.4	23.80	.0334	.0863	36.9	.0054	.0139	65.6	.00109	.00282	122.7	.00049	.0077	28.5	67.5	.0286	.0829
2.6	23.58	.0341	.0881	34.4	.0062	.0161	58.4	.00138	.00357	122.4	.00049	.0077	27.4	67.1	.0288	.0833
2.8	23.35	.0347	.0896	32.4	.0070	.0181	52.6	.00169	.00437	122.2	.00049	.0078	26.6	66.4	.0289	.0837
3.0	23.20	.0352	.0907	30.8	.0078	.0200	48.0	.00203	.00525	122.0	.00049	.0078	25.9	65.6	.0289	.0838
4.0	22.80	.0364	.0940	26.6	.0104	.0268	35.3	.00376	.00971	121.5	.00050	.0079	24.2	63.8	.0293	.0848
5.0	22.60	.0370	.0955	24.9	.0119	.0306	30.0	.00523	.01348	121.3	.00050	.0079	23.5	63.0	.0298	.0863
6.0	22.50	.0373	.0963	24.0	.0127	.0328	27.4	.00627	.01618	121.2	.00050	.0079	23.1	62.5	.0304	.0876
7.0	22.46	.0375	.0968	23.6	.0133	.0343	25.8	.00706	.01821	121.1	.00050	.0079	22.9	62.3	.0308	.0886
8.0	22.42	.0376	.0971	23.2	.0136	.0352	24.8	.00761	.01964	121.1	.00050	.0079	22.7	62.1	.0311	.0894
9.0	22.40	.0377	.0973	23.0	.0139	.0358	24.2	.00794	.02050	121.0	.00050	.0079	22.6	62.0	.0312	.0899
10.0	22.40	0.0378	0.0974	22.8	0.0141	0.0364	23.8	0.00822	0.02120	121.0	0.00050	0.0079	22.6	61.9	0.0314	0.0901
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

*Positive sign in the case of the clamped plate and negative sign in the case of the simply supported plate.

tained, according to the proposed design procedure, by adding A_1, A_2, A_3, A_4 in the case of deflection and B_1, B_2, B_3, B_4 in the case of stress. The four-term approximation to the response should be adequate for pulses in which the first mode makes the primary contribution to the deflection and stress. It can be shown that the results obtained by adding these amplitudes are in most cases greater than the static results obtained from the type of solution assumed here. For a few cases, however (for example, the stress for a fixed plate with aspect ratios of 1, 1.2) the more exact solution given in Reference 4 gives greater values than are obtained by summation of the first four terms given in columns 3, 6, 9, and 12 of Table 1.

The following example is presented to illustrate the procedure.

Calculate the frequency of the first four symmetrical modes, the estimated maximum tensile stress, and the maximum deflection in a plate subjected to a triangular pulse load of 0.005-sec duration. The 24- by 48- by ½-in. plate is made of steel and has fixed edges. The magnitude of the load is 15 psi.

Natural Frequencies (for $\frac{b}{a} = 2$):

First Symmetrical Mode $F_1 = \frac{9730 \times 0.5}{(24)^2} \times 24.60 = 208$ cps

Second Symmetrical Mode $F_2 = \frac{9730 \times 0.5}{(24)^2} \times 45.00 = 380$ cps

Third Symmetrical Mode $F_3 = \frac{9730 \times 0.5}{(24)^2} \times 87.50 = 739$ cps

Fourth Symmetrical Mode $F_4 = \frac{9730 \times 0.5}{(24)^2} \times 123.00 = 1043$ cps

Natural Periods:

$T_1 = 0.0048$ sec hence $\frac{t_1}{T_1} = \frac{0.0050}{0.0048} = 1.04$

$T_2 = 0.0026$ sec $\frac{t_1}{T_2} = \frac{0.0050}{0.0026} = 1.92$

$T_3 = 0.0014$ sec $\frac{t_1}{T_3} = \frac{0.0050}{0.0014} = 3.57$

$T_4 = 0.0010$ sec $\frac{t_1}{T_4} = \frac{0.0050}{0.0010} = 5.00$

Load Factors:

Referring to Figure 2, the following load factors are obtained:

$L_1 = 1.5$

$L_2 = 1.0$

$L_3 = 1.1$

$L_4 = 1.1$

Maximum Deflection:

$$w_{max} = \frac{P_0 a^4}{Eh^3}[0.0313 \times 1.5 + 0.0036 \times 1.0 + 0.00061 \times 1.1 + 0.00048 \times 1.1]$$

$$= \frac{15 \times (24)^4}{30 \times 10^6 \times (0.5)^3} \times 0.0517 = 0.0686 \text{ in.}$$

Maximum Stress:

$$\sigma_{max} = \frac{6}{h^2} P_0 a^2 \; [0.0807 \times 1.5 + 0.0094 \times 1.0 + 0.00158 \times 1.1 + 0.0076 \times 1.1]$$

$$= \frac{6 \times 15 \times (24)^2}{(0.5)^2} \times 0.1405 = 29{,}134 \text{ psi*}$$

EVALUATION OF ACCURACY OF PROPOSED METHOD

As there were no experimental data or exact theoretical values with which to compare the dynamic deflections and stresses, the static values are compared in Table 2 with those taken from Reference 4. The frequencies are compared with values obtained from Reference 5.

Table 3 gives the formulas for the frequency numbers for the first mode for a number of plates including the ones considered in Table 1. The results are compared with those obtained from Reference 5.

Table 4, like Table 3, gives results for other plates beside those given in Table 1. The cases considered in Table 1 are included as Cases 1 and 2. The results in Table 4 were computed assuming that the total deflection and stress is represented by the first mode. Again only the uniform load is considered, and results are compared with those taken from Reference 4.

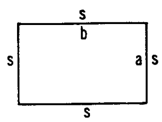
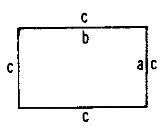
DISCUSSION

The methods developed in the appendixes provide the means for computing the responses of rectangular plates for a great number of cases. Table 1 may be utilized for the specific case of the simply supported or clamped plate under uniform impulsive load. By the use of this table, the designer will be able to obtain approximate frequencies, deflections, and stresses in a matter of minutes. A great amount of work plus a good knowledge of plate theory would be necessary if the designer desired to compute precise answers to his dynamic design problems. When finished, he would then probably apply a safety factor to his results. It is true that the methods given here do not give an exact solution to the problem; however, they permit an estimate of the responses and frequencies of plates under varying time and space load conditions. They will give answers to various problems which might otherwise

*The very high value of stress is due to the fact that the duration of the load is almost equal to the fundamental period of the plate.

TABLE 2

Comparison of Frequency Parameter, Static Deflection, and Stress for Uniform Load ($\nu = 0.3$)

Sketch	$\frac{b}{a}$	Value of K Table 1				Value of K Reference 5				Deflection			Stress		
		1st Mode	2nd Mode	3rd Mode	4th Mode	1st Mode	2nd Mode	3rd Mode	4th Mode	γ from Table 1	γ from Reference 4	Percent Difference	δ from Table 1	δ from Reference 4	Percent Difference
	1.0	19.70	49.20	79.00	99.00	19.74	49.34	78.96	98.69	0.0443	0.0443	insignificant	0.0466	0.0479	2.7
	1.2	16.70								.0616	.0616	↓	.0614	.0626	1.9
	1.4	14.86								.0772	.0770	↓	.0741	.0753	1.6
	1.6	13.70								.0905	.0906	↓	.0848	.0862	1.6
	1.8	12.90								.1017	.1017	↓	.0934	.0948	1.5
	2.0	12.30				12.34				.1106	.1106	↓	.1004	.1017	1.3
	3.0	10.95				10.97				.1339	.1336	↓	.1179	.1189	0.8
Plate with all edges simply supported	4.0	10.46								.1410	.1400	0.7	.1235	.1235	
	5.0	10.25								.1437	.1416	1.5	.1255	.1246	0.7
	10.0	9.95													
	∞					9.87					.1422			.1250	
	1.0	36.00	73.80	109.00	132.5*	35.99	73.41	108.3	132.3	.0138	.0138	0	.0432	.0513	15.8
	1.2	30.80								.0188	.0188	0	.0567	.0639	11.3
	1.4	28.00								.0226	.0226	0	.0668	.0726	8.0
	1.6	26.35								.0251	.0251	0	.0734	.0780	5.9
	1.8	25.35								.0268	.0267	insignificant	.0778	.0812	4.2
	2.0	24.60				24.57				.0278	.0277		.0805	.0829	2.9
	3.0	23.20				23.19				.0289			.0838		
Plate with all edges clamped	4.0	22.80								.0293			.0848		
	5.0	22.60								.0298			.0863		
	10.0	22.40								0.0314			0.0901		
	∞					22.37					0.0284			0.0830	

*The approximation in this case gives the correct value of frequency but not the correct nodal pattern.

Note: The following formulas give the frequency, deflection, and stress respectively:

$$F = \frac{K}{2\pi a^2}\sqrt{\frac{D}{\rho h}}; \quad w = \gamma \frac{P_0 a^4}{E h^3}; \quad \sigma = \delta \frac{6 P_0 a^2}{h^2}$$

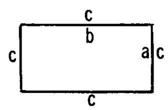
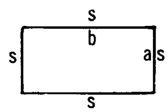
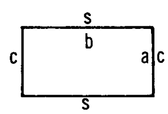
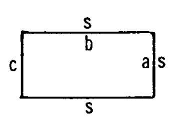
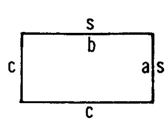
remain unsolved, and these answers will at least be of the right order of magnitude.

For cases other than that of the rectangular plate, the appendixes gives the general formulas for the determination of the response and frequency. Of course, the mode function must be known to calculate any particular response or frequency.

Further work is now underway to obtain actual pressure magnitudes, pulse shapes, and times of duration as well as stresses and deflections in the bottom plating of ships under service conditions. These results and comparisons should be available within the next year. Further work is also in progress for obtaining plastic deformations of panels for possible criteria applicable to limit design.

TABLE 3

Comparison of First Mode Frequency Parameter for Plates with Combinations of Clamped and Simply Supported Edges ($\nu = 0.3$)

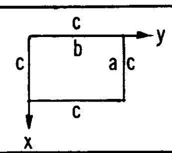
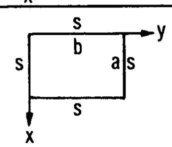
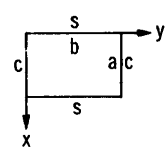
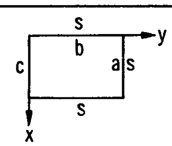
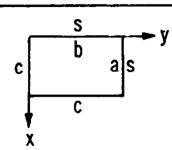
Case	Sketch*	Frequency Parameter	$\frac{b}{a}$	K from Formula	K from Reference 5	K' from Formula	K' from Reference 5
I		$K' = \sqrt{500(1+\eta^4) + 303\eta^2}$	1.0	36.00	35.98	36.00	
			1.5	27.10	27.00	60.90	
			2.0	24.60	24.57	98.50	
			2.5	23.71	23.77	148.00	
			3.0	23.20	23.19	208.50	
			∞	22.37	22.37		
II		$K' = \sqrt{97.4(1+\eta^4) + 194.8\eta^2}$	1.0	19.70	19.74	19.70	
			1.5	14.24	14.26	32.00	
			2.0	12.33	12.34	49.30	
			2.5	11.44	11.45	71.50	
			3.0	10.91	10.97	98.50	
			∞	9.87	9.87		
III		$K' = \sqrt{97.4\eta^4 + 500 + 243\eta^2}$	0			22.40	22.37
			0.333	207.00		23.00	22.99
			0.400	145.00		23.30	23.27
			0.500	95.20		23.80	23.82
			0.667	56.30		25.00	25.05
			1.000	29.00		29.00	28.95
			1.500	17.40	17.37	39.20	
			2.000	13.80	13.69	55.00	
			2.500	12.20	12.13	76.30	
			3.000	11.40	11.36	102.50	
			∞	9.87	9.87		
IV		$K' = \sqrt{97.4\eta^4 + 238 + 227\eta^2}$	0			15.35	15.43
			0.333	145.80		16.20	16.26
			0.400	103.80		16.60	16.63
			0.500	69.20		17.30	17.33
			0.667	42.50		18.90	18.90
			1.000	23.70	23.65	23.70	23.65
			1.500	15.7	15.57	35.33	
			2.000	13.00	12.92	52.00	
			2.500	11.80	11.75	73.80	
			3.000	11.10	11.14	100.50	
			∞	9.87	9.87		
V		$K' = \sqrt{238(1+\eta^4) + 265\eta^2}$	1.0	27.20		27.20	
			1.5	20.00		45.00	
			2.0	17.90		71.50	
			2.5	16.80		105.00	
			3.0	16.40		148.00	
			∞	15.35			

*C indicates clamped edge,
S indicates simply supported edge.

$$F = \frac{K}{2\pi a^2}\sqrt{\frac{D}{\rho h}} = \frac{K'}{2\pi b^2}\sqrt{\frac{D}{\rho h}}\ ;\ \eta = \frac{b}{a},\ K = \frac{a^2}{b^2}K'$$

TABLE 4

Comparison of Static Deflection and Stress for Uniform Load Taking Into Account Only First Term ($\nu = 0.3$)

Case	Sketch	$\frac{b}{a}$	Location of Deflection	Value from Equations in Appendix 1 $w = \gamma \frac{P_0 a^4}{Eh^3}$	Value from Reference 4 $w = \gamma \frac{P_0 a^4}{Eh^3}$	Location of Stress	Value from Equations in Appendix 1 $\sigma = \frac{6}{h^2} \delta P_0 a^2$	Value from Reference 4 $\sigma = \frac{6}{h^2} \delta P_0 a^2$
				γ	γ		δ	δ
I		1.0		0.0146	0.0138		0.0376	0.0513
		1.5	$x = 0.5a$	.0260	.0240	$x = 0$	.0669	.0757
		2.0	$y = 0.5b$	.0313	.0277	$y = 0.5b$	.0807	.0829
		∞		.0288*	.0284	(σ_x)	.0740*	.0830
II		1.0	$x = 0.5a$	.0454	.0443	$x = 0.5a$	.0533	.0479
		1.5	$y = 0.5b$	.0874	.0843	$y = 0.5b$	.0874	.0812
		2.0		.1163	.1106		.1130	.1017
		∞		.1430*	.1420	(σ_x)	.1290*	.1250
III**		0.50		.0020	.0018		.0208	.0210
		0.67	$x = 0.5a$	.0058	.0053	$x = 0.5a$	.0335	.0370
		1.00		.0218	.0209		.0563	.0700
		1.50		.0603	.0582		.0692	.1050
		2.00	$y = 0.5b$	.0969	.0987	$y = 0$	.0625	.1190
		∞		.1430*	.1422	(σ_y)	.1290*	.1250
IV**		1.0	$x = 0.5a$	.0307	.0300	$x = 0.5a$	.0600	.0840
		1.5		.0704	.0700		.0612	.1120
		2.0	$y = 0.5b$	.1022	.1010	$y = 0$	.0499	.1220
		∞		.1430*	.1420	(σ_y)	0.1290*	0.1250
V		1.0		.0227				
		1.5	$x = 0.5a$	.0419				
		2.0	$y = 0.5b$	.0528				
		∞		0.0570*	0.0570			

*This value was computed by using equations in Appendix 2.

**There seems to be no consistency to the one term approximations in calculating the stresses in these cases.

Examination of Tables 1, 2, and 3 reveals the following:
As the aspect ratio b/a increases, the rate at which the frequency numbers as well as the deflection and stress amplitudes change becomes small rather quickly. This fact is very significant since it can be used to simplify many problems by using the infinite aspect ratio formulas derived in Appendix 2 for plates with aspect ratios greater than 5.

The frequencies and deflections in clamped and simply supported plates under uniform static load are in good agreement with the more accurate values in Reference 4,* even though only four terms are considered in the series which is used to represent the deflection. The stresses, on the other hand, do not agree as well. For the simply supported plate, the stresses agree reasonably well; however, for the fixed plate, the error exceeds 10 percent for aspect ratios less than 1.2.

The one-term approximation for static values is reasonably good for deflection computations but must be used with caution for obtaining stresses. For large aspect ratios, however, the infinite aspect ratio formulas given in Appendix 2 should give good results. Since the contributions of the several modes are added, the magnitudes of deflection and stress are expected to be on the conservative side (overestimated) under the action of impulsive loads.

Although the accuracy of the method could only be compared for static load applications, it is believed that the same order of accuracy will be obtained for dynamic load applications.

The formulas given throughout the report hold only for small deflections (up to about 0.7 of the thickness of the plate) of flat plates. The more complicated shell theory must be used for plates with large initial curvature.

If the designer knows the type of loading to which the plate is subjected, if he knows the type of edge support, and if he knows that the stresses will always be within the elastic limit (or if he wants the stresses to be always within the elastic limit) he can select a minimum thickness of plate with the information given in this report. The type of loading can be obtained from experimental records. Such records are now being taken for ship plating. As far as the edge conditions of the plate are concerned, they could probably best be approximated by fixed edges if the plate is one panel of a larger plate containing stiffeners.

ACKNOWLEDGMENTS

The author wishes to express his thanks to Mr. N.H. Jasper of the David Taylor Model Basin for his valuable criticism and suggestions.

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*For aspect ratios greater than 4 Table 1 gives larger values for stress and deflection than the values for infinite aspect ratio predicted by the more exact theory.

APPENDIX 1

THEORY OF FINITE PLATES

GENERAL THEORY

The general theory of small vibrations of plates is given in References 6 and 7. The plate problem consists of finding a solution of the plate equation [1] which satisfies given conditions on the boundaries of the plate:

$$D \nabla^4 w + \rho h \frac{\partial^2 w}{\partial t^2} = P(x, y, t) \ldots \qquad [1]$$

where D, ρ, and h are the plate modulus, mass density, and thickness, respectively, and P is the load per unit area applied normal to the plate (see the notation). If $\partial/\partial n$ denotes differentiation along the outward normal and $\partial/\partial s$ differentiation along the tangent, the boundary conditions for clamped, simply supported, and free plates can be written as follows (see Reference 4, Chapter IV):

(A) Clamped boundary:

$$w = 0 \text{ along } s$$

$$\frac{\partial w}{\partial n} = 0 \text{ along } s$$

(B) Simply supported boundary:

$$w = 0 \text{ along } s$$

$$M_n = 0 \text{ along } s$$

(C) Free boundary:

$$M_n = 0 \text{ along } s$$

$$V_n = Q_n - \frac{\partial M_{ns}}{\partial s} = 0 \text{ along } s$$

The terms w, $\partial w/\partial n$, M_n, V_n, Q_n, and M_{ns} are defined in the notation and explained completely in Reference 4. When a plate is loaded by a lateral dynamic load as considered here, the deflection w can be expressed as follows, provided stresses remain within the elastic limit:

$$w = \sum_{r=1}^{\infty} w_r q_r \ldots \qquad [2]$$

where w_r is the normal-mode function depending only on space and is taken as dimensionless,

q_r is a function depending only on time and has the dimension of length, and

r corresponds to the mode number.

If Equation [2] is substituted into Equation [1], the following expression is obtained

$$\frac{D}{\rho h} \nabla^4 \sum_{r=1}^{\infty} w_r q_r + \sum_{r=1}^{\infty} w_r \frac{d^2 q_r}{dt^2} = \frac{P}{\rho h} \ldots \qquad [3]$$

Now multiply both sides of this equation by one of the normal mode functions w_m and integrate over the area A_p of the plate:

$$\frac{D}{\rho h}\int_{A_p} w_m\left[\nabla^4\sum_{r=1}^{\infty} w_r q_r\right]dA + \int_{A_p} w_m\left[\sum_{r=1}^{\infty} w_r \frac{d^2 q_r}{dt^2}\right]dA = \int_{A_p}\frac{P w_m}{\rho h}\,dA \ldots \quad [4]$$

The first term in this equation contains integrals of the form

$$\int_{A_p} w_m \nabla^4 w_r dA,$$

and the second term contains integrals of the form

$$\int_{A_p} w_m w_r dA.$$

Rayleigh[6], has demonstrated that

$$\int_{A_p} w_m w_r\, dA$$

is zero if r is not equal to m and if the boundary of the plate is wholly or partly clamped, simply supported, or free.

It can also be demonstrated that

$$\int_{A_p} w_m \nabla^4 w_r dA$$

is zero if r is not equal to m. If the plate is vibrating freely in one of its modes, then

$$w = w_r \sin p_r t \ldots \quad [5]$$

Substituting this into the differential equation for free vibrations, multiplying through by w_m, and integrating over the area of the plate, the following expression is obtained:

$$\frac{D}{\rho h}\int_{A_p} w_m \nabla^4 w_r dA = p_r^2 \int_{A_p} w_m w_r dA \ldots \quad [6]$$

Since $\int_{A_p} w_m w_r dA = 0$, $\int_{A_p} w_m \nabla^4 w_r dA = 0$

Now Equation [4] can be written

$$\frac{D}{\rho h}\int_{A_p} w_m \nabla^4 w_m q_m dA + \int_{A_p} w_m{}^2 \frac{d^2 q_m}{dt^2}\, dA = \int_{A_p}\frac{P w_m dA}{\rho h}$$

or

$$\frac{d^2 q_m}{dt^2} + \frac{D}{\rho h}\,\frac{\int_{A_p} w_m \nabla^4 w_m dA}{\int_{A_p} w_m^2\, dA}\, q_m = \frac{1}{\rho h}\,\frac{\int_{A_p} P w_m dA}{\int_{A_p} w_m^2 dA} \ldots \quad [7]$$

or written in more compact notation

$$\ddot{q}_m + p_m^2 q_m = A_m(t) \ldots \qquad [8]$$

where the frequency of the mth mode of vibration

$$p_m = \sqrt{\frac{D}{\rho h}} \left[\sqrt{\frac{\int_{A_p} w_m \nabla^4 w_m \, dA}{\int_{A_p} w_m^2 \, dA}} \right] \ldots \qquad [9]$$

and

$$A_m(t) = \frac{1}{\rho h} \frac{\int_{A_p} P w_m \, dA}{\int_{A_p} w_m^2 \, dA} \ldots \qquad [10]$$

The solution of Equation [8] is

$$q_m = C_1 \sin p_m t + C_2 \cos p_m t + \frac{1}{p_m} \int_0^t A_m(\tau) \sin p_m (t - \tau) \, d\tau \ldots \qquad [11]$$

where C_1 and C_2 are constants of integration. The first two terms correspond to the free vibration and the third to the forced vibration. The complete expression for the deflection is then given as

$$w = \sum_{m=1}^{\infty} w_m \left[C_1 \sin p_m t + C_2 \cos p_m t + \frac{1}{p_m} \int_0^t A_m(\tau) \sin p_m (t - \tau) \, d\tau \right] \ldots \qquad [12]$$

IMPULSIVE LOADING

Assume that the plate is subjected to an impulsive load $G(x, y) f(t)$ where G is the space load distribution and $f(t)$ is the time load distribution called the shape of the pulse. The initial conditions for this type of loading at $t = 0$ are

$$w = 0, \frac{dw}{dt} = 0 \ldots \qquad [13]$$

Under these conditions $C_1 = C_2 = 0$ so the deflection can be written as

$$w = \sum_{m=1}^{\infty} w_m \left[\frac{1}{p_m} \int_0^t A_m(\tau) \sin p_m (t - \tau) \, d\tau \right] \ldots \qquad [14]$$

where

$$A_m(\tau) = \frac{1}{\rho h} f(\tau) \frac{\int_{A_p} G w_m \, dA}{\int_{A_p} w_m^2 \, dA} \ldots \qquad [15]$$

so

$$w = \sum_{m=1}^{\infty} w_m \left[\frac{1}{\rho h} \frac{\int_{A_p} G w_m dA}{p_m^2 \int_{A_p} w_m^2 dA} \right] \left[p_m \int_0^t f(\tau) \sin p_m (t - \tau)\, d\tau \right] \ldots \qquad [16]$$

Both Frankland[8] and Salvadori[9] have studied the response of a single-degree-of-freedom system to pulses of various shapes and have evaluated the integral $p_m \int_0^t f(\tau) \sin p_m (t - \tau) d\tau$ for different pulses $f(\tau)$. If the load is gradually applied (i.e., the time of application of the load is much greater than the fundamental period), then $p_m \int_0^t f(\tau) \sin p_m (t - \tau)\, d\tau$ has a value of unity. Thus the static deflection under a given load distribution G is

$$w_{\text{static}} = \sum_{m=1}^{\infty} w_m \left[\frac{1}{\rho h} \frac{\int_{A_p} G w_m dA}{p_m^2 \int_{A_p} w_m^2 dA} \right] \ldots \qquad [17]$$

or written more compactly

$$w_{\text{static}} = \sum_{m} (w_{\text{static}})_{m\text{th mode}} \ldots \qquad [18]$$

Since the stresses are space derivatives of the deflection, it may be concluded from Equations [16] and [17] that the total dynamic response, deflection or stress, is equal to the sum over all modes of the static deflection or stress in the mth mode multiplied by the response factor of the mth mode; the response factor is defined by the following equation

$$R_m(t) = p_m \int_0^t f(\tau) \sin p_m (t - \tau)\, d\tau \ldots \qquad [19]$$

The maximum value of the response factor is known as the load factor and is designated by L.

RECTANGULAR PLATES

To calculate the frequency and response (deflection or stress) of the plate, the normal modes of vibration must be known. Young[11] has employed the beam functions to approximate the normal modes of vibration of rectangular plates by using the Ritz method for his calculations. In the calculations which follow, the normal modes of the plate are approximated by the product of two beam functions, i.e., $w_m = X_i Y_j$.* The functions X_i and Y_j depend on the boundary conditions of the plate.** For example, if the plate has two opposite edges clamped, a third edge simply supported, and a fourth edge free, then X_i is the function corresponding to

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*The numbers i and j are connected with the mode, e.g., for the first mode $i = 1$, $j = 1$, for the second mode $i = 1$, $j = 2$, etc. It should also be stated at this time that the product of the beam functions is not the exact expression for the modes of a plate since this combination does not, in general, satisfy the plate equation. However, it can be seen by comparison of frequencies deflections, and stresses with those available in the literature that this assumption gives reasonable results.

**The product of beam functions constitutes an exact solution to the problem only in the case of a plate simply supported on all edges.

a clamped-clamped beam and Y_j to a simply supported free beam. Using this value of w_m and substituting it into Expressions [9] and [17], the following formulas are obtained for the frequency and deflection in the mth mode for plates with free, simply supported, or fixed edges.

$$p_{ij} = \sqrt{\frac{D}{\rho h}} \sqrt{\frac{(\beta_i)^4}{a^4} + \frac{(\beta_j)^4}{b^4} + \frac{2 \int_0^a \int_0^b X_i X_i'' Y_j Y_j'' \, dxdy}{\int_0^a \int_0^b X_i^2 Y_j^2 \, dxdy}} \quad \ldots \qquad [20]$$

$$w_{ij} = X_i Y_j \frac{\int_0^a \int_0^b G(x, y) X_i Y_j \, dxdy}{\rho h p_m^2 \int_0^a \int_0^b (X_i Y_j)^2 \, dxdy} R_m \quad \ldots \qquad [21]$$

The expressions for the bending moment and bending stress are as follows:

$$(M_x)_{ij} = D\left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}\right) = D(X_i'' Y_j + \nu X_i Y_j'') \frac{\int_0^a \int_0^b G(x, y) X_i Y_j \, dxdy}{\rho h p_m^2 \int_0^a \int_0^b (X_i Y_j)^2 dxdy} (R_m) \quad \ldots \qquad [22]$$

$$(M_y)_{ij} = D\left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}\right) = D(X_i Y_j'' + \nu X_i'' Y_j) \frac{\int_0^a \int_0^b G(x, y) X_i Y_j \, dxdy}{\rho h p_m^2 \int_0^a \int_0^b (X_i Y_j)^2 \, dxdy} (R_m) \ldots \qquad [23]$$

and the bending stresses are given by

$$\sigma_x = \frac{6 M_x}{h^2}, \qquad \sigma_y = \frac{6 M_y}{h^2} \quad \ldots \qquad [24]$$

In the above expressions for the bending moments, the primes denote differentiation with respect to x or y, e.g., $X_i'' = d^2 X_i / dx^2$, $Y_j'' = d^2 Y_j / dy^2$.

The values of X_i and Y_j are given below for several types of beams. These expressions were obtained from References 10 and 11.

$$X_i = \cosh \frac{\beta_i x}{a} - \cos \frac{\beta_i x}{a} - \alpha_i \left(\sinh \frac{\beta_i x}{a} - \sin \frac{\beta_i x}{a}\right) \ldots \qquad [25]$$

$$Y_j = \cosh \frac{\beta_j y}{b} - \cos \frac{\beta_j y}{b} - \alpha_j \left(\sinh \frac{\beta_j y}{b} - \sin \frac{\beta_j y}{b}\right) \ldots \qquad [26]$$

$$X_i = \sin \frac{\beta_i x}{a} \ldots \qquad [27]$$

$$Y_j = \sin \frac{\beta_j y}{b} \ldots \qquad [28]$$

Formulas [25] and [26] hold for a beam whose edges are clamped-clamped, clamped-free, or clamped-supported. Formulas [27] and [28] hold for a beam whose edges are supported-supported. Expressions [27] and [28] are the exact expressions for a plate all of whose edges are simply supported. The values of β and α as well as the integrals $\int_0^a X_i X_i'' dx$, $\int_0^a X_i^2 dx$ and the values of X_i and X_i'' are contained in References 10, 11, and 12. These references were used to compute the tables given in the report. The values used in the calculation are given in Table 5

TABLE 5

Beam Function Values

Here a or b is the length of the beam, and the origin $x = 0$ or $y = 0$ is located at one end.

Type of Beam	i or j	α_i, α_j	β_i, β_j	$b\int_0^b Y_j Y_j'' dy$ or $= a\int_0^a X_i X_i'' dx$	$\frac{\int_0^b Y_j^2 dy}{b}$ or $= \frac{\int_0^a X_i^2 dx}{a}$	*Value of X_i	Point at Which this Value of X_i Occurs	*Value of $\frac{a^2}{\beta_i^2} X_i''$	Point at Which this Value of $\frac{a^2}{\beta_i^2} X_i''$ Occurs	$\frac{\int_0^b Y_j dy}{b}$ or $= \frac{\int_0^a X_i dx}{a}$
Clamped-Clamped	1	0.9825	4.7300	– 12.3026	1	1.5882	$x = 0.5\ a$	2	$x = 0$	0.8309
	2	1.0008	7.8532	– 46.0501	1	0	$x = 0.5\ a$	2	$x = 0$	0
	3	1.0000	10.9956	– 98.9048	1	–1.4060	$x = 0.5\ a$	2	$x = 0$	0.3638
	5	1.0000	17.2788	– 263.9980	1	1.4146	$x = 0.5\ a$	2	$x = 0$	0.2315
Supported-Supported	1		3.1416	– 4.9349	0.5	1	$x = 0.5\ a$	–1	$x = 0.5\ a$	0.6366
	2		6.2832	– 19.7396	0.5	0	$x = 0.5\ a$	0	$x = 0.5\ a$	0
	3		9.4248	– 44.4141	0.5	–1	$x = 0.5\ a$	1	$x = 0.5\ a$	0.2122
	5		15.7080	–123.3725	0.5	1	$x = 0.5\ a$	–1	$x = 0.5\ a$	0.1273
Clamped-Free	1	0.7341	1.8751	0.8582	1	2	$x = a$	2	$x = 0$	0.7830
Clamped-Supported	1	1.0008	3.9266	– 11.5128	1	1.4449	$x = 0.5\ a$	2	$x = 0$	0.8599

*The tabulations will remain valid if X_i is replaced by Y_j.

APPENDIX 2

THEORY OF SEMI-INFINITE PLATE AND FINITE BEAM

In this problem, let $b/a = \infty$; then the problem reduces to one in which x is the only space variable. The equation of motion is

$$D \frac{\partial^4 w}{\partial x^4} + \rho h \frac{\partial^2 w}{\partial t^2} = P(x, t) \ldots \qquad [29]$$

assuming that the load varies only in x and t.

Now letting

$$w = \sum_{r=1}^{\infty} X_r(x)\, q_r(t)$$

and going through the same procedure as outlined for the finite plate, the following formula is obtained for the deflection:

$$w = \sum_{m=1}^{\infty} X_m \left[\frac{a^4 \int_0^a G(x) X_m dx}{D K_m^2 \int_0^a X_m^2 dx} \right] \left[p_m \int_0^t f(\tau) \sin p_m (t - \tau) d\tau \right] \ldots \qquad [30]$$

where K_m is the frequency number for the mth mode of an infinite plate. This is equal to the corresponding frequency number of a beam with the same boundary conditions.

The first-mode static deflection and moment are

$$w = X_1 \frac{a^4 \int_0^a G(x) X_1 dx}{D \beta_1^4 \int_0^a X_1^2 dx} \ldots \qquad [31]$$

$$M_x = D \frac{\partial^2 w}{\partial x^2} = X_1'' \frac{a^4 \int_0^a G(x) X_1 dx}{\beta_1^4 \int_0^a X_1^2 dx} \ldots \qquad [32]$$

the stress

$$\sigma_x = \frac{6 M_x}{h^2} \ldots \qquad [34a]$$

and the first-mode frequency

$$p_1 = \sqrt{\frac{D}{\rho h}} \frac{\beta_1^2}{a^2} \ldots \qquad [33]$$

The deflection, stress, and frequency of a beam of length a, moment of inertia I, density ρ, cross-sectional area A_1 and modulus E can be written down from these expressions by noting that the beam equation is mathematically analogous to the plate equation for infinite aspect ratio.

$$EI\frac{\partial^4 w}{\partial x^4} + \rho A_b \frac{\partial^2 w}{\partial t^2} = P(x, t) \ldots \qquad [34]$$

where $P(x, t)$ is the load per unit length. For the beam, the first mode expressions are:

$$w = X_1 \frac{a^4 \int_0^a G(x) X_1 \, dx}{EI \beta_1^4 \int_0^a X_1^2 \, dx} \ldots \qquad [35]$$

$$M_x = EI\frac{\partial^2 w}{\partial x^2} = X_1'' \frac{a^4 \int_0^a G(x) X_1 \, dx}{\beta_1^4 \int_0^a X_1^2 \, dx} \ldots \qquad [36]$$

$$p_1 = \sqrt{\frac{EI}{\rho A_b}} \frac{\beta_1^2}{a^2}$$

and the usual formula for the stress in a uniform beam is

$$\sigma_x = \frac{M_x c}{I}$$

where c is the distance of the element from the neutral axis for which the stress is desired.

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