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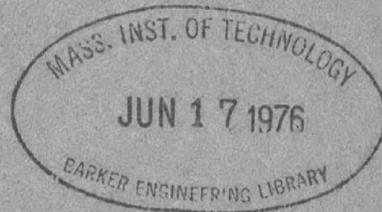
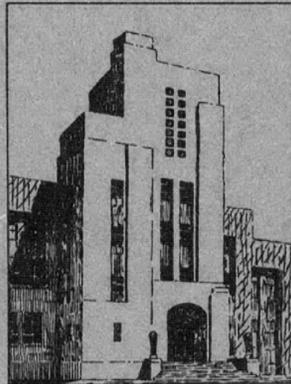
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THE DAVID W. TAYLOR MODEL BASIN

UNITED STATES NAVY

ANALYSIS OF MOTIONS AND STRESSES IN A
HIGH-IMPACT SHOCK TESTING MACHINE

BY B. L. MILLER, Ph.D.



RESTRICTED

JULY 1944

REPORT 536

NAVY DEPARTMENT
DAVID TAYLOR MODEL BASIN
WASHINGTON, D. C.

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DAVID TAYLOR MODEL BASIN

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PERSONNEL

The strain measurements described in this report were made by Dr. B.L. Miller and F. Harper of the David Taylor Model Basin during calibration tests conducted by the Engineering Experiment Station, Annapolis, on a shock testing machine for medium-weight equipment. Engineering Experiment Station personnel assisted in making the electronic measurements and in analyzing the records. This report is the work of Dr. Miller. Helpful suggestions were made by Captain W.P. Roop, USN, W.J. Sette, and G.E. Hudson.

ANALYSIS OF MOTIONS AND STRESSES IN A HIGH-IMPACT
SHOCK TESTING MACHINE

ABSTRACT

Dynamic strains produced in the table holding-down bolts and in other parts of the Engineering Experiment Station H-I* Shock Testing Machine for Medium-Weight Equipment during its operation were measured with SR-4 metallic strain gages. The stresses set up in the bolts by the most severe hammer impacts were found to be less than 50,000 pounds per square inch. This is about one-half the value calculated on the assumption that all the energy of the hammer blow is absorbed by the bolts. Possible reasons for the low stresses are discussed. It is concluded that the reduction is caused by a loss of energy at hammer impact and by the distribution of the remaining energy among the bedplate, table, and bolts.

INTRODUCTION

A warship engaging in battle need not be sunk to be deprived of its fighting effectiveness. The explosion of bombs and the impact of shell fire cause the propagation throughout the ship's structure of sudden rapid movements which may result in the failure of installations vital to the operation of the ship during combat. Such damage is termed damage from "shock" and it is of major importance to minimize these effects as much as possible.

Although it is not yet possible to specify by a single number or parameter the degree of shock that may be encountered in service, it is evident that a great deal can be accomplished by testing equipment under conditions approximating service shock as nearly as possible. Therefore, a means of generating shock in the laboratory becomes necessary for a study of shock protection.

The Navy H-I Shock Stand (1)** for testing lightweight equipment weighing less than 400 pounds partially meets this need; a number of these units have been in operation for some time. Recently, however, a machine has been constructed to test heavier installations. This was designed by the Westinghouse Electric and Manufacturing Company to test equipment weighing approximately 500 to 2500 pounds and has been installed at the U.S. Naval Engineering Experiment Station at Annapolis. It is referred to as an H-I Shock Testing Machine for Medium-Weight Equipment. A diagram of the machine is shown in Figure 1.

* "H-I" is the abbreviation adopted for "High Impact."

** Numbers in parentheses indicate references on page 16 of this report.

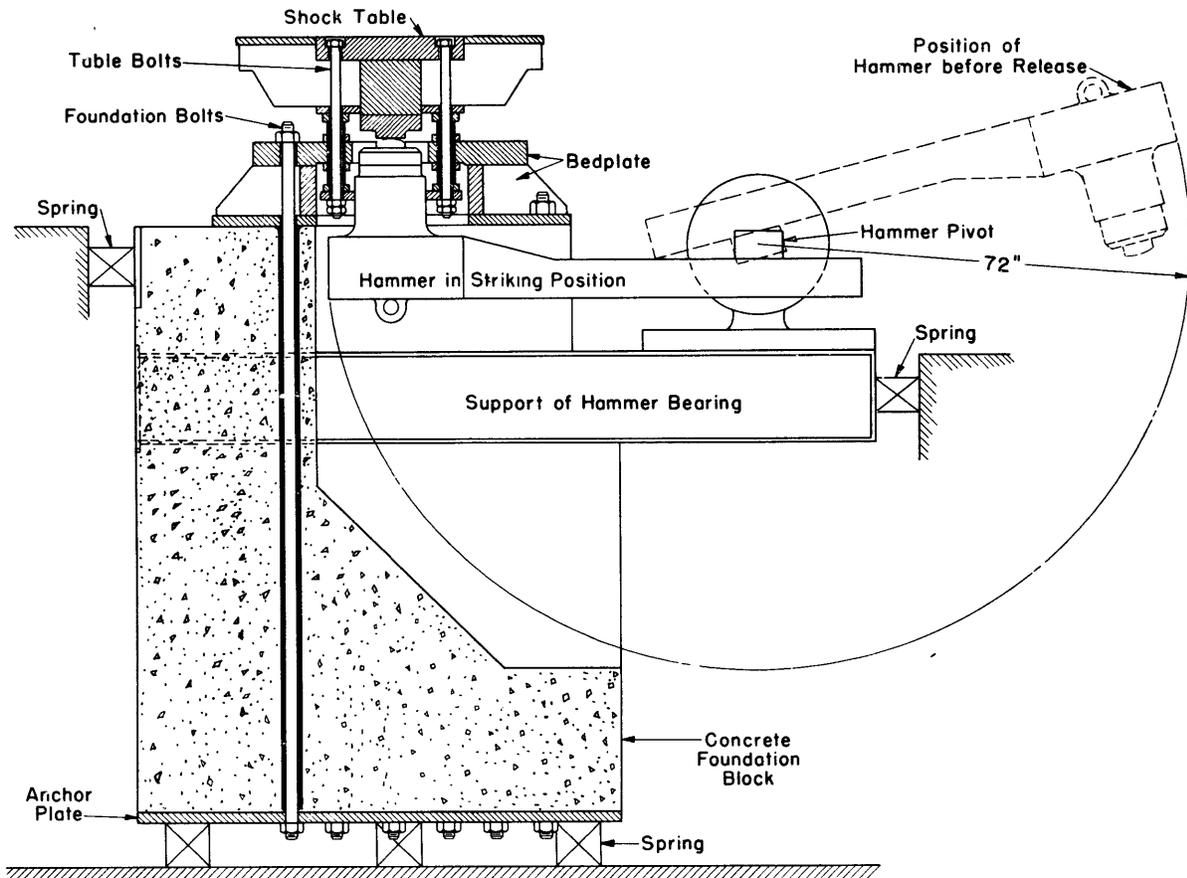


Figure 1 - The H-I Shock Machine for Medium-Weight Equipment

The machine is composed of four main parts, the foundation, the bedplate, the shock table, and the hammer, assembled as shown.

The foundation is a massive block of concrete weighing about 100,000 pounds, which is mounted on heavy springs to insulate the surrounding building from shock.

The bedplate is a steel weldment weighing 7000 pounds. It is secured on the foundation by 13 long bolts each 3 inches in diameter.

The shock table is a steel structure weighing 4000 pounds, consisting of a solid core with a horizontal plate on top of it, and 12 vertical stiffening webs extending radially out from the core to the extremities of the plate. Equipment to be tested is secured on the platform provided by the horizontal plate. The table can move 3 inches vertically, guided by 12 bolt assemblies which slide through holes in the bedplate. These assemblies consist essentially of 12 holding-down bolts, referred to as table bolts, extending from the platform down between the radial stiffeners and finally through hollow sleeves about each bolt; they serve mainly to arrest the vertical motion of the table after it has traversed the allowed clearance.

The hammer consists of a 3000-pound steel weight on the end of a box-girder arm which is adjustable to a radius of either 5 or 6 feet. The hammer swings through a channel in the concrete block in the final portion of its arc.

Before the machine was placed in operation, the Electrical Section of the Bureau of Ships calculated that the stresses in the holding-down bolts of the shock table would exceed 100,000 pounds per square inch during the high-energy impacts. The bolts were made of a chromium-molybdenum steel, SAE

4140, having a specified minimum yield strength of 85,000 pounds per square inch, and a specified minimum ultimate strength of 105,000 pounds per square inch. Since the calculated maximum stresses exceeded the specified yield strength of the bolts, and since conditions were envisaged whereby they might also exceed the ultimate strength, the Bureau of Ships requested (2) that an experimental determination of the actual stresses be made by the David Taylor Model Basin before placing the machine in routine operation.

GENERAL CONSIDERATIONS

The simple measurement of these stresses by metaelectric gages was a task which was easily and promptly performed with equipment which had been developed for other similar uses at the Taylor Model Basin. The recording of these stress values might have furnished some reassurance as to the capacity of the members to which the gages were attached for withstanding the transient loads put upon them. It would not, however, have given the same assurance for other members of the machine, or even for other parts of the same member inaccessible for application of the gages. In a word, it was necessary to give the data some *interpretation*.

In the course of a long evolution of practice in strain gage work at the Taylor Model Basin, it has become clear that the main object of this work is not, as was at first supposed, simply to explore the stress distribution in a search for concentrations. Other methods have served that purpose and the nature of these concentrations has become a part of rather common engineering knowledge.

Accurate strain measurement is undertaken, rather, at stations where design calculations can be depended upon to give a reasonably good approximation, mainly in order to obtain a check on the accuracy of these calculations.

Since no such calculations were available in connection with the tests described here, an independent analysis of the action of the machine was undertaken. The main features of this analysis are outlined in Appendix 1.

OPERATION OF THE H-I SHOCK MACHINE

Prior to the impact of the hammer the shock table is raised $1 \frac{5}{8}$ inch above the bedplate where it is held by auxiliary supports as shown in Figure 2. The hammer is then raised to any desired height up to 6 feet above the impact level. This height is measured from the level of the center of gravity of the hammer in the "up" position to its level at the instant of impact. The hammer is released from rest, swings through an arc greater than 180 degrees, and strikes the underside or core of the shock table. This blow propels the table upward the remaining $1 \frac{3}{8}$ inch of its 3-inch vertical travel, at the end of which the surfaces A on the table-bolt assemblies strike

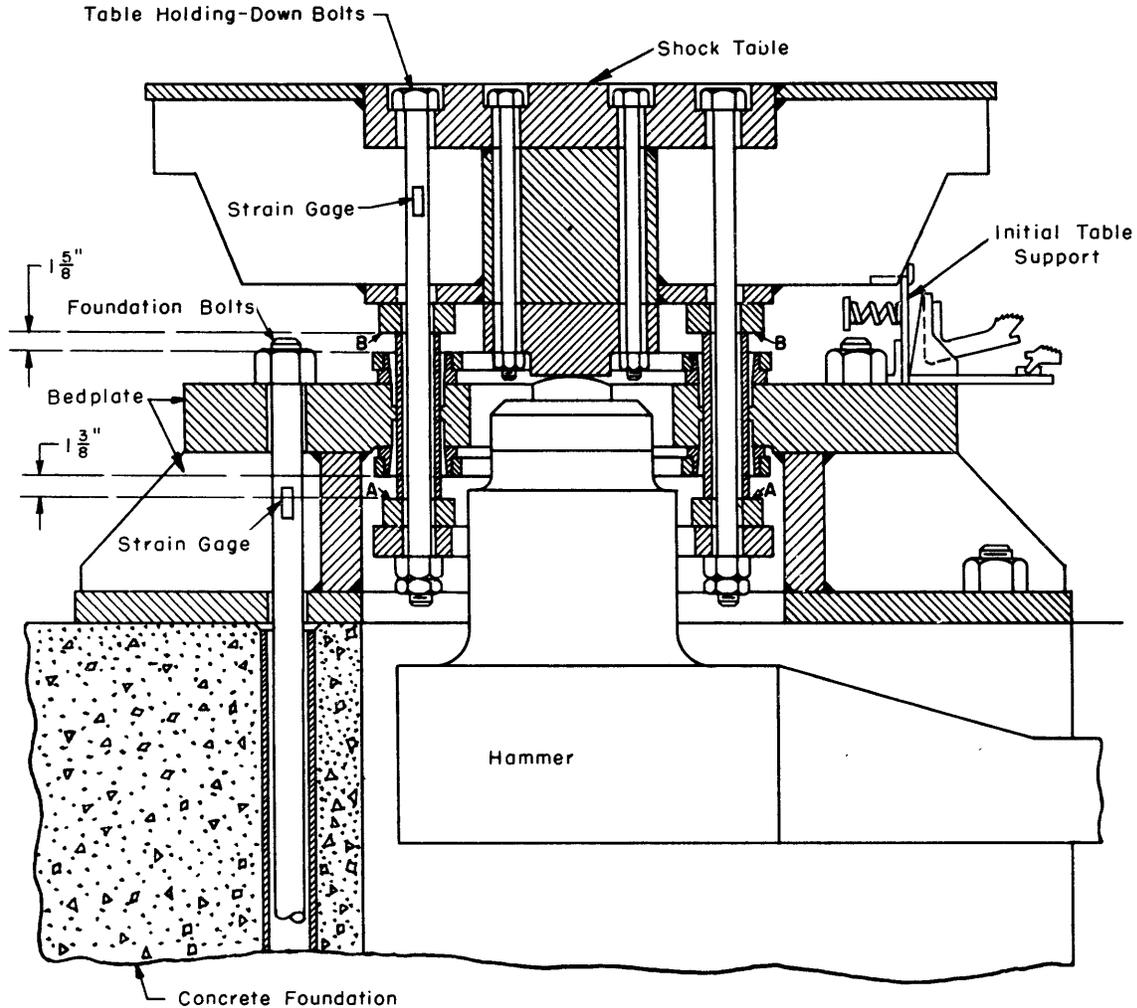


Figure 2 - Detailed Arrangement of H-I Shock Machine for Medium-Weight Equipment

the stops on the underside of the bedplate. The bolts then stretch as the moving table pulls on them, until the loading of the 12 holding-down bolts stops the table and starts it downward. The table drops the entire 3 inches because the initial supports are pulled aside by springs during the upward movement of the table. The table finally rests with the surfaces B of the table-bolt assemblies against the stops on the top of the bedplate.

Equipment which is to be shock-tested is secured to the top of the shock table and undergoes the following successive shocks:

1. The sudden upward propulsion caused by the impact of the hammer on the table.
2. The abrupt stopping of the table's upward movement when the ends of

the holding-down bolt assemblies at the points A are suddenly arrested by striking the stops.

3. The final shock when the falling table strikes the bedplate.

PRELIMINARY CALCULATION OF STRESSES IN THE TABLE BOLTS

The calculated value of the stress in the table bolts during stopping of the upward motion of the table depends on the choice of the idealized mechanical system used to represent the action of these bolts. The simplest idealization is to consider that the 12 table bolts all act as ideal massless springs, each exerting a twelfth part of the force required to stop the table. Such an arrangement is illustrated in Figure 3 and will be referred to as System 1.

The table assembly is imparted an upward velocity v by the impulse of the hammer blow. At the end of a movement of $1 \frac{3}{8}$ inch, the free ends of the springs at A are suddenly arrested by encountering rigid restraints representing the bedplate.

The kinetic energy of the motion is then converted into strain energy in the springs as they stretch, a process which represents the stressing of the bolts. This strain energy is related to the maximum tensile stress σ_m in the bolts, by the formula*

$$\text{Strain energy} = \frac{1}{2} \frac{\sigma_m^2}{E} V \quad [1]$$

in which V is the volume of the 12 table bolts and E is the modulus of elasticity of steel. The maximum possible stress would occur if the potential energy of the hammer in the raised position were converted without loss into kinetic energy of the table by the impact and then into strain energy in the bolts when they stop the table. Under these conditions Equation [1] becomes

$$Wh = \frac{1}{2} \frac{\sigma_m^2 V}{E} \quad [2]$$

in which W is the weight of the hammer and h is the height above the striking position from which the hammer is released.

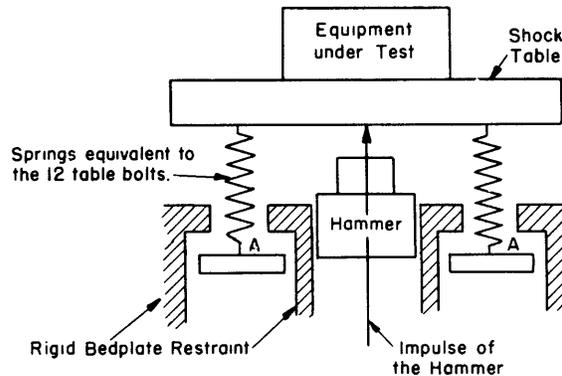


Figure 3 - System 1: A Model for Illustrating the Stopping of the Table by the Holding-Down Bolts

* The detail analysis of System 1 is carried out in Appendix 2.

Actually there are losses of energy in these transformations, but because of the lack of definite information as to their magnitude prior to the operation of the machine, these losses were neglected in calculating the maximum possible stresses in the table bolts. Thus in this calculation it was assumed that there was no frictional loss in the hammer bearing as the hammer dropped, and that there was no loss of energy during the impact of the hammer on the table. It was recognized that there would also be some energy left in the rebounding hammer, but as shown in Appendix 1 for the case in which the table is loaded with the minimum design load of 500 pounds, this may amount to only 4 per cent of the energy of the hammer prior to impact.*

Assuming that all the energy goes into straining the table bolts and that the load is shared equally among the twelve bolts, the peak magnitude of the stress was calculated from Equation [2] as 107,000 pounds per square inch if the hammer is dropped from a height of 6 feet.** Furthermore it was considered possible that the table might tip slightly while rising, a condition which would result in unequal division of the load among the bolts; consequently there was a possibility that the stresses in some of the bolts would appreciably exceed this estimate of 107,000 pounds per square inch.

TEST APPARATUS

Metaelectric strain gages, SR-4 Type C, were cemented to the exposed parts of the bolts as shown in Figure 2. The strains experienced by these gages during operation of the machine were recorded by an oscillograph (3). The "sweep," i.e., the electronic time axis on the oscillograph, was initiated just before impact by contact between the hammer arm and a bronze wire attached to the bedplate. Four electronic channels, each including a battery box, an amplifier, an oscillograph, and a camera, were available for taking simultaneous records on four selected gages, together with calibrating equipment comprising a test set, a microvolter, and an oscillator.

TEST PROCEDURE

A series of tests were conducted with three different weights on the shock table, namely, 575 pounds, 1435 pounds, and 2500 pounds. In the first test the load consisted of a steel elbow mounted on Fabreeka pads; in

* It is shown in Appendix 1 that the energy imparted to the loaded table will be 11 per cent less for the maximum design load of 2500 pounds than for a load of 500 pounds, if the coefficient of restitution is the same for all impacts. Since strain energy is proportional to the square of the stress, the bolt stresses would be expected to be only 5 per cent lower.

** It is shown in Appendix 2 that the stress-time relation in the bolts is predicted to be the positive half-cycle of a sine wave lasting 1.7 millisecond for the table loaded with 500 pounds.

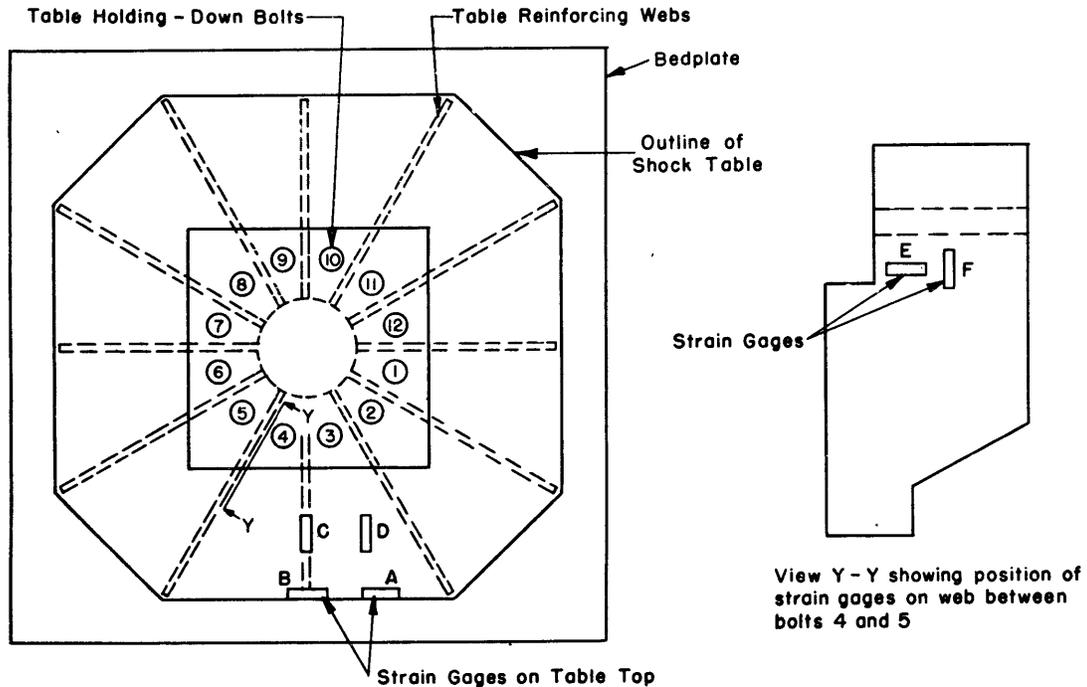


Figure 4 - Plan View of Shock Machine Showing Numbering of Table Bolts and Positions and Orientation of Strain Gages

the second and third tests the loads were steel masses clamped directly on the table. The center of gravity of each load was always over the center of the table.

In the second and third tests special attention was given to the bedplate. To clamp it more securely, the nuts on the foundation bolts were tightened until a tensile stress of 35,000 pounds per square inch existed in the bolts, as determined by strain gages attached to those bolts. Also, a lateral restraint was built around the bedplate to keep it from shifting sidewise.

Identification of the table bolts is shown in Figure 4; the foundation bolts were numbered in the same manner. In the first test, strain measurements were made simultaneously on four of the twelve table bolts, and in a few cases also on Foundation Bolt 12; strain gage measurements were also made during the first test on selected positions in the table assembly at positions A to F shown in Figure 4. In the second and third tests with the two heavier table loads, simultaneous records were made of strain in two table bolts and two foundation bolts.

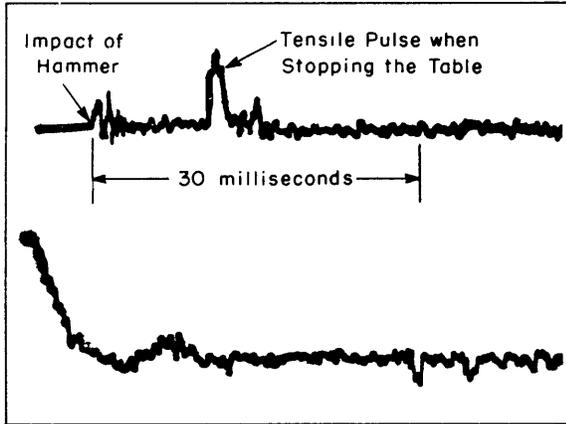


Figure 5a - 4-Foot Drop, 575-Pound Load on Table, Table Bolt 12

The maximum corresponding stress was 41,000 pounds per square inch.

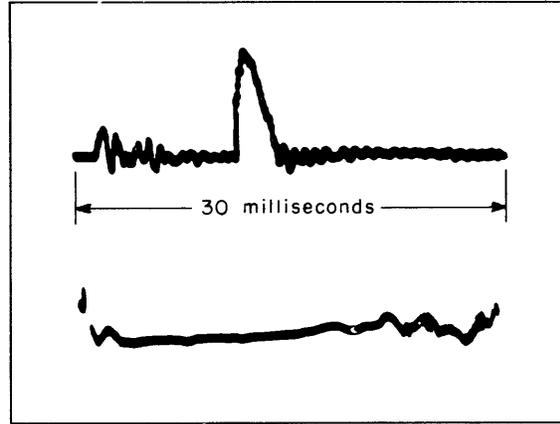


Figure 5b - 5.5-Foot Drop, 2500-Pound Load on Table, Table Bolt 4

The maximum corresponding stress was 43,600 pounds per square inch.

Figure 5 - Typical Records of Strains in Table Bolts

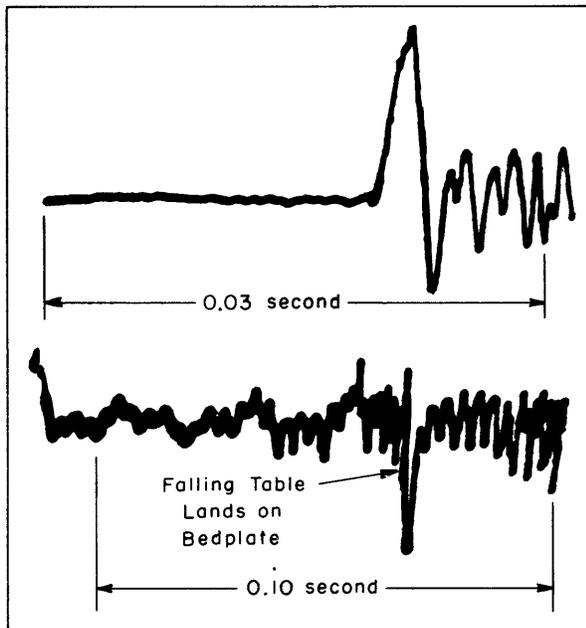


Figure 6 - Oscillogram of Strain in Foundation Bolt

This record was made with a 2-foot hammer drop and a 2500-pound load. The maximum corresponding stress is 5300 pounds per square inch.

TEST RESULTS

Figures 5a and 5b show typical oscillographic records of the strain in the table bolts* during operation of the machine. The peak values of the tensile stress and the durations of the stress pulses in the holding-down bolts are listed in Table 1 for all of the tests performed. It is evident that there is considerable scatter in these values but the greatest stress observed is less than 50,000 pounds per square inch.

Stresses in a foundation bolt were measured in only two cases with the 575-pound load; these values are given in Table 1. In the tests with 1435- and 2500-pound loads, the dynamic tensile stresses measured in the foundation bolts were superimposed on the initial tensile stress of

35,000 pounds per square inch mentioned in the foregoing; the peak values of these dynamic stresses also appear in Table 1. A typical record of stress in a foundation bolt appears in Figure 6.

* The vibration occurring in the table bolts prior to the main strain pulse is discussed in Appendix 3.

TABLE 1

Stresses* in the Table Bolts and Foundation Bolts

Load of 575 Pounds†										
Hammer Drop feet	Table Bolt 12		Table Bolt 4		Table Bolt 8		Table Bolt 2		Foundation Bolt 12	
	Peak Stress kips/in ²	Duration milliseconds								
2			7.2	1.9	8.0					
3	27.4	2.5	20.0	1.9	13.0					
3	25.4	1.9					29.0			
4	41.0	2.1	30.0	2.0	27.0	1.8	33.0		16.4	
5	40.0	1.8	32.0	1.8	Bolt 1 32.0	1.7	28.5		17.0	
5	32.0	2.0	29.0	1.9	42.0	1.8	Bolt 10 35.0	1.7		
Load of 1435 Pounds										
Hammer Drop feet	Table Bolt 12		Table Bolt 4		Foundation Bolt 2		Table Bolt 2		Foundation Bolt 12	
	Peak Stress kips/in ²	Duration milliseconds								
3			27.0	2.8						
4	44.8	2.7			5.9				4.0	
4			31.6	2.7	3.0				4.5	
4	33.2	2.4	24.2	2.9						
5	48.6	2.6	40.8	3.0	7.4				5.3	
Load of 2500 Pounds										
Hammer Drop feet	Table Bolt 12		Table Bolt 4		Foundation Bolt 4		Table Bolt 2		Foundation Bolt 12	
	Peak Stress kips/in ²	Duration milliseconds								
1	18.5	3.1	8.8	2.4					3.4	
1			8.2	2.6					3.3	
2			18.4	3.7	1.9				5.3	
3			19.6	2.9	2.0				6.7	
3	25.4	3.4	31.7	3.3	2.0				7.0	
4			30.1	3.0	3.3				8.6	
4	36.9		30.8	2.5	1.7				9.4	
5	36.6	4.0			5.5				10.6	
5	44.7	4.7	34.1	3.3	3.4					
5.45	46.3	4.4	36.5	3.3	4.2				11.7	
5.5	45.4	3.3	37.9	3.3	4.4					
5.5	41.3	3.4	43.6	3.3	4.2				12.4	

† A number of duration values are missing for the 575-pound load because the sound of the impact was picked up on two of the amplifiers on this series and distorted the latter part of the record.

The peak stresses obtained at Stations A to F in the table assembly are listed in Table 2. These were caused mainly by vibrations at a frequency of about 700 cycles per second, as illustrated in Figure 7.

Values of the maximum velocity imparted to the shock table when loaded with 575 pounds and when loaded with 2500 pounds are plotted in Figure 8. These were found by dividing the observed time between the hammer impact

* These stresses were calculated from observed strains by multiplying by an assumed modulus of 30×10^6 pounds per square inch.

TABLE 2

Peak Stresses in Table Structure with 575-Pound Load on Table

For gage positions see Figure 4.

Hammer Drop feet	Peak Stresses* in Kips per Square Inch					
	Gage A on Table	Gage B on Table	Gage C on Table	Gage D on Table	Gage E on Web	Gage F on Web
3	± 12.0**	± 13.3	± 4.6	+ 5.6	± 4.0	- 15.4 + 11.6
4		± 26.4		+ 6.8	± 5.2	- 21.0 + 15.0
5		± 19.2		+ 8.2	± 7.0	± 19.0

* These stresses were calculated from observed strains by multiplying by an assumed modulus of 30×10^6 pounds per square inch.
 ** + represents tension, - represents compression.

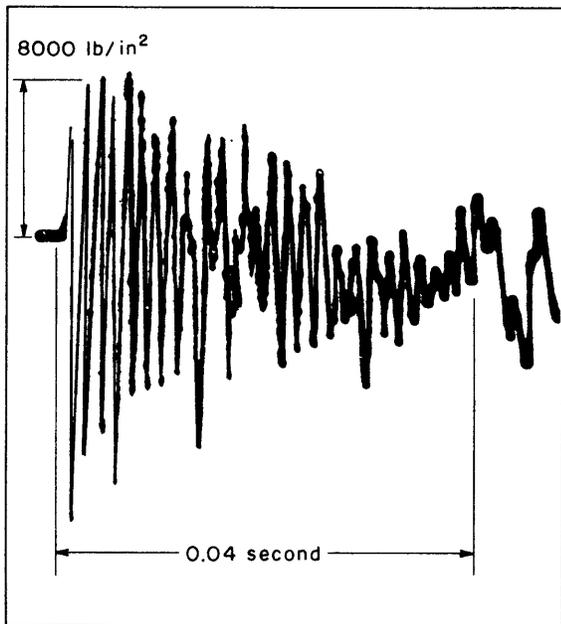


Figure 7 - Oscillogram of Strain in Table Top

This record was made at Station B with a 3-foot hammer drop. The maximum corresponding stress was 13,300 pounds per square inch. The frequency was about 700 cycles per second.

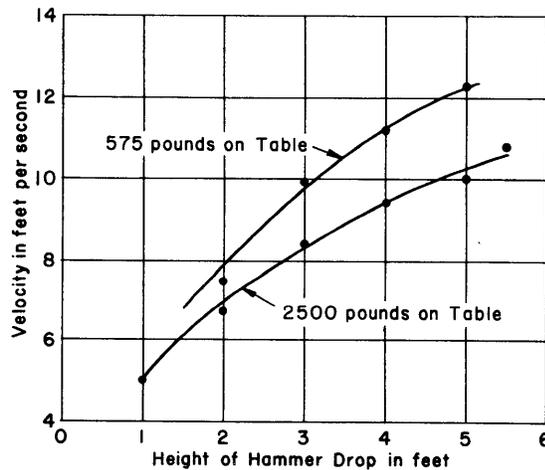


Figure 8 - Velocity Imparted to Table by Hammer Plotted on Height of Hammer Drop

and the start of the main tensile pulse in the holding-down bolts, into the 1 3/8-inch upward motion of the table. Correction for the period of acceleration of the table during the impact of the hammer and

for the effect of gravity during rising of the table were applied as set forth in Appendix 4.

DISCUSSION OF RESULTS

The maximum stresses in the table bolts showed differences among the bolts for the same drop and a great deal of scatter on repeated tests. These variations may be caused by a slight tipping* of the table when it is propelled upward, resulting in a variation in the loading of the bolts. There is no significant trend in the maximum stresses with an increase in the table load; this is in accordance with the estimate** that the variation in table-bolt stresses for different table loads in the design range should not exceed 5 per cent.

The most significant result, however, is that the table-bolt stresses were much lower than expected. For example, the highest table-bolt stress observed was 48,600 pounds per square inch, which occurred during a 5-foot drop of the hammer; the predicted maximum was 92,000 pounds per square inch for a drop of this height. Thus the highest observed value was only 53 per cent of the value predicted on the basis of System 1. Furthermore the average of all the observed table-bolt stresses for the 5-foot drop was only 44 per cent of the calculated value. It will be the purpose of the remainder of this section to explain why these stresses were so much lower than the calculated values.

To begin with, it is shown in Appendix 1 that as the hammer impact is not perfectly elastic, the kinetic energy imparted to the table is less than the value used in the theoretical calculation. From the plot of velocity imparted to the table, Figure 8, and the known weight of table and load, the kinetic energy imparted to the loaded table by the hammer can be obtained. These values of energy are listed in Table 3 for the different drops, together with the potential energy of the raised hammer relative to the impact level, and a correction for the table rising against gravity for a distance of $1 \frac{3}{8}$ inch. It is seen that the energy remaining to strain the table bolts averages about 68 per cent of the potential energy of the hammer, whereas the predicted maximum stresses were based on utilizing 100 per cent of the hammer energy in this operation. Since the energy stored in the bolts is proportional to the square of the stress, the calculated stresses should thus be reduced to 82 per cent of their estimated values. Additional causes which were not contemplated in the simple System 1 are evidently acting to lower the observed stresses.

For the simple model of System 1, Figure 3, the assumption is made that the lower ends of the table bolts at the points A are rigidly held by

* Rotation of the table by the hammer blow is discussed in Appendix 2.

** See the first footnote on page 6.

TABLE 3

Kinetic Energy Transferred to the Shock Table by the Hammer Impact

These values were obtained with loads of 575 pounds and 2500 pounds on the table.

Hammer Drop feet	Weight of Loaded Table pounds	Potential Energy of Hammer in Raised Position foot-pounds	Velocity Imparted to Table by Hammer feet per second	Kinetic Energy Imparted to Table by Hammer foot-pounds	Per Cent of Hammer Energy Imparted to Table	Loss of Energy in Rising 1 3/8 Inch foot-pounds	Kinetic Energy in Table After Rising 1 3/8 Inch foot-pounds	Per Cent of Potential Energy of Hammer Available for Straining Bolts
2	4575	6000	7.8	4310	72	520	3790	63
3	4575	9000	9.7	6650	74	520	6130	68
4	4575	12000	11.2	8900	74	520	8380	70
5	4575	15000	12.2	10600	71	520	10080	67
2	6500	6000	6.7	4550	76	740	3810	63
3	6500	9000	8.3	6995	78	740	6250	69
4	6500	12000	9.4	8910	74	740	8170	68
5	6500	15000	10.2	10500	70	740	9770	65

the bedplate during the stopping of the table. That this assumption is incorrect for the first test under a 575-pound load is indicated by the stress recorded in the foundation bolts; for example, for a 5-foot hammer drop the dynamic stress in a foundation bolt reached 17,000 pounds per square inch. This would stretch these bolts 0.08 inch with a corresponding movement of the bedplate.

A system in which the motion of the bedplate is taken into account may better represent the true conditions. System 2, in which the bedplate, instead of being represented as a rigid restraint, is shown held by springs representing the foundation bolts, is illustrated in Figure 9. The analysis of this model is carried out in Appendix 5 for a load of 575 pounds on the table.

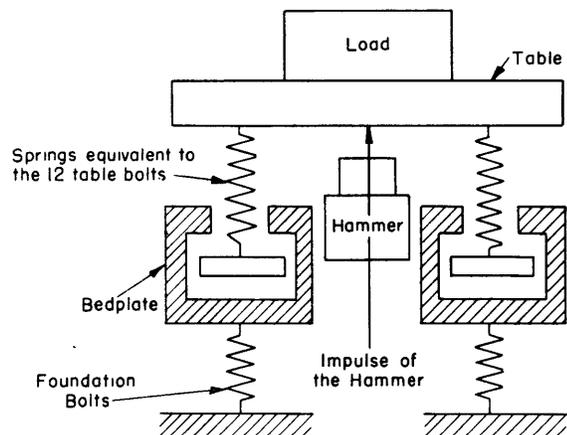


Figure 9 - System 2: A Model Which Includes the Movement of the Bedplate

The stresses in the table bolts as calculated for this system are found to be 83 per cent of those given by System 1. When this effect is taken together with the energy lost on impact, it accounts for a reduction to 68 per cent of the stresses

initially expected, but the experimental values for the 5-foot drop are on the average only 44 per cent of the value originally calculated, thus still leaving an appreciable discrepancy. Moreover the analysis in Appendix 4 predicts a stress pulse lasting only 1.4 millisecond, whereas the mean duration on the experimental records for the 575-pound load is 1.9 millisecond. The cushioning action of the foundation bolts thus does not suffice completely to account for the characteristics of the stress observed in the table bolts.

As mentioned before, in the two tests with 1435- and 2500-pound loads, the bedplate was clamped more tightly by turning down the nuts on the foundation bolts until the bolts were pre-stressed to 35,000 pounds per square inch. This was done to prevent the bedplate from lifting off and pounding down on the concrete foundation. With the bedplate thus securely held System 1 would be expected to approximate the action in the table bolts better than it did for the first test when little pre-tension existed in the foundation bolts, but it must be remembered that the foundation bolts can still stretch, relieving some of the pre-compression. It is not surprising, therefore, that the peak stresses in the later two tests actually averaged about as low as on the first test. Also instead of a sinusoidal stress pulse lasting less than 2.1 milliseconds, as calculated from System 1 for these loads, the pulses, as shown in Figure 5b, were more nearly of triangular form and lasted from 2.5 to 4.5 milliseconds.

It appears that neither of the idealized models, System 1, Figure 3, and System 2, Figure 9, is adequate to explain the results. Reconsidering the problem from the beginning, it is known that the shock table starts upward with a definite momentum and energy as a result of the hammer blow. The external forces acting on the table to stop it are gravity and the forces exerted by the 12 holding-down bolts. The impulse supplied by these forces must account for the change in momentum of the table as its motion is stopped and reversed; gravity can be neglected for the time being. However, as far as impulse is concerned, it does not matter whether the force is great and the time short, or the force small and the time long; if the product of the two or, more exactly if $\int f dt$ is a constant, the same impulse is applied and the same velocity change occurs. Consequently whatever acts to increase the duration of the forces over the values predicted by Systems 1 and 2 reduces the peak stresses.

Some figures are given in Table 4 showing approximate values of the stopping impulse, $\int f dt$, exerted by the table bolts. These values were obtained by estimating the area under the stress-time curves, multiplying by the combined cross section of the 12 bolts and averaging all records for the same height of drop. The values thus obtained are of the order of the initial upward momentum of the table as estimated from the product of the mass

TABLE 4

Momentum Imparted to the Table by the Hammer as Compared
with Stopping Impulse Supplied by the Table Bolts

Hammer Drop feet	Weight of Loaded Table pounds	Product of Mass and Initial Upward Velocity of Table pound-seconds	Average Stopping Impulse Imparted by Table Bolts pound-seconds
3	4575	1380	1420
4	4575	1590	1900
5	4575	1760	2000
3	5435	1510	1470
4	5435	1720	1680
5	5435	2230	2360
1	6500	1000	630
2	6500	1350	1280
3	6500	1670	1550
4	6500	1900	1570
5	6500	2060	2750
5.5	6500	2180	2670

and the initial upward velocity. Thus the experimental stress-time curves indicate sufficient impulse for the reversal of the observed upward motion of the table. However, the peak stresses are associated with the maximum strain energy stored in the bolts and account for only about 1/3 of the kinetic energy of the table. What becomes of the rest of this energy?

In the representations of Systems 1 and 2 previously described, the table and the bedplate have been taken as rigid masses incapable of storing elastic strain energy. This is not true; Table 2 shows that very appreciable high-frequency stresses occurred in the table assembly when the table was struck by the hammer. It thus appears that the table must be thought of as an elastic body capable of storing considerable vibrational energy, and no doubt the same is true of the bedplate, even though it is much more rigid.

It follows that the extremely sudden application of forces to both the table and the bedplate when the bolt assemblies strike the bedplate must cause elastic vibrations in both bodies, as represented in Figure 10. There is evidence on the oscillograms of increased vibration in the table from the instant when the table bolts come into action. For example, for a 3-foot drop of the hammer the bolts strike the bedplate at 0.012 second after the hammer impact; reference to Figure 7 on page 10 shows a sudden large negative strain

peak at Station B at this time. Consequently there is probably a considerable conversion of kinetic energy of bodily upward motion into vibrational energy when the table-bolt assembly strikes on the bedplate; this can account for the missing energy. Instead of passing wholly into elastic energy of strain in the bolts, part of the energy of bodily motion of the table is converted into vibration of the table and the bedplate.

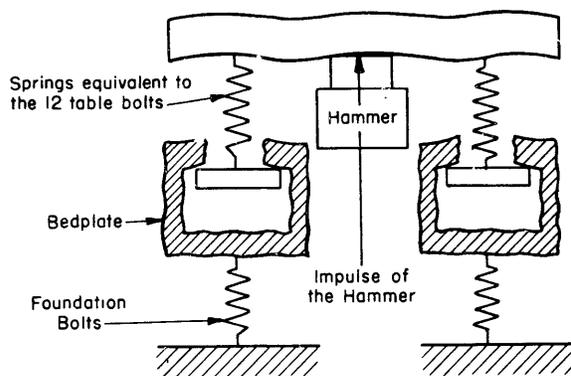


Figure 10 - Sketch Illustrating Elastic Vibration in the Table and Bedplate during the Stopping of the Table

This illustrates the oversimplification of Systems 1 and 2, the predictions of which are based on storing all the kinetic energy of the table in the holding-down bolts. In System 1 the kinetic energy of the shock-table mass goes entirely into strain energy of the springs representing the table bolts. In System 2 this strain energy is reduced by the amount of kinetic energy imparted to the bedplate and of the strain energy imparted to the foundation bolts, but this reduction is not great at the early instant of peak stress in the table bolts because of the high inertia of the bedplate mass.

The explanation thus offered for the further reduction in table-bolt stresses beyond that caused by the loss of energy on hammer impact may now be summarized. The bolts must transmit sufficient impulse to reverse the motion of the table. Since they connect two elastic bodies, namely the bedplate and shock table, it is possible to do this without experiencing the high stress which would be associated with storing the full kinetic energy of the table. The kinetic energy of the table is thus distributed among the elastic bodies of the bedplate, the table, and the holding-down bolts when the table is stopped.

It becomes clear that the primary purpose of the holding-down bolts is to exert an *impulse* equal to the full momentum of the table regardless of how the kinetic energy of the table distributes itself; but this impulse is not necessarily associated with large forces. That this primary function is satisfactorily performed has been pointed out in the foregoing.

This point may be illustrated by an extreme example: If the table bolts were infinitely rigid they could store none of the energy though they would still exert the necessary impulse to stop the table. The elastic deflections in the bedplate and table, would then alone determine the duration of the stopping impulse and consequently the magnitude of the maximum stress occurring in the table bolts.

CONCLUSIONS

1. The tensile stresses in the holding-down bolts of the table in the H-I Shock Machine for medium-weight equipment, derived from observed strains, did not exceed 48,600 pounds per square inch when the machine was loaded over the design range and the hammer was dropped from heights up to 5 1/2 feet above impact level. This value of stress is well below the yield strength of 85,000 pounds per square inch of the steel in these bolts.

2. Some of the energy of the hammer blow is lost in the impact, and some is used up in lifting the table 1 3/8 inch; when the table strikes against the stops, the kinetic energy of the table has been reduced to about 68 per cent of the energy of the hammer blow in a typical case.

3. Another loss occurs by conversion into internal vibrational energy when the table bolts act suddenly on the table and bedplate. The net result is that the table-bolt stresses are much lower than the values derived on the assumption that the original hammer energy is converted entirely into tensile energy in the bolts when they stop the table.

4. The table bolts, while exerting a smaller maximum force* than anticipated, transmit the required impulse to reverse the table by acting for a longer time than previously calculated.

5. The maximum stresses in the table top and webs, derived from observed strains, were 26,400 and 21,000 pounds per square inch respectively; these were associated with elastic vibrations at a frequency of approximately 700 cycles per second.

REFERENCES

(1) Bureau of Ships Ad Interim Specification 17E13(Int), Electrical Equipment, Navy Class HI (High Impact) Shockproof, 1 March 1942.

(2) Bureau of Ships letter S60-(2)(350) dated 7 December 1942 to TMB.

(3) "Electronic Methods of Observation at the David W. Taylor Model Basin - Part 1 - Dynamic Actions in Hull Structures," TMB Report R-45, October 1941.

(4) "Elementary Rigid Dynamics," by E.J. Routh, MacMillan and Company, Ltd., New York, 1905, Part 4, Article 181, page 151.

* From the average of the peak stresses developed in the different table bolts the maximum deceleration of the table during the stopping of the table has been calculated and a list of the values for the different hammer drops appears in Appendix 6.

APPENDIX 1

SIMPLE THEORY OF THE IMPACT OF THE HAMMER ON THE SHOCK TABLE

TRANSLATIONAL FEATURES OF THE IMPACT

Let the constants of the moving hammer be denoted by subscripts 1 and those of the table by subscripts 2. Thus the hammer of mass m_1 and striking velocity u_1 hits the table of mass m_2 and of zero initial velocity; see Figure 11.

On contact, the pressure between the two bodies increases until their relative velocity is reduced to zero; at this instant, by the conservation of momentum, their common velocity v is given by

$$(m_1 + m_2)v = m_1 u_1$$

$$v = \frac{m_1 u_1}{m_1 + m_2} \quad [3]$$

If the two bodies are inelastic, like putty, both of them continue on at this same speed. If they are perfectly elastic bodies and if the force-time curve is symmetrical, as much velocity is imparted to the struck body as they separate as was imparted in the compression stage; the struck body goes off with a speed v_2 equal to twice the velocity v , i.e.,

$$v_2 = 2 \frac{m_1 u_1}{m_1 + m_2} \quad [4]$$

If the impact is partly elastic, partly inelastic, the imparted speed v_2 is somewhere between the values given by Equations [3] and [4], i.e.,

$$v_2 = (1 + e) \frac{m_1 u_1}{m_1 + m_2} \quad [5]$$

where e is known as the coefficient of restitution and has a value between zero and unity. These limiting values correspond to the two extreme cases of a completely inelastic and perfectly elastic impact, respectively.

In the present case the hammer weighs 3000 pounds, the shock table 4000 pounds. The loaded table weighs from 4500 to 6500 pounds as the load on the table is varied over the design range.

A comparison will now be made of the speed and kinetic energy imparted to the table with the two extreme loads 500 pounds and 2500 pounds. From Equation [5], if e has a constant value, the ratio of speeds is given by

$$\frac{3000 + 6500}{3000 + 4500} = 1.26$$

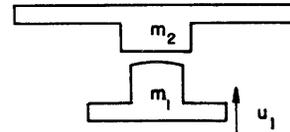


Figure 11 - Diagram Showing Impact of Hammer on Table

i.e., 26 per cent greater speed is imparted to the loaded table for the smaller load.

The kinetic energy imparted to the table is

$$E_2 = \frac{1}{2} m_2 v_2^2$$

and by Equation [5]

$$E_2 = \frac{1}{2} \frac{(1 + e)^2 m_2 m_1^2 u_1^2}{(m_1 + m_2)^2} \quad [6]$$

from which it is calculated that 11 per cent more kinetic energy is imparted to the table when loaded with 500 pounds than when loaded with 2500 pounds.

Finally the portion of the energy of the hammer blow that can be transferred to the table in a perfectly elastic impact for which $e = 1$ is readily determined. The kinetic energy of the hammer prior to impact is

$$E_1 = \frac{1}{2} m_1 u_1^2 \quad [7]$$

Thus Equation [6] may be written

$$E_2 = E_1 \frac{(1 + e)^2}{4} \frac{4 m_1 m_2}{(m_1 + m_2)^2}$$

and for $e = 1$

$$E_2 = E_1 \frac{4 m_1 m_2}{(m_1 + m_2)^2} \quad [7a]$$

If the table load is 500 pounds Equation [7a] yields

$$E_2 = 0.96 E_1$$

If the table load is 2500 pounds

$$E_2 = 0.87 E_1$$

Thus at the most 96 per cent of the hammer energy can be transferred to the loaded table, 4 per cent remaining in the rebounding hammer. Under the actual conditions of imperfect elasticity the percentage of energy transferred will be reduced by the factor $(1 + e)^2/4$.

ROTATIONAL FEATURES OF THE IMPACT

At the instant of impact the striking face of the hammer is not moving vertically upward in pure translation, but rather it is rotating about the bearing B. Thus every point in the hammer is moving in a direction perpendicular to the radius joining the bearing to the point of impact. This

condition is represented in Figure 12; in particular the point of impact is moving at an angle of approximately 17 degrees to the vertical.

If the hammer is dropped from a height h , the angular velocity ω at impact, assuming no loss due to friction in the bearing, is given by

$$\frac{1}{2} I \omega^2 = Wh \quad [8]$$

in which I is the polar moment of inertia of the hammer about the axis B and W is the weight of hammer. Now

$$I = I_c + Mr^2$$

in which I_c is the moment of inertia of the hammer relative to its center of gravity,

M is the mass of the hammer, and

r is the distance between the center of gravity of the hammer and the bearing point B.

Equation [8] becomes

$$\frac{1}{2} (I_c + Mr^2) \omega^2 = Wh$$

or

$$\frac{1}{2} I_c \omega^2 + \frac{1}{2} Mu_1^2 = Wh \quad [9]$$

in which u_1 is the vertical translational velocity $r\omega$ of the hammer. The first term $\frac{1}{2} I_c \omega^2$ is the rotational energy in the hammer, and the second, $\frac{1}{2} Mu_1^2$, the translational energy; their ratio is

$$\frac{I_c \omega^2}{Mu_1^2} = \frac{M \rho_1^2 \omega^2}{Mr^2 \omega^2} = \left(\frac{\rho_1}{r}\right)^2$$

in which ρ_1 is the radius of gyration of the hammer relative to its center of gravity. $(\rho_1/r)^2$ is calculated to be about 0.05 so that about 5 per cent of the total energy of the hammer is in the form of rotation about its own center of gravity.

The impact is therefore one between a body at rest, the table, and another body, the hammer, which although moving mainly in translation is also rotating; see Figure 13.

The surface of the hammer near the point of impact moves horizontally as well as vertically, because of the rotation, with the resultant

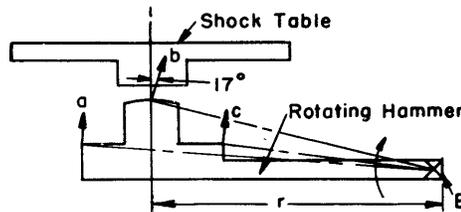


Figure 12 - Diagram Showing Hammer Rotation about the Bearing B

The arrows a, b, and c are the vector velocities of the points shown.

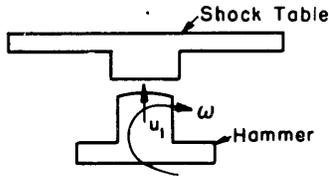


Figure 13 - Diagram Showing Translational and Rotational Motion of the Hammer

The hammer is represented as rotating about its center of gravity with angular velocity $\omega = u_1/r$ as well as moving upward with velocity u_1 .

$\tan^{-1} 0.1$, which is approximately 6 degrees. Thus sliding would be expected to occur between the hammer and the table during this contact and the net impulse exerted on the table would be inclined only 6 degrees to the vertical as shown in Figure 14.

The effect of the inclined impulse acting on the table will be to cause its center of mass to move in the direction of the impulse and also to impart a counterclockwise rotation to the table. Since the table moves upward $1 \frac{3}{8}$ inch, the 6-degree inclination will correspond to a sidewise motion of the table toward the hammer bearing B of $1 \frac{3}{8}$ inch \times $\sin 6$ degrees = 0.14 inch. This tendency of the table to move sidewise will result in a lateral pull on the bedplate, when the table bolts act. This may account for the fact that the bedplate shifted laterally toward the hammer bearing B during the first test, a situation which led to tightening the foundation bolts and adding a lateral restraint to the bedplate. It may be, however, that the rotation of the table causes the bedplate to shift, since on account of the rotation the lower table-bolt assembly moves toward the hammer and may apply a force to the bedplate in this direction.

The amount of rotational energy E_r , imparted to the table is given by

$$E_r = \frac{1}{2} I_2 \omega_2^2$$

motion inclined 17 degrees to the vertical as mentioned previously. If there were sufficient friction between the two bodies, the struck surface on the table would be carried along in this inclined direction.

Handbooks list a variety of values for the coefficient of sliding friction* between steel surfaces; for this discussion it will be assumed about 0.1. Consequently a force could not be exerted between the two surfaces at an inclination greater than

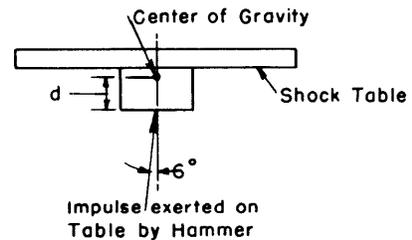


Figure 14 - Sketch Showing Direction of Impulse Acting on Table during Impact

* There is some experimental evidence that the coefficient of friction during an impact is the same as for sliding friction (4). This coefficient is commonly designated by μ .

in which I_2 is the moment of inertia of the table with respect to its center of gravity about an axis parallel to the axis about which the hammer turns, and ω_2 is the imparted angular velocity. Then

$$E_r = \frac{1}{2} \frac{(I_2 \omega_2)^2}{I_2} \quad [10]$$

Now $I_2 \omega_2$ is the angular momentum imparted and must equal the impulsive torque acting during the impact. The impulsive torque acting equals the normal impulse or translational momentum imparted, multiplied by the coefficient of friction μ and by the lever arm, d , as in Figure 14. Thus

$$I_2 \omega_2 = m_2 v_2 \mu d \quad [11]$$

so that from Equations [10] and [11]

$$E_r = \frac{1}{2} \frac{\mu^2 d^2 m_2^2 v_2^2}{I_2} \quad [12]$$

The translational energy imparted is

$$E_t = \frac{1}{2} m_2 v_2^2$$

$$\frac{E_r}{E_t} = \frac{\mu^2 d^2 m_2}{I_2} = \frac{\mu^2 d^2}{\rho_2^2} \quad [13]$$

in which ρ_2 is the radius of gyration of the table relative to its center of gravity. Now, d is approximately 12 inches and ρ_2 about 10 inches,* so that E_r/E_t will be somewhat less than μ^2 . If μ is about 0.1, the rotational energy imparted to the table will be less than 1 per cent of the imparted translational energy.

Because of the clearance about the bolt assemblies the table may tilt during rising. If the table is entirely free to rotate, then during the time t of rising, it will tilt by an angle θ , where

$$\theta = \omega_2 t$$

and from Equation [11]

$$\theta = \frac{\mu d m_2 v_2 t}{I_2}$$

Since $v_2 t$ is an upward displacement of $1 \frac{3}{8}$ inch

$$\theta = \frac{\mu d}{\rho_2^2} \times 1 \frac{3}{8} \text{ inch}$$

* This applies to the unloaded table. With load the assembly will have a higher value.

$$\begin{aligned}\theta &= \frac{0.1 \times 12 \times 1.375}{10^2} \text{ radians} \\ &= 0.016 \text{ radians} \\ &= 0.9 \text{ degree}\end{aligned}$$

so that only about 1-degree tilt of the table during its $1 \frac{3}{8}$ inch rise may be expected even if the table bolts are unrestrained against rotation.

Inssofar as the hammer is concerned an impulsive reaction is called into play at the pivot during the impact and it is this reaction, together with the impulsive reaction exerted by the table on the hammer, which cancels the rigid-body rotation.

APPENDIX 2

ANALYSIS OF SYSTEM 1 FOR STRESS IN TABLE HOLDING-DOWN BOLTS

The loaded table is represented by the mass m ; the 12 bolts by springs having a constant k as in Figure 15. The springs are held at the lower end when the points A are arrested by the bedplate. If a total force F is exerted by the 12 bolts, then Hooke's Law relates it to the extension x , assumed the same in all the springs. Hence

$$\frac{F}{A} = \sigma = E \frac{x}{l} \quad [15]$$

in which A is the total cross section of the 12 bolts or 37.8 square inches, σ is the tensile stress in the bolts in pounds per square inch, l is the length of bolts in inches or 30 inches, and E is the modulus of elasticity of steel assumed as 30×10^6 pounds per square inch.

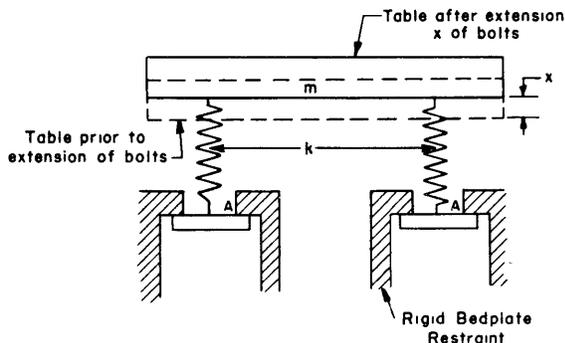


Figure 15 - System 1: Sketch Illustrating the Straining of the Table Bolts during Stopping of the Table

Thus

$$F = \frac{AE}{l} x \quad [16]$$

Since from the spring law $F = kx$, comparison shows that the spring constant must have the value

$$k = \frac{AE}{l} = 38 \times 10^6 \quad [17]$$

pounds per inch

in order to represent the action of the 12 bolts.

The strain energy E_s in a spring when extended a distance x is

$$E_s = \int_0^x F dx = \int_0^x kx dx$$

$$E_s = \frac{1}{2} kx^2$$

This may be expressed in terms of the stress in the bolts by the use of Equations [16] and [17] as

$$E_s = \frac{1}{2} \frac{F^2}{k} = \frac{1}{2} \frac{\sigma^2 A^2}{\frac{AE}{l}} = \frac{1}{2} \frac{\sigma^2}{E} Al$$

so that

$$E_s = \frac{1}{2} \frac{\sigma^2}{E} V \quad [18]$$

where $V = Al$, the combined volume of steel in all the bolts.

Assuming that the total energy of the hammer blow is finally converted into strain energy in the table bolts when the table is stopped, Equation [18] becomes

$$Wh = \frac{\sigma_m^2 V}{2E} \quad [19]$$

in which W is the hammer weight, 3000 pounds,

h is the height from which the hammer is dropped, and
 σ_m is the maximum stress attained in the bolts.

From Equation [19] it is predicted that the maximum stress in the bolts when the hammer is dropped from a height of 6 feet is 107,000 pounds per square inch. The maximum stresses for other heights of drop were calculated; the stresses are set down in Table 5.

TABLE 5
 Maximum Stress in Table Bolts as Predicted by System 1

Height of Drop feet	Maximum Stress pounds per square inch
1	43,600
2	61,700
3	75,600
4	87,300
5	97,600
6	107,000

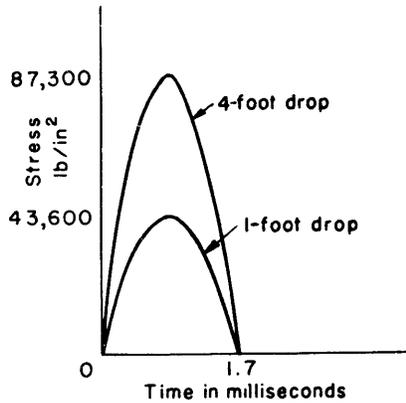


Figure 16 - Stress in Table Bolts Predicted by System 1 for Two Different Heights of Drop

The load on the table was taken as 575 pounds.

To obtain the variation of bolt stress as a function of the time during stopping, it is noted that System 1 is the familiar single-degree-of-freedom mass-spring system. Thus it must execute a simple harmonic vibration with a period independent of the initial velocity v_0 of the mass. This period T is given by

$$T = 2\pi \sqrt{\frac{m}{k}}$$

Since the mass-spring system will part from the bedplate when the mass is pulled back to its starting point by the spring, the extension in the springs will execute only one-half of a sinusoidal cycle; the duration of this pulse will be

$$t = \frac{T}{2} = \pi \sqrt{\frac{m}{k}}$$

For the case of a 575-pound load it is found that

$$t = 1.7 \text{ millisecond}$$

The stress in the table bolts as predicted by System 1 will thus appear as in Figure 16.

APPENDIX 3

OSCILLATING STRAIN IN TABLE BOLTS PRIOR TO THE MAIN PULSE

It will be noted that prior to the stopping-stress pulse an oscillating stress appears on the table-bolt records; see Figure 5. The hammer strikes the shock table, which in turn pulls the bolt assembly upward. An

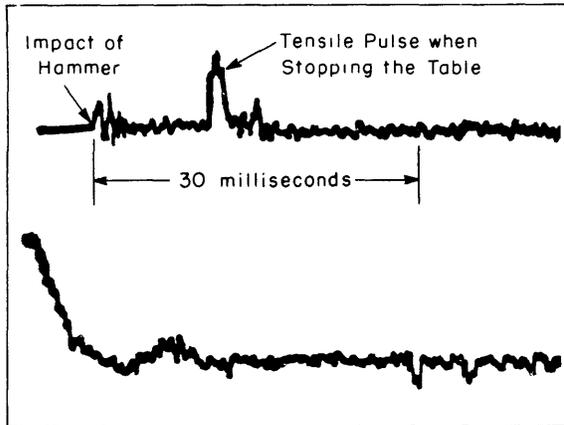


Figure 5a - 4-Foot Drop, 575-Pound Load on Table, Table Bolt 12

The maximum corresponding stress was 41,000 pounds per square inch.

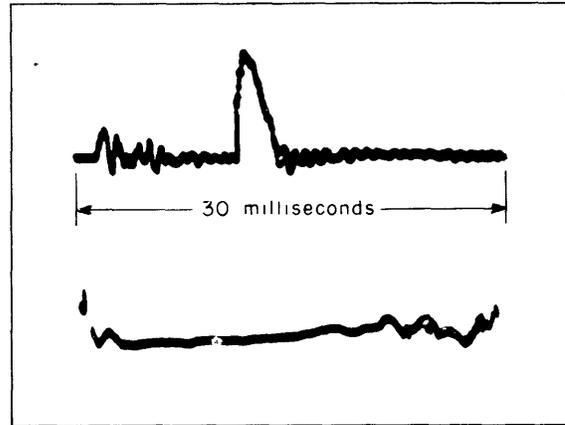


Figure 5b - 5.5-Foot Drop, 2500-Pound Load on Table, Table Bolt 4

The maximum corresponding stress was 43,600 pounds per square inch.

Figure 5 - Typical Records of Strains in Table Bolts

elastic wave travels through the table and along the bolt and initiates a vibration. The action may be visualized by reference to Figure 17. The large mass m_2 represents the shock table. The small mass m_3 represents the weight of the lower part of the bolt assembly. The table bolts are represented by

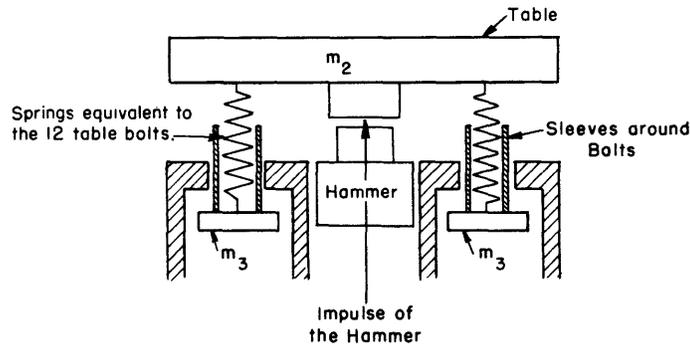


Figure 17 - Diagram of Table after Impact

When the table is propelled upward the inertia of the lower portion of the table-bolt assembly causes strain and subsequently vibration in the bolts.

springs. The hammer propels m_2 upward with a velocity v_2 while inertia detains m_3 so as to introduce a stretch in the springs and then oscillation in them. It is this oscillation which shows up as the damped vibration at the start of the record. The damping is probably introduced by the sleeve about each bolt.

The amplitude and period of this damped vibration can be simply compared with the amplitude and period of the main stress pulse. Since $m_2 \gg m_3$ this initial vibration can be considered as though m_2 were held fixed and m_3 were projected downward with the speed v_2 . In contrast, the main stress pulse in the bolts occurs later when m_3 is held fixed by the bedplate and m_2 continues up with the velocity v_2 as in System 1 of Appendix 2.

If m represents any such mass as m_2 or m_3 then the maximum stretch Δl caused in the springs by stopping the mass is given by equating the kinetic energy to the strain energy

$$\frac{1}{2} k (\Delta l)^2 = \frac{1}{2} m v^2$$

or

$$\Delta l = v \sqrt{\frac{m}{k}}$$

Thus the maximum extension is proportional to \sqrt{m} since v is the same in both these cases. Since the period T of such a vibration is

$$T = 2\pi \sqrt{\frac{m}{k}}$$

the period is also proportional to \sqrt{m} .

The maximum stress in the oscillating portion and that in the main stopping pulse of the record should thus be in ratio $\sqrt{m_3/m_2}$; the same ratio holds for the periods and thus also for the ratio of the duration of the first positive pulse in the oscillating portion to the duration of the main stress pulse.

The weight of each bolt is approximately 40 pounds, totaling 480 pounds for 12 bolts. The weight of the table varies from 4575 to 6500 pounds. Therefore $\sqrt{m_3/m_2}$ ranges from 0.32 to 0.27.

The data have not been carefully analyzed on this point but reference to Figure 5 shows that the maximum stress and duration of the first positive pulse in the oscillating stress are roughly from 0.25 to 0.50 of the values for the main stress pulse.

APPENDIX 4

VELOCITY IMPARTED TO THE TABLE BY THE HAMMER BLOW

The strain records on the table bolts show the time elapsed between the hammer impact and the bottoming of the bolt assemblies at the points A after 1 3/8 inch of upward motion. From this information there is obtained the average velocity \bar{v} of the table while rising. To obtain the peak upward velocity, i.e., the velocity imparted to the table by the impact, this average speed is to be corrected for the period of acceleration during contact with the hammer and for deceleration by gravity forces during the 1 3/8-inch rise. The time of acceleration is of the order of 1 millisecond, according to acceleration measurements made by the Naval Research Laboratory. The velocity is assumed to increase linearly with time during contact between the hammer and the table. Thus the graph of velocity is as shown in Figure 18. The distance traversed during acceleration is $\frac{1}{2} v t_0$ where v is the peak velocity. During the time of deceleration due to gravity $t - t_0$, the displacement is

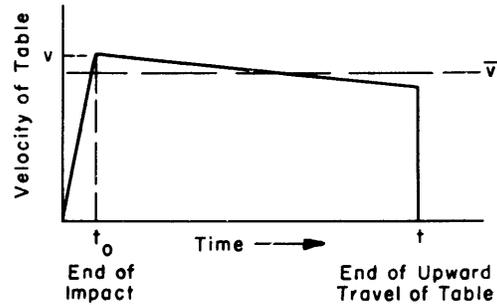


Figure 18 - Velocity of Table during 1 3/8-Inch Upward Travel

$$\frac{1}{2} (t - t_0) \left[v + v - g(t - t_0) \right]$$

The total displacement is 1 3/8 inch, which is equal to $\bar{v} t$; accordingly

$$\bar{v} t = \frac{1}{2} v t_0 + (t - t_0) \left[v - \frac{1}{2} g(t - t_0) \right]$$

$$v \left(t - \frac{1}{2} t_0 \right) = \bar{v} t + \frac{1}{2} g(t - t_0)^2$$

$$v = \frac{\bar{v} t \left[1 + \frac{1}{2} g \frac{(t - t_0)^2}{\bar{v} t} \right]}{t - \frac{1}{2} t_0}$$

$$v \doteq \bar{v} \left(1 + \frac{1}{2} \frac{t_0}{t} \right) \left[1 + \frac{1}{2} g \frac{(t - t_0)^2}{\bar{v} t} \right]$$

Of the two terms, the first, not involving g , $1 + \frac{1}{2} \frac{t_0}{t}$, gives the correction factor to be applied to \bar{v} in order to allow for the finite time of

acceleration, provided that t is sufficiently larger than t_0 . It ranges from 2 per cent for a drop of 1 foot to 4 per cent for a drop of 5 feet. The second term is the correction for gravity and varies from 8 per cent down to 1 per cent. Thus the average upward speed of the table must be increased from 10 per cent on the 1-foot drop of the hammer to 5 per cent for the 5-foot drop to find the top speed imparted to the table by the blow.

APPENDIX 5

STRESSES IN TABLE BOLTS AND FOUNDATION BOLTS FOR 575-POUND LOAD AS PREDICTED FOR A SYSTEM OF TWO DEGREES OF FREEDOM

This system differs from the single-degree-of-freedom system of Appendix 2 in that the bedplate is not taken to be rigidly held but rather the elasticity of the foundation bolts is taken into consideration. The mass of the loaded table is represented by m_1 , the combined spring constant of the table bolts by k_1 . The bedplate is treated as a rigid mass m_2 and the foundation bolts are replaced by springs of resultant constant k_2 .

The lower ends of the foundation-bolt springs may be thought of as fixed because they are held by the weight of the 100,000-pound block of concrete. In the present discussion it is assumed that the foundation bolts are not pre-stressed; this is approximately the condition for the first of the three tests. Measuring displacements from the initial positions of the masses as in Figure 19 the displacements of m_1 and m_2 are governed by the equations

$$\begin{aligned} m_1 \frac{d^2 x_1}{dt^2} &= k_1 (x_2 - x_1) \\ m_2 \frac{d^2 x_2}{dt^2} &= -k_1 (x_2 - x_1) - k_2 x_2 \end{aligned} \quad [20]$$

with the initial conditions

$$x_1 = x_2 = 0; \quad \frac{dx_1}{dt} = v_0 \quad \text{and} \quad \frac{dx_2}{dt} = 0 \quad \text{at} \quad t = 0$$

Let $y = x_1 - x_2$ be the extension in the table bolts. Writing

$$\omega_1^2 = \frac{k_1}{m_1}; \quad \omega_2^2 = \frac{k_2}{m_2}; \quad \omega_3^2 = \frac{k_1}{m_2} \quad [21]$$

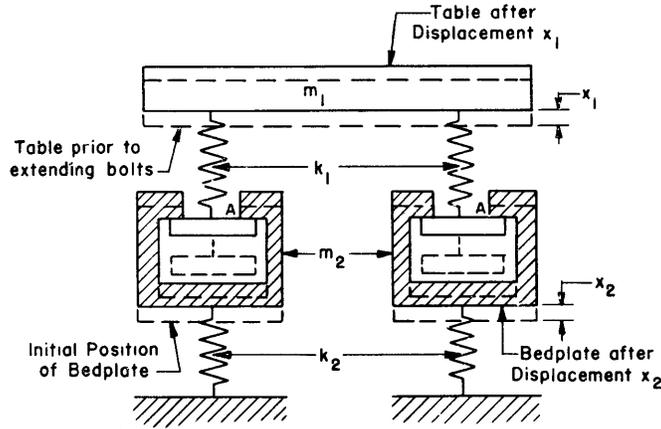


Figure 19 - System 2: Sketch Illustrating the **Stopping** of the Table by Straining both Table Bolts and Foundation Bolts

The solution for y is found in the form

$$y = \frac{v_0}{p_1^2 - p_2^2} \left[\frac{p_1^2 - \omega_2^2}{p_1} \sin p_1 t + \frac{\omega_2^2 - p_2^2}{p_2} \sin p_2 t \right] \quad [22]$$

in which p_1 and p_2 are roots of the frequency equation

$$p^4 - (\omega_1^2 + \omega_2^2 + \omega_3^2) p^2 + \omega_1^2 \omega_2^2 = 0 \quad [23]$$

For the table loaded with 575 pounds, $W_1 = m_1 g = 4000 + 575 = 4575$ pounds.

$$W_2 = m_2 g = 7000 \text{ pounds}$$

$$k_1 = \frac{A_1 E}{l_1} = 38 \times 10^6 \text{ pounds per inch}$$

There are 13 foundation bolts each $\frac{1}{2}$ inches in diameter of which eleven are 141 inches long and two are 126 inches long; from this k_2 is found to be 13.8×10^6 pounds per inch.

By Equation [21] the values of ω_1^2 , ω_2^2 , and ω_3^2 are found to be, in radians per second squared

$$\omega_1^2 = 3.19 \times 10^6; \quad \omega_2^2 = 0.76 \times 10^6; \quad \omega_3^2 = 2.09 \times 10^6 \quad [24]$$

from the frequency equation

$$p_1 = 2370 \text{ radians per second, } p_2 = 660 \text{ radians per second}$$

Thus the extension y becomes

$$y = v_0 \times 10^{-4} (3.95 \sin 2370 t + 0.97 \sin 660 t) \quad [25]$$

The stress in the table bolts is then

$$\sigma = \frac{y}{l} E = v_0 (395 \sin 2370 t + 97 \sin 660 t) \quad [26]$$

where v_0 is in inches per second and t is in seconds.

For the case considered in Appendix 2, i.e., when all the energy of the hammer blow is considered to be converted into kinetic energy of the table, then

$$\frac{1}{2} m_1 v_0^2 = Wh$$

in which W is weight of hammer and h is height of hammer drop from which, for a 6-foot drop

$$v_0 = 191 \text{ inches per second} = 15.9 \text{ feet per second}$$

and

$$\sigma = 75400 \sin 2370t + 18500 \sin 660t \text{ pounds per square inch} \quad [27]$$

The two sine terms are plotted in their proper proportion in dotted lines and their sum, i.e., the stress pulse in the holding-down bolts, by a solid line

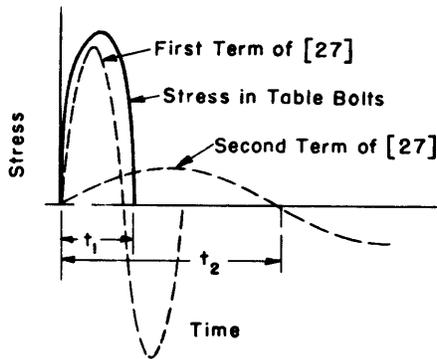


Figure 20 - Graphical Representation of Equation [27]

in Figure 20. The curve ends at time t_1 , because the stress in the bolts returns to zero when the table system parts from the bedplate at points A.

It is found that the peak table-bolt stress is $\sigma_m = 83,400$ pounds per square inch, i.e., about 83 per cent of the prediction of the one-degree-of-freedom System 1 of Appendix 2 for the 6-foot drop. This reduction may be attributed to the cushioning effect of the springs representing the foundation bolts. The duration t_1 is found $t_1 = 1.4$ millisecond, a value independent of the height of hammer drop because v_0 affects

only the magnitude of the stress as shown by Equation [25].

The solution of Equation [20] for the extension x_2 in the foundation bolts may be placed in the form

$$x_2 = \frac{\omega_3^2 v_0}{p_1^2 - p_2^2} \left[\frac{\sin p_2 t}{p_2} - \frac{\sin p_1 t}{p_1} \right] \quad [28]$$

and for the 6-foot drop

$$x_2 = 0.117 \sin 660t - 0.033 \sin 2370t \text{ inches} \quad [29]$$

This solution is valid only for $t \leq t_1 = 1.4$ millisecond at which time the $m_1 k_1$ system, traveling down, parts from the $m_2 k_2$ system traveling up. At this time t_1 , $x_2 = 0.099$ inch and $dx_2/dt = 124$ inches per second. After

$t = t_1$, the mass m_2 continues to move on the end of the springs k_2 ; the subsequent displacement x_2 , subject to the initial conditions at t_1 , is given by

$$x_2 = \frac{124}{\omega_2} \sin \omega_2 (t - t_1) + 0.099 \cos \omega_2 (t - t_1) \quad [30]$$

from which

$$x_{2, \max} = \sqrt{\left(\frac{124}{\omega_2}\right)^2 + (0.099)^2} = 0.14 \text{ inch for the 6-foot drop}$$

Thus

$$\sigma_{2, \max} = \frac{x_2}{l_2} E = 34,000 \text{ pounds per square inch}$$

The time t_2 at which the foundation-bolt stress reached a maximum is given by setting $dx_2/dt = 0$ which gives

$$\tan \omega_2 (t_2 - t_1) = 1.44$$

$$t_2 = 2.5 \text{ milliseconds}$$

a value independent of the height of hammer drop since v_0 affects only the magnitude of the stress as shown by Equation [28].

The observed time of peak stress in Foundation Bolt 12 for the 5-foot drop on the first test was 2.1 milliseconds.

Thus the maximum stress in the foundation bolts occurs after the stress in the table bolts has ended. Physically this happens because the table bolts pulling up on the bedplate accelerate it the entire time that stress exists in the table bolts; thus, when the stress in the table bolts ends, the bedplate has acquired a definite upward velocity which causes the foundation-bolt stress to continue to increase until the bedplate is stopped.

Recapitulating, this analysis predicts for the 6-foot drop, 575-pound load,

Maximum stress in table bolts = 83,400 pounds per square inch

Duration of pulse = 1.4 millisecond

Maximum stress in foundation bolts = 34,000 pounds per square inch

Time to reach maximum stress = 2.5 milliseconds.

APPENDIX 6

DECELERATION OF TABLE BY HOLDING-DOWN BOLTS

It has been mentioned that the external force exerted on the loaded table in stopping its upward motion is the sum of the loads in the 12 bolts. This total force divided by the weight of the loaded table will give the stopping acceleration in multiples of g.

The measured values of the maximum stress were averaged for repeated drops of the same height and the average was used as a basis for calculating the net elastic force acting on the table. The force exerted by the bolts was divided by the weight of the table to give the maximum deceleration as listed in Table 6.

TABLE 6

Maximum Deceleration during the Stopping of the Shock Table
by the Holding-Down Bolts

Hammer Drop feet	Maximum Deceleration in Multiples of g		
	575-Pound Load on Table	1500-Pound Load on Table	2500-Pound Load on Table
1			70
2	67		105
3	190	190	150
4	270	270	190
5	280	300	230
5.5			240

The method is, of course, a rough one; there is scatter in the data and measurements were made on only a few of the 12 bolts during each drop. If the stress pulses in all the bolts were identical, however, each one could represent the deceleration-time pulse faithfully, provided the stress were multiplied by the area of the 12 bolts and divided by the weight of the table. As it is, the main stress-time pulse in a single bolt, as in Figure 5, can give a rough idea of the shape of the deceleration-time pulse.

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