# THE DAVID W. TAYLOR
MODEL BASIN

UNITED STATES NAVY

DESIGN AND PERFORMANCE OF THE TMB PLANING FLOAT
AS A TOWED POSITION INDICATOR

BY L. LANDWEBER AND ENSIGN P. EISENBERG, USNR
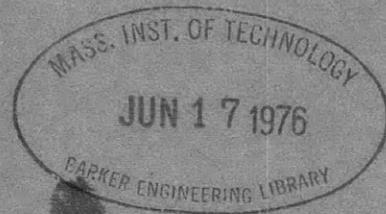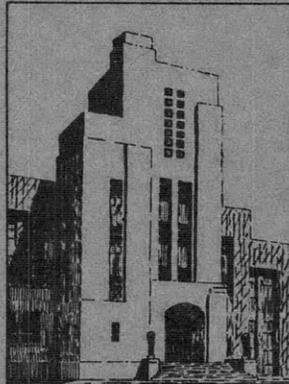
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PERSONNEL

The TMB planing float was designed and developed by Ensign P. Eisenberg, USNR, L. Landweber, and J. Plum, of the David Taylor Model Basin. Mr. Plum did the greater part of the development and testing on a series of earlier models; the developable shape and equiangular bridle were contributed by Mr. Landweber; Ensign Eisenberg conducted the towing tests of full-scale floats both in the model basins and at sea, and contributed numerous practical features to the final design.

The authors collaborated closely in writing the report; the analytical part was developed by Mr. Landweber and applied by Ensign Eisenberg. The authors checked each other's work.

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ABSTRACT

The development, characteristics, and performance of the TMB planing float, and its application for indicating the position of a subsurface body, are discussed in detail.

By analysis of the forces acting on this float, a graphical method is developed for determining a suitable range of float sizes and of cable weights within specified restrictions, and making use of the same general design. The converse problem of determining whether a float of this type and a cable of an assumed size are suitable for a given purpose is solved by direct substitution into the expressions given. Both problems are illustrated by numerical examples.

Drawings of the float and data on its performance are presented in sufficient detail to permit a size to be chosen and a float to be built to meet particular requirements.

INTRODUCTION

When it is necessary to follow visually the path and location of a subsurface body moving in a fluid, this can best be done by observation of the position of a surface buoy or float towed by the submerged body. Such a float is especially useful as an indicator in submarine-tracking exercises in which the float is towed by the submarine and is sighted from the attacking vessel. In certain minesweeping operations, paravanes towed by the minesweeper can be located by observing a float whose towline is secured to the submerged paravane. The problem to be solved is to keep the float "watching" on top of the water while it is traveling in choppy water or rough seas, and to accomplish this with a towline short enough so that the float will maintain position not too far astern of the submerged towpoint.

Specifically, it is necessary to meet the following detailed requirements:

a. Roll and pitch of the float should be a minimum to assure stability of towing and ease of observation.

b. Seaworthiness must be inherent in the design for successful operation in rough water.

c. Strength of the float must be sufficient to prevent collapse if it happens to be pulled under water to a reasonable depth.

d. Size and shape must permit the float to be easily handled
and stowed on board ship.

e. Materials composing the float should be easily fabricated
and of noncritical character to facilitate replacement in the event
of loss or damage.

The performance and the range of operation of a float are dependent
upon its load-carrying and drag characteristics. Choice of the size of the
float and of the diameter and length of cable for towing at a given depth and
speed is subject to two restrictions:

1. The strength of the cable must be great enough in comparison with
the maximum upward pull of the float so that the tension in the cable will
not exceed its safe working load.

2. The float must be large enough in relation to the length and diam-
eter of the cable so that the total load on the float, due to its own weight
and to the downward component of the fluid forces on the cable, will not ex-
ceed the maximum load the float can carry without submerging.

To satisfy these restrictions most efficiently the float should have
low drag and be capable of carrying a large load in comparison with its
weight.

The TMB planing float was developed at the David Taylor Model Basin
at the request of the Bureau of Ships (1)* for use as a paravane-position
indicator. This type of float was later successfully used as a submarine-
position indicator by the Naval Ordnance Laboratory (2).

To make the TMB planing float design available for other problems
in which it may be useful, this report provides methods of analysis and in-
formation from which a complete design to meet particular specifications can
be prepared. A graphical method is developed for determining a suitable
range of float sizes and cable weights to meet specified conditions. The
converse problem of determining whether an assumed float and cable size are
suitable for a given purpose is solved by direct substitution into derived
expressions. Although the analysis is developed for the TMB float, it can be
applied with slight modifications to any float for which certain character-
istics are known.

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* Numbers in parentheses indicate references on page 21 of this report.

DEVELOPMENT OF THE TMB PLANING FLOAT

Many of the floats used by the forces afloat during the past few years had been improvised affairs, rather crudely constructed and by reason of their form inherently not capable of "watching" on the surface (3) or of carrying relatively heavy towline loads.

To satisfy the requirements of high load-carrying capacity with accompanying low drag the development was limited, from the start, to floats with planing surfaces. A series of 10 floats, with variations of some, were tested. The TMB planing float, Model 7 of this series, is the culmination of this work.

MODEL 0

When the present project was initiated the design of a planing float, designated as Model 0 and shown in Figure 1, was on hand. This model had been developed for towing with a long towline from a surface vessel.

The principal features of Model 0 are the varying V-bottom or dead-rise angle, increasing from zero at the tail to a maximum of 10 degrees at the bow, the wedgeshaped runner at the stern, and the partly rounded bow and deck. It was towed by a 2-point lateral bridle with the attachment points near the chine lines at the midlength of the float.
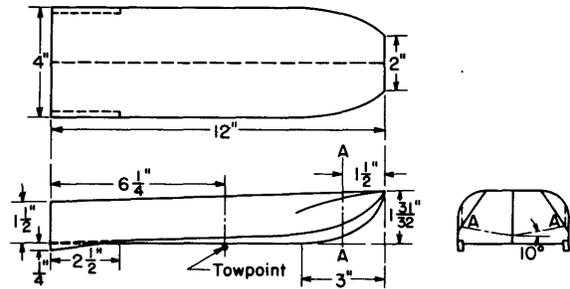
Figure 1 - Sketch of Model 0

In the project for which this design was developed it was necessary to lay great emphasis on the elimination of yawing, pounding, and porpoising. The wooden-wedge runners eliminated yawing and porpoising. The bow was rounded to reduce the punishment when it entered waves. The deck was partly rounded to enable the float to shed water. This float was highly successful for its original purpose.

To determine its behavior when towed from an underwater body, a 12-inch model was towed by a line attached to the bottom of a strut, immersed to a depth of 3 feet. When towed on the surface in quiet water the model would normally follow behind its towpoint, planing in a satisfactory manner. If, however, it was displaced some distance to one side from the vertical fore-and-aft plane through the submerged towpoint, the float assumed a stable attitude in which it was heeled and yawed away from the towpoint, and it continued to run in a laterally displaced position. This behavior was eliminated

when the 2-point bridle was replaced by a single towpoint on the keel. If
the bow of the model became submerged the float would dive and remain below
the surface, towing stably at a small trim angle. This was very undesirable,
since in a heavy sea a small planing float would be submerged frequently.

MODEL 1

Model 1, shown in Figure 2, was designed to correct the diving
characteristics exhibited by Model 0. The rather complicated bottom surface
of Model 0 was replaced by a flat V-bottom with a constant dead-rise angle
of 10 degrees, since it was thought that pounding and porpoising would be
less likely to occur with a submerged towpoint. The float was towed by a
single line to the keel at the mid-length. It planed well but was un-stable at low speeds because of its large, immersed frontal area.
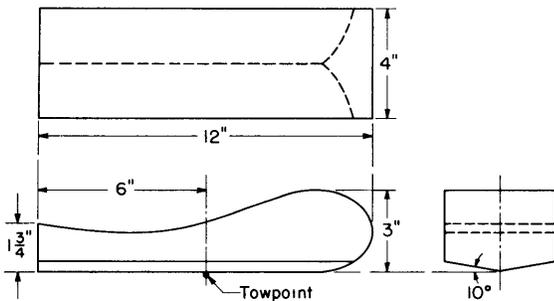
Figure 2 - Sketch of Model 1

MODELS 2 AND 3

Sketches of Models 2 and 3 are shown in Figure 3. Model 2 had the
same bottom lines as Model 0, but the profile was modified at the top to add
buoyancy at the bow and to provide dynamic lift as a hydrofoil when sub-merged. Both these features eliminated the diving tendency of Model 0. Model
3 differed from Model 2 in that its bottom had a constant dead-rise angle of
10 degrees and the wedge runners were eliminated. Both Models 2 and 3 had
watertight compartments to reduce the deadweight, especially at the bow. To
eliminate yawing they were tested with flat vertical fins of various lengths
extending aft from the stern.
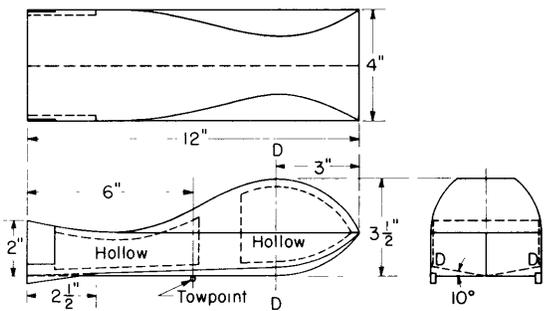
Figure 3a - Sketch of Model 2
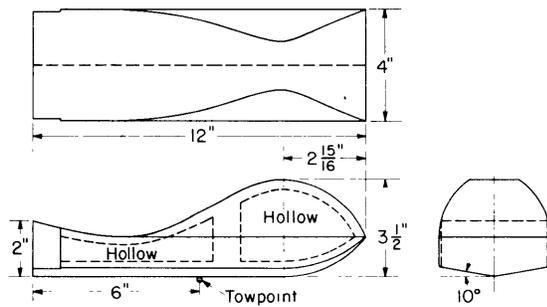
Figure 3b - Sketch of Model 3

The floats were towed by a single line to the keel at the mid-length. A depressor was attached at the towpoint to simulate the presence of an underwater towing body. With sufficiently large fins, the models were stable on the surface throughout the speed range, but they stalled when towed completely submerged. Their underwater behavior was more satisfactory than that of Model 0, since the floats would return to the surface after submerging, but the stalling was considered undesirable because it caused excessive towline tensions.

MODELS 4, 5, AND 6

Since the special longitudinal profile of Models 2 and 3 had proved to be unsatisfactory, it was discarded in favor of a simpler profile. Models 4, 5, and 6, shown in Figure 4, were tried in turn. The models were stabilized against yawing on the surface, and against both yawing and stalling when submerged, by a retractable outrigger tail. All were towed by a single line attached to the keel at the midlength.

Model 4 had a flat V-bottom planing surface with a constant deadrise angle of 10 degrees. Its bow consisted of a pair of frustums of elliptical cones. The top surface was rounded in longitudinal section but also
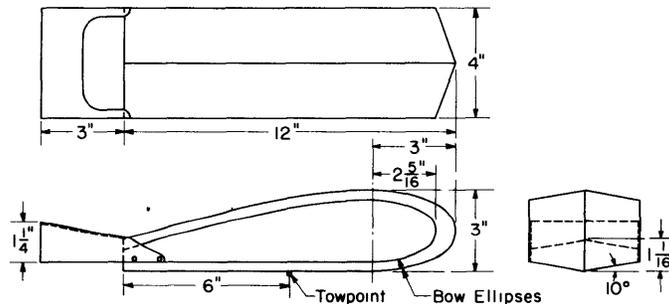
Figure 4a - Sketch of Model 4
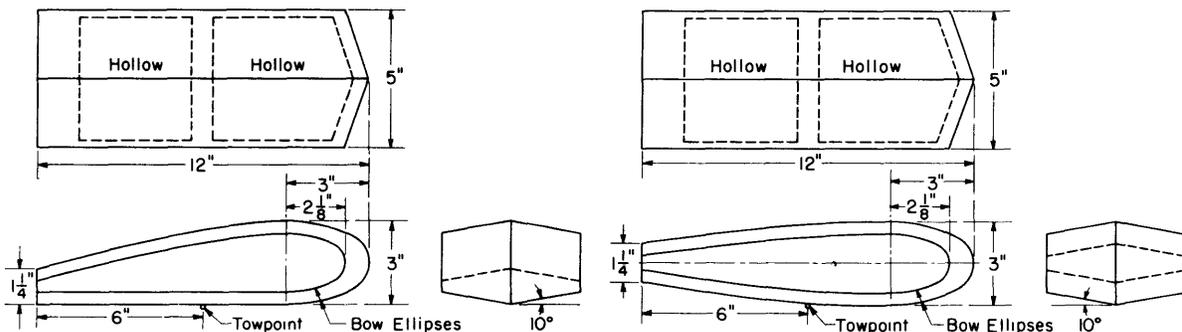
Figure 4b - Sketch of Model 5

Figure 4c - Sketch of Model 6

had a constant dead-rise angle of 10 degrees. The introduction of the elliptical bow and V-top made the entire float surface developable with simple lines, so that the float could be constructed easily out of sheet metal. This feature was maintained in all subsequent designs. The float planed stably at all speeds and assumed a moderate angle of attack when towed submerged.

Model 5 had a higher aspect ratio than Model 4, 0.42 against 0.33, but was otherwise identical. The former appeared to run more cleanly, that is, to throw less spray.

It was thought that the planing could be improved further in this direction by increasing the rate of rise at the bow. Consequently this change was made in Model 6. Its aspect ratio was the same as that of Model 5 but its longitudinal profile was symmetrical with a circular planing surface. However, as a consequence of its longitudinal curvature, its initial planing speed was higher than that of Model 5.

MODELS 7, 8, AND 9

As a result of the tests of Models 4, 5, and 6, the flat V-bottom planing surfaces and the symmetrical shape were retained, and a series of models was designed to investigate more fully the effects of aspect ratio and varying dead rise. In Model 7, shown in Figure 5a, the simplicity of the symmetry of Model 6 was maintained, but the top and bottom surfaces were made flat in longitudinal section. The rate of rise at the bow was increased over that in Model 5 by interposing a circular arc between the bottom line and the bow ellipse in a longitudinal section, instead of making the straight line tangent directly to the bow ellipse as in Model 5; see Figures 13 and 14 of Appendix 2.

For comparative tests two other models, 8 and 9, shown in Figures 5b and 5c, were also prepared. Model 8 is similar to Model 7, but with 5 degrees dead rise at the stern, increasing uniformly to 15 degrees dead rise at the maximum section. Model 9 is similar to Model 8 but with increased aspect ratio.

Of these three floats, Model 7 exhibited the best overall towing characteristics. Under the same loads, Model 7 began porpoising at speeds slightly lower than those at which Model 8 behaved similarly. As the loads were increased, the speeds at which porpoising began decreased with each float. However, with increasing load, the speed at which porpoising began decreased more rapidly with Model 8 than with Model 7. Furthermore, Model 7 was more stable below planing speeds than was Model 8, which possessed a yawing instability in the lower speed range. Under equivalent loads, however, the drag of Model 7 was, on the average, 5 per cent higher than that of Model 8.

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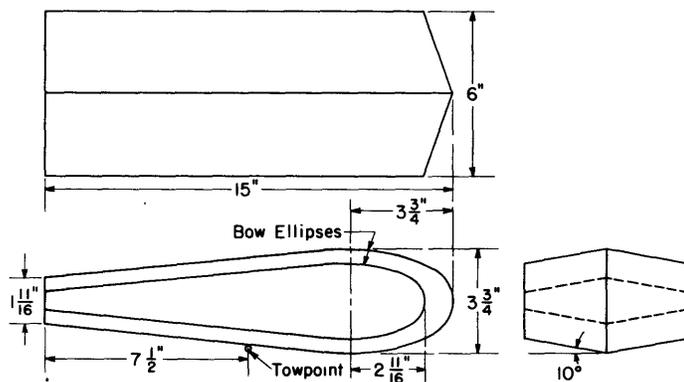
Figure 5a - Sketch of Model 7
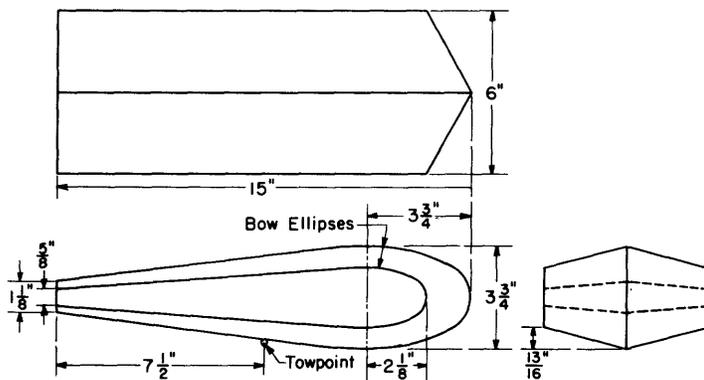
Figure 5b - Sketch of Model 8
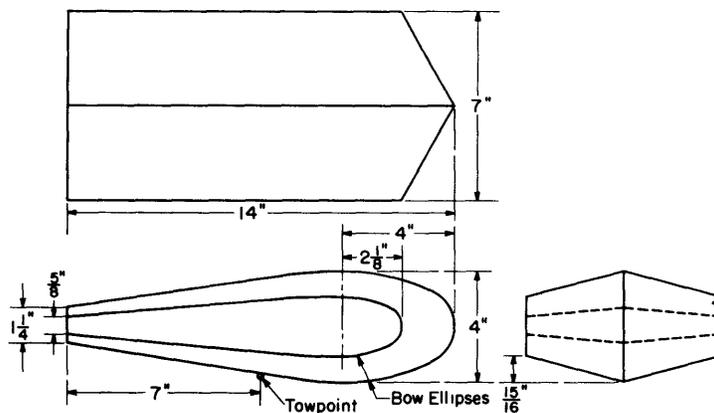
Figure 5c - Sketch of Model 9
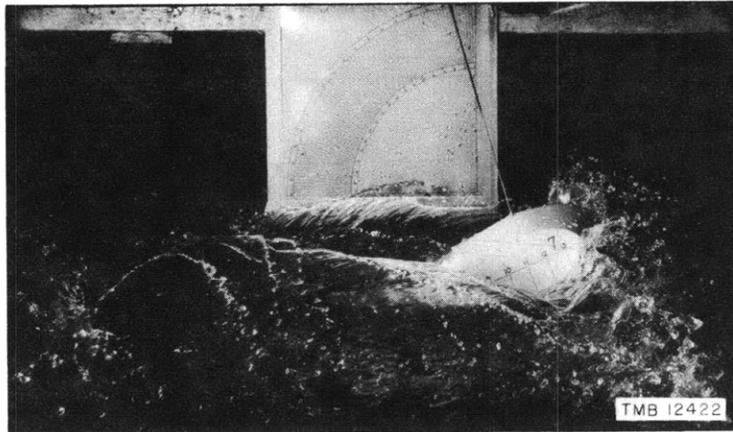
Figure 6a – Model 7, V = 5.00 knots
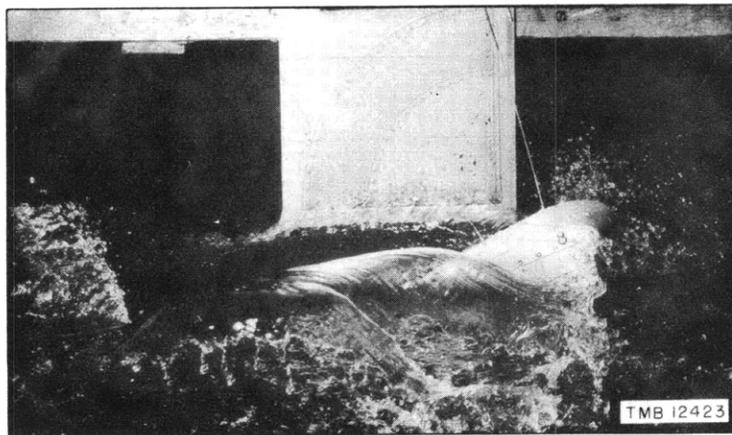
Figure 6b – Model 8, V = 5.10 knots
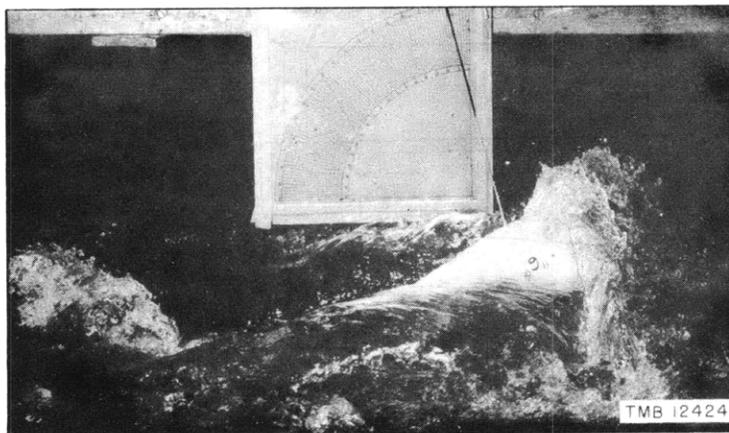
Figure 6c – Model 9, V = 4.10 knots

Figure 6 - Models 7, 8, and 9 Towed under Equal Loads

For these tests the floats were equipped with the outrigger stabilizer shown in Figure 4a and were towed with a single line attached to the midlength of the keel. Only the bow of the models can be seen in the photographs. A light pole attached to the float and an adjacent quadrant in the background were used to measure the trim angle. The direction of motion is to the right.

Although the drag of Model 9 was also lower than that of Model 7, it began porpoising at speeds lower than the porpoising speed of either Model 7 or Model 8. Models 7, 8, and 9 are shown in Figure 6 during tests under the same load.

As a result of this final series of tests, the form of Model 7 was selected as the most promising and was standardized as the "TMB Planing Float."

DEVELOPMENT OF THE EQUIANGULAR BRIDLE

Throughout the development of this series of floats, the outrigger stabilizer was used. However, for most applications in a seaway, the outrigger was thought to be too vulnerable to damage. Instead double tail fins were provided for directional stability, as shown in Figures 8 and 9. A lead ballast plate was added across the stern to keep the longitudinal center of gravity aft. To provide pitching stability, a two-legged equiangular bridle was substituted for the outrigger.
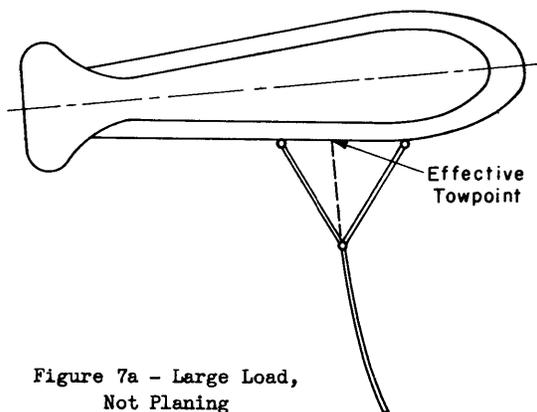
Figure 7a - Large Load, Not Planing
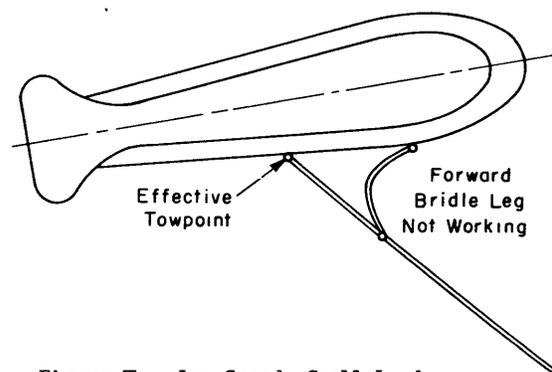
Figure 7b - Low Speed, Small Load and High-Speed Planing
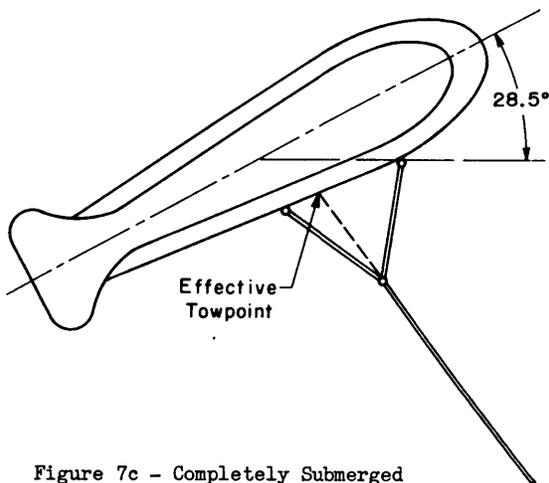
Figure 7c - Completely Submerged

Figure 7 - Illustration of the Effective Towpoint with an Equiangular Bridle

The function of the bridle is to permit stable towing submerged as well as on the surface. When the float is towed by the bridle on the surface, both legs function when the float is carrying very large loads on the line. When the float is planing or carrying small loads, the forward leg is slack; the after leg carries the entire load. When the float is towed submerged, the line of action of the towline at the apex of the bridle passes between the two legs, so that the effective point of attachment of the cable is moved forward of the rear leg. The float stalls if towed with a single line to the after bridle point of attachment. The positions of the effective point of tow on the float under the various conditions discussed are illustrated in Figure 7. The equiangular bridle was selected as giving good performance both on the surface and completely submerged. The effect of bridles other than equiangular is discussed in Appendix 3.

DESCRIPTION OF THE TMB PLANING FLOAT

A solid wood model, 45 inches long, of the TMB planing float is shown in Figure 8. A drawing of the float and the development of its surfaces are given in Figure 9, in which all dimensions are expressed in terms of the beam. When the proper float beam b has been selected, as described in subsequent sections, the other dimensions of the float can be readily calculated. If a flagstaff is desired it should be placed in the position indicated in Figure 9.

The float can be built of solid wood or of hollow metal, with vertical metal stabilizing fins in either case. If made of wood, it should be built of laminated pine or some other light wood, made up with waterproof resin glue, and held together with wooden dowels glued in place. The vertical fins should be of some corrosion-resistant material, held in place with wood screws.

If the float is to be made of sheet steel, the float parts may be welded together, using internal lightened transverse bulkheads, to retain the shape and prevent collapse. Hollow steel buoys, however, are frequently pulled to considerable depths under water and in several cases they have collapsed under these circumstances.

It is recommended that white pine, sugar pine, or Western pine be used for the hull wherever possible. Wood for the hull construction has the advantage of being easily worked and the total buoyancy is then very nearly twice the weight of the float. Moreover, a float built of wood is not likely to be damaged by hydrostatic pressure in the event that it becomes deeply submerged by fouling of the towline.
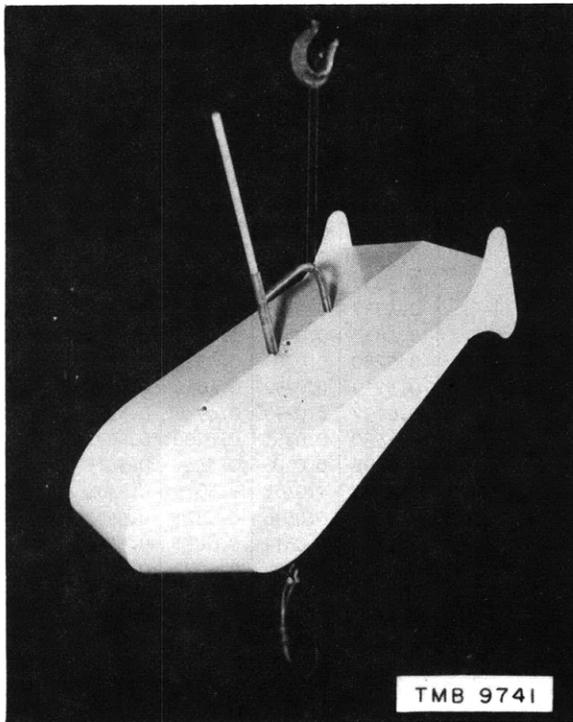
Figure 8a – Bow View

The hull has a uniform 10-degree V-top and
V-bottom and is symmetrical about a horizontal
plane. The bow is formed by elliptical cones
so that the curved surfaces are completely
developable. The flagstaff and the lifting
bail are shown on the top of the float.
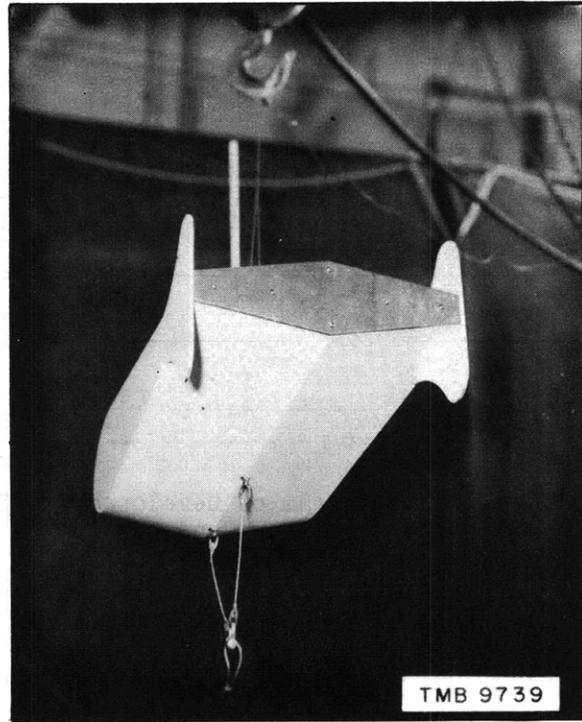
Figure 8b – Quarter View

Tail fins are used as stabilizers, with lead
plates as trimming ballast across the stern.
The equiangular towing bridle is shown on the
keel line of the float.

Figure 8 - TMB Towed Indicator Float

PERFORMANCE OF THE FLOAT

Unlike floats of the displacement type which have a maximum load-
carrying capacity equal to their reserve buoyancy, the TMB float has a
maximum capacity equal to its reserve buoyancy plus its dynamic lift. This
increase in capacity due to dynamic lift enables the TMB float to carry eas-
ily the additional load imposed upon a towed body by the vertical component
of the towline tension. For equivalent conditions, therefore, the TMB float
will be capable of towing at higher speeds and with a shorter scope than a
displacement float of equal reserve buoyancy.

Under very light loads such as those obtained with a towline of
small diameter and with the towpoint at shallow depths, the TMB float planes
like a high-speed motorboat. Model tests have shown that the float planing
characteristics in this case, are in agreement with those obtained by
Shoemaker (4) (5). However, the applied loads for towed position-indicator

Developed Curve on Centerline		Developed Curve on Chines	
x/b	y/b	x'/b	y'/b
0	0	0	0.5082
0.1175	0.0009	0.1484	0.5082
0.1722	0.0021	0.1872	0.5104
0.2262	0.0035	0.2257	0.5118
0.2798	0.0054	0.2631	0.5134
0.3320	0.0076	0.3002	0.5151
0.3837	0.0100	0.3362	0.5176
0.4337	0.0135	0.3706	0.5200
0.4817	0.0170	0.4039	0.5229
0.5280	0.0209	0.4354	0.5260
0.5719	0.0252	0.4654	0.5294
0.6137	0.0299	0.4936	0.5331
0.6530	0.0351	0.5200	0.5371
0.6898	0.0406	0.5447	0.5415
0.7241	0.0468	0.5675	0.5464
0.7560	0.0536	0.5891	0.5517
0.7860	0.0611	0.6091	0.5578
0.8139	0.0695	0.6279	0.5645
0.8402	0.0788	0.6461	0.5721
0.8655	0.0894		

Figure 9a - Diagram of Developed Top Surface Showing Layout for Construction of a Hollow Metal Float

Bow Ellipse on Centerline		Bow Ellipse on Chines	
x/b	y/b	x'/b	y'/b
0	0.3125	0	0.2243
0.10	0.3088	0.10	0.2189
0.20	0.2963	0.15	0.2115
0.30	0.2744	0.20	0.2009
0.40	0.2403	0.25	0.1862
0.45	0.2170	0.30	0.1665
0.50	0.1875	0.35	0.1404
0.55	0.1484	0.40	0.1016
0.60	0.0875	0.4486	0
0.625	0		
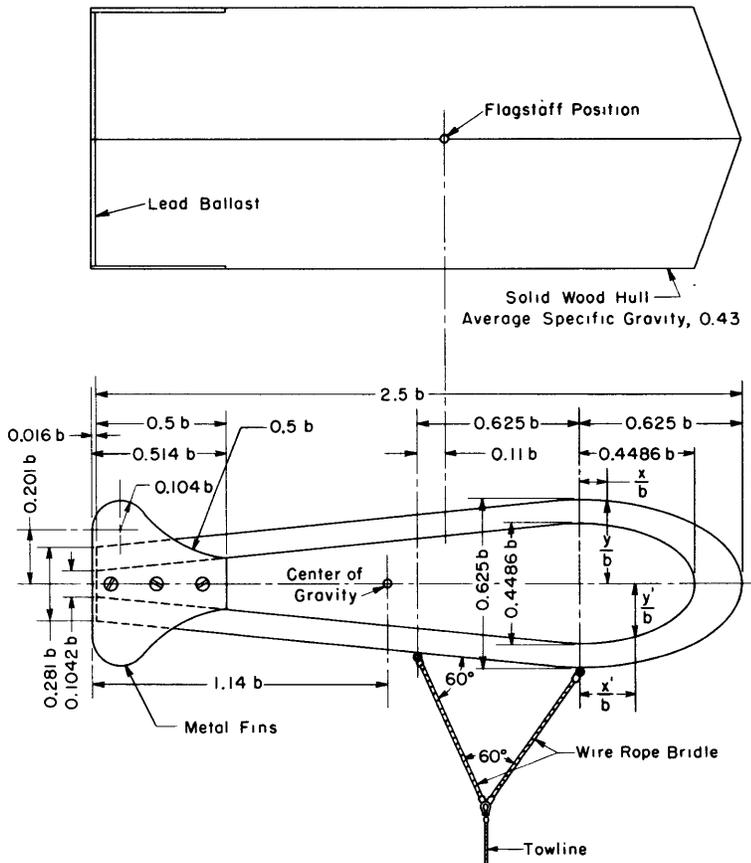
Figure 9b - Dimensioned Sketch of TMB Planing Float, Referred to a Selected Beam b

First b is selected, and then all the other dimensions are derived.
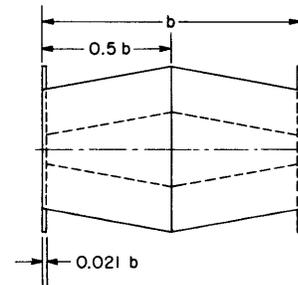
floats are much greater than those encountered in the usual applications of planing surfaces.

As the load is increased, the float submerges slightly so that a sheet of water flows over the top. This induces additional dynamic lift. Under these conditions, the lift-drag ratio has been found to be 3.0, where the buoyancy is included in the lift, and the average trim angle is about 15 degrees.

By actual test it was found that the float developed a total dynamic lift, in addition to the buoyancy corresponding to its own weight W, of $0.625\,qb^2$, where q is equal to $1/2\,\rho v^2$. Here ρ is the mass density of the fluid, v is the towing speed in feet per second, and b is the beam of the float in feet. Hence a safe limiting lifting capacity of the TMB float as a surface buoy is

$$L = 0.625\,qb^2 + W \qquad\qquad [1]$$

The float has also been tested as a fully-submerged hydrofoil. In this case the trim angle is 28.5 degrees, and the drag D is

$$D = 0.83\,qb^2 \qquad\qquad [2]$$

The total vertical fluid reaction, dynamic and buoyant, is

$$L = 1.44\,qb^2 + B \qquad\qquad [3]$$

where B is the total buoyancy of the float. If the average specific gravity of the float is 0.5, as is the case when the hull is built solid of white pine or its equivalent, then $B = 2W$.

BEHAVIOR IN A SEAWAY

The TMB float has been used successfully in several trials at sea to indicate the position of an underwater body. In each case, stable towing and excellent behavior in waves was observed, with only slight roll and pitch in choppy water.

The 45-inch model, Figure 8, has been used in submarine-attack trials. The float was towed with an aircraft cable 1/8 inch in diameter and 260 feet long, from the conning tower of a submarine at towpoint depths up to 130 feet and speeds up to 6 knots. Over this speed range, the float was observed to be running with a thin sheet of water flowing over the top. Although this condition does not greatly impair observation since the bow spray and the wake are clearly visible in a smooth sea, a flag and pole were added to permit observation at distances up to several thousand feet in 4- or 5-foot waves. In long swells, the float had only a very slight roll, and did

not slide laterally on the slope of a swell. As far as could be determined, the trim angle remained fairly constant except as the float entered the wake of the attacking surface vessel. Even under this condition no severe pitching was observed, and any severe rolling which occurred as the float rode through the wake of the vessel was quickly damped out.

On one occasion during these trials, the towline evidently became entangled in some fitting on the submarine and the float could be observed towing about 10 feet below the water surface. Although a flagstaff 3/4 inch in diameter and 4 feet long was firmly secured to the float, its drag did not put an excessive overturning moment on the float. The float characteristics while running submerged in this manner seemed not to be adversely affected.

A 30-inch model of the TMB float has also been used as a paravane-position indicator. Full details of these sea trials are not available at the time of writing, but from observations on a half-scale paravane towing a 15-inch model of the float, generally successful behavior was expected.

It has been discovered in tests at the Taylor Model Basin and substantiated in the various sea trials that to avoid an oscillatory instability in the direction of motion, the towing body must have considerably more mass than the float being towed, as is the case for a submarine or paravane.

RELATIVE POSITIONS OF FLOAT AND TOWPOINT

In most applications it is required that the position of the underwater towpoint be known from observations made on the towed surface float. Figure 10 is a dimensionless plot of x, the float position abaft the towpoint, for various ratios y/s of towpoint depth to towline length. The values are based on the lift-drag ratio obtained when towing with the equiangular bridle. Over the speed range for which a particular float is designed, the position of the float relative to the towpoint is independent of speed within plus or minus 3 per cent.

ANALYSIS OF FORCES ON THE FLOAT AND TOWING CABLE

Suppose a float of weight W in air, of buoyancy $2W$ when totally submerged, and of beam b to be towed by a body at depth y and by a towline of diameter d and length s. Let T be the tension in the towline and ϕ_0 the angle of the towline with the direction of towing at the float, as shown in Figure 11. Let L and D be the total lift and the drag acting on the float due to the fluid reaction. Let R be the drag per unit length of the cable when it is tested at a position normal to the direction of motion.

$\frac{x}{y} = 0.8$

0.9 1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 2.0 2.2 2.5 3.0 4.0 5.0 6.0

Float

Waterline

Towing Bridle

Towline

x

y

s

Direction
of Motion

All dimensions are
in the same units.

Towpoint

Ratio $\frac{y}{s}$

Ratio $\frac{x}{s}$
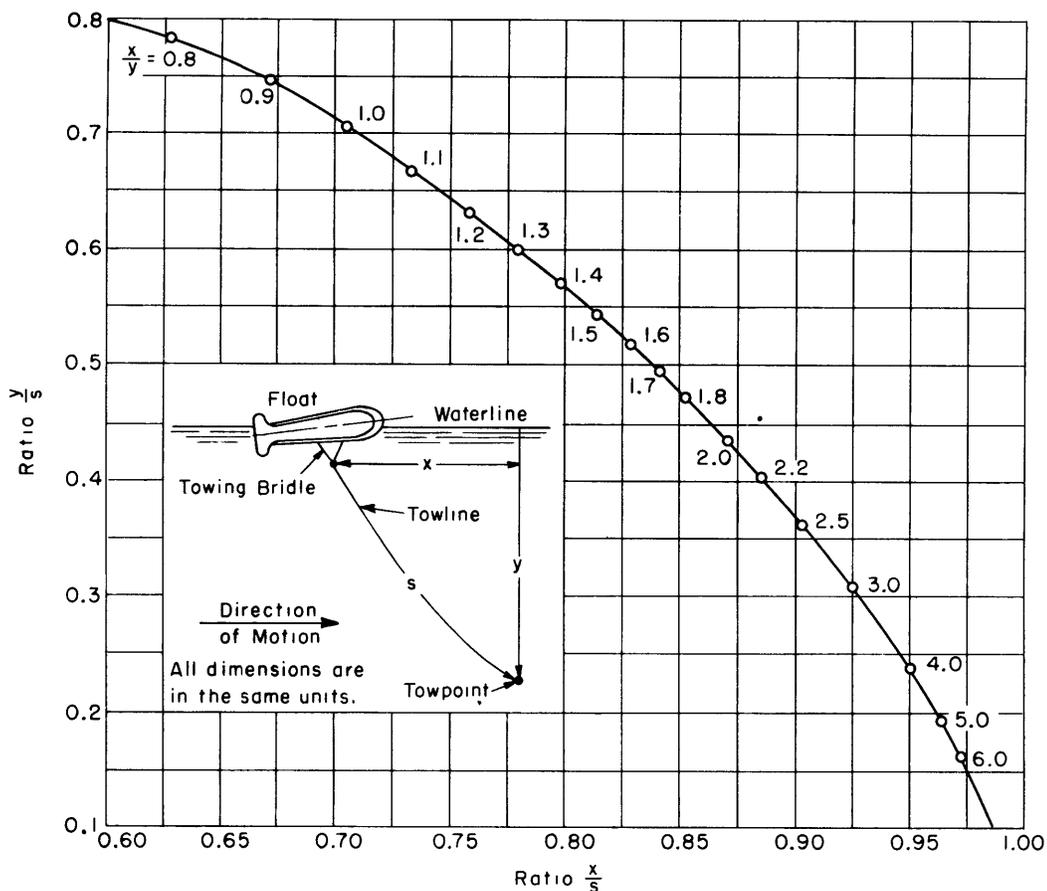
Figure 10 - Graph for Determining the Float Displacement
Downstream from the Towpoint

The equilibrium equations at the float are

$$L = T \sin\phi_0 + W \qquad [4]$$

and

$$D = T \cos\phi_0 \qquad [5]$$

SOLUTION FOR A FLOAT TOWED ON THE SURFACE

It will be assumed that the lift-drag ratio L/D, designated by a,
is constant when the float is towed on the surface; for the TMB planing float,
a is equal to 3.0. Put

$$t = \frac{T}{Rs}, \quad \Delta = t\cos\phi_0, \quad \Lambda = t\sin\phi_0, \quad \eta = \frac{y}{s} \qquad [6]$$

It is shown in TMB Report 533 (6) that Λ is a function of Δ and η; this func-
tion is reproduced in Figure 11 as curves of Λ plotted on a base of Δ for
various values of η.
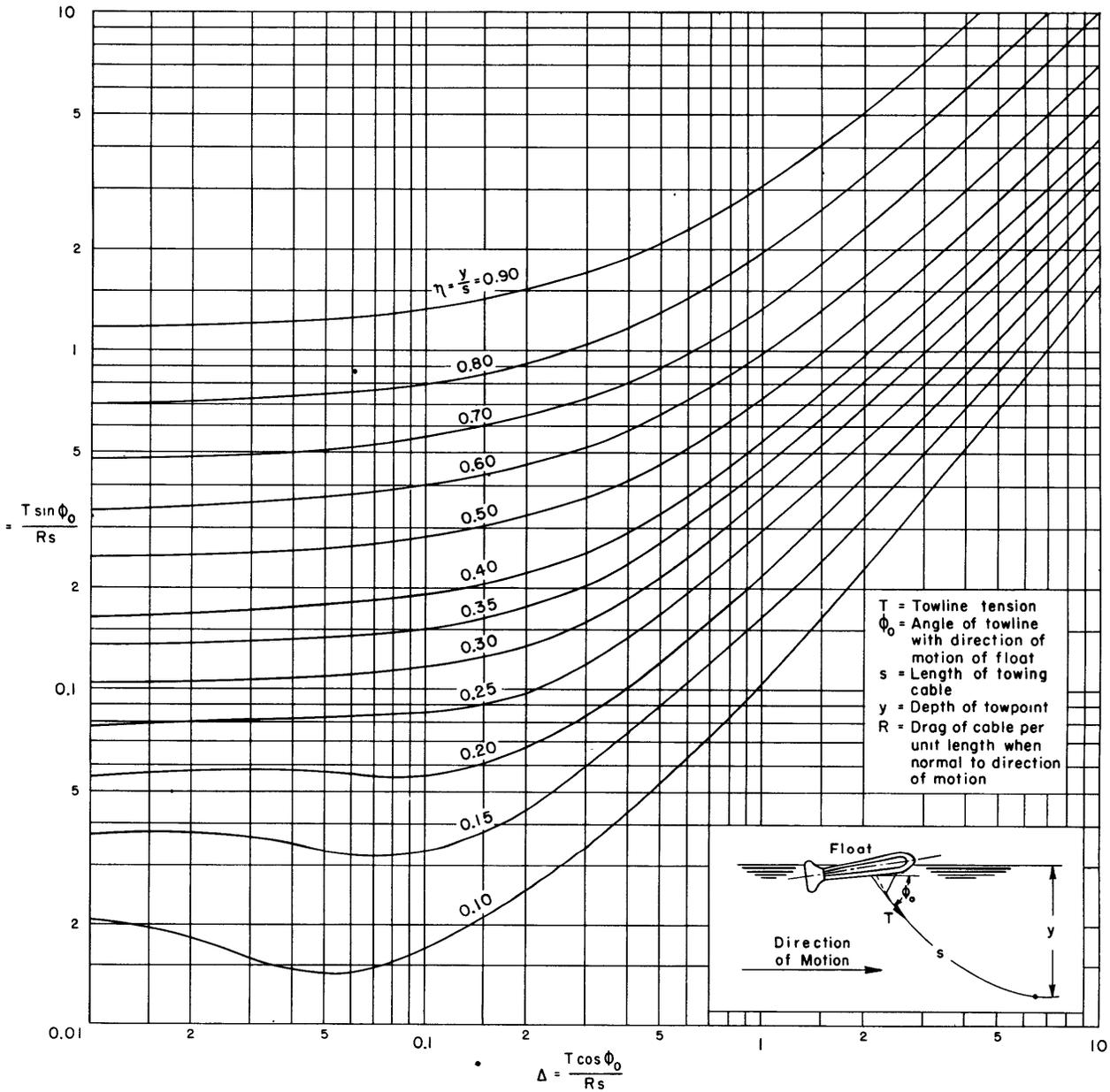
Figure 11 - Diagram for Determining the Angle of and the Tension
in a Float Towing Cable with the Float on the Surface

From Equations [4], [5], and [6], and dividing by Rs

$$\Lambda = a\Delta - \Omega \qquad [7]$$

where

$$\Omega = \frac{W}{Rs} \qquad [8]$$

Equation [7] can be plotted on a diagram such as Figure 11 as a series of curves of Λ against Δ for various values of Ω. When this is done, the intersections of the Ω and η contours give values of Δ, Λ and hence also of $t = \sqrt{\Lambda^2 + \Delta^2}$, which may be considered as functions of η and Ω. These values are given in Table 1 for the TMB planing float for L/D equal to 3.0. The use of this table in determining a float size for particular specifications is outlined in the following sections.

As stated previously in the Introduction, the design of a towed float is subject to two restrictions, namely, the safe working load for the towing cable selected, and the maximum load that the tentative design of planing float can carry at a given speed. The former condition can be expressed by the inequality

$$T \leq md^2 \, (= T_m) \qquad\qquad [9]$$

where T_m is the maximum safe working load for the cable and m is a coefficient* whose value depends upon the type of cable and the factor of safety to be used. The factor of safety must provide a margin of strength over the steady-state towing conditions to take care of suddenly applied loads. As was discussed in the section on Performance, the maximum load to be carried by the float, including its own weight, can be expressed by the inequality

$$L \leq nqb^2 + W \qquad\qquad [10]$$

where n is a coefficient whose value depends upon the shape of the float and the position of the towpoint. From Equation [1], $n = 0.625$ for the TMB planing float.

The following relations are also required;

$$W = wb^3 \qquad\qquad [11]$$

where w, a coefficient for geometrically similar floats of the same material and construction, is 29.0 pounds per cubic foot for the TMB planing float if made of white pine, and

$$R = C_D q d \qquad\qquad [12]$$

where C_D is the drag coefficient for the cable, when tested normal to the direction of motion. Dividing by Rs and using Equation [12], the inequality

---

* For any type of cable the value of m can be determined from the data for breaking strength given in the handbooks. Since the breaking strength varies very nearly as the square of the cable diameter, m can be determined from the data for any cable size. See Appendix 1.

[9] becomes

$$\frac{md}{C_D qs} \geq t, \text{ for the surface cable condition} \qquad [13]$$

Also from [4], [10], and [12],

$$\frac{nb^2}{C_D ds} \geq \Lambda, \text{ for the surface float condition} \qquad [14]$$

From [8], [11], and [12],

$$\Omega = \frac{wb^3}{C_D qds} \qquad [15]$$

TABLE 1

Values of Λ and t for Various Values of η and Ω for $\frac{L}{D} = 3.0$

$$\Lambda = a\Delta - \Omega, \quad t = \sqrt{\Lambda^2 + \Delta^2}$$

Ω \ η		0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.50	0.60	0.70	0.80	0.90
0.02	Λ	0.020	0.038	0.057	0.081	0.109	0.142	0.183	0.284	0.428	0.658	1.170	3.255
	t	0.024	0.043	0.063	0.088	0.117	0.152	0.195	0.301	0.454	0.695	1.234	3.430
0.05	Λ	0.017	0.037	0.057	0.082	0.110	0.144	0.186	0.287	0.432	0.670	1.200	3.390
	t	0.029	0.047	0.067	0.093	0.122	0.158	0.201	0.309	0.461	0.714	1.268	3.440
0.10	Λ	0.015	0.034	0.057	0.084	0.113	0.147	0.190	0.296	0.445	0.693	1.230	3.480
	t	0.041	0.056	0.078	0.104	0.134	0.169	0.213	0.323	0.481	0.738	1.305	3.702
0.20	Λ	0.015	0.032	0.055	0.085	0.118	0.155	0.199	0.312	0.470	0.733	1.305	3.695
	t	0.073	0.083	0.101	0.128	0.159	0.195	0.244	0.354	0.520	0.792	1.392	3.802
0.50	Λ	0.023	0.041	0.063	0.097	0.136	0.182	0.233	0.360	0.548	0.845	1.510	4.250
	t	0.175	0.186	0.200	0.221	0.251	0.288	0.333	0.458	0.649	0.954	1.650	4.537
1.00	Λ	0.039	0.068	0.096	0.138	0.186	0.240	0.302	0.454	0.694	1.055	1.910	5.120
	t	0.350	0.362	0.380	0.408	0.436	0.480	0.528	0.666	0.890	1.257	2.142	5.510
2.00	Λ	0.072	0.121	0.165	0.232	0.303	0.378	0.465	0.673	0.985	1.490	2.760	7.280
	t	0.696	0.718	0.738	0.778	0.823	0.874	0.943	1.115	1.400	1.894	3.175	7.905
5.00	Λ	0.193	0.283	0.382	0.505	0.657	0.790	0.975	1.310	1.855	2.910	5.185	
	t	1.745	1.777	1.829	1.903	2.000	2.085	2.266	2.474	2.938	3.920	6.195	
10.00	Λ	0.437	0.596	0.778	0.963	1.225	1.460	1.770	2.350	3.290	5.300	9.200	
	t	3.506	3.579	3.672	3.776	3.925	4.090	4.300	4.740	5.520	7.350	11.21	
20.00	Λ	1.020	1.290	1.620	1.960	2.370	2.810	3.320	4.360	6.150			
	t	7.070	7.210	7.380	7.580	7.800	8.080	8.430	9.210	11.67			

An alternate expression for the surface-float condition may be derived as follows. From [14] and [15]

$$\frac{nb^2}{C_D ds}\,\Omega \geq \frac{wb^3}{C_D q ds}\,\Lambda$$

or simplifying

$b \leq \dfrac{nq}{w}\,\dfrac{\Omega}{\Lambda}$, for the surface-float condition, alternate form   [16]

The values of t and Λ occurring in [13], [14], and [16] are given as functions of η and Ω in Table 1.

SOLUTION FOR A FLOAT TOWED SUBMERGED

The float intended to tow normally on the surface may accidentally become submerged if the towing body runs at a greater depth than was planned, if the towline becomes fouled, or if the float is used in a heavy sea. In this case the float behaves as a submerged hydrofoil and its drag D may be written as

$$D = kqb^2 \qquad [17]$$

or, from Equation [5],

$$T\cos\phi_0 = kqb^2 \qquad [18]$$

where k is the drag coefficient of the hydrofoil when fully submerged. From Equation [2], $k = 0.83$ for the TMB planing float.

Also the total lift L, dynamic and buoyant, may be written as

$$L = a_0 D + B \qquad [19]$$

where a_0 is the lift-drag ratio of the dynamic forces on the submerged hydrofoil, a_0 is 1.73 for the TMB planing float, and where B may be assumed approximately as $2W$ for a white pine float. Hence from Equations [4], [11], [17], and [19]

$$T\sin\phi_0 = a_0 kqb^2 + wb^3 \qquad [20]$$

As for the surface float, Condition [9] for the limiting tension in the towing cable must be satisfied. Hence, from Equations [18] and [20], the cable condition becomes

$$(kqb^2)^2 + (a_0 kqb^2 + wb^3)^2 \leq (md^2)^2 \qquad [21]$$

FLOAT PROBLEMS

Two main problems arise in connection with floats towed by subsurface bodies. First it may be required to determine in advance whether an

available float can be towed by an underwater body at a given depth and speed by a towline of given length and diameter and whether it will remain on the surface as required. On the other hand, for purposes of design and procurement, it may be desirable to determine the permissible range of float sizes and cable size when the type of float and cable, the depth of the towpoint, the length of the towline, and the towing speed are prescribed.

For convenience, the equations to which reference is made in this section are repeated in the following.

SURFACE OPERATION

The criterion for the cable condition is

$$\frac{md}{C_D q s} \geq t \tag{13}$$

Criteria for the float condition are

$$\frac{nb^2}{C_D ds} \geq \Lambda \tag{14}$$

or

$$b \leq \frac{nq}{w} \frac{\Omega}{\Lambda} \tag{16}$$

where

$$\Omega = \frac{wb^3}{C_D q ds} \tag{15}$$

The quantities t and Λ are given as functions of η and Ω in Table 1.

SUBMERGED OPERATION

The criterion for the cable condition is

$$(kqb^2)^2 + (a_0 kqb^2 + wb^3)^2 \leq (md^2)^2 \tag{21}$$

Problem 1: To determine whether a given float and cable will perform satisfactorily under prescribed conditions.

First compute Ω from Equation [15]. From this value of Ω and the given value of η read t and Λ from Table 1. Using these values check whether the Inequalities [13] and [14] or [16] are satisfied. If they are satisfied, the float will be adequate for surface operation.

For the submerged condition the given values are used to check whether the Inequality [21] is satisfied. If this condition is satisfied the tension in the cable will not exceed the safe working load when the float becomes submerged. The solution is illustrated by Example 1 of Appendix 1.

Problem 2: To find the range of float beam b and diameter of towing cable d.

The quantities other than b and d in Conditions [13] and [14] are assumed to be known. Since η is given, the equality in Condition [13] gives values of d on a basis of Ω from Table 1, and hence corresponding values of b from Equation [15]. If d is plotted as an ordinate with b as an abscissa, values of d and b above the curve satisfy [13]; see Figure 12, Appendix 1.

Also the equality in [16] gives values of b on a basis of Ω from Table 1, and hence values of d from Equation [15]. If b is plotted as an ordinate with d as an abscissa, values to the right of the curve satisfy [14]; see Figure 12, Appendix 1.

For the float acting as a submerged hydrofoil the equality in [21] gives values of b against d. If d is plotted as an ordinate with b as an abscissa, values above the curve satisfy [21]; see Figure 12, Appendix 1.

The area on the graph, Figure 12, Appendix 1, simultaneously satisfying the hydrofoil and the two float conditions, gives the required range of permissible values of b and d. Since the cable tensions are higher for the submerged condition than for the surface-float condition, the latter need be considered only if the assumption is made that the float will not become submerged. The solution is illustrated by Example 2 of Appendix 1.

REFERENCES

(1) Bureau of Ships CONFIDENTIAL letter S81-2(681-350) of 29 October 1941.

(2) TMB CONFIDENTIAL Memorandum C-S65-5 of 13 September 1943.

(3) "Aircraft Float Design," by Capt. H.C. Richardson, 1928.

(4) "Tank Tests of Flat and V-Bottom Planing Surfaces," by James M. Shoemaker, NACA Technical Note 509, November 1934.

(5) "Characteristic Curves for Planing Surfaces," by P. Eisenberg and L. Landweber, TMB Report R-80, January 1943.

(6) "The Shape and Tension of a Light, Flexible Cable in a Uniform Current," by L. Landweber and M.H. Protter, TMB Report 533, October 1944.

(7) "Preliminary Tests in the NACA Tank to Investigate the Funda-
mental Characteristics of Hydrofoils," by Kenneth E. Ward and Norman S. Land,
NACA CONFIDENTIAL Report, September 1940.

This project carries the Bureau of Ships Symbol X68. All corre-
spondence relating to it may be found on TMB file C-S65-5.

The following groups of plans are listed for convenience.

TMB Plan	Title	Date
	Solid Wood Model, 15 Inches Long	
S-2161	Sea Sled, TMB 7	25 May 1942
	Sheet-Metal Float, 30 Inches Long	
	This float has internal bulkhead reinforcement.	
D-304, Alt. III	Sea Sled, TMB 10 - Assembly.	6 Jul 1942
D-305, Alt. III	Sea Sled, TMB 10 - Details.	6 Jul 1942
D-306, Alt. IV	Sea Sled, TMB 10 - Details.	6 Jul 1942
	Solid Wood Float, 30 Inches Long	
S-2521, Alt. I	Model of Sea Sled, TMB 10	28 Dec 1942
	Solid Wood Float, 45 Inches Long	
	This float is designed to be towed by a submarine.	
A-7209	Submarine Indicator Float, General Assembly and Details.	13 Sep 1942
A-7210	Submarine Indicator Float, Bridle Details.	13 Sep 1942
	Solid Wood Model, 30 Inches Long	
	This float is equipped with a scoop for providing an indicating spout.	
A-7457, Alt. I	Paravane Indicator Float, General Assembly and Details, E99-2.	27 Dec 1943
A-7458	Paravane Indicator Float, Bridle Assembly and Details, E99-2.	27 Dec 1943
A-8469	Paravane Indicator Float, Scoop Details, E99-2.	18 Aug 1944
	Hollow Sheet-Metal Float, 41 Inches Long	
	This float was prestressed by internal air pressure.	
A-7598, Alt. VII	Float for Catenary Sweep, Assembly, E195.	13 Jan 1944
A-7599, Alt. II	Float for Catenary Sweep, E195.	13 Jan 1944
A-7600, Alt. V	Float for Catenary Sweep, Details, E195.	13 Jan 1944

Hollow Sheet-Metal Float, 41 Inches Long

This float is equipped with a pipe across the stern for securing a grappling net and is towed by a line in the surface of the water. It was prestressed by internal air pressure.

A-8190, Alt. I	Messenger Float for Refueling Operations, Assembly, E231.	6 Jun 1944
A-8191	Messenger Float for Refueling Operations, Details, E231.	6 Jun 1944
A-8192, Alt. I	Messenger Float for Refueling Operations, Details, E231.	6 Jun 1944
A-8193, Alt. III	Messenger Float for Refueling Operations, Details, E231.	6 Jun 1944
A-8258	Messenger Float for Refueling Operations, Alternate Towing Bridle, Assembly and Details, E231.	17 Jun 1944

Solid Wood Float, 18 Inches Long

This float is designed to be towed by a paravane.

A-8293	Float for Paravane Diverter, Assembly, S81.	30 Jun 1944
A-8294	Float for Paravane Diverter, S81.	30 Jun 1944

Solid Wood Float, 30 Inches Long

This float is designed to be towed by a paravane.

A-8391	Float for Paravane Diverter, Assembly and Details, S81.	8 Aug 1944
A-8392	Float for Paravane Diverter, Bridle Details and Assembly, S81.	8 Aug 1944
A-8408	Float for Paravane Diverter, Bridle Details, S81.	8 Aug 1944

SOLUTION OF FLOAT PROBLEMS

The following examples are worked out for the two float problems for which the solutions are outlined in the section entitled "Float Problems." These examples are selected from actual applications for which floats were designed at the Taylor Model Basin. Although it will usually be desired to check float sizes or design floats on the basis of the distance x of the float astern of the towing point, it is more convenient to use the length of cable s. If a value x is first selected, the corresponding value of s for a given length of towline s can be found from Figure 10 and the ratio y/s used as in the following examples.

EXAMPLE 1

A TMB planing float, 1 foot in beam and weighing 29 pounds, is to be towed in sea water at a maximum speed of 18 knots. The towline consists of 80 feet of stainless steel cable 1/8 inch in diameter whose breaking strength is 2550 pounds. A factor of safety of 4 will be assumed for the cable, so that the maximum working tension is 637.5 pounds. A cable drag coefficient $C_D = 1.7$ will be assumed. The maximum towpoint depth y is 40 feet.

It is required to determine whether the float will perform satisfactorily under the foregoing conditions.

The values $b = 1$ foot, $W = 29$ pounds, $d = 0.0104$ foot, $T_m = 637.5$ pounds, $V = 18$ knots, $y = 40$ feet, $s = 80$ feet, and $C_D = 1.7$ are given.

At the given speed, the dynamic pressure q is $q = \frac{1}{2}\rho v^2 = 918$ pounds per square foot.

From Equation [11],

$$w = \frac{W}{b^3} = 29 \text{ pounds per cubic foot}$$

The dynamic float coefficients n, k, and a_0, as defined in Equations [10], [17], and [19], have the values $n = 0.625$, $k = 0.83$, and $a_0 = 1.73$.

The cable coefficient m, defined by Equation [9], is

$$m = \frac{T_m}{d^2} = \frac{637.5}{(0.0104)^2} = 5.89 \times 10^6 \text{ pounds per square foot}$$

The ratio η has the value

$$\eta = \frac{y}{s} = \frac{40}{80} = 0.5$$

From Equation [15],

$$\Omega = \frac{wb^3}{C_D q\, ds} = \frac{29 \times 1.0}{1.7 \times 918 \times 0.0104 \times 80} = 0.0223$$

SURFACE OPERATION - CABLE CONDITION

In order not to exceed the cable strength, Condition [13], $t \leqq \dfrac{md}{C_D qs}$, must be satisfied.

From Table 1 at $\Omega = 0.0223$ and $\eta = 0.5$, we have $t = 0.302$ and $\Lambda = 0.285$. But

$$\frac{md}{C_D qs} = \frac{5.89 \times 10^6 \times 0.0104}{1.7 \times 918 \times 80} = 0.491$$

This is greater than the factor t, 0.302, and the cable condition is satisfied.

SURFACE OPERATION - FLOAT CONDITION

To prevent overloading the float, either Condition [14] or Condition [16] must be satisfied. Consider Condition [16],

$$\frac{bw}{nq} \leqq \frac{\Omega}{\Lambda}$$

But

$$\frac{bw}{nq} = \frac{1 \times 29}{0.625 \times 918} = 0.0506$$

and

$$\frac{\Omega}{\Lambda} = \frac{0.0223}{0.285} = 0.0782$$

This is greater than the preceding quotient, 0.0506, and the float condition is satisfied.

SUBMERGED OPERATION - CABLE CONDITION

Condition [21], $(kqb^2)^2 + (a_0 kqb^2 + wb^3)^2 \leqq (md^2)^2$, must be satisfied.

By substitution we obtain

$$kqb^2 = 0.83 \times 918 \times 1.0 = 762 \text{ pounds}$$

and

$$a_0 kq b^2 + w b^3 = 1.73 \times 762 + 29 \times 1.0 = 1339 \text{ pounds}$$

Hence

$$T = \sqrt{(762)^2 + (1339)^2} = 1540 \text{ pounds}$$

Also

$$m d^2 = 5.89 \times 10^6 \times (0.0104)^2 = 637 \text{ pounds}$$

so that Condition [21] is not satisfied. Since a safety factor of 4 is in-cluded in the assumed value for m, the cable will not break until the tension exceeds $4 m d^2$ or 2550 pounds. Thus Condition [21] may be considered as satis-fied but with a safety factor of $\frac{2550}{1540} = 1.65$.

If the new safety factor is considered insufficient, a larger cable must be used and the various criteria must again be checked for the larger cable.

EXAMPLE 2

A TMB planing float is to be towed at a maximum speed of 9 knots by a cable 200 feet in length from a depth of 100 feet. It will be supposed that the cable is stainless steel and that a safety factor of 2 is acceptable.

It is required to determine the range of float sizes and cable dia-meters for satisfactory performance.

The value $V = 9$ knots, $s = 200$ feet, $y = 100$ feet are given. The cable coefficient m is 11.78×10^6, double the value used in Example 1, be-cause of the difference in safety factors. As in Example 1 the dynamic float coefficients n, k, and a_0 have the values $n = 0.625$, $k = 0.83$ and $a_0 = 1.73$. Also $w = 29$ pounds per cubic foot. The ratio η is 0.5, and, at the given speed, $q = 229.8$ pounds per square foot.

SURFACE OPERATION

From Condition [13], values of d will first be obtained at various values of Ω for $\eta = 0.5$, using Table 1. At these values of Ω and d, b will be obtained from Equation [15]. The constants

$$\frac{m}{C_D qs} = \frac{11.78 \times 10^6}{1.7 \times 229.8 \times 200} = 150.7$$

and

$$\frac{w}{C_D q s} = \frac{29}{1.7 \times 229.8 \times 200} = 3.71 \times 10^{-4}$$

are used in this calculation. For example, from Table 1 when $\Omega = 0.10$ and $\eta = 0.5$, $t = 0.323$. Then, from Condition [13],

$$d \geqq \frac{0.323}{150.7} = 0.00214 \text{ foot} = 0.026 \text{ inch}$$

Hence from Equation [15],

$$b \geqq \sqrt[3]{\frac{0.10 \times 0.00214}{3.71 \times 10^{-4}}} = 0.832 \text{ foot} = 10 \text{ inches}$$

The value of b and d are given in Table 2 and were plotted in Figure 12 to define the curve marked "Cable Condition - Surface Float."
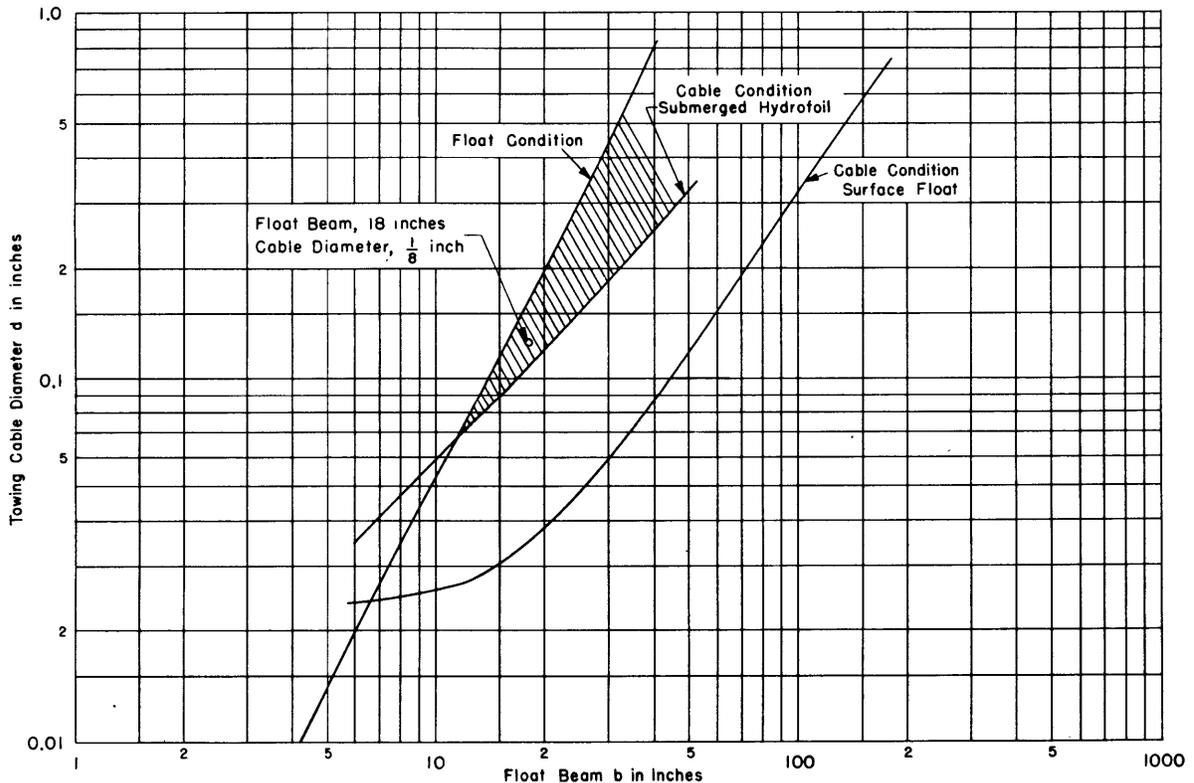

Figure 12 - Graph for Selection of Cable and Float Sizes

The maximum speed is 9 knots, the maximum depth of towpoint is 100 feet, the towline length is 200 feet, and the factor of safety on the breaking strength of the towing cable is 2.

The shaded area shows the permissible range of float beam and towing-cable diameter to meet these specifications.

TABLE 2

Calculation of Limiting Cable Size for Various Sizes of Float -
Surface Condition

Ω	t	d inches	b inches	Ω	t	d inches	b inches
0.02	0.301	0.024	5.71	1.00	0.666	0.053	27.40
0.05	0.309	0.025	7.82	2.00	1.115	0.089	41.01
0.10	0.323	0.026	10.00	5.00	2.474	0.197	72.60
0.20	0.354	0.028	12.99	10.00	4.740	0.377	113.60
0.50	0.458	0.037	19.21	20.00	9.210	0.734	178.60

SURFACE OPERATION - FLOAT CONDITION

From Condition [16] values of b will first be obtained at various values of Ω for $\eta = 0.5$. At these values of b and Ω, d will be obtained from Equation [15].

The constant

$$\frac{nq}{w} = \frac{0.625 \times 229.8}{29} = 4.95$$

is used in this calculation. For example, from Table 1, when $\Omega = 0.10$ and $\eta = 0.5$, $\Lambda = 0.296$. Then, from Condition [16],

$$b \leqq \frac{nq}{w}\frac{\Omega}{\Lambda} = 4.95 \times \frac{0.10}{0.296} = 1.67 \text{ foot} = 20.06 \text{ inches}$$

Hence, from Equation [15],

$$d \leqq \frac{wb^3}{C_D q s}\frac{1}{\Omega} = 3.71 \times 10^{-4} \times \frac{(1.67)^3}{0.10} = 0.0173 \text{ foot} = 0.208 \text{ inch}$$

The values of b and d are given in Table 3 and were plotted in Figure 12 to define the curve marked "Float Condition."

TABLE 3

Calculation of Limiting Float Size for Various Cable Sizes -
Surface Condition

Ω	Λ	b inches	d inches	Ω	Λ	b inches	d inches
0.02	0.284	4.19	0.010	1.00	0.454	130.8	5.765
0.05	0.287	10.34	0.057	2.00	0.673	176.5	7.080
0.10	0.296	20.06	0.208	5.00	1.310	226.5	5.470
0.20	0.312	38.08	0.710	10.0	2.350	252.6	4.150
0.50	0.360	82.42	2.875	20.0	4.360	272.3	2.600

SOLUTION FOR SUBMERGED HYDROFOIL

For the condition in which the float becomes submerged, the values of b will be assumed and d will then be found from Equation [21].

For example, assume b = 6 inches. The total lift developed by the submerged float at 18 knots is

$$a_0 kqb^2 + wb^3 = 1.73 \times 0.83 \times 229.8 \times \frac{36}{144} + 29 \times \frac{216}{1728}$$

$$= 82.5 + 3.6 = 86.1 \text{ pounds}$$

The drag of the submerged float is

$$kqb^2 = 0.83 \times 229.8 \times \frac{36}{144} = 47.7 \text{ pounds}$$

Hence,

$$md^2 = \sqrt{(86.1)^2 + (47.7)^2} = 98.4$$

and

$$d = \sqrt{\frac{98.4}{11.78 \times 10^6}} = 0.0029 \text{ foot} = 0.035 \text{ inch}$$

The values are given in Table 4 and were plotted on Figure 12 to define the curve marked "Cable Condition - Submerged Float."

The shaded area on Figure 12 shows the permissible range in which a float and cable may be selected. For the particular project from which these specifications were taken, a float of 18-inch beam towed with a 1/8-inch cable was selected. This solution is shown by the small circle in the shaded portion of Figure 12.

TABLE 4

Calculation of Limiting Cable Size for Various Float Sizes -
Submerged Condition

Beam b, inches	6	12	15	18	24	30	36	48
kqb^2, pounds	47.7	191	298	429	763	1192	1715	3100
$a_0 kqb^2 + wb^3$, pounds	86.1	359	572	840	1552	2515	3751	7220
Cable Tension T, pounds	98.4	406	645	944	1728	2781	4123	7853
Cable Diameter d, inches	0.035	0.071	0.089	0.107	0.145	0.185	0.225	0.310

APPENDIX 2

DIMENSIONS, OFFSETS, AND DEVELOPMENT OF THE TMB PLANING FLOAT

A. LONGITUDINAL MIDSECTION

The longitudinal midsection consists of a quadrant of an ellipse from A to B, a circular arc of radius ρ with center at E from B to C, and a line $\overline{CD}$ tangent at C, as shown in Figure 13. Tables 5 and 6 give the coordinates of the principal points and the offsets of the ellipse relative to X and Y axes along the major and minor axes of the ellipse, respectively. The major axis of the ellipse is twice the minor axis.


$\overline{CD}$ = 1.7650 b

β = 5.40 degrees

Radius of Curvature
At A: 0.1563 b
At B: 1.250 b
where b is the width of beam
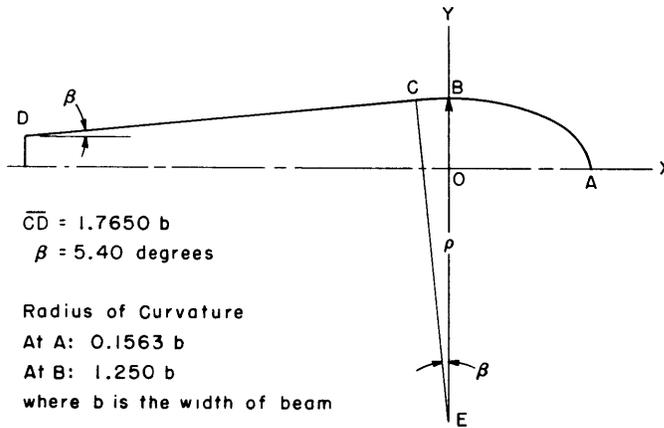
Figure 13

TABLE 5

Coordinates of Principal Points

	$\dfrac{x}{b}$	$\dfrac{y}{b}$
A	0.6250	0
B	0	0.3125
C	- 0.1177	0.3070
D	- 1.875	0.1405
E	0	- 0.9375

TABLE 6

Offsets of Ellipse

$\dfrac{x}{b}$	$\dfrac{y}{b}$
0	0.3125
0.10	0.3088
0.20	0.2963
0.30	0.2744
0.40	0.2403
0.45	0.2170
0.50	0.1875
0.55	0.1484
0.60	0.0875
0.625	0

B. SIDE CONTOUR

The side contour consists of a semi-ellipse from A' to B', a circular arc of radius ρ' with center at E' from B' to C', and a line $\overline{C'D'}$ tangent to the circular arc at C', as shown in Figure 14. The length $\overline{C'D'}$ is 1.7985 b, where b is the beam of the float.

Tables 7 and 8 give the coordinates of the principal points and the offsets of the ellipse relative to X' and Y' axes along the major and minor axes of the ellipse, respectively.


$\overline{C'D'}$ = 1.7985 b
β = 5.40 degrees
Radius of Curvature
At A': 0.1122 b
At B': 0.8973 b
where b is the width of beam

Figure 14

TABLE 7

Coordinates of Principal Points

	$\dfrac{x'}{b}$	$\dfrac{y'}{b}$
A'	0.4486	0
B'	0	0.2243
C'	- 0.0844	0.2204
D'	- 1.8750	0.0508
E'	0	- 0.6729
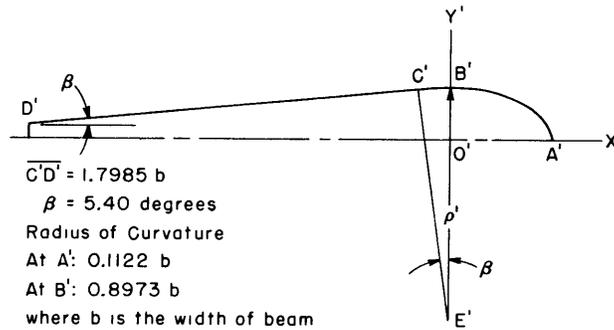
TABLE 8

Offsets of Ellipse

$\dfrac{x'}{b}$	$\dfrac{y'}{b}$
0	0.2243
0.10	0.2189
0.15	0.2115
0.20	0.2009
0.25	0.1862
0.30	0.1665
0.35	0.1404
0.40	0.1016
0.4486	0

C. TOP AND BOTTOM SURFACES

The top or bottom surfaces consist of the surface of a frustum of an elliptical cone from AA' to BB', the surface of an oblique circular cone from BB' to CC', and the tangent plane at CC' from CC' to DD', as shown in

Figures 15 and 16. The notation is the same as in Figures 13 and 14. P is the apex of the elliptical and circular cones. The surface is symmetrical about the XY-plane, so that a dihedral angle is formed at the centerline contour ABCD.
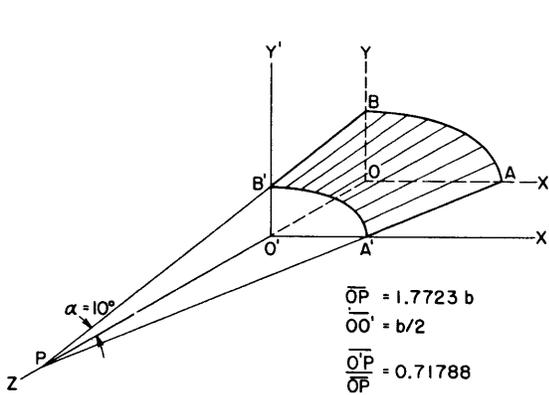

Figure 15
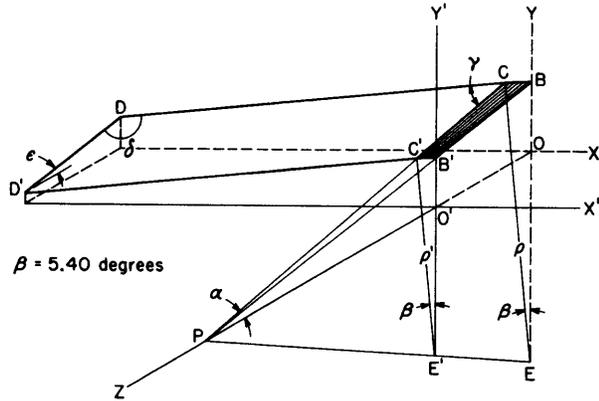
Figure 16

In connection with the development of the float surfaces, it is necessary to evaluate the angles γ and δ in Figure 16. The angle of the plane CC′DD′ with the Z-axis, the dead-rise angle, will differ from the angle α since the cone BB′CC′ is oblique; this angle will also be derived.

The coordinates of C in the XY-plane are $\left[-\rho\sin\beta,\ \rho\left(\cos\beta-\frac{3}{4}\right)\right]$. Since $\overline{DC}$ is perpendicular to $\overline{EC}$, its equation in the XY-plane is

$$y - \rho\left(\cos\beta - \frac{3}{4}\right) = (x + \rho\sin\beta)\tan\beta$$

or

$$y - x\tan\beta = \rho\left(\sec\beta - \frac{3}{4}\right) \qquad [22]$$

Hence, at D,

$$y_D = \rho\left(\sec\beta - \frac{3}{4}\right) - 1.875\tan\beta$$

Also, at D′,

$$y'_D = \rho'\left(\sec\beta - \frac{3}{4}\right) - 1.875\tan\beta$$

Hence the angle ϵ between $\overline{DD'}$ and the Z-axis, shown in Figure 16 is given by

$$\tan \epsilon = \frac{y_D - y_D'}{\overline{OO'}} = \frac{2}{b}(\rho - \rho')\left(\sec \beta - \frac{3}{4}\right) \qquad [23]$$

But

$$\frac{\rho'}{\rho} = \frac{\overline{O'P}}{\overline{OP}} = 1 - 1.6 \tan \alpha \qquad [24]$$

or

$$\rho - \rho' = 1.6\rho \tan\alpha = 2.0\, b \tan \alpha \qquad [25]$$

since

$$\rho = 1.250\, b$$

Hence Equation [23] becomes

$$\tan \epsilon = (4 \sec \beta - 3) \tan \alpha \qquad [23a]$$

Substituting $\alpha = 10$ degrees, $\beta = 5.40$ degrees, we obtain

$$\tan \epsilon = 0.1794; \quad \epsilon = 10.17 \text{ degrees} \qquad [23b]$$

The angle δ can now be computed. The direction cosines of $\overline{D'C'}$ or $\overline{DC}$, relative to the XYZ-axes, are $(\cos\beta, \sin\beta, 0)$. The direction cosines of $\overline{DD'}$ are $(0, -\sin\epsilon, \cos\epsilon)$. Hence

$$\cos \delta = -\sin\epsilon \sin\beta \qquad [26]$$

Substituting $\epsilon = 10.17$ degrees, $\beta = 5.40$ degrees, we obtain

$$\delta = 90.95 \text{ degrees} \qquad [26a]$$

The angle γ will now be evaluated. Direction numbers of $\overline{PC}$ are

$$\left[-\sin\beta, \left(\cos\beta - \frac{3}{4}\right), -\frac{1}{4}\cot\alpha\right]$$

those of $\overline{DC}$, are

$$[\cos\beta, \sin\beta, 0]$$

Hence

$$\cos \gamma = \frac{-\sin\beta \cos\beta + \sin\beta \left(\cos\beta - \frac{3}{4}\right)}{\sqrt{\sin^2\beta + \left(\cos\beta - \frac{3}{4}\right)^2 + \frac{1}{16}\cot^2\alpha}} = -\frac{3\sin\beta \sin\alpha}{\sqrt{1 + 4\delta \sin^2\alpha \sin^2\frac{\beta}{2}}}$$

or, substituting α = 10 degrees, β = 5.40 degrees, we obtain

$$\gamma = 92.80 \text{ degrees} \qquad [27]$$

To find the dead-rise angle α' of the plane $C'CDD'$, the direction cosines of the normal to the plane will first be obtained. The angle between this normal and the Z-axis will be the complement of the dead-rise angle. The direction of the normal is given by the vector cross product of the vectors $\vec{PC}$ and $\vec{DC}$, whose direction numbers are

$$[- \sin\beta \cot\alpha, \quad \cot\alpha \cos\beta, \quad 4 - 3 \cos\beta]$$

Hence

$$\sin\alpha' = \frac{4 - 3 \cos\beta}{\sqrt{\cot^2\alpha + (4 - 3 \cos\beta)^2}} = \frac{(4 - 3 \cos\beta) \tan\alpha}{\sqrt{1 + (4 - 3 \cos\beta)^2 \tan^2\alpha}}$$

and

$$\tan\alpha' = (4 - 3 \cos\beta) \tan\alpha \qquad [28]$$

$$\alpha' = 10.13 \text{ degrees} \qquad [28a]$$

The various angles, in degrees, are summarized as follows:

Cone Angle	α = 10	γ = 92.80
Dead-Rise Angle	α' = 10.13	δ = 90.95
Circular Arc	β = 5.40	ϵ = 10.17

D. DEVELOPMENT OF AN ELLIPTICAL CONE

A right elliptical cone of height h with semi-major and minor axes a and b at the base is shown in Figure 17. Let R be the length of a generator
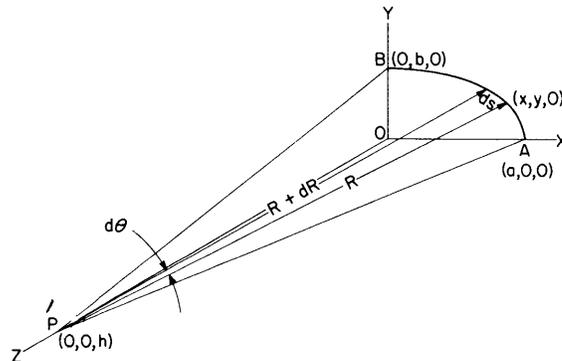

Figure 17

of the cone corresponding to a point $(x, y, 0)$ of the base ellipse. Let ds be an element of arc of the base ellipse; $d\theta$ the angle between successive generators, subtended by the element of arc at the base. Then R, θ are the polar coordinates of the developed curve of the base ellipse. We have

$$R\,d\theta = \sqrt{(ds)^2 - (dR)^2} \qquad [29]$$

The equation of the base ellipse may be written as

$$x = a\cos\phi, \quad y = b\sin\phi \qquad [30]$$

where ϕ is a parameter. Then

$$(ds)^2 = (dx)^2 + (dy)^2 = (a^2\sin^2\phi + b^2\cos^2\phi)(d\phi)^2 \qquad [31]$$

$$R^2 = h^2 + a^2\cos^2\phi + b^2\sin^2\phi \qquad [32]$$

$$2R\,dR = -(a^2 - b^2)\sin 2\phi\,d\phi \qquad [33]$$

$$(dR)^2 = \frac{(a^2 - b^2)^2\sin^2 2\phi\,(d\phi)^2}{4(h^2 + a^2\cos^2\phi + b^2\sin^2\phi)} \qquad [34]$$

Hence, from Equations [29], [31], and [34],

$$R\,d\theta = \sqrt{a^2\sin^2\phi + b^2\cos^2\phi - \frac{(a^2 - b^2)^2\sin^2 2\phi}{4(h^2 + a^2\cos^2\phi + b^2\sin^2\phi)}}\ \ d\phi$$

$$= \sqrt{\frac{h^2(a^2\sin^2\phi + b^2\cos^2\phi) + a^2 b^2}{h^2 + a^2\cos^2\phi + b^2\sin^2\phi}}\ \ d\phi$$

and from Equation [32],

$$\theta = \int_0^\phi \frac{1}{R^2}\sqrt{h^2(a^2\sin^2\phi + b^2\cos^2\phi) + a^2 b^2}\ \ d\phi \qquad [35]$$

For purposes of calculation, this may be put in the form

$$\theta = \int_0^\phi \frac{1}{R^2}\sqrt{h^2(a^2 + b^2 + h^2 - R^2) + a^2 b^2}\ \ d\phi \qquad [35a]$$

Equations [32] and [35] give the polar coordinates R, θ of the developed curve of the base ellipse.

For the TMB planing float $a = \frac{\rho}{2}$, $b = \frac{\rho}{4}$, $h = \frac{\rho}{4} \cot\alpha$, where $\rho = 1.25$ and $\alpha = 10$ degrees. Hence, from Equation [32],

$$R = \frac{\rho}{4} \sqrt{\cot^2\alpha + 4\cos^2\phi + \sin^2\phi} = \frac{\rho}{4}\sqrt{\csc^2\alpha + 3\cos^2\phi} \qquad [32a]$$

also

$$\sqrt{h^2(a^2 + b^2 + h^2 - R^2) + a^2 b^2} = \frac{\rho^2}{16}\sqrt{4\csc^2\alpha - 3\cot^2\alpha\cos^2\phi}$$

Hence

$$\theta = \int_0^\phi \frac{\sqrt{4\csc^2\alpha - 3\cot^2\alpha\cos^2\phi}}{\csc^2\alpha + 3\cos^2\phi} \qquad [35b]$$

The elliptic integral in Equation [35b] was evaluated by Simpson's rule. The values of R and θ against ϕ are given in Table 9.

E. DEVELOPMENT OF OBLIQUE CIRCULAR CONE

Since the circular arc $\overset{\frown}{BC}$ is small, the triangular segment PBC will be developed as a plane element. From the coordinates of P and C given in Figure 18,

$$R_1 = \overline{PC} = \sqrt{\frac{\rho^2}{16}\cot^2\alpha + \rho^2\sin^2\beta + \rho^2\left(\cos\beta - \frac{3}{4}\right)^2}$$

$$= \frac{\rho}{4}\csc\alpha\sqrt{1 + 48\sin^2\alpha\sin^2\frac{\beta}{2}}$$

$$\overline{OB} = \frac{\rho}{4} = 0.3125\,b$$

$$\overline{OP} = \frac{\rho}{4}\cot\alpha = 1.7723\,b$$

$$R_0 = \overline{PB} = \frac{\rho}{4}\csc\alpha = 1.7995\,b$$

$$C: \left[-\rho\sin\beta,\ \left(\rho\cos\beta - \frac{3}{4}\right),\ 0\right]$$


$\alpha = 10$ degrees
$\beta = 5.40$ degrees
$= 0.0942$ radians
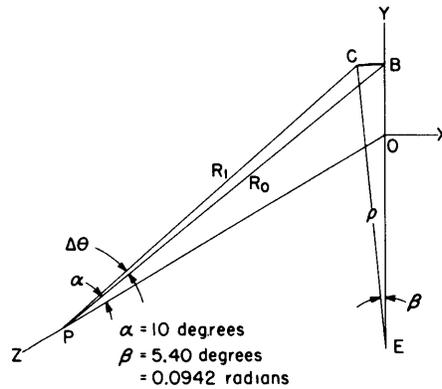
Figure 18

Since the second term under the radical is very small, we have with negligible error,

$$R_1 = \frac{\rho}{4} \csc\alpha + 6\rho \sin\alpha \sin^2 \frac{\beta}{2}$$

$$= 1.7995\,b + 0.326\,\beta^2 b \qquad [36]$$

where β is in radians.

Angle PBC is a right angle. Hence

$$\tan\Delta\theta = \frac{\widehat{BC}}{R_0} = \frac{\rho\beta}{\frac{\rho}{4}\csc\alpha} = 4\beta \sin\alpha = 0.6944\,\beta \qquad [37]$$

When $\beta = 0.0942$ radians,

$$R_1 = 1.8024\,b, \quad \Delta\theta = 3.74 \text{ degrees} \qquad [38]$$

F. OFFSETS FOR COMPLETE DEVELOPMENT SURFACE

The polar coordinates of the developed curve from A to C have been derived in Sections D and E. The curve will now be expressed in rectangular coordinates with C as origin and $\overline{DC}$ as the X-axis.

From Figure 19, denoting the coordinates of a point on the curve by $(x,\ y)$, corresponding to the polar coordinates $(R,\ \theta)$ relative to $\overline{PA}$, the coordinates of P by $(x_0,\ y_0)$, and the angle between $\overline{AP}$ and the Y-axis by θ_0, we have

$$x = x_0 + R \sin(\theta_0 - \theta) \qquad [39]$$

$$y = y_0 - R \cos(\theta_0 - \theta) \qquad [40]$$

$\overline{PC} = R_1 = 1.8024\,b$

$\overline{PC}' = R_1' = 1.2939\,b$

$\gamma = 92.80$ degrees

$\theta_0 = 24.44$ degrees

$x_0 = 0.0880\,b$

$y_0 = 1.8002\,b$
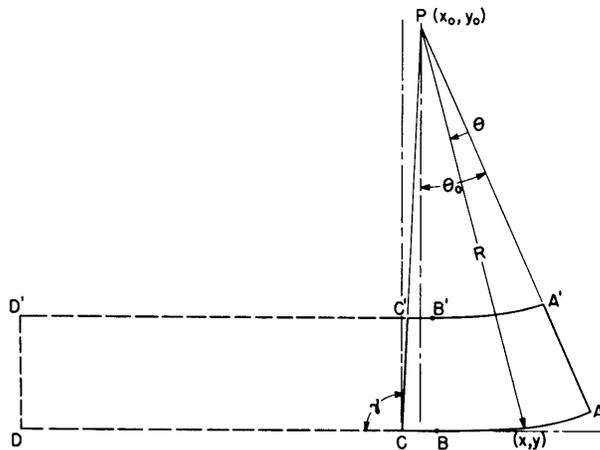
Figure 19

Hence $x_0 = R_1 \sin \left(\gamma - \frac{\pi}{2}\right)$; $y_0 = R_1 \cos \left(\gamma - \frac{\pi}{2}\right)$. Entering the value of R_1 from Section E and the value of γ from Section C,

$$x_0 = 0.0880 \, b, \quad y_0 = 1.8002 \, b \qquad [41]$$

Also, from Table 9, Figure 20, and Section E,

$$\theta_0 = \angle APB + \angle BPC - \left(\gamma - \frac{\pi}{2}\right) = 23.50 + 3.74 - 2.80 = 24.44 \text{ degrees} \qquad [42]$$


Figure 20

TABLE 9

Polar Coordinates of Developed
Bow Cones

θ degrees	$\dfrac{R}{b}$	$\dfrac{R'}{b}$
0	1.8792	1.3490
0.8348	1.8786	1.3486
1.6868	1.8768	1.3473
2.572	1.8740	1.3453
3.51	1.8700	1.3424
4.499	1.8652	1.3390
5.56	1.8596	1.3349
6.69	1.8533	1.3304
7.90	1.8467	1.3257
9.19	1.8398	1.3207
10.55	1.8329	1.3158
11.99	1.8262	1.3110
13.49	1.8198	1.3064
15.06	1.8141	1.3023
16.69	1.8091	1.2987
18.34	1.8050	1.2957
20.04	1.8020	1.2936
21.76	1.8001	1.2922
23.50	1.7995	1.2918
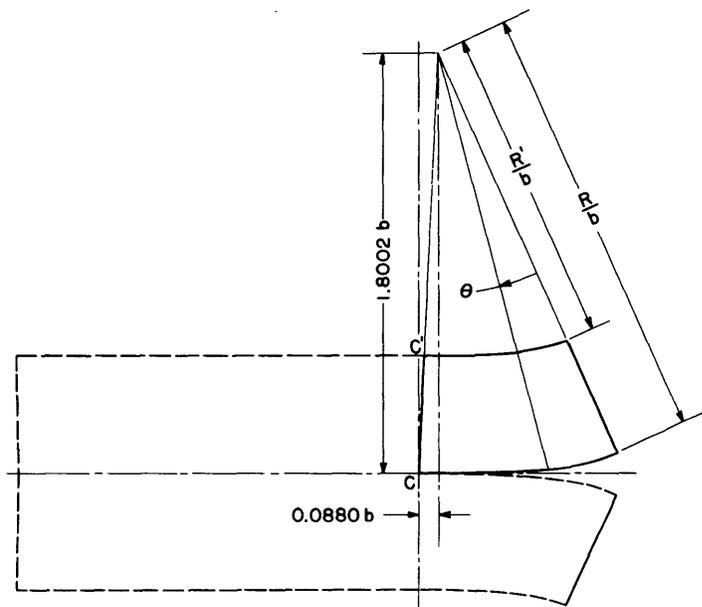
The (x, y) coordinates of the curve ABC can now be computed from Equations [39], [40], [41], and [42], using the values for R and θ in Table 9. The offsets for the developed curve of the chine, or side contour, A'B'C', are similarly given by

$$x = x_0 + R' \sin(\theta_0 - \theta) \qquad [43]$$

$$y = y_0 - R' \cos(\theta_0 - \theta) \qquad [44]$$

where, by Equation [24],

$$R' = R(1 - 1.6 \tan \alpha) = 0.71788 R$$

The polar coordinates of the developed point C' are $(R_1 - R_1'$, $\pi - \gamma)$. But $R - R' = 1.6 R \tan \alpha = 0.5085 b$. Hence the rectangular coordinates of C' are

$$x_C' = 0.5085 \sin\left(\gamma - \frac{\pi}{2}\right) = 0.0248 b$$

$$y_C' = 0.5085 \cos\left(\gamma - \frac{\pi}{2}\right) = 0.5082 b$$

From Section A, the distance $\overline{CD}$ is $1.7650 b$.

From Section B, the distance $\overline{C'D'}$ is $1.7985 b$. Hence the rectangular coordinates of D' are $x_D' = x_C' - \overline{C'D'} = 0.0248 b - 1.7985 b = 1.7737 b$

$$y_D' = y_C' = 0.5082 b$$

The offsets of the developed curves are tabulated in Figure 9, together with the other dimensions necessary for the construction of a TMB planing float.

APPENDIX 3

EFFECT OF BRIDLE MODIFICATIONS ON THE CHARACTERISTICS
OF THE TMB PLANING FLOAT

The characteristics of the TMB planing float when submerged depend primarily upon the position of the apex of the two-legged towing bridle. To investigate the variation in towline tensions with different positions of the apex, the float was towed at a depth of about two float lengths, using a fixed underwater towpoint, and the towline tensions were measured for three different bridles. The bridles were varied by lengthening the rear bridle leg, as is shown in Figure 21 and Table 10, to give angles of 60, 52.5, and 45 degrees between the rear bridle leg and the keel line.

The test results are summarized in Table 10. Towline tensions are given in terms of the dimensionless tension coefficient

$$C_T = \frac{T}{qb^2}$$

where T is the towline tension,

q is the dynamic stream pressure $\frac{1}{2}\rho v^2$, and

b is the beam of the float.
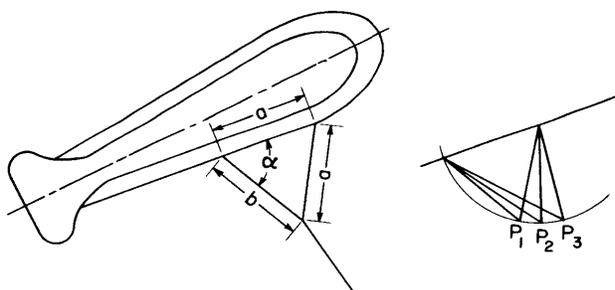
Figure 21

TABLE 10

Tension Coefficients for Various
Positions of the Bridle Apex

Bridle Apex	$\frac{a}{b}$	α degrees	C_T
P_1	1.0	60.0	1.50
P_2	0.86	52.5	0.97
P_3	0.71	45.0	0.60

The value of $C_T = 1.50$ is 11 per cent lower than the value of $C_T = \sqrt{(1.44)^2 + (0.83)^2} = 1.66$, derived from the lift and drag coefficients given in Equations [2] and [3]. This variation is due to the difference in lift and drag developed at various depths of submergence by the float acting as a lifting wing. The per cent difference agrees fairly well with the results at lift coefficients above 1.0 reported in Reference (7).

The 60-degree bridle is best suited for carrying heavy loads at relatively low speeds, since the load-carrying capacity increases rapidly with a slight submergence. At high speeds, under relatively light loads, the float also planes most cleanly with this bridle.

However, under conditions where the float alternates between the surface and submerged conditions, as in a heavy sea, large and sudden changes in the float characteristics between these conditions may cause shock loads in the towline, and failure of the line or fittings may result. In some cases such loads may harm the equipment supported by the float. In these cases it is recommended that the 52.5- or 45-degree bridle be used.

Although the analysis in the text of this report assumes the use of the 60-degree bridle, it can be applied to the other bridles with only slight modification.