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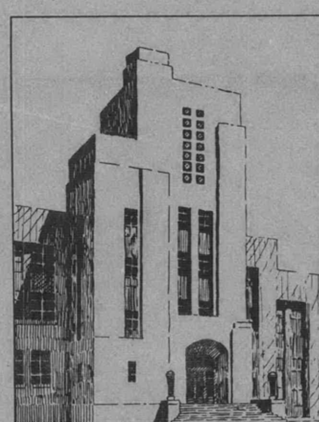
UNITED STATES NAVY

THE PRESSURE AND THE VELOCITY FIELDS AROUND A TNT CHARGE DETONATED IN FREE WATER

BY E. H. KENNARD, Ph. D.



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REPORT 544

PERSONNEL

The calculations for the curves and diagrams in this report were made by Miss Wilhelmina Schafer, under the direction of Dr. E.H. Kennard, who wrote the report.

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ABSTRACT

A spherical charge detonated in free water generates in the surrounding water pressures and radial particle velocities which are functions both of the time from the instant of detonation and of the distance from the center of the charge. Curves are presented to give a general idea of this field of pressure and velocity, both in the shock wave and between the shock wave and the gas globe, up to the time of maximum expansion of the globe. The curves are based upon a combination of observations with approximate calculations.

INTRODUCTION

In considering the effects of underwater explosions upon ship and other structures, information is needed concerning the general field of pressure and particle velocity in the water. In many cases the damaging effects on a target are produced chiefly by the high-pressure shock wave that is generated by the detonation of the charge. The passage of the shock wave is followed, however, by a much longer regime of lower pressures, alternately above and below hydrostatic pressure; these pressures are associated in an intimate way with the outrush and inrush of the water that occurs as the gas globe formed out of the exploded material oscillates or pulsates. In some cases appreciable damaging effects may be produced on a structure in the vicinity, not only by sharp pulses of positive pressure emitted as the gas globe contracts repeatedly to a minimum size, but also by the intervening and more enduring lower pressures, or by inequalities of pressure due to the blocking of the flow of the water by an obstacle.

As a basis for such considerations, an approximate estimate will be made of the pressures and velocities produced by a spherical charge of cast TNT surrounded by unlimited sea water and detonated at its center, with such accuracy as can readily be obtained on the basis of existing information.

It will be assumed that the gas globe remains spherical and that its center does not move. The motion of the water will be considered, however, only from the instant of detonation up to the time at which the radius of the gas globe first attains a maximum value.

The results may also be used for the earlier part of the subsequent contraction, since the pressure and velocity pass in reverse order through nearly the same sequence of numerical values as they do during the latter part of the first expansion.

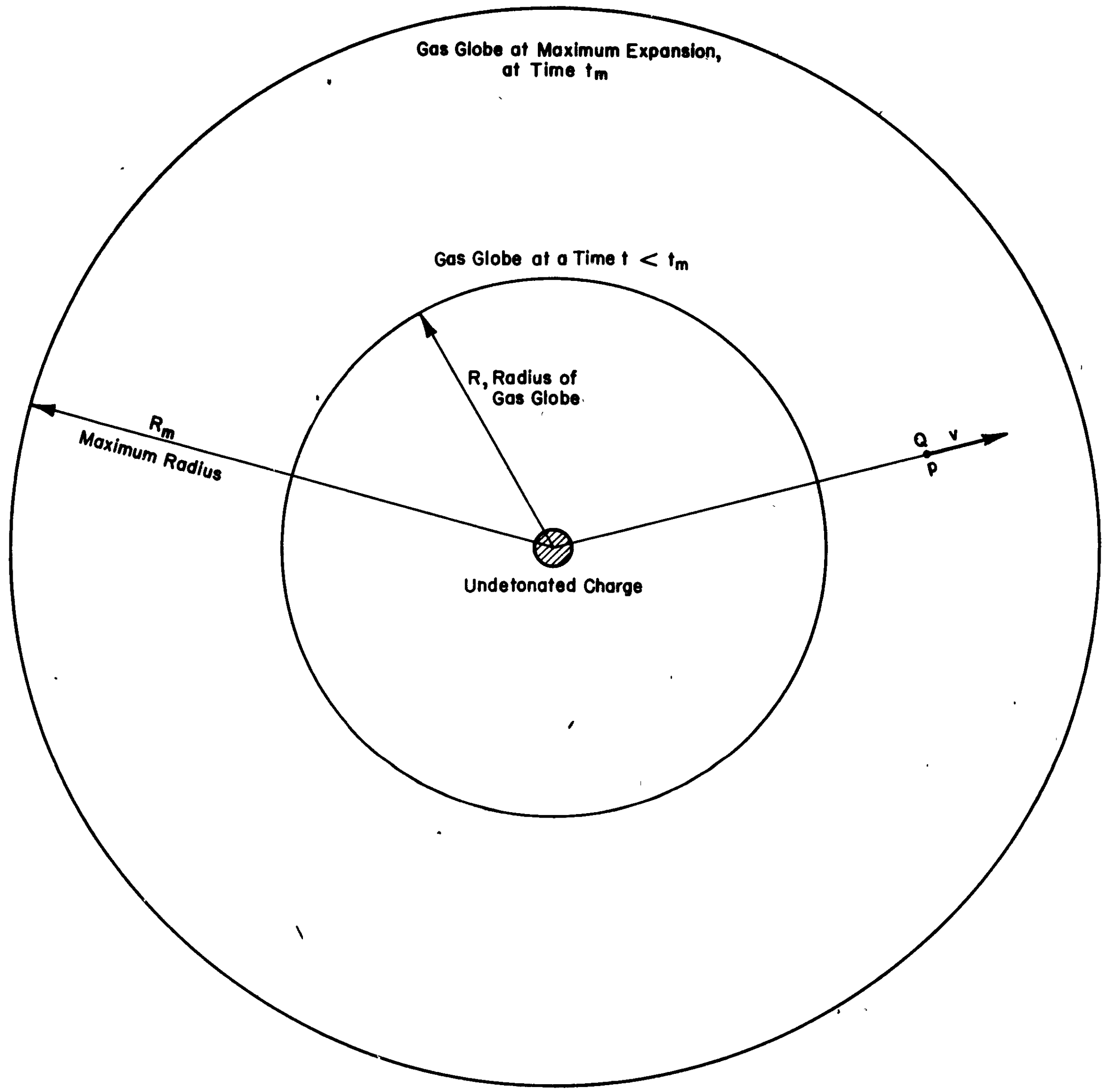

Figure 1 - Diagram Illustrating the Meaning of Certain Symbols

In actual cases, the motion following the passage of the shock wave will usually be more or less distorted by the action of gravity, and perhaps also owing to the influence of neighboring surfaces. Estimates of these effects may be found in TMB Report R-182 (1).* If migration of the gas globe occurs, the pressures and the particle velocity in the water as described in the present report should not be greatly affected so long as the displacement of the gas globe during the first oscillation does not exceed one-half of its minimum radius.

The manner in which data have been obtained for drawing the curves in the following section will be explained in subsequent sections of the report.

THE PRESSURE AND THE PARTICLE VELOCITY

The situation that is created when a charge is detonated under water is illustrated graphically in Figure 1. At any instant during the first expansion the gas globe has expanded to a certain radius R. The water outside the gas globe has been more or less compressed, and at any point such as Q it is under a certain pressure p; at the same time the water at Q has a certain velocity v, directed radially outward. Both the pressure p and the particle velocity v are functions of the time t and of the distance r from the center.

In order to exhibit the values of p and v graphically, curves may be drawn showing their values as a function of the time t at a given point, or curves may be drawn showing p and v as functions of the distance r from the center of the charge at a given time. The latter type of curve, referring to a selected instant of time, will be used chiefly in this report.

To assist in visualizing the pressure and velocity field, two early stages of the field are first illustrated pictorially in Figure 2. In the first stage, the detonation has occurred at the center of the charge and the detonation wave has arrived at the boundary; the charge retains its original shape and volume, but it is now compressed gas instead of a solid. In the second stage, the shock wave has progressed only to a short distance from the center, and the water between it and the gas globe is compressed under a pressure ranging from 40,000 to 70,000 pounds per square inch. In the lower part of Figure 2 a later third stage is shown in which appreciable compression of the water exists only near the shock front. A curve is drawn to show the variation of the pressure with distance from the gas globe as it exists at this instant, and arrows indicate the relative magnitude of the particle velocity at several points.

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* Numbers in parentheses indicate references listed on page 20 of this report.

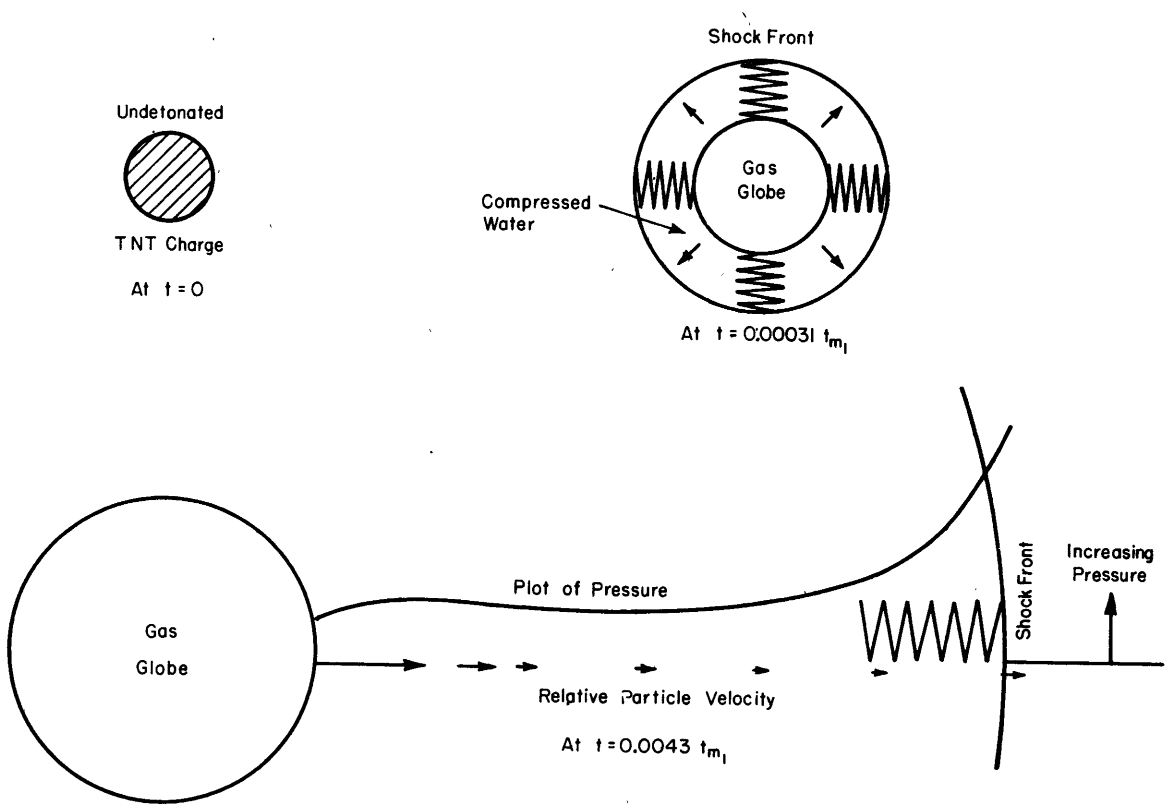

Figure 2 - Diagram Illustrating Three Stages in the Emission of the Shock Wave

At the time $t = 0.00031 t_{m1}$, where t_{m1} is the time of the maximum first expansion of the gas globe, under a hydrostatic pressure of 1 atmosphere, the water between the shock front and the gas globe is under a pressure of 40,000 to 70,000 pounds per square inch, and is moving outward at a velocity of roughly 1000 feet per second. At $t = 0.0043 t_{m1}$, a maximum pressure of 6000 pounds per square inch occurs at the shock front; the outward water velocity is 450 feet per second just outside the gas globe and 85 feet per second at the shock front. Here $t_{m1} = 0.118 W^{1/3}$ second, where W is the weight of the TNT charge in pounds.

Quantitative plots are given in Figures 3 to 7, showing the estimated values of p and v as a function of r at each of a series of selected times. The pressure p refers to the excess of pressure, sometimes negative, above the total hydrostatic pressure at the level of the center of the charge. The charge is assumed to be surrounded by unlimited water.

In order to make the curves applicable to charges of all sizes, in accordance with the familiar principle of similitude, the abscissa is taken to represent the ratio r/R_{m1}, where R_{m1} is the maximum radius attained by the gas globe in its first expansion under a hydrostatic pressure of 1 atmosphere. The chosen times to which the successive plots refer are similarly specified in terms of a ratio, either t/t_{m1} or t/t_{m8}, where t is the time that has elapsed since detonation of the charge; t_{m1} is the time required for the gas globe to attain its maximum size when expanding under a total hydrostatic pressure of 1 atmosphere, whereas t_{m3} is the same time when the total hydrostatic pressure, including atmospheric pressure, is 3 atmospheres. The same values of p and v will then hold for all charges at the same values of r/R_{m1}, and of t/t_{m1} or t/t_{m3} as may be appropriate.

The value of R_m, the maximum radius of the gas globe, and the value of t_m, the time to reach maximum radius on the first expansion, as found in the calculations made for this report, are closely given by the formulas

$$R_m = 4.1\left(\frac{W}{p_A}\right)^{\frac{1}{3}} \text{ feet and } \quad t_m = 0.118\,\frac{W^{\frac{1}{3}}}{p_A^{\frac{5}{6}}} \text{ seconds}$$

where W is the weight of the charge in pounds,

p_A is the total hydrostatic pressure at the level of its center, atmospheric pressure included, expressed in atmospheres, and

t_m is the time required from detonation up to the occurrence of the first maximum radius R_m.

Thus, for a hydrostatic pressure of 1 atmosphere, $R_m = R_{m1} = 4.1W^{1/3}$ feet, $t_m = t_{m1} = 0.118W^{1/3}$ second. The value of R_m under a hydrostatic pressure of 3 atmospheres would be denoted by R_{m3}, where $R_{m3} = 4.1(W/3)^{1/3} = 2.84W^{1/3}$ feet; and $t_{m3} = 0.047W^{1/3}$ second.

Figures 3a to 3e show successive early stages in which the shock front has advanced to a distance from the center less than $r = 2R_{m1}$. Each pair of curves has reference to a particular instant of time, as indicated; the simultaneous radius of the gas globe is shown by a heavy horizontal bar.

Figure 3a shows at the top of the figure the radius of the undetonated charge and an estimate of the pressure and particle velocity in the adjacent water just after the detonation wave has reached the interface between the charge and the water. The two pairs of curves on this figure are extended toward the left to show the calculated pressures and particle velocities inside as well as outside the gas globe, whereas in all later figures only values in the water are shown.

The values of pressure p and velocity v shown in Figures 3a to 3e are little affected by hydrostatic pressure. Hence the time t at which each figure represents conditions in the water is stated for simplicity in terms of t_{m1}, the time occupied by the expansion of the gas globe to its maximum radius when the hydrostatic pressure is 1 atmosphere. It is assumed, however, that the charge is surrounded by unlimited water.

At later times the influence of the hydrostatic pressure becomes noticeable; accordingly, separate plots are shown in Figures 4 through 7 for hydrostatic pressures p_A of 1 atmosphere and 3 atmospheres. The curves for $p_A = 1$ are of interest primarily in connection with the small charges used in

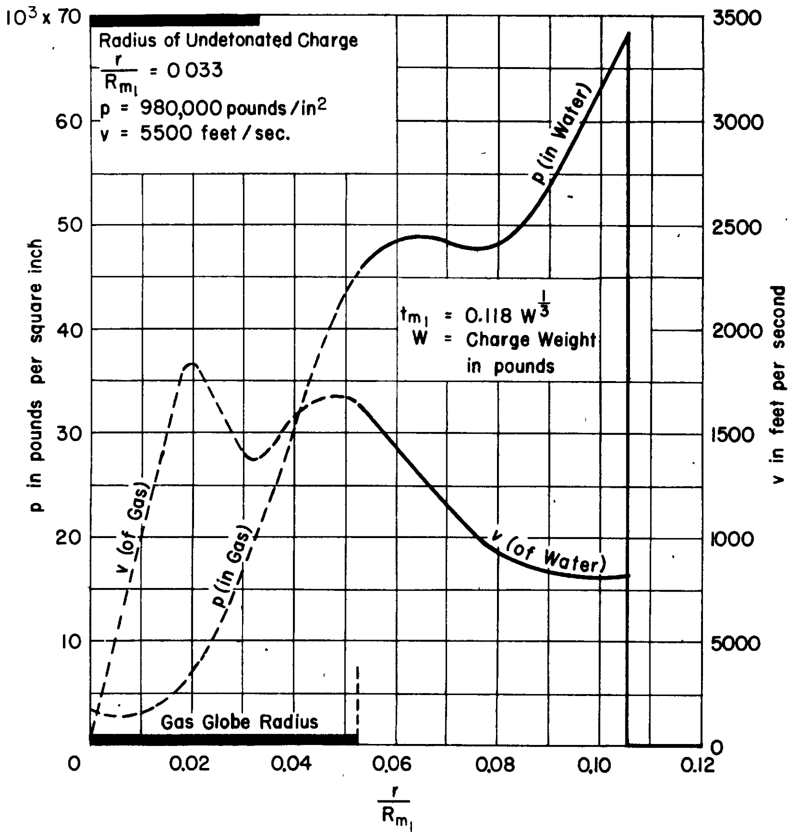

Figure 3a
Time t = $0.00031\, t_{m1}$

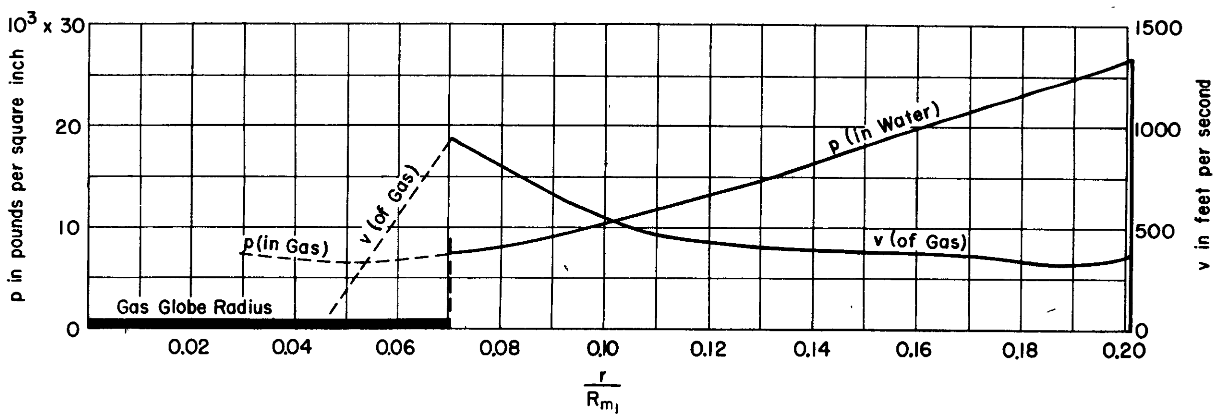

Figure 3b - Time t = $0.00087\, t_{m1}$

Figure 3 - Excess of Pressure above Hydrostatic Pressure p and Particle Velocity v at Successive Selected Times t

The first diagram at the top represents the start of the phenomenon, just after the completed detonation of the charge. The curves are extended to show p and v inside the gas globe itself. Here t_m is the time for detonation to the maximum radius of the gas globe on the first expansion, under the actual total hydrostatic pressure of p_A atmospheres at the level of the charge, and t_{m1} is the value of t_m when $p_A = 1$. The radius r is the distance from the center of the charge; R_{m1} is the maximum radius when $p_A = 1$. The pressure p_A may range from 1 to 5 atmospheres.

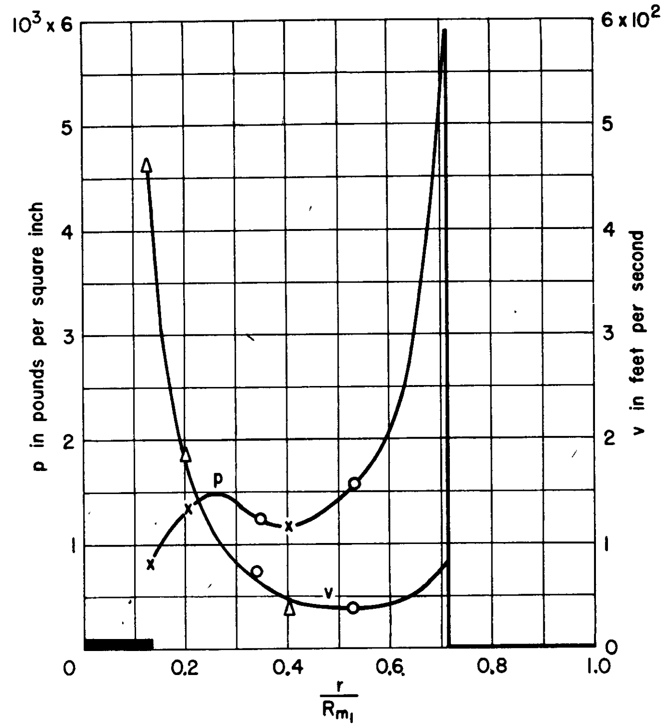

Figure 3c - Time $t = 0.0043\,t_{m1}$

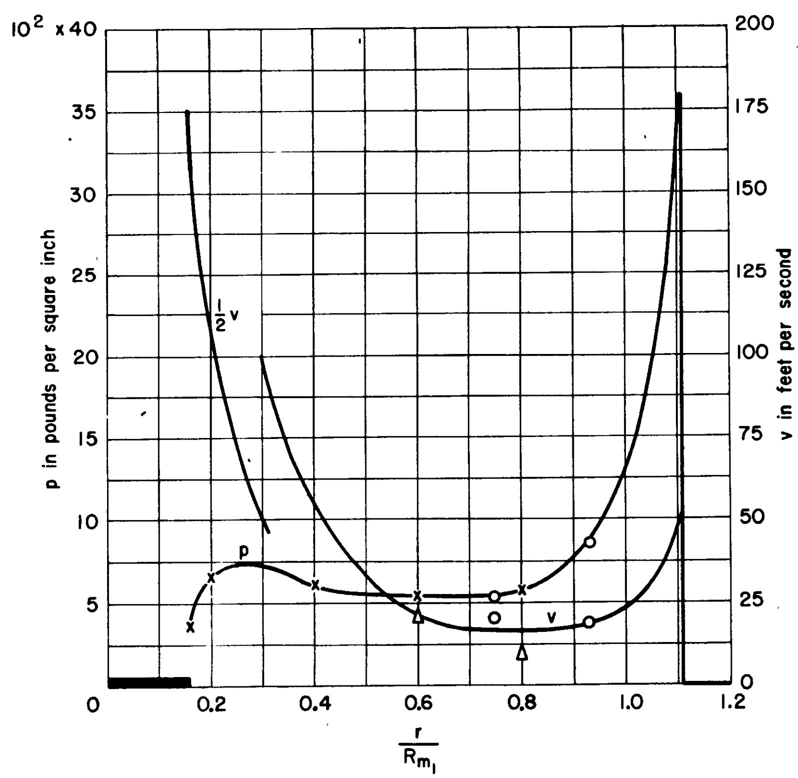

Figure 3d - Time $t = 0.0069\,t_{m1}$

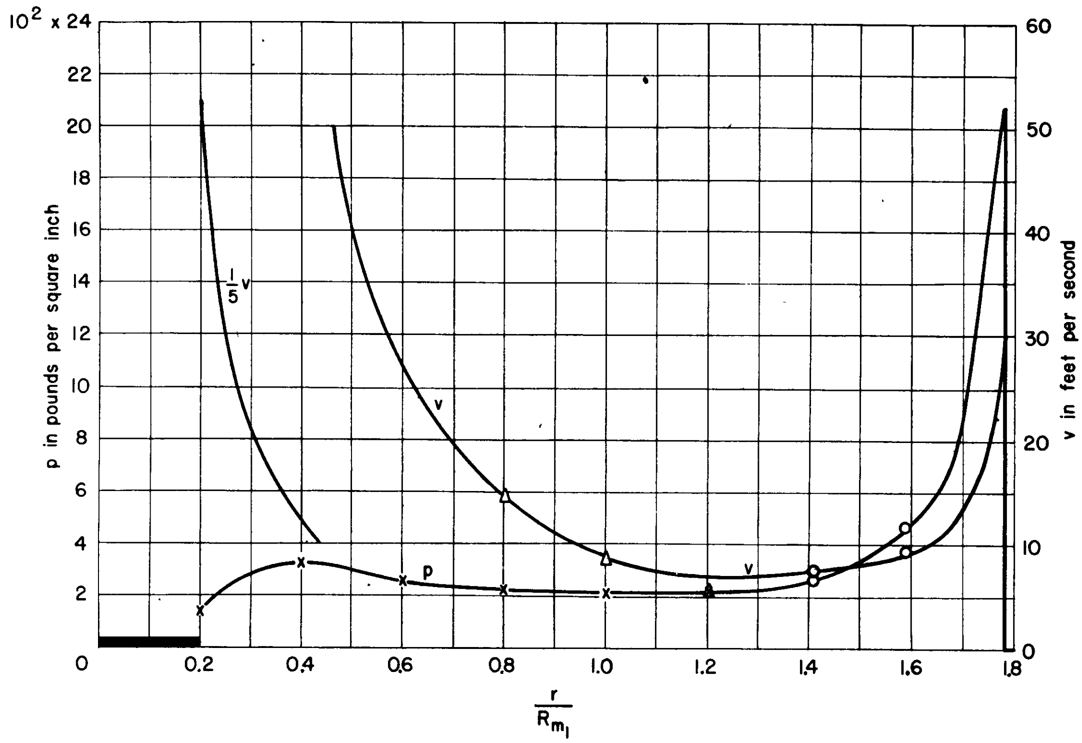

Figure 3e - Time $t = 0.0116\, t_{m1}$

experimental tests. A hydrostatic pressure of 3 atmospheres corresponds to a depth of 66 feet under the surface of the sea. A rough idea of the field for other values of the hydrostatic pressure can be formed by interpolation or extrapolation.

In Figures 4 through 7 the abscissa again represents r/R_{m1} and is proportional to the actual distance from the charge. Curves for a given value of t/t_{m3} refer, however, to a much earlier actual time than would a curve for the same value of t/t_{m1}, since $t_{m3} = 0.40\, t_{m1}$. To facilitate comparison between the curves for $p_A = 1$ and those for $p_A = 3$, the curves for $p_A = 3$ are also labeled with values of t/t_{m1} in brackets. All curves labeled with a given value of t/t_{m1} represent pressure or velocity fields at the same actual time t.

The curves showing p and v for $p_A = 3$ and $t_{m3} = 0.061$, or $t/t_{m1} = 0.0243$, are similar to those for $p_A = 1$ and $t/t_{m1} = 0.030$, corresponding to a time t just a little later. Subsequent pressure curves for $p_A = 3$ and 1, respectively, are, however, quite dissimilar.

The curves in Figures 4 through 7 may be extended with fair accuracy to larger values of r/R_{m1} than those shown by decreasing all pressures numerically in proportion to $1/(r/R_{m1})$, and all velocities in proportion to $1/(r^2/R_{m1}^2)$. The time should also be increased by the interval required for

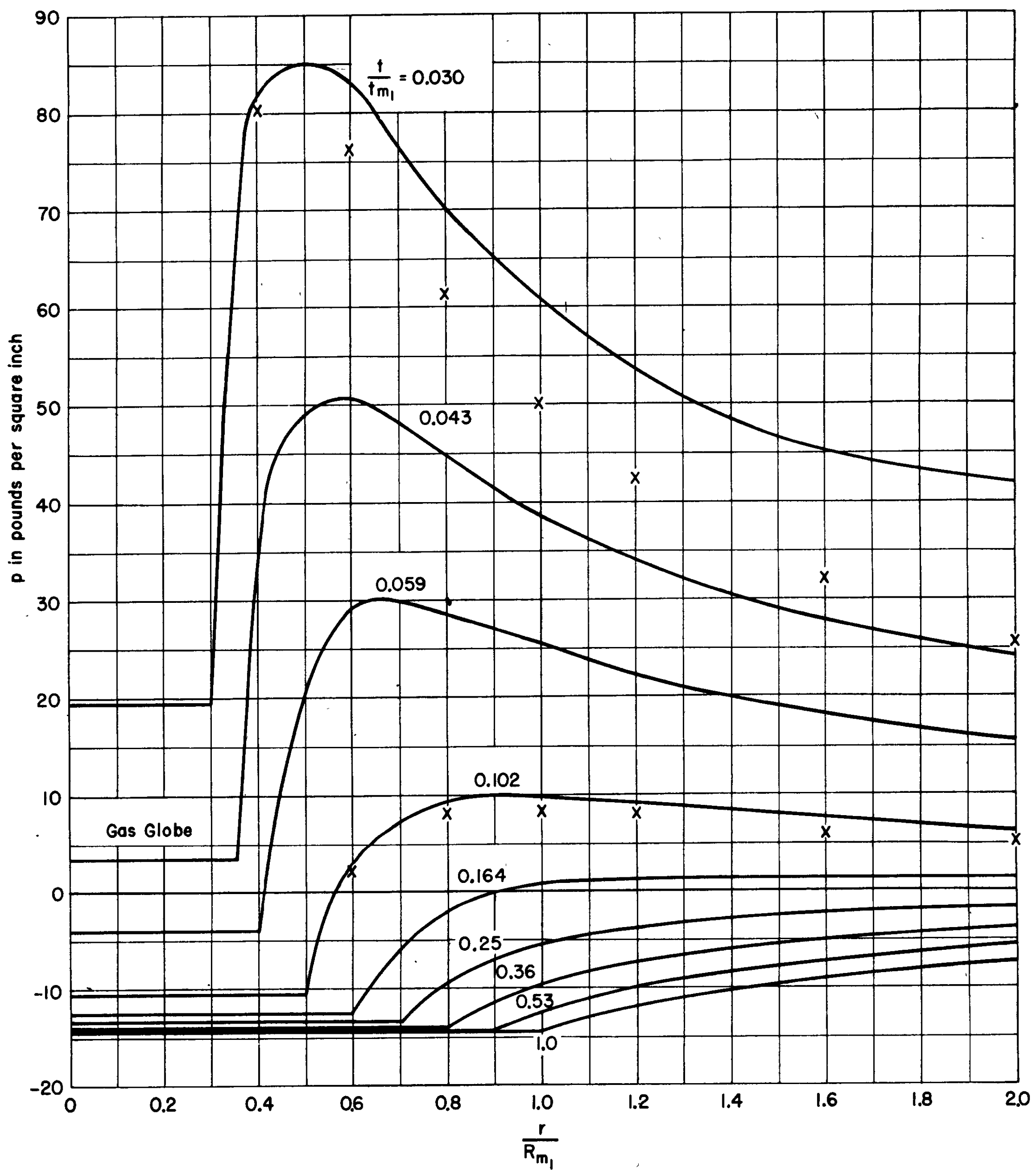

Figure 4 - Curves of Excess Pressure p When p_A = 1 Atm

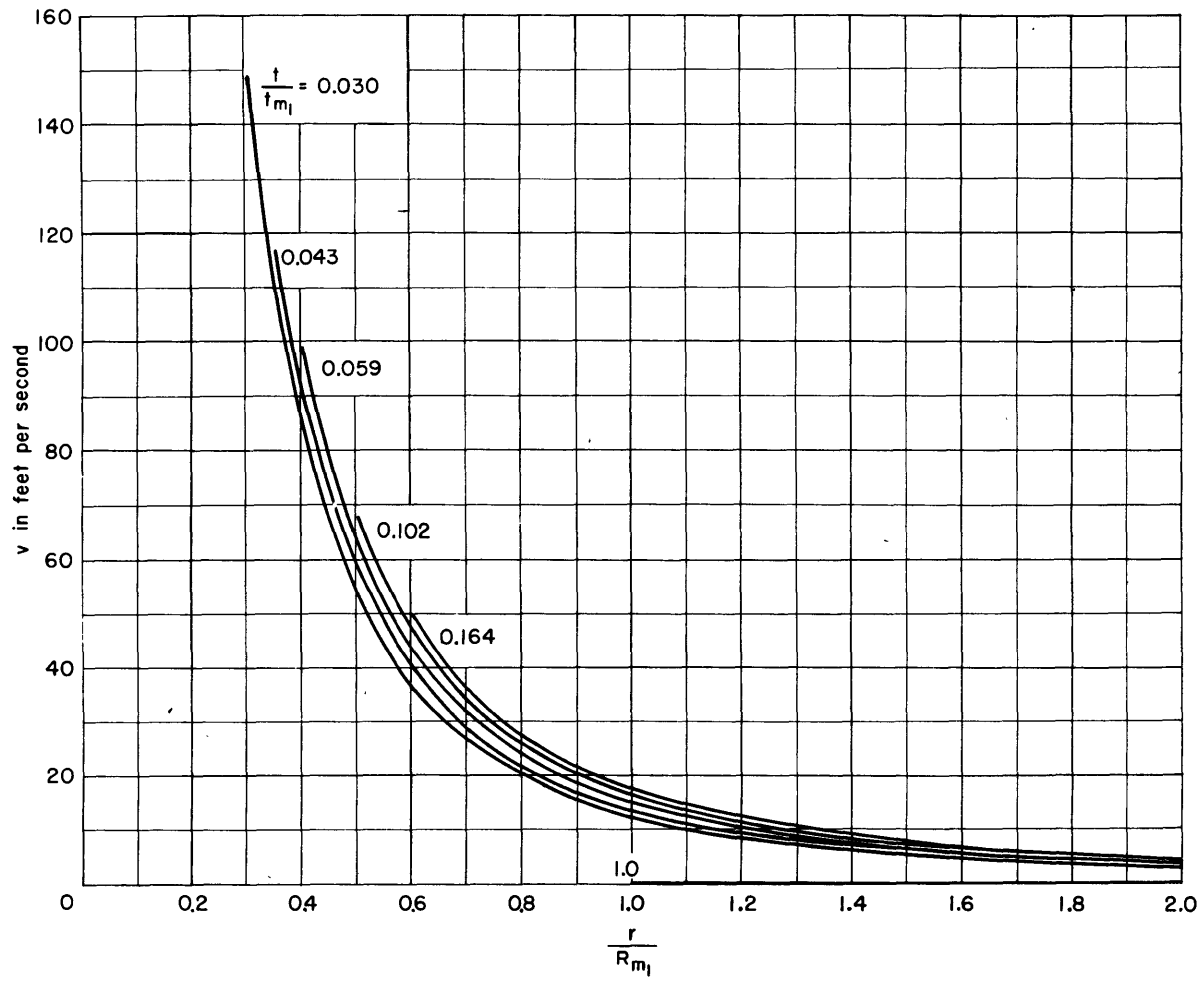

Figure 5 - Water Velocity v When p_A = 1 Atm

The curve for t/t_{m1} = 0.25 almost coincides with a segment of that for t/t_{m1} = 0.164, t/t_{m1} = 0.36 similarly with 0.102, t/t_{m1} = 0.53 similarly with t/t_{m1} = 0.030. The left end of each curve refers to a position at the surface of the gas globe.

sound in water to travel the extra distance as represented by the increase in r.

From the plots it is evident that the highest pressure in the water is always located at the shock front, and the lowest at the surface of the gas globe. This results from the fact that the pressure of the gas sinks very rapidly in consequence of its tremendous rate of expansion during the early stages. For a time, a second low maximum of pressure is in evidence at a moderate distance from the surface of the gas globe. This maximum results from the inertia of the outrushing water, which crowds upon the water farther out and so raises its pressures.

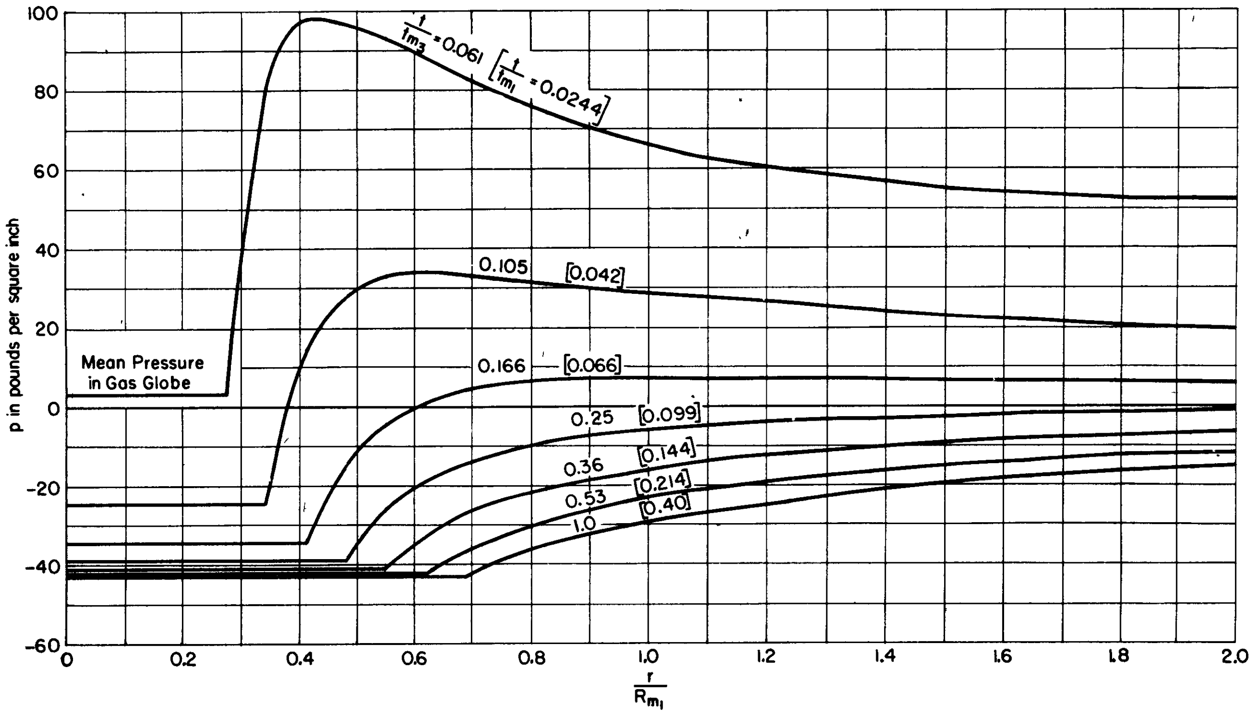

Figure 6 - Excess Pressure p When p_A = 3 Atm

Except in the earliest stages, a low maximum in the particle velocity always accompanies the shock front, but the highest particle velocity soon comes to occur at the surface of the gas globe, where it is associated with the rapid expansion of the gas. After the departure of the shock wave to considerable distances, the particle velocity soon becomes nearly inversely proportional to the square of the distance r from the center. This is in harmony with the law that holds for the spherically symmetrical outflow of an incompressible fluid.

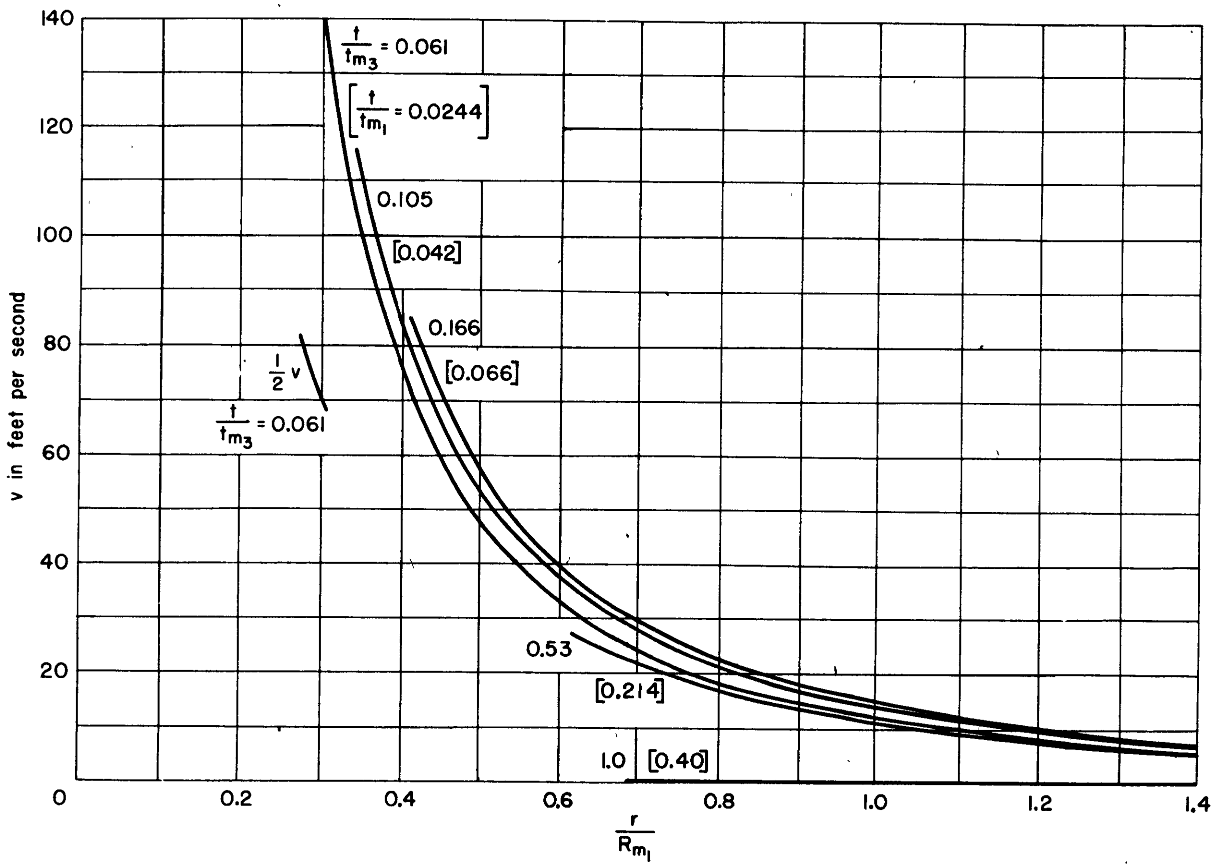

Figure 7 - Particle Velocity v When p_A = 3 Atm

The curve for t/t_{m3} = 0.25 almost coincides with that for t/t_{m3} = 0.166 beyond r/R_{m1} = 0.48, and that for t/t_{m3} = 0.36 with that for t/t_{m3} = 0.105 beyond r/R_{m1} = 0.55. The left end of each curve refers to a position at the surface of the gas globe.

THE PRESSURE AT A GIVEN POINT

In Figure 8 is shown one example of the pressure p as a function of the time t at a fixed point in space. Only the shock-wave part of the pressure-time curve is shown. The curve was drawn for the distance r = $0.72R_{m1}$, but according to existing data this part of the pressure-time curve should have nearly the same shape when observed at any value of r between $0.5R_{m1}$ and $0.9R_{m1}$; all pressures would, however, be changed in proportion to the peak pressure. This part of the pressure-time curve should also not vary much as the hydrostatic pressure is varied in the range from 1 atmosphere to 3 atmospheres.

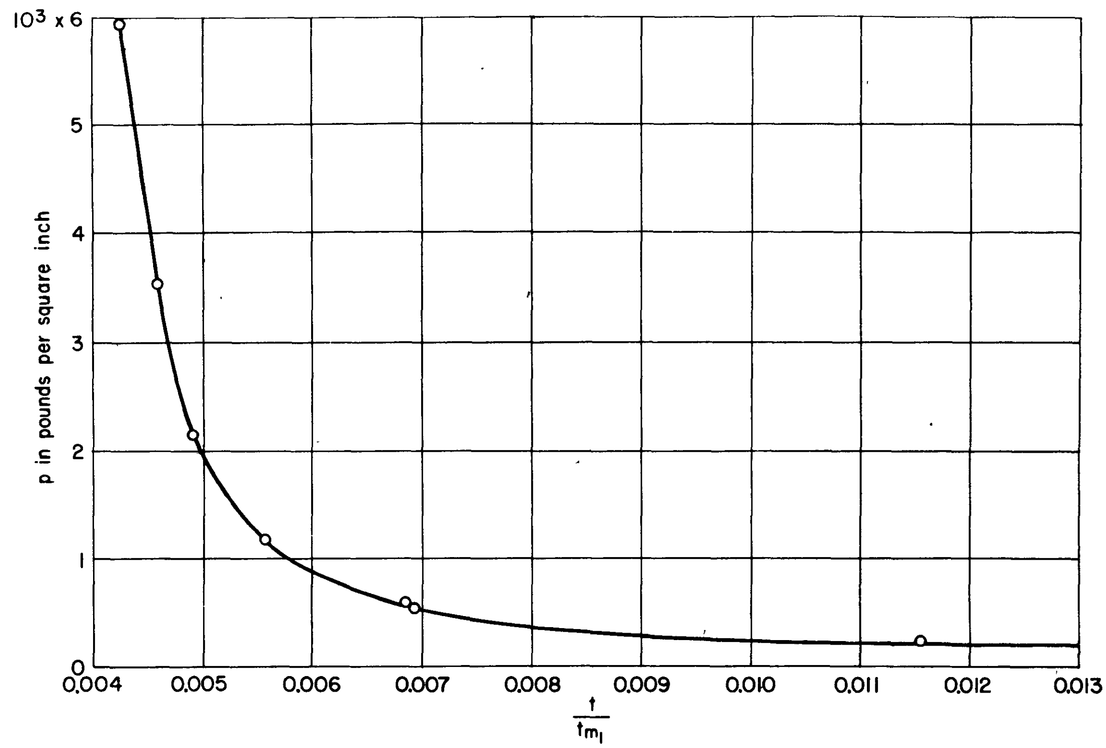

Figure 8 - Pressure-Time Curve at a Certain Point

Shock-wave pressure at $r = 0.72R_{m1}$, as a function of time t where r is the distance from the center of the charge and t_{m1} is the time of expansion to the maximum radius R_{m1} under hydrostatic pressure of 1 atmosphere. Circles show the calculated points.

ORIGIN OF THE CURVES

The data shown on the plots in Figures 3 through 8 preceding have been put together from various sources.

For the very early stages the calculated values given by Penney and Dasgupta (2) were adopted, with all pressures and particle velocities increased by 15 per cent in accordance with a later suggestion by Penney (3).

For the peak pressure in the shock wave at later times, good observations have been reported from the NDRC Group, Division 8, working at the Woods Hole Oceanographic Institute (4). The peak pressure p_m above hydrostatic pressure is closely given by the formula

$$p_m = 20{,}700\left(\frac{W^{\frac{1}{3}}}{r}\right)^{1.15} \text{ pounds per square inch}$$

where W is in pounds of weight and r is the distance from the center of the charge in feet. This formula is plotted in contour style, in Figure 9, on a basis of W and r on logarithmic scales. The initial charge radius is also shown on the assumption of spherical form and specific gravity of 1.5. To

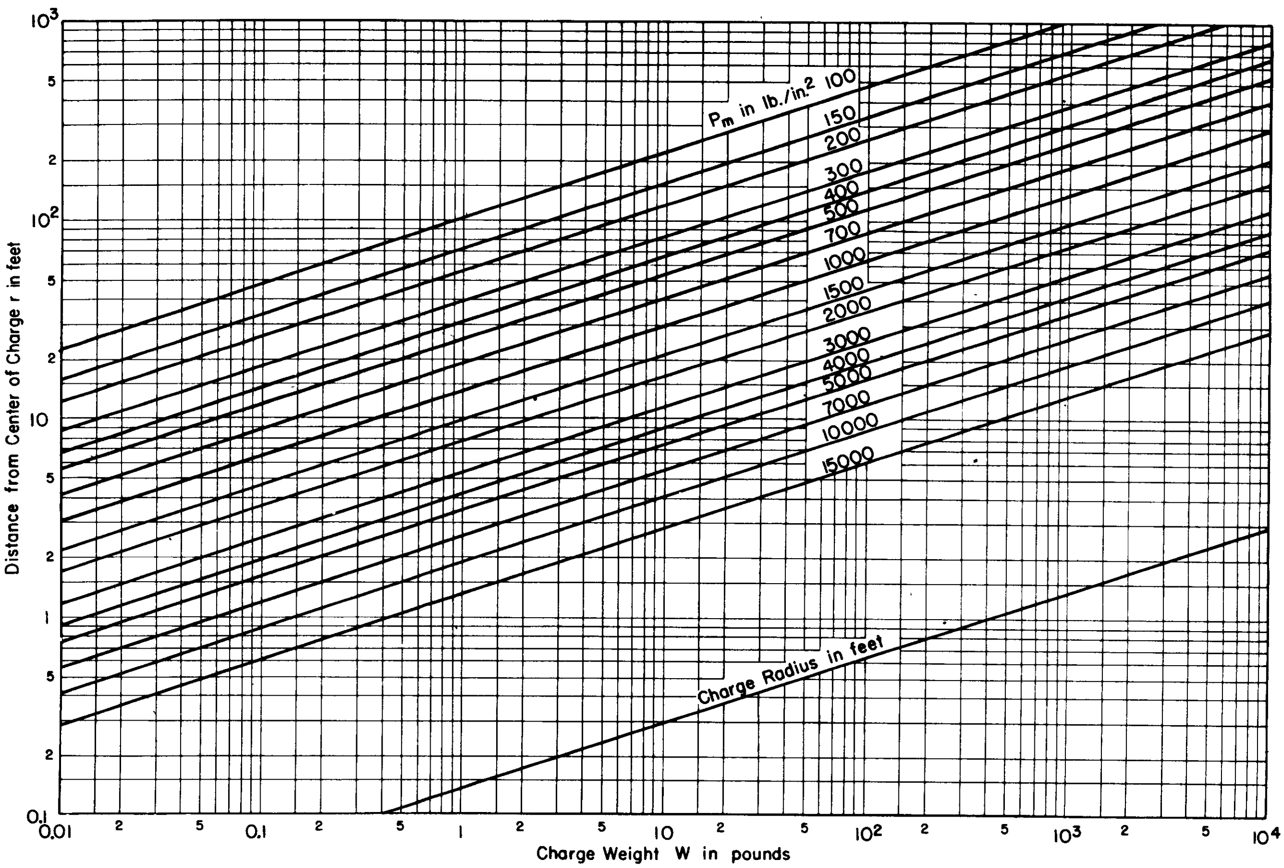

Figure 9 - Maximum Pressure p_m of Shock Wave as a Function of Charge Weight W and Distance r from Center of Charge, for Cast TNT

Each straight line refers to a certain value of p_m as indicated on the plot. The scale is log-log.

illustrate the use of the plot, the oblique line labeled 3000 shows, for example, that a shock-front pressure of 3000 pounds per square inch occurs at 54 feet from a 1000-pound charge, or at 5.4 feet from a 1-pound charge.

The pressure at a given point begins decreasing with time roughly according to the equation

$$p = p_m e^{-\frac{t'}{\theta}}$$

where e = 2.718....,

t' is the elapsed time since the arrival of the shock front, and

θ is a constant.

After p has sunk to about $p_m/4$, however, it decreases less rapidly than this equation implies. When an experimental curve is used, θ is commonly taken off as the value of t' at which $p = p_m/e$.

The value of θ is peculiarly subject to instrumental error and is perhaps also more variable than p_m. Some data from the explosion of about 220 pounds of TNT, obtained at the Underwater Explosives Research Laboratory, Woods Hole, were kindly supplied by Mr. J.S. Coles. After averaging for various orientations of the slightly asymmetric charge, these data give for 1000 pounds: θ = 0.76, 0.73, and 1.05 millisecond at distances of 33, 49.6, and 165 feet, respectively. These are somewhat larger values than those previously published from Woods Hole but are considerably below those indicated by curves obtained 20 years ago in England. Accordingly, for Figures 3c, 3d, and 3e, θ was taken to be 0.76, 0.74, 0.76 millisecond at 29.8, 45.9, and 73.7 feet, respectively, from a 1000-pound charge.* Then, as suggested with fair consistency by the curves, p was assumed to equal $p_m/5$ at $t' = 2\theta$ and $p_m/10$ at $t' = 4\theta$.

These data were used as a basis for the shock-wave portion of Figures 3c, 3d, and 3e on pages 7 and 8. In constructing a space curve referring to a particular time t_1, it was assumed that the spatial distribution of pressure in the wave generates the values of the pressure as observed in succession at a given point by the following process: Each part of the space wave advances at the speed of sound to the position occupied by the shock front at the initial time t_1, and in doing so decreases in inverse proportion to the distance from the center of the charge.

The particle velocity v was calculated from the usual acoustic formula for a diverging spherical wave, or, as stated on page 38 of TMB Report 480 (6)

$$v = \frac{p}{\rho c} + \frac{1}{\rho r}\int p\,dt \qquad [1]$$

where ρ is the density of sea water and c the speed of sound in it. The integral $\int p\,dt$ is easily found for the exponential part of the wave, and it was estimated roughly for the subsequent part. Values of p and v obtained from the points $p = p_m/5$ and $p = p_m/10$ are shown on the plots by circles.

After the departure of the shock wave, the effects of compressibility of the water should soon become rather small, and the formulas of noncompressive hydrodynamics are then applicable. Accordingly, an independent numerical calculation of the motion with neglect of compressibility was made.

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* After this report had been prepared for publication, better values of θ were published in Reference (4). The new values for 1000 pounds, at 29.8, 45.9, and 73.7 feet, respectively, are 0.74, 0.80, and 0.88 millisecond. The effect on Figures 4 through 6 of substituting these values of θ in the calculations would be slight.

The time required for the gas globe to expand to successive fractions of its maximum radius was calculated, and also the associated distribution of pressure and particle velocity in the water.

The exploded TNT was assumed to expand adiabatically according to the calculations of Booth (7). The total amount of work done by it on the water in expanding to zero pressure was calculated to be 1.48×10^6 foot-pounds per pound of TNT. Penney's estimate (3) was accepted that about 25 per cent of this energy is carried away by the shock wave, 30 per cent is wasted in heating the water near the charge, and 45 per cent is left, except for a small residue in the gas, as kinetic energy in the water. This kinetic energy becomes converted into work against hydrostatic pressure at the instant of maximum expansion of the gas globe, and the value of R_m, the maximum radius attained by the gas globe, was calculated on this assumption.

A comparison was made between the values of the pressure and of the particle velocity at the surface of the gas globe as found from the noncompressive calculation and the values found by Penney and Dasgupta, at the last stage to which their calculations were extended, in which the gas-globe radius is $R = 0.0706R_m$. The noncompressive values were found to be, respectively, 30 per cent higher and 9 per cent lower than the values of Penney and Dasgupta. Relying on this rough fit, and in default of anything better, it was assumed that from this point on the gas-globe radius would vary with time as it was found to do in the noncompressive calculation.

A partial correction for compressibility of the water was then introduced into the noncompressive results, consisting merely of an allowance for the finite rate of propagation of effects outward through the water. Some of the values obtained in this manner for p are shown by the crosses in Figures 3c, 3d, and 3e, and some of those for v by the triangles. These values of p obviously join on smoothly with those obtained from the shock-wave data. The curves for v were drawn through the region of junction by estimation.

Values of the pressure at two later times, uncorrected for compressibility, are shown by crosses in Figure 4.

Great accuracy cannot be claimed for curves constructed by such procedures, but it is hoped that the curves given here may be of some use.

FURTHER DETAILS OF THE MODE OF CALCULATION

The values of the absolute temperature, pressure, and volume as found by Booth (7) are shown in Table 1, converted into pounds per square inch and cubic inches per pound of TNT, as well as the values of the energy W in the gas as calculated from these data by numerical integration of $\int p_g dt$.

TABLE 1

T degrees K	p_g pounds per square inch	V cubic inches per pound	W foot-pounds per pound of charge
3128	111.317×10^4	18.45	1480.0×10^3
3000	96.512	19.57	
		20.33	1325.2
2800	75.139	21.58	
2600	57.029	23.82	
		24.39	1091.4
2400	41.746	26.40	
2200	28.768	29.74	
		30.49	898.5
2000	18.285	34.48	
		40.64	750.6
1800	101.602×10^3	42.84	
		60.97	627.3
1600	45.965	61.95	
1500	30.102	78.35	
1400	19.155	103.64	
		121.94	496.5
1300	117.784×10^2	144.08	
1200	69.687	211.20	
1100	41.238	313.81	
		385.61	354.9
1000	24.012	476.20	
900	1349.515	747.76	
		1.219×10^3	255.2
800	699.915	1.264	
700	345.100	2.224	
		3.856	181.2
600	160.225	4.086	
500	66.280	8.204	
		12.19	126.4
400	24.940	17.42	
		38.56	87.4
300	7.543	43.17	84.9

The pressure of the gas is denoted here by p_g. The energy is expressed in foot-pounds per pound of TNT and is assumed to be zero at zero pressure. For purposes of interpolation as a basis for the use of Simpson's rule in integrating, more nearly horizontal curves were obtained by plotting $p_g V^2$ against $1/V$ or $p_g V$ against $\log V$. At very low pressures, the law of ideal gases was assumed in order to calculate p_g, with $\gamma = 1.32$ as suggested by Booth's last points; here γ is as usual the ratio of the two specific heats.

The radial velocity of the surface of a spherical gas globe of radius R, surrounded by incompressible fluid of density ρ, can conveniently be written

$$\frac{dR}{dt} = \sqrt{\frac{2p_0}{3\rho}}\left[(1 + W_m - W)\frac{1}{y^3} - 1\right]^{\frac{1}{2}}, \quad y = \frac{R}{R_m}$$

where p_0 is the total hydrostatic pressure or the pressure in the water at infinity,

W is the energy in the gas,

R_m is the maximum value of R, at which $dR/dt = 0$, and

W_m is the simultaneous value of W.

From this formula values of dR/dt were calculated and plotted as a function of y. Intervals of time were then determined by integrating

$$t = \int \left(\frac{dR}{dt}\right)^{-1} dR = R_m \int \left(\frac{dR}{dt}\right)^{-1} dy$$

From $y = 0.9$ to $y = 1$, however, an expansion in powers of $u = \sqrt{1 - y}$ was integrated term by term.

The integration was extended only down to $R/R_m = 0.0706$, corresponding to the last point given by Penney and Dasgupta, and their value of the time from detonation to this point was adopted, changed in scale in proportion to $W^{1/3}$. The use of noncompressive increments of time from this point onward undoubtedly introduces a considerable relative error into some of the small earlier times, but the relative error in the times to large radii will be small.

The excess pressure p above hydrostatic pressure and the particle velocity v were then calculated at several distances r from the formulas

$$v = \left(\frac{r}{R}\right)^2 \frac{dR}{dt}$$

$$p = \frac{R}{r}\left[p_g - p_0 + \frac{1}{2}\rho\left(\frac{dR}{dt}\right)^2\right] - \frac{1}{2}\rho v^2$$

where p_0 is the total hydrostatic pressure, including atmospheric pressure, at the level of the gas globe. These formulas are readily obtained from those given on pages 46 and 47 of TMB Report 480 (6).

The correction for compressibility was based on the following considerations. The familiar equations of motion for spherically symmetrical motion may be written

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial r}$$

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 v) = 0$$

For slow motion and small changes of pressure, second-order terms may be dropped, so that $v\ \partial v/\partial r$ may be dropped and ρ may be treated as a constant except in its derivative $\partial\rho/\partial t$; also, $\Delta p = c^2\Delta\rho$ where $c = \sqrt{dp/d\rho}$ and represents the speed of sound. It is then found that the field around an expanding gas globe can be represented by an outwardly traveling sound wave, and in it the values of p and v differ from those given by noncompressive theory in just two respects:

(a) They are retarded in time, so that those values of p and v which, according to the noncompressive calculation, occur at a given point P at a time t, when the gas-globe radius has attained a certain value R, actually occur at P at the time at which a sound wave leaving the surface of the gas globe at time t reaches P;

(b) The particle velocity v contains an additional term $p/\rho c$.

In the present calculations, the noncompressive calculations were not corrected for (b), because this correction is appreciable only where high values both of p and of the velocity occur, so that the dropping of the second-order terms is not really justified.

As to the velocity of sound, data given by Penney and Dasgupta (2) for pure water show a fractional increase with pressure p, up to p = 140,000, almost exactly equal to p/100,000, where p is in pounds per square inch. Thus if p = 10,000, the increase is 10 per cent. The velocity of a shock front relative to uncompressed water ahead of it exceeds that of an infinitesimal sound wave by almost exactly 2/3 of the fraction p/100,000, so that a shock front in which p = 10,000 travels 6.7 per cent faster than the ordinary speed of sound. Theoretical considerations indicate that it is more accurate to allow for the increase in the speed of sound c even in using the acoustic formula [1] to calculate v.

The calculations were actually made for a charge of 1000 pounds, and the charge was assumed to be surrounded by sea water, in which at small pressure c = 4930 feet per second. In default of better data, c was assumed to vary in the same manner as has just been described for pure water. The calculations underlying Figure 3 were likewise made for pure water rather than sea water, but the difference should be slight.

Only a moderate number of points were calculated, since the accuracy of the results is limited in any case. Interpolation was then resorted to in applying the correction for retardation; for the pressure, curves representing calculated values of the product pt at a given distance from the charge were drawn.

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