NAVY DEPARTMENT

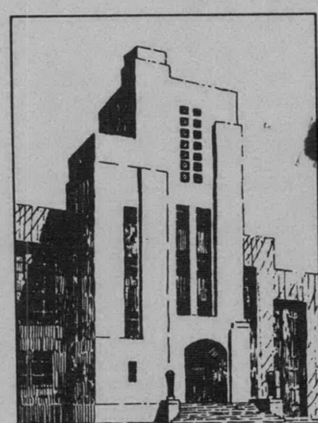
THE DAVID W. TAYLOR MODEL BASIN

WASHINGTON 7, D.C.

MEASUREMENT OF INTENSITY OF TURBULENCE IN WATER BY DYE DIFFUSION

by

M.S. Macovsky, W.L. Stracke, and J.V. Wehausen



Second Printing

November 1952

Report 700

NS 713-049

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ABSTRACT

This report describes exploratory experiments which were made to develop a means for measuring intensity of turbulence in water by a method of dye diffusion analogous to the method developed by Schubauer using heat diffusion in air. Although the method was dropped before the experimental technique was refined, the work completed indicates that the method can be used under certain circumstances to give fairly reliable results. Some of the restrictions on the method, some necessary improvements in the technique, and some methods of data correction are discussed.

INTRODUCTION

This report describes some exploratory experiments which were made in order to develop a method for measuring turbulence intensity in water by a method of dye diffusion analogous to the heat diffusion method used by Schubauer.[1] Although the experiments were carried out somewhat crudely, they have perhaps served their purpose in that they indicate that the method can be used successfully and they point out the places where greater care needs to be taken. The two values of intensity obtained as a result of the experiments are in fairly good agreement with results obtained under the same circumstances with a hot-wire velocity meter for water. The method was not pursued further since a hot-wire velocity meter had been developed which was satisfactory for water and which was much more versatile in use.

It has been known for some time that the rate of diffusion of a substance from a source in a turbulent stream can be used to measure the intensity of the transverse component of the turbulent fluctuations and also at least one of the several scales of turbulence which may be defined. The theoretical justifications for the method was given by G.I. Taylor[2,3] and the

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[1]References are listed on page 14.

method was used by Schubauer[1] to measure turbulence in a wind tunnel from the measured temperature distribution behind a heated wire. Diffusion methods have also been used in water by Kalinske, Robertson, and van Driest[4,5,6] in order to determine the intensity of turbulence. In these experiments the diffusing substance was small particles and their meanderings had to be recorded photographically and then analyzed statistically. The present method has the advantage that the statistical analysis is bypassed since average quantities are measured directly. On the other hand, it has some disadvantage in that the operation of measurement alters the quantities being measured and makes substantial correction necessary.

OUTLINE OF THE METHOD

The idea of the method will be briefly described before proceeding to a more detailed description of experimental procedure. A schematic diagram of the equipment is given in Figure 1. In a turbulent stream of water a small element of dye emitted from the injector will tend to meander from the axis of the injector in an erratic fashion following the erratic motion of the water. Elements leaving the injector at different times, not too close together, will generally follow quite different paths. If the sampler is now fixed in some definite position and a sample of the water passing by withdrawn through it at a uniform rate over a sufficiently long-time interval, the resulting sample will be more or less strongly colored depending upon how frequently elements of dye are carried as far off the axis as the sampler is situated. One expects the greatest density of dye if the sampler is right on the axis and steadily decreasing amounts as it is moved off the axis. If the sample is taken over too short a time interval, the sample may give a concentration of dye deviating too much from the true average; it is conceivable that such a sample might be almost pure water or almost pure dye, depending upon the short time interval in which the sample was taken. The actual concentration of dye was measured by means of an electrophotometer which had been calibrated for the dye being used. However, it is not difficult to think of other schemes for doing the same thing.

If the turbulence of the water is very intense, one will expect the dye elements to wander farther from the axis, on the average, in a given distance downstream, than if the turbulence is not very intense. Consequently, the rate of spreading of the average dye wake gives a measure of the intensity of turbulence near the end of the injector. The discussion of the method for obtaining a quantitative measure of the turbulence is described in the section dealing with the analysis of the data.

In the case of heat diffusion behind a heated wire it is necessary to make a correction for the molecular diffusion of heat which is taking place as well as the turbulent diffusion. In the case of dye diffusion the molecular diffusion can be completely neglected, for the amount is very small during the time intervals involved.

DESCRIPTION OF PROCEDURE

The experiments were carried out in the 1/22-scale model of the circulating water channel of the David Taylor Model Basin (Figure 2). The test section is 12 inches across and the water was about 6 inches deep during the experiments. The velocity used was about 1.6 ft/sec.

At the entrance to the test section a brass grid was placed as a source of turbulence. The mesh width was 0.75 in. and the wire diameter 0.15 inch. A length of brass tubing, inside diameter 0.035 in., outside diameter 0.060 in., was then placed in the test section in the manner shown schematically in Figure 1. The downstream end of the horizontal section of the tubing was located 15 and 22 mesh lengths from the grid. The other end of the tubing eventually led to a jar containing the dye to be used. In order to be able to control the rate of flow of dye through the tubing the jar was connected with a flask of compressed air equipped with a valve. Measurement of the discharge rate showed that the average velocity through the release tube was about 2.2 ft/sec.

To measure the average concentration of dye at positions downstream from the end of the dye release tube, a 3-in. length of the same brass tubing was attached to the movable sampler shown in Figure 3. This probe was rigidly clamped to the channel carriage in a position such that the sampler lay in the same vertical plane as the release tube. By means of the probe micrometer screw, this sampler could be traversed vertically in any desired increment with an accuracy of 0.001-in. to a total displacement of 3 in. In order to extract a sample of water from some position downstream of the release tube a 25 cc pipette was connected to the sampling tube by means of a short length of rubber tubing and a sample sucked out. Tests with a stop watch showed that the average velocity of the water passing through the end of the sampling tube was about 5 ft/sec.

The dye solution used in these experiments was made by dissolving about 1 gram of Ponceau red, a water-soluble food dye, in 4 liters of water. The concentration of a sample was measured by means of a Fisher Electrophotometer. Such measurements are generally considered to be accurate to about 1 percent. No series of experiments to determine the accuracy of this instrument was made since the exploratory nature of the setup did not seem to warrant them. In order to determine whether the length of time over which a given sample was taken was sufficient to give a concentration close to the average concentration at that point, several of these samples were taken twice. The maximum changes were of the order of 8 percent. In order to take account of the gradual coloring of all the water in the channel, a sample of the channel water was also taken after every set of five measurements. The correction for the darkening of the channel water was surprisingly little considering how noticeable the coloring was to the eye.

Surveys of the distribution of average dye concentration across-stream in the vertical plane were made at distances downstream of 1/2, 3/4, 1, and 1 1/2 in.

ANALYSIS OF RESULTS

In Figures 4 and 5 are plotted the points obtained from these surveys at the various stations downstream from the injector. Note that the y-coordinate in these figures is measured from a displaced origin, not from the axis of symmetry. A smooth curve has been faired through each set of data. The ordinates give the ratio of the concentration of the measured sample to the concentration of the original solution. These curves will be called concentration curves. Actually, they are traces in a vertical plane of a concentration surface. Although it was not verified, it will be assumed that these surfaces are axially symmetric so that the surface could be generated by revolving the measured curve about its symmetry axis.

Measurements of other experimenters indicate that one can expect the concentration surface to be an axially symmetric Gaussian error surface. One would then expect the concentration curves to fit an equation of the form

$$f(r) = \frac{A}{2\pi\sigma^2} e^{-\frac{r^2}{2\sigma^2}}$$

where r is the distance from the axis and A and σ are constants depending upon the station under consideration. The constant A gives a measure of the total amount of dye passing a given station; σ is a measure of the width of the dye wake, the distance from the axis to the inflection of the concentration surface. Actually, however, the effect of the finite size of opening of the sampler will be to deform this curve, the deformation being greater, the greater the size of opening of the sampler relative to the quantity σ. Indeed, in case the rate of flow of water entering the sampler is greater than the free-stream velocity the effective size of the opening is further increased. In the present experiments the average velocity of water in the sampler was about three times the free-stream velocity. This corresponds to an effective diameter of the sampler about 1.7 times its actual diameter. Since the actual sampler diameter was 0.035 in., we shall take as effective sampler diameter 0.060 in. The effective radius of the sampler will hereafter be denoted by R.

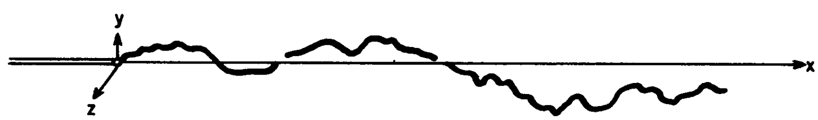
The chief difficulty in analyzing the data is in determining from the measured, deformed curves the constants A and σ of the true curves. The theory for the method of correction is developed in the Appendix and is based upon the assumption that the true concentration surface is a circular Gaussian surface. The procedure for finding σ was as follows. First, the experimental curves were treated as if they were Gaussian curves. In this case the area under the curve between the values $r = \sigma$ and $r = -\sigma$ divided by the total area under the curve is 0.68. If g(r) denotes the curve faired through the measured points, this entailed measuring the areas $\int_{-r}^{r} g(s)ds$ for enough values of r to plot a curve, which will be denoted by 2G(r). A value σ_0 was then determined from $G(\sigma_0) = 0.68G(\infty)$. This will not, of course, be the true value of σ. The true value, denoted by σ_1, was then determined by the method described in the Appendix. Two methods for determining A are described in the Appendix. Tests with two different curves gave nearly the same value for A for each method so that it was decided to use the second method which involved somewhat less labor. The results of this analysis are shown in the following table. The meanings of the letters X, L, and M are shown in Figure 1.

$\frac{L}{M} = 15$				$\frac{L}{M} = 22$			
X inches	σ_0 inches	σ_1 inches	A sq. inches	X inches	σ_0 inches	σ_1 inches	A sq. inches
0.75	0.017			0.50	0.023	0.016	0.84×10^{-3}
1.00	0.027	0.022	1.30×10^{-3}	0.75	0.030	0.025	1.21×10^{-3}
1.25	0.040	0.037	1.42×10^{-3}	1.00	0.036	0.032	1.46×10^{-3}
1.50	0.052	0.050	2.22×10^{-3}	1.25	0.041	0.038	1.38×10^{-3}

The values of σ_1 and A are omitted for the station L/M = 15, X = 0.75 in. because a corrected value of σ_0 did not seem to exist. This would seem to indicate that the measured value of $\sigma_0 = 0.017$ in. was impossible, at least under the assumption that the effective value of the sampler diameter was 0.060 in. Discrepancies of this sort could probably be avoided in a more carefully controlled experiment in which the rate of removal of fluid through the sampler was adjusted to be equal to or less than the free-stream velocity.

Since the total amount of dye passing in unit time each station downstream of the injector should be the same, one will expect A to be a constant. The constancy of A provides a sort of check upon the accuracy of the experiments, providing, of course, that the rate of dye release has been kept constant during all the measurements. Inspection of the values of A in the table shows that the variation is not large with the exception of two points; the value of A for L/M = 15 and X = 1.50 in. is too large by almost a factor of 2, whereas the value for L/M = 22 and X = 0.50 in. is too small by a substantial amount. This discrepancy may have arisen from the general crudeness of the experiments or perhaps was caused by an actual error.

The transverse intensity of turbulence may now be computed from the values of σ. Let X be the distance measured downstream from the end of the dye injector and let Y be the distance measured vertically. If a particle starts at the end of the injector tube and moves downstream in a turbulent flow its trajectory might appear as follows:



Let the vertical velocity component of the particle at time t after leaving the injector be v(t). Then the transverse displacement y(t) of the particle at time t is given by $y(t) = \int_0^t v(\tau)d\tau$. The mean square displacement is then given by

$$\overline{y^2(t)} = \int_0^t \int_0^t \overline{v(\tau)v(\sigma)}\ d\tau d\sigma$$

or, introducing the correlation coefficient

$$R(\tau,\sigma) = \overline{v(\tau)v(\sigma)}/v'(\tau)v'(\sigma),\quad v' = \sqrt{\overline{v^2}}$$

$$\overline{y^2(t)} = \int_0^t \int_0^t v'(\tau)v'(\sigma)R(\tau,\sigma)\ d\tau d\sigma$$

It should be noted that the means are not time averages of the functions $v^2(t)$ and $y^2(t)$, but are ensemble averages taken over a large number of particles. From the last equation one easily derives

$$\frac{d^2}{dt^2}\overline{y^2(t)} = 2v'^2(t) + 2\frac{dv'(t)}{dt}\int_0^t v'(\sigma)R(t,\sigma)d\sigma + 2v'(t)\int_0^t v'(\sigma)\frac{\partial}{\partial t}R(t,\sigma)d\sigma$$

and, setting t = 0,

$$\frac{d^2}{dt^2}\overline{y^2(0)} = 2v'^2(0)$$

One may recast this formula as follows:

$$\frac{d^2}{dt^2}\overline{y^2(0)} = \frac{d}{dt}\left[2\sqrt{\overline{y^2(0)}}\,\frac{d}{dt}\sqrt{\overline{y^2(0)}}\right] = 2\left[\frac{d}{dt}\sqrt{\overline{y^2(0)}}\;\right]^2 + 2\sqrt{\overline{y^2(0)}}\,\frac{d^2}{dt^2}\sqrt{\overline{y^2(0)}} = 2v'^2(0)$$

or, finally,

$$\frac{d}{dt}\sqrt{\overline{y^2(0)}} = v'(0)$$

If one now makes the assumption that the fluctuations of velocity in the longitudinal direction are small compared with the free-stream velocity U, one may write, approximately, X = Ut and write the last equation above

$$\frac{d}{dx}\sqrt{\overline{y^2(0)}} = \frac{v'(0)}{U}$$

The standard deviations, $\sigma(x)$, of the concentration curves are just the quantities $\sqrt{\overline{y^2(x)}}$. Consequently, the last formula becomes:

$$v'(0) = U\,\frac{d\sigma}{dx}(0),$$

i.e., the transverse intensity of turbulence at the end of the injector tube may be obtained from the slope of the curve $\sigma(x)$ at that point. Figure 6 shows σ as a function of x together with the points determining the curves for the two cases under consideration, L/M = 15 and L/M = 22. In the case L/M = 15 it appears that the curve would not extend back through the origin. It seems likely that this is an effect of the dye injection method used. The finite size of opening of the injection tube will have the effect of starting the dye wake with a positive value of σ right at the exit. However, this is in the wrong direction to explain the behavior mentioned above. A discrepancy of this type could also be caused by the following fact. The rate of dye injection used was equivalent to an average velocity at the exit of 2.2 ft/sec, that is, about 1.4 times the free-stream velocity. The higher velocity of the jet will tend to postpone, as a function of x, its diffusion. In particular, the substitution x = Ut used above will no longer be true, even approximately, near the injector. Although correction for the finite size of the injector would be relatively easy to carry out, correction for the velocity discrepancy would be more troublesome and could easily be avoided by more careful control of the injection rate. In the present case, the slope was computed from the lower end of the curve. Corrections would apparently have the effect of changing the value of L/M somewhat but perhaps not very greatly the actual slope at the sampling stations. In any case, it is clear that more careful experimentation is needed. The two slopes so obtained were used to compute the values of U/v' shown in Figure 7. On the same figure are shown values of U/u' obtained with a hot-wire velocity meter in the same channel with the same grid (M = 3/4 in.) and with a smaller grid (M = 1/2 in.). The agreement seems better than the crudeness of the experiment leads one to expect.

CRITIQUE OF METHOD

It seems clear from the analysis of the data obtained in these experiments that the method will be more reliable as the necessary corrections become smaller. This can only be accomplished by having the dimensions of the injector and sampler tubes small in comparison with the scale of the turbulence. This will generally be attained if the tube diameters are small compared with the mesh width M. From dimensional considerations, increasing the mesh width M while preserving the mesh Reynolds number UM/ν will not change the slope $d\sigma/dx$ but will increase the distance from the end of the injector in which σ is nearly a linear function of x. Thus, by allowing measurements to be made farther away from the injector, increasing M has the effect of making R, the effective radius of the sampler smaller relative to the useful values of σ. In the present set of experiments (R = 0.03, M = 0.75 in.) the effective value of R/M = 0.040 seems to be still too large to allow great confidence in the values of σ after the very substantial corrections were made. Since, however, the corrections necessary to obtain σ_1 were less than 10 percent when σ_1 was as large as 0.04, one can perhaps conclude that the scale of the experiment should be so arranged that $\sigma_1/R > 1.5$ over a sufficient range in which $\sigma(x)$ is linear to compute a slope $d\sigma(x)/dx$ fairly accurately. If R is taken as 0.0175 in., corresponding to the sampler actually used but with velocity matching the stream velocity one must have $\sigma_1 > 0.026$ in. With a slope $d\sigma/dx = 0.04$, corresponding to L/M = 22, the first sampling station would then be at x = 0.6 in. If one wishes to have at least another inch in the linear range, a total of 1.6 in., the mesh of the grid should be about double that in the present experiment, where the linear range was about 3/4 in. long.

In the present experiments no effort was made to control the rate of discharge of dye from the injector or the rate of removal through the sampler except to try to keep them constant. The rate of discharge should be adjusted to be equal to the free-stream velocity and the rate of removal to be less than or equal to it. The effect upon the flow of having a source and sink at the injector and sampler, respectively, should again be small as long as the diameters of the tubes are small compared with M.

The corrections applied to obtain the true values of σ from the experimental curves were based upon the assumption that the true concentration curves were Gaussian curves. This has not been tested. However, a test could be made by comparing the experimental curves with theoretical curves obtained from a Gaussian curve by local averaging as described in the Appendix (i.e., by comparing the experimental $g(r)$ curves with the theoretical $g(r)$ curves).

APPENDIX

DISCUSSION OF CORRECTIONS

It has been mentioned in the body of the report that the finite size of the sampling tube will have an effect upon the measured values of the dye concentration curves. This effect may be estimated as follows if one assumes that the true dye concentration curve is a generator of a circular Gaussian error surface. Then the true curve will have the equation

$$f(r) = \frac{A}{2\pi\sigma^2} e^{-\frac{r^2}{2\sigma^2}}$$

where r is the distance from the axis and A is some constant depending upon x, the distance from the injector to the sampler. If one takes the radius of the opening of the sampler as R, the curve actually obtained by measurement will be

$$g(r) = \frac{A}{\pi R^2} \int_0^{2\pi} \int_0^R \frac{1}{2\pi\sigma^2} e^{-\frac{r^2+\rho^2+2r\rho\cos\theta}{2\sigma^2}} \rho\, d\rho\, d\theta$$

which can be rewritten as

$$g(r) = \frac{A}{\pi R^2} e^{-\frac{r^2}{2\sigma^2}} \int_0^R \frac{1}{\sigma^2} e^{-\frac{\rho^2}{2\sigma^2}} I_0\left(\frac{r\rho}{\sigma^2}\right) \rho\, d\rho$$

where I_0 is the Bessel function of zero order with pure imaginary argument. The procedure described for obtaining σ was to compute the indefinite integral $G(r) = \int_0^r g(s)ds$ and determine σ from the condition $G(\sigma)/G(\infty) = 0.68$. Although this would be correct if the measured curve were f(r), it tends to overestimate σ in the actual case. This correction could be computed from the expression for g(r) without a large amount of labor for a sufficient range of values of r/σ and R/σ. However, one may substitute for the circular opening of the sampler a square opening of equal area without altering too greatly the correction and with substantially less labor.

For the case of the square sampler of side 2R one replaces g(r) by

$$\tilde{g}(r) = \frac{A}{4R^2} \int_{r-R}^{r+R} d\rho \int_{-R}^{R} \frac{1}{2\pi\sigma^2} e^{-\frac{\rho^2+\eta^2}{2\sigma^2}} d\eta$$

$$= A \left[\frac{1}{2R} \int_{-R}^{R} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{\eta^2}{2\sigma^2}} d\eta\right] \times \frac{1}{2R} \int_{r-R}^{r+R} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{\rho^2}{2\sigma^2}} d\rho$$

The integrals occurring here have been well tabulated. For convenience, let

$$p(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} , \quad P(x) = \int_0^x p(s)ds, \quad \tilde{G}(r) = \int_0^r \tilde{g}(s)ds$$

Then, after an interchange of order of integration, one obtains

$$\tilde{G}(r) = \frac{A}{\sigma^2}\left[\frac{\sigma}{R} P\left(\frac{R}{\sigma}\right)\right] \left\{\frac{r+R}{2R} P\left(\frac{r+R}{\sigma}\right) - \frac{|r-R|}{2R} P\left(\frac{|r-R|}{\sigma}\right) - \frac{\sigma}{2R}\left[p\left(\frac{r-R}{\sigma}\right) - p\left(\frac{r+R}{\sigma}\right)\right]\right\}$$

Also,

$$\tilde{G}(\infty) = \frac{A}{2\sigma^2} \frac{\sigma}{R} P\left(\frac{R}{\sigma}\right)$$

Consequently,

$$\frac{\tilde{G}(r)}{\tilde{G}(\infty)} = 2\left\{\frac{r+R}{2R} P\left(\frac{r+R}{\sigma}\right) - \frac{|r-R|}{2R} P\left(\frac{|r-R|}{\sigma}\right) - \frac{\sigma}{2R}\left[p\left(\frac{r-R}{\sigma}\right) - p\left(\frac{r+R}{\sigma}\right)\right]\right\}$$

which holds for all values of r and R. Values for the functions occurring here may be found in tables of the normal probability function and the function $\tilde{G}(r)/\tilde{G}(\infty)$ tabulated as a function of r/σ for various values of R/σ. Figure 8 shows a graph of these functions for R/σ = 0, 1, 2, 3, 4. A cross-plot may now be obtained from these curves for the particular value $\tilde{G}(r/\sigma, R/\sigma)/\tilde{G}(\infty), R/\sigma) = 0.68$. This gives a curve for R/σ against r/σ shown in Figure 9. The correction to the value of σ, say σ_0, obtained from the experimental curve may now be made as follows: First, compute $R' = R\sqrt{\pi}/2$ in order to obtain the square opening of area equal to that of the circular opening of radius R. Then find the point $(R'/\sigma_0, 1)$ on the graph. The line passing through the origin and this point will intersect the curve at some point $(R'/\sigma_1, \sigma_0/\sigma_1)$. The value of σ_1 can then be found immediately, and is the corrected value. This correction was made to all the standard deviations taken from data. In one exceptional case, for the data taken 3/4 in. downstream from the injector when this in turn was 15 mesh lengths from the grid, the line through the point $(R'/\sigma_0, 1)$ and the origin did not intersect the curve at all. This fact is difficult to explain, for the indication is presumably that the observed value of σ_0 is impossible. The dilemma was resolved by ignoring the point.

It has been mentioned in the report that the total amount of dye passing any one station should be a constant. However, since the concentration curves obtained from the experiments have been altered by the effect of the finite size of the sampler, one might expect that the volume under the rotated measured curves would no longer be constant. It is possible to compute this effect for the case of a circular Gaussian error surface and a circular sampler. It has been shown above that the altered distribution curve will be

$$g(r) = \frac{A}{\pi R^2} e^{-\frac{r^2}{2\sigma^2}} \int_0^R \frac{1}{\sigma^2} e^{-\frac{\rho^2}{2\sigma^2}} I_0\left(\frac{r\rho}{\sigma^2}\right)\rho d\rho$$

Then

$$\int_0^\infty g(r)\ 2\pi r dr = 2\pi A\frac{\sigma^2}{\pi R^2}\int_0^\infty e^{-\frac{\alpha^2}{2}}\alpha d\alpha \int_0^{\frac{R}{\sigma}} \beta e^{-\frac{\beta^2}{2}} I_0(\alpha\beta)d\beta$$

$$= A\frac{2\sigma^2}{R^2}\int_0^{\frac{R}{\sigma}} e^{-\frac{\beta^2}{2}} \beta d\beta \int_0^\infty \alpha e^{-\frac{\alpha^2}{2}} I_0(\alpha\beta)d\alpha$$

$$= A\frac{2\sigma^2}{R^2}\int_0^{\frac{R}{\sigma}} e^{-\frac{\beta^2}{2}} \beta\ e^{\frac{\beta^2}{2}}\ d\beta$$

$$= A$$

Hence, the volume under the intensity surface is not changed by the sampling procedure. It is of some interest to note that this is not the case with a square sampler. A simple calculation shows that

$$\int_0^\infty \tilde{g}(r)\ 2\pi r dr = \pi A P\left(\frac{R}{\sigma}\right) \frac{R}{\sigma}\left[\left(\frac{R}{\sigma} + \frac{\sigma}{R}\right) P\left(\frac{R}{\sigma}\right) + p\left(\frac{R}{\sigma}\right)\right]$$

where $p(x)$ and $P(x)$ are the functions defined above. This function has a very flat maximum of one at $R/\sigma = 0$, has scarcely deviated from this value at $R/\sigma = 1$ and approaches $\pi/4$ as $R/\sigma \to \infty$.

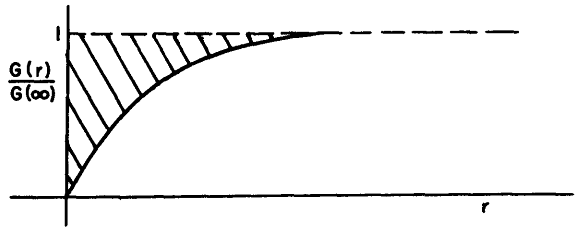
In order to find the value σ_0 it was necessary to plot from the experimental data the curves $G(r)/G(\infty)$. It would be convenient if this work could be used to estimate the constant A and to verify if possible its constancy as the value x, the distance downstream from the injector, varies. Integrating $\int_0^r sg(s)ds$ by parts, one gets

$$rG(r) - \int_0^r G(s)ds = \int_0^r \left[G(r) - G(s)\right] ds = G(r)\int_0^r \left[1 - \frac{G(s)}{G(r)}\right]ds$$

Consequently,

$$A = \int_0^\infty 2\pi s g(s)ds = 2\pi G(\infty)\int_0^\infty \left[1 - \frac{G(s)}{G(\infty)}\right] ds$$

This allows computation of A directly from the data. In the process of finding the curve $G(r)/G(\infty)$, it was necessary to find the total area $G(\infty)$ under the intensity curve $g(r)$. The curves appear somewhat as follows:



The integral $\int_0^\infty [1 - G(s)/G(\infty)]ds$ is then the shaded area. The value of A may also be obtained directly from $G(\infty)$ and the corrected value of σ in the following manner:

$$\int_0^\infty g(r)dr = \frac{A}{\pi R^2}\int_0^\infty dr\, e^{-\frac{r^2}{2\sigma^2}} \int_0^R \frac{1}{\sigma^2} e^{-\frac{\rho^2}{2\sigma^2}} I_0\left(\frac{r\rho}{\sigma}\right)\rho\, d\rho$$

$$= \frac{A}{\sigma}\,\frac{1}{\pi(R/\sigma)^2}\int_0^{\frac{R}{\sigma}} d\beta\,\beta\, e^{-\frac{\beta^2}{2}} \int_0^\infty d\alpha\, e^{-\frac{\alpha^2}{2}} I_0(\alpha\beta)$$

$$= \frac{A}{\sigma}\,\frac{1}{\pi(R/\sigma)^2}\int_0^{\frac{R}{\sigma}} d\beta\,\beta\, e^{-\frac{\beta^2}{2}} \sqrt{\frac{\pi}{2}}\, e^{\frac{\beta^2}{4}}\, I_0\left(\frac{\beta^2}{4}\right)$$

$$= \frac{A}{\sigma}\,\frac{\sqrt{\pi/2}}{\pi(R/\sigma)^2}\int_0^{\frac{R}{\sigma}} e^{-\frac{\beta^2}{4}} I_0\left(\frac{\beta^2}{4}\right)\beta\, d\beta$$

$$= \frac{A}{\sigma}\,\frac{1}{4}\sqrt{\frac{2}{\pi}}\,\frac{1}{(R/2\sigma)^2}\int_0^{\left(\frac{R}{2\sigma}\right)^2} e^{-\gamma}\, I_0(\gamma)\, d\gamma$$

If the indefinite integral $(R/2\sigma)^{-2}\int_0^{\left(\frac{R}{2\sigma}\right)^2} e^{-\gamma}\, I_0(\gamma)d\gamma$ is denoted by $S(R/\sigma)$, one obtains the equation

$$A = 4\sqrt{\frac{\pi}{2}}\,\frac{\sigma G(\infty)}{S(R/\sigma)}$$

In order to compute A it is necessary to have a table or graph of the function $S(R/\sigma)$. This is relatively easy to compute from existing tables of the function $e^{-x} I_0(x)$. A graph of $S(R/\sigma)$ for values of R/σ between 0 and 4 is shown in Figure 10. On the same graph, for comparison, is shown the corresponding function for a square sampler, $\widetilde{S}(R/\sigma) = (\sigma/R)\int_0^{\frac{R}{\sigma}} e^{-\frac{\mu^2}{2}}\, d\mu$. Since intensity data, in this report, are given nondimensionally as fractions of the original dye solution. A will have the dimension of square inches.

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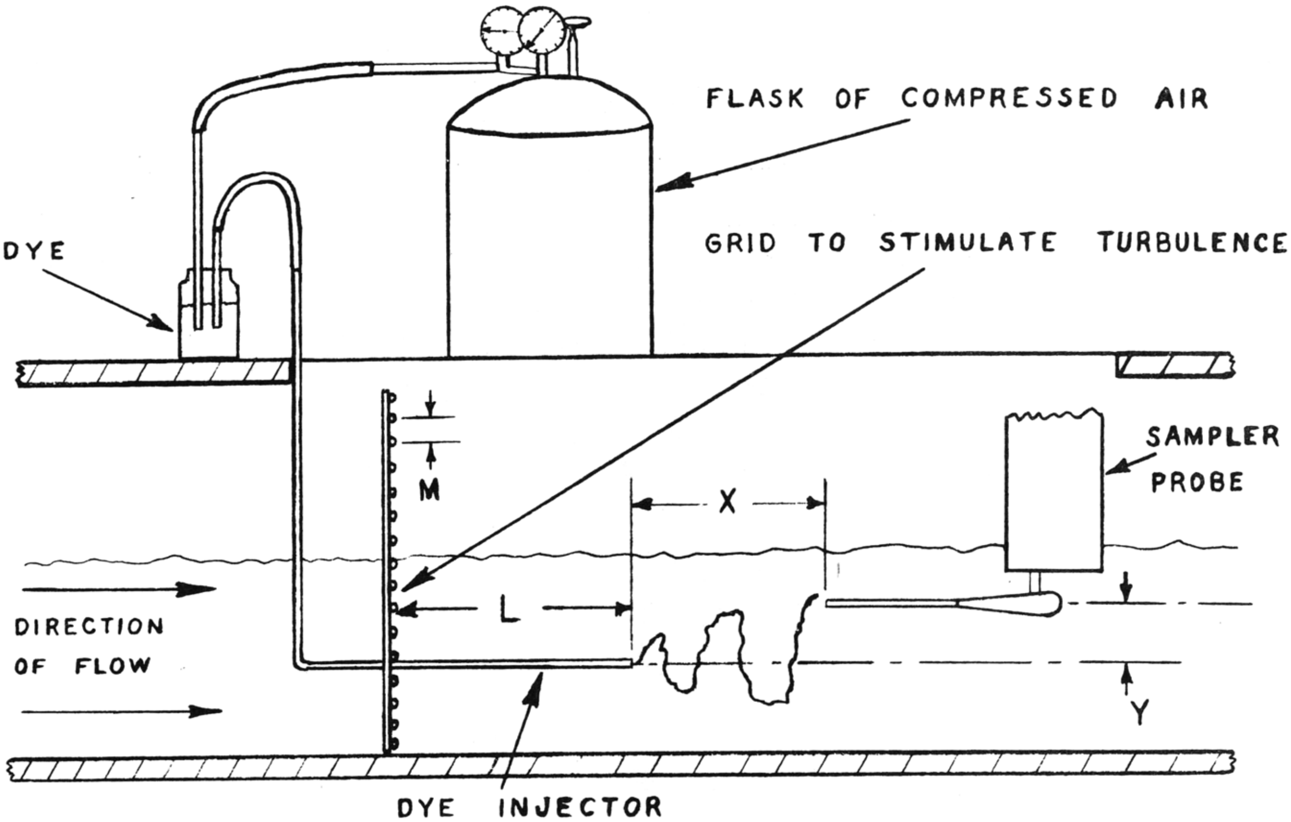

Figure 1 - Schematic Diagram of the Equipment Used in Dye Diffusion Measurements of Turbulence

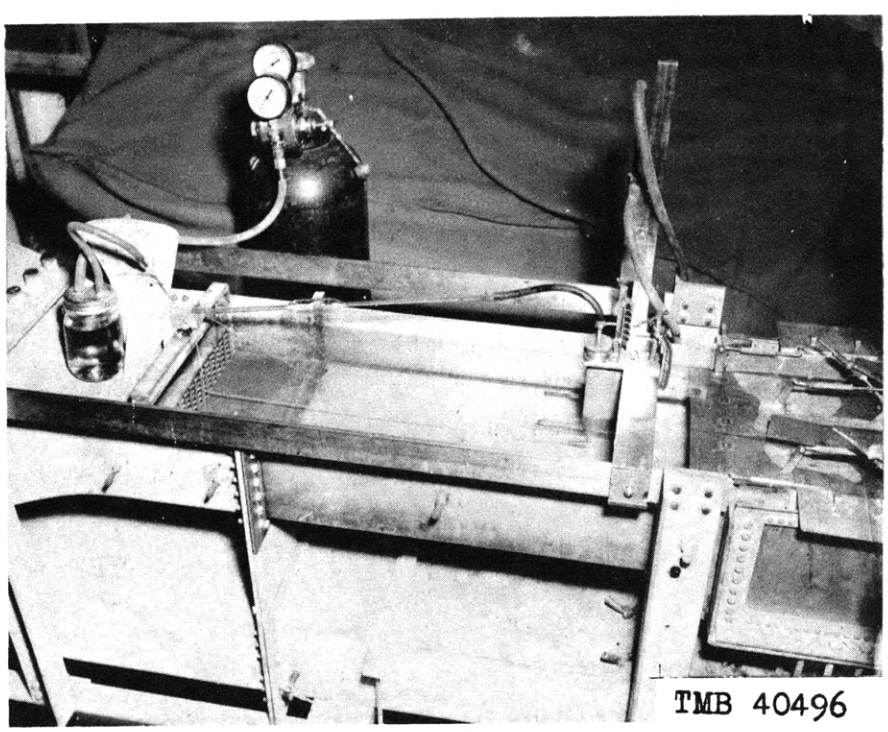

Figure 2 - Photograph Showing Test Section of the 1/22-Scale Model of the TMB Circulating Water Channel and the Arrangement of Apparatus

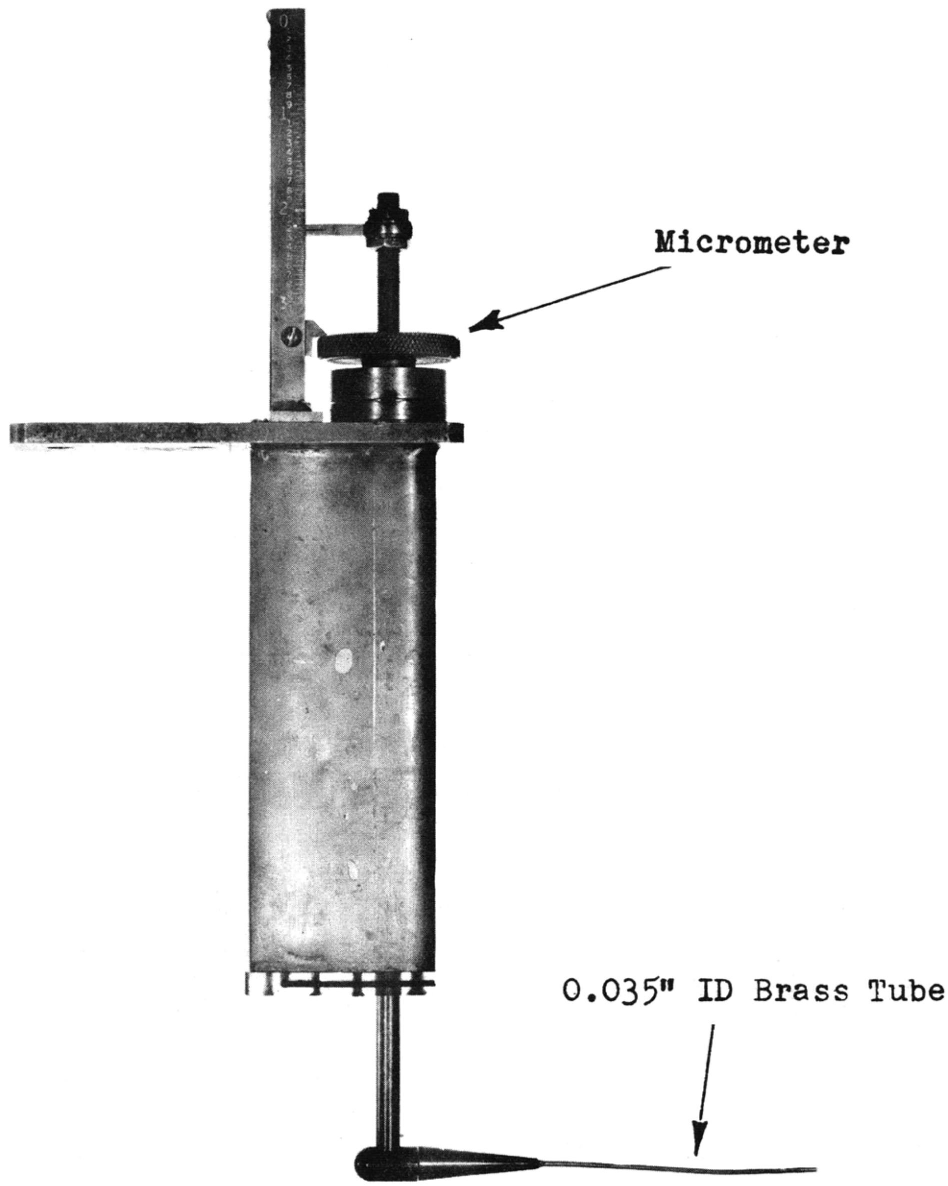

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Figure 3 - Sampler Used in Dye Diffusion Measurements of Turbulence

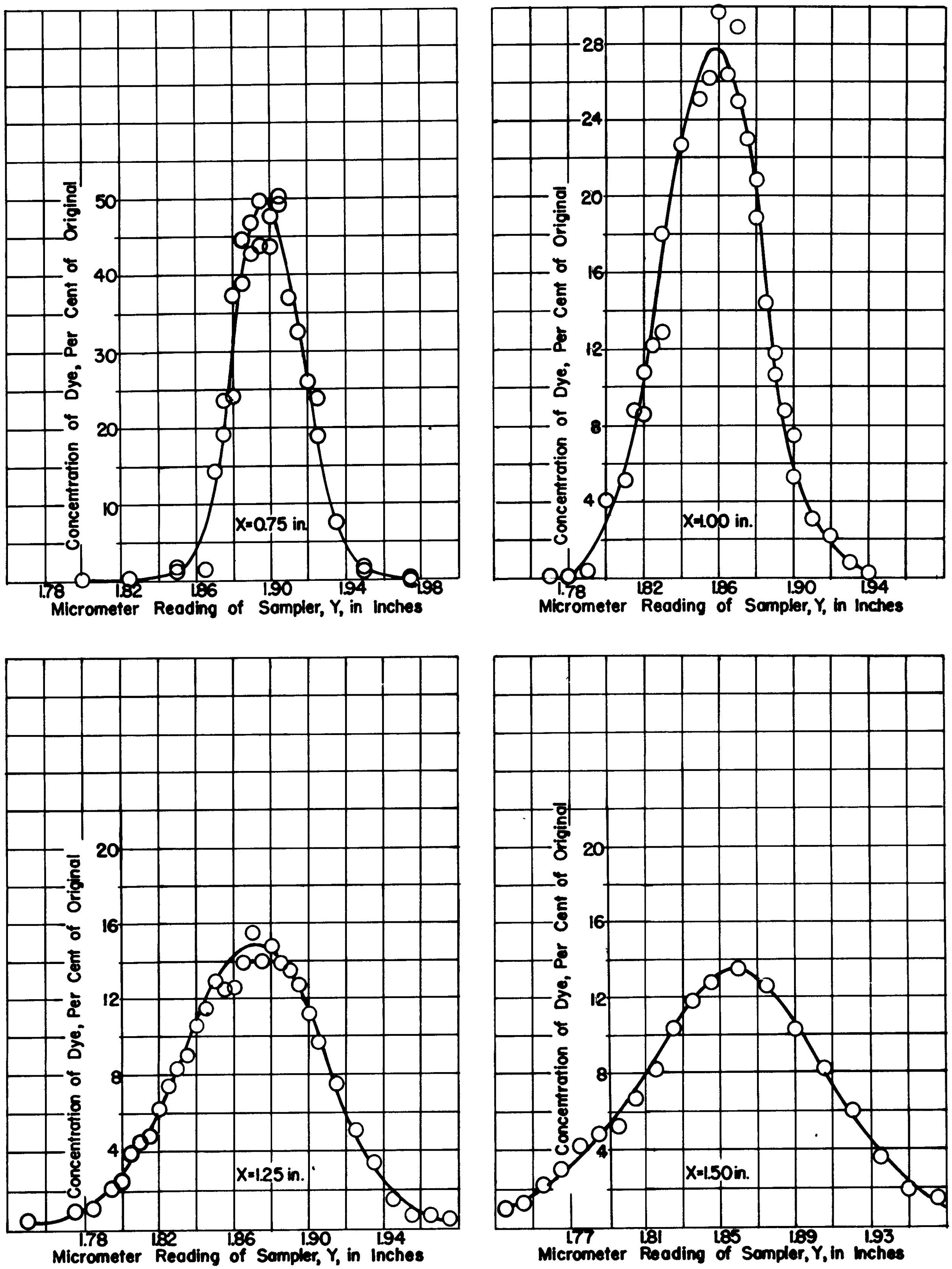

Figure 4— Cross-Stream Distributions of Dye Concentration 15 Mesh-Lengths Behind a 3/4-Inch Mesh at Various Positions, X, Downstream from Injector
Mean Stream Speed U=1.6 Ft/Sec

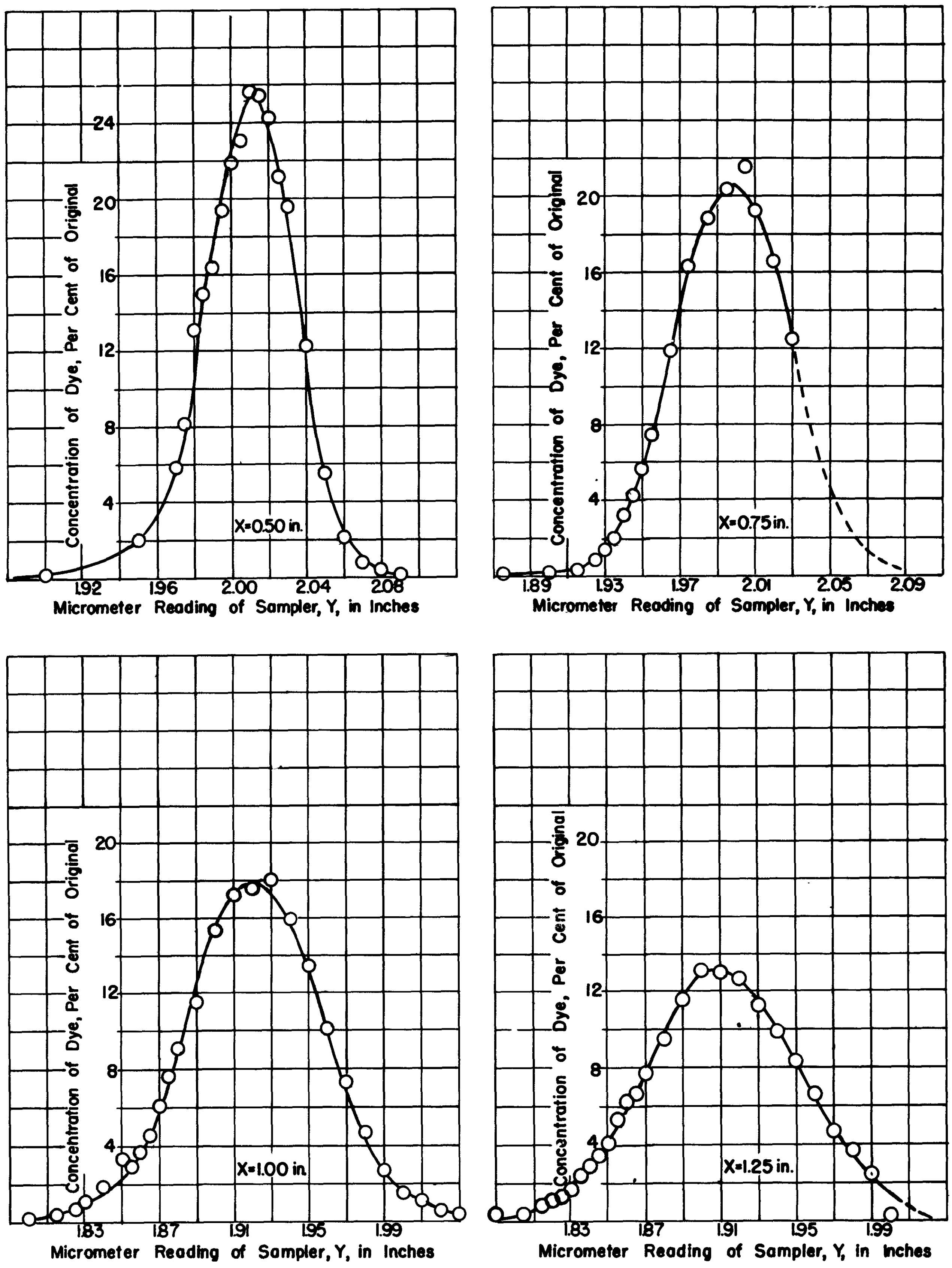

Figure 5— Cross-Stream Distributions of Dye Concentration 22 Mesh-Lengths Behind a 3/4-Inch Mesh at Various Positions, X, Downstream from Injector

Mean Stream Speed U=1.6 Ft/Sec

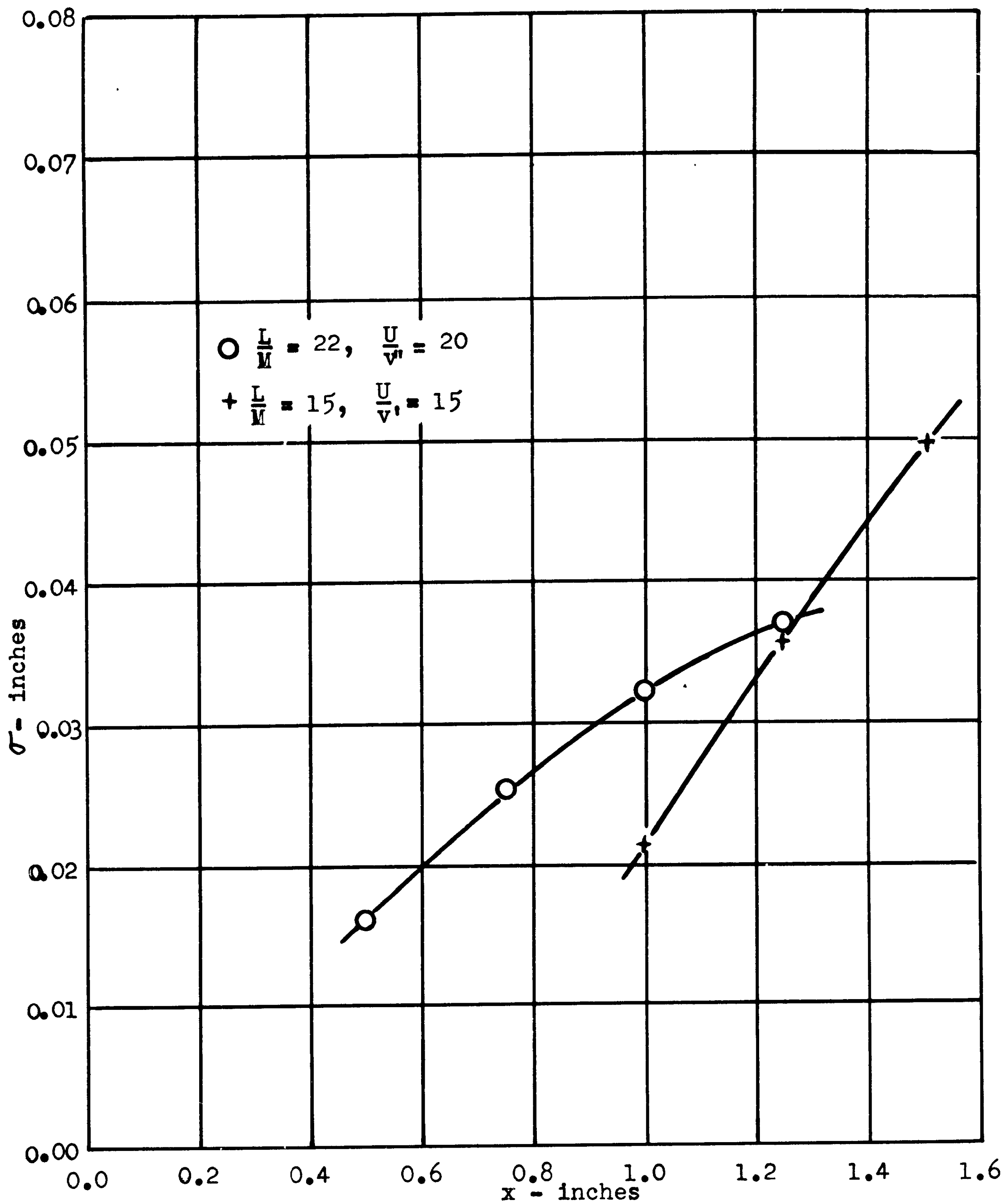

Figure 6 - Corrected Standard Deviations for Dye Concentration Surveys

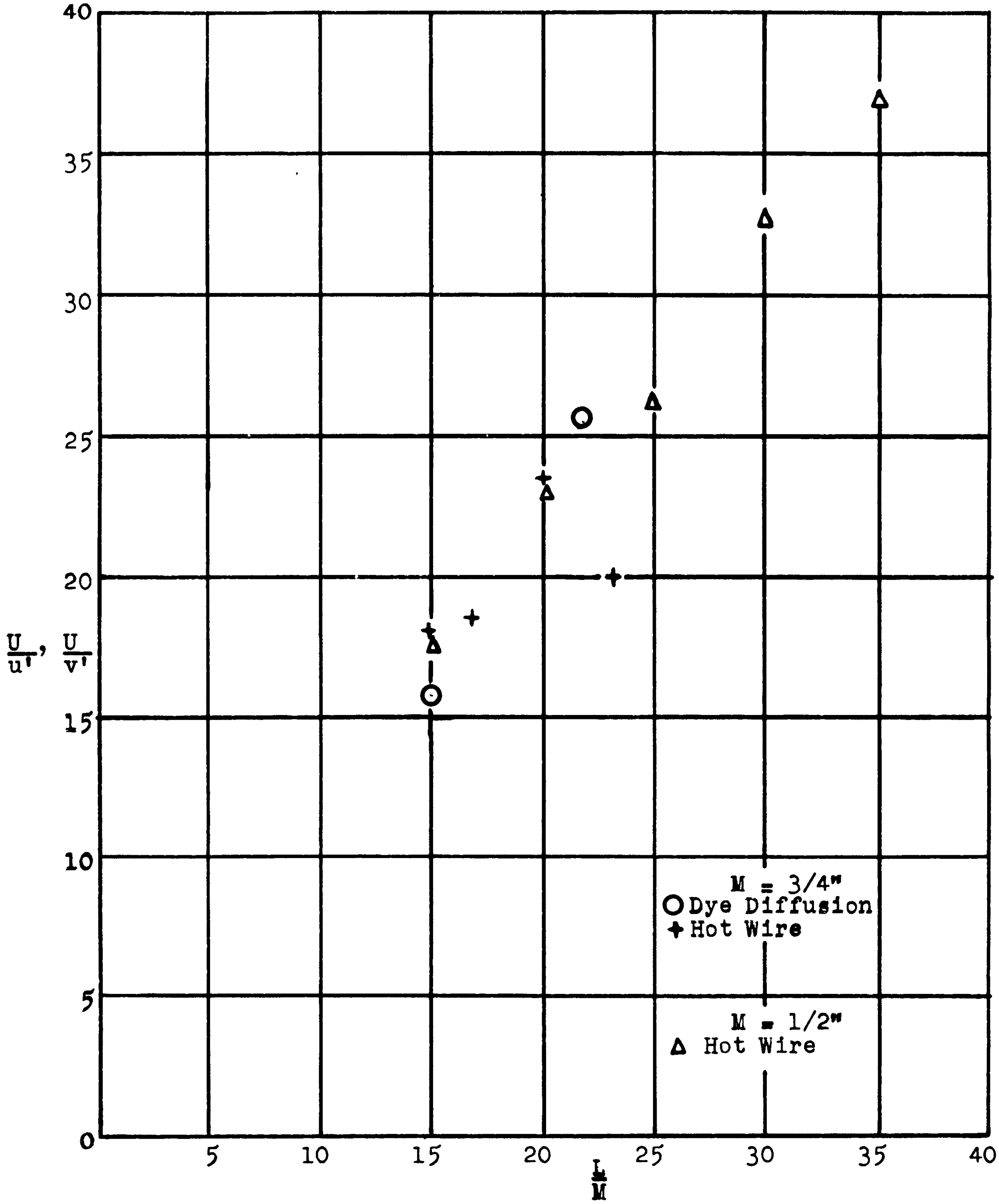

Figure 7 - Decay of Intensity of Turbulence in Water as Measured by the Dye Diffusion and Hot-Wire Methods

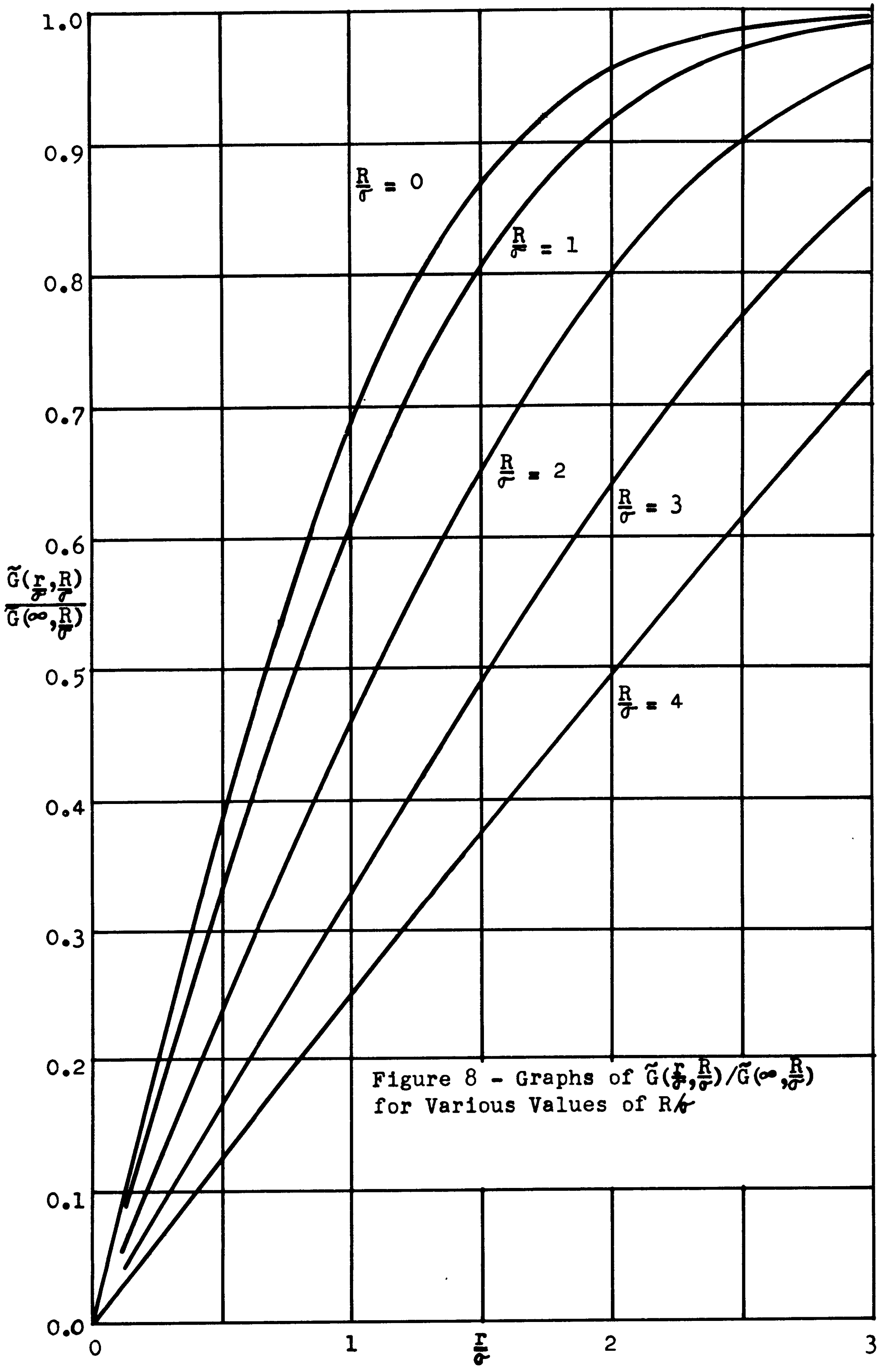

Figure 8 - Graphs of $\tilde{G}(\frac{r}{\sigma},\frac{R}{\sigma})/\tilde{G}(\infty,\frac{R}{\sigma})$ for Various Values of R/σ

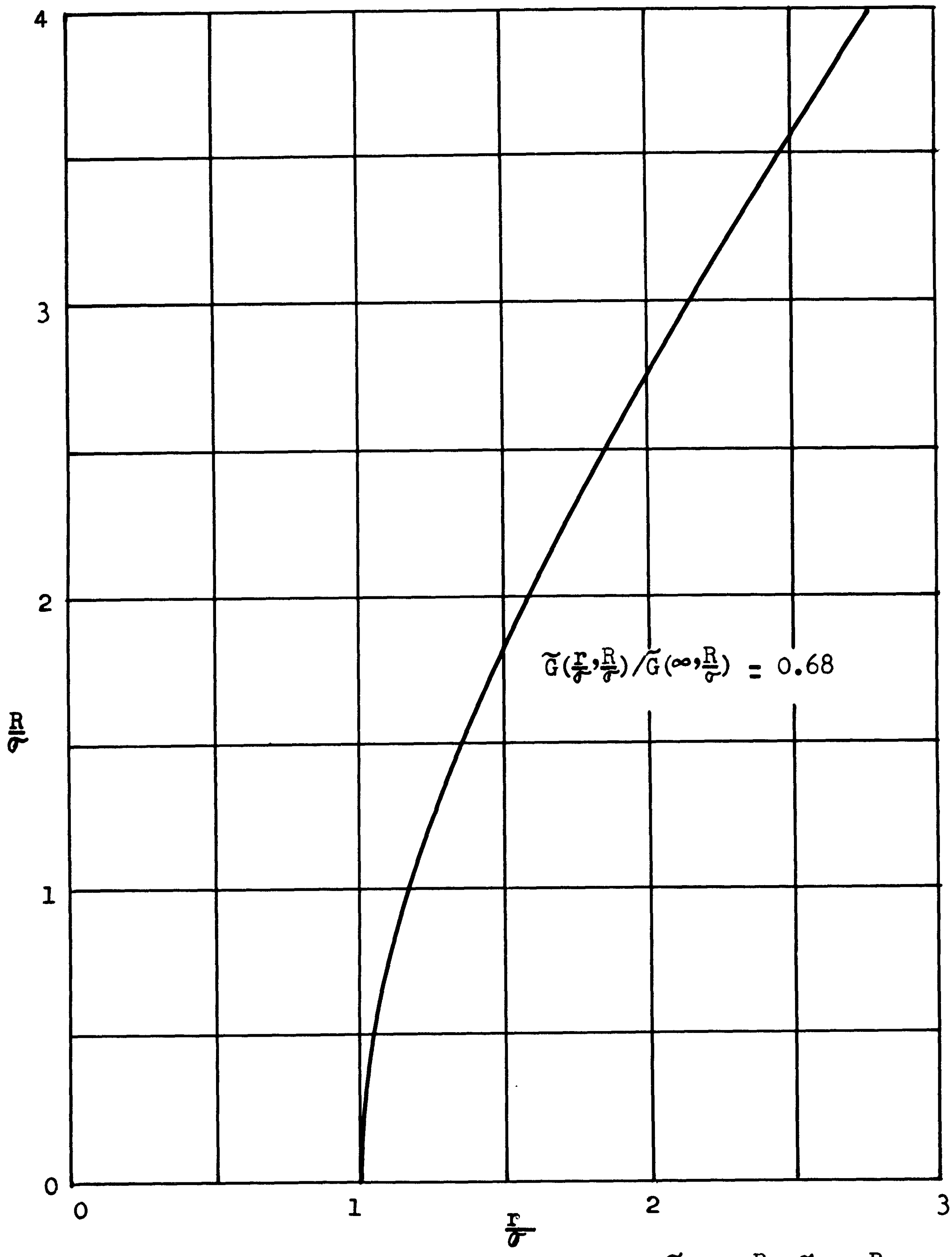

Figure 9 - Graphs of R/σ against r/σ for $\tilde{G}\ (\frac{r}{\sigma}, \frac{R}{\sigma})/\tilde{G}(\infty, \frac{R}{\sigma}) = 0.68$

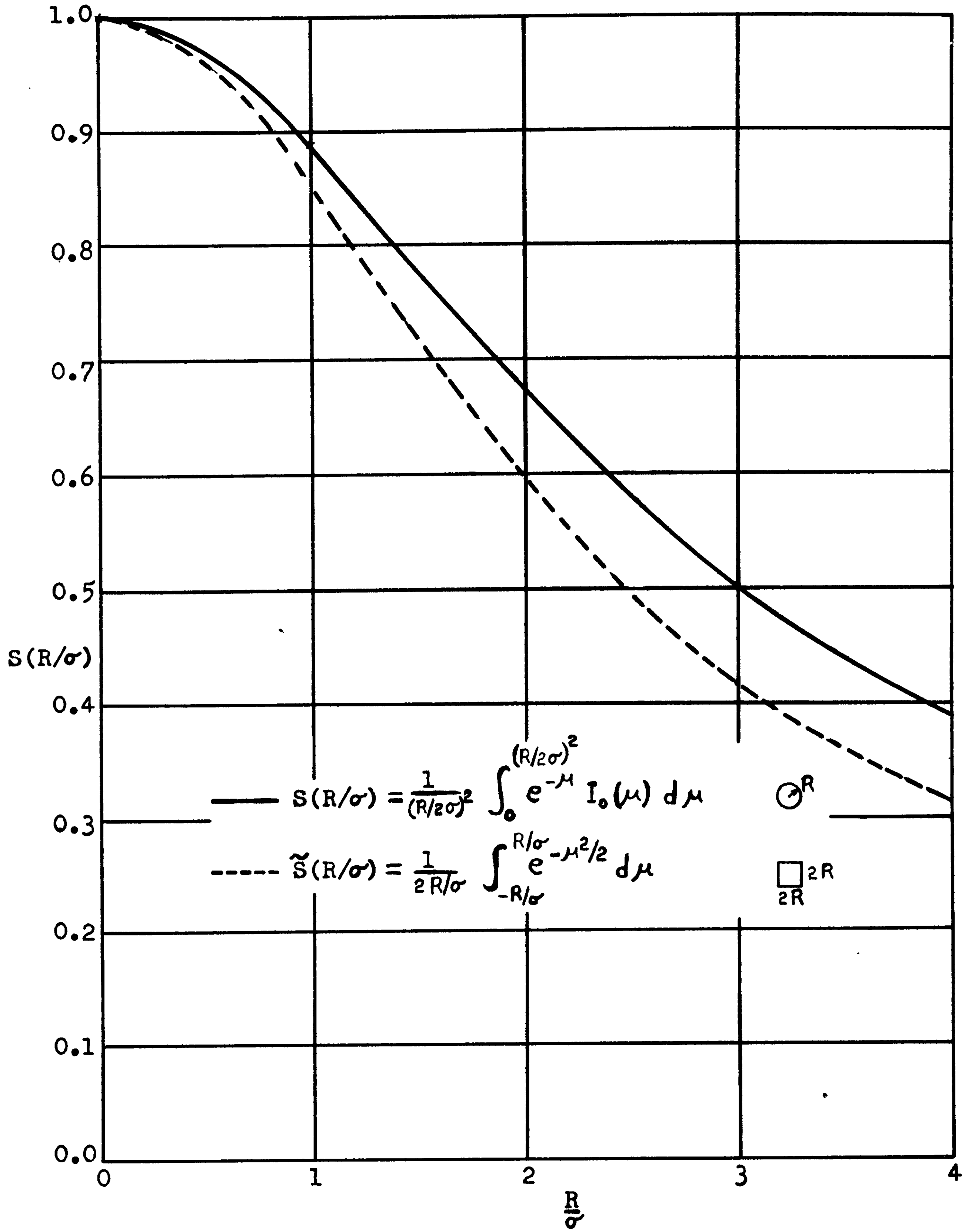

Figure 10 - Graphs of $S(R/\sigma)$ and $\tilde{S}(R/\sigma)$