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THE DETERMINATION OF FRICTIONAL RESISTANCE

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One of the most useful acts that the Fifth International Conference of Ship Tank Superintendents could perform would be to agree upon a uniform method of estimating the skin-friction resistance of models and ships. There is at present a large output of test results from model basins in Europe and North America which, when translated into ship figures for publication, are not at once comparable because of the different skin-friction formulations used by the two groups concerned.

As stated in the introductory remarks to this session, the two methods in use are respectively those due to the Froudes and to Schoenherr, and it is believed that the choice before this Conference in order to achieve uniformity of practice can be limited from the outset to one or other of these methods.

The Froude frictional coefficients, as used by the European model basins, are based on the classic experiments of W. Froude, described by him in reports to the British Admiralty in 1872 and 1874 (1).** He towed smooth planks of different lengths, the longest being 50 ft; the maximum speed was 650 fpm. In order to apply the results to ships, the derived skin-friction coefficients had to be extrapolated to very much greater lengths and speeds. W. Froude did not give these extrapolation figures in his reports, but he suggested two methods which might be used for the purpose, taking into account the experimental fact that the friction per unit area decreased with increasing length of surface. He also made an effort to obtain more information to assist him in this project by towing the 172 ft 6 in. sloop GREYHOUND (2). The full-scale results gave resistances considerably higher than those derived from the model, but, while Froude attributed this fact to the poor condition of the ship's copper sheathing, he did not state the value of the friction coefficient used for the ship in predicting the resistance from that of the model. In fact, he never did publish any such ship coefficients at all, but Payne has reproduced the curve Froude used at Torquay in 1876 for ships up to 500 ft in length (3). This curve did not agree with the curve which would be obtained by either of the methods suggested by W. Froude in his original reports.

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**Numbers in parentheses indicate references on page 6.

In 1887 R.E. Froude, after careful reconsideration of the original resistance experiments made by his father, drew new curves which differed from the curves previously in use for lengths between 22 ft and 250 ft. These curves were converted into the '0' values and published in 1888 (4).

By 1904 it was necessary to extend the curves to still greater lengths. This was done by finding an empirical formula which would give the existing values of O_s at 275 ft and 535 ft and which would fit the existing curve as closely as possible between these two points, and then using this formula to extend the results to 1200-ft lengths. It is interesting to notice that these extended curves had no experimental basis beyond the 50-ft plank tests made in 1872.

These '0' values are still in use today, so far as is known, in all European tanks, in accordance with the agreements made at previous meetings of this Conference.

In 1883 Osborne Reynolds published the results of his experiments on flow in pipes, and distinguished between laminar and turbulent flow (5). Since that day it has become generally recognized that skin-friction resistance coefficients depend upon the parameter VL/ν, which is now universally known as Reynolds number. Curiously enough, R.E. Froude does not seem to have made any attempt to examine his father's skin-friction experiments in the light of this new conception.

Since Froude's time, many experiments have been made extending the measurement of friction coefficients to higher speeds and greater lengths, and many attempts made to extrapolate the resulting figures on the basis of Reynolds number. In recent years the rapid development of aerodynamic theory and experiment have greatly increased our knowledge of the subject of skin friction.

In 1932 Schoenherr (6) collected most of the results then available and plotted them as ordinates of $C_F = \left(\frac{R_f}{1/2\,\rho S V^2}\right)$ to a base of Reynolds number VL/ν. He included the results of experiments on 20-ft and 30-ft planks towed at the Washington tank, and at the lower Reynolds numbers some original work on 6-ft catamarans with artificially induced turbulence. At the higher Reynolds numbers he was guided largely by the results given by Kempf for smooth varnished plates (7). Kempf's measurements were made on small plates inserted at intervals along a 252-ft pontoon, which was towed in the Hamburg tank. They gave local specific resistances, which Schoenherr integrated to obtain the total resistance of surfaces of different lengths.

Any skin-friction formulation, to be generally acceptable, must fulfill the following requirements:

1. It should conform to any physical laws which may be known to control the phenomena of skin-friction drag between liquids and solid smooth surfaces.

2. It must lead to satisfactory correlation between the measured resistances of different-scaled models of the same hull form.

3. It must provide a basis for the calculation of the resistance of a smooth hull surface, in order that the necessary roughness allowances for different types of vessels can be assessed as reliable trial and service data are accumulated.

The Schoenherr mean line is shown on Figure 1, together with the lines of C_F derived from Froude's O_s coefficients for ships of different lengths. It should be remembered in this connection that, whereas Schoenherr's line is applicable to smooth varnished surfaces, Froude's O_s values were given as applying to painted surfaces of ships.

An examination of the Froude lines in Figure 1 shows that the O_s coefficients do not conform to the general picture of present-day knowledge and so do not fulfill Requirement 1 above. The corresponding C_F coefficients, for example, do not lie on a unique line varying only with Reynolds number. For a given length of ship, C_F decreases with increase of speed, but somewhat more rapidly than suggested by modern experiment results. For a given speed, moreover, moderate increases in length cause much less reduction in C_F than is indicated by such results. As pointed out, the O_s coefficients were intended to apply to ship surfaces, and it is not known whether any allowance for roughness was consciously added by the Froudes when they decided on the ship-friction coefficients. On the evidence of Figure 1, however, such an allowance is virtually present, and it is one which increases with length of ship. Since Froude's formulation includes such an allowance, and does not apply to a smooth surface, it does not fulfill Requirement 3 above. Moreover, the variation in roughness allowance with length of ship would appear to be in the wrong direction (8)(9).

The Schoenherr line is not a strictly theoretical one, being based on experiment results, even as were Froude's O_m values. On the other hand, it is in agreement with modern concepts that the frictional coefficient C_F is a function of the Reynolds number, and the numerical values of C_F are based only on turbulent-flow data. The absolute values of C_F given by the Schoenherr line were obtained from a statistical average of a large number of experiments, weighted with respect to known experimental defects, whereas Froude's ship values were based on only one set of experiments, the maximum

Reynolds number reached being 4.6×10^7. Schoenherr, on the other hand, had data available from Kempf's work going up to a Reynolds number of 4.5×10^8, so that the slope of his line in the upper regions should be more reliable.

The Schoenherr line has also given satisfactory correlation of the results of resistance tests on geometrically similar models of various sizes, provided satisfactory turbulence-promoting devices were used. This correlation suggests that at least the slope of the curve is correct, which is of great importance when correlating the results for models of lengths ranging from 4 ft to 30 ft tested in the different model basins.

Further evidence of the correctness of slope is provided by the fact that the C_T curves* for surface models at low speeds and for totally submerged models at all speeds lie parallel to the Schoenherr line. The fact that the C_R values obtained for the latter bodies by deducting Schoenherr's C_F frictional coefficients from the total-resistance coefficients agree reasonably well with the results of pressure-distribution tests made in wind tunnels is evidence that the absolute values of the Schoenherr coefficients are also reasonably correct.

From time to time objections have been raised to the use of the Schoenherr line. The equation to it is

$$\frac{0.242}{\sqrt{C_f}} = \log_{10}(R \times C_f)$$

which is an implicit rather than an explicit function of C_F. This is no drawback to its use in practice, since all values of R and C_F are lifted from tables. This latter fact also disposes of the objection that, being a curve and not a straight line, extrapolation is more difficult.

A more serious objection is that it gives too small values of C_F for very small models. This fact has been suggested by the results of towing tests on 4- and 5-ft models, and attributed to the effect of curvature in increasing specific resistance. L. Landweber, of the David Taylor Model Basin, has recently shown that such an increase is to be expected on purely theoretical grounds, and it may be necessary to make such an allowance for models in this range of size. This effect would be present, however, whatever formulation were used and is in no sense specific to Schoenherr's.

In view of the above reasoning, it is believed that the most satisfactory formulation to adopt at the present time is Schoenherr's curve to obtain the frictional resistance of a smooth ship, and to associate with it

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* $C_T = \frac{R_T}{1/2\rho SV^2}$ and $C_R = \frac{R_R}{1/2\rho SV^2}$ where R_T and R_R are respectively the total and residuary resistance.

an allowance for roughness depending upon the speed and length of ship and type and condition of surface.

In Figure 2 are plotted the results of a number of full-scale trials in which the propeller thrust was measured, or the vessel towed for resistance. Assuming that there is no scale effect on the residuary resistance or thrust-deduction factors, it is possible to calculate from the thrust the value of C_F for the ships. The results shown suggest that the roughness allowance decreases with length of ship, contrary to the trend of Froude's coefficients (8)(9), but the variation is not thought to be sufficiently definite as yet to justify the use of a varying allowance. The American Towing Tank Conference has therefore adopted a standard roughness allowance of 0.0004, which is shown on Figure 2, with the clear understanding that this figure will probably have to be altered as more knowledge is gained from trial- and service-performance data for ships.

The David Taylor Model Basin delegates to the Fifth International Conference of Ship Tank Superintendents would therefore submit the following resolutions for the consideration of the Conference:*

1. Analyses of model tests will be based on the Schoenherr mean line. Any correction allowances applied to the Schoenherr mean line are to be clearly stated in the report.

2. Ship EHP calculations will be based on the Schoenherr mean line, with an allowance that is ordinarily to be +0.0004 for clean new vessels, to be modified as desired for special cases and in any event to be clearly stated in the report.

3. All model data shall be corrected to a standard temperature of 59° F through the use of the correct value of ν in the Reynolds number.

4. The length to be used in determining the frictional coefficient for ship or model is to be that on the load waterline.

5. The wetted surface to be used is that calculated without any correction for obliquity in the fore and aft direction.

6. A reporter to be appointed to continue to collect relevant information regarding skin-friction resistance and roughness allowances. He would be responsible for presenting a report embodying the latest knowledge on the subject to the next Conference.

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*Resolutions 1 and 2 are identical with those passed by the American Towing Tank Conference at Newport News, Virginia, U.S.A., in 1947 and now in use in that country.

The attached table shows the results obtained by the use of the Schoenherr method and the Froude method for a few typical ships. The agreement is very close except for short lengths, where the Froude estimate is, for example, some 7 per cent lower for a 130-ft ship.

Tables of the values of the Schoenherr C_F coefficients have been calculated at close intervals of Reynolds number by M. Gertler, of the Taylor Model Basin (10). Copies of these tables will be gladly furnished to the Conference for inclusion in the Proceedings should the above resolutions be carried.

Such a change in the method of making the skin-friction correction would lead to small differences in Ⓒ and to that extent would make comparisons between new and old tests somewhat unreliable. To correct for the change in method, Mr. Gertler has also produced a chart from which the Ⓒ values obtained by using the Schoenherr line can be quickly converted to give those which would have been obtained by using Froude's O_m and O_s values, or vice versa, and copies of this chart will also be furnished to the Conference if desired (11).

The author's thanks are due to M. Gertler, of the David Taylor Model Basin staff, who has checked the figures in the attached table, and devised and prepared the very neat and quick method referred to above for converting the Froude and Schoenherr Ⓒ values.

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TABLE 1

Comparison of C_F Coefficients by Froude and Schoenherr for Some Typical Ships

No.	Type of Ship	L feet	V knots	$\frac{VL}{\nu} \times 10^{-8}$	C_F Schoenherr +0.0004	C_F Froude	$\frac{C_F \text{ (Froude)}}{C_F \text{ (Schoenherr } +0.0004)}$	$\frac{C_F \text{ (Froude)}}{C_F \text{ (Schoenherr)*}}$
1	Trawler	130	9	1.51	0.002357	0.00223	0.94	1.14
			11	1.84	0.002305	0.00215	0.93	1.13
2	Coaster	200	10	2.57	0.002222	0.00215	0.97	1.18
			12	3.08	0.002178	0.00209	0.96	1.18
3	Cargo	400	10	5.13	0.002065	0.00211	1.02	1.27
			12	6.16	0.002026	0.00204	1.01	1.25
			14	7.20	0.001995	0.00199	1.00	1.25
4	Passenger and Cargo Liner	600	14	10.8	0.001917	0.001962	1.02	1.29
			17	13.1	0.001881	0.001897	1.01	1.28
5	Atlantic Liner	1000	25	32.1	0.001737	0.001749	1.01	1.31
			30	38.5	0.001704	0.001695	0.99	1.30
6	T.B.D.	350	20	9.0	0.001951	0.00187	0.96	1.21
			30	13.5	0.001876	0.00175	0.93	1.19
			40	18.0	0.001826	0.00166	0.91	1.16
7	Battle-ship	850	20	21.9	0.001793	0.00182	1.02	1.31
			25	27.3	0.001757	0.00175	1.00	1.29
			33	36.1	0.001714	0.00167	0.97	1.27
8	Model	20	3	0.084	0.003019*	0.003051		1.011
			4	0.112	0.002880*	0.002903		1.008

ν = 1.3160 x 10^{5} ft²/sec for salt water at 59° F.

$C_F = \frac{R_F}{1/2 \rho S V^2}$

* Without +0.0004 allowance.

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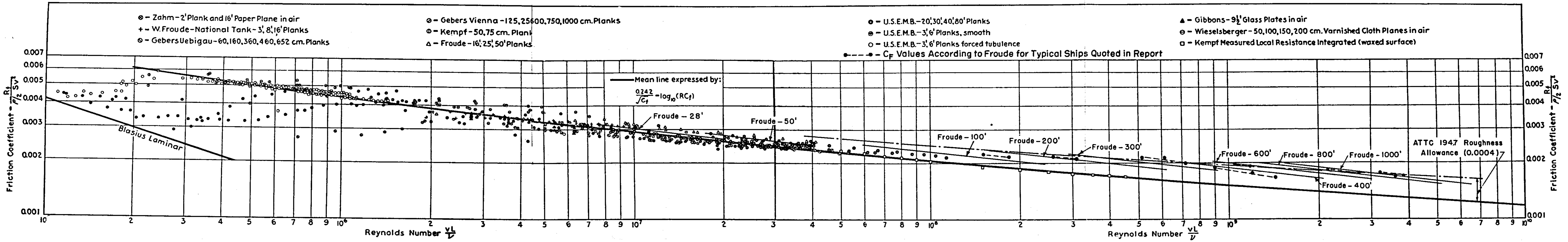

Figure 1 - Schoenherr Line, Together with C_F Lines Derived from Froude's O_s Values

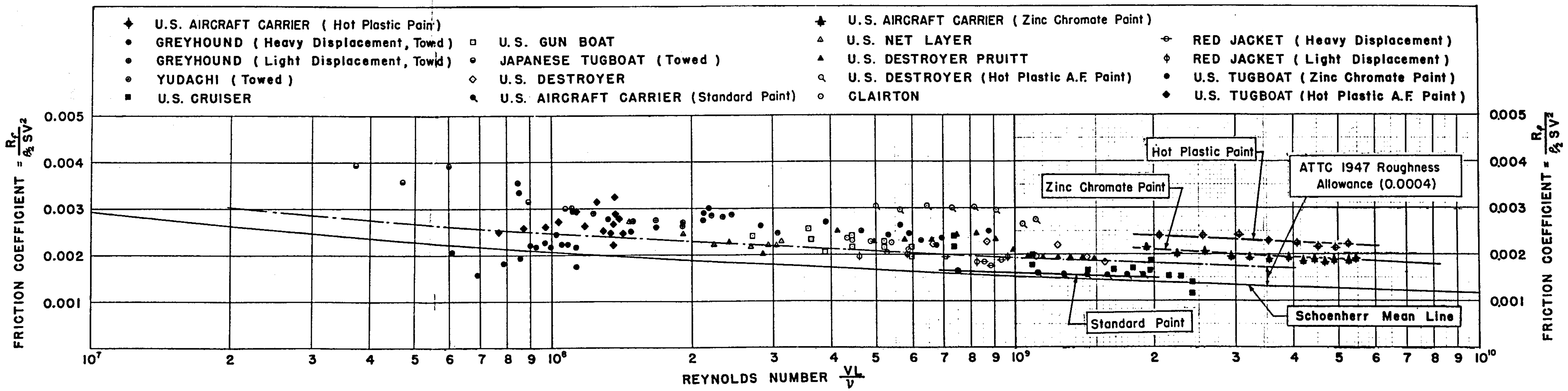

Figure 2 - Plot of C_F Derived from Full-Scale Thrust Measurements Showing Schoenherr Line with and without 0.0004 Allowance