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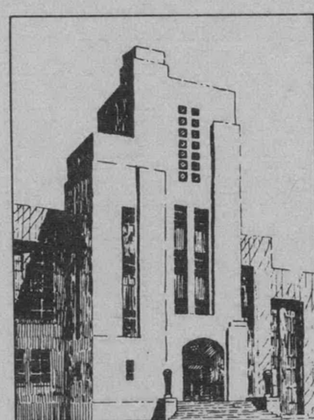
BUCKLING OF MULTIPLE-BAY RING-REINFORCED CYLINDRICAL SHELLS SUBJECT TO HYDROSTATIC PRESSURE

by

W. A. Nash



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April 1954

Report 785

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Reprint from
Journal of Applied Mechanics
Vol. 20, No. 4, pp. 469-474 (1953)

April 1954 Report 785

FOREWORD

In the strength analysis of submarine structures consideration has been given to the possible elastic buckling of the shell plating between ring stiffeners. Re-evaluation of the existing theoretical and experimental results has indicated that the desired accuracy of calculation is not often obtained. As a consequence further research was initiated at the Taylor Model Basin under a project designated "Buckle."

The experimental phase of this research program has been discussed in TMB Reports C-439 and 822. Theoretical phases have been under study based on preliminary analyses developed by Salerno and Levine, working at the Polytechnic Institute of Brooklyn.

In this report a mathematical treatment is presented for the shell instability of ring-reinforced cylindrical shells which is believed more general and more in accord with recent experimental observations of buckling configuration than the previously available theory of von Mises. At the same time, the following restrictions inherent in recent analyses of Salerno and Levine have been removed:

(a) The expressions for the work done by the external forces acting upon the shell during the buckling process appear to be incorrect. Although the importance of these errors upon the numerical results obtained for the particular buckling configurations considered in their analyses appears to be small, it is of course desirable to correct the analysis, particularly if other buckling configurations are to be considered.

(b) The reinforcing ring was considered to be a line element. This leads to some ambiguities regarding the unsupported length of the cylindrical shell. Further, the energy contained in that portion of the shell directly under the frame is not considered.

(c) The treatment applies only to rings consisting of thin-walled "open" cross sections. It is not strictly applicable to rings of rectangular cross sections.

(d) The assumed displacement configurations did not permit any radial displacement of the rings during buckling.

The analysis presented herein is considered significant primarily in reporting a rigorous treatment of the shell with fixed ends as contrasted with the condition of hinged ends assumed by von Mises. Results computed by applying this theory to several special geometries are found in even poorer agreement with experimental observations than were the less valid von Mises analysis. There are new and definite indications, however, that discrepancies result from initial imperfections in the test specimens. This analysis should thus be considered as applicable only to a perfect structure and thus in need of further extension to accommodate initial out-of-roundness and residual welding stresses.

Paper No.
53—APM-29

Buckling of Multiple-Bay Ring-Reinforced Cylindrical Shells Subject to Hydrostatic Pressure

By W. A. Nash,[1] Washington, D. C.

An analytical solution is presented for the problem of the elastic instability of a multiple-bay ring-reinforced cylindrical shell subject to hydrostatic pressure applied in both the radial and axial directions. The method used is that of minimization of the total potential. Expressions for the elastic strain energy in the shell and also in the rings are written in terms of displacement components of a point in the middle surface of the shell. Expressions for the work done by the external forces acting on the cylinder likewise are written in terms of these displacement components. A displacement configuration for the buckled shell is introduced which is in agreement with experimental evidence, in contrast to the arbitrary patterns assumed by previous investigators. The total potential is expressed in terms of these displacement components and is then minimized. As a result of this minimization a set of linear homogeneous equations is obtained. In order that a nontrivial solution to this system of equations exists, it is necessary that the determinant of the coefficients vanish. This condition determines the critical pressure at which elastic buckling of the cylindrical shell will occur.

Introduction

The first analytical treatment of the problem of the buckling of a cylindrical shell of infinite length subject to external hydrostatic pressure was carried out by Bresse (1)[2] in 1859. In 1888 Bryan (2) published his classical paper deriving the same expression as did Bresse. Bryan's work was based upon the energy criterion for stability. In 1913 Southwell (3) published the first of a series of three papers treating the elastic instability of a geometrically perfect cylindrical shell. In the first of these papers he rederived Bryan's expression for the tube of infinite length in a manner different from that of the original author. In this same paper Southwell discussed the effect of circumferential reinforcing rings upon the hydrostatic buckling pressure of a shell of finite length and derived an expression for the minimum length of tube for which the effect of the rings can be neglected. He used an energy method to obtain the buckling pressure for a shell of finite length, neglecting the effect of the reinforcing rings. This solution considered only radial pressure.

This paper also presented for the first time an analytical treatment of the problem of the number of lobes that would form on a shell subject to external hydrostatic pressure. In 1914 von Mises (4) published his classical analysis of the buckling of a thin elastic shell of finite length subject to uniform radial pressure. However, this study failed to take into account any axial component of pressure. In 1929 von Mises (5) extended his original analysis so as to cover the case of hydrostatic pressure applied to the ends as well as to the curved wall of the shell. In neither of his analyses was there considered the effect of the elastic restraint of the ends (or any reinforcing rings in the case of a ring-reinforced shell) upon the structure.

In 1934 Windenburg (6) presented a simplification of von Mises equation for the buckling pressure of a cylinder subject to hydrostatic pressure. This result is independent of the number of lobes formed upon buckling and differs on the average from the von Mises value by about 1 per cent. Batdorf (7) in 1947 presented a new method of determining the buckling stresses of cylindrical shells under various loading conditions. By this method he obtained a solution to the problem of the collapse of a cylinder of finite length loaded by hydrostatic pressure on all surfaces. His solution is almost identical with that presented by von Mises.

In a recent series of four papers Salerno and Levine (8, 9, 10, 11) treated the elastic instability of a circular cylindrical shell reinforced by evenly spaced circumferential rings having an I-type cross section. The scope of their work is restricted by the assumption that the rings must be thin-walled open sections.

All the foregoing investigations, as well as that presented here, are predicated upon the classical small deformation theory of elastic thin shells as presented by Love (12). This theory assumes:

1 The shell is composed of a material which is elastically homogeneous and isotropic.

2 The material follows Hooke's law.

3 The thickness of the shell at any point is small compared to either of the principal radii of curvature at that point.

4 The normals to the middle surface of the shell before deformation also are normal to the middle surface after deformation.

In addition to these theoretical analyses of the buckling of a cylindrical shell subject to hydrostatic pressure numerous experimental investigations have been conducted within the past hundred years. The principal investigators were Fairbairn (13), Carman (14), Steward (15), Carman and Carr (16), Southwell (3), Windenburg (17), Sturm (18), and Kirkby (19).

Analysis

Fundamental Equations. The problem of the buckling of a multiple-bay ring-reinforced cylindrical shell subject to hydro-

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[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

For presentation at the National Conference of the Applied Mechanics Division, Minneapolis, Minn., June 18–20, 1953, of The American Society of Mechanical Engineers.

Discussion of this paper should be addressed to the Secretary, ASME, 29 West 39th Street, New York, N. Y., and will be accepted until one month after final publication of the paper itself in the Journal of Applied Mechanics.

 Manuscript received by ASME Applied Mechanics Division, July 2, 1952. Paper No. 53—APM-29.

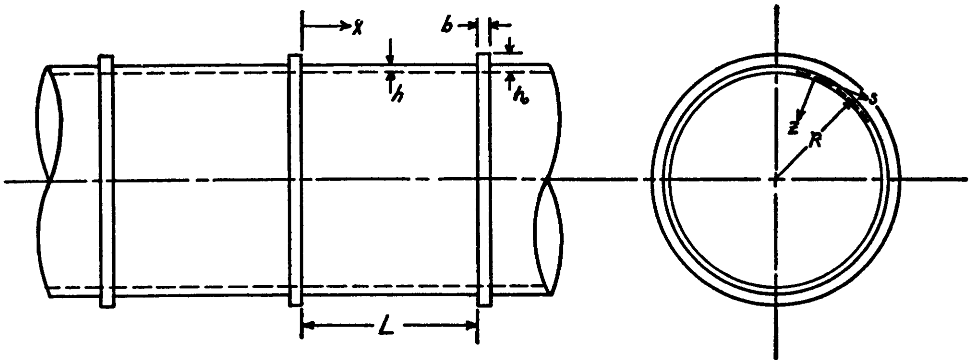

FIG. 1

static pressure may be attacked by the method of minimum potential described by Timoshenko (20). Let us consider a cylinder of mean radius R and thickness h with stiffening rings spaced a distance L apart. Such a shell is shown in Fig. 1. The rings under consideration here are assumed to be spaced equally along the axis of the cylinder and of uniform cross section around the circumference of any ring. Further, it is assumed that all rings are identical. In the first part of the analysis rings of infinite rigidity are discussed and they, of course, may be of arbitrary cross section. In the second part of the analysis, rings of finite rigidity are discussed and their cross sections must be rectangular.

Let x, s, and z be orthogonal co-ordinates in the axial, tangential, and radial directions, respectively. The positive directions of these co-ordinates are shown in Fig. 1. The shell is subject to a hydrostatic pressure p acting both radially and axially. Let u_0, v_0, and w_0 be the components of displacement (in the x, s, and z-directions) of any point in the middle surface of the shell at an instant before buckling and further let u, v, and w denote the additional displacements during the buckling process.

For the purpose of this analysis it is sufficient to isolate and study one bay of the shell, i.e., the portion of the structure between the centers of two adjacent rings. In the minimum potential method it is first necessary to obtain values of the additional extensional, bending, and shearing elastic strain energies stored in one bay of the shell during the buckling process. If the axial and tangential strains existing in the shell immediately before buckling are denoted by ϵ_{x0} and ϵ_{s0}, then, for the case of uniform radial pressure p and axial pressure P, these quantities are given by

$$\epsilon_{x0} = \frac{1}{E}\left(-P + \frac{\nu pR}{h}\right) \quad \text{[1]}$$

$$\epsilon_{s0} = \frac{1}{E}\left(-\frac{pR}{h} + \nu P\right) \quad \text{[2]}$$

where E denotes the modulus of elasticity of the material and ν represents Poisson's ratio. In Equations [1] and [2] the effect of the rings on the strains near the ends of the bay are neglected. The axial and tangential strains existing immediately after buckling are given by

$$\epsilon_x = \frac{[(1 + \epsilon_{x0} + u_x)^2(dx)^2 + w_x^2(dx)^2]^{1/2} - dx}{dx}$$
$$= \epsilon_{x0} + u_x + \frac{1}{2} w_x^2 \quad \text{[3]}$$

and

$$\epsilon_s = \frac{[(1 + \epsilon_{s0} + v_s - w/R)^2(ds)^2 + (v/R + w_s)^2(ds)^2]^{1/2} - ds}{ds}$$
$$= \epsilon_{s0} + v_s - \frac{w}{R} + \frac{1}{2}\left(\frac{v}{R} + w_s\right)^2 \quad \text{[4]}$$

The increase in the extensional elastic energy during the buckling process is given by

$$\Delta U_e = \frac{Eh}{2(1-\nu^2)} \int_0^{2\pi R} \int_0^L [(\epsilon_x^2 + 2\nu\epsilon_x\epsilon_s + \epsilon_s^2) - (\epsilon_{x0}^2 + 2\nu\epsilon_{x0}\epsilon_{s0} + \epsilon_{s0}^2)]dx\, ds \quad \text{[5]}$$

With the values of these strains given by Expressions [1] through [4], the increase in extensional energy is found to be

$$\Delta U_e = \frac{Eh}{2(1-\nu^2)} \int_0^{2\pi R} \int_0^L \left[u_x^2 + v_s^2 + \frac{w^2}{R^2} + 2\nu u_x v_s - \frac{2\nu u_x w}{R} - \frac{2v_s w}{R}\right] dx\, ds + pR \int_0^{2\pi R} \int_0^L \left[\frac{w}{R} - \frac{1}{2}\left(\frac{v}{R} + w_s\right)^2\right] dx\, ds - \frac{Ph}{2} \int_0^{2\pi R} \int_0^L w_x^2 dx\, ds - Ph \int_0^{2\pi R} [u_L - u_0]ds \quad \text{[6]}$$

The bending energy immediately before buckling is given by Love (12) and Salerno and Levine (8) to be

$$U_{b0} = \frac{Eh^3}{24R^2(1-\nu^2)} \int_0^{2\pi R} \int_0^L \left[R^2(w_{0xx})^2 + \frac{1}{R^2}(R^2 w_{0ss} + w_0)^2 + 2\nu w_{0xx}(R^2 w_{0ss} + w_0) + 2(1-\nu)\left(R w_{0xs} + \frac{v_{0x}}{2} - \frac{u_{0s}}{2}\right)^2\right] dx\, ds \quad \text{[7]}$$

The bending energy immediately after buckling is obtained by replacing u_0 by $(u_0 + u)$, v_0 by $(v_0 + v)$, and w_0 by $(w_0 + w)$. Consequently the increase in strain energy of bending during the buckling process is

$$\Delta U_b = \frac{Eh^3}{24(1-\nu^2)} \int_0^{2\pi R} \int_0^L \left[w_{xx}^2 + w_{ss}^2 + 2\nu w_{xx} w_{ss} + 2(1-\nu)(w_{xs})^2 + \frac{w^2}{R^4} + \frac{2}{R^2} w_{ss} w\right] dx\, ds \quad \text{[8]}$$

This expression for the increase in bending energy was obtained by neglecting certain terms differing by h^2/R^2, $h^2 \frac{\partial^2}{\partial x^2}$, or $h^2 \frac{\partial^2}{\partial s^2}$ from corresponding terms appearing in ΔU_e.

Prior to buckling the cylindrical shell is in a state of uniform compression and the membrane shear stresses are zero. After buckling, the shear stress is given by Love (12) to be

$$U_s = \frac{Eh}{2(1-\nu^2)} \int_A \left(\frac{1-\nu}{2}\right)(\gamma_{x\phi})^2 dA \quad \text{[9]}$$

where $$\phi = \frac{s}{R}$$

But
$$\gamma_{x\phi} = \frac{u_\phi}{R} + v_x \qquad [10]$$

Hence
$$U_s = \frac{Eh}{4(1+\nu)}\int_0^{2\pi R}\int_0^L (u_s + v_x)^2 dx\, ds \qquad [11]$$

Knowing the elastic strain energy stored in the shell, it is next necessary to calculate the work done by the external forces acting on the shell. The total work done by the external forces acting upon the cylinder may be calculated as the hydrostatic pressure p multiplied by the decrease of volume of the cylinder. This decrease of volume may be considered to consist of a shortening of the radius in any fixed radial plane together with a sliding of the ends of the cylinder along a plate. Any radius is shortened by an amount.

$$w - uw_x - vw_s - \frac{v^2}{2R}$$

(in a fixed radial plane) and hence the decrease in volume between the initial end planes is

$$\int_0^L \pi R^2 dx - \frac{1}{2R}\int_0^{2\pi R}\int_0^L \left[R - w + uw_x + vw_s + \frac{v^2}{2R}\right]^2 dx\, ds$$
$$= \int_0^{2\pi R}\int_0^L \left[w - \frac{v^2}{2R} - \frac{w^2}{2R} - uw_x - vw_s\right] dx\, ds \qquad [12]$$

where terms of the third and higher orders have been neglected. The correction to the decrease of volume in the vicinity of the ends of the cylinder is of the form

$$\int_0^{2\pi R} \frac{u}{2}(R-w)^2 \frac{ds}{R} = \int_0^{2\pi R} u\left(\frac{R}{2} - w\right) ds$$

Accordingly, the correction for both ends of the cylinder is

$$\frac{R}{2}\int_0^{2\pi R}(u_L - u_0)\, ds - \int_0^{2\pi R}[(uw)_L - (uw)_0]\, ds$$

The work done by the hydrostatic pressure p is thus given by

$$p\int_0^{2\pi R}\int_0^L \left(w - \frac{v^2}{2R} - \frac{w^2}{2R} - uw_x - vw_s\right) dx\, ds$$
$$-\frac{pR}{2}\int_0^{2\pi R}(u_L - u_0) ds + p\int_0^{2\pi R}[(uw)_L - (uw)_0]\, ds \qquad [13]$$

For the cylindrical shell, then, the total potential (which is defined to be the algebraic sum of the strain energies and the negative of the work done by the external loads) may be found by use of Equations [6], [8], [11], and [13]. Equation [6] may be simplified by use of the statics relation

$$P = \frac{pR}{2h}$$

Also, Equation [13] may be simplified by using the following result obtained by integration by parts

$$\int_0^L uw_x dx = (uw)_L - (uw)_0 - \int_0^L u_x w\, dx$$

Let us first consider the case of a cylindrical shell reinforced by infinitely rigid reinforcing rings. In this case the rings contain no elastic strain energy and the total potential (i.e., the sum of the strain energies and the potential of the external loads) is

$$U_T = \frac{Eh}{2(1-\nu^2)}\int_0^{2\pi R}\int_0^L \left[u_x^2 + v_s^2 + \frac{w^2}{R^2} + 2\nu u_x v_s - \frac{2\nu u_x w}{R} - \frac{2v_s w}{R}\right] dx\, ds + \frac{Eh^3}{24(1-\nu^2)}\int_0^{2\pi R}\int_0^L \left[w_{xx}^2 + w_{ss}^2 + 2\nu w_{xx}w_{ss} + 2(1-\nu)(w_{xs})^2 + \frac{w^2}{R^4} + \frac{2}{R^2}w_{ss}w\right] dx\, ds$$
$$+ \frac{Eh}{4(1+\nu)}\int_0^{2\pi R}\int_0^L (u_s + v_x)^2\, dx\, ds$$
$$+ pR\int_0^{2\pi R}\int_0^L \left[-\frac{w_s^2}{2} + \frac{w^2}{2R^2} - \frac{u_x w}{R} - \frac{w_x^2}{4}\right] dx\, ds \qquad [14]$$

Displacement Pattern. For the case of infinitely rigid reinforcing rings let us take the additional displacements during buckling to be

$$\left.\begin{aligned} u &= A\cos\frac{ms}{R}\sin 2\delta x \\ v &= B\sin\frac{ms}{R}(1-\cos 2\delta x) \\ w &= C\cos\frac{ms}{R}(1-\cos 2\delta x) \end{aligned}\right\} \qquad [15]$$

where A, B, and C are arbitrary constants, m is the number of waves (lobes) in the circumferential direction, L/n represents the period of the function defining the deformed generator in one bay of the shell, and $\delta = n\pi/L$. The elastic strain energies and the potential due to the external loads may now be expressed in terms of the constants A, B, and C by substituting the displacements, Equations [15], in the energy expressions and integrating. For brevity, let us introduce the following notation

$$\left.\begin{aligned}
k_1 &= \frac{Eh}{2(1-\nu^2)} & k_{12} &= \frac{4(1-\nu)\delta^2 m^2\pi L}{R} \\
k_2 &= 2\delta^2\pi RL & k_{13} &= \frac{Eh}{4(1+\nu)} \\
k_3 &= \frac{3m^2\pi L}{2R} & k_{14} &= \frac{m^2\pi L}{2R} \\
k_4 &= \frac{3\pi L}{2R} & k_{15} &= 2\delta m\pi L \\
k_5 &= 2\nu\delta m\pi L & k_{16} &= 2\delta^2\pi RL \\
k_6 &= 2\nu\delta\pi L & k_{17} &= \frac{3pm^2\pi L}{4} \\
k_7 &= \frac{3m\pi L}{R} & k_{18} &= \frac{3p\pi L}{4} \\
k_8 &= \frac{Eh^3}{24(1-\nu^2)} & k_{19} &= pR\delta\pi L \\
k_9 &= 8\delta^4\pi RL & k_{20} &= \frac{p\delta^2\pi R^2 L}{2} \\
k_{10} &= \frac{3m^4\pi L}{2R^3} & k_{21} &= \frac{3\pi L}{2R^3} \\
k_{11} &= \frac{4\nu\delta^2 m^2\pi L}{R} & k_{22} &= \frac{3m^2\pi L}{R^3}
\end{aligned}\right\} \qquad [16]$$

Minimization of Potential. The variation of the total potential with respect to each of the constants A, B, and C must vanish for equilibrium. This means that

$$\frac{\partial U_T}{\partial A} = 0; \quad \frac{\partial U_T}{\partial B} = 0; \quad \frac{\partial U_T}{\partial C} = 0 \ldots\ldots [17]$$

When the differentiations indicated in Equations [17] are carried out they lead to the three linear homogeneous equations

$$k_{23}A + k_{24}B + k_{25}C = 0 \ldots\ldots [18]$$

$$k_{24}A + k_{26}B + k_{27}C = 0 \ldots\ldots [19]$$

$$k_{25}A + k_{27}B + k_{28}C = 0 \ldots\ldots [20]$$

where

$$\left.\begin{aligned} k_{23} &= 2k_1k_2 + 2k_{13}k_{14} \\ k_{24} &= -k_1k_5 - k_{13}k_{15} \\ k_{25} &= k_1k_6 + k_{19} \\ k_{26} &= 2k_1k_3 + 2k_{13}k_{16} \\ k_{27} &= -k_1k_7 \\ k_{28} &= 2k_1k_4 + 2k_8k_9 + 2k_8k_{10} + 2k_8k_{21} \\ &\quad - 2k_8k_{22} + 2k_8k_{11} + 2k_8k_{12} - 2k_{17} + 2k_{18} - 2k_{20} \end{aligned}\right\} \ldots [21]$$

The trivial solution $A = B = C = 0$ is of no consequence. For a nontrivial solution to exist, the determinant of the coefficients of the unknowns must vanish. When this is the case, there exist deformed equilibrium configurations in addition to the original one. Such a situation corresponds to neutral equilibrium and the loads under which it exists are the buckling loads. Here, the stability determinant is

$$\begin{vmatrix} k_{23} & k_{24} & k_{25} \\ k_{24} & k_{26} & k_{27} \\ k_{25} & k_{27} & k_{28} \end{vmatrix} = 0 \ldots\ldots [22]$$

The expansion of this determinant leads to a quadratic equation in the unknown buckling pressure p. The buckling pressure p predicted by this theory is consequently the minimum positive root of this quadratic equation. It is of interest to note that for the case of an infinitely long cylindrical shell, the buckling pressure obtained by this analysis is

$$p = \frac{Eh^3}{4(1-\nu^2)R^3} \ldots\ldots [23]$$

which agrees with the well-known Bryan-Bresse result (12).

Other Displacement Patterns. It might be thought that the somewhat more general displacement pattern

$$\left.\begin{aligned} u &= \left(A \sin\frac{ms}{R} + B \cos\frac{ms}{R}\right) \sin 2\delta x + \frac{Cx}{L} \\ v &= D \sin\frac{ms}{R}(1 - \cos 2\delta x) \\ w &= \left(F \sin\frac{ms}{R} + G \cos\frac{ms}{R}\right)(1 - \cos 2\delta x) \end{aligned}\right\} \ldots [24]$$

would lead to a lower buckling pressure since it contains six constants instead of three as used in Equations [15]. However, by the minimum potential method it may be shown that the same stability determinant, Equations [22], is obtained as in the case of the configuration given by Equations [15].

It is of interest to investigate a slight variation in the configuration given by Equations [15]. Let us assume that the additional displacements during buckling are given by

$$\left.\begin{aligned} u &= A \cos\frac{ms}{R} \sin 4\delta x \\ v &= B \sin\frac{ms}{R}(1 - \cos 2\delta x) \\ w &= C \cos\frac{ms}{R}(1 - \cos 2\delta x) \end{aligned}\right\} \ldots\ldots [25]$$

Using the minimum potential method, three equations analogous to Equations [18], [19], and [20] may be written and a stability determinant formed. However, this configuration will usually indicate a slightly higher buckling pressure than that found by use of the pattern, Equations [15]. This is illustrated in the numerical example given later for one particular geometry. For an infinitely long cylindrical shell the buckling pressure found by use of the Configuration [25] again agrees with the Bryan-Bresse expression.

The displacement configurations discussed thus far are for the case of infinitely rigid reinforcing rings. In all previous analyses investigations have assumed displacement components corresponding to arbitrarily selected boundary conditions without any reference to experimental evidence. As the configuration occurring during buckling of a cylindrical shell reinforced by rings having finite rigidity let us take

$$\left.\begin{aligned} u &= A \sin\frac{ms}{R} \sin 2\delta x + B \sin\frac{ms}{R} \cos \delta x \\ &\quad + C \cos\frac{ms}{R} \sin 2\delta x + D \cos\frac{ms}{R} \cos\delta x + \frac{Fx}{L} \\ v &= \sin\frac{ms}{R}(G + H \sin 2\delta x + J \cos 2\delta x) \\ w &= \left(K \sin\frac{ms}{R} + M \cos\frac{ms}{R}\right)(1 - \cos 2\delta x) + N \end{aligned}\right\} \ldots [26]$$

This configuration permits the ring to (*a*) bend out of its plane, (*b*) undergo a uniform radial compression, (*c*) undergo tangential displacements, and (*d*) translate along the axis of the cylinder. Here, the form of the w-component of displacement was obtained from radial-displacement measurements taken shortly before and immediately after the formation of lobes in two ring-stiffened cylinders each subject to hydrostatic pressure. (The reinforcing rings were of rectangular cross section.) These measurements indicated that the ring did not undergo any bending in its plane and further that the profile of the shell along a generator was given very closely by the expression $(1 - \cos 2\delta x)$ with $n = 1$. This experimental evidence has been incorporated into the expressions given in Equations [26] in such a way that all other types of deformation of the ring, namely, (*a*) through (*d*), are possible.

In calculating the total potential of a cylindrical shell reinforced by rings of finite rigidity it is necessary to compute the potential of the rings. This may be done in a manner analogous to that used for the derivation of the potential of the shell and the result, for a ring of rectangular cross section is

$$U_R = \frac{bh_0E}{2}\int_0^{2\pi R_1}\left[\left(v_s\frac{R_1}{R}\right)^2 + \frac{w^2}{R_1^2} - \frac{2v_sw}{R}\right]_{x=0} ds + pbR_1\int_0^{2\pi R_1}\left[\frac{w^2}{2R_1^2}\right]_{x=0} ds + \frac{EI_{z0}}{2}\int_0^{2\pi R_1}[u_{ss}^2]_{x=0}\, ds \ldots\ldots [27]$$

where b denotes the width of the ring as shown in Fig. 1, h_0 denotes the depth of the composite ring-shell section (i.e., he ring and the portion of the shell immediately contiguous to t is regarded as monolithic), R_1 represents the radius to the entroid of the composite ring-shell section, and I_{z0} denotes the noment of inertia of the ring-shell cross section with respect to n axis through the centroid of the cross section and coinciding vith a diameter of the shell.

This potential must be added to the potential of the shell iven in Equation [14]. The elastic strain energies and the otential of the external loads may now be expressed in terms f the constants $A, B, C, \ldots N$ by substituting the Displacements [26] in the energy expressions and integrating. Let us ntroduce the notation

$$
\left.\begin{array}{ll}
h_1 = \dfrac{Eh}{2(1-\nu^2)} & h_{23} = \dfrac{3m^2\pi L}{R^3} \\[2ex]
h_2 = 2\delta^2\pi RL & h_{24} = \dfrac{4(1-\nu)\delta^2 m^2\pi L}{R} \\[2ex]
h_3 = \dfrac{\delta^2\pi RL}{2} & h_{25} = \dfrac{Eh}{4(1+\nu)} \\[2ex]
h_4 = \dfrac{2\pi R}{L} & h_{26} = \dfrac{8m^2 L}{3R} \\[2ex]
h_5 = \dfrac{8\delta\pi R}{3} & h_{27} = 2\delta^2\pi RL \\[2ex]
h_6 = \dfrac{m^2\pi L}{R} & h_{28} = 2\delta m\pi L \\[2ex]
h_7 = \dfrac{m^2\pi L}{2R} & h_{29} = \dfrac{16m\pi}{3} \\[2ex]
h_8 = \dfrac{3\pi L}{2R} & h_{30} = \dfrac{3m^2\pi Lp}{4} \\[2ex]
h_9 = \dfrac{2\pi L}{R} & h_{31} = \dfrac{3\pi Lp}{4} \\[2ex]
h_{10} = 4\nu\pi m & h_{32} = \pi Lp \\[2ex]
h_{11} = 2\delta m\pi L\nu & h_{33} = \delta\pi LpR \\[2ex]
h_{12} = \dfrac{4\nu\pi m}{3} & h_{34} = \dfrac{8\delta pRL}{3} \\[2ex]
h_{13} = 2\delta\pi\nu L & h_{35} = 2\pi Rp \\[2ex]
h_{14} = \dfrac{16\nu\pi}{3} & h_{36} = \dfrac{\pi R^2 L\delta^2 p}{2} \\[2ex]
h_{15} = \dfrac{2\pi L}{R^3} & h_{37} = \dfrac{bh_0 E}{2} \\[2ex]
h_{16} = \dfrac{2m\pi L}{R} & h_{38} = \dfrac{\pi m^2 R_1^3}{R^4} \\[2ex]
h_{17} = \dfrac{m\pi L}{R} & h_{39} = \dfrac{2m^2\pi R_1^3}{R^4} \\[2ex]
h_{18} = \dfrac{Eh^3}{24(1-\nu^2)} & h_{40} = \dfrac{2\pi}{R_1} \\[2ex]
h_{19} = 8\delta^4\pi RL & h_{41} = b\pi p \\[2ex]
h_{20} = \dfrac{3m^4\pi L}{2R^3} & h_{42} = \dfrac{EI_{z0}}{2R^4} \\[2ex]
h_{21} = \dfrac{4\nu\delta^2 m^2\pi L}{R} & h_{43} = m^4\pi R_1 \\[2ex]
h_{22} = \dfrac{3\pi L}{2R^3} &
\end{array}\right\} \quad \ldots [28]
$$

Again, the variation of the total potential with respect to each of the constants $A, B, \ldots N$ must vanish for equilibrium. This condition leads to the system of equations

	A	B	C	D	F	G	H	J	K	M	N		
1	h_{44}	h_{45}							h_{46}			= 0	
2	h_{45}	h_{47}							h_{48}			= 0	
3			h_{44}	h_{45}				h_{49}		h_{46}		= 0	
4			h_{45}	h_{47}		h_{51}		h_{50}		h_{48}		= 0	
5					h_{52}						h_{35}	= 0	.. [29]
6				h_{51}		h_{53}		h_{54}		h_{55}		= 0	
7							h_{60}					= 0	
8			h_{49}	h_{50}		h_{54}		h_{56}		h_{57}		= 0	
9	h_{46}	h_{48}							h_{58}			= 0	
10			h_{46}	h_{48}		h_{55}		h_{57}		h_{58}		= 0	
11					h_{35}						h_{59}	= 0	

where

$$
\left.\begin{array}{l}
h_{44} = 2h_1h_2 + 2h_{25}h_7 \\
h_{45} = h_1h_5 + h_{25}h_{26} \\
h_{46} = h_1h_{13} + h_{33} \\
h_{47} = 2h_1h_3 + 2h_{25}h_7 + 2h_{42}h_{43} \\
h_{48} = h_1h_{14} + h_{34} \\
h_{49} = h_1h_{11} + h_{25}h_{28} \\
h_{50} = h_1h_{12} + h_{25}h_{29} \\
h_{51} = -h_1h_{10} \\
h_{52} = 2h_1h_4 \\
h_{53} = 2h_1h_6 + 2h_{37}h_{38} \\
h_{54} = h_{37}h_{39} \\
h_{55} = -h_1h_{16} \\
h_{56} = 2h_1h_7 + 2h_{25}h_{27} + 2h_{37}h_{38} \\
h_{57} = h_1h_{17} \\
h_{58} = 2h_1h_8 + 2h_{18}h_{19} + 2h_{18}h_{20} + 2h_{18}h_{21} + 2h_{18}h_{22} \\
\qquad - 2h_{18}h_{23} + 2h_{18}h_{24} - 2h_{30} + 2h_{31} - 2h_{36} \\
h_{59} = 2h_1h_9 + 2h_{15}h_{18} + 2h_{32} + 2h_{37}h_{40} + 2h_{41} \\
h_{60} = 2h_1h_7 + 2h_{25}h_{27}
\end{array}\right\} \quad \ldots [30]
$$

These equations show that $H = 0$ for all geometries of ring-reinforced cylindrical shell. Also, the constants A, B, and K and no others each appear in only three equations and the stability determinant of these three may be solved to obtain a value of p. Also, the constants C, D, G, J, and M and no others each appear in five equations and the stability determinant of these five may be solved to determine another value of p. Lastly, a stability determinant may be formed of the two equations containing F and N and a third value of p determined. (This last corresponds to the case of axial symmetric buckling.) The critical buckling pressure is, of course, the minimum of these three values of p. For the particular geometries investigated in this paper the minimum value of p is given by the second of these determinants.

Numerical Example

Let us consider a multiple-bay ring-reinforced cylinder having the following dimensions:

$$
\begin{aligned}
2R &= 26.753 \text{ in.} \\
h &= 0.065 \text{ in.} \\
L &= 5.08 \text{ in.} \\
b &= 0.1875 \text{ in.} \\
h_0 &= 0.71875 \text{ in.}
\end{aligned}
$$

If the rings are first assumed to be infinitely rigid and the displacement pattern, Equations [15], adopted, the stability determinant, Equations [22], becomes

$$\begin{vmatrix} \begin{matrix} 3.497362 \times 10^{8} \\ +4.476262 \times 10^{5}m^{2} \end{matrix} & -1.374697 \times 10^{7}m & \begin{matrix} 6.344761 \times 10^{6} \\ +1.320211 \times 10^{2}p \end{matrix} \\ -1.374697 \times 10^{7}m & \begin{matrix} 3.836798 \times 10^{6}m^{2} \\ +1.224077 \times 10^{8} \end{matrix} & -3.836799 \times 10^{6}m \\ \begin{matrix} 6.344761 \times 10^{6} \\ +1.320211 \times 10^{2}p \end{matrix} & -3.836799 \times 10^{6}m & \begin{matrix} 4.024995 \times 10^{6} \\ +1\ 361312 \times 10^{3}m^{2} \\ +7.549691m^{4} \\ -2.395072 \times 10m^{2}p \\ -1.067641 \times 10^{3}p \end{matrix} \end{vmatrix} = 0$$

In this form n has been taken equal to unity, as found by the tests of the two ring-reinforced cylinders, but it is necessary to minimize the value of p with respect to the variable m. It is found that the minimum value of p occurs with $m = 17$ and is 189 psi. It is of interest to note that if the p^2-term of the quadratic equation is neglected (with $m = 17$) the same pressure is obtained.

If the displacement pattern, Equations [25], be adopted it is found that the minimum value of p occurs when $m = 17$ and is 199 psi. If the displacement pattern, Equations [26], is adopted it is found that the fifth-order determinant with $m = 16$ yields a critical buckling pressure of 179 psi, which is a lower value than may be obtained by use of either the second or the third-order determinants that also arose in conjunction with configuration, Equations [26]. Consequently, for a cylinder having the geometry stated it is apparent that the deformations of the ring permitted by the buckling configuration, Equations [26], reduce the buckling pressure 5.3 per cent from that given for a cylindrical shell reinforced by infinitely rigid rings. (It is of interest to note that the theory of Reference (10) predicts a buckling pressure of 175 psi for a model having this geometry and reinforced by infinitely rigid rings.)

Conclusions

1 A theoretical analysis is presented for the problem of determining the pressure necessary to cause an elastic instability mode of failure of a geometrically perfect thin cylindrical shell reinforced by equally spaced rings and subject to hydrostatic pressure.

2 The buckling pressures given by this, as well as by all other existing theories, are higher than those found during test. Possible causes of this are initial out-of-roundness or residual welding stresses.

Acknowledgment

The author expresses his appreciation to Dr. E. H. Kennard of the David Taylor Model Basin for suggesting Equations [6] and [13]. Further, the author would like to express his thanks to Mrs. L. V. Peugh, Mr. R. C. Slankard, and Mr. M. Borg of the David Taylor Model Basin staff, for checking the mathematical analysis and for carrying out the calculations occurring in this paper.

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An analytical solution is presented for the problem of the elastic instability of a multiple-bay ring-reinforced cylindrical shell subject to hydrostatic pressure applied in both the radial and axial directions. The method used is that of minimization of the total potential. Expressions for the elastic strain energy in the shell and also in the rings are written in terms of displacement components of a point in the middle surface of the shell. Expressions for the work done by the external forces acting on the cylinder likewise are written in terms of these displacement components. A displacement configuration for the buckled shell is

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