NAVY DEPARTMENT

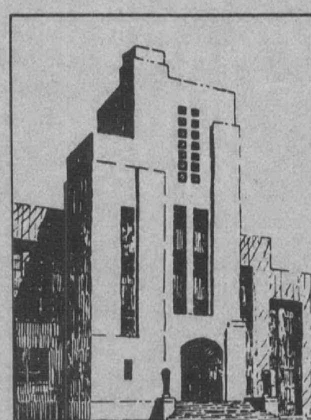
THE DAVID W. TAYLOR MODEL BASIN

WASHINGTON 7, D.C.

A METHOD OF DETERMINING OPTIMUM LENGTHS OF TOWING CABLES

by

Leonard Pode



April 1950

Report 717

NS 880-019

INITIAL DISTRIBUTION

Copies	
10	Chief, BuShips, Project Records (Code 362), for distribution: 5 Project Records 1 Technical Assistant to Chief of the Bureau (Code 106) 1 Research (Code 330) 1 Technical Information Disclosure (Code 367) for: National Research Establishment, Halifax, Canada, Attn: Mr. Richard Parr Blake 1 Minesweeping (Code 620) 1 Magnetic Defense (Code 660M)
2	Commanding Officer and Director, U.S. Navy Electronics Laboratory, Point Loma, San Diego 52, Calif.
1	Commanding Officer and Director, U.S. Navy Underwater Sound Laboratory, Fort Trumbull, New London, Conn.
1	Commander, Puget Sound Naval Shipyard, Bremerton, Wash.
1	Development Contract Officer, Kellex Corporation, Silver Spring Laboratory, 936 Wayne Ave., Silver Spring, Md.
1	Director, Woods Hole Oceanographic Institution, Woods Hole, Mass.
1	Chief, Bureau of Aeronautics Attn: Mr. John S. Attinello (Code RS3) Rm 2W87, Navy Department Washington, D.C.

A METHOD OF DETERMINING OPTIMUM LENGTHS OF TOWING CABLES

by

Leonard Pode

INTRODUCTION

A problem that arises frequently in connection with the design of towing arrangements is that of choosing the design variables so that the size of equipment and the magnitude of the forces involved are kept to a minimum. Usually the preliminary choice of the design variables has been merely a guess and the improvement of the guess has depended upon the results of extensive exploratory calculations. Tables and charts are presented here which, for the most frequent design problems, will help to reduce the labor of such calculations and to enable the designer to determine optimum conditions in a straightforward manner.

STATEMENT OF PROBLEM

Suppose that it is desired to tow a body at a stated depth, y, using a specified cable. Since the hydrodynamic behavior of the cable may be assumed to be known, the length of cable needed to reach the required depth and the tension at the upper end of the cable are determined when the direction and magnitude of the force that the towed body applies to the lower end of the cable are known. The question of interest to the designer is the manner in which the tension at the upper end of the cable, which is the greatest tension in the cable, will vary with the direction and magnitude of the force applied by the towed body.

A force of given magnitude may best be used to attain depth by orienting it as nearly as possible in the direction of gravity because the component of force perpendicular to the direction of gravity increases the tension in the cable without contributing to the attainment of the required depth. The angle, ϕ_0, that the force applied by the towed body makes with the direction of stream—which is equal, for equilibrium, to the angle that the cable makes with the stream at the point where the cable meets the body—is therefore made as close to $\pi/2$ as possible. If the downward force of the body is developed by means of lifting surfaces, the angle ϕ_0 is limited by the lift-drag ratios which such surfaces can attain; if the downward force is derived from the weight of the body, the angle ϕ_0 is limited by the relationship of the weight of the body to its drag. Since the weight of the body is constant, whereas its drag increases with the square of the speed, the weight required to obtain a given value of ϕ_0 increases very rapidly with speed. Hence the limitation on ϕ_0 becomes more severe as the speed increases. The value of ϕ_0 for a body employing lifting surfaces is not affected appreciably by speed or by scaling its dimensions. However, the magnitude of the force obtained from such a body varies as the square of the speed and as the square of the factor to which its dimensions may be scaled.

Let it be assumed that the direction of the force applied by the body is known. The question is then how should the magnitude of the force be adjusted. It is clear that if the magnitude of the force is very small, the length of cable that is required will be exceedingly long so that the hydrodynamic force acting along the cable will cause the tension at the upper end of cable to become very large. On the other hand the length of the cable is shortest only when the magnitude of the force applied by the towed body grows exceedingly large and then the tension in cable is also very large. Between these extremes there must lie an optimum.

ANALYSIS

The calculation of this optimum configuration depends upon the specific assumptions made regarding the forces acting on the cable. It can be shown from very general considerations, however, that regardless of these specific assumptions the solution of the cable configuration can be expressed by equations of the following parametric form.

$$\frac{T}{T_0} = \frac{\tau}{\tau_0} \qquad [1]$$

$$\frac{Rs}{T_0} = \frac{\sigma - \sigma_0}{\tau_0} \qquad [2]$$

$$\frac{Ry}{T_0} = \frac{\eta - \eta_0}{\tau_0} \qquad [3]$$

$$\frac{Rx}{T_0} = \frac{\xi - \xi_0}{\tau_0} \qquad [4]$$

where T is the tension at the upper end of the cable,

s is the length of the cable,

y is the depth of the body,

x is the distance of the body aft of the upper end of the cable,

T_0 is the tension in the cable at the point where the cable meets the body, i.e., the magnitude of the force applied by the towed body,

R is the drag of a unit length of the cable when the cable is normal to the stream,

τ, σ, η, and ξ, are certain functions of ϕ which depend upon the specific assumptions that are made regarding the forces acting on the cable, where

ϕ is the angle between the cable and the direction of the motion at the upper end of the cable, and

τ_0, σ_0, η_0, and ξ_0, are the values of these functions for $\phi = \phi_0$. See Figure 1.

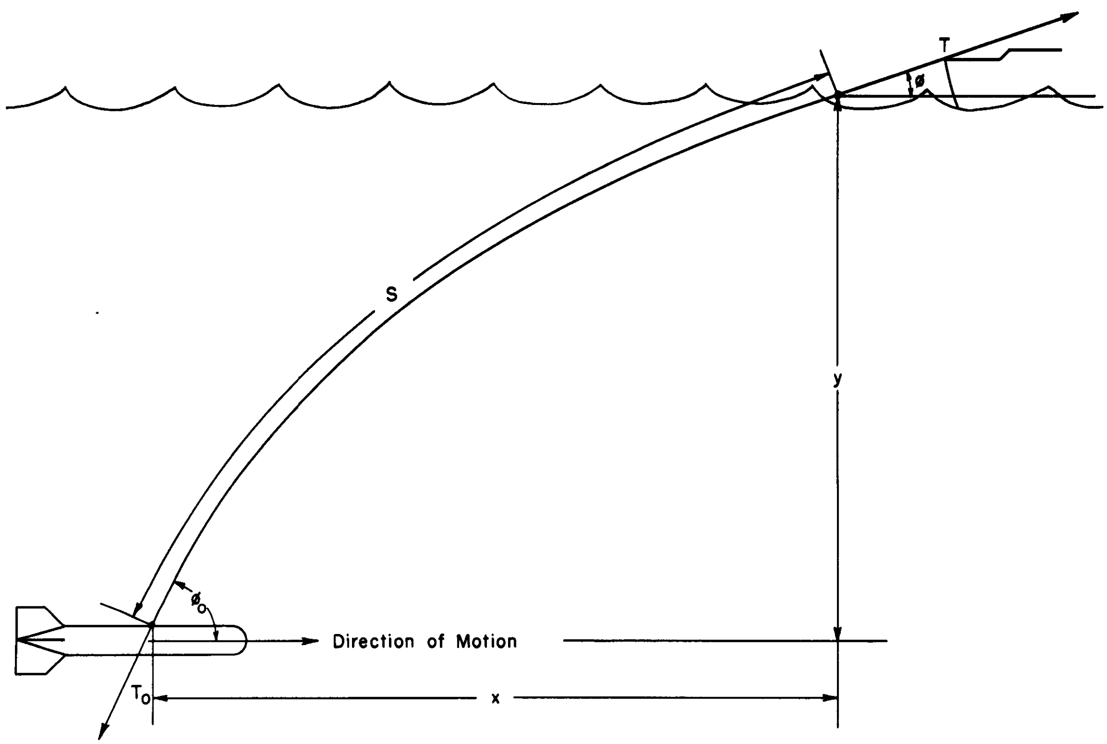


Figure 1 - Cable Configuration For a Towed Body

A general expression for the optimum configuration can be obtained through the use of these equations. Since y, R, and ϕ_0 are fixed, Equation [3] gives ϕ as an implicit function of T_0, or T_0 as an explicit function of ϕ. In Equation [1] T can therefore be considered to be a function of either T_0, ϕ, or any function of ϕ. The optimum configuration is obtained by minimizing T. This can be done most easily from the differential forms of Equations [1] and [3] taken simultaneously, which can be written

$$d\left(\frac{T}{Ry}\right) = \frac{d\tau}{\left(\frac{Ry\ \tau_o}{T_o}\right)} - \frac{\tau d\ \frac{Ry\ \tau_o}{T_o}}{\left(\frac{Ry\ \tau_o}{T_o}\right)^2} \qquad [1a]$$

$$d\left[\frac{\tau_o\ Ry}{T_o}\right] = d\eta \qquad [3a]$$

For minimum T, dT must vanish. Hence from Equations [1a], [3a], and [3]

$$\frac{Ry\ \tau_o}{T_o} = \frac{\tau d\eta}{d\tau} = \eta - \eta_o \qquad [5]$$

or

$$\eta - \tau\frac{d\eta}{d\tau} = \eta_o \qquad [5a]$$

This equation may be used to determine the value of ϕ that obtains at the optimum configuration. From Equation [3] the appropriate value of T_o may then be found; thence T, s, and x can be calculated from [1], [2], and [4].

The designer is usually interested in the optimum conditions for high-speed operation. If the speed of towing is sufficiently high so that the weight of the cable can be neglected, the functions τ, σ, η, and ξ may be taken as those given in TMB Report C-122, Appendix I, p. 27, i.e.,

$$\tau = 1 + f\ \csc\phi;\ f = \frac{F}{R} \qquad [6]$$

$$\sigma = \cot\phi \qquad [7]$$

$$\eta = \ln\cot\frac{\phi}{2} \qquad [8]$$

$$\xi = \csc\phi - 1 \qquad [9]$$

where F is the drag per unit length of the cable when the cable is parallel to the stream. Equation [5a] then becomes

$$\ln\cot\frac{\phi}{2} - \frac{1 + f\ \csc\phi}{f\ \cot\phi} = \ln\cot\frac{\phi_o}{2} \qquad [5b]$$

Thus far only the case of towed bodies has been considered so that the maximum value of ϕ_0 is $\pi/2$—since negative drag cannot be realized in this case. There are, however, some cable configurations in which the values of ϕ_0 greater than $\pi/2$ are possible; i.e., configurations where the cable forms a loop. An example of such a configuration is a cable joining two self-powered bodies such as two airplanes or two submarines. When the speed of towing is high, so that the effect of weight is negligible, the plane in which the cable lies need not include the direction of gravity; so that a line strung between two surface vessels may also present such a problem (see Figure 2). In such cases, if the angle of the cable at one end may be considered fixed this end may be referred to as the lower end, the other end of the cable may be called the upper end and the fixed angle can still be designated as ϕ_0. Then the foregoing analysis still applies and Equation [5b] gives the condition that the tension at the upper end of the cable is minimal.

It is found that when the angle ϕ_0 is increased beyond $\pi/2$ the tension at the upper end of the cable will continue to be reduced. Nevertheless there is a limiting condition beyond which minimizing the tension at the upper end of the cable is no longer a reasonable procedure: When further increase in ϕ_0 will cause the tension at the lower end to become greater than that at

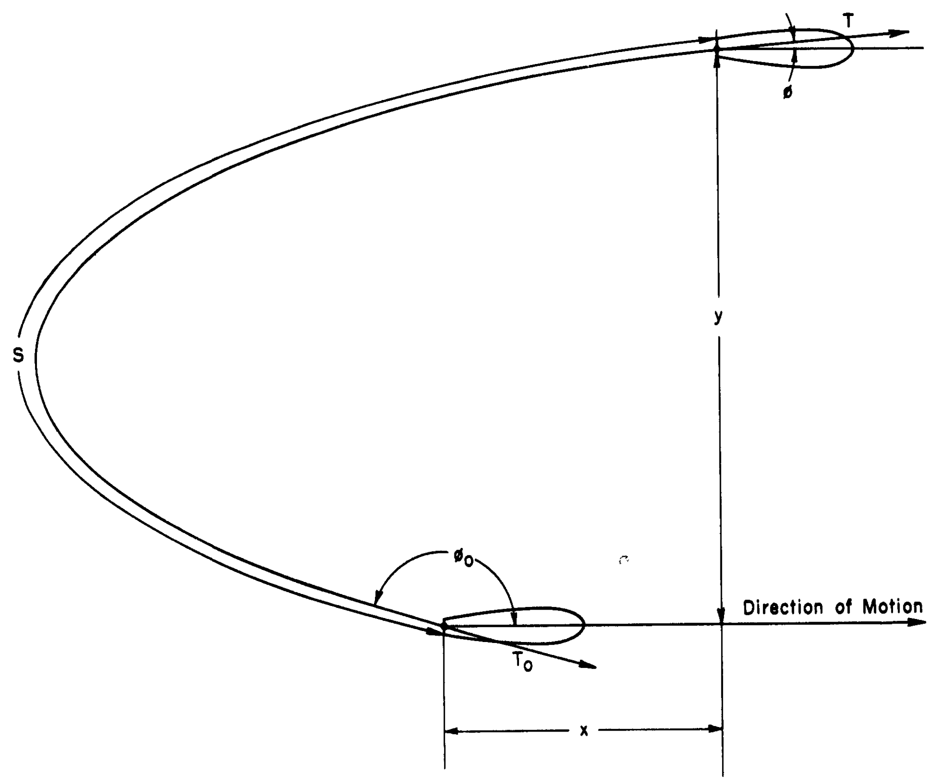


Figure 2 - Cable Configuration for Two Powered Vessels

the upper end of the cable. From the symmetry of the functions τ, σ, η, ξ about $\phi = \pi/2$, it is clear that this condition will occur when the tensions at the two ends of the cable are equal and when the configuration of the cable is completely symmetric about a line parallel to the direction of motion.

Solutions of Equation [5b], and pertinent values obtained therefrom are listed in Table 1 and are graphically presented in Figure 3.

The cable configurations that are found by solution of Equation [5b] are optimum configurations with respect to tension in one end of the cable

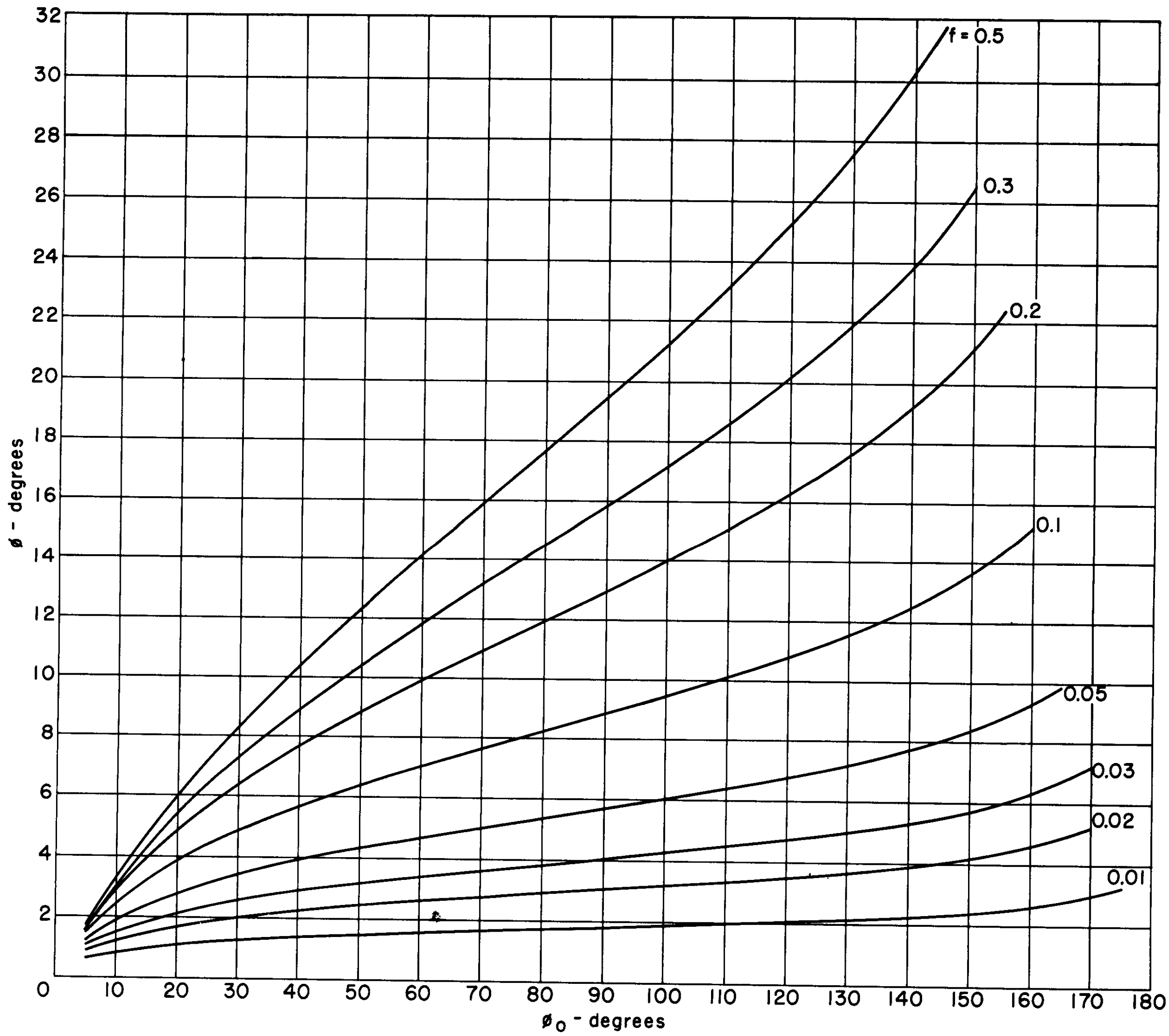


Figure 3a - Variation of ϕ with ϕ_0 and f.

Figure 3 - Values Obtained for Optimum Cable Configurations for High-Speed Towing

when the angle at the other end of the cable and the depth of towing are specified. Other types of optimum problems may occur. For example it may be desired that the drag at the upper end of the cable be a minimum instead of the tension; also, instead of the angle ϕ_0 being specified at the lower end of the cable the drag of the body might be specified or the ratio of the depth to the distance aft might be fixed. The tables presented here do not give the solution to such problems. However in many cases these tables may be used to get a first approximation to a solution for such problems.

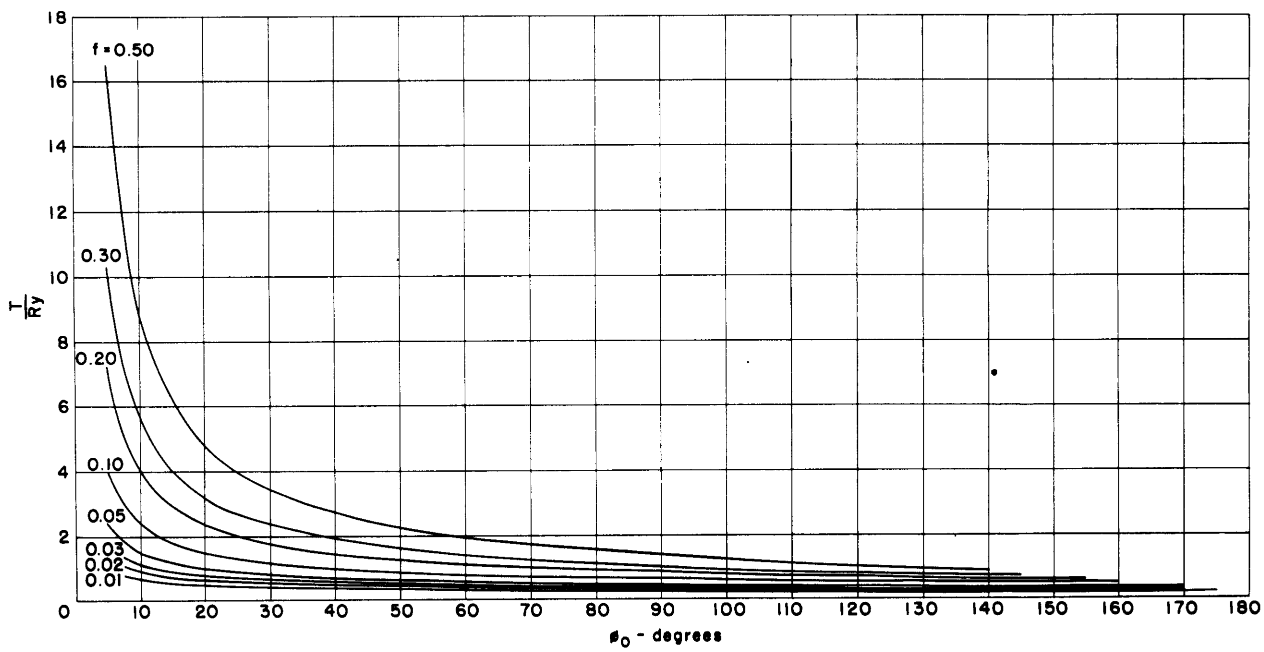


Figure 3b - Variation of T/Ry with ϕ_0 and f.

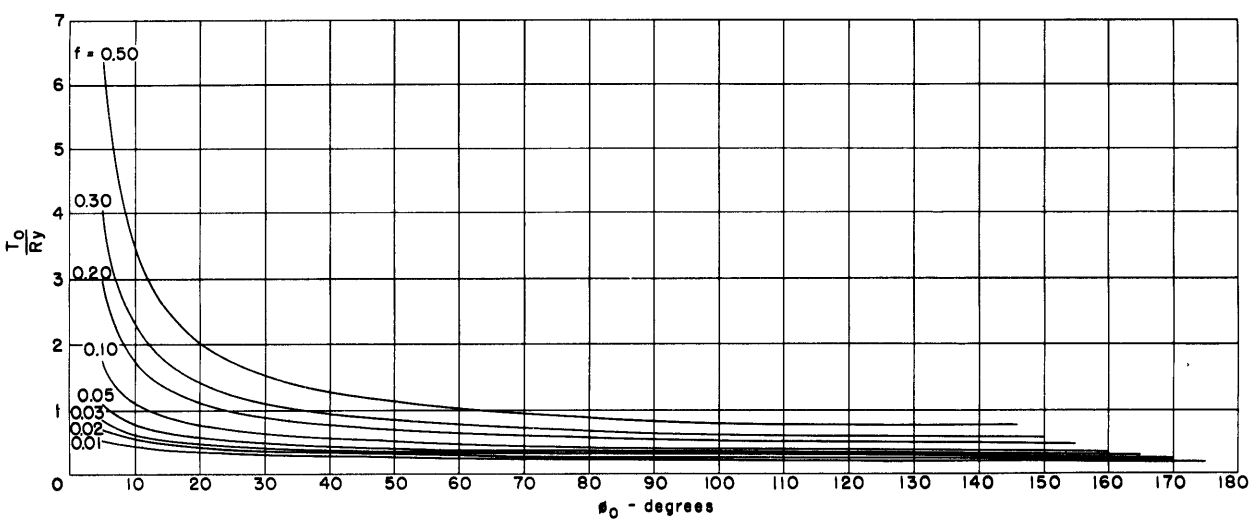


Figure 3c - Variation of T_0/Ry with ϕ_0 and f.

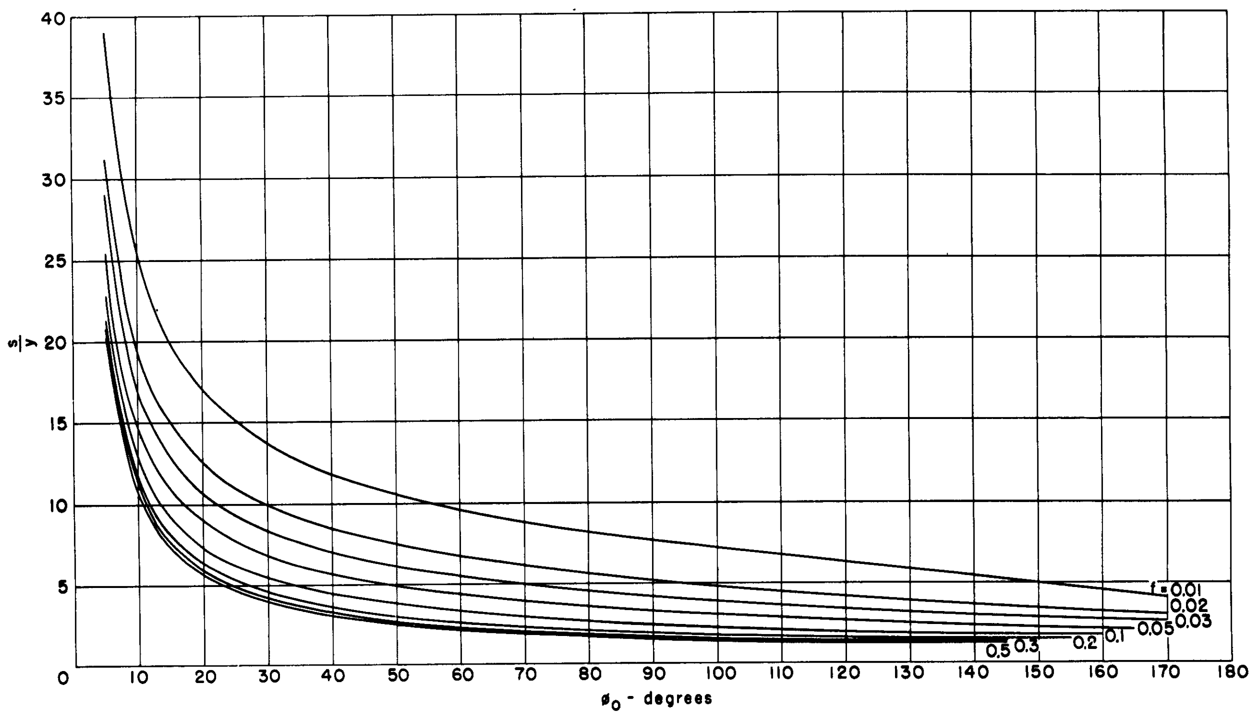


Figure 3d - Variation of s/y with ϕ_0 and f.

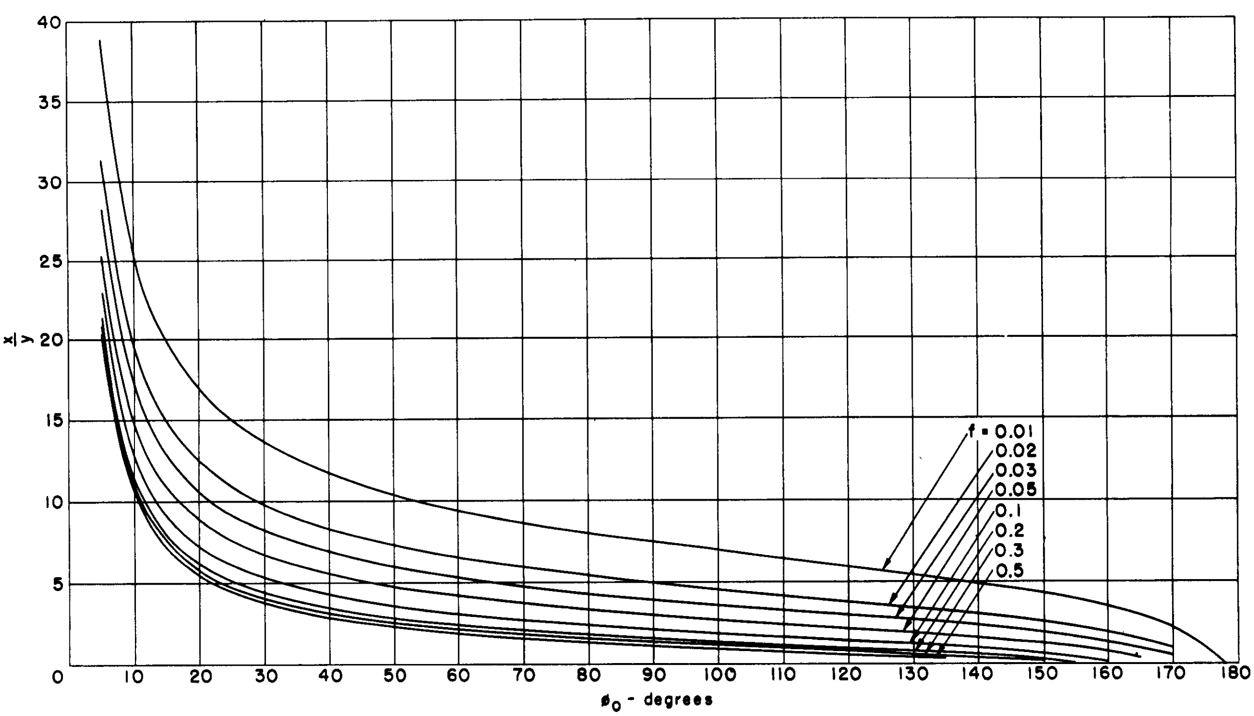


Figure 3e - Variation of x/y with ϕ_0 and f.

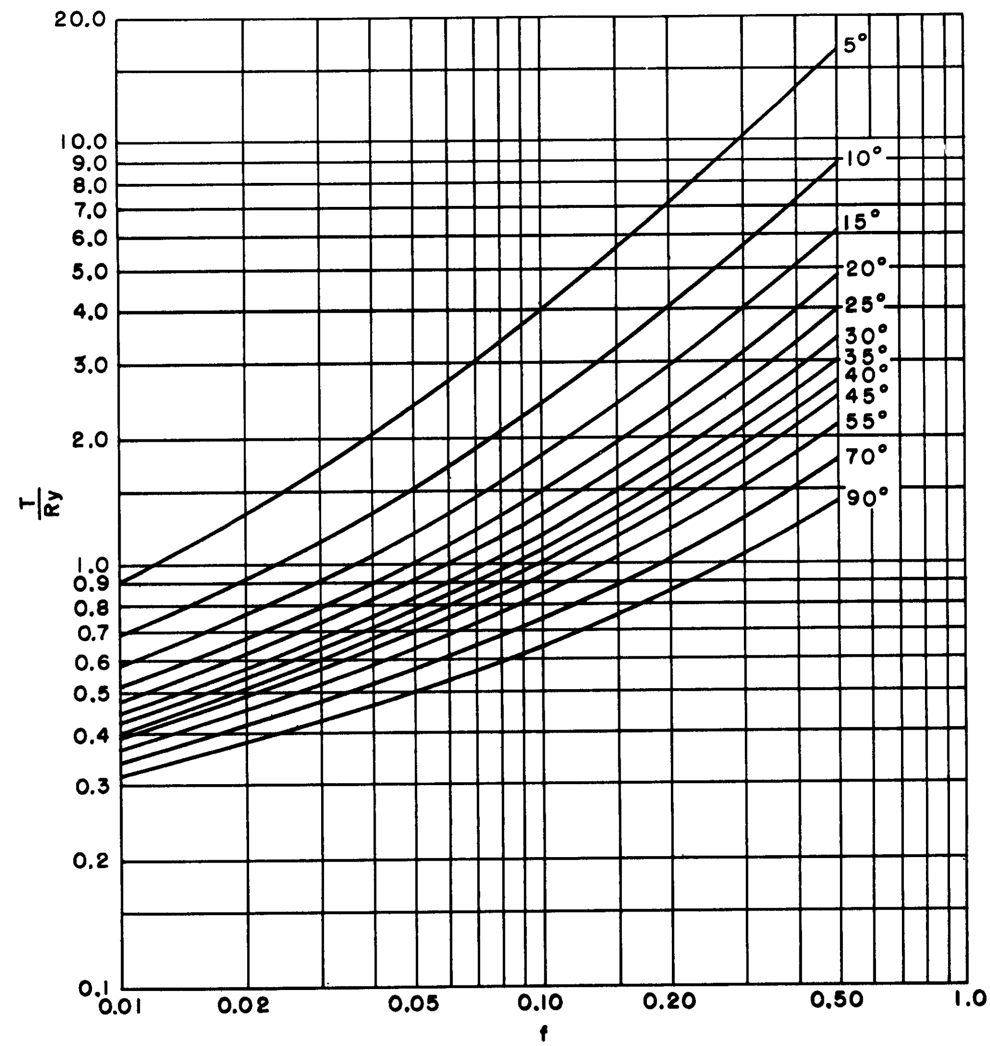


Figure 3f - Variation of T/Ry with ϕ_0 and f.

TABLE 1 - Optimum Cable Configurations for High-Speed Towing

TABLE 1a — $f = 0.01$

ϕ_0 degrees	ϕ degrees	$\frac{T}{Ry}$	$\frac{T_o}{Ry}$	$\frac{s}{y}$	$\frac{x}{y}$
5	0.622	0.922	0.535	38.71	38.70
10	0.844	.678	.427	25.13	25.10
15	0.988	.580	.381	19.90	19.85
20	1.096	.523	.353	16.99	16.93
25	1.184	.484	.334	15.08	15.02
30	1.257	.456	.319	13.72	13.64
35	1.322	.433	.308	12.67	12.58
40	1.380	.415	.298	11.83	11.73
45	1.432	.400	.290	11.14	11.02
50	1.481	.387	.283	10.55	10.43
55	1.527	.375	.276	10.05	9.908
60	1.570	.365	.270	9.600	9.448
65	1.611	.355	.265	9.200	9.037
70	1.651	.347	.260	8.839	8.662
75	1.690	.339	.256	8.508	8.318
80	1.729	.331	.251	8.202	7.996
85	1.767	.324	.247	7.915	7.695
90	1.804	.317	.243	7.646	7.651
95	1.843	.311	.239	7.391	7.136
100	1.881	.304	.236	7.149	6.874
105	1.920	.298	.232	6.912	6.617
110	1.961	.292	.228	6.684	6.360
115	2.003	.286	.225	6.462	6.117
120	2.047	.280	.221	6.243	5.868
125	2.093	.274	.217	6.028	5.620
130	2.143	.267	.214	5.813	5.365
135	2.196	.261	.209	5.599	5.104
140	2.255	.254	.206	5.383	4.831
145	2.321	.247	.201	5.165	4.541
150	2.396	.239	.197	4.946	4.229
155	2.483	.231	.192	4.725	3.876
160	2.590	.221	.186	4.498	3.476
165	2.729	.210	.180	4.287	2.973
170	2.926	.196	.173	4.136	2.268
175	3.258	0.176	0.167	4.334	0.915

TABLE 1b — $f = 0.02$

ϕ_0 degrees	ϕ degrees	$\frac{T}{Ry}$	$\frac{T_o}{Ry}$	$\frac{s}{y}$	$\frac{x}{y}$
5	0.865	1.325	0.701	31.22	31.20
10	1.244	0.921	.534	19.35	19.31
15	1.499	.764	.467	14.93	14.88
20	1.694	.676	.427	12.54	12.47
25	1.853	.618	.400	10.99	10.91
30	1.989	.576	.380	9.890	9.799
35	2.108	.543	.364	9.059	8.955
40	2.216	.517	.351	8.401	8.283
45	2.314	.495	.340	7.862	7.731
50	2.405	.476	.331	7.408	7.264
55	2.491	.460	.323	7.019	6.862
60	2.572	.445	.315	6.678	6.507
65	2.650	.432	.308	6.375	6.190
70	2.726	.420	.302	6.102	5.902
75	2.800	.409	.296	5.854	5.639
80	2.873	.399	.291	5.626	5.395
85	2.945	.389	.285	5.415	5.166
90	3.017	.379	.280	5.217	4.949
95	3.090	.371	.276	5.030	4.742
100	3.163	.362	.271	4.853	4.544
105	3.238	.353	.266	4.683	4.351
110	3.316	.345	.262	4.516	4.142
115	3.396	.337	.258	4.362	3.974
120	3.481	.329	.253	4.209	3.788
125	3.570	.321	.249	4.059	3.600
130	3.655	.312	.244	3.909	3.407
135	3.768	.304	.239	3.767	3.213
140	3.883	.295	.235	3.623	3.006
145	4.009	.285	.230	3.483	2.787
150	4.154	.275	.224	3.345	2.548
155	4.324	.265	.219	3.214	2.278
160	4.532	.252	.212	3.095	1.960
165	4.798	.238	.207	3.009	1.556
170	5.176	0.221	0.202	3.020	0.962

TABLE 1c f = 0.03

ϕ_0 degrees	ϕ degrees	$\frac{T}{Ry}$	$\frac{T_o}{Ry}$	$\frac{s}{y}$	$\frac{x}{y}$
5	1.024	1.689	0.844	28.71	28.15
10	1.521	1.130	.622	16.96	16.92
15	1.869	0.919	.534	12.89	12.83
20	2.139	.803	.484	10.70	10.63
25	2.362	.727	.451	9.305	9.220
30	2.553	.673	.426	8.322	8.224
35	2.722	.631	.407	7.583	7.470
40	2.874	.598	.391	7.002	6.875
45	3.014	.570	.378	6.527	6.387
50	3.144	.546	.367	6.131	5.976
55	3.267	.526	.357	5.791	5.621
60	3.384	.507	.348	5.495	5.310
65	3.498	.491	.340	5.236	5.033
70	3.605	.476	.333	4.999	4.783
75	3.712	.462	.326	4.786	4.554
80	3.817	.450	.319	4.592	4.338
85	3.921	.438	.313	4.412	4.146
90	4.026	.426	.308	4.244	3.956
95	4.130	.415	.302	4.086	3.777
100	4.238	.405	.297	3.938	3.605
105	4.346	.395	.292	3.796	3.438
110	4.459	.385	.286	3.661	3.275
115	4.576	.375	.281	3.530	3.114
120	4.698	.365	.276	3.405	2.953
125	4.828	.355	.271	3.283	2.791
130	4.967	.345	.266	3.165	2.626
135	5.118	.335	.261	3.048	2.455
140	5.284	.324	.256	2.936	2.276
145	5.463	.313	.251	2.829	2.085
150	5.682	.302	.245	2.727	1.875
155	5.932	.289	.240	2.635	1.636
160	6.232	.275	.234	2.562	1.353
165	6.623	.258	.238	2.531	0.986
170	7.177	0.238	0.225	2.615	0.431

TABLE 1d f = 0.05

ϕ_0 degrees	ϕ degrees	$\frac{T}{Ry}$	$\frac{T_o}{Ry}$	$\frac{s}{y}$	$\frac{x}{y}$
5	1.207	2.369	1.107	25.34	25.32
10	1.899	1.508	0.774	14.72	14.68
15	2.399	1.196	.649	10.95	10.89
20	2.796	1.024	.580	8.966	8.889
25	3.128	0.915	.534	7.716	7.622
30	3.416	.838	.501	6.841	6.733
35	3.673	.779	.476	6.190	6.067
40	3.906	.732	.455	5.683	5.544
45	4.121	.694	.438	5.272	5.117
50	4.322	.662	.424	4.930	4.759
55	4.512	.634	.411	4.638	4.463
60	4.694	.609	.400	4.386	4.183
65	4.868	.587	.390	4.165	3.945
70	5.039	.567	.381	3.968	3.730
75	5.205	.549	.372	3.789	3.534
80	5.370	.532	.364	3.627	3.352
85	5.534	.516	.357	3.478	3.183
90	5.698	.501	.350	3.340	3.024
95	5.863	.487	.343	3.212	2.872
100	6.032	.473	.337	3.091	2.726
105	6.204	.460	.331	2.977	2.584
110	6.382	.447	.325	2.869	2.446
115	6.567	.434	.319	2.766	2.309
120	6.762	.422	.313	2.668	2.172
125	6.968	.409	.307	2.573	2.034
130	7.189	.396	.302	2.483	1.894
135	7.428	.383	.296	2.398	1.748
140	7.692	.370	.290	2.317	1.594
145	7.987	.356	.285	2.242	1.429
150	8.328	.342	.279	2.175	1.246
155	8.724	.326	.290	2.123	1.036
160	9.212	.308	.270	2.095	0.781
165	9.837	0.288	0.266	2.120	0.444

TABLE 1e f = 0.1

ϕ_0 degrees	ϕ degrees	$\frac{T}{Ry}$	$\frac{T_o}{Ry}$	$\frac{s}{y}$	$\frac{x}{y}$
5	1.433	3.998	1.717	22.84	22.81
10	2.416	2.370	1.108	12.67	12.62
15	3.179	1.800	0.890	9.167	9.101
20	3.809	1.502	.775	7.356	7.270
25	4.352	1.314	.701	6.234	6.131
30	4.831	1.183	.649	5.463	5.341
35	5.264	1.085	.610	4.895	4.755
40	5.662	1.009	.579	4.457	4.299
45	6.031	0.946	.553	4.105	3.929
50	6.380	.894	.532	3.816	3.622
55	6.711	.850	.514	3.571	3.359
60	7.030	.811	.498	3.362	3.131
65	7.338	.777	.484	3.179	2.929
70	7.639	.746	.471	3.017	2.748
75	7.935	.717	.459	2.873	2.583
80	8.229	.691	.448	2.743	2.431
85	8.522	.667	.438	2.624	2.289
90	8.816	.645	.429	2.516	2.156
95	9.112	.623	.420	2.416	2.029
100	9.416	.603	.412	2.323	1.907
105	9.728	.592	.404	2.236	1.790
110	10.05	.564	.397	2.155	1.674
115	10.38	.545	.390	2.079	1.560
120	10.74	.527	.383	2.008	1.446
125	11.11	.509	.376	1.942	1.330
130	11.51	.491	.370	1.880	1.212
135	11.95	.473	.364	1.824	1.088
140	12.43	.454	.358	1.775	0.957
145	12.97	.434	.353	1.734	.815
150	13.58	.414	.348	1.704	.656
155	14.31	.392	.345	1.694	.469
160	15.19	0.368	0.346	1.715	0.238

TABLE 1f f = 0.2

ϕ_0 degrees	ϕ degrees	$\frac{T}{Ry}$	$\frac{T_o}{Ry}$	$\frac{s}{y}$	$\frac{x}{y}$
5	1.600	7.161	2.890	21.38	21.36
10	2.867	3.994	1.719	11.42	11.38
15	3.927	2.914	1.317	8.053	7.980
20	4.845	2.359	1.110	6.340	6.246
25	5.661	2.018	0.982	5.294	5.180
30	6.400	1.783	.893	4.584	4.448
35	7.080	1.610	.828	4.067	3.912
40	7.713	1.477	.777	3.672	3.496
45	8.309	1.369	.737	3.359	3.163
50	8.877	1.281	.703	3.103	2.886
55	9.421	1.205	.675	2.890	2.652
60	9.948	1.140	.650	2.708	2.449
65	10.46	1.083	.629	2.551	2.270
70	10.96	1.032	.610	2.414	2.110
75	11.46	0.986	.593	2.293	1.965
80	11.96	.944	.578	2.184	1.832
85	12.45	.906	.564	2.087	1.708
90	12.95	.870	.551	1.998	1.591
95	13.46	.836	.540	1.918	1.480
100	13.97	.804	.529	1.844	1.374
105	14.51	.773	.519	1.777	1.271
110	15.06	.744	.510	1.715	1.170
115	15.63	.715	.501	1.658	1.070
120	16.23	.687	.493	1.607	0.970
125	16.87	.659	.486	1.561	.869
130	17.56	.632	.479	1.520	.764
135	18.30	.605	.474	1.486	.654
140	19.12	.577	.470	1.460	.536
145	20.05	.548	.467	1.444	.407
150	21.08	.519	.467	1.443	.260
155	22.29	0.488	0.471	1.464	0.0862

TABLE 1g $f = 0.3$

ϕ_0 degrees	ϕ degrees	$\frac{T}{Ry}$	$\frac{T_o}{Ry}$	$\frac{s}{y}$	$\frac{x}{y}$
5	1.669	10.29	4.046	20.85	20.83
10	3.078	5.579	2.310	10.95	10.90
15	4.304	3.986	1.722	7.622	7.547
20	5.397	3.175	1.423	5.940	5.842
25	6.388	2.680	1.240	4.920	4.800
30	7.300	2.342	1.115	4.232	4.090
35	8.149	2.095	1.024	3.735	3.570
40	8.948	1.905	0.954	3.356	3.170
45	9.707	1.754	.899	3.058	2.850
50	10.44	1.629	.853	2.815	2.585
55	11.14	1.524	.816	2.615	2.362
60	11.82	1.434	.783	2.445	2.169
65	12.48	1.355	.755	2.299	1.999
70	13.14	1.285	.731	2.172	1.848
75	13.79	1.222	.709	2.060	1.710
80	14.44	1.165	.690	1.961	1.584
85	15.09	1.113	.673	1.872	1.467
90	15.74	1.064	.657	1.793	1.357
95	16.41	1.019	.643	1.689	1.253
100	17.09	0.976	.630	1.657	1.153
105	17.79	.935	.618	1.597	1.056
110	18.51	.896	.608	1.544	0.961
115	19.26	.858	.598	1.496	.866
120	20.06	.822	.590	1.453	.772
125	20.81	.786	.583	1.414	.674
130	21.80	.750	.577	1.386	.576
135	22.77	.715	.573	1.362	.471
140	23.84	.679	.572	1.346	.358
145	25.03	.643	.573	1.342	.233
150	26.37	0.605	0.578	1.354	0.0909

TABLE 1h $f = 0.5$

ϕ_0 degrees	ϕ degrees	$\frac{T}{Ry}$	$\frac{T_o}{Ry}$	$\frac{s}{y}$	$\frac{x}{y}$
5	1.732	16.54	6.350	20.42	20.39
10	3.282	8.718	3.475	10.54	10.49
15	4.692	6.093	2.511	7.242	7.164
20	5.988	4.767	2.026	5.584	5.482
25	7.194	3.961	1.732	4.584	4.458
30	8.326	3.417	1.534	3.914	3.764
35	9.396	3.021	1.392	3.432	3.259
40	10.42	2.720	1.284	3.068	2.871
45	11.40	2.480	1.199	2.783	2.562
50	12.34	2.285	1.131	2.553	2.308
55	13.27	2.121	1.074	2.363	2.093
60	14.17	1.981	1.027	2.203	1.908
65	15.05	1.859	0.986	2.067	1.746
70	15.93	1.752	.951	1.950	1.602
75	16.80	1.657	.921	1.848	1.483
80	17.66	1.570	.894	1.758	1.352
85	18.54	1.491	.870	1.678	1.241
90	19.42	1.418	.850	1.607	1.137
95	20.32	1.351	.831	1.543	1.039
100	21.23	1.287	.815	1.487	0.944
105	22.17	1.227	.801	1.436	.852
110	23.14	1.170	.789	1.392	.762
115	24.15	1.115	.779	1.338	.665
120	25.21	1.062	.771	1.320	.583
125	26.34	1.010	.765	1.292	.491
130	27.52	0.960	.762	1.271	.396
135	28.81	.909	.762	1.258	.295
140	30.20	.859	.766	1.254	.186
145	31.73	0.809	0.776	1.262	0.0656

ILLUSTRATIVE EXAMPLE

To provide an illustrative example of an application of the method, the following problem has been worked out.

It is desired to tow a body 25 feet deep at 30 knots. The body is to have a lift-drag ratio of tan 70°. It is required that the safety factor on the breaking strength of the cable be no less than 2. What is the minimum size cable diameter that may be used, if the strongest wire-strand cable is used?

For such cable the following empirical data apply: Where d is the cable diameter in inches and V is the speed in knots.

Cable drag normal to the stream: $R \approx 0.34\ V^2 d$ pound per foot,

Cable drag parallel to the stream: $F \approx 0.02\ R$,

Breaking strength of cable: $S = 80{,}000\ d^2$ pounds.

Since the safety factor must be no less than two, $40{,}000\ d^2 \geqq T$; where T is the greatest tension in the cable. From Table 1b, for f = 0.02 and $\phi_o = 70$ we have the optimum value of T = (0.420 Ry). Hence

$$d^2 \geqq \frac{(0.420)(0.34)(900)(25)d}{40{,}000}$$

$$d \geqq 0.08$$

The cable must therefore be at least 0.08 inch in diameter. From Table 1b it is seen that the optimum length of this cable is (6.102)y or 525 feet. With this length of cable, the tension at the upper end of the cable will be a minimum and equal to 2560 pounds.

NAVY-DPPO PRNC. WASH., D.C.